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ASTRON PROGRAM FINAL REPORT

R. J. Briggs
R. E. Hester
G. H. Porter
W. A. Sherwood
R. Spoerlein
B. W. Stallard
J. Taska
P. B. Weiss

August 25, 1975

MASTER

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University of California/Livermore, California 94550

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R. Spoerlein
B. W. Stallard
J. Tasse
P. B. Weiss*

MS. date: August 25, 1975

*Permanent address: Los Alamos Scientific Laboratory,
Los Alamos, New Mexico 87544

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ASTRON PROGRAM FINAL REPORT

Abstract

This report describes important experimental results obtained in the last two years of the Astron Program, an LLL controlled nuclear

fusion program which terminated in 1973. Little theoretical work is included, but an extensive bibliography is given.

Introduction

This final report documents the status of the Astron Program at its termination in February 1973. The objective of this program was to establish a strong circulating current of relativistic electrons (the E-layer) to be utilized in confining and heating a thermonuclear plasma for use in controlled nuclear fusion. We compile here several reports, written by the Astron Group, which cover the major highlights of the

experimental effort in the last several years. The bulk of the theoretical work, in particular the results obtained from computer simulation codes, is not included. However, this work is adequately documented in the published literature. We include an extensive bibliography as a guide to this theoretical work and to other aspects of the program, including earlier experimental results.

Outline of the Report

The Astron Program took a significantly new direction early in 1972. Prior to this time, the buildup of the E-layer to field reversal (closed magnetic lines) by multiple-pulse injection was to be accomplished in the presence of a plasma generated by the ionization of background gas. It was found that the addition of a relatively weak toroidal magnetic field opened

up a new option for the program, namely, buildup of the layer by multiple-pulse injection in a relatively good vacuum where the layer space charge is only partially neutralized by positive ions. The rationale and supporting experimental evidence for this new approach are given here in the first chapter, which contains a report written early in 1972 by N.C. Christofilos.

A key feature of the "vacuum buildup" approach is the axial compression of the E-layer as it slowly becomes space-charge neutralized. This compression can obviate a predicted difficulty with E-layer buildup in the electrically neutralized case, where incoming pulses cause a stretching of the layer axially and a saturation in the layer's current density. This aspect of the vacuum buildup approach is discussed in Ch. 2, which briefly describes the essential ideas involved in the vacuum buildup approach.

The "resistors" mounted on the inner and outer tank walls are discussed in Ch. 2. They perform two crucial functions fundamental to the vacuum trapping and buildup process, namely, the slowing-down of the layer by eddy current losses, and the axial focusing provided by the wires when the magnetic field has penetrated them. Axial focusing calculations are given in Ch. 3, while both calculations and measurements of the resistive trapping process are presented in Ch. 4. The quantitative understanding in these areas in the later stages of the program resulted in an ability to design new resistor geometries with relatively high confidence in how they would perform in practice.

As mentioned above, addition of an external toroidal field was required before the vacuum stacking approach could be implemented. This was required because an unstable precessional mode causes a

"dump" of the unneutralized layer unless a toroidal field (B_θ) is applied. This unstable electrostatic precession was avoided previously in the experiments without toroidal field by having a prompt enough charge neutralization (high enough gas pressure). This explanation for the inability to sustain E-layers in a good vacuum was suggested by Christofilos as early as 1965, when stable "magnetic" precessions were first observed. Theoretical and experimental results on the electrostatic precession mode, including B_θ , are given in Chs. 5 and 6. Here again, the quantitative understanding of the phenomena was such that we could predict beforehand the stable parameter regimes.

Chapter 7 summarizes the experimental results obtained in the studies of trapping and buildup by multiple-pulse injection in high vacuum. As noted above, many facets of the important physical processes in the experiments were felt to be understood quite well, such as the precessional mode stability, axial focusing and adiabatic compression, and the resistor eddy current losses for various resistor geometries. The buildup of the E-layer by multiple-pulse injection was generally *unsuccessful*, however, and the understanding of the precise nature of the limiting processes was incomplete at the termination of the experiments. In short, the buildup of an E-layer in vacuum turned out to be a harder task than

was originally hoped, even though the relative simplicity of the system (i.e., lack of plasma interactions) allowed a much more rapid and quantitative understanding of the observed phenomena.

At the end of the program, a brief period was spent assessing the regime of high plasma density (high pressure) combined with strong toroidal fields ($B_0 \sim B_z$) for formation of the E-layer. The

results, as described in Ch. 8, were extremely encouraging; single pulses were efficiently trapped in short E-layers, leading to self-fields about half that required for field reversal. These experiments suggest that high current injection of single pulses in high density plasmas, with strong toroidal fields, is an excellent direction for any future Astron-type experiments.

Late 1971-Early 1972 Experimental Results on Astron*

N. C. Christofilos, W. C. Condit, Jr., R. E. Hester,
G. D. Porter, B. W. Stallard, and P. B. Weiss

RECENT EXPERIMENTAL RESULTS, JANUARY 9-FEBRUARY 3, 1972[†]

When the precessional instability in E-layers was eliminated by a weak toroidal field in May 1971, we were able to trap E-layers in high vacuum for the first time.¹ In September 1971, 10 small pulses were stacked in high vacuum (base pressure 3 μ Torr). As the background gas pressure was increased, we observed a saturation phenomenon. At higher pressure (e.g., 0.1 to 0.2 mTorr of hydrogen), the same E-layer amplitude was achieved in less pulses with a limit of approximately 10 A/cm. However, when a gas puff

was added during any of the small pulses, an additional 15 to 20 A/cm was added in the E-layer.

Next, in the fall of 1971, resistors were added to the cantilever in the trap region, after an analytical solution for the resistor focusing in cylindrical geometry was obtained.* Following the addition of these resistors, we observed for the first time E-layer amplitudes up to 30 A/cm in a background pressure of 0.1 to 0.2 mTorr in two-pulse stacking. At the same time an abrupt loss was observed 50 to 100 μ sec after trapping of the second pulse. It was observed that gas puffing eliminated the losses restoring E-layer lifetime. The cause of the loss was not clear at that time.

*Adapted from Lawrence Livermore Laboratory Rept. UCID-15991 (1972).

[†]Note: This report, written in February 1972, summarizes Astron experimental results for late 1971 to early 1972. The use of "present" means February 1972.

