VARIOUS HOLOGRAPHIC SCANNING CONFIGURATIONS
FOR UNDER-SODIUM VIEWING

By

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H. Dale Collins

SUMMARY

The purpose of this research was to investigate various holographic scanning configurations that could be used for acoustic imaging in liquid sodium. Different scanning configurations were analyzed and experiments were performed to verify the unique properties of each configuration. This report presents a general analysis of acoustical scanned holography with phase-shifting the electronic reference beam and compares various scanning configurations for viewing in liquid sodium. The optimum holographic imaging configuration in liquid sodium would consist of scanning either a focused or point source and receiver transducer and phase shifting the electronic reference. The optimum frequency used for illumination will be determined by the required resolution and range of the imaging system. Either system should be compatible for viewing in liquid sodium.
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VARIABLE HOLOGRAPHIC SCANNING CONFIGURATIONS FOR UNDER-SODIUM VIEWING

H. Dale Collins

INTRODUCTION

The objective of this research was to analyze and experimentally verify the various holographic scanning configurations that could effectively be used for acoustic imaging in liquid sodium. A number of different scanning configurations with unique signal processing techniques were analyzed and experiments performed to verify the unique imaging capability of each configuration. In this report we present a generalized theoretical analysis of acoustical scanned holography employing an off-axis electronically simulated acoustic reference signal. By comparing our test results, we show the advantages and disadvantages of each configuration with respect to imaging in liquid sodium.

The conventional acoustic scanned holograms using an on-axis electronically simulated reference require imaging the objects off-axis (i.e., outside the projected aperture). Off-axis imaging presents serious acoustic illumination problems and decreases the hologram resolution as a result of decreasing the effective scanning aperture. The optimum holographic imaging technique for viewing in liquid sodium should have the following characteristics: 1) ability to image objects directly in the projected scanning aperture, 2) simultaneous source-receiver scanning for maximum resolution, and 3) efficient illumination of the objects in liquid sodium for optimum signal-to-noise ratios.
CONCLUSIONS

The analysis of the various holographic configurations shows that simultaneous point or focused source-receiver scanning with an electronically simulated off-axis reference is the most promising configuration for imaging in liquid sodium.

The focused source-receiver configuration with off-axis reference has the following unique characteristics:

- Optimum resolution and efficient illumination of the objects
- Ability to image objects directly in the projected scanning aperture
- Ability to construct a C-scan image with phase information at the focal plane of the transducer

This system utilizes the best properties of C-scan and that of holography which provides complete depth information. The main disadvantage of this technique is it must be operated pulse echo.

The simultaneous scanning of a point source and receiver with a simulated off-axis reference is the other configuration that should also be considered for imaging in liquid sodium. This configuration has the following unique characteristics:

- Optimum resolution and efficient illumination of the objects
- Ability to image objects directly in the projected scanning aperture
- Ability to operate continuous wave or pulse echo

Either scanning configuration will provide excellent images in liquid sodium, but the focused transducer configuration has the inherent restriction of operating pulse echo. The pulse repetition rate determines the acceptable phase shift rate of the simulated off-axis reference, which determines the scan rate and hologram generation time. The pulse operation is essentially a sampling technique and the sampling rate must be compatible with the phase shift rate if adequate diffraction grating information is to be recovered in the recorded hologram. The sampling frequency (i.e., pulse frequency) must be at least twice the phase shift control frequency to insure the minimum sampling rate.
The point source-receiver configuration can be operated continuous wave and this eliminates the conflict of under sampling the diffraction grating information. The hologram generation time is unrestricted and this may be of importance when considering the final choice for an imaging system in liquid sodium. The main disadvantage of the configuration is that it lacks the capability of generating a C-scan image.
This section presents a detailed analysis of acoustic holography employing simultaneous focused (or point) source receiver scanning. The analysis used is similar to that of Hildebrand and Haines. The image location equations, resolution, lateral and longitudinal magnifications are derived for the various scanning techniques that are used in holography.

The hologram construction and reconstruction geometry used in the analysis is illustrated in Figure 1. The phase at the receiver point \((x,y,z)\) during the hologram construction is

\[
\phi(x,y,z) = \phi_0(x,y,z) - \phi_r(x,y,z) \tag{1}
\]

and

\[
\phi(x,y,z) = \frac{2\pi}{\lambda_S} \left[ r_0 + r_1 - r_2 \right] \tag{2}
\]

The phase at the receiver point \((x,y,z)\) after illumination of the hologram by the reconstruction source is

\[
\phi_1(x,y,z) = \pm \frac{2\pi}{\lambda_S} \left[ r_0 + r_1 - r_2 \right] - \frac{2\pi}{\lambda_L} r_a \tag{3}
\]

where

\[
\lambda_S = \text{acoustic wavelength in construction medium}
\]

\[
\lambda_L = \text{reconstruction (light) wavelength, and}
\]

\(\pm\) refers to the conjugate image,

\(-\) refers to the true image.

