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MODELS FOR DIFFRACTION SCATTERING
WITH FIXED POLE AND SHIELDING CUT

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and the Department of Physics
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ABSTRACT

It is proposed that the nonshrinking πp diffraction peak is due to a fixed Pomeron, t-channel unitarity being preserved by a shielding cut, which also explains the constancy of high energy total cross sections. Several models are discussed, and used to fit πp and pp data; in one, a new pole is assumed near the Pomeron. A hypothetical "scaling" relation among $\pi\pi$, πp , and pp elastic scattering amplitudes is discussed.

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I. INTRODUCTION

For many years, the existence of a nonshrinking diffraction peak in πp scattering has posed a problem for Regge theorists. A number of explanations have been given, involving either fixed singularities¹ or cancellation effects among several moving singularities.² All of these solutions, so far as we are aware, predicted a falling total cross section in the energy region above that then accessible. Such predictions are sharply contradicted by the recent Serpukhov data, which show the total cross section constant, or perhaps even slightly rising, for incident pion momenta of 25 to 65 GeV/c.

In this paper, we discuss models for πp scattering in which the mechanism used to achieve a non-shrinking forward peak not only is consistent with the Serpukhov results, but explains them. This mechanism is a fixed Pomeron plus a moving shielding cut. It is applied not only to πp scattering, but also to $\pi\pi$ and pp scattering, which are linked to πp by low energy unitarity in the crossed channel.

II. DATA SELECTION, TERMINOLOGY, AND OTHER PRELIMINARIES

We use data for π^+p , pp , $\bar{p}p$ and $\pi\pi$ total cross sections, π^+p , pp and $\bar{p}p$ differential cross sections. Our πp total cross sections come from the Foley³ data and from Serpukhov.⁴ We use the same sources for pp and $\bar{p}p$ total cross sections, except that the pp high energy total cross sections, which are not available experimentally in the Serpukhov region, are estimated from the $\bar{p}p$ cross sections, on the assumption that their difference continues to fall slowly. $\pi\pi$ total cross sections are taken from OPE⁵ calculations. The phase of πp forward scattering comes from Coulomb interference experiments.

For πp differential cross sections we use the data of Foley, Lindenbaum, et. al.⁶, and for pp differential scattering the results of Beznogikh et. al.⁷

We also make use of the cosmic ray results on very high energy pp inelastic total cross sections from Jones et. al. (Echo Lake Experiment⁸) and Grigoy et. al.⁹

We will frequently write scattering amplitudes in the form $F(s,t)$. Since we must go back and forth between crossed reactions, interchanging the physical significance of s and t , we establish the following convention. s is always the energy variable in the channel for which we examine actual physical data, and t the momentum transfer variable in that channel. Thus in the crossed channel, whose unitarity condition is crucial to our arguments but for which no experimental numbers are presented, t is the energy variable and s the momentum transfer variable.

In discussing the diffraction peak, we deal with expressions of the form $|F(s,t)|^2 = f(s)e^{at}$. The peak associated with such an amplitude is described as nonshrinking (in some range of s) if a does not change as s changes. If a increases with increasing s , the peak is shrinking; if it decreases, the peak is broadening. a is called the slope parameter.

$F(s,t)$ may be a sum of terms $T_i(s,t)$. Each such term is described as nonshrinking, shrinking, or broadening, according to the behaviour of $|T(s,t)|^2 = t(s)e^{bt}$.

A singularity in $F(t,\lambda)$ which does not change its position in the λ plane as t changes is called a fixed singularity, and must give rise to a nonshrinking peak in $F(s,t)$. A single moving singularity in $F(s,\lambda)$ must generate a shrinking peak. But a peak may be nonshrinking for some range of s due to cancellations among several moving singularities; thus we cannot use "fixed term" and "nonshrinking term" as synonyms.

It should be noted that we use the word nonshrinking, in defiance of the rules of English usage, to mean neither shrinking nor broadening. In this we believe that we follow established practice.

Since the models we propose concern only positive signature amplitudes, we will, whenever possible, write experimental data in the form of positive signature quantities,

$$\sigma_{\pi p}^+ \equiv \frac{\sigma_{\pi^+ p} + \sigma_{\pi^- p}}{2}$$

for instance, and fit the data in that form, ignoring the negative signature quantities. Where it is impossible to calculate a positive signature quantity directly from the data (pp slope parameter, for instance) we fit the negative signature amplitude with a single Regge term having an intercept of .5. None of our results are sensitive to such fits.

We do not assume exchange degeneracy between the residues of our positive and negative signature Regge poles. In most of our models, the only contribution to the total cross section which falls with energy comes from the ρ' . If this is completely exchange degenerate with the ρ ,^(w) then they cancel in $\sigma_{\pi p}$ (σ_{pp}) which must therefore be monotonically increasing. It is not.

Throughout this paper, we will deal only with spin nonflip amplitudes. We will define our amplitude such that:

$$\sigma_{tot} = \frac{16\pi}{2g(s)\sqrt{s}} \operatorname{Im} F(0, s)$$

We will use μ for the pion mass and M for the nucleon mass.

III. UNITARITY CONSTRAINTS

Some years ago, Oehme demonstrated¹⁰ that shielding cuts are required in certain amplitudes in order to satisfy crossed channel unitarity in the presence of a fixed pole. His argument, as applied to $\pi\pi$ scattering, goes as follows:

The continued unitarity condition for $\pi\pi$ scattering at the 2π threshold is

$$F^{-1}(t, \lambda) = F_{II}^{-1}(t, \lambda) - i\rho(t); \quad \rho(t) = \sqrt{\frac{t - 4m_\pi^2}{t}}$$

Thus if there is a fixed pole at λ_0 in F , there is no such pole in F_{II} , the continuation of the amplitude through the unitarity cut onto the second sheet. But since a fixed pole is, by definition, present at the same value of λ for all t on the physical sheet, it can be continued through the unitarity cut onto the second sheet, unless there is a cut which, at $\lambda = \lambda_0$, exactly covers the unitarity cut, preventing the continuation. The relationship between this "shielding cut" and the unitarity cut can be seen by considering:

$$\phi^{-1}(t, \lambda) \equiv F^{-1}(t, \lambda) + i\rho(t)$$

which has no unitarity cut. Obviously,

$$\phi^{-1}(t, \lambda_0) = i\rho(t)$$

if there is a fixed pole at $\lambda = \lambda_0$. So ϕ^{-1} contains a cut which exactly covers the unitarity cut at $\lambda = \lambda_0$. It must be a moving cut, since otherwise unitarity would be violated everywhere, instead of only at a single point. It therefore must be in $F(t, \lambda)$, since ϕ^{-1} and F^{-1} differ only by a fixed cut. Two models showing how such a cut could be incorporated in an amplitude are discussed in this paper.