*Refer to Ch. 3.

Since January 9, 1972, systematic measurements in high vacuum (0.5 μ Torr base pressure) and in a range of hydrogen gas pressures revealed the following:

- The abrupt loss occurs at a time two to three times the neutralization time of the E-layer and it lasts approximately a time equal to the neutralization time. During this loss, the E-layer amplitude drops to approximately 5 or 10 A/cm where the loss stops. Experimental evidence indicates (Fig. 1) that the ratio of the time of occurrence to the time required for neutralization increases slightly as vacuum improves. At a base pressure of 1- μ Torr, when the neutralization time is approximately 300 μ sec, the loss occurs 1.5 to 2 msec

later. At 0.5 μ Torr, the loss occurs 4 to 5 msec later.

- The abrupt loss is not observed when the E-layer amplitude is less than 5 A/cm. This amplitude threshold may explain the saturation observed in September 1971, in many-pulse stacking.
- There is no obvious rf associated with the loss, although the investigation is not yet conclusive.
- The mystery of this loss became still greater when it was observed that 10% increase of the toroidal field, applied much before the loss occurs, postpones the initiation of the loss by 0.5 msec (Fig. 2).

Although the cause of the loss is not yet clear, the experimental observation is that when trapping occurs in the presence of gas (e.g., 10 to 20 mTorr of hydrogen, where a strong hybrid instability lasting 0.5 μ sec produces a plasma density approximately 100 times the E-layer density) the loss does not occur.

Therefore, from what we know at this time, the E-layer has two stable regions:

- (1) Plasma density $n_p < (2 \text{ to } 3) n_e$
- (2) Plasma density $n_p > (50 \text{ to } 100) n_e$,

where n_e is the E-layer electron density.

We recently trapped an E-layer having the same amplitude and current

Fig. 1. Loss time after injection as a function of pressure.

distribution at two different pressures. The pressures were 0.1 mTorr and 17 Torr. In the first case, the abrupt loss occurs in 40 to 60 μ sec following neutralization (which requires 20 μ sec at 0.1 mTorr) while in the second case no loss

Fig. 2. Effect of toroidal field on occurrence of abrupt loss, where base pressure is 1.1 μ Torr, and E-layer amplitude is 0.2 V/div (~ 5 A/cm); (a) Loss occurs at 2 msec after injection. (b) Toroidal field is increased by approximately 10% at 0.5 msec after E-layer formation. Initiation of loss delayed to 2.5 msec.

occurs. The differences in the high gas pressure are the fast plasma generation and the fact that the strong hybrid mode probably causes some energy spread in the E-layer electrons. To find out whether the energy spread alone can cure the loss, such energy spread should be generated with other means than a microinstability. A rf wave at 120 MHz with a modest electric field of 100 V/cm, e.g., the E-layer electron gyrofrequency staying in phase with the E-layer electrons for a few revolutions, can cause an energy spread of several tens of keV. Fortunately, the dimensions of the Astron tank allow the excitation of the first TE mode (E_0 , E_r , B_z) at 120 MHz. Due to the presence of the resistors, the Q of the tank is only of the order of 100 so that about 200 kW of rf power must be applied in short pulses of 100 μ sec or less to each new E-layer. We installed the pulsed power in the tank, excited the mode, and plan to try it on the E-layer soon.

In the meantime, however, we discovered that the most interesting regime for E-layer stacking is the regime where the plasma density does not yet exceed the E-layer electron density, e.g., much before the observed loss occurs. Therefore, the finding of the cause and cure of the loss, although of extreme interest, is not needed to proceed with E-layer stacking and buildup. Nevertheless, we do not intend to leave this question unanswered.

Furthermore, the problem which loomed large at the first meeting of the Panel, that is the role of the plasma in E-layer stacking, is now eliminated by the simple observation that there is no need for plasma during the stacking phase. What we discovered is that not only is there no need for plasma during stacking, but the delay of complete neutralization of the E-layer until after the stacking is over can yield a much higher field reversal parameter ($\zeta = \Delta B/B_0$) if saturation in E-layer (ζ) as discovered in the numerical experiment does really exist.

During the last three weeks (January-February 1972), we have observed the trapping in high vacuum (no hydrogen background gas) of large amplitude E-layers up to 34 A/cm in single pulses. The amplitude of the E-layer upon trapping is independent of the pressure. The neutralization time and the time of occurrence of the fast loss depend on the pressure, however. In order to observe the effect of the self-forces of the E-layer, we trap the E-layer in a very shallow mirror which, within the E-layer length, is of the order of 1%, e.g., much less than the self-mirror of the E-layer. On trapping, the self-forces, which govern the E-layer length, are the radial magnetic field and axial electric field. The latter, however, is substantially reduced by the presence of the resistors so that $E_z/\beta B_r \ll 1$. An analytical expression for the

resistor focusing giving the ratio ($E_z/\beta B_r$) in cylindrical geometry was obtained in the summer of 1971.² Then under these conditions the E-layer is axially focused as if it were partially neutralized with an effective degree of neutralization (η_0)

$$\eta_0 = 1 - \frac{E_z}{\beta B_r}, \quad (1)$$

where the ratio ($E_z/\beta B_r$) is calculated for several resistor configurations in Ref. 3.

When the E-layer is then actually neutralized by the ions, η_0 increases to unity as the axial electric field vanishes. As the E-layer is neutralized, its self-forces increase slowly, causing an axial compression of the E-layer. Since the neutralization takes place in tens or even hundreds of microseconds, e.g., a time much longer than the frequency of the axial oscillations of the E-layer, the compression should be adiabatic. Then Liouville's theorem applies and

$$v_z L = \text{const.} \quad (2)$$

From the conservation of the canonical angular momentum we find, for $v_z^2/c^2 \ll 1$,

$$\frac{v_z^2}{c^2} = 2\alpha\zeta\eta_0 \quad (3)$$

where α is a geometric factor of the order of one-half. This is a relation for axial force balance

Table 1. Observed parameters of five E-layers.

