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AN APPROACH TO MULTIPARAMETRIC
EDDY CURRENT NONDESTRUCTIVE TESTING
USING PULSE TYPE WAVEFORMS

I. L. Barman

September 1970

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By

I. L. Barman*

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ABSTRACT

The vector space concept as applied to the identification of test specimen parameters in nondestructive tests is reviewed. The analysis of the response of an exciting coil encircling a metal cylinder to a step function driving current is made to gain insight into the use of such a signal in an eddy current nondestructive test. A closed form approximate expression for this response function is obtained by starting with the classical series solution for this cylindrical case and applying the Euler-McLaurin summation formula. The closed form approximation was checked experimentally using a coil encircling a stainless steel bar, and the measured and calculated results agreed within explainable experimental error. The principles of orthogonal filters are reviewed and their application to the analysis of the test coil response function is discussed.

CONTENTS

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INTRODUCTION

Any nondestructive testing (NDT) device can be considered as a measuring system composed of a driving transducer to which a driving signal is applied, the specimen to be tested, a response transducer that picks up the response signal and the instrumentation required to analyze that response in order to obtain information related to the properties (physical properties, integrity, dimensional changes, etc.) that characterize the specimen. These properties can be given the general name of parameters of the specimen, and the test is performed for their detection or measurement.

Regardless of the particular NDT method used (i.e., the type of energy used to analyze the specimen), the fundamental goal of an NDT system is to deliver the information required about the parameters of interest (out of a number of them) in such a way that the nature and "size" of each can be determined with the accuracy needed and without ambiguity. Eddy current nondestructive tests can be performed using various test coil driving current waveforms such as sinusoids or pulses. The purpose of this study is to analyze the properties of the response of an actual test coil-test specimen system to a step function driving current in order to gain insight of the possibilities of using such a driving signal in NDT.

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SUMMARY

Vector space concepts as applied to the identification of test specimen parameters in nondestructive tests are reviewed. An analysis of the response of an exciting coil encircling a metal cylinder to a step function driving current is made to gain insight into the use of such a signal in an eddy current nondestructive test. A closed form approximate expression for the response is obtained by analysis and checked experimentally. The calculated and measured results agreed within explainable error. Orthogonal filter principles are reviewed and their application to the analysis of the test coil response function is discussed.

REVIEW OF VECTOR SPACE THEORY APPLIED TO NDT

The response signal can be represented in the algebraic sense as a vector from the space spanned by the parameters, as above defined, so that:

$$\overline{R} = a_i \overline{p_i} \quad (1)$$

where the $\overline{p_i}$ are vectors in the parameters space {P} and the a_i are scalars measuring the projection of R into each $\overline{p_i}$ (the a_i will be scalars only if a linear operation is considered).

Now depending on the method used, the $\overline{R}$ vector will contain information about all those parameters that affect the response in some way, that is if they produce any observable change in the response as they change. Let us suppose that their number is n. Then by Equation (1)

$$\overline{R} = \sum_1^n a_k \overline{p_k} = a_1 \overline{p_1} + a_2 \overline{p_2} + \dots + a_k \overline{p_k} + \dots + a_n \overline{p_n} \quad (2)$$

we see that for the relation between a given response and a given set of parameters values to be univocally determined, the $\overline{p_i}$ vectors can be thought of as forming an orthogonal basis in

the parameter space, since only in such a case the description of R (whether in the parameter space or in some other suitable domain) will be given by a definite set of coefficients of the type of Equation (2).

The response in the time domain can be expressed then as

$$r(t) = \sum_{k=1}^n c_k \phi_k(t) \quad (3)$$

where the c_k are scalars called signal descriptors and the $\phi_k(t)$ form an orthogonal basis in the time domain. It should be noticed that $\text{sign} =$ in Equations (2) and (3) stands for $r(t)$ "expressed by" rather than "equal to" in the sense we are using the summation here.

As the passage from Equation (2) to Equation (3) involves only a linear transformation (rotation of axes), the coefficients c_k will be given by

$$c_1 = a_{11} \overline{p_1} + a_{12} \overline{p_2} + a_{13} \overline{p_3} + \dots + a_{1n} \overline{p_n}$$

$$c_2 = a_{21} \overline{p_1} + a_{22} \overline{p_2} + a_{23} \overline{p_3} + \dots + a_{2n} \overline{p_n}$$

.

$$c_n = a_{n1} \overline{p_1} + a_{n2} \overline{p_2} + a_{n2} \overline{p_2} + a_{n3} \overline{p_3} + \dots + a_{nn} \overline{p_n}$$

which can be written as a matrix equation:

$$\begin{aligned} c_k] &= [a_{kj}] p_j] & j &= 1, 2, \dots \\ & & k &= 1, 2, \dots \end{aligned} \quad (4)$$

where $[a_{jk}]$ is a matrix of $n \times n$ order that can be obtained from the signal descriptors once the former had been experimentally determined, by instrumenting Equation (4) in such a way that

$$p_j] = [b_{jk}] c_k] \quad (4^1)$$

is obtained.