This argument applies, not only to the 2π threshold in $\pi\pi$ scattering, but to any threshold in any reaction such that a fixed pole would vanish going through the unitarity cut generated by that

threshold. Such a threshold Oehme calls "relevant." If there is a fixed pole in the amplitude at $\lambda=\lambda_0$, there must be a moving branch point associated with each relevant threshold; as λ goes to λ_0 , each such branch point goes to its threshold.

The essential feature of our models is that they explain the nonshrinking diffraction peak in πp scattering by a fixed pole in the πp amplitude, and explain the Serpukhov total cross section as reflecting the influence on the amplitude of the shielding cut associated with the two pion threshold in the crossed channel $\pi\pi \rightarrow p\bar{p}$.

We assume that the branch point trajectory has a slope comparable to the slopes of Regge poles and Regge cuts. The fixed pole it is shielding, the Pomeron, occurs at $\lambda=1$; thus at $\lambda=1$ the branch point of the shielding cut must be at its relevant threshold, $t=4\mu^2$. Therefore the branch point should have a $t=0$ intercept slightly to the left of 1.

Under the same assumptions, the cut associated with the $p\bar{p}$ threshold will have an intercept well to the left of 0, and may therefore be ignored in high energy calculations. Multi-pion thresholds modify the discontinuity along the $\pi\pi$ cut, since the intercepts of their shielding cut branch points are lower than the intercept of the branch point associated with the $\pi\pi$ threshold (see Figure 1). Since we have no theoretical grounds for predicting the details of the discontinuity, and since our fittings and predictions are not very sensitive to them, the effects of multi-pion thresholds are ignored.

It remains to determine whether the $\pi\pi$ threshold is a relevant threshold for the reaction $\pi\pi \rightarrow p\bar{p}$. As we shall show, it is possible to assume singularity structures in the other amplitudes connected to $\pi\pi \rightarrow p\bar{p}$ by low t unitarity such that $\pi\pi$ is not a relevant amplitude; in such a situation, a fixed pole can exist in πp without a

shielding cut. However, it is much more natural to assume singularity structures such that $\pi\pi$ is a relevant threshold, and a fixed pole in $\bar{\pi}\pi \rightarrow \bar{p}p$ thus requires a moving shielding cut connected with the $\pi\pi$ threshold.

Consider the three amplitudes $F(\bar{\pi}\pi \rightarrow \bar{\pi}\pi)$, $G(\bar{\pi}\pi \rightarrow \bar{p}p)$ and $N(\bar{p}p \rightarrow \bar{p}p)$. All three are positive signature, and the plus sign is therefore omitted. In all $\bar{p}p$ states the two particles have opposite helicities in the center of mass, corresponding to helicity non-flip scattering in the crossed channel. F_{II} , G_{II} , and N_{II} are the continuations of F , G , and N through the unitarity cut just above the $\pi\pi$ threshold onto the second sheet. The unitarity conditions at the $\pi\pi$ threshold can then be

written:

$$\begin{aligned} F_{II} &= F - \frac{2ipF^2}{1+2ipF} = \frac{F}{1+2ipF} \\ G_{II} &= G - \frac{2ipGF}{1+2ipF} = \frac{G}{1+2ipF} \\ N_{II} &= N - \frac{2ipG^2}{1+2ipF} \end{aligned}$$

The $\pi\pi$ threshold is relevant for G if these conditions imply that a fixed pole in G must vanish in G_{II} ; this depends on the singularity structure in F and N . For instance, there could be a fixed pole in both G and G_{II} provided that there was no such pole in F or F_{II} and a double pole in N or N_{II} . In such a case, there need be no shielding cut in G , since the pole need not vanish when the amplitude is continued through the unitarity cut.

Table I lists the various combinations of singularities in F , G , and N consistent with a fixed pole in G . One may reduce the

number of possibilities in several ways. The simplest is to require that the three amplitudes, all of which correspond to "vacuum" channels, should have the same singularities; this immediately reduces us to case 1 of Table I. Alternatively, one can assume that there are no double poles in physical amplitudes, which reduces us to cases 1-4. Either of these assumptions can be defended on the basis of observed similarities among total cross sections for the crossed reactions of these amplitudes; such similarities are discussed at greater length in part 3. If we reduce ourselves to cases 1-4, we then note that 1, 2 and 3 require a $\pi\pi$ shielding cut in G (for 3, Oehme's argument can be applied to $(\lambda - \lambda_0) G$), and that in 4 either N has a double pole, or there is a shielding cut in at least one amplitude. This is seen by considering the K matrix element $N/(FN - G^2) + 2i\rho(t)$, which, like $\phi(t, \lambda)$ in $\pi\pi$ scattering, has no unitarity cut. If N is not a double pole, this goes to $2i\rho(t)$ as λ goes to λ_0 . Therefore the expression, and thus at least one of its component amplitudes, must have a moving shielding cut.

We see that although unitarity alone does not absolutely force a fixed pole in G to be accompanied by a moving cut shielding the $\pi\pi$ threshold, it is reasonable to expect it to be.

If, as mentioned above, we assume similar singularity structures in all three amplitudes, then the moving cut will be present in N as well (case 1). Alternatively, in case 4, the shielding cut might be in N rather than in G . It is thus possible to have a shielding cut for only πp scattering, for only $p p$ scattering, or for both; the third alternative seems most attractive.

IV. MODELS AND FITTING

Two models for shielding cuts are developed in later sections.

In one the shielding cut has the form of a square root cut, in the other an inverse square root cut. As is well known, a square root cut with its tip at $\lambda=1-a$ gives a contribution to the total cross section at high energies of the form:

$$\text{Im } G_{\text{cut}}(0,s) = \frac{C s^{1-a}}{[\ln(2\cos\theta(0,s))]^{3/2}} = \frac{C s^{1-a}}{[\ln s/\mu M]^{3/2}}$$

An inverse square root cut similarly gives a contribution to the high energy total cross section of the form:

$$\text{Im } G_{\text{cut}}(0,s) = \frac{C s^{1-a}}{[\ln(2\cos\theta(0,s))]^{1/2}} = \frac{C s^{1-a}}{[\ln s/\mu M]^{1/2}}$$

The reader will note that the energy term inside the logarithm is of the form $s/\mu M$, as a result of our starting with the energy term $2 \cos \theta(0,s)$; $\cos \theta(0,s)$ is, of course, the cosine of the scattering angle in the crossed channel; it is in this form that the energy enters the Sommerfeld-Watson transform that takes us from the one channel to the other. Such justification as we have for using this particular scaling of the energy is discussed in a later section. At infinite energy, of course, the particular scaling does not matter; but infinity is, in this respect, very far away.