E-layer	Injection Parameters					
	J_1 (A/cm)	J_2 (A/cm)	L_1 (cm)	L_2 (cm)	Momentum (MeV/c)	Beam current (A)
1	14.5	20	58	29	3.6	270
2	27.6	34.5	68	40	6.2	480
3	24.4	26.8	72	47	6.2	480
4	25.4	28.5	86	49	to	to
5	22.0	22.8	96	67	6.5	520

in equilibrium. Hence, it follows that the adiabatic invariant is

$$\epsilon L^2 \eta_0. \quad (4)$$

Since ϵ is proportional to J , the peak E-layer current per cm, the invariant can be written $JL^2\eta_0$. If J_1 and L_1 are the peak current per cm and length of the E-layer, respectively, upon trapping, and J_2 , L_2 after neutralization ($\eta_0 = 1$), then

$$L_2 = L_1 \left(\frac{\eta_0 J_1}{J_2} \right)^{1/2}. \quad (5)$$

The most important result of the last three weeks is that we indeed observe the E-layer to be adiabatically compressed. Table 1 lists the observed parameters of five E-layers. In the last experimental run the E-layers are trapped 5 m from the inflector where the resistor geometry yields a higher value (η_0), hence better focusing. The value of η_0 as a function of L_1 is shown in Fig. 3. Then with the measured values of L_1 , J_1 , J_2

and the computed value of η_0 , the length L_2 is calculated from Eq. (5) and plotted in Fig. 4. This figure also shows the actual measured values of L_2 . We observe that the deviation from the theoretical value is only a few percent, which is within the error of observation. Hence, we may conclude that the E-layer undergoes adiabatic compression during its neutralization phase.

Fig. 3. The value of η_0 as a function of length L_1 .

Approximately 20-30% of the electrons are gradually lost during the compression phase due to the presence of a 150-G toroidal field required temporarily because the cantilever is not concentric with the tanks. The increase of the E-layer thickness during the compression phase is approximately

$$\frac{\delta_2 - \delta_1}{2R} = \frac{v_{z2} - v_{z1}}{c} \frac{B_0}{B_z} \quad (6)$$

With the observed E-layer parameters, the change of thickness should be approximately 3 cm. If the electron velocity distribution remained completely unchanged during the compression, 15% of the electrons should be lost. (The dis-

tance between inner and outer resistors is 20 cm.) The actual loss is somewhat higher, indicating the presence initially of electrons with large axial velocity.

To determine whether adiabaticity is observed continuously during the compression phase, one E-layer was analyzed at three intermediate points where the length and the current per cm were calculated and plotted in Fig. 5. In Fig. 6, (L/L_1) is plotted as calculated from

Fig. 4. Calculated and experimental values for the length ratio L_2/L_1 . The calculated value for $(L_2/L_1)_{\text{theory}} = (n_0 j_1 / j_2)^{1/2}$. The average error for the five E-layers is $\left| \frac{L_{2\text{th}} - L_{2\text{ex}}}{L_1 - L_2} \right| = 6.5\%$.

Fig. 5. E-layer length (a) and current density (b) compression during neutralization.

Fig. 6. Comparison of experiment and theory for compression during neutralization.

Eq. (5) with η_0 appropriately corrected for neutralization (assumed to occur linearly with time in 100 μsec). Then the actually measured L is shown. We observe again that the deviation from the theoretical line is very small. The conjecture that electrons with large axial velocity are initially present is backed up by the rate of change of total current as a function of time which is shown for E-layer No. 1 in Fig. 7. We observe that the loss is fast at the beginning of the compression phase and very slow at the end. We also observe that the loss is continuous and that nothing abrupt occurs during the compression phase. The last docu-

Fig. 7. Circulating current as a function of time for E-layer No. 1.

mentation needed is that the losses occur radially, e.g., that the electrons are lost by intercepting the resistor cans. In Fig. 8, the results of an x-ray scan are shown. The field of view of the x-ray detectors is aimed at a radius of 40 cm with a vertical field of view from 25 to 55 cm. The detector is mounted in a trolley which travels along the Astron solenoid. Figure 8a shows a 10-m scan. The loss appears to be centered at coil 20, which is the center of the trap. We observe that the losses are occurring in the region of the E-layer. In Fig. 8b, the scan is limited to ± 120 cm about the peak of the E-layer. In Fig. 8, the negative signals should be discarded. We observe that the x-ray intensity follows the E-layer profile but is somewhat accentuated since more losses occur when electrons pass through the midplane.

Thus, we believe that the losses occurring during the compression by neutralization are an accidental consequence of the larger than usual toroidal field needed to overcome the present misalignment of the tank. Later, the mysterious losses, possibly due to plasma,

occur. The entire sequence is shown in Figs. 9-13. The measured field of the E-layer as a function of time at its peak value (coil 20) and at ± 25 cm (coils 19 and 21) away from the peak are shown in Figs. 9 and 10. The decay of the signals to a constant value in

(a)

(b)

Fig. 8. X-ray scans of radial E-layer loss. The amplitude of the signal is proportional to the integral of the x rays seen by the PM tube over 100 μ sec during the compression phase. (a) Scan along 10 m. (b) Scan of ± 120 cm about the E-layer peak.

Fig. 9. Measured field of the E-layer as a function of time at its peak value (coil 20).

Fig. 10. Measured field of the E-layer as a function of time at ± 25 cm from its peak value (coils 19 and 21).

Fig. 11. Measured field distribution of the E-layer at $t = 0$ and $100 \mu\text{sec}$.

Fig. 13. Actual current distributions of the E-layer for $t = 0$ and $120 \mu\text{sec}$.

Fig. 12. Determination of E-layer length and diamagnetic loop calibration from measured magnetic field width. Assumptions: (1) Loops have 62-cm radii, (2) E-layer has 40-cm radius, and (3) $j(z) \propto 1/z$
 $(1 - \cos \frac{2\pi}{L} z)$,
 $0 < z \leq L$
 $= 0 < z' < 0, z > L$.

coils 19 and 21 in $100 \mu\text{sec}$ indicates the completion of neutralization phase. This time is in agreement with the pressure measured at the time, which was

$3.2 \mu\text{Torr}$. At approximately $400 \mu\text{sec}$, we observe the beginning of the mysterious fast loss. Before this fast loss process starts, however, we observe that the amplitude of the E-layer is constant for $300 \mu\text{sec}$. The measured field distribution of the E-layer is shown in Fig. 11 at $t = 0$ and $100 \mu\text{sec}$. Due to the short length of presently achieved E-layers, the field distribution is corrected to yield the actual current per cm and actual length. This is obtained through a Green's function calculation.⁴ The correction curves are shown in Fig. 12. The actual current distributions of the E-layer (which is E-layer No. 2 in Table 1) are shown in Fig. 13 for $t = 0$ and $100 \mu\text{sec}$.

