FEBRUARY 1976

MATT-1211

EXCITATION OF FAST WAVES
BY SLOW WAVES
NEAR THE LOWER-HYBRID FREQUENCY

BY

R. L. BERGER AND LIU CHEN

PLASMA PHYSICS
LABORATORY

MASTER

PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY

This work was supported by U. S. Energy Research and Development Administration Contract E(11-1)-3073. Reproduction, translation, publication, use and disposal, in whole or in part, by or for the United States Government is permitted.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

Printed in the United States of America.

Available from
National Technical Information Service
U. S. Department of Commerce
5285 Port Royal Road
Springfield, Virginia 22151

Price: Printed Copy \$ * ; Microfiche \$1.45

<u>*Pages</u>	<u>NTIS Selling Price</u>
1-50	\$ 4.00
51-150	5.45
151-325	7.60
326-500	10.60
501-1000	13.60

Excitation of Fast Waves by Slow Waves
Near the Lower-Hybrid Frequency

R.L. Berger and Liu Chen

Plasma Physics Laboratory, Princeton University,
Princeton, New Jersey 08540

ABSTRACT

NOTICE
This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

Resonant and non-resonant decays of short wavelength lower-hybrid waves into long wavelength whistler waves and ion acoustic waves are considered. It is shown that the dominant coupling to the ion acoustic mode arises from the magnetic force producing a pressure variation along the magnetic field lines. The growth rate and the threshold condition for this decay instability compare favorably with other decay instabilities near the lower-hybrid frequency.

MASTER

I. INTRODUCTION

Parametric instabilities near the lower-hybrid frequency have been studied extensively in the electrostatic limit.¹

In this work, we consider the decay of an electrostatic pump wave with frequency ω_0 near the lower-hybrid frequency ω_{lh} into a whistler wave and an ion acoustic wave. Here,

$\omega_{lh}^2 = \omega_{pi}^2 / (1 + \omega_{pe}^2 / \omega_{ce}^2)$, $\omega_{pe}(\omega_{pi})$ is the electron (ion) plasma frequency, and ω_{ce} is the electron cyclotron frequency.

We believe that this process is important for the following reason. Numerical solutions of the wave kinetic equation for parametrically excited lower-hybrid waves have shown condensation of wave energy in the lowest frequency electrostatic mode ($\omega \sim \omega_{lh}$) for pump powers only a few times the threshold value.² This energy can be dissipated by nonlinear decay of this electrostatic mode via the process considered here.

Because we are considering decay into long-wavelength modes, the magnetic component of the electromagnetic field cannot be neglected. In fact, the dominant coupling to the ion acoustic mode is through the pressure perturbation parallel to the background magnetic field caused by the magnetic force. This magnetic force is due to the $\underline{E} \times \underline{B}$ drift induced by the pump wave coupling to the perpendicular magnetic field of the fast wave.

In Sec. II, we derive the basic equations describing the decay. In Sec. III, we find the threshold conditions and the growth rates for resonant and non-resonant decays. The implications of our results are discussed in Sec. IV.

II. THEORY

We assume an infinite, spatially homogeneous plasma embedded in a uniform magnetic field $\underline{B}_0 = B_0 \underline{e}_z$. The pump field is of the form

$$\underline{E}_0(x, t) = 2 (E_{0\perp} \underline{e}_x + E_{0\parallel} \underline{e}_z) \cos(k_{0x}x + k_{0\parallel}z - \omega_0 t). \quad (1)$$

We assume that $\omega_{pe} \sim \omega_{ce}$ and, therefore, $\omega_{pi} \gg \omega_{ci}$. The pump wave can be either a dipole pump for which $k_0 = 0$ or a self-consistent electrostatic lower-hybrid wave for which

$$\left| E_{0\parallel} / E_{0\perp} \right| = \left| k_{0\parallel} / k_{0x} \right| \sim (m_e / m_i)^{1/2} \quad \text{and}$$

$\omega_0 = \omega_{lh} (1 + m_i k_{0\parallel}^2 / m_e k_0^2)^{1/2}$. The latter case is of primary interest here.

To analyze the parametric interactions, we assume that the pump electric field $\underline{E}_0$ is sufficiently weak that only interactions up to $O|\underline{E}_0|^2$ (i.e., four wave-wave interactions) need be considered. The four waves are the pump $(\omega_0, \underline{k}_0)$ and $(-\omega_0, -\underline{k}_0)$, the lower sideband $(\omega_-, \underline{k}_-) = (\Omega - \omega_0, \underline{k} - \underline{k}_0)$, the upper sideband $(\omega_+, \underline{k}_+) = (\Omega + \omega_0, \underline{k} + \underline{k}_0)$, and the ion acoustic mode $(\Omega, \underline{k})$. The ion acoustic wave and the pump are taken to be electrostatic waves but the sidebands are electromagnetic waves which we designate whistlers.