To determine a , we must know the slope of the moving branch point trajectory. We assumed that the slope was either 1 BeV^{-2} , the universal Regge slope, or $1/2 \text{ BeV}^{-2}$, the slope of a cut generated by the exchange of two Regge poles, and did fittings on both assumptions.

As, shown on figure 1, the first assumption leads to $a=.08$, the second to $a=.04$.

The amplitude was also assumed to contain contributions from the Pomeron itself and from the P' , with an intercept of .5 and a slope of 1.

All residues were assumed exponential in t ; this was sufficient for our purposes, since we were interested in diffraction peaks only for very small t . Thus our amplitude had the form:

$$G(s,t) = s A e^{at} + B s^{t+.5} e^{bt} + \left(\frac{s^{\alpha'_{cut}(0) [- .08 + t]}}{[\ln \frac{1}{4M}]^b} \right) e^{ct} \quad (2)$$

$$\alpha'_{cut}(0) = 1 \text{ or } 1/2$$

$$b = 1/2 \text{ or } 3/2$$

We fitted

$$\sigma^+ \equiv \frac{\sigma_{\pi^+p} + \sigma_{\pi^-p}}{2} = \frac{16\pi}{2g(s)\sqrt{s}} \text{Im} G(s,0)$$

$$\text{Re}^+ G(s,0) \equiv \frac{\sigma_{\pi^+p} \frac{\text{Re} G_{\pi^+p}}{\text{Im} G_{\pi^+p}} + \sigma_{\pi^-p} \frac{\text{Re} G_{\pi^-p}}{\text{Im} G_{\pi^-p}}}{2} = \frac{16\pi}{2g(s)\sqrt{s}} \text{Re} G(s,0)$$

We also fitted the slope parameter for the positive signature

amplitude, b^+ , defined by:

$$|G(s,t)|^2 \sim e^{b^+ t}$$

for t small. Since $G^-(s,t)$ is small compared to G^+ , and since the slope parameters for π^+p and π^-p scattering are close together, we can approximate b^+ by

$$b^+ \approx \frac{b_{\pi^+p} + b_{\pi^-p}}{2} = b_{\text{exp}}^+$$

We then calculate b^+ for our model by a two point fit ($t=0, t=-.1$) of our amplitude to the form Ae^{b^+t} , thus

$$b_{\text{exp}}^+ = 10 \times \ln \frac{|G(s,0)|^2}{|G(s,-.1)|^2}$$

Having done all of these fits for πp , we then used the same form of amplitude for pp scattering (with different coefficients). Thus we wrote

$$N(s,t) = SD e^{dt} + F s^{t+.5} e^{ft} + SK \frac{s^{\alpha'_{\text{cut}}(0)[-0.08+t]}}{[\ln(s/M^2)]^b} e^{kt};$$

note that for pp scattering, $-\cos \theta(s,0) = \frac{s}{2M^2}$. We fit the pp data in the same way as the πp , except that, being unable to approximate b_{exp}^+ in any simple way, we instead fitted b_{pp} . To do this, we wrote:

$$N_{pp}(s,t) = N^+(s,t) + N^-(s,t) = N^+(s,t) + H s^{.5+t} e^{ht}$$

we calculated H from the difference between σ_{pp} and $\sigma_{\bar{p}p}$, and h from the slope parameters for the two reactions in the energy range where they have been measured. We then fitted b_{pp} to

$$b_{pp} = 10 \times \ln \frac{|N_{pp}(s,0)|^2}{|N_{pp}(s,-.1)|^2}$$

The results of these fits are shown on figures 3.

All of these fits gave qualitatively similar results: an asymptotic total cross section near the Serpukhov cross sections, a cut term with negative sign and a magnitude well under half that of the Pomeron term, and a pole term with positive sign and similar magnitude.

While such successful fits demonstrate that the form of the amplitude suggested by our model is consistent with existing πp and pp high energy data, one cannot take them as very convincing evidence for our model. We have so far shown little more than our ability to fit additional data by adding one more parameter to the traditional $P+P'$ model. In the later parts of this paper, we attempt to go a little further, fitting the new data without additional parameters.

V. MODELS WITH CUT-GENERATING REGGE POLES

A very simple explicit form for an amplitude in which a fixed pole vanishes in going through the unitarity cut is:

$$F(t, \lambda) = \gamma(t, \lambda) \frac{\sqrt{t_c - t} + \sqrt{4\mu^2 - t}}{\lambda - 1} + B(t, \lambda) \quad (3)$$

Here γ and B are regular at $(0,1)$, and B is assumed to have no moving cut. Unitarity at the pole is satisfied by constraints on B and

B_{II} , its continuation through the unitarity cut. We assume that $t_c(1) = 4\mu^2$, so that

$$F_{II}(t, \lambda) = \gamma_{II}(t, \lambda) \frac{\sqrt{t_c - t} - \sqrt{4\mu^2 - t}}{\lambda - 1} + B_{II}(t, \lambda)$$

does not contain the fixed pole.

If we simplify matters by assuming that $\gamma(t, \lambda)$ is constant in λ between the fixed pole and the tip of the cut, we can calculate the cut contribution to $F(s, t)$ in terms of the fixed pole's contribution.

Unfortunately, as Ross Amann¹¹ has pointed out, this assumption gives cut and Pomeron terms of the same sign, and thus cannot explain the Serpukhov πp data. We would therefore like some simple form for $\gamma(0, \lambda)$ that changes sign between the pole ($\lambda=1$) and the tip of the cut ($\lambda=\lambda_0$; $t_c(\lambda_0)=0$).

So far we have not discussed the question of where the shielding cut comes from. If it is produced by the exchange of two Regge poles with trajectory $\alpha(t)$, then the cut trajectory is $\alpha_c(t) = 2(\alpha(t) - 1)$.

If $\alpha(t)$ has the universal Regge slope of 1 BeV^{-2} , α_c will have a slope of $1/2$. Since $\alpha_c(4\mu^2) = 1$ for a shielding cut, this gives

us $\alpha_c(0) = 1 - 2\mu^2 = .96$. Since $\alpha_c(0) \neq \alpha(0) - 1$, we obtain $\alpha(0) = 1 - \mu^2 = .98$.