If the phase front Eq. (2) is to focus at the image point \((x_b, y_b, z_b)\), then

\[
\phi_1(x,y,z) = \frac{2\pi}{\lambda_L} r_b \tag{4}
\]
Figure 1. Geometry for Scanned Acoustic Holography
which is termed the Gaussian-image sphere. The usual procedure is to expand the distance terms \((r_a, r_b, r_1, r_2, r_0)\) in a binomial series and equate coefficients of \(x, y\) and \(z\). We expand the distance terms about the origin of the \((x,y,z)\) system and the distance \(r_0\) is expanded about the \(\alpha, \beta, \gamma\) system. The area in which the receiver scans is assumed small with respect to the distances and is centered at the \((x,y,z)\) origin. A similar restriction holds for the source motion. Then we have

\[
\phi_1(x,y,z) = -\frac{2\pi}{\lambda_s} \left\{ r_1 - \frac{xy_1 r_1}{r_1} - \frac{y r_0}{2r_1} + \frac{y^2}{2r_1} + r_0 - \frac{r_0}{r_0} \right\}
\]

\[
+ \frac{\alpha^2}{2r^2} + \frac{\beta^2}{2r^2} - r_2 + \frac{x^2 y_2}{2r_2} - \frac{x^2}{2r_2} - \frac{y^2}{2r_2} + \ldots \right\} = \frac{2\pi}{\lambda_s} \left\{ r_a - \frac{x_a x}{r_a} \right\}
\]

\[
- \frac{y_a y}{r_a} + \frac{y^2}{2r_a} + \frac{x^2}{2r_a} + \ldots \right\} = \frac{2\pi}{\lambda_s} \left\{ r_b - \frac{x_b x}{r_b} + \frac{x^2}{2r_b} + \ldots \right\} .
\]  

(5)

The simulated receiver position \((\xi, \eta)\) can be defined in terms of the actual and simulated velocities \(V_x, V_y, V_\xi, V_\eta\), where

\[
\xi = \frac{V_\xi}{V_x} x
\]  

(6)

and

\[
\eta = \frac{V_\eta}{V_y} x
\]  

(7)

After substituting Eqs. (6) and (7) into Eq. (8) and retaining only the first two terms of the expansion, we have:
If we allow only parallel motion of the source and receiver, then the
expressions for the source position are:

\[ \alpha = a_0 + a_1 x \quad , \quad \beta = b_0 + b_1 y \quad . \]

The velocities of the source and receiver are related by

\[ \frac{d\alpha}{dt} = a_1 \frac{dx}{dt} \quad , \quad \frac{d\beta}{dt} = b_1 \frac{dy}{dt} \quad . \]
where $\frac{da}{dt}$, $\frac{d\beta}{dt}$ and $dx$, $dy$ are the velocity components of the source and receiver, respectively. The final expressions for $\alpha$ and $\beta$ (assuming $a_0 = b_0 = 0$) are

$$\alpha = \frac{V_{0\alpha}}{V_x} \xi$$

$$\beta = \frac{V_{0\beta}}{V_y} \eta$$

**IMAGE LOCATION EQUATIONS**

After substituting Eqs. (12) and (13) into Eq. (8) and equating coefficients, we obtain the following expressions for image location:

$$\frac{x_b}{r_b} = \frac{\lambda_L}{\lambda_S} \frac{V_x}{V_\xi} \left\{ \frac{x_1}{r_1} + \left( x_1 - x_0 \right) \frac{V_{0\alpha}}{V_x} \frac{1}{r_0} - \frac{x_2}{r_2} \right\} - \frac{x_a}{r_a}$$

$$\frac{y_b}{r_b} = \frac{\lambda_L}{\lambda_S} \frac{V_y}{V_\eta} \left\{ \frac{y_1}{r_1} + \left( y_1 - y_0 \right) \frac{V_{0\beta}}{V_y} \frac{1}{r_0} - \frac{y_2}{r_2} \right\} - \frac{y_a}{r_a}$$

$$\frac{1}{r_b} = \frac{\lambda_L}{\lambda_S} \left( \frac{V_x}{V_\xi} \right)^2 \left\{ \frac{1}{r_1} + \left( \frac{V_{0\alpha}}{V_x} \right)^2 \frac{1}{r_0} - \frac{1}{r_2} \right\} - \frac{1}{r_a}$$

$$\frac{1}{r_b} = \frac{\lambda_L}{\lambda_S} \left( \frac{V_y}{V_\eta} \right)^2 \left\{ \frac{1}{r_1} + \left( \frac{V_{0\beta}}{V_y} \right)^2 \frac{1}{r_0} - \frac{1}{r_2} \right\} - \frac{1}{r_a}$$

In order for $r_b$ to be the same for each coordinate (i.e., stigmatic), then the following conditions must be satisfied:
\[ \frac{V_x}{V_\xi} = \frac{V_x}{V_\eta} \quad (18) \]

and

\[ \frac{V_{0x}}{V_x} = \frac{V_{0y}}{V_y} \quad (19) \]