Finally, some preliminary results on stacking two pulses in high vacuum ($5 \mu\text{Torr}$, neutralization time of the first pulse, $\tau_1 \sim 600 \mu\text{sec}$) are shown in Figs. 14, 15, and 16.

In Fig. 14, the measured field of the E-layer at location 20 is

Fig. 14. Two-pulse stacking at high vacuum. The E-layer current density at the center of the E-layer is shown. The vertical scale is 0.2 V/div and the horizontal scale is 0.2 msec/div.

Fig. 15. The measured field distribution at several times before and after injection of a second pulse.

Fig. 16. Current distribution of the E-layer at times given in Fig. 15.

shown as a function of time. In Fig. 15, the measured field distribution is shown for $t = 0$, $t = t_1 - \epsilon$, $t = t_1 + \epsilon$, and $t = t_1 + 300 \mu\text{sec}$, where $t_1 = 1250 \mu\text{sec}$ is the time the second pulse is injected. In Fig. 16, the current distribution of the E-layer is shown at the same times.

CONCLUSIONS

The discovery that the E-layer compresses itself adiabatically during the neutralization phase, provided that neutralization takes place in many tens of microseconds, is of extreme importance. The reason is twofold:

- (1) If the saturation demonstrated in the computer run is real, judicious use of the adiabatic compression can produce an increase of ζ proportional to the square of the injected beam current. In the computer it has been shown that the saturation level increases linearly with the injected current.
- (2) By trapping short E-layers with FWHM of 60 to 70 cm before neutralization, the E-layer can be compressed to an almost circular cross section following neutralization. Thus, the E layer can do in part what was expected to be done with the toroidal compression. Then the toroidal field can further compress the E-layer to increase both ζ and its current density.

The results of two computer runs, assuming 10 mTorr of gas with injection current of 600 A and 1800 A, respectively, are shown in Table 2. Whether the saturation observed in the computer, e.g., that the achievable ζ is approximately three times the ζ of the

first pulse, will occur in the experiment is not known yet because in the computer run, the E-layers are much longer than those observed experimentally. In the computer, the trapping occurs in 10 mTorr of hydrogen gas, while in the experiment the trapping occurs either in high vacuum or in the presence of a hydrogen background at pressure of a fraction of a mTorr. The mirror shape is different, etc.

Assuming, however, that a saturation level exists, we can show that the use of adiabatic compression of the E-layer can increase the ζ quadratically with increase of injection current. A more accurate calculation of the adiabatic invariant (valid from any value of v_z/c) yields

$$\zeta = \frac{\zeta_0}{\eta_0 + \frac{\alpha}{2} \zeta_0}, \quad (7)$$

where ζ_0 and η_0 are the E-layer strength and degree of neutralization, respectively, at the end of the stacking phase, and ζ is the final value following complete neutralization. The forces of mutual interaction during electron injection and formation of the E-layer are proportional to $(\eta_0 I_{inj})$. Therefore, if the injection current is increased and η_0 decreased so that $\eta_0 I_{inj} = \text{const}$, it is expected that the initial length of the E-layer will not change. Hence, ζ_0 will increase linearly with the injected current. At an injection current of 500 A, the ζ_0 achieved experimentally is

Table 2. Computer results.

	Injection current	
	600 A	1800 A
ζ at the end of first pulse	0.15	0.57
ζ at the end of fifth pulse	0.42	1.20

0.066. Hence, it is expected that at higher current

$$\zeta_0 = 0.066 \left(\frac{I_{inj}}{500} \right). \quad (8)$$

Since $n_0 I_{inj}$ should be kept constant and $n_0 = 0.4$ in present experiments,

$$n_0 = 0.4 \left(\frac{500}{I_{inj}} \right). \quad (9)$$

Equations (7), (8), and (9) yield

$$\zeta = \frac{0.165 \left(\frac{I_{inj}}{500} \right)^2}{1 + 0.082\alpha \left(\frac{I_{inj}}{500} \right)^2}. \quad (10)$$

The accelerator presently yields up to 700 A. Due to some perturbation in the transport system we presently inject only 500 A in the Astron. When the transport system is restored, we plan to inject 700 A into the Astron tank.

In Table 3, the expected values of ζ_0 and ζ_1 are listed before and after neutralization of one pulse for a range of currents from 500 to 1500 A and $\alpha = 0.4$. The total

Table 3. Compression during neutralization for several injected currents ($\alpha = 0.4$).

I_{inj} (A)	n_0	ζ_{01}	ζ_1	I_c (A)
500	0.40	0.066 ^a	0.09 ^a	2000
700	0.28	0.092	0.31	2800
1000	0.20	0.132	0.58	4000
1200	0.17	0.158	0.80	4800
1500	0.14	0.185	1.02	5600

^a Achieved experimentally.

circulating current is shown in the last column. If we assume that saturation will occur in the experiment at a level three times the level of the first pulse, the values shown in the table will increase by a factor of three, provided that the degree of neutralization will change by a small amount during stacking. In stacking (n) pulses of equal amplitude, the degree of ionization (n_1) upon trapping of the n^{th} pulse is

$$n_1 = \frac{(n-1)}{2} \frac{\tau_1}{\tau_n}, \quad (11)$$

where $\tau_1 = 1$ msec is the time between injection of successive pulses and τ_n is the neutralization time of one pulse. τ_n equals 3.2 msec at a base pressure (air background) p of 0.1 μ Torr. The presently achieved vacuum in the Astron is 0.3 μ Torr. The vacuum must be improved by another factor of six to reach a base pressure of 0.05 μ Torr. This will allow us to stack several pulses and maintain a small degree of neutralization during stacking, and thus take full advantage of the self-compression of the E-layer following the stacking phase. At this pressure, neutralization time is 6 msec. Complete neutralization will occur following the injection of 13 pulses.