If we now insert this new pole as a term multiplying the fixed pole in Eqn. 3, we get:

$$F(t, \lambda) = \gamma'(t, \lambda) \frac{\sqrt{t_c(\lambda) - t} + \sqrt{4\mu^2 - t}}{(1 - 1)(\lambda - \alpha(t))} + B(t, \lambda)$$

(3')

Here $\gamma(t, \lambda) = \frac{\gamma'(t, \lambda)}{\lambda - \alpha(t)}$

Since the new pole is between the Pomeron and the tip of the cut, it provides just the term we need to reverse the relative sign of pole and cut contribution. We can now assume that $\gamma'(\lambda, 0)$ is essentially constant in λ for the small range of λ between the Pomeron and the tip of the cut ($\lambda = 1$ to $\lambda = 1 - 2\mu^2$), and the ratio of pole to cut terms at high energy is fixed.

Of course, the new pole itself may contribute to the amplitude. If it is present only in the first term of Eqn 3' and not in $B(\lambda, t)$, its contribution is fixed, and the amplitude has the form

$$\begin{aligned} F(s, t) = & 2i \gamma'(t, 1) \frac{\Gamma(1/2)}{\Gamma(2)} \frac{\sqrt{4\mu^2 - t}}{u^2 - t} \frac{s}{\mu M} + \\ & + \left[\frac{1 + \cos \pi \alpha(t)}{\sin \pi \alpha(t)} - i \right] \gamma'(t, \alpha(t)) \frac{\Gamma(5/2 - \mu^2)}{\Gamma(2 - \mu^2)} \\ & \times \frac{\sqrt{t_c(\alpha(t)) - t} + \sqrt{4\mu^2 - t}}{1 - \alpha(t)} \left(\frac{s}{\mu M} \right)^{1 - \mu^2} + \\ & + \frac{1}{\pi} \int_{t_c(\lambda)=t}^{\lambda=\infty} d\lambda \left[\frac{1 + \cos \pi \lambda}{\sin \pi \lambda} - i \right] \gamma'(t, \lambda) \frac{\Gamma(\lambda + 3/2)}{\Gamma(\lambda + 1)} \frac{\sqrt{t - t_c(\lambda)}}{(\lambda - 1)(\lambda - \alpha(t))} \left(\frac{s}{\mu M} \right)^\lambda \end{aligned} \quad (4)$$

This form, with $\gamma'(0, \lambda)$ assumed constant, was used to fit the same data fitted in the previous section in the same way. No assumption was made about the behaviour of $\gamma'(t, \lambda)$ for $t \neq 0$; thus the ratio of each term at $t = -.1$ to the same term at $t = 0$ was treated as a free fitting parameter as in models I. The results are shown as model IIa on figure 3.

Instead of assuming that our new pole is absent from B in Eqn. 3', we could assume that it is present with just the strength necessary to cancel its forward contribution to the amplitude from the first term of Eqn. 3'. The new pole would thus be a sort of useful fiction (at least in the forward direction), giving no contribution to the total cross section but generating the cut and determining its strength. Thus,

$$B(t, \lambda) \approx -\gamma'(t, \lambda) \frac{\sqrt{2\mu^2 + t} + \sqrt{4\mu^2 - t}}{(t - \mu^2)(\lambda - \alpha(t))} + B'(t, \lambda) \quad \text{for } -.1 \leq t \leq 0$$

$$\text{and } B(0, \lambda) = \frac{(2 + \sqrt{2})\gamma'(0, 1 - \mu^2)}{\mu(\lambda - \alpha(0))} + B'(0, \lambda)$$

The amplitude produced on this assumption is the same as the amplitude shown in Eqn. 4, but with the second term missing. We call it model IIb. Since we are applying this model for a range of small t , we are assuming that the cancellation is virtually perfect for $0 > t > -.1$.

A third alternative is that the new pole contributes to B a term of arbitrary strength. Thus,

$$B(t, \lambda) = \frac{B'(t, \lambda)}{\lambda - \alpha(t)} + B''(t, \lambda)$$

This again gives us the amplitude of Eqn. 4, but this time with the second term multiplied by an arbitrary coefficient. This we call model IIc. Obviously, it has one more free parameter than the previous models, which themselves correspond to special cases of model IIc.

One may reasonably ask for a physical interpretation of the new pole. The question is beyond the scope of this paper, but is discussed at some length by Arnold,¹² who assumes much the same set of singularities as we do, for somewhat different reasons.

He argues that the pole must have an exchange degenerate negative signature partner, and that the residue near $t=0$ should be small, since a zero for small positive t is required to prevent the observation of a physical particle on negative signature trajectories where λ goes through 1.

Fits using all three models are shown in figs. 3. Of particular interest are Figures 3-e, f, which show the slope parameters.

In model IIa there is a large shrinking singularity interfering destructively with the Pomeron. One way this can generate a shrinking pp peak is by being broader than the Pomeron peak, so that by $t=-.1$ the shrinking term is larger than the fixed term. Such a fit is shown in fig. 3. But there must then be a zero of the imaginary part of the amplitude between $t=0$ and $t=-.1$, implying a substantial dip in the differential cross section. Such a dip is not observed in the data of Beznogikh et. al.

Alternatively, the new pole could have a peak so narrow that its contribution, even by $t=-.1$, was small. One could thus confine the

broadening effect to an arbitrarily small neighborhood of $t=0$. But there is no obvious reason why the new pole should have a residue peaked so much more sharply than that of other poles.

In model IIb, we assume that the effective cancellation of the new pole extends throughout the near forward direction; observed pp and πp slope parameters can then be fitted, as shown in figures 3e, f

In IIc, there exist fits for which the new pole interferes constructively with the Pomeron in pp scattering. In πp it still interferes destructively, but the effect is not great enough to prevent us from fitting the slope parameter successfully. Such a fit is shown in figure 3.

Looking at figures 3, we see reasonably good fits to all of the data except for the real parts - especially the real part of the pp amplitude. This is not surprising. The singularities that dominate high energy total cross sections are at or near $\lambda = 1$; their contribution to the amplitude is almost pure imaginary. Consequently the real part of the amplitude will be much more sensitive than the imaginary part to lower trajectories, and we can expect serious deviations from the predictions of a high energy model. Since σ_{pp}^+ is falling somewhat faster than $\sigma_{\pi p}^+$, the contribution to it of secondary trajectories is presumably greater.