Then we have the final expressions for the image location equations:

\[ \frac{x_b}{r_b} = + \frac{\lambda_L}{\lambda_S} g \left\{ \frac{x_1}{r_1} + (x_1 - x_0) \frac{f}{r_0} - \frac{x_2}{r_2} \right\} - \frac{x_a}{r_a} \quad , \quad (20) \]

\[ \frac{y_b}{r_b} = + \frac{\lambda_L}{\lambda_S} g \left\{ \frac{y_1}{r_1} + (y_1 - y_0) \frac{f}{r_0} - \frac{y_a}{r_2} \right\} - \frac{y_a}{r_a} \quad , \quad (21) \]

\[ \frac{1}{r_b} = + \frac{\lambda_L}{\lambda_S} g^2 \left\{ \frac{1}{r_1} + \frac{f^2}{r_0} - \frac{1}{r_2} \right\} - \frac{1}{r_a} \quad , \quad (22) \]

where

\[ g = \frac{V_x}{V_\xi} = \frac{V_y}{V_\eta} \quad (23) \]

and

\[ f = \frac{V_{0x}}{V_x} = \frac{V_{0y}}{V_y} \quad (24) \]
MAGNIFICATIONS

Lateral Magnification:

\[ M_L(x) = \frac{\partial y_b}{\partial x_1} = \pm \frac{\lambda_L}{\lambda_S} g \left\{ 1 + \frac{r_1 f}{r_0} \right\} \left( \frac{r_b}{r_1} \right) \text{,} \tag{25} \]

\[ M_L(y) = \frac{\partial y_b}{\partial y_1} = \pm \frac{\lambda_L}{\lambda_S} g \left\{ 1 + \frac{r_1 f}{r_0} \right\} \left( \frac{r_b}{r_1} \right) \text{,} \tag{26} \]

where

\[ z_1 \gg x_1 \text{ or } y_1 \]

\[ z_1 - z_0 \gg x_1 - x_0 \text{ or } y_1 - y_0 \text{.} \]

If the source and the receiver are located at the same position (i.e., \( r_1 = r_0 \text{ and } f = 1 \)), then the lateral magnification can be expressed by

\[ M_L(x) = M_L(y) = 2 \frac{\lambda_L}{\lambda_S} \frac{r_b}{r_1} g \text{.} \tag{27} \]

Radial Magnification:

\[ M_R = \frac{\lambda_L}{\lambda_S} \left( \frac{r_b}{r_1} g \right)^2 \left[ 1 + f^2 \left( \frac{r_1}{r_0} \right)^2 \right] \text{.} \tag{28} \]

If the magnification of the hologram is to be undistorted, then the ratio \( M_R / M_L \) must equal unity.
The ratio of the magnifications can be controlled by the velocity ratios

\[
\left( f = \frac{V_0 \alpha}{V_x} = \frac{V_0 \beta}{V_y} \right)
\]  

(30)

Scanning the receiver and source together the ratio reduces to

\[
\frac{M_R}{M_L} = g \frac{r_b}{r_1} \left[ 1 + f^2 \left( \frac{r_1}{r_0} \right) \right]
\]  

(31)

**Lateral Resolution**

The resolution in the object space is defined as the incremental distance \( \Delta x_1 \) through which an object point can be displaced before the phase of the finest fringe arising at the hologram plane is deviated \( \pi \) radians during the recording process. The resolution can be expressed approximately by the following equation if simultaneous point source receiver scanning is employed:

\[
\Delta x_1 \approx \frac{\lambda S r_1}{2L}
\]  

(32)

where \( L \) is the aperture length.
The resolution is decreased (i.e., $\Delta x_1$ increased) by a factor of two if stationary source or plane wave source scanning is employed. The resolution is defined for objects located near the "z" axis and if the object distances are large compared with the aperture dimensions.

**Radial Resolution**

The radial or depth resolution is more difficult to define mathematically than the lateral resolution. The radial resolution as stated by Hildebrand\(^2\) can be related to a decrease in intensity of the image as a result of a incremental change in $r_b$. The object point can then be moved an incremental distance $\Delta r_1$ to achieve the same decrease in intensity of the image. The radial resolution can then be expressed as:

$$\Delta r_1 = 1.8 \lambda S \left( \frac{r_1}{L} \right)^2 \quad (33)$$
SIMULATED ACOUSTIC OFF-AXIS REFERENCE BEAMS

If we assume an acoustic plane wave reference beam, then the reference beam can be expressed as

$$S_R(x,y) = P_R(x,y) \cos \left( \omega t - \frac{2\pi}{\lambda_S} x \sin \alpha_{RS} \right)$$  \hspace{1cm} (34)

where $$\phi_{RS}(x,y) = \frac{2\pi}{\lambda_S} x \sin \alpha_{RS}$$.