The ion acoustic wave satisfies the dispersion relation

$$\Omega = \omega_A - i\gamma_A, \quad \text{where}$$

$$\omega_A = kC_s (1 + 3T_i/T_e)^{1/2}$$

and

$$\gamma_A = (\omega_A/2) (\pi/2)^{1/2} (1 + 3T_i/T_e)^{1/2}$$

$$\left\{ \frac{k}{k_{||}} \left(\frac{m_e}{m_i} \right)^{1/2} + \left(\frac{T_e}{T_i} \right)^{3/2} \exp \left(-\frac{3}{2} - \frac{T_e}{2T_i} \right) \right\},$$

provided the conditions $\omega_{ci} < kv_i < \omega < k_{||} v_e$ are satisfied.

For the pump and the whistler sidebands, we assume that $k_{||} v_e \ll \omega \ll \omega_{ce}$, $k_{||} \ll k$, and $k_{\perp} v_e \ll \omega_{ce}$. These assumptions indicate that we are considering cold plasma waves propagating almost perpendicular to B_0 . The electrons $c\vec{E} \times \vec{B}$ drift across the field lines and oscillate along the field lines with velocity $eE_{||}/m_e \omega$. The components of the electric field of the sidebands obey the following wave equations:

$$\left(-n_{||}^2 - n_y^2 - \frac{\omega_{pi}^2}{\omega^2} \right) + 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} E_x + \left(n_x n_y + \frac{i\omega_{pe}^2}{\omega \omega_{ce}} \right) E_y + n_x n_{||} E_{||} = 0, \quad (2)$$

$$\left(n_y n_x - \frac{i\omega_{pe}^2}{\omega \omega_{ce}} \right) E_x + \left(-n_{||}^2 - n_x^2 - \frac{\omega_{pi}^2}{\omega^2} + 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} \right) E_y + n_y n_{||} E_{||} \quad (3)$$

$$= -4\pi i J_Y^{(2)}/\omega,$$

$$n_x n_{||} E_x + n_y n_{||} E_y - \left(n_{\perp}^2 + \frac{\omega_{pe}^2}{\omega^2} \right) E_{||} = 0. \quad (4)$$

Here, $n = ck/\omega$ is the refractive index; $J_Y^{(2)}$ is the nonlinear part of the current due to coupling between the pump and the ion acoustic (Ω, k) mode. Equations (2) - (4) can be solved in the limit $J_Y^{(2)} = 0$ to find the normal-mode dispersion relations. In the limit $c^2 k^2 \gg \omega_{pe}^2$, we recover the well-known dispersion for the electrostatic lower-hybrid waves, i.e., $\omega = \omega_e$, where

$$\omega_e = \omega_{lh} (1 + m_i k_{||}^2 / m_e k^2)^{1/2} . \quad (5)$$

The damping of these waves in the limit $\omega_e \gg k_{||} v_e$, $k v_i$ is mainly due to electron-ion collisions and is given by

$$\gamma_e = \frac{v_{ei}}{2} \left(\frac{\omega_{lh}^2}{\omega_e^2} + \frac{\omega_{pe}^2}{\omega_{pe}^2 + \omega_{ce}^2} \right) . \quad (6)$$

The ratio of the parallel to perpendicular electric field is

$$|E_{||} / E_{\perp}| \simeq |k_{||} / k_{\perp}| \ll 1.$$

In the limit $c^2 k^2 \ll \omega_{pe}^2$, we find the whistler mode with frequency $\omega = \omega_f - i\gamma_f$, where

$$\omega_f = \frac{ck}{\omega_{pe}} \frac{\omega_{ce}}{\omega_{pe}} \left(\omega_{pi}^2 + c^2 k_{||}^2 \right)^{1/2} , \quad (7)$$

and

$$\gamma_f = \frac{v_{ei}}{2} \frac{c^2 k_{\perp}^2}{\omega_{pe}^2} \left(1 + \frac{m_i}{m_e} \frac{k_{||}^2}{k^2} \right) \frac{\omega_{pi}^2}{(\omega_{pi}^2 + c^2 k_{||}^2)} . \quad (8)$$

The ratio of parallel to perpendicular electric field for this mode is $|E_{||} / E_{\perp}| \simeq (c^2 k^2 / \omega_{pe}^2) |k_{||} / k_{\perp}| \ll |k_{||} / k_{\perp}| \ll 1$.