BUILDUP OF THE E-LAYER IN HIGH VACUUM

Since it has been demonstrated that the E-layer can compress itself

adiabatically, we may conclude that it can be compressed by external fields or even transferred from one trap location to another. The idea is to trap a large pulse at a region where n_0 is high (like coil 20, Fig. 17) and then (before it is neutralized), transfer the E-layer to its permanent trap (at coil 40, Fig. 17) by a time-varying magnetic field. An example of a magnetic field profile of most recent traps is shown in Fig. 17. The E-layer is trapped in coil 20 as shown in the figure. The mirror just adjacent to the E-layer is an independently adjustable hump located over the coils of the plasma sweeper. This hump can be eliminated by properly phasing the 3200-Hz ac mag-

netic field of the sweeper coils and allowing the E-layer to slide down towards its final trap at coil 40, approximately 20 μ sec after formation. This scheme is advantageous as long as the E-layer trapped at coil 20 is strong enough so that the radial component of its self-magnetic field is much larger than that of the external mirror field it has to go over. If such is the case, the E-layer will move slowly towards the trap in coil 40 without additional blow-off losses. At the present level of trapping (30 A/cm), however, the E-layer strength appears to be marginal for this operation.

By improving the vacuum and the injection current, it appears now

Fig. 17. Magnetic field trap for the E-layer. Inset shows current distribution of trapped E-layer.

that E-layer buildup to field reversal can be achieved in a few milliseconds, while the initiation of the abrupt loss can be delayed more than 10 msec following neutralization. During this interval, the E-layer can be compressed by the poloidal or toroidal field, or both, to enhance both the degree of field reversal and its current

density. If the energy spread introduced by the 120 MHz wave does not cure the abrupt loss, then a plasma density equal to 100 times the E-layer density should be introduced in the E-layer in less than 1 usec. More plasma through gas ionization can be added depending on the plasma parameters desired for each particular experiment.

2. Axial Compression of the Astron E-Layer During Neutralization*

N. C. Christofilos and T. K. Fowler

INTRODUCTION

This chapter shows that, despite processes that tend to elongate the Astron E-layer as it is built up by injecting successive pulses of electrons, in a good vacuum this tendency is overcome by the slow turn-on of focusing forces that compress the E-layer as it slowly neutralizes.

DISCUSSION

The objective of the Astron experiment is to establish a strong circulating current of relativistic electrons (the E-layer) to be utilized in confining a plasma.³ Recently it was observed that an E-layer established in a good vacuum is compressed in length as its space charge is slowly neutralized. This chapter discusses

the importance of this effect in building up the E-layer by injecting a series of electron pulses stacked one upon the other.¹

The E-layer electrons are constrained to roughly circular orbits by an external magnetic field and produce a solenoidal current distribution (Fig. 18). The E-layer is held together axially by the attractive force of adjacent current loops. We write the axial force balance as

$$\begin{aligned} T &= e \int_0^{L/2} dz \left[\frac{v_0}{c} B_r - E_z(1 - \eta_1) \right] \\ &= e n g(L)I, \end{aligned} \quad (12)$$

where we have neglected the weak external mirror and

$$n = 1 - \frac{\int_0^{L/2} dz E_z(1 - \eta_1)}{\int_0^{L/2} dz \frac{v_0}{c} B_r} \quad (13)$$

*Originally published in Phys. Fluids, Res. Note 16, 713 (1973).

and

$$g(L) = \int_0^{L/2} dz \frac{v_0}{c} \frac{B_r}{I} \quad (14)$$

Here, $T = \frac{1}{2} m \bar{v}_z^2$ is the mean axial energy of E-layer electrons; L is the length; I is the current; E_z and B_r are fields due to the E-layer charge and current; and η_1 is the ratio of the ion density to the electron density.

In a good vacuum, before neutralization, the repulsive space-charge force would overcome the magnetic attraction, except for the difference in location of the images of the E-layer charge and current. In steady state, the positive electrostatic images lie in the closely spaced resistor wires at $r = R_{wr}$ and R_{cr} (Fig. 18), thereby weakening the defocusing field E_z . The current images lying in the aluminum walls behind the resistors similarly weaken the B_r field, but to a lesser extent, since the walls are farther away. The effect is that of a partial neutralization to an extent

$$\eta_0 = 1 - \frac{\int_0^{L/2} dz E_z}{\int_0^{L/2} dz \frac{v_0}{c} B_r} \quad (15)$$

which is η in Eq. (13) with $\eta_1 = 0$; η_0 is positive and its magnitude depends on the resistor geometry.*

The axial compression is a consequence of the increase in the axial focusing force during the slow

accumulation of ions formed by collisions between E-layer electrons and the residual gas (the slow electrons are expelled up to $\eta_1 + 1$). Because neutralization occurs slowly, we assume that the equilibrium force balance is approximately preserved during compression and that changes in equilibrium parameters occur adiabatically. If we neglect coupling between r and z oscillations, the relevant constraint is the average axial adiabatic invariant for trapped electrons,*

$$\bar{J} = \int dz p_z = (m \eta T L^2)^{1/2} \quad (16)$$

Introducing T from Eqs. (12)-(14) and setting $\bar{J}$ equal to its initial value gives the change of L as η increases during neutralization. For a long E-layer, for example, $g(L) \propto L^{-1}$ so that, if no current is lost in the process, a single pulse trapped in a good vacuum would contract in length by a factor η_0^{-1} as η increases from its initial value of η_0 to $\eta = 1$ when neutralization is complete.

To apply these ideas to E-layer buildup, it is necessary to determine $\bar{J}$ after the addition of a new pulse. An electron pulse injected at one end of the tank executes coherent axial oscillations in the trap. These oscillations are damped by eddy currents induced in the resistive wires shown in Fig. 18.

*The adiabatic invariants are derived, with and without an external toroidal field, by M. E. Rensink in Ref. 4.

*Refer to Ch. 3.