No loss of generality results in only considering two dimensions with the object located on the (x-z) plane. The signal contribution to the acoustic receiver by the reference beam is

$$P_R(x,y) \cos \left( \omega t - \frac{2\pi}{\lambda_S} \sin \alpha_{RS} V_x t \right)$$  \hspace{1cm} (35)

where $$V_x$$ is the scanning velocity of the acoustic receiver and

$$\omega_{RS} = \frac{2\pi}{\lambda_S} V_x \sin \alpha_{RS}$$  \hspace{1cm} (36)

Equation (35) represents a sinusoidal wave whose phase ($$\phi_{RS}$$) is a function of the scanning velocity ($$V_x$$) and the inclination angle ($$\alpha_{RS}$$). Thus, the acoustic plane wave reference beam can be simulated with an electrical signal of this form and combined with the object signal in a balanced mixer or multiplier. If the direction of propagation of the plane wave reference is perpendicular to the x axis (i.e., $$\alpha_{RS} = 0$$), then $$\omega_{RS} = 0$$ and the reference signal is simply $$P_R(x,y) \cos \omega t$$. This electrical signal when combined with the object signal in a mixer would simulate an on-axis plane wave acoustic reference beam. Employing an on-axis reference beam requires imaging objects outside of the projected scanning aperture to provide separation of nondiffracted light and the
images in the reconstruction. Now if the inclination angle is not zero, then the electrical reference signal must be phase shifted to simulate an off-axis acoustic reference beam. The simulated inclination angle is a function of the scanning velocity, wavelength of sound, and the phase shifter control voltage frequency. Naturally, in three dimensions skewed beams are possible with phase shifting and time delay circuits. The phase of the electronic reference can be shifted with respect to time by the following voltage waveform shown in Figure 2. For a more detailed analysis of the phase shifter and control circuits, see Appendix A.

The control voltage radian frequency ($\omega_p$) can be expressed in terms of $\omega_{RS}$:

$$\omega_p = 2\pi f_p = \frac{2\pi}{\lambda_S} V_x \sin \alpha_{RS}$$

\hspace{1cm} (37)
If the simulated plane wave is inclined with respect to the hologram plane (i.e., $\alpha_{RS} > 0$), then a linear diffraction grating will be imposed on the hologram. The grating spacing is a function of the scanning velocity ($V_x$) phase shifter frequency ($f_p$) and the hologram magnification ($m$). The grating spacing on the hologram is

$$d = \frac{V_x T_p}{m} = \frac{V_x}{m f_p} = \frac{\lambda_S}{\sin \alpha_{RS}}$$

where

$$\alpha_{RS} = \sin^{-1} \left( \frac{\lambda_S m f_p}{V_x} \right).$$

If $\alpha_{RS} = \pi/2$, then the finest grating imposed on the hologram has a spacing of $\lambda_S$ and the maximum frequency is

$$f_{p_{max}} = \frac{V_x}{m^{\lambda_S}}.$$

It should be obvious that any grating spacing can be imposed on the hologram by proper adjustment of the phase shift control voltage frequency. The grating spacing can be less than a wavelength which indicates that electronic simulation is more versatile than using an acoustic reference beam. The grating lines are usually constructed perpendicular to the scanning lines in rectilinear scanning. This results in two sets of gratings. Figure 3 is a typical grating imposed on the hologram (rectilinear scanning). Figure 4 is a typical grating imposed on the curvilinear scanned hologram. Figure 5 is the acoustic hologram and the reconstruction of a point source located directly in the center of the projected scanning aperture. The phase shift control-voltage-frequency was 78.1 Hz and the acoustic frequency was 3.26 MHz. The hologram shows the familiar zone plate located directly in the center of the aperture and also shows the diffraction grating as a result of phase shifting the reference. The reconstruction shows the image of the point source in the center of the undiffracted light and the first order diffracted images directly above
Acoustic Frequency: 3 MHz  
Scan Rate: 6.1 cm/sec  
Phase Shift Frequency: 83.5 Hz  
Vertical Grating Spacing: 0.46 mm

Figure 3. Diffraction Gratings Imposed on the Acoustic Hologram (Rectilinear Scanning)

Acoustic Frequency: 3.2 MHz  
Scan Rate: 1.61 rev/sec  
Phase Shift Frequency: 555 Hz  
Grating Spacing: 0.348 mm

Figure 4. Diffraction Gratings Imposed on the Acoustic Hologram (Curvilinear Scanning)
Figure 5. Acoustic Hologram and the Reconstruction of a Point Object Using Off-Axis Reference
and below it. There are six diffracted images present and the outer two on each side of the undiffracted light indicate skewed gratings are present in the hologram. These gratings result from cross terms in the vertical and horizontal gratings.

The image was reconstructed using a helium-neon laser (6328 Å) at 10 m and the image was diffracted approximately 3.2 cm from the undiffracted light. The lateral image displacement from the undiffracted light was calculated using either of the following equations:

\[ \delta = \frac{\lambda_L}{\lambda_S} m n r_b \sin \alpha_{RS} \]  

or

\[ \delta = \frac{\lambda_L m n f_p}{V} r_b \]  

where
- \( \lambda_L \) = reconstruction wavelength
- \( \lambda_S \) = construction wavelength
- \( m \) = hologram magnification
- \( n \) = order of the diffracted light
- \( r_b \) = hologram to image distance
- \( \alpha_{RS} \) = inclination angle (with respect to the horizontal) of the simulated reference beam
- \( f_p \) = phase shift control voltage frequency
- \( V \) = scan velocity of the acoustic receiver.