The number of parameters that could be present in a given specimen may be larger than the signal descriptors available; then $[a_{jk}]$ will not be a square matrix and this means that the instrumentation of Equation (4) must involve statistical methods, if no ambiguity is desired in the determination of the parameters of interest.

As far as the driving signal is concerned, it is clear that its dimension (also thought of as a vector) must be at least equal to the number of parameters; then a driving signal suitable to be used to separate n parameters could be formed with $n/2$ sinusoids of different $n/2$ frequencies, as each of them will have a dimension equal to two (its magnitude and phase).

On the other hand, a pulse-like type of signal will have the required dimensionality, and it is the purpose of this study to analyze the properties of the response of an actual system to a step function in order to gain insight of the possibilities of such a driving signal in NDT.

Hereafter, the structure of the response will be studied for an eddy-current method and a step-type driving signal.

ANALYTICAL DEVELOPMENT

The system is defined as having:

1. Specimen: Cylindrical bar with a radius a , conductivity σ , permeability $\mu = 4\pi \times 10^{-7}$ Hy/meter (non-magnetic) and of infinite length.

2. Driving transducer: Single layer coil of N_1 turns, of sufficient length to produce a magnetic field with only a component parallel to the surface of the bar.
3. Pickup transducer: Single layer coil of N_2 turns placed underneath the driving coil, with length neglectible with respect to the driving coil and placed equidistant from the driving coil.

The driving signal is a constant current I_0 interrupted at $t = 0$, so it is defined as:

$$E(t) = I_0 \quad t \leq 0$$

$$E(t) = 0 \quad t > 0.$$

The response signal will be given by (per turn of pickup coil)

$$r(t) = \frac{d\phi}{dt} = - \frac{d}{dt} \int_A \mu H \, dA \quad (5)$$

where A = area encircled by the pickup coil,
 H = magnetic field, expressed as a function of time and geometry for $t > 0$
 $dA = 2\pi r dr$.

To find H , the Maxwell equations for our geometry and initial conditions must be solved. This problem has been solved by several authors and the solution is given by:

$$H = H_0 \sum_{\gamma=1}^{\infty} \frac{2J_0(\beta_{\gamma} \frac{r}{c})}{\beta_{\gamma} J_1(\beta_{\gamma})} \exp\left(-\frac{\beta_{\gamma}^2 t}{c^2 \sigma \mu}\right). \quad (6)$$

where c = radius of the bar
 H_0 = applied magnetic field, as defined previously
 J_0 = Bessel first kind, zero order function

J_1 = Bessel first kind, first order function
 β_γ = γ 'th argument of $J_0(\beta_\gamma) = 0$ (successive zeros of the $J_0(\beta)$ function)
 r = radius $0 \leq r \leq c$
 t = time.

Equation (6) also represents the magnetic density B . Then, if the coils are supposed to have the same diameter of the bar

$$\phi(t; r) = \int_A B dA = \int_0^c B 2\pi r dr \quad (7)$$

$$\phi(t; r) = \int_0^c B_0 \left[\sum_{\gamma=1}^{\infty} \frac{2 J_0\left(\frac{r}{c} \beta_\gamma\right)}{\beta_\gamma J_1(\beta_\gamma)} \exp\left(-\frac{\beta_\gamma^2 t}{\epsilon^2 \sigma_\mu}\right) \right] \cdot 2\pi r dr$$

which can be written as

$$\phi(t; r) = B_0 \sum_{\gamma=1}^{\infty} \frac{4\pi \exp\left(-\frac{\beta_\gamma^2 t}{\epsilon^2 \sigma_\mu}\right)}{\beta_\gamma J_1(\beta_\gamma)} \int_0^c J_0\left(\frac{r}{c} \beta_\gamma\right) r dr. \quad (8)$$

To evaluate the integral in Equation (8), the following change of variable is made:

$$r = u \cdot \frac{c}{\beta_\gamma}. \quad (9)$$

Then

$$dr = \frac{c}{\beta_\gamma} \cdot du \quad \text{and} \quad r dr = \left(\frac{c}{\beta_\gamma}\right)^2 u du.$$

