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Ventilation Systems Analysis During Tornado Conditions

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by

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VENTILATION SYSTEMS ANALYSIS DURING TORNADO CONDITIONS

by

Gloria A. Bennett, William S. Gregory,
and
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ABSTRACT

The principal concern of this investigation is to develop the capability to simulate the dynamic effects of a tornado depressurization on a ventilation system. The basic formulation and solution of the two-zone series model ventilation subsystem is based on lumped parameter component response equations, the isothermal compression of air, and the conservation of mass. Solutions based on these assumptions are also presented for the two-zone series model with natural bypass, the two-zone series model with recirculation, and the natural branching model. A parameter study is presented comparing the effects of changes in system resistance, system capacitance, and variable tornado depressurization rates. This report examines the adaptability of the basic formulation to adiabatic compression of air and the addition of duct resistance.

A quasi-steady formulation is introduced and preliminary considerations of the importance of inertia are presented. Preliminary conclusions in this area indicate that inertial effects can be neglected. For relatively long ducts slow shock development appears possible. Work on the effect of tornado depressurization rates as related to shock development and on the importance of inertia effects is continuing.

I. INTRODUCTION

A. General

The operation of a ventilation system is highly dependent upon atmospheric pressure to maintain proper pressure differentials between containment zones and to prevent possible release of contamination to the atmosphere. A tornado causes a drop in atmospheric pressure which in turn causes a pressure transient to occur within the ventilation system. The objective of this investigation is to develop an analytical tool which will simulate the dynamic effects of a tornado on a facility ventilation system and be useful for evaluating methods for minimizing tornado-induced pressure and flows in proposed systems.

This report follows the initial report (LA-5894-PR) which covered the time period July through December 1974. The time period for this report is January through June 1975. A brief review of the initial report is given below.

B. Review of Initial Report

The principal concern of the initial report consisted of familiarization with planned and existing plutonium reprocessing and fabrication plants. From that review, ventilation system components and basic subsystems common to ventilation systems were identified. A search for pressure versus flow data at high flow rates on ventilation system components proved fruitless and it was concluded that experimental data in this area is required.

Architect-engineering firms were contacted to solicit their inputs to the investigation. A general idea of their interest and computing capability was learned from a telephone survey. Their interest was generally favorable and the knowledge gained regarding their computing capability will allow development of a computer code that can be easily used by a majority of the firms.

Formulation of fluid dynamic system equations was initiated. The dynamic response of a "straight through" two-zone system was analyzed using a "lumped parameter" approach. This approach was called the basic formulation and relied on an isothermal assumption for fluid expansion and compressibility. Duct resistance, inertial forces, recirculation, and branching flow were neglected.

C. Program Plan Schedule

The program plan schedule is shown in Table I. As shown, the major areas for work during the time period from January to June 1975 consisted of a) formulation of the fluid dynamic system equations, b) examination of data preparation requirements, and c) identification of digital solution techniques. The major emphasis of this report consists of the fluid dynamic equation formulations and their associated assumptions and simplifications. Data preparation and solution techniques are inherently included for each formulation as they are necessary before a satisfactory solution can be achieved. They will be discussed only in passing and not as separate topics. Considerable attention will be directed into these areas as the generalized computer code is written.

This report examines the adaptability of the initial "basic" formulation to include adiabatic conditions and duct resistance. This formulation is extended to the other subsystem models identified in the initial report which are 1) two-zone series model with natural bypass, 2) two-zone series model with recirculation, and 3) parallel flow natural branching model. A parameter study is reported comparing the effect of changes in the capacitance and resistance of the system. Also the effect of variable tornado depressurization rates is presented.

A "quasi-steady" formulation is introduced. Magnitudes of duct velocities attained using this approach are illustrated.

This report considers the importance of inertia in a rather preliminary way. Work on the effect of tornado depressurization rates related to shock development and on the importance of inertia effects is continuing.

II. BASIC FORMULATION

The review of facility ventilation systems allowed identification of several typical subsystems that are repeated in ventilation system design. Each subsystem is modeled using lumped parameter components and is a combination of rooms, ducts, filters, valves or dampers, and a fan. The component models used are valid under design conditions since response data is not available for high flow rates. The techniques and methods developed for the transient response of ventilation systems are quite accurate under normal conditions and are operative under high flow and backflow conditions. Completion of appropriate experiments will allow the use of more accurate component response models under conditions encountered during a depressurization transient.

A. Component Response Models

Filters are modeled as linear components whose response is given by:

$$\dot{m} = K \Delta P \quad (1)$$

where

$\dot{m}$ = mass rate of flow
 K = resistance coefficient
 ΔP = pressure drop.

Algebraic manipulation of Eq. (1) to describe the filter response in conventional heating and ventilating variables gives

$$\Delta H = RF \cdot Q \quad (2)$$

where

ΔH = pressure head loss
 RF = filter resistance coefficient
 Q = volume rate of flow.

The filter resistance coefficients used in this study are the recommended design values for various sizes of HEPA filters.¹ Previous tests on 8 in. HEPA filters have shown that their behavior is nonlinear at high flow rates.² The behavior of other standard size HEPA filters at high flow rates is unknown. Figure 1 illustrates the filter model used in this study.