The results were in excellent agreement with the theoretical calculations. This technique of electronic phase shifting the reference beam provides the necessary conditions for imaging directly in the projected scanning aperture and for separation of the images with the undiffracted light. The ability to image directly in the scanning aperture increases the lateral and radial resolution by increasing the effective aperture. Use of this technique and point source-receiver scanning provides an additional increase in the resolution by a factor of two and reduces the object illumination problem.
ACOUSTICAL SCANNED HOLOGRAPHY SYSTEMS

RECTILINEAR SCANNER

The system used in the construction of the rectilinear scanned holograms is shown in Figure 6. The apparatus is basically an x-y scanner with some unique signal conditioning features. The acoustic transmitters were either plane wave, focused or spherical PZT-4 ceramics. The acoustic receiver was a PZT-5 ceramic (0.254 mm in diameter). The objects were located outside of the periphery of the projected scanning aperture when stationary source illumination was employed, which allowed complete separation of the two images (true, conjugate) and the undiffracted light. Objects were placed inside the projected aperture and the reference signal was phase-shifted to provide the necessary conditions for image separation. Another technique for imaging objects in the aperture is scanning an inclined plane wave source with the receiver. Either technique imposes a diffraction grating on the image which shifts the image from the undiffracted light.

The field of sound scattered or generated by the objects is scanned over a plane area by either or both the receiver and source transducers, depending on the scanning configuration. The scanned area, (i.e., aperture) was usually about 10 cm by 10 cm with a line separation of 0.457 cm. The scanner is capable of varying the aperture, line density, and the scan rate. The receiver signals are amplified by a preamplifier located in close proximity to the receiving transducer. They are amplified again and then mixed with the electronic reference signal in a balanced mixer or multiplier. The output signal of the mixer is time-averaged, and the large voltage peaks are clipped before the signal is used to modulate the intensity of a glow modulator tube. The average level is adjusted to the center of the dynamic range of the light source and film sensitivity. The varying intensity of the modulator tube is coupled optically by a fiber light-pipe to the top of the receiving transducer. A mirror located on the transducer reflects the light to a camera mounted above the scanning plane. The hologram is then constructed on 46L Polaroid film.
Figure 6. Ultrasonic Hologram Construction
The circular scanning system operates in the same way as the rectilinear system, except the receiving transducer scans a circular path as it is being translated the length of the aperture. The system consists of three sections: the rotational disk, the translation platform table, and the electronic instrumentation unit. The circular scanning system is shown in Figure 7. The rotational disk on which the receiving transducer is located is 16 cm in diameter and can be driven from 0 to 1500 rpm. The translation platform on which the rotation system is located can be driven at various speeds 0 to 100 cm/min. The hologram generation time and the translational velocity determines the aperture length. The angular velocity of the disk and the translational velocity determine the number of scan lines generated per second (i.e., sampling density). For example, if a total of 150 lines are required in the hologram and the generation time is 1 sec, the disk must rotate at 150 rps to generate the minimum sampling density.

The signal conditioning features are the same as the x-y system. The basic difference between the two systems is the raster pattern imposed on the hologram. The x-y scanner imposes a straight line grating which produces multiple images in the reconstruction. The circular scanner imposes a curved line grating which does not produce multiple images in the reconstruction, but spreads the information over the entire hologram. The minimum sampling density is then dependent on the scanning configuration used to prevent image overlap between the diffracted orders.
Figure 7. Circular Scanning System
ACOUSTIC HOLOGRAM RECONSTRUCTION

The hologram reconstruction technique in acoustical holography differs from its optical counterpart as shown in Figure 8. In optical holography, the reconstruction distances (i.e., image-to-hologram distances) are the same as the construction distances (i.e., object-to-hologram distances) if the same source is used in the construction and reconstruction of the hologram. In acoustical holography, the hologram is constructed using sound and it is reconstructed optically. The sound wavelength is much greater than the optical wavelength and at 10 MHz the sound-to-optical wavelength ratio is approximately 250, assuming a helium-neon laser reconstruction source. This large difference in wavelengths means the image distance will be much greater than the object distance. The approximate image distance \( r_b \) is given by

\[
r_b \approx \frac{\lambda_S}{\lambda_L} r_I
\]  

(41)

Figure 8. Hologram Reconstruction Geometry
where the hologram magnification is unity. The object distance \( r_1 \) is multiplied by the wavelength ratio and at 1 MHz the image would reconstruct at 2500 times the object distance.

In the reconstruction a positive or converging lens is used to effectively reduce the image distances, as shown in Figure 8. A helium-neon laser (6328 Å) is used and the beam is expanded with a microscope objective to completely illuminate the hologram. The lens is adjusted to bring the true (real) image into focus at approximately 10 m from the hologram. The zero-order (i.e., undiffracted light) and the conjugate (real) image are then located between the true image and the hologram.