Then the integral in Equation (8) becomes

$$\int_0^c J_0\left(\frac{r}{c} \beta_\gamma\right) r \cdot dr = \left(\frac{c}{\beta_\gamma}\right)^2 \int_0^{\beta_\gamma} J_0(u) u du. \quad (10)$$

But for the Bessel functions of the first kind

$$\int u^n J_{n-1}(u) du = u^n J_n(u) \quad n = \text{order of the function.}$$

Then if $n = 1$

$$\int_0^u J_0(u) du = u J_1(u). \quad (11)$$

Application of Equation (11) to the right hand member of Equation (10) yields to

$$\left(\frac{c}{\beta_\gamma}\right)^2 \int_0^{\beta_\gamma} J_0(u) u du = \left(\frac{c}{\beta_\gamma}\right)^2 u J_1(u) \Big|_0^{\beta_\gamma} = \frac{c^2}{\beta_\gamma} J_1(\beta_\gamma). \quad (12)$$

Then by Equations (12) and (10), we have

$$\begin{aligned} \phi(t) &= B_0 \sum_{\gamma=1}^{\infty} \frac{4\pi}{\beta_\gamma J_1(\beta_\gamma)} \cdot \frac{c^2}{\beta_\gamma} J_1(\beta_\gamma) \exp\left(-\frac{\beta_\gamma^2 t}{c^2 \sigma_\mu}\right) \\ \phi(t) &= B_0 \sum_{\gamma=1}^{\infty} 4\pi \left(\frac{c}{\beta_\gamma}\right)^2 \exp\left(-\frac{\beta_\gamma^2 t}{c^2 \sigma_\mu}\right). \end{aligned} \quad (13)$$

The response will then be

$$\begin{aligned} r(t) &= -\frac{d\phi}{dt} = -4\pi B_0 \sum_{\gamma=1}^{\infty} \left(\frac{c}{\beta_\gamma}\right)^2 \left(-\frac{\beta_\gamma^2}{c^2 \sigma_\mu}\right) \exp\left(-\frac{\beta_\gamma^2 t}{c^2 \sigma_\mu}\right) \\ r(t) &= 4\pi \mu H_0 \sum_{\gamma=1}^{\infty} \frac{1}{\sigma_\mu} \cdot \exp\left(-\frac{\beta_\gamma^2 t}{c^2 \sigma_\mu}\right) \\ r(t) &= \frac{4\pi H_0}{\sigma} \sum_{\gamma=1}^{\infty} \exp\left(-\frac{\beta_\gamma^2 t}{\epsilon^2 \theta_\mu}\right) \end{aligned} \quad (14)$$

We make

$$b_{\gamma} = \frac{\beta_{\gamma}^2}{c^2 \sigma_{\mu}} = (M \beta_{\gamma})^2 \quad M = \frac{1}{c \sqrt{\mu \sigma}} \quad (15)$$

From Equation (14) we see that $r(t)$ is bounded (except for $t = 0$ where it is infinite). To check the results, the summation will be approximated with a closed function.

The Euler-McLaurin summation formula states:

$$\begin{aligned} \sum_{\gamma=1}^{\infty} F(\gamma) &= \int_0^{\infty} F(\gamma) d\gamma - \frac{1}{2} [F(0) + F(\infty)] \\ &+ \sum_{p=1}^{\infty} (-1)^{(p-1)} \frac{B_p}{(2^p)!} \cdot \left[F^{(2p-1)}(\infty) - F^{(2p-1)}(0) \right] \quad (16) \end{aligned}$$

where $\gamma = 1, 2, \dots \dots \infty$ parameter of the summation

$F(\gamma)$ = the function of the summation with all the variables but γ considered constants.

$F(0)$ = value of the function for $\gamma = 0$

$F(\infty)$ = value of the function for $\gamma = \infty$

$F_{(2p-1)}^{(2p-1)} = (2p-1)$ the derivative of $F(\gamma)$ where $p = 1, 2, 3, \dots \infty$

B_p = p'th Bernoulli Number.

As the function of Equation (14) is of exponential type, its value for $\gamma = \infty$ ($b_{\gamma} \rightarrow \infty$) is zero and so will all its derivatives; then only the values for $\gamma = 0$ have to be considered in application of Equation (16) to Equation (14).