Since the (Ω, k) mode is an ion acoustic wave, the dominant contribution to the nonlinear current $J_y^{(2)}$ of the sidebands is due to the coupling between the electron density perturbation associated with the ion acoustic wave, δn_e , and the electron $\underline{E} \times \underline{B}$ drift induced by the pump; that is,

$$4\pi i J_{y\pm}^{(2)} / \omega_{\pm} = i(\omega_{pe}^2 / \omega_{\pm} \omega_{ce}) (\delta n_e / n_o) \begin{pmatrix} E_{o\perp} \\ E_{o\perp}^* \end{pmatrix}. \quad (9)$$

Substituting Eq. (9) into Eq. (2) , we obtain

$$(E_y)_{\pm} = -i \frac{\omega_{\pm}^3 \omega_{ce}}{\omega_{pe}^2} \frac{\delta n_e}{n_o} \begin{pmatrix} E_{o\perp} \\ E_{o\perp}^* \end{pmatrix} \frac{(n^2 + n_y^2)_{\pm}}{D_{\pm}}, \quad (10)$$

where

$$D_{\pm} = \omega_{\pm}^2 - \omega_{fe}^2 (k_{\pm} \pm k_o). \quad (11)$$

Since $\omega \ll \omega_{ce}$ for the ion acoustic wave, electron dynamics is mainly in the parallel direction. Thus, the dominant nonlinearity is the electron parallel pressure variation caused by the ponderomotive forces $-m_e (\underline{v} \cdot \nabla) v_{\parallel}$ and $-\frac{e}{c} (\underline{v}_{\perp} \times \underline{b}_{\perp})_{\parallel}$. If we define an effective potential, Ψ , corresponding to these nonlinear forces, we can use it along with the self-consistent potential, ϕ , in the electron one-dimensional Vlasov equation to obtain the charge density perturbation ρ_e given by

$$4\pi \rho_e = -k^2 \chi_e (\phi + \Psi), \quad (12)$$

where

$$\chi_e = - \frac{\omega_{pe}^2}{k^2} \int dv_{\parallel} \frac{\partial f_{oe}}{\partial v_{\parallel}} \bigg/ (v_{\parallel} - \Omega/k_{\parallel}), \quad (13)$$

and $\Psi(\Omega, \underline{k})$ is given by

$$\Psi(\Omega, \underline{k}) = \frac{ic}{B_0 \omega_0} \frac{k_{O||}}{k_{||}} \left[E_{O\perp} E_{Y-} \left(1 - \frac{k_{-||} \omega_0}{k_{O||} \omega_-} \right) + E_{O\perp}^* E_{Y+} \left(1 - \frac{k_{+||} \omega_0}{k_{O||} \omega_+} \right) \right] \quad (14)$$

We note in deriving Eq (14) , we have used the following relations:

$\underline{v}_{10}^e = -(cE_{Ox}/B_0) \underline{e}_Y$, $\underline{v}_{O||}^e = -ie E_{O||} / m_e \omega_0$, $(\underline{v}_{||}^e)_{\pm} \approx 0$, and $(\underline{b}_{\perp})_{\pm} = (ck_{||}/\omega)_{\pm} (E_x \underline{e}_Y - E_Y \underline{e}_x)_{\pm}$. Substituting Eq. (12) into Poisson's equation and assuming a linear ion response, we find

$$\phi = - \chi_e \Psi / \epsilon, \quad (15)$$

and, hence,

$$\frac{\delta n_e}{n_0} = \left[k^2 \chi_e (1 + \chi_i) / 4\pi n_0 e \epsilon \right] \Psi, \quad (16)$$

where $\epsilon = 1 + \chi_i + \chi_e$ and χ_i is the linear ion susceptibility in an unmagnetized plasma. Combining Eqs. (10) , (14), and (16), we obtain the dispersion relation,

$$\epsilon(\Omega, \underline{k}) + \frac{\omega_{ce}^2}{\omega_{pe}^2} \left(\frac{kc|E_{O\perp}|}{B_0 \omega_{pe}} \right)^2 \chi_e(\Omega, \underline{k}) \left[1 + \chi_i(\Omega, \underline{k}) \right] \left(1 - \frac{k_{||0}}{k_{||}} \frac{\Omega}{\omega_0} \right) \left[\frac{c^2(k_Y^2 + k_{||+}^2)}{D_+} + \frac{c^2(k_Y^2 + k_{||-}^2)}{D_-} \right] = 0. \quad (17)$$

III. DECAY INSTABILITIES

Equation (17) has solutions describing both purely growing and decay instabilities. We shall neglect the upper sideband and limit our discussion to the decay process, which can either be a resonant process if $T_e \gg T_i$ so that the ion acoustic wave is weakly damped or a nonresonant process if $T_e \sim T_i$ or $k_{||} \sim (m_e/m_i)^{1/2} k$ so that the ion acoustic wave is highly damped. In either case, the threshold for decay is easily exceeded because the whistler wave is damped weakly by collisions only.