The hologram is inserted in a liquid gate containing a solution of decahydronaphthalene (6 parts) and tetrahydronaphthalene (1 part) between two optically flat (5x) glass plates. The two chemicals form a solution with an index of refraction approximating that of the hologram film (i.e., Polaroid 46L). The liquid gate essentially eliminates the undesired effects of film thickness variations.
EXPERIMENTAL RESULTS WITH THE VARIOUS SCANNING CONFIGURATIONS

RECTILINEAR SCANNER EXPERIMENTS

Stationary Source Receiver Scanning and Phase Shifting The Electronic Reference

Figure 9 shows the acoustic hologram and the reconstruction of a styrofoam object. The object was located directly in the projected scanning aperture 30.5 cm from the aperture plane. The rectilinear scanner shown in Figure 6 was used to construct the hologram. The poor quality image of the letter "F" is the result of inadequate illumination from the stationary source. The electronic reference signal was phase-shifted which imposed a diffraction grating (i.e., 0.183 mm) on the hologram. The image was diffracted in the lateral direction approximately 2.75 cm from its original position in the undiffracted light. The raster produced a vertical diffraction grating and multiple images appear in the reconstruction (see Figure 9). The first order diffracted image to the left of the undiffracted light shows the position of the "F" in the scanning aperture. The phase shifted reference produced a horizontal diffraction grating and the diffracted image is easily seen in Figure 9.

Simultaneous Point Source Receiver Scanning and Phase Shifting The Electronic Reference

Figure 10 shows the acoustic hologram and reconstruction of the true image. The object was located directly in the projected scanning aperture 19.1 cm from the aperture plane. The image resolution of the letter "F" is better by a factor of two for the same object-to-hologram distance as compared with the stationary source hologram. Hildebrand proved theoretically and experimentally that scanning a point source and receiver together increased the resolution by a factor of two compared with the stationary source hologram and made the image appear closer to the hologram. (2)
Acoustic Hologram
Illumination: Stationary Source
Frequency: 3.1 MHz
Film Magnification: 2
Line Density: 22 lines/cm
Phase Shift Frequency: 83.5 Hz
Scanner: x-y

True Image at 10 m
Object: Styrofoam "F"
Aperture: 12.5 cm x 12.5 cm
Lateral Magnification: 0.174
Lateral Resolution: 1.21 mm
Grating Spacing: 0.183 mm

Figure 9. Scanned Receiver Hologram with Phase Shifting and the Reconstruction
Acoustic Hologram
Illumination: Scanning Point Source
Frequency: 3.2 MHz
Scan Rate: 6.1 cm/sec
Film Magnification: 2
Line Density: 22 lines/cm
Phase Shift Frequency: 133 Hz
Scanner: x-y

Figure 10. Simultaneous (Point) Source Receiver Scanned Hologram with Phase Shifting and the Reconstruction

True Image at 4 m
Object: Styrofoam "F"
Aperture: 10 cm x 10 cm
Lateral Magnification: 0.266
Lateral Resolution: 0.46 mm
Grating Spacing: 0.114 mm
The image-to-hologram distance \( r_b \) reduces to the following expression if simultaneous source receiver scanning is employed and if the reference and reconstruction sources are plane waves:

\[
\bar{r}_b = \pm \frac{\lambda S}{\lambda L m^2} \frac{r_1}{2}
\]

(42)

where

\[
m = \frac{V_x}{V_z} = \frac{V_y}{V_n} = \frac{L_x}{L_z} = \frac{L_y}{L_n}
\]

If stationary source or scanning plane wave source is employed, then the image location equation reduces to

\[
\bar{r}_b = \pm \frac{\lambda S}{\lambda L m^2} r_1
\]

(43)

The ratio of the stationary source to scanning source image-to-hologram distance (i.e., \( r_b / \bar{r}_b \)) is approximately two. The simulated off-axis reference (i.e., phase shifting) imposed a diffraction grating and shifted the image from the undiffracted light. The phase shift frequency was 133 Hz and the scan rate 6.1 cm/sec. The grating spacing was 0.114 mm, and the electronic phase-shifted reference imposed a grating spacing that was less than could be achieved with an off-axis reference acoustic beam. This means finer gratings can be generated electronically than acoustically for any given source frequency.

Simultaneous Focused Source Receiver Scanning with Phase Shifted Electron Reference

Figure 11 is the acoustic hologram and the reconstruction of the milled "F" in the aluminum block. A focused transducer acting as the source and receiver (pulse-echo operation) scanned a 12.5 cm by 12.5 cm aperture approximately 10 cm above the aluminum block using the rectilinear scanner.
The diameter of the transducer was 2.54 cm with a 10 cm focal length. The object was located 11.4 cm from the block surface and 21.6 cm from the scanned aperture. The focused transducer simulated a point source and receiver at the block surface. The electronic reference was phase shifted with respect to the object signal which imposed a linear grating on the hologram. The image is of excellent quality and shows this is one of the optimum holography scanning configurations for imaging flaws or defects in metallic objects.

**Simultaneous Source Receiver Scanned Hologram with Phase Shifting and The Reconstruction**

Figure 12 shows the acoustic hologram and the reconstruction of the FFTF fueling cap. The object was located directly in the projected scanning aperture 44 cm from the aperture plane. The hologram was constructed using the rectilinear scanner and the line density was approximately 22 lines/cm. The image quality of the metal FFTF fuel duct socket for a specular reflector was excellent. The electronic reference signal was phase shifted and imposed a diffraction grating on the hologram. The image was diffracted out of the zero order light and is shown clearly in the reconstruction. A spherical source and point receiver was scanned together to construct the acoustic hologram. The image lateral magnification was approximately 0.287 and the lateral resolution was 1.81 mm. This scanning configuration satisfies the three requirements for optimum imaging in liquid sodium but it does not satisfy C-scan requirements.