But it is also necessary to have some type of relationship between the β_γ and γ . Here the following recurrence relation between the zeros of the Bessel function of first kind, zero order will be used

$$\beta_\gamma = (\gamma - \frac{1}{4}) \pi. \quad (17)$$

Then from Equation (15)

$$b_\gamma = (M\beta_\gamma)^2 = (M\pi)^2 (\gamma^2 - \frac{\gamma}{2} + \frac{1}{16})$$

and

$$\begin{aligned} b_{\gamma t} &= (M\pi\sqrt{t})^2 (\gamma^2 - \frac{\gamma}{2} + \frac{1}{16}) \\ &= A^2 \gamma^2 + \frac{A^2}{16} - \frac{A^2}{2} \gamma = a\gamma^2 - b_\gamma + c \end{aligned} \quad (18)$$

if

$$\begin{aligned} A &= M\pi \sqrt{t} & b &= \frac{A^2}{2} \\ a &= A^2 & c &= \frac{A^2}{16} = \left(\frac{A}{4}\right)^2. \end{aligned}$$

Then substituting Equation (18) in Equation (14) and **apply-**ing Equation (16)

$$\begin{aligned} \sum_{\gamma=1}^{\infty} \exp [-(a\gamma^2 - b_\gamma + c)] &= \int_0^{\infty} \exp [-(a\gamma^2 - b_\gamma + c)] d\gamma - \frac{1}{2} \exp \left[\left(-\frac{A^2}{16}\right) \right] \\ &- \frac{A^2}{12 \cdot 2!} \exp \left(-\frac{A^2}{16} \right) = \int_0^{\infty} \exp [-(a\gamma^2 - b_\gamma + c)] d\gamma \\ &- \frac{1}{2} \exp \left[\left(-\frac{A}{4}\right)^2 \right] \left(1 + \frac{A^2}{12}\right) \end{aligned} \quad (19)$$

where only terms up to the first derivative have been considered. The integral of Equation (19) is given by:

$$\int_0^\infty \exp -(a\gamma^2 - b\gamma + c) d\gamma = \frac{1}{2} \sqrt{\frac{\pi}{a}} \exp \left(\frac{b^2}{4a} - c \right) \operatorname{erfc} \frac{b}{2\sqrt{a}}.$$

Replacing a,b and c by the values of Equation (18)

$$\operatorname{erfc} \frac{b}{2\sqrt{a}} = \operatorname{erfc} \left(-\frac{A^2}{4} \right) = 1 + \operatorname{erf} \frac{A^2}{4} \quad (\text{where } \operatorname{erf} \frac{A^2}{4} \text{ is the error}$$

$$\text{function of } \frac{A^2}{4}), \exp \left(\frac{b^2}{4a} - c \right) = 1$$

$$\text{and. } \frac{1}{2} \sqrt{\frac{\pi}{a}} = \frac{\sqrt{\pi}}{2A}.$$

Going back to Equation (19) and Equation (14), we have

$$r(t) \approx \frac{4\pi H_0}{\sigma} \left[\frac{\sqrt{\pi}}{2A} \left(1 + \operatorname{erf} \frac{A}{4} \right) - \exp \left[\left(-\frac{A}{4} \right)^2 \right] \left(1 + \frac{A^2}{12} \right) \right] \quad (20)$$

making

$$A = 4a\sqrt{t} = M\pi\sqrt{t} \rightarrow a = \frac{M\pi}{4}$$

$$r(t) \approx \frac{4\pi H_0}{\sigma} \left\{ \frac{\pi}{8(a\sqrt{t})} (1 + \operatorname{erf} a\sqrt{t}) - \frac{\exp (-a\sqrt{t})^2}{2} \left[1 + \frac{16}{12} (a\sqrt{t})^2 \right] \right\}. \quad (21)$$

Both Equations (14) and (21) can then be written as

$$r(t) = A_f \sum_{\gamma=1}^{\infty} \exp(-b_{\gamma} t) \quad (14a)$$

$$r(t) = A_f \left\{ \frac{\sqrt{\pi}}{8(a\sqrt{t})} (1 + \operatorname{erf} a\sqrt{t}) \frac{\exp(-a\sqrt{t})^2}{2} \left[1 + \frac{16}{12} (a\sqrt{t})^2 \right] \right\} \quad (21a)$$

where

$$A_f = \frac{4\pi H_o}{\sigma} \quad (22)$$

is an amplitude factor.

Equation (21) shows that the response function is analytic which yields the important property that no matter at what particular instant of time the function is sampled (measured) the information content will be the same. This property of the response function permits considerable latitude in the selection of the time of sampling of the outputs of the orthogonal filter, which will be described in the next section.