Ideally, this damping extracts axial energy from the electrons until $J \rightarrow 0$ and the electrons are well trapped. However, once their velocities approach those of electrons already trapped, resonant Landau damping with the trapped electrons takes over. This is inevitable as resistive damping takes its course unless some other process washes out the coherent oscillations first. In the latter case, the average J of already trapped electrons is little changed, but the newly trapped electrons have a higher value, so that the new average is higher. Let the new electrons, with current $\delta I \ll I$, have $J = (1 + f)\bar{J}$. Then the average J is increased by an amount

$$\delta J = \frac{\delta I (1 + f)\bar{J} + I\bar{J}}{I + \delta I} - \bar{J}$$

and

$$\delta J = f \frac{\delta I}{I} \bar{J}. \quad (17)$$

Conversely, if resistive damping persists until Landau damping on

the trapped electrons occurs, the new electrons continue to cool axially, but, in so doing, they nonadiabatically heat the trapped electrons, whose J -value increases. Let the nonadiabatic transfer of axial energy be fT per new electron, corresponding to $(\delta I/I) fT$ per trapped electron on the average. Again, the increase in average J is given by Eq. (17).⁵

Using this model, one can calculate the new $\bar{J}$ in terms of the interaction parameter f by integrating Eq. (17).⁵ For the simple case of constant f ,

$$\bar{J} = \bar{J}_0 \left(\frac{I}{I_0} \right)^f, \quad (18)$$

where the subscript (0) denotes the value for the first pulse. Squaring and substituting $\bar{J}$ and T from Eqs. (18), (16), and (12) gives, after some algebra,

$$I g(L) = I_0 g(L_0) \frac{\eta}{\eta_0} \left(\frac{I_0}{I} \right)^{2(1-f)}. \quad (19)$$

For a long E layer, $g(L) \propto L^{-1}$, whereby

$$\frac{I}{L} = \frac{I_0}{L_0} \frac{\eta}{\eta_0} \left(\frac{I_0}{I} \right)^{2(1-f)} \quad (20)$$

and

$$\frac{L}{L_0} = \frac{\eta_0}{\eta} \left(\frac{I}{I_0} \right)^{2f-1}. \quad (21)$$

The parameter f actually varies from pulse to pulse. According to computer simulation studies,⁶ f is less than unity for the first two or three pulses but then tends to

Fig. 18. Cross section of E-layer.

approach unity. We expect $f \sim 1$ to be a rough upper limit by the following argument. Since the existing E-layer has little influence outside its own dimensions, each injected pulse should behave the same (its axial excursion being steadily damped by the resistors) until the excursions damp to the order of E-layer dimensions. At that point, the axial energy of the new electrons is about that of the most energetic electrons already trapped, that is, typically about $2T$, or twice the average, corresponding to $J \sim 2\bar{J}$ ($J \propto \omega L^2 \propto T/\omega$). Thus, if some competing process stops resistive damping at that point, the newly trapped electrons would have $J \sim 2\bar{J}$, or $1 + f \sim 2$ and $f \sim 1$. If, instead, Landau damping on the trapped electrons sets in, the non-adiabatic energy transfer, as the new electrons with energy $2T$ con-

tinue to cool to about the mean energy T , is at most $\sim T$. Again, $f \leq 1$. Berk and Pearlstein have calculated f by Landau damping for simple models.⁵ An experimental determination of f is in progress.

At high pressures where $\eta_0 = \eta = 1$, the model indicates saturation of the buildup if $f \rightarrow 1$, as reported in Ref. 5; the length merely increases in proportion to the current added with no increase in I/L . However, for trapping in a vacuum, the model predicts buildup of I/L at least by a factor η_0^{-1} , even if $f \rightarrow 1$. Predictions of I/L from Eq. (20) for various values of f and for values of η_0 and other parameters that can be achieved in the Astron experiment are given in Table 4. In the table, N is the number of pulses injected; p is the tank pressure such that neutralization would be completed ($\eta_1 = 1$

Table 4. Predicted results of injecting and stacking N pulses in the Astron experiment prior to neutralization of the E-layer: $\zeta = (4\pi I/B_0 L)$, where B_0 is the external magnetic field and p is the maximum allowed tank pressure (air) for each case. In the first case, the injection current is 600 A for 100 msec and, in the second case, 1200 A for 100 msec.

f	N	p (10^{-7} Torr)	I (10^4 A)	$\frac{I}{L}$ (A/cm)	ζ
Case 1 - $I_0 = 2500$ A, $L_0 = 60$ cm, $\eta_0 = 0.25$					
1.0	4	1	1	167	0.42
0.9	6	1.2	1.5	250	0.60
0.8	10	0.6	2.5	417	1.10
Case 2 - $I_0 = 5000$ A, $L_0 = 60$ cm, $\eta_0 = 0.14$					
1.0	7	1	3.5	583	1.46
0.9	11	0.6	5.5	916	2.29
0.8	26	2.3	13	2160	5.40

and $n \approx 1$) after N pulses; and it is assumed that each pulse adds a current I_0 so that $I = NI_0$. The tabulated quantity ζ is $\zeta = \frac{4\pi}{B_0} \frac{I}{L}$, which must exceed unity to achieve the "field reversal" condition required for plasma containment. One interesting feature of the model is the possibility of buildup at constant length by programming the pressure, and hence

n , so as to maintain L constant according to Eq. (21).

Thus we conclude that, despite processes that tend to elongate the E-layer during pulse stacking, in a good vacuum this tendency is overcome by the slow turn-on of focusing forces (due to progressive neutralization) that compress the E-layer as it neutralizes, and I/L should build up as desired.

3. Resistor Focusing of the Injected Electrons in the Astron*

N. C. Christofilos, A. B. Langdon, G. D. Porter, and M. E. Rensink

In this chapter, we calculate the net axial focusing force for a non-neutral cylindrical E-layer in various Astron resistor configurations. There is strong focusing for cases in which there are resistors on the cantilever.

Several years ago Christofilos⁷ showed that the presence of resistor wires between the walls of the evacuated chamber and the E-layer electron bunch provides axial focusing, thus removing the electrostatic defocusing which tends to expand and spread the electron bunch. At that time, Christofilos⁷ solved the problem in the approximation of plane geometry where the electrostatic force (Fig. 19) is

$$eE_z = eB_x/\beta, \quad (22)$$

where $\beta = (\gamma^2 - 1)^{1/2}/\gamma$ and γ is the electron energy expressed in

rest mass units. Therefore, the net defocusing force is

$$e(E_z - \beta B_x) = e(1 - \beta^2)E_z, \quad (23)$$

which is γ^{-2} times smaller than the axial magnetic or electric force.

Fig. 19. Resistor configuration for the plane geometry calculation.

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Fig. 20. Resistor configuration for the cylindrical geometry calculation.