Model IIc, in spite of its additional parameter, does not fit the data substantially better than IIa or IIb. IIb fits the Re_{pp}^+ substantially better than IIa, and in addition avoids the slope parameter difficulties of IIa discussed above; it thus seems to be the most

attractive of the models. In addition, by giving the new pole a residue of 0. very near $\lambda=1$, it explains the failure of its hypothetical exchange degenerate parity partner to appear at $\lambda=1$.

VI. s-SCALING HYPOTHESIS

Before going on to our next model, we must first develop a speculation connecting the amplitudes $F(\pi\pi \rightarrow \pi\pi)$, $G(\pi p \rightarrow \pi p)$, and $N(pp \rightarrow pp)$. This will allow us to use the simple unitarity condition of $\pi\pi$ elastic scattering to construct our model, and then apply the model to the other two reactions. The reader should remember that, as in our previous analysis, all amplitudes are positive signature.

From the unitarity conditions shown in part III one can derive the following:

$$F/F_{II} = G/G_{II}$$

A sufficient condition for this to hold is

$$F = \gamma G$$

where γ is any function of t and λ that is regular at the unitarity cut. We speculate that, since the two reactions proceed through the "same" (vacuum) intermediate state, perhaps, in some high energy approximation, they are the "same", and γ is a constant. Were this the case, the scattering amplitudes $F(s,t)$, $G(s,t)$ would be proportional for values of s corresponding to the same value of $\cos \theta(s,\lambda)$, the angular variable that appears in the Sommerfeld-Watson transformation taking us from $F(t,\lambda)$ to $F(s,t)$. Since $\cos \theta(s,0)$ is proportional, at high energies, to s/M_μ for πp scattering (s/μ^2 for $\pi\pi$, s/M^2 for pp) this implies, among other things, that the total cross sections for πp scattering and $\pi\pi$ scattering should be proportional, at energies related by $s_{\pi\pi} = (\mu/M) s_{\pi p}$. Applying

factorization at P , P' , and the tip of the cut (for models such as that discussed in the next section, where the branch point is associated with a pole) we find that the total cross sections for πp and pp scattering should also be proportional, again with the energies scaled by the same factor.

If, in addition, we accept the idea that, at infinite energies, all elementary particle scattering reduces to scattering of the component quarks, we can choose for our constants of proportionality the quark counting ratios, 4:6:9.

$$F(s, t) = \frac{2}{3} G(s \frac{M}{\mu}, t) = \frac{4}{9} N(s \frac{M^2}{\mu^2}, t) \quad (5)$$

It may reasonably be objected that by choosing a different form for γ one may with equal validity get any other scaling factor, or none at all. This is true. We respond, first, that if there is a "natural" scaling factor relating the different amplitudes, it seems plausible that it should involve the expression $\cos\theta(s, t)$, since it is that expression which links the angle in one channel to the energy in the other. In some sense, we can say that, at infinite energy, all scattering by a given angle "looks the same", and that by equating the different energies which, in different reactions, correspond to the same physical angle in the crossed channels of those reactions, we have in some sense a "natural" scaling.

Second, we can point to figure 4a which shows positive signature total cross sections for $\pi\pi$, πp , and pp scattering, with their energies scaled by the ratio of pion to nucleon mass, and the

total cross sections divided by their appropriate quark counting factors. Considering the limitations in the data, the pseudo-experimental nature of the $\pi\pi$ points, and the rate at which Serpukhov high energy pp points have been changing in recent months, we consider this graph strikingly suggestive.

Figures 4b and 5h show real parts and slope parameters for the two reactions; here our assumption is far less successful.

From 4b, it is clear that the assumption does not work for real parts. Analyzing 5h is more difficult, since we have no direct measure of the positive signature pp slope parameter in the range of pp energies (48 Bev+) corresponding to the πp high energy region (8 Bev+). We do, however, have pp slope parameters in this range, and $\bar{p}p$ slope parameters at lower energies. If we assume that the negative signature amplitude can be approximated by a single Regge term, we can estimate its slope parameter in the lower energy region, and so calculate the negative signature contribution to the slope parameter. On such assumptions, the positive signature pp slope parameter cannot possibly be the same as the positive signature πp slope parameter, at corresponding energies.

Furthermore, in order for the pp negative signature amplitude to explain the large discrepancy between pp and πp slope parameters, its slope parameter at high energies would have to be much lower than at lower energies; the negative signature amplitude would have to broaden radically between 8 Bev and 60 Bev. This would require that that

amplitude be made up of several Regge poles of comparable importance in the high energy region, an assumption hostile to the spirit of Regge analysis.

So we conclude that Equation 2 does not hold for real parts, nor for $t \neq 0$. This is not surprising. Our plausibility argument for scaling involves extrapolating backwards from infinite energy; at infinite energy, real parts are expected to vanish. And the slope parameter involves t as well as s ; nothing in our argument specifies what values of t in πp correspond to what values in pp . We might speculate that some variant of Equation 5, with both s and t "scaled", would explain both cross sections and slope parameters; since t is small, and s large, they would presumably be scaled in different ways. We have not found any such more general scaling relation.

We do note, looking at fig. 5h, in the energy region corresponding to that where πp has a nonshrinking peak, that the pp peak is also essentially nonshrinking. The experimental pp slope parameter shrinks by only .19 from 48 Bev to 69 Bev; this is less than the experimental error for the nonshrinking πp slope parameter. Indeed, the pp data have a maximum at 62.5 Bev. It is impossible to tell from the data whether this is a real phenomenon or a result of experimental error. So the data is consistent with the assumption that pp and πp positive signature amplitudes are non-shrinking in the same region, although they have different values; this we call qualitative scaling. If this

assumption is correct, then the pp slope parameter should be approximately constant in the energy range above that where it has been measured. For many of our fits, this is the case, as can be seen on figures 2f and 3f.

Having now established the nature and status of the scaling relationship which, we hope, links $\pi\pi$, πp , and pp amplitudes, we will derive a model for the shielding cut in $\pi\pi$ scattering, and apply it to the other amplitudes.