Figure 13 shows the acoustic hologram and the reconstruction of three metal FFTF duct sockets. The restricted aperture shows the three intersection points of the end caps in the reconstruction. The image quality is excellent and shows the clear distinct features of the intersections of the three duct sockets. The theoretical lateral resolution was approximately 0.81 mm and the image magnification 0.237. The image was shifted out of the zero order light and the grating spacing imposed on the hologram was approximately 1.68 mm. Figure 14 shows two FFTF duct sockets separated approximately 11 cm apart in depth. The reconstruction shows clearly that holography has the capability of displaying complete depth
Acoustic Hologram
Illumination: Focused Source
Frequency: 3.4 MHz
Film Magnification: 2
Line Density: 22 lines/cm
Phase Shift Frequency: 91 Hz
Scanner: x-y

Figure 11. Simultaneous Focused Source-Receiver Scanned Hologram with Phase Shifting and the Reconstruction
Acoustic Hologram
Illumination: Spherical Source
Frequency: 3.27 MHz
Film Magnification: 2
Line Density: 22 lines/cm
Phase Shift Frequency: 83.5 Hz

Optical Image
Top View

Reconstruction
True Image at 10 m
Object: Metal FFTF Fuel Duct Socket
Aperture: 12.5 cm x 12.5 cm
Lateral Magnification: 0.287
Lateral Resolution: 1.81 mm
Grating Spacing: 0.183 mm

Figure 12. Simultaneous Source Receiver Scanned Hologram with Phase Shifting and the Reconstruction
Acoustic Hologram
Illumination: Spherical Source
Frequency: 3.27 MHz
Film Magnification: 2
Line Density: 22 lines/cm
Phase Shift Frequency: 100 Hz
Scanner: X-Y

Optical Image
Top View

Reconstruction
True Image at 10 m
Object: Metal FFTF Fuel Duct Sockets
Aperture: 12.5 cm x 12.5 cm
Lateral Magnification: 0.237
Lateral Resolution: 1.81 mm
Grating Spacing: 0.168 mm

Figure 13. Simultaneous Source Receiver Scanned Hologram with Phase Shifting and the Reconstruction
Acoustic Hologram
Illumination: Spherical Source
Frequency: 3.27 MHz
Film Magnification: 2
Line Density: 22 lines/cm
Phase Shift Frequency: 91 Hz
Scanner: X-Y

Optical Image

Reconstructions
True Images at 10 m
Object: Metal FFTF Fuel
Duct Socket
Aperture: 12.5 cm x 12.5 cm
Lateral Magnification: 0.25
Lateral Resolution: 2 mm
Upper Object: 0.55 mm
Lower Object: 0.76 mm
Grating Spacing: 0.19 mm

Figure 14. Simultaneous Source Receiver Scanned Hologram with Phase Shifting and the Reconstruction
information about the objects. This is an important advantage over the conventional C-scan which displays information on one plane only in depth. The two pictures show the upper duct socket in focus and the other picture shows the lower end cap in focus. This geometrical arrangement of duct sockets shows the advantages of holography over conventional C-scan which is incapable of imaging other than at the focal point of the transducer.

C-Scan Images Using a Focused Transducer

Figure 15 shows typical C-scan images of the FFTF fuel duct sockets. The C-Scan images show excellent resolution, but the image distance was approximately 5 cm. Imaging with C-scan techniques require the object to be located approximately parallel with the scanning plane and this requires proper placement of the objects with respect to the scanning aperture. The optimum imaging system for under-sodium viewing would incorporate the best qualities of C-scan and holography. The two techniques can be combined into one system and this system should be unique for imaging in liquid sodium.

CURVILINEAR SCANNER EXPERIMENTS

Stationary Spherical Source and Scanning Point Receiver

Figure 16 shows the acoustic hologram and the reconstruction of the true image at 10 m. The translational circular scanner was used without phase shifting the electronic reference (see Figure 7). The reference signal simulated an on-axis plane wave. The object was located outside of the periphery of the projected scanning aperture 34 cm from the aperture plane. This scanning configuration has the disadvantage of reduced resolution and off-axis imaging. The effective aperture is decreased as the object moves off-axis. This decrease in the effective aperture decreases the lateral resolution.