If we consider the $r(t)$ being normalized (i.e., being divided by $4\pi H_o$) the form of it will be

$$R(t) = \frac{r(t)}{4H_o\pi} = \frac{1}{\sigma} F(a\sqrt{t}) \text{ with } a\sqrt{t} = f(\sigma; c; t)$$

$$\frac{dR(t)}{d\sigma} = -\frac{F(a\sqrt{t})}{\sigma^2} + \frac{1}{\sigma} \frac{dF(a\sqrt{t})}{d(a\sqrt{t})} \frac{\partial(a\sqrt{t})}{\partial\sigma} \quad (23)$$

while

$$\frac{dR(t)}{dc} = \frac{1}{\sigma} \frac{dF(a\sqrt{t})}{d(a\sqrt{t})} \frac{\partial(a\sqrt{t})}{\partial c} \quad (24)$$

From Equations (23) and (24) we see that the structure of the response will be different for changes in conductivity and/or diameter, and those changes will be apparent in $r(t)$ and could be separated with appropriate analyzing circuitry, as stated in the introduction.

TEST RESULTS

To check the results of Equations (14a) and (21a) an experimental set up was arranged consisting of

1. Bar of stainless steel with:

Radius $c = 0.564$ in.

Conductivity $\sigma = 0.135 \times 10^7 \frac{\text{mhos}}{\text{meter}} \pm 5\%$

Magnetic permeability assumed to be $\mu = 4\pi \times 10^{-7}$ Hy/meter

Length = 8 in.

2. Driving coil:

Number of turns $N_1 = 269$

Length $L_1 = 5$ in.

3. Pickup coil:

Number of turns $N_2 = 55$

Length $L_2 = 1$ in.

Both coils were wound directly over the bar with layers of paper between them and the bar. The pickup coil was underneath the driving coil and centered with respect to it.

The driving coil was excited through a power transistor which in turn was driven by a General Radio 1219-A pulser, with a pulser with ON time of approximately 600 μsec and a repetition rate of 500 pps.

The response signal was recorded at the trailing edge of the pulse, when the transistor went to OFF condition and then presented a high impedance to the coil, resembling a current source. The results are shown in Figure 1 which is a picture taken from the screen of a 547 Tektronix Scope. Both the response signal and the driving step current are shown.

FIGURE 1. Step Function Drive Current and Measured Response Function

Four points were selected to check:

Point 1: $t = 1.41 \times 10^{-6}$ sec

Point 2: $t = 5.645 \times 10^{-6}$ sec

Point 3: $t = 11.06 \times 10^{-6}$ sec

Point 4: $t = 24.9 \times 10^{-6}$ sec

For these times the values of $a\sqrt{t}$ are:

Point 1: $a\sqrt{t} = 0.05$

Point 2: $a\sqrt{t} = 0.10$

Point 3: $a\sqrt{t} = 0.14$

Point 4: $a\sqrt{t} = 0.21$

(25)

For the same times, Equation (14a) was calculated and successive exponentials were considered up to the point where the absolute error was of the same order for all the four points and in the order of 0.5 parts per thousand. The results were (see Table 1):