VII. MODEL III

Assume that there exists a fixed pole at $\lambda=1$ in the $\pi\pi \rightarrow \pi\pi$ amplitude

$$F(t, \lambda) \xrightarrow{\lambda \rightarrow 1} \frac{\beta(t)}{\lambda-1}$$

Define

$$i\rho(t, \lambda) \equiv \varphi^{-1}(t, \lambda) = F^{-1}(t, \lambda) + i\rho(t)$$

where $\rho(t) = \frac{\sqrt{t-4\mu^2}}{\sqrt{t}}$ as usual

since

$$F^{-1}(t, 1) \equiv 0,$$

$$\rho(t, 1) = \rho(t)$$

Note that $\rho(t, \lambda)$ does not contain the unitarity cut since the unitarity condition can be written:

$$F^{-1}(t, \lambda) = F_{\pi}^{-1}(t, \lambda) - 2i\rho(t)$$

which implies that

$$F^{-1}(t, \lambda) + i\rho(t) \equiv i\rho(t, \lambda) = F_{\pi}^{-1}(t, \lambda) - i\rho(t) = i\rho_{\pi}(t, \lambda)$$

But $\rho(t, \lambda)$ must have a cut, since $\rho(t, 1) = \rho(t)$ which has a branch point at $t=4\mu^2$. We therefore write:

$$\rho(t, \lambda) = \frac{\sqrt{t-t_c(\lambda)}}{\sqrt{t}} \chi(t, \lambda)$$

where

$$t_c(1) = 4\mu^2$$

We require that:

$$F(0, \lambda) \neq 0$$

be some total cross section and also

$$F(0, 1) \neq \infty$$

and

$$\chi(t, 1) = 1$$

in order that there should

Since

$$F(t, \lambda) = \frac{-i\sqrt{t}}{\sqrt{t-t_c} \chi(t, \lambda) - \sqrt{t-4\mu^2}}$$

this requires that

$$\chi(t, \lambda) = \frac{2\mu i}{\sqrt{t-t_c}} + \sqrt{t} g(t, \lambda)$$

where g is neither 0 nor ∞ at $t=0$. But

$$\chi(t, 1) = 1 + \sqrt{t} g(t, 1) \equiv 1$$

$$\therefore g(t, \lambda) = (\lambda - 1) f(t, \lambda); \quad f(t, 1) \neq 0$$

(provided that g does not have a fixed branch point at $\lambda=1$). Plugging

this all back into F , and assuming that $t_c(\lambda)$ is a linear trajectory:

$$t_c(\lambda) = 4\mu^2 + C(\lambda - 1)$$

we get:

$$F(t, \lambda) = \frac{\sqrt{1 - \frac{t}{t_c(\lambda)}} + \sqrt{1 - \frac{t}{4\mu^2}}}{(\lambda - 1) \left(\frac{-C\sqrt{t}}{2\mu t_c(\lambda)} + i\sqrt{t-t_c(\lambda)} f(t, \lambda) [\sqrt{1 - \frac{t}{t_c(\lambda)}} + \sqrt{1 - \frac{t}{4\mu^2}}] \right)} \quad (6)$$

From Eqn. 6 we can show (with due attention to limiting processes)

that:

$$F(s, t) = F_{\text{pole}} + F_{\text{cut}} + F_{\text{other singularities}} \quad (7)$$

where

$$F_{\text{pole}}(s, 0) = -i \left(\frac{s}{4\mu} \right) \frac{3\pi}{4\mu f(0, 1)} \quad (7a)$$

and

$$F_{cut}(s,0) = -\frac{1}{2\pi i} \int_{\lambda_0}^{-\infty} d\lambda \frac{2\lambda+1}{\lambda-1} \frac{1+e^{-i\pi\lambda}}{2\sin\pi\lambda} \left(\frac{s}{\mu M}\right)^{\lambda} \frac{\Gamma(\lambda+1/2)}{\Gamma(\lambda+1)} \frac{\sqrt{\pi}}{i\sqrt{-\frac{1}{2}\lambda}} f(0,\lambda) \quad (7b)$$

provided f is such that these expressions exist.

as $s \rightarrow \infty$

$$F_{cut}(s,0) \rightarrow \frac{c^{1/2}}{4} \frac{[3 - \frac{8\mu^2}{c}]}{-4\mu^2} \frac{1+e^{-i\pi(1-\frac{4\mu^2}{c})}}{\sin\pi(1-\frac{4\mu^2}{c})} \frac{\Gamma(1.5-\frac{4\mu^2}{c})}{\Gamma(2-\frac{4\mu^2}{c})} \times$$

$$\times \frac{[s/\mu M]^{1-4\mu^2/c}}{f(0,1-\frac{4\mu^2}{c}) \ln^{1/2}(s/\mu M)}$$

Thus if we make the ∞ energy approximation, the cut term has one free parameter, $f(0,1-\frac{4\mu^2}{c})$. We may choose a value for c on theoretical grounds. In order to eliminate the free parameter, we must find some way of predicting the behaviour of $f(0,\lambda)$, for $1 - \frac{4\mu^2}{c} < \lambda < 1$.

So far, we have made no assumptions beyond unitarity, analyticity, the presence of the fixed pole, the absence of a fixed cut at $\lambda=1$, and the existence of a finite, non zero, cross section. We have merely rewritten the amplitude in a form that incorporates these assumptions, but is otherwise entirely general. All our ignorance has been shoved onto the unknown function f .

We must now make some simplifying approximation to f . Since neither we, nor anyone else, can write a satisfactory explicit amplitude that satisfies all theoretical constraints, such an approximation will necessarily have something wrong with it. The simplest assumption would be that f is constant. This would at once determine the ratio between pole and cut contribution, and thus eliminate one parameter. It is, however, an entirely arbitrary assumption. We choose, instead, to parameterize f by a simple form having some relation to the observed diffraction peak, and one that allows us to set approximate limits to the parameters. We therefore write

$$f(t, \lambda) = A e^{at+b\lambda} \quad (8)$$

near $t=0$, $\lambda=1$.

For $t \neq 0$, this form has various difficulties, including a left hand cut in f , a real part of the Pomeron residue, and several moving poles at zero's of the denominator of Eqn. 3 (at $t=0$, these vanish). Some, but not all, of these problems could be eliminated by more complicated forms for f . Since we are concerned primarily with very small values of t , we will use the simple form of Eqn. 8, and limit ourselves in our fitting to those terms in $F(s, t)$ that are non-zero at $t=0$.

We can make rough guesses at the proper values of a and b in several ways. If we assume qualitative scaling, and assume further that the change in slope parameter from F to G is the same as from G to N , we get a $\pi\pi$ elastic slope parameter of about 7. This suggests an a of about 3.5.