TABLE I
FY 1975 PROGRAM PLAN SCHEDULE

TABLE II

TYPICAL VENTILATION SUBSYSTEMS

SCHEMATIC	DESCRIPTION
Single Zone Model
Two-Zone Series Model
Two-Zone Series Model With Natural Bypass
Two-Zone Series Model With Recirculation
Natural Branching Model

Fig. 1. Hepa filter models.

Valves and dampers are modeled as nonlinear components whose response is given by

$$\dot{m} = K_1 \Delta P^n \text{sgn } \Delta P \quad (3)$$

where n = exponent
 $\text{sgn } \Delta P$ = notation for sign of ΔP .

Algebraic manipulation gives

$$\Delta H = RV \cdot Q \cdot |\Delta H|^{1/2} \text{sgn } \Delta H \quad (4)$$

where RV = valve resistance coefficient
 $\text{sgn } \Delta H$ = notation for sign of ΔH .

The valve resistance coefficients used in this study are typical of parallel and opposed blade dampers at various angles of closure. Figure 2 illustrates the model used for valves and dampers.

Fig. 2. Valve and damper models.

Fans are the only active components in the ventilation subsystems. Their response has been modeled as linear and parabolic and is given by

$$\text{linear } \Delta H = A - XM \cdot Q, \quad (5)$$

$$\text{parabolic } \Delta H = a + b \cdot Q + c \cdot |Q| \cdot Q, \quad (6)$$

where

A, a, b, c, XM = constants.

The linear and parabolic models resulted in only slightly different responses to the depressurization transient for flow in the normal operation quadrant. Altering the fan model in the backflow quadrant caused wide variations in solution of the system equations. Experimental information for fan response to backflow conditions is necessary before the numerical model will accurately reflect the response of a ventilation system to depressurization transients. Figure 3 illustrates several fan curves used in the analysis. The solution of the system is presented using a linear fan model.

Rooms and cells are modeled as passive energy storage components that offer no resistance to flow. The initial formulation assumed isothermal expansion and compression of air in each volume. The compressibility of the system is based on the equation of state for a perfect gas and on the continuity equation. During the transient, the equation of state is

$$\frac{dp}{dt} = \frac{dm}{dt} \cdot \frac{RT}{V} \quad (7)$$

where

$\frac{dp}{dt}$ = rate of change of pressure

$\frac{dm}{dt}$ = rate of change of mass

Fig. 3. Fan models.

R = gas constant for air
T = room temperature
V = room volume.

Algebraic manipulation of Eq. (7) gives

$$\frac{dH}{dt} = CK(Q_{in} - Q_{out}) \quad (8)$$

where

$\frac{dH}{dt}$ = rate of change of pressure head

Q_{in}, Q_{out} = volume flow rates in and out

CK = volume capacitance coefficient.

The continuity equation for the room is given by

$$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out} \quad (9)$$

where

$\dot{m}_{in}, \dot{m}_{out}$ = mass rates of flow in and out.

Proper combination of the above algebraic and ordinary differential equations results in a system of simultaneous equations for each ventilation subsystem. Application of the appropriate boundary conditions and use of conventional numerical techniques results in unique solutions.

B. Adiabatic Compressibility

Thermodynamic considerations must be utilized in the analysis for the compression or expansion of air within the system. The air within the system may compress or expand isothermally or adiabatically. Typically, slow compressions or expansions of a gas tend to take place isothermally, whereas rapidly changing compression and expansion must be looked upon as an adiabatic process.

The basic formulation has utilized the isothermal assumption for compression and expansion of air within the system. Application of the adiabatic assumption requires use of the following relationships³

$$p_2 = p_1 \left(\frac{p_2}{p_1} \right)^{1/k} \quad (10)$$

$$T_2 = T_1 \left(\frac{p_2}{p_1} \right) \quad (11)$$

where

p_2, p_2, T_2 = new air density, pressure, and temperature

p_1, p_1, T_1 = old air density, pressure, and temperature

K = specific heat ratio.

These relationships are incorporated into the analysis such that at each time step new densities and temperatures for the air are calculated. A comparison of the pressures within the system for both the isothermal and adiabatic assumptions is shown in Fig. 4. The change in temperature and density during a tornado transient are shown in Figs. 5 and 6.

C. Duct Resistance

The basic formulation is based on a ventilation subsystem composed of various components connected by frictionless ducts. Introducing duct friction into the basic formulation requires an extra unknown and an extra equation for each duct but adds accuracy to the steady state solution. The duct friction model provides for a comparison to be made with the air friction charts used in design and reflects the effects of friction during the tornado depressurization.

Fig. 4. Comparison of pressures, adiabatic vs isothermal compression.

Fig. 5. Temperature vs time, adiabatic case.

Fig. 6. Air density vs time, adiabatic case.