TABLE 1. Evaluation of Equation (14a) for Four Selected Points

			Point 1: $t_1 = 1.411 \times 10^{-6}$, sec	Point 2: $t_2 = 5.645 \times 10^{-6}$, sec	Point 3: $t_3 = 11.06 \times 10^{-6}$, sec	Point 4: $t_4 = 24.9 \times 10^{-6}$, sec	
γ	β_γ	β_γ^2	b_γ	$\frac{b_\gamma t_1}{\exp(-b_\gamma t_1)} \quad \sum \exp(-b_\gamma t_1)$	$\frac{b_\gamma t_2}{\exp(-b_\gamma t_2)} \quad \sum \exp(-b_\gamma t_2)$	$\frac{b_\gamma t_3}{\exp(-b_\gamma t_3)} \quad \sum \exp(-b_\gamma t_3)$	$\frac{b_\gamma t_4}{\exp(-b_\gamma t_4)} \quad \sum \exp(-b_\gamma t_4)$
1	2.40482	5.78316	16,611.26	0.023438 0.97683	0.09377 0.91049	0.18372 0.83217	0.413620 0.66125
2	5.52007	30.47117	87,523.86	0.12345 0.88386 1.86069	0.49407 0.61014 1.52063	0.96801 0.37984 1.21201	2.17934 0.11311 0.77436
3	8.65372	74.88687	215,100.3	0.30350 0.73822 2.59891	1.21425 0.29693 1.81756	2.37902 0.09264 1.30465	5.35602 0.00472 0.77908
4	11.79153	139.04018	399,372.0	0.56351 0.56920 3.16811	2.25445 0.10493 1.92249	4.41705 0.01207 1.31672	9.94436 0.00005
5	14.9309	222.93207	640,339	0.90352 0.40514 3.57325	3.61471 0.02692 1.94941	7.08125 0.00084 1.31756	
6	18.07106	326.5632	938,003	1.37352 0.26620 3.83945	5.29503 0.00502 1.95443	10.37432 0.00003	
7	21.21163	449.93325	1,292,365	1.82353 0.16146 4.00091	7.29540 0.00068 1.95511		
8	24.35247	593.04279	1,703,426	2.40353 0.09040 4.09131	9.61584 0.00007		
9	27.49348	775.89144	2,228,627	3.14460 0.04308 4.13439			
10	30.63461	938.47933	2,695,636	3.80354 0.02229 4.15668			
11	33.77582	1,140.80602	3,276,788	4.62350 0.00982 4.16650			
12	36.91710	1,362.87227	3,914,643	5.52336 0.00399 4.17049			
13	40.05	1,604.677013	4,609,190	6.50357 0.001498 4.17199			
14	43.19979	1,866.22186	5,360,538	7.56358 0.00052 4.17251			
15	46.34119	2,147.5059	6,168,384	8.70359 0.00017 4.17268			
16	49.48261	2,448.52869	7,033,026	9.92360 0.00005			
				4.173	1.956	1.318	0.779
				$E < 0.5 \times 10^{-3}$	$E < 0.5 \times 10^{-3}$	$E < 0.5 \times 10^{-3}$	$E < 0.2 \times 10^{-3}$

TABLE 1. Evaluation of Equation (14a) for Four Selected Time Points

β_Y	β_Y^2	b_Y	Point 1: $t_1 = 1.411 \times 10^{-6}$, sec			Point 2: $t_2 = 5.645 \times 10^{-6}$, sec			Point 3: $t_3 = 11.06 \times 10^{-6}$, sec			Point 4: $t_4 = 24.9 \times 10^{-6}$, sec		
			$b_Y t_1$	$\exp(-b_Y t_1)$	$\int \exp(-b_Y t_1)$	$b_Y t_2$	$\exp(-b_Y t_2)$	$\int \exp(-b_Y t_2)$	$b_Y t_3$	$\exp(-b_Y t_3)$	$\int \exp(-b_Y t_3)$	$b_Y t_4$	$\exp(-b_Y t_4)$	$\int \exp(-b_Y t_4)$
2.40482	5.78316	16,611.26	0.023438	0.97683		0.09377	0.91049		0.18372	0.83217		0.413620	0.66125	
5.52007	30.47117	87,523.86	0.12345	0.88386	1.86069	0.49407	0.61014	1.52063	0.96801	0.37984	1.21201	2.17934	0.11311	0.77436
8.65372	74.88687	215,100.3	0.30350	0.73822	2.59891	1.21425	0.29693	1.81756	2.37902	0.09264	1.30465	5.35602	0.00472	0.77908
11.79153	139.04018	399,372.0	0.56351	0.56920	3.16811	2.25445	0.10493	1.92249	4.41705	0.01207	1.31672	9.94436	0.00005	
14.9309	222.93207	640,339	0.90352	0.40514	3.57325	3.61471	0.02692	1.94941	7.08125	0.00084	1.31756			
18.07106	326.5632	938,003	1.37352	0.26620	3.83945	5.29503	0.00502	1.95443	10.37432	0.00003				
21.21163	449.93325	1,292,365	1.82353	0.16146	4.00091	7.29540	0.00068	1.95511						
24.35247	593.04279	1,703,426	2.40353	0.09040	4.09131	9.61584	0.00007							
27.49348	775.89144	2,228,627	3.14460	0.04308	4.13439									
30.63461	938.47933	2,695,636	3.80354	0.02229	4.15668									
33.77582	1,140.80602	3,276,788	4.62350	0.00982	4.16650									
36.91710	1,362.87227	3,914,643	5.52336	0.00399	4.17049									
40.05	1,604.677013	4,609,190	6.50357	0.001498	4.17199									
43.19979	1,866.22186	5,360,538	7.56358	0.00052	4.17251									
46.34119	2,147.5059	6,168,384	8.70359	0.00017	4.17268									
49.48261	2,448.52869	7,033,026	9.92360	0.00005										
				4.173			1.956			1.318			0.779	
				E < 0.5 × 10 ⁻³			E < 0.5 × 10 ⁻³			E < 0.5 × 10 ⁻³			E < 0.2 × 10 ⁻³	