The circular scanner is capable of constructing holograms very rapidly. The translational circular scanner developed at Battelle-Northwest has constructed acoustic holograms in 15 sec. The variable circle scanner (not described in this report) has constructed holograms in essentially real time (i.e., milliseconds).
Figure 15. C-Scan Images of FFTF Fuel Duct Sockets Located 5 cm from the Transducer
Figure 16. Curvilinear Scanned Receiver Hologram and the Reconstruction

Acoustic Hologram
Illumination: Spherical Source
Frequency: 2.92 MHz
Line Density: 13.1 lines/cm
Scan Rate: 100 rpm
Scanner: Circular

True Image at 10 m
Object: Styrofoam "F"
Aperture: 15 cm x 15 cm
Lateral Magnification: 0.127
Lateral Resolution: 1.17 mm
Stationary Source, Scanning Point Receiver and Phase Shifting The Electronic Reference

The translational circular scanner (see Figure 7) was used to construct the acoustic hologram. Figure 17 is the hologram and the reconstructed image. The receiver scans a series of circular paths as the rotational disk is translated in one direction. This scanning configuration imposes a curved diffraction grating on the hologram which distorts the higher diffracted order images that are always present in the rectilinear scanned holograms. The blurred patches above and below the undiffracted light are the diffracted light and image. The image quality is rather poor as a result of unequal spacing between the diffraction grating lines generated by phase shifting the electronic reference. The phase-shifted reference signal imposed a diffraction grating that was parallel to the translational scan direction (i.e., vertical lines). The image was diffracted approximately 1.12 cm from the undiffracted light at 10 m. The image of the letter "F" is visible in the undiffracted light if one looks closely at the reconstruction. The object was located directly in the projected scanning aperture 44 cm from the aperture plane.

Simultaneous Focused Source Receiver Scanning and Phase Shifting The Electronic Reference

Figure 18 is the acoustic hologram and the reconstruction of a point object located approximately 40 cm from the scanning aperture. The hologram was constructed at 3.4 MHz using a focused 2.54 cm diameter PZT-4 transducer. The focal length was approximately 10 cm. The sound pulse repetition rate was 1 msec and the phase shift control frequency about 500 Hz. The reconstruction of the hologram shows the point object in the center of the hologram (i.e., in the undiffracted light) and also diffracted light to the right of the undiffracted light. The simulated off-axis reference imposed a diffraction grating on the hologram which provided the necessary conditions for diffraction of the image. The sampling rate (i.e., repetition rate) has to be compatible with the phase shift rate if adequate grating information is to be retained in the hologram. In practice the sampling rate should be approximately
Acoustic Hologram
Illumination: Spherical Source
Frequency: 3.25 MHz
Line Density: 9.3 lines/cm
Phase Shift Frequency: 555 Hz
Scan Rate: 150 rpm
Scanner: Circular

Object: Styrofoam "F"
Aperture: 20 cm x 20 cm
Lateral Magnification: 0.167
Lateral Resolution: 1 mm
Grating Spacing: 0.52 mm

Figure 17. Curvilinear Scanned Receiver Hologram with Phase Shifting and the Reconstruction
Acoustic Hologram
Illumination: Focused Source
Frequency: 3.4 MHz
Line Density: 9.4 lines/cm
Phase Shift Frequency: 500 Hz
Scan Rate: 100 rpm
Scanner: Circular

Figure 18. Curvilinear Scanned Focused Source Receiver Hologram with Phase Shifting and True Image at 10 m
Object: Glass Hemisphere
Aperture: 10 cm x 10 cm
Lateral Resolution: 2 mm
Grating Spacing: 0.49 mm

Figure 18. Curvilinear Scanned Focused Source Receiver Hologram with Phase Shifting and The Reconstruction
five to ten times the phase shift rate. The image of the point object shows that it is feasible to use the simultaneous focused source receiver curvilinear scanning technique for viewing in liquid sodium. If electronic phase shifting is used, then the sampling rate must be compatible with the phase shift rate of the reference. This restricts the hologram generation time of this technique.

REFERENCES


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APPENDIX A

THE ELECTRONIC REFERENCE PHASE SHIFTER

The phase shifter consists of six basic units each capable of phase shifts exceeding 170°, and linear to approximately 65°. The phase of the electronic reference signal is shifted by the FET-capacitance network as shown in Figure A.1. The FET's drain to source resistance is varied by the voltage applied between the gate and the source.

![Figure A.1. Schematic of the Phase Shifter Unit](image-url)
The equivalent circuit of the phase shifter is shown in Figure A.2. Writing the equations for the equivalent circuit shows

\[ I = \frac{2E_{in}}{R-jX_C} \]

\[ \frac{E_o}{E_{in}} = 1 - \frac{2R}{R-jX_C} \frac{R+jX_C}{R-jX_C} = \frac{e^{j\phi}}{e^{-j\phi}} = e^{j(\pi + 2\theta)} \]

where \( \theta = \tan^{-1}\left(\frac{1}{\omega RC}\right) \).

The gain of the phase shifter (i.e., \(|E_o/E_{in}|\)) is unity and the output amplitude \(E_o\) is independent of either \(R\) or \(C\). If \(R\) or \(C\to 0\) the phase is shifted 180°. Thus, each section theoretically is capable of shifting the phase 180°. In practice, with 2N3820 FET's the linear phase shift range was approximately 65° per unit and the complete system consisted of six units to obtain 360° phase shift.

**Figure A.2.** Equivalent Circuit of the Phase Shifter Unit
The six units are connected in series and controlled by a variable frequency sawtooth generator. The system provides essentially constant output amplitude of the signal over the entire phase shift range. Each unit is followed by an inverting amplifier to provide adjustable gain and impedance matching. Figure A.3 shows the complete system with the external variable frequency sawtooth generator.
Figure A.3. Electronic Phase Shifter and Associated Circuits
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