Ducts are modeled as nonlinear components whose response is given by

$$\dot{m} = K|\Delta P|^n \text{sgn } \Delta P. \quad (12)$$

The Darcy-Weisbach equation expresses pressure head loss due to frictional flow as⁴

$$\Delta H = f \cdot \frac{L}{D} \cdot \frac{V^2}{2g} \quad (13)$$

where
 ΔH = pressure head loss
 f = friction factor
 L = duct length
 D = duct diameter
 V = average air velocity
 g = acceleration due to gravity.

Continuity is expressed as

$$Q = AV \quad (14)$$

where
 Q = volume rate of flow
 A = duct area
 V = average air velocity.

Algebraic manipulation of Eqs. (12), (13), and (14) results in

$$\Delta H = RD \cdot Q |\Delta H|^{1/2} \text{sgn } \Delta H \quad (15)$$

RD = duct resistance coefficient.

Using a trial value of a duct resistance, the code iterates on the system pressures and flow rates to a preset convergence limit, then calculates a duct resistance coefficient from Eq. (15). Results are compared and the process is repeated at each time step until the trial value and the calculated duct resistance coefficients converge. An approximation of the Claybrook equation is used as the basis for recalculating friction factors and duct resistance coefficients in this duct model.⁵ Head loss results from this model check closely with air friction charts.

$$\Delta P = 0.03 \cdot f \cdot \frac{L}{d^{1.22}} \cdot \left(\frac{V}{1000} \right)^{1.82} \quad (16)$$

where

ΔP = friction loss
 f = friction factor
 L = duct length
 d = duct diameter
 V = air velocity.

Equations (12), (14), and (16) are the basis for the second duct friction model. Algebraic manipulation of these equations results in

$$\Delta H = RD \cdot Q \cdot |\Delta H|^{0.549} \text{sgn } \Delta H. \quad (17)$$

Again, the code iterates until duct resistance coefficients converge.

D. Subsystem Models

The basic formulation based on lumped mass components, isothermal compressibility, and frictionless ducts is used to solve the ventilation subsystems shown in Table II.

1. Two-Zone Series Model with Natural Bypass.

This subsystem is a slight modification of the two-zone series model that allows natural, unregulated flow around a cell or room. Figure 7 illustrates a labeled schematic ventilation subsystem. The following represents the problem formulation for solution of this particular ventilation subsystem.

Component Response Equations:

Filters:

$$H1 - H2 = RF1 \cdot Q1 \quad (18)$$

$$H4 - H5 = RF2 \cdot Q2 \quad (19)$$

$$H5 - H6 = RF4 \cdot Q3 \quad (20)$$

$$H5 - H6 = RF3 \cdot Q5 \quad (21)$$

Valves:

$$H2 - H3 = RV1 \cdot Q1 \cdot |H2 - H3|^{1/2} \cdot \text{sgn } \Delta H_{2-3} \quad (22)$$

$$H3 - H4 = RV2 \cdot Q2 \cdot |H3 - H4|^{1/2} \cdot \text{sgn } \Delta H_{3-4} \quad (23)$$

$$H6 - H7 = RV3 \cdot Q3 \cdot |H6 - H7|^{1/2} \cdot \text{sgn } \Delta H_{6-7} \quad (24)$$

Fan:

$$H8 - H7 = C5 - XM \cdot Q6 \quad (25)$$

Equations of State:

$$\frac{dH3}{dt} = CK3 \cdot (Q1 - Q2) \quad (26)$$

$$\frac{dH5}{dt} = CK5 \cdot (Q4 - Q5) \quad (27)$$

Continuity Equations:

$$Q2 = Q3 + Q4 \quad (28)$$

$$Q6 = Q3 + Q5 \quad (29)$$

Problem Formulation Summary

Unknown		Equations	
pressures	8	component	8
flows	6	state	2
	<u>14</u>	continuity	<u>2</u>
boundary	-2		
conditions	<u>12</u>

Fig. 7. Two-zone series model with natural bypass.

This system of equations is composed of seven linear and three nonlinear algebraic equations, two first order ordinary differential equations, and twelve unknowns. Figures 8, 9, and 10 show the pressure changes and the flows that occur during a tornado depressurization imposed on the supply side of the system.

2. Two-Zone Series Model With Recirculation.

The recirculation subsystem allows forced, regulated recirculation around a room. A labeled schematic of this ventilation subsystem is shown in Fig. 11.

Fig. 8. Pressure vs time.

Fig. 9. Flow vs time.

Fig. 10. Flow vs time.

Fig. 11. Two-zone series model with recirculation.