Since, in G, the diffraction peak is essentially nonshrinking for $-t$ as large as -1 , the peaks of the shrinking singularities cannot be much broader than the Pomeron peak; otherwise they would dominate for large $-t$. This gives us the approximate limit:

For $6 < p < 22$ Bev in πN scattering:

$$b \lesssim \ln 2 \cos \theta(s, t)$$

therefore

$$b \lesssim 4$$

We have attempted to get additional information on b by considering that the P' trajectory must correspond to a zero of the denominator of Eqn. 6. By pushing Eqn. 8 beyond its legitimate range of applicability, to the region $\lambda < .5$, $t < -4\mu^2$, we can get an approximate equation which yields:

$$2.8 \lesssim b \lesssim 5.0 \quad \text{for } C=1$$

$$2.3 \lesssim b \lesssim 4.1 \quad \text{for } C=2$$

Obviously, such an argument is highly heuristic; we have arbitrarily chosen to consider only the simple features of our amplitude corresponding to $t = 0$, and have then tried to get numerical estimates for that simplified model using argument from the range $-1 < t < 0$. Our purpose is not to construct a precise model, but only to make plausible the naive assumption that f does not change radically between $\lambda = 1$ and $\lambda = 1 - \frac{4\mu^2}{C}$; once that assumption is accepted, our freedom to vary the ratio of cut term to Pomeron term is drastically reduced. Plugging Eqn. 8 into Eqns. 7, we get

$$F(s, 0) = D \left[i - \frac{e^{4\mu^2/C [b - \ln t/\mu M]}}{3\pi^{3/2}} \mu \right] \times \int_0^{-\infty} \frac{(2\lambda+1) \Gamma(\lambda+1/2)}{[z-4\mu^2] \Gamma(\lambda+1)} \frac{e^{z/C (\ln t/\mu M - b)}}{\sqrt{-z}} dz + E \left(\frac{t}{\mu M} \right)^{\alpha_{P', (0)} - 1}$$

here $\lambda \equiv \frac{z-4\mu^2}{C} + 1$; D and E are free parameters

Because of the approximate nature of our estimate of b , we have chosen to reverse the usual fitting procedure in fitting data with this model. Instead of varying b within our allowed range and looking for the best fit to the data, we have done fits for several fixed values of b within our range of values, and then chosen the worst fit among them. We thus estimate an outer bound for how badly our model can fit the data, given our ignorance about the value of b . In practice, the fits are not very sensitive to b ; this justifies, to some degree, the use of so approximate a model.

The model developed in this and the previous section makes two predictions. One is that $\pi\pi$, πp , and pp positive signature amplitudes should be described by one amplitude, such that:

$$\text{Im } F(s, 0) = \frac{2}{3} \text{Im } G(sM/\mu, 0) = \frac{4}{9} \text{Im } N(sM^2/\mu^2, 0) \quad (5')$$

The other prediction is that that amplitude has a certain form, implying a particular relationship between the Pomeron and cut terms in $F(s, t)$.

The first prediction was tested by using model I to fit, simultaneously, πp and pp total cross sections and pp slope parameters, relating the πp and pp cross sections by Equation 5'.

The second prediction was tested by using the expression of Eqn. 9 to fit, separately, the data for πp and pp scattering. This is our model IIIa.

In order to learn whether the relation between pole and cut terms predicted by the arguments of the previous section was reasonably

optimal, we also fitted the data for πp and pp , separately, using the same model but with the ratio between pole and cut terms free to vary. This was model IIIb.

We then tested both of our predictions simultaneously by using models IIIa and IIIb to fit πp and pp total cross sections, and real parts, and pp slope parameters, simultaneously.

The results of these fits are shown in figures 5.

VIII. COSMIC RAY DATA

Recently, data have become available on inelastic total cross sections for pp scattering at cosmic ray energies. In principle, such data can be used to test theories for high energy pp scattering, provided that they predict both the total cross section and the shape of the diffraction peak. For, as pointed out by Jones et. al.⁸ If

$$\frac{d\sigma}{dt} = Ke^{At},$$

then

$$\sigma_{tot} = B/2 - ((B/2)^2 - B\sigma_I)^{1/2}$$

where

$$B = 16\pi h^2 A$$

Thus, if our theory predicts A over some range of s, we can calculate total cross sections from the inelastic total cross sections measured by the cosmic ray groups, and compare them with the total cross sections predicted by our theory.

Figures 6a, b and c show total cross sections calculated in this way. For figure 6a, A was assumed to be fixed, for high energies, at 9, its value for πp scattering; this corresponds to the strongest form of the scaling hypothesis, in which the narrow pp peak is blamed on the negative signature amplitude. For figure 6b, A was assumed to remain fixed at 11.4, corresponding to the assumption that pp scattering is produced by a fixed pole, and that the pp slope parameter reaches its asymptotic value at the high energy end of the Serpukhov region. For figure 6c, A was set equal to $11.5 + \ln(E/69)$; this corresponds to the assumption that the Pomeron is a moving pole, and that it dominates scattering for all energies above the Serpukhov range.

On each figure, we have graphed the experimental total cross sections and the predictions of total cross sections (Eqn. 2') predictions are graphed along with the high energy predictions of models III on figures 6a. The fits of models II and III are graphed on figure 5b; models I give results similar to those of models III. And on fig. 6c we have shown the predictions of the Frautschi Margolis Eikonal model, as given in a recent pre-print by Hamer and Ravndal.¹²

As will be seen by examining the figures, the experimental errors in the cosmic ray data are still too large to allow us to eliminate any of the alternative models. If future experimenters can substantially reduce those errors, that will no longer be true. It would also be useful if we knew which of the two cosmic ray experiments to believe, since their results are not entirely consistent. If one rejects the Russian results and accepts the Echo lake data, then the predictions of the Eikonal model, although not ruled out, agree rather poorly with the data. The predictions of our scaling hypothesis, on the other hand, fit rather well.

IX. COMPARISON WITH OTHER MODELS

The work most nearly parallel to this one is the paper of Hamer and Ravndal¹³ fitting essentially the same data to the Eikonal Regge model of Frautschi and Margolis. If we compare their model with our models IIa, IIb and IIIa we see that they have available one parameter we do not have-- α'_p , the slope of the Pomeron trajectory, which we have assumed to be 0--and one constraint we do not have, exchange degeneracy in pp scattering. They thus fit πp total cross sections and phase with the same number of parameters as we do, πp slope parameters with one more (although that same parameter must also be used, with the same value, for pp and kp), pp total cross section with one less parameter, and pp slope parameters with yet another less (since the P' residue is assumed exchange degenerate with the residue of the negative signature amplitude). Their results are better than ours for pp phase, but worse for πp total cross sections. They also, as mentioned above, are less consistent with the Echo Lake cosmic ray data.