$$\begin{aligned}
\text{Point 1: } r(t) &= A_{\perp} \times 4.173 \text{ with } \xi < 0.5 \times 10^{-3} \\
\text{Point 2: } r(t) &= A_f \times 1.956 \text{ with } \xi < 0.5 \times 10^{-3} \\
\text{Point 3: } r(t) &= A_{\perp} \times 1.318 \text{ with } \xi < 0.5 \times 10^{-3} \\
\text{Point 4: } r(t) &= A_f \times 0.779 \text{ with } \xi < 0.2 \times 10^{-3}
\end{aligned} \tag{26}$$

With the values of $a\sqrt{t}$ given by Equation (25), Equation (21a) was applied with the results:

$$\begin{aligned}
\text{Point 1: } r(t) &= A_f \quad 4.245 \\
\text{Point 2: } r(t) &= A_f \quad 1.994 \\
\text{Point 3: } r(t) &= A_f \quad 1.328 \\
\text{Point 4: } r(t) &= A_f \quad 0.797.
\end{aligned} \tag{27}$$

Comparing Equations (26) and (27), the relative error is well below 2.5%, and Equation (21) is an approximation of Equation (14) with an error of less than 2.5% for at least times up to:

$$a\sqrt{t} = 0.20.$$

To check for the absolute values of Figure 1, we calculate the amplitude factor from Equation (22) and for N_2 turns in the pickup coil:

$$A_f = \frac{4\pi H_o}{\sigma} N_2 \tag{28}$$

where

$$H_o = \frac{N_1 I_o}{L_1} \left[1 - 2 \frac{(c)^2}{L_1^2} \right] = \frac{N_1 I_o}{L_1} \times 0.98.$$

Then in the present circumstances, H_o will have an error at least of 2.5%; then:

$$A_f = \frac{4\pi N_1 N_2}{\sigma L_1} \cdot I_o \text{ (amp)} = 1.0884 \frac{\text{volts}}{\text{amp}} I_o \text{ (amp)}$$

with a relative error at least equal to the sum of the errors in H_o and σ which is $\approx 7.5\%$.

With these considerations, the values of Figure 1 check with those of Equations (21a) or (14a) well within this error.

ORTHOGONAL FILTER

In the Introduction it was stated that to get suitable signal descriptors, the response signal could be expressed in terms of orthogonal components. Therefore, the $\phi_k(t)$ functions must satisfy the orthogonality condition which states:

$$\int_0^{\infty} \phi_i(t) \phi_j(t) w(t) dt = S_{ij}$$

$$\text{and } S_{ij} \begin{cases} = 0 & \text{for } i \neq j \\ = 1 & \text{for } i = j = k \text{ (normalizing condition)} \end{cases}$$

and $w(t)$ = weighting function, to which the $\phi_k(t)$ are orthogonal if we have

$$w(t) = e^{-2at}.$$

Then

$$\int_0^{\infty} \phi_i(t) \phi_j(t) e^{-2at} dt = \int_0^{\infty} \theta_i(t) \theta_j(t) dt \quad (30)$$

$$\text{where } \theta_i(t) = \phi_i(t) e^{-at} \quad \theta_j(t) = \phi_j(t) e^{-at}$$

$$\text{and the Laplace transforms: } \theta_i(s) = \Phi_i(s + a)$$

$$\theta_j(s) = \Phi_j(s + a).$$

Then Equation (30) can be evaluated as follows:

$$\int_0^{\infty} \theta_i(t) \theta_j(t) dt = 1 \int_{C-j\infty}^{C+j\infty} \phi_i(s+a) \phi_j(s-a) ds. \quad (31)$$