Component Response Equations:

Filters:

$$\begin{aligned} H1 - H2 &= RF1 \cdot Q1 & (30) \\ H4 - H5 &= RF2 \cdot Q2 & (31) \\ H5 - H6 &= RF3 \cdot Q3 & (32) \\ H6 - H9 &= RF4 \cdot Q3 & (33) \end{aligned}$$

Valves:

$$\begin{aligned} H2 - H3 &= RV1 \cdot Q1 \cdot |H2 - H3|^{1/2} \cdot \text{sgn } \Delta H_{2-3} & (34) \\ H3 - H4 &= RV2 \cdot Q2 \cdot |H3 - H4|^{1/2} \cdot \text{sgn } \Delta H_{3-4} & (35) \\ H6 - H7 &= RV3 \cdot Q6 \cdot |H6 - H7|^{1/2} \cdot \text{sgn } \Delta H_{6-7} & (36) \\ H9 - H10 &= RV4 \cdot Q3 \cdot |H9 - H10|^{1/2} \cdot \text{sgn } \Delta H_{9-10} & (37) \end{aligned}$$

Fans:

$$\begin{aligned} H8 - H7 &= C5 - XM \cdot Q6 & (38) \\ H5 - H10 &= C6 - XM \cdot Q3 & (39) \end{aligned}$$

Equations of State:

$$\frac{dH3}{dt} = CK3 \cdot (Q1 - Q2) \quad (40)$$

$$\frac{dH5}{dt} = CK5 \cdot (Q4 - Q5) \quad (41)$$

Continuity Equations:

$$Q4 = Q2 + Q3 \quad (42)$$

$$Q5 = Q3 + Q6 \quad (43)$$

Problem Formulation Summary

Unknown		Equations	
pressures	10	component	10
flows	6	state	2
	16	continuity	2
boundary conditions	-2		14
	14		

Pressure changes and flows that occur during a tornado depressurization of this system are illustrated in Figures 12, 13, and 14.

3. Natural Branching Model. The natural branching model consists of parallel two-zone series models. Figure 15 is a labeled schematic of the natural branching subsystem.

Filters:

$$\begin{aligned} H1 - H2 &= RF1 \cdot Q2 & (44) \\ H4 - H5 &= RF2 \cdot Q3 & (45) \\ H5 - H6 &= RF3 \cdot Q4 & (46) \\ H1 - H8 &= RF4 \cdot Q5 & (47) \\ H10 - H11 &= RF5 \cdot Q6 & (48) \\ H11 - H12 &= RF6 \cdot Q7 & (49) \end{aligned}$$

Valves:

$$\begin{aligned} H2 - H3 &= RV1 \cdot Q2 \cdot |H2 - H3|^{1/2} \cdot \text{sgn } \Delta H_{2-3} & (50) \\ H3 - H4 &= RV2 \cdot Q3 \cdot |H3 - H4|^{1/2} \cdot \text{sgn } \Delta H_{3-4} & (51) \\ H6 - H7 &= RV3 \cdot Q4 \cdot |H6 - H7|^{1/2} \cdot \text{sgn } \Delta H_{6-7} & (52) \\ H8 - H9 &= RV4 \cdot Q5 \cdot |H8 - H9|^{1/2} \cdot \text{sgn } \Delta H_{8-9} & (53) \\ H9 - H10 &= RV5 \cdot Q6 \cdot |H9 - H10|^{1/2} \cdot \text{sgn } \Delta H_{9-10} & (54) \\ H12 - H7 &= RV6 \cdot Q7 \cdot |H12 - H7|^{1/2} \cdot \text{sgn } \Delta H_{12-7} & (55) \end{aligned}$$

Fan:

$$H13 - H7 = C5 - XM \cdot Q8 \quad (56)$$

Fig. 12. Pressure vs time.

Fig. 13. Flow vs time.

Fig. 14. Flow vs time.

Fig. 15. Natural branching model.

Equations of State:

$$\frac{dH3}{dt} = CK3 \cdot (Q2 - Q3) \quad (57)$$

$$\frac{dH5}{dt} = CK5 \cdot (Q3 - Q4) \quad (58)$$

$$\frac{dH9}{dt} = CK9 \cdot (Q5 - Q6) \quad (59)$$

$$\frac{dH11}{dt} = CK11 \cdot (Q6 - Q7) \quad (60)$$

Continuity Equations:

$$Q1 = Q2 + Q5 \quad (61)$$

$$Q8 = Q4 + Q7 \quad (62)$$

Problem Formulation Summary

Unknowns		Equations	
pressures	13	component	13
flows	8	state	4
	<u>21</u>	continuity	<u>2</u>
boundary conditions	<u>-2</u>		<u>19</u>

Again the formulation summary reflects a determinant set of simultaneous equations. Figures 16, 17, 18, and 19 illustrate pressure changes and flows that occur in the subsystem during a tornado depressurization at the supply.

E. Parameter Study

A previous report illustrated the changes that occur in the two-zone series model for a tornado depressurization imposed at the supply, at the exhaust, at supply and exhaust simultaneously, and at supply and exhaust with a time lag. By fixing the tornado at the supply and varying the system characteristics one at a time, flow rate and pressure time histories can be readily compared.

Fig. 16. Pressure vs time.

Fig. 17. Flow vs time.

Fig. 19. Flow vs time.

Fig. 18. Pressure vs time.

A parameter study using the basic formulation and the two-zone series model is the basis for these comparisons. System response is illustrated for various system resistances, various system capacitances, and four different pressure ramps.

1. System Resistance. Significant changes in system resistance are achieved by varying the valve resistance coefficients while keeping all other coefficients constant. Changing valve resistance coefficients simulates adjustments that are made to rebalance ventilation systems with loaded filters. Figure 20 shows the various system resistance curves

Fig. 20. System resistance curves and fan curve.

and the system fan curve used in the comparison. In each case, the steady flow conditions are calculated by the code. These conditions are then used as the input for calculations performed during the pressure transient. Figures 21-25 illustrate the

Fig. 21. Building pressure vs time.