Another comparable paper is that of Barger and Phillips.¹⁴ Their model has two more parameters than our model I, since they parameterize their cut. They do not consider slope parameters, but otherwise fit the same data as we do with similar success.

SUMMARY

We have started with the idea of using a shielding cut to permit a fixed Pomeron, and using the contribution of that shielding cut to the total cross section to explain the Serpukhov πp data. We have concretized that idea in several models. In addition, we have introduced a scaling hypothesis that seems to hold quantitatively for total cross sections and may hold qualitatively for differential cross sections.

Our shielding cut hypothesis appears to be consistent with available data. The models we have constructed, with the exception of those containing a new shrinking singularity near the Pomeron (IIa and c) predict relatively little shrinking for the πp peak in the Serpukhov region, and the pp peak in the "post Serpukhov" region. At the worst, πp shrinkage between 18 and 65 Bev should be about .6, not much more than twice the experimental error in present πp slope parameter measurements; in most fits it is more like .15. The pp slope parameter, by contrast, increases about 1. in that range.

For pp scattering, most of our models predict relatively little shrinkage above the range where the peak has been measured. The model with the greatest shrinkage gives a slope parameter, at 1000 Bev, of about 13.4; for most models it is under 12. At the highest energies where it has been measured, the slope parameter is about 10.5.

If, to take the extreme example of the moving Pomeron hypothesis, the peak shrank at the rate corresponding to a one Regge pole amplitude, the slope parameter at 1000 Bev would be about 16.

Thus the most immediate tests of our models will be the measurement of πp slope parameters at Serpukhov energies and the measurement, in storage rings or at Batavia, of pp slope parameters at Batavia energies. Improved experiments at cosmic ray energies may also allow tests against alternative models.

It should be noted that our models include several different, and to some degree independent, hypotheses, which can be separately tested. The assumption of a fixed pole in a particular channel stands or falls according to the shrinkage of the diffraction peak. If there is a fixed pole, shrinkage should become slower and slower as energy increases; if there is an ordinary Regge pole, shrinkage should increase towards the rate corresponding to a single Regge pole amplitude.

The scaling assumption will be directly testable for pp vs. πp as soon as higher energy pp data becomes available from Batavia. More accurate O.P.E. calculations, or, in the more distant future, direct measurements of $\pi\pi$ cross sections from colliding beam experiments, will allow it to be tested for $\pi\pi$ as well.

Only one of our models attempts to give a dynamical explanation for our moving branch cut; even in that model, only a very limited connection is made between the cut and the pole generating it. Our position is that, although such an explanation may exist, it is not a

necessary part of our approach. The nonshrinking πp peak seems to indicate a fixed pole. A fixed pole requires a shielding cut. What generates that cut is beside the point. "Je n'ai pas besoin de cette hypothèse-1a."

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TABLE I

#	No Pole	Pole	Double Pole	Triple Pole	Any of These
1	F_{II}, G_{II}, N_{II}	F, G, N			
2	F_{II}, G_{II}	F, G, N, N_{II}			
3	F	F_{II}, G	G_{II}	N_{II}	$N(0,1,2 \text{ or } 3)$
4	F, F_{II}	G, G_{II}	N_{II}		$N(0,1, \text{ or } 2)$
5	F, F_{II}	G, G_{II}	N		$N_{II}(0,1, \text{ or } 2)$
6	F	F_{II}, G	G_{II}	N	$N_{II}(0,1, \text{ or } 2)$

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FIGURE CAPTIONS

Figure 1:

Trajectories of poles and branch points assumed in our models.

Figure 2:

Fit of models I to Re^+G , Re^+N , $\sigma_{\pi p}^+$, σ_{pp}^+ , $b_{\text{exp}}^+(\pi p)$ and b_{pp} .

Ia: $\alpha'_{\text{cut}}(0)=1/2$, $b=1/2$

Ic: $\alpha'_{\text{cut}}(0)=1$, $b=1/2$

Ib: $\alpha'_{\text{cut}}(0)=1/2$, $b=3/2$

Id: $\alpha'_{\text{cut}}(0)=1$, $b=3/2$

In Figs. 2b and 2d

I: Serpukov data

I: pre-Serpukhov data

Figure 3:

Fit of models II to the same data fitted in figs. 2.

Figure 4a:

$\sigma_{\pi\pi}^+$ as a function of $2 \cos\theta(s,0) \approx s/\mu^2$

$\sigma_{\pi p}^+$ as a function of $2 \cos\theta(s,0) \approx s/\mu M$

σ_{pp}^+ as a function of $2 \cos\theta(s,0) \approx s/M$

Figure 4b:

Re^+G as a function of $2 \cos\theta(s,0) \approx s/\mu M$

Re^+N as a function of $2 \cos\theta(s,0) \approx s/M^2$

Figure Captions (continued)

Figures 5a-f:

Models III fitted to the same data as in figs. 2

For IIIa, IIIb, $\alpha'_{\text{cut}}(0) = 1$

For IIIa', IIIb', $\alpha'_{\text{cut}}(0) = 1/2$

Figure 5g:

Models I fitted simultaneously to:

$\frac{1}{2} : \sigma_{\pi p}^+$ as a function of $2 \cos \theta(s, 0) \equiv s/\mu M$

$\frac{1}{3} : (\frac{2}{3})\sigma_{pp}^+$ as a function of $2 \cos \theta(s, 0) \equiv s/\mu^2$

Figure 5h:

Models I fitted to b_{pp} . The same curve, lowered to fit $b_{\text{exp}}^+(\pi p)$, is shown as an insert.

Figure 5i, j:

Models III fitted as in figs. 5g, h.

Figure 6a:

σ_{total} for $p+p \rightarrow p+p$, calculated from cosmic ray experiments, with high energy predictions of models III

Figures 6b, c:

The same as fig. 6a, calculated on different assumptions. The predictions of models II and III are shown in 6b, the predictions of the eikonal model in 6c.

Fig.1

Fig.2(a)

Fig.2(b)

Fig.2(c)

Fig.2(d)

Fig.2 (e)

Fig.2 (f)

Fig.3(a)

Fig.3(b)

Fig.3(c)

Fig.3(d)

Fig.3(e)

Fig.3(f)

Fig.4(a)

Fig.4(b)

Fig.5(a)

Fig.5(b)

Fig.5(c)

Fig.5 (d)

Fig.5(e)

Fig.5(f)

Fig.5(g)

Fig.5(h)

Fig.5 (i)

Fig.5 (j)

Fig.6(a)

Fig. 6(b)

Fig.6(c)