If the $\phi_k(t)$ are formed from exponential functions of the type

$$\exp(-\alpha_k t), \alpha_k: \text{ real and positive}$$

then the $\phi_k(t)$ will only contain real poles over the left hand complex plane and Equation (31) yields, with conditions of Equation (29) to:

$$\phi_1(s) = \sqrt{2(a+\alpha_1)} \frac{1}{s+\alpha_1}$$

$$\phi_2(s) = \sqrt{2(a+\alpha_2)} \frac{1}{(s+\alpha_1)} \frac{(s-2a-\alpha_1)}{(s+\alpha_2)} = \phi_1(s) \frac{(s-2a-\alpha_1)}{(s+\alpha_2)}$$

$$\phi_3(s) = \sqrt{2(a+\alpha_3)} \frac{(s-2a-\alpha_1)(s-2a-\alpha_2)}{(s+\alpha_1)(s+\alpha_2)(s+\alpha_3)} = \phi_2(s) \frac{(s-2a-\alpha_2)}{(s+\alpha_3)},$$

and in general

$$\phi_n(s) = \sqrt{2(a+\alpha_n)} \frac{(s-2a-\alpha_1)(s-2a-\alpha_2)\dots(s-2a-\alpha_{n-1})}{(s+\alpha_1)(s+\alpha_2)\dots(s+\alpha_n)}. \quad (32)$$

If we choose $a = 0$, that is $w(t) = 1$ and the $\phi_k(s)$ are built around a single exponential in a cyclic way:

$$\alpha_1 = \alpha_2 = \alpha_3 = \dots \alpha_k = \alpha,$$

from Equation (32) we have

$$\phi_1(s) = \sqrt{2\alpha} \frac{1}{s + \alpha}$$

$$\phi_2(s) = \sqrt{2\alpha} \frac{1 \cdot (s - \alpha)}{(s + \alpha)(s + \alpha)(s + \alpha)} = \phi_1(s) \cdot \frac{(s - \alpha)}{(s + \alpha)}$$

$$\phi_3(s) = \sqrt{2\alpha} \frac{(s - \alpha)(s - \alpha)}{(s + \alpha)(s + \alpha)(s + \alpha)} = \phi_2(s) \cdot \frac{(s - \alpha)}{(s + \alpha)}$$

Then, the basis will be given by:

$$\frac{\phi_{(n+1)}(s)}{\sqrt{2\alpha}} = \frac{(s - \alpha)^n}{(s + \alpha)^{n+1}} \quad n = 0, 1, \dots, k, \dots, \infty. \quad (33)$$

If a filter is realized with a transfer function given by Equation (33) (see Figure 2) and $r(t)$ is applied at its input, the

FIGURE 2.

output at the k^{th} terminal will be given by:

$$L^{-1} [r(s) \cdot \phi_k(s)] = \int_0^T \phi_k(t) \cdot r(T - t) dt = c_k(T) \quad (34)$$

which represents the magnitude of the projection of $r(s)$ onto

$\phi_k(s)$ when measured at a time $t = T$, $T > 0$; therefore, the $c_k(T)$ will form a set that describes $r(t)$ in terms of an orthonormal basis in the sense defined by Equation (3).

Equation (34) actually shows that the $c_k(T)$ are the projections of $r(T - t)$ rather than $r(t)$. However as $r(t)$ does not change its structure if reversed in time, it does not make any difference if $r(T - t)$ is analyzed rather than $r(t)$; the only difference is that the $\phi_k(t)$ are functions that decrease with time so the product:

$$\phi_k(t) \cdot r(T - t) \quad 0 \leq t \leq T,$$

will be smaller than the one defined by

$$\phi_k(T - t) \cdot r(T - t) \quad 0 \leq t \leq T,$$

which means that the ϕ_k are formed by functions that also increase with time.

From the above, it does not seem necessary to have time increasing functions as the basis, as this could only change the magnitude of the $c_k(t)$'s but will not change their nature as signal descriptors.

From Equation (14) it seems that a natural choice for the time constant α of Equation (33) could be the exponent of the first exponential of the summation. Then if in Equations (15) and (17) we put $\gamma = 1$:

$$\beta_1 = \frac{3}{4}\pi$$

$$b_1 = \left(\frac{M}{4} \frac{3}{4} \pi \right)^2 = 9 \left(\frac{M}{4} \frac{\pi}{4} \right)^2 = 9a^2. \quad (35)$$

As for sampling time T , it has been stated that it could be any, and from the filter operation we see that it cannot be $t = 0$ if $r(t) = 0$, $t < 0$. From a practical point of view it should be chosen where a good signal-to-noise ratio is available. A good starting point for trial and error could be where

$$a\sqrt{T} \approx 0.10 \div 0.20 \quad 0.10 < a\sqrt{T} < 0.20, \quad (36)$$

as for that time, in relative terms, the signal has enough amplitude left.

It should be noticed that Equations (35) and (36) are only indicative as starting points for design. In a practical system, only relative changes in $r(t)$ are of interest rather than absolute values, and even though the basic philosophy is not changed, the circuitry will be more involved.

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