Fig. 23. Building supply vs time.

Fig. 22. Cell pressure vs time.

Fig. 24. Cell supply vs time.

effects of changing system resistance on the flows and pressures that occur during the transient.

Increases in system resistance cause decreases in the pressure and flow response curves in both volumes. Large system resistances cause significant attenuation of the flow and pressure but increase the corresponding pressure differentials across the

valve and filter nearest the tornado. Loss of the first valve in the system causes building supply flows to reach magnitudes of -22800 cfm and +20000 cfm. Increasing the building volume to 35000 cubic feet without the benefit of the first valve induces building flow rates to -99000 cfm and a corresponding pressure of 83 in. of H_2O across the final filter.

2. System Capacitance. The capacitance of the system is directly proportional to the volume of the rooms and cells. Changing the volume of the room nearest the depressurization illustrates the effectiveness of using a large volume to annuete the

Fig. 25. Cell exhaust vs time.

Fig. 27. Cell pressure vs time.

Fig. 26. Building pressure vs time.

Fig. 28. Building supply vs time.

pressure pulse. Again, the steady flow conditions are calculated by the code and used as the input for the calculations performed during the pressure transient. Figures 26 through 30 show the effects on the flows and pressures that occur during the transient from changes made to system capacitance.

Flow at steady state through the normal two-zone series model is 300 cfm. Tornado depressurization of this system shows building supply flow rate

maximums of -9600 cfm and +7200 cfm. Increases in volume by a factor of seven cause those flow rate maximums to change only to -11000 cfm and +4700 cfm. Flow rates during a tornado depressurization reach magnitudes thirty-six times larger than the design values. Increases in system volume cause significant decreases in the pressure drops experienced in both volumes.

Fig. 29. Cell supply vs time.

Fig. 30. Cell exhaust vs time.

3. Depressurization Ramps. The two-zone series model was used to evaluate the effect of four different tornado depressurization transients. Three of the transients used are those established as Region I, Region II, and Region III tornadoes;⁷ the other transient was valid before publication of the referenced document. Figures 31, 32, 33 and 34 define the total pressure drop, the rate of pressure drop, and the period of each of the four transients. Hold time for each transient is calculated from information in the referenced document. The radius of maximum rotational windspeed is given as 150 feet for typical intense tornadoes. The hold

Fig. 31. Previous Region I Tornado.

$0 < t < 3, P_1 = -1 \text{ psi-sec} \cdot \text{Time} \quad (63)$
 $3 < t < 6, P_1 = -3.0 \text{ psi}$
 $6 < t < 9, P_1 = -1 \text{ psi/sec} \cdot (9.0 - \text{Time})$
 $t > 9, P_1 = 0.0$

Fig. 32. Region I Tornado.

$0 < t < 1.5, P_1 = -2 \text{ psi/sec} \cdot \text{Time} \quad (64)$
 $1.5 < t < 4.5, P_1 = -3.0 \text{ psi}$
 $4.5 < t < 6.0, P_1 = -2 \text{ psi/sec} \cdot (6.0 - \text{Time})$
 $t > 6.0, P_1 = 0.0$

time at maximum pressure is calculated by dividing the typical intense tornado diameter by the maximum translational windspeed.

Each point in the system displayed the same shape pressure and flow response curves to the four transients. The response curves differed only in amplitude and in the time required to decay back to steady state conditions. None of the ramps imposed on the system led to velocities that would require

Fig. 33. Region II Tornado.

$$0 < t < 1.875, P_I = 1.2 \text{ psi/sec} * \text{Time} \quad (65)$$

$$1.875 < t < 5.275, P_I = -2.25 \text{ psi}$$

$$5.275 < t < 7.15, P_I = -1.2 \text{ psi/sec} * (7.15 - \text{Time})$$

$$t > 7.15, P_I = 0.0$$

Fig. 34. Region III Tornado.

$$0 < t < 2.5, P_I = -0.6 \text{ psi/sec} * \text{Time} \quad (66)$$

$$2.5 < t < 6.6, P_I = -1.5 \text{ psi}$$

$$6.6 < t < 9.1, P_I = -0.6 \text{ psi/sec} * (9.1 - \text{Time})$$

$$t > 9.1, P_I = 0.0$$

better physical description of compressibility or momentum effects. Figures 35 through 42 illustrate system response to the four imposed transients.

III. QUASI-STEADY FORMULATION

This approach to formulation of the fluid dynamic equations consists of utilizing the momentum equation along with the energy and the continuity equation. The formulation is called "quasi-steady" because the energy and continuity equations are

Fig. 35. Pressure vs time.

Fig. 36. Flow vs time.

simplified so that they are not time dependent. The formulation is developed in the following way: Consider a simple system consisting of a room and a single duct as shown in Fig. 43.