However, the results are significantly different when the calculation is done in cylindrical geometry.⁸ As in plane geometry, it is assumed that the electric field is terminated on the resistors, while the magnetic field terminates at the walls (Fig. 20). In cylindrical geometry, then, the volume occupied by these fields is a rather strong function of the radii of the walls and resistors. Hence the net force on the E-layer depends strongly on these radii.

In this chapter we calculate the ratio of $(\delta B_r/E_z)$ assuming that the E-layer current distribution is proportional to $\sin^2(kz/2)$, where $k = 2\pi/L$ and L is the length of the E-layer. We also assume that the time dependence of the solution for the vector potential A_θ and the electrostatic potential ϕ can be omitted. This assumption is discussed in more detail later. The E-layer or the injected electron bunch is located at radius (R_e) .

Its radial thickness is assumed negligible and the distribution of the E-layer current is

$$j(z) = \frac{1}{2} j_0 [1 - \cos(kz)] \text{ A/m.} \quad (24)$$

The radius of the inner and outer walls of the tank are R_c and R_w , respectively. The inner and outer radius of the resistive wires (Fig. 20) are R_{cr} and R_{wr} , respectively.

Under the above assumptions the solution for the vector potential is²

$$A_\theta(R_e, z) = - \frac{2\pi j_0 R_e}{c} \left[\frac{(R_e^2 - R_w^2)(R_e^2 - R_c^2)}{2R_e^2(R_w^2 - R_c^2)} - \frac{f_1(R_e, R_w)f_1(R_e, R_c)}{f_1(R_w, R_c)} \cos kz \right], \quad (25)$$

where

$$f_n(a, b) = I_n(ka)K_n(kb) - K_n(ka)I_n(kb) \quad (26)$$

and I_n and K_n are modified Bessel functions. Equation (25) is consistent with the boundary conditions that A_θ vanishes at $r = R_c$ and $r = R_w$, B_r is continuous at $r = R_e$, and B_z is discontinuous at $r = R_e$. The jump in B_z is proportional to the linear current density of the E-layer.

The solution for the electrostatic potential ϕ is obtained in a similar fashion. The boundary conditions are that ϕ vanishes at both the inner and outer resistor

radii, and that it be continuous at $r = R_e$. In addition, E_r is discontinuous at $r = R_e$, the change in its value being proportional to the linear charge density of the E-layer. The result is:

$$\phi(R_e, z) = -\frac{2\pi j_0 R_e}{\beta c} \times \left[\frac{\ln(R_e/R_{cr}) \ln(R_e/R_{wr})}{\ln(R_{wr}/R_{cr})} + \frac{f_o(R_e, R_{cr}) f_o(R_e, R_{wr})}{f_o(R_{cr}, R_{wr})} \cos kz \right]. \quad (27)$$

The net force on the E-layer can be investigated by examining the ratio

$$\frac{\beta B_r}{E_z} = \beta^2 \frac{f_l(R_e, R_w)}{f_o(R_e, R_{wr})} \frac{f_l(R_e, R_o)}{f_o(R_e, R_{cr})} \times \frac{f_o(R_{cr}, R_{wr})}{f_l(R_w, R_c)}. \quad (28)$$

If $\beta B_r/E_z > 1$, there is a net focusing force on the E-layer and $\beta B_r/E_z < 1$ implies a defocusing force.

In the absence of walls and for $kR_e = 1$, the ratio of the focusing magnetic force to the axial electrostatic defocusing force is 0.637 indicating strong defocusing.

Although the above solutions are for periodic E-layers, it has been shown² that for $kR_c \approx 1$ or smaller the magnetic flux of the E-layer is practically confined within its length. Therefore, for sufficiently long E-layers, the above solution applies with good approximation to

Fig. 21. Focusing force for several resistor geometries on the Astron machine.

a single E-layer with a $\sin^2(kz/2)$ current distribution.

Equation (28) has been evaluated for several different configurations of the Astron experiment. The results are plotted in Fig. 21 as a function of the FWHM (π/k) of the E-layer. Clearly, there is considerable focusing for both cases in which there are resistors on the cantilever while there is only a very small focusing force with no cantilever resistors.

These results are not always applicable, however, because there is a time lapse of a few nanoseconds until the electrostatic charges reach their full value at the surface of the resistors and a time

lag until the magnetic field of the injected bunch penetrates in the space between the resistors and the wall. Thus, these calculations

overestimate the focusing on the injected electron bunch, while they are quite accurate for an E-layer that has settled down in the trap.

4. E-Layer Trapping in Astron by Resistor Interaction: Theory and Experiment*

B. W. Stallard and T. J. Fessenden

This paper studies theoretically and experimentally the interaction of a moving E-layer with the resistor structures that line the walls of the Livermore Astron apparatus. It is shown that for a weak interaction, no change in mean layer radius results from the energy loss in the resistors. Using the approximation of a long thin layer in axisymmetric geometry, curves are given for calculating the energy dissipated in the resistors per meter traveled or, equivalently, the axial force on the moving layer.

Results of experiments are reported showing transport of the incoming layer in the Astron tank and trapping in the magnetic mirrors. Measurements of the damping of E-layer axial motion in several resistor geometries show typical damping times of 0.5 μsec . Comparison of experiment and theory show agreement within a factor of 2.

INTRODUCTION

In the Astron approach to controlled fusion,⁹ a relativistic electron beam is trapped inside a magnetic mirror to provide a circulating current of sufficient magnitude to reverse the external magnetic field and thus efficiently confine a thermonuclear plasma. The beam is injected into a solenoidal field and allowed to travel down a magnetic ramp to the bottom of a magnetic well. As the electron layer travels down the ramp it loses a small amount of its energy to resistive structures which line the walls of the vacuum vessel. In this way the E-layer is trapped because it has insufficient energy to return to the injector after reflecting from the far wall of the magnetic mirror. The purpose of the paper is to elucidate experimentally and theoretically the interactions of the

*Originally published in Phys. Fluids 17 (4), 575 (1974).

moving E-layer with the resistive structures.

The original analysis of resistor/E-layer interaction is contained in an LLL internal report by Christofilos.⁷ His calculation was performed in cartesian coordinates by assuming the Larmor radius of the E-layer electrons could be considered infinite and the resistive structure was contained on one side of the layer. The E-layer consisted of a large number of line currents moving perpendicular to the direction of their currents across a resistor structure. Nebenzahl¹⁰ at Cornell University has considered the interaction of a rotating electron beam passing through a loop of resistive material. He showed that the interaction slows the beam motion through the loop but does not change the rotational energy of the beam.