If we assume that the growth rate γ satisfies $\gamma_f \ll \gamma < \gamma_A$ and use the frequency selection rule $\omega_o = \omega_A + \omega_f$, we find the growth rate γ is given by

$$\gamma = \frac{\omega_o - \omega_A}{2} \left| \frac{C E_o + \omega_{pe} \omega_{pi}}{B_o C \omega_{ce} C k_{||}} \right|^2 \frac{1 + \alpha R_i (\pi/2)^{1/2} (m_e/m_i)^{1/2} (k_{ox}/k_{||})}{R_i + (\pi/2)^{1/2} (m_e/m_i)^{1/2} (k_{ox}/k_{||}) \alpha}, \quad (18)$$

where $R_i = (\pi/2)^{1/2} (3 + T_e/T_i)^{1/2} \exp(-T_e/T_i - 3/2)$ and

$\alpha = (1 + 3 T_i/T_e)^{1/2}$. In obtaining Eq. (18), we set

$k_x = k_{ox} \gg k_y$, $k_{||o} \gg \omega_{pi}/c$ and $k_{||} \gg k_{||o}$. These relations follow from the fact that $|k - k_o| \ll \omega_{pe}/c$ but $k_{ox} \gg \omega_{pe}/c$.

Also, if $\omega_o < \sqrt{2} \omega_{lh}$ as assumed here, then $k_{||o} < (m_e/m_i)^{1/2} k_{xo}$.

However, ion acoustic waves propagate only if

$(m_i/m_e)^{1/2} k_{||} > k_x = k_{ox}$. It is obvious from Eq. (18) that the

maximum growth rate obtains when $k_{||}$ is as small as possible

which we take to be $k_{||} = (m_e/m_i)^{1/2} k_{ox}$. Thus the growth rate

is maximum when the ion acoustic wave is highly electron Landau damped and is given by

$$\gamma_{NR} = \frac{1}{2} \left| \frac{CE_{O\perp}}{BC_s} \right|^2 \left(\frac{\omega_{pe}}{\omega_{ce}} \frac{\omega_{pe}}{c k_{ox}} \right)^2 \frac{1 + (\pi/2)^{1/2} R_i}{R_i + (\pi/2)^{1/2}} (\omega_o - \omega_A). \quad (19)$$

If $T_e \gg T_i$ and $k_{||} \gg k_{ox} (m_e/m_i)^{1/2}$, then it is possible for $\gamma_A < \gamma < \omega_A$. Then, we find the growth rate is given by

$$\gamma_{RES} = \frac{1}{2} \frac{C|E_{O\perp}|}{R_o C_s} \frac{\omega_{pe}}{\omega_{ce}} \frac{\omega_{pi}}{ck_{||}} \left[(\omega_o - \omega_A) \omega_A \right]^{1/2}, \quad (20)$$

provided

$$\left| \frac{CE_{O\perp}}{B_o C_s} \right|^2 > \frac{\pi}{2} \frac{c^2 k_{ox}^2}{\omega_{ce}^2} \frac{k_{ox} c_s}{\omega_o - k_{ox} c_s}. \quad (21)$$

For most cases of practical interest, $\gamma_{NR} > \gamma_{RES}$ when Eq. (21) is satisfied.

IV. DISCUSSION

The growth rate given in Eq. (19) and the threshold for this decay instability are comparable to those for decay into quasimodes¹; that is, nonlinear ion and electron Landau damping of electrostatic lower-hybrid waves. However, the threshold for decay is most likely determined by convective losses and finite spatial extent of the pump electric field. These effects may favor decay into short wavelength sidebands, which have smaller group velocities, rather than the long wavelength modes considered here. The latter process may be important, however, in extracting energy from short-wavelength, parametrically-excited daughter waves with frequency so close to the lower-hybrid frequency that further nonlinear decay into electrostatic waves is impossible. That is, this decay process could prevent the condensation of wave energy in low-frequency lower-hybrid modes that was observed in the numerical solution of the wave kinetic equation describing the nonlinear evolution of parametrically excited lower-hybrid waves².

ACKNOWLEDGMENT

This work was supported by U.S. Energy Research and Development Administration Contract E(11-1)3073.

REFERENCES

¹M. Porkolab, Phys. Fluids 17, 1432 (1974); R.L. Berger and F.W. Perkins, Princeton University Report No. PPL-MATT-1130 (to be published in Phys. Fluids).

²Liu Chen and R.L. Berger, Princeton University Report No. PPL-MATT-1169 (submitted to Phys. Rev. Letters); A. Hasegawa, and Liu Chen, Phys. Fluids 18, 1321 (1975).