It is assumed that the flow is one-dimensional and initially at rest. At time $t = 0$ the pressure at the outlet of the duct begins to decrease linearly with time in the following way:

2 psi/second ramp;
3 psi maximum;
3 second hold

Fig. 37. Pressure vs time.

1.2 psi/second ramp;
2.25 psi maximum,
3.4 second hold

Fig. 39. Pressure vs time.

Fig. 38. Flow vs time.

Fig. 40. Flow vs time.

Fig. 41. Pressure vs. time.

Fig. 42. Flow vs. time.

$$P_o = P_{amb} - at \quad (67)$$

where

- P_o = pressure at the outlet
 P_{amb} = ambient pressure at normal conditions
 t = time
 a = depressurization rate due to a tornado.

Fig. 43. Room with duct.

- V_r = Volume of the room
 T_r, T_o = Absolute temperature of the room and outlet air
 P_r, P_o = Pressure of the room and outlet air
 ρ_r, ρ_o = Density of the room and outlet air
 D = Duct diameter
 L = Duct length
 x = Horizontal distance
 $u(x,t)$ = Duct velocity as a function of distance and time

The expansion at the outlet is assumed isentropic, so:

$$T_o = T_{amb} \left(\frac{P_o}{P_{amb}} \right)^{\frac{k-1}{k}} \quad (68)$$

where

- T_{amb} = absolute temperature of ambient air
 k = specific heat ratio.

The momentum equation, continuity equation, energy equation and equation of state for the air flowing through the duct are, respectively:

$$\begin{aligned} \frac{\partial u(x,t)}{\partial t} + u(x,t) \frac{\partial u(x,t)}{\partial x} \\ = - \frac{1}{\rho(x,t)} \frac{\partial P(x,t)}{\partial x} + \frac{f u^2(x,t)}{2D} \end{aligned} \quad (69)$$

$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial}{\partial t} (\rho(x,t) u(x,t)) = 0, \quad (70)$$

$$\begin{aligned} \rho(x,t) C_p \left[\frac{\partial T(x,t)}{\partial t} + u(x,t) \frac{\partial T(x,t)}{\partial x} \right] \\ = \frac{\partial P(x,t)}{\partial t} + u(x,t) \frac{\partial P(x,t)}{\partial x} + u \frac{4}{3} \left(\frac{\partial u(x,t)}{\partial x} \right)^2, \end{aligned} \quad (71)$$

and

$$P(x,t) = \rho(x,t) R T(x,t) \quad (72)$$

where

- f = friction factor
- C_p = heat capacity
- μ = kinematic viscosity
- R = universal gas constant.

For the room, the energy equation, the continuity equation, and the equation of state are:

$$C_p \frac{d}{dt} \left(\rho_r(t) T_r(t) \right) V_r = \rho_r(t) \left(C_p T_r(t) + \frac{u^2(L,t)}{2} \right) u(L,t) A, \quad (73)$$

$$\frac{V_r d\rho_r(t)}{dt} = \rho_r(t) A u(L,t), \quad (74)$$

$$P_r(t) = \rho_r(t) R T_r(t) \quad (75)$$

where

A = duct cross-sectional area.

The above equations can be simplified by assuming that the energy equation and continuity equation are not time dependent. Then, the energy equation becomes:

$$T \left(1 + \frac{k-1}{2} M^2 \right) = T_0, \quad (76)$$

where

- M = mach number
- T_0 = constant

and the continuity equation becomes

$$\rho u A = \text{constant}. \quad (77)$$

Eq. (69) can be reduced to the following neglecting the mach number squared, M^2 , compared to unity:

$$\frac{\partial M^2}{\partial t^2} - \left(\sqrt{\frac{RT_0}{k}} \frac{1}{M} \right) \frac{\partial M^2}{\partial x} + \left(\frac{f}{D} \sqrt{kRT_0} M \right) M^2 = 0. \quad (78)$$

Simplifying Eq. (78) further by neglecting the inertial term yields:

$$\frac{dM^2}{dM^2} = \frac{f}{D} k dx \quad (79)$$

$$\frac{1}{M^2} \left| \frac{M_1^2}{M_0^2} \right| = \frac{f}{D} k x \left| \frac{L}{0} \right| \quad (80)$$

or

$$\frac{1}{M_0^2} - \frac{1}{M_1^2} = \frac{f}{D} k L \quad (81)$$

where

- M_1 = mach number in room
- M_0 = mach number at outlet.

It can be shown that:

$$\frac{dP}{P} = -1/2 \left(\frac{1 + (k-1) M^2}{1 + k-1 M^2} \right) \frac{dM^2}{M^2} \quad (82)$$

or, assuming $M^2 \ll 1$,

$$\frac{dP}{P} = -\frac{1}{2} \frac{dM^2}{M^2}, \quad (83)$$

and, upon integration

$$\ln P = -\frac{1}{2} \ln M^2 + \ln B$$

or

$$\ln P = \ln \frac{B}{M}$$

or

$$P = B/M. \quad (84)$$

At $x = 0$, $M = M_0$, $P = P_0$, so

$$B = P_0 M_0$$

and

$$P = \frac{P_0 M_0}{M}. \quad (85)$$

Since the flow is adiabatic, then

$$T = T_0 \frac{1 + \frac{k-1}{2} M_0^2}{1 + \frac{k-1}{2} M^2} \quad (86)$$

and, in keeping with the small mach number approximation

$$T = T_0 = \text{constant}.$$

The density is then

$$\rho = \frac{P}{RT} = \frac{P_0 M_0 / M}{RT_0} = \rho_0 \frac{M_0}{M}. \quad (87)$$

As the pressure, P_0 , at the outlet is reduced, pressure disturbances are propagated from the outlet toward the room at the speed of sound in the upstream air. If the air upstream of the disturbance is labeled 1 and the air downstream 2, as shown in Fig. 44, then from a momentum balance

$$P_2 A - P_1 A = \rho_1 (w + V_1) A [(V_1 + w) - (V_2 + w)]$$

or

$$V_2 = V_1 + \frac{P_1 - P_2}{\rho_1 (w + V_1)} \quad (88)$$

where

w = speed of the pressure disturbance.

Then, for quasi-steady flow at $x = 0$,

$$V_0(t + \Delta t) = V_0(t) + \frac{P_0(t) - P_0(t + \Delta t)}{\rho_0(t)(w + V_0(t))} \quad (89)$$

where

Δt = time increment.

Now, using Eqs. (67), (68), (72), (81), (85), (86), (87), and (89) for the duct and Eqs. (73), (74), and (75) for the room, then the pressures, temperatures, mach numbers, and velocities throughout the system can be solved for, as a function of time.

A computer code along with the following information leads to the output in Table III.

$a = 1 \text{ psi/sec}$
 $D = 1 \text{ ft}$
 $L = 100 \text{ ft}$
 $P_{\text{amb}} = 14.7 \text{ psi}$
 $T_{\text{amb}} = 530^\circ\text{R}$
 $f = 0.02$
 $V_r = 500 \text{ ft}^3$
 $w = \text{speed of sound for } T_0(t)$

Fig. 44. Pressure disturbance.

Solution to the quasi-steady formulation of a room and duct has been obtained. This model must be expanded to include the same ventilation system components as the two-zone series model before an adequate comparison of solutions between formulation techniques can be made.

IV. INERTIAL EFFECTS

Consideration of inertial effects would require solution of Eq. (78) which was:

$$\frac{\partial M^2}{\partial t} - \left(\sqrt{\frac{RT_0}{k}} \frac{1}{M} \right) \frac{\partial M^2}{\partial x} + \left(\frac{f}{D} \sqrt{kRT_0} M \right) M^2 = 0. \quad (78)$$

The first term of Eq. (78) represents the inertia, while the second and third terms represent convection and friction, respectively. As shown, the coefficient on the inertial or first term is one. The coefficients of the second and third terms are on the order of 1000 and 20, respectively. Further, since the second term is divided by mach number, which is very small if the flow starts from rest, then the rate of change of M^2 with respect to time must be on the order of 1000/M if the inertia term is to be of the same relative magnitude as the convective term. Suppose $M = 0.1$; then the time interval in which the transient must take place, in order for the inertial term to be significant, is 1/10000 seconds.

Although Eq. (78) would seem to indicate that inertial effects are unimportant, one must remember that this equation was derived assuming a steady state energy equation and a steady state continuity equation. Therefore, before inertial effects can be completely ignored, the full set of equations, Eqs. (69) through (75) should be solved numerically. This study is now in progress, but has proven difficult because of numerical instabilities.

Three methods of solution have been tried:

- (1) a discrete-discrete marching technique,
- (2) discrete in x and integration in time by Runge-Kutta technique, and
- (3) discrete in x and integration by Milne's technique in time.

V. SHOCK DEVELOPMENT

The possibility of shock wave formation was investigated in the following way. Consider a pressure falling as shown in Fig. 45 below.

TABLE III
QUASI-STEADY FORMULATION RESULTS

Time (secs)	Outlet Pressure (psia) P_o	Outlet Temperature (°R) T_o	Outlet Mach Number M_o	Inlet Mach Number M_i	Room Pressure (psia) P_R	Room Temperature (°R) T_R	Outlet Velocity (t + Δt) V_o
0.00	14.7	530.0	0.0	0.0	14.7	530.0	5.474
0.20	14.5	527.9	0.009709	0.009707	14.69	529.8	16.43
0.40	14.29	525.8	0.01947	0.01946	14.67	529.5	27.42
0.60	14.09	523.7	0.02930	0.02926	14.64	529.0	38.44
0.80	13.89	521.5	0.03920	0.039090	14.60	528.3	49.49
1.00	13.69	519.4	0.04917	0.04896	14.56	527.4	60.58
1.20	13.49	517.2	0.05920	0.05884	14.50	526.4	71.67
1.40	13.29	515.0	0.06932	0.06873	14.43	525.2	82.82
1.60	13.09	512.8	0.07951	0.07862	14.35	523.8	94.02
1.80	12.89	510.5	0.08978	0.08852	14.27	522.2	105.2
2.00	12.69	508.3	0.1001	0.09839	14.17	520.5	116.5
2.20	12.49	506.0	0.1106	0.1082	14.07	518.5	127.8
2.40	12.29	503.6	0.1211	0.1180	13.96	516.5	139.1
2.60	12.09	501.3	0.1317	0.1278	13.84	514.2	150.6

Fig. 45. Tornado depressurization.

Consider a sound wave moving to the left into the region of falling pressure with a velocity, c_1 , as shown in Figure 46.

Fig. 46. Sound wave.

The pressure and temperature in front of the wave are falling so the waves generated at some later time are traveling at a higher speed than the ones first sent out. Hence, the later waves tend to overtake the early waves and create a shock wave. Now, using Eqs. (67) and (68), the following result can be obtained

$$T_1 = T_2 \left(\frac{P_1}{P_2} \right)^{\frac{k-1}{k}} = T_2 \left(1 - \frac{at}{P_{amb}} \right)^{\frac{k-1}{k}} \quad (90)$$

where

P_1, T_1 = pressure downstream of shock wave

P_2, T_2 = pressure upstream of shock wave.

Using Eq. (90), c_1 can be calculated as follows

$$c_1(t) = 49.1 \sqrt{T_2} \left(1 - \frac{at}{P_{amb}} \right)^{\frac{k-1}{2k}} \quad (91)$$

The average sonic velocity during the period that the pressure drops is

$$\hat{c}_1 = \frac{1}{t_f} 49.1 \sqrt{T_2} \int_0^{t_f} \left(1 - \frac{at}{P_{amb}} \right)^{\frac{k-1}{2k}} dt \quad (92)$$

Equation (91) can be approximated, using a binomial expansion as

$$\hat{c}_1 = \frac{49.1 \sqrt{T_2}}{t_f} \int_0^{t_f} \left\{ 1 - \frac{at}{P_{amb}} \left(\frac{k-1}{2k} \right) + \left(\frac{k-1}{2k} \right) \left(\frac{k-1}{2k} - 1 \right) \frac{a^2 t^2}{2 P_{amb}^2} + \dots \right\} dt \quad (93)$$

$$\approx \frac{49.1 \sqrt{T_2}}{t_f} \left\{ 1 - \frac{a}{P_{amb}} \left(\frac{k-1}{2k} \right) \frac{t_f}{2} + \left(\frac{k-1}{2k} \right) \left(\frac{k-1}{2k} - 1 \right) \frac{a^2 t_f^2}{6 P_{amb}^2} \right\} \quad (94)$$

If $P = at_f = 3$ psi, then

$$\begin{aligned} \hat{c}_1 &= 49.1 \sqrt{T_2} \left\{ 1 - \frac{3}{14.7} \left(\frac{k-1}{4k} \right) + \left(\frac{k-1}{2k} \right) \left(\frac{k-1}{2k} - 1 \right) \frac{3}{2 (14.7)^2} \right\} \\ &= 49.1 \sqrt{T_2} (0.98457). \end{aligned} \quad (95)$$

The distance traveled by a wave generated at $t = t_f$ in a time Δt is

$$x = c_2 \Delta t. \quad (96)$$

If the wave, c_2 overtakes the initial wave generated at that point, a shock wave develops. Thus,

$$x = \hat{c}_1 t_f + c_1 (t > t_f) \Delta t. \quad (97)$$

Eliminating x gives

$$c_2 \Delta t = \hat{c}_1 t_f + c_1 (t > t_f) \Delta t \quad (98)$$

or

$$\begin{aligned} 49.1 \sqrt{T_2} \Delta t &= 49.1 \sqrt{T_2} (0.98457) t_f \\ &+ 49.1 \sqrt{T_2} \left(\frac{11.7}{14.7} \right)^{\frac{k-1}{2k}} \Delta t \end{aligned} \quad (99)$$

or

$$\Delta t = 0.98457 t_f + \left(\frac{11.7}{14.7} \right)^{\frac{k-1}{2k}} \Delta t. \quad (100)$$

Thus, for a given value of t_f , Δt can be calculated. The time needed for the shock wave to form is $t_f + \Delta t$. Values of $\Delta t + t_f$ are located in Table IV for various values of t_f .

TABLE IV
SHOCK DEVELOPMENT TIME FOR
SEVERAL DEPRESSURIZATION RATES

t_f	$\Delta t + t_f$
seconds	seconds
1	31.69
0.5	15.85
0.1	3.17
0.05	1.58

From Table IV one can see that if the total duration of the pressure drop is 3.17 seconds, then the initial drop in pressure must occur within 0.1 seconds if a shock wave is to form. Assuming an average wave speed of 1000 ft/sec, this would require a duct length of approximately 3000 ft.

IV. OBJECTIVES FOR NEXT WORK PHASE

- Solve full set of equations necessary to determine the importance of inertial effects.
- Review codes developed primarily for simulation of "loss of coolant" accidents. The code structure used in these developments could be directly applicable to transients in ventilation systems.
- Develop a large hypothetical ventilation system to be used as a "test case".
- Initiate examination of the utility of the "quasi-

steady" formulation for modeling ventilation subsystems.

- Initiate code development necessary for implementation of the basic formulation.

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