The analysis presented in this paper considers the weak interaction of a circular E-layer with resistive structures both inside and outside the layer.

THE RESISTIVE STRUCTURE

The geometry of Astron and the location of the resistive structures is shown in Fig. 22. The resistors consist of many fine wires oriented in the azimuthal (θ) direction. The structures have surface resistances of 5 to 20 Ω /square. Currents are induced in the resistors by the magnetic fields of the

Fig. 22. Cross section perpendicular to the axis of the Astron experimental chamber showing placement of the resistive wires.

moving E-layer and, as will be shown, the resistive loss in the wires causes a reduction of the axial E-layer velocity. A qualitative understanding of this interaction can be obtained from Fig. 23. Here a cross section of Astron is shown with an E-layer moving along the axis. The axial (z) velocities of the layer are approximately $c/10$. The resistive structures initially inhibit the penetration of the magnetic field of the E-layer into the volume adjacent to the walls and cantilever. As the currents induced in the wires relax, the fields penetrate as shown. The field lines cutting the resistors as the layer

Fig. 23. Schematic of the moving E-layer showing diffusion of the magnetic field lines through the resistive layer.

approaches and again as the layer leaves cause a drag on the layer which slows the motion of the layer along the axis.

We next show that the interaction of the E-layer with the resistor structure to lowest order causes no change in the E-layer radius.

FORCE ANALYSIS

In the Livermore Astron device, the interaction of the E-layer with the resistor structure is rather weak and the E-layer travels many times its length before stopping due to resistor action. The layer falls into a magnetic well and oscillates a few times before being trapped in the bottom of the well due to the resistor interaction or other loss processes. Typical mirror ratios are 1.05 to 1.15.¹ Furthermore, in the experiment the layer may be only partially neutralized during the trapping processes. However, for purposes

of analysis, we shall make the following simplifying assumptions. We will consider a layer moving with constant speed in the z -direction in a uniform magnetic field. Furthermore, we shall assume that the layer is charge neutral and that no electrostatic fields are present. Thus, the layer is completely characterized by a magnetic vector potential

$$\vec{A} = A(r, z - v_z t) \hat{z} \quad (29)$$

From the Lorentz force law, the net forces in the θ and z directions are

$$F_\theta = e[E_\theta + v_z B_r],$$

and

$$F_z = -ev_\theta B_r. \quad (30)$$

Since the induction electric field and the magnetic field are related to the vector potential $\vec{A}$ according to

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t}, \quad \vec{B} = \nabla \times \vec{A}, \quad (31)$$

we find by direct substitution

$$F_\theta = 0; \quad F_z = e \frac{v_\theta}{v_z} E_\theta. \quad (32)$$

Consequently, the only unbalanced force generated by the resistors is in the axial direction. Using Newton's equations, we find the well-known relation

$$v_z \frac{d}{dt} \gamma m v_z = ev_\theta E_\theta,$$

where γm is the relativistic mass. For weak interactions, $\gamma = \text{constant}$ and defining $U_z \equiv \frac{1}{2} \gamma m v_z^2$,

$$\frac{dU_z}{dt} \approx e \vec{v} \cdot \vec{E}. \quad (33)$$

Thus, even though the resistors produce an electric field in the θ direction, because of the θ symmetry this results in an unbalanced force only in the z -direction which tends to slow the E-layer with no change in E-layer radius.

METHOD OF ANALYSIS

We assume that the interaction of the E-layer with the resistor structure is weak. That is, for purposes of analysis, we assume that somehow work is done on the system to maintain v_z constant. We then calculate how much work must be done and attribute that to resistor loss. Furthermore, we assume that the current of the E-layer is given by

$$K_e(r, z, t) = K_0 \sin^2[\pi(z - v_z t)/L] \times \delta(r - r_e), \quad 0 \leq \arg \leq \pi \\ = 0 \quad \text{otherwise.} \quad (34)$$

Here v_z is the velocity of the E-layer in the z -direction and L is the length of the layer. Experimental observation of the moving E-layer shows this to be a highly reasonable assumption.

The problem is further simplified by assuming that the layer is

Fig. 24. Sketch of Astron showing notational identification.

long compared to the separation between the wall and the cantilever. In this limit the magnetic fields generated by the moving E-layer are independent of r . This approximation has been examined by Porter and Weiss.² Referring to Fig. 24 for the geometry, this yields a linear system of six equations in six unknowns as follows:

1. Flux conservation,

$$a_1 H_1 + a_2 H_2 + a_3 H_3 + a_4 H_4 = 0, \quad (35)$$

2. Boundary conditions,

$$h_1 - H_2 = K_1, \\ H_2 - H_3 = K_e, \\ H_4 - H_3 = K_2, \quad (36)$$

3. And from Maxwell's first equation

$$K_1 = -\tau_1 \frac{\partial H_1}{\partial t}, \quad K_2 = \tau_2 \frac{\partial H_2}{\partial t}. \quad (37)$$

Here a_1 denotes the cross-sectional area in which the magnetic induction is H_1 ; $\tau_{1,2}$ is the L/R time constant of the outer, inner resistors; K_e is the E-layer current density, and $K_{1,2}$ are the surface currents flowing in the outer, inner resistors.

The power dissipated per unit length in these resistors is

$$P_{d1}(t,z) = 2\pi r_1 R_1 K_1^2(t,z), \quad (38)$$

$$P_{d2}(t,z) = 2\pi r_2 R_2 K_2^2(t,z). \quad (39)$$

Here $r_{1,2}$ is the outer, inner resistor radius and $R_{1,2}$ is the outer, inner ohms/square of the resistors. It is convenient to define a parameter $x = (z - v_z t)/v_z \tau_1$. One finds the energy dissipated per unit length as

$$\begin{aligned} E_{d1} &= 2\pi r_1 R_1 \tau_1 \int_{-\infty}^{\infty} K_1^2(x) dx \\ &= \mu_0 a_1 \int_{-\infty}^{\infty} K_1^2(x) dx \end{aligned} \quad (40)$$

and

$$\begin{aligned} E_{d2} &= 2\pi r_2 R_2 \tau_2 / \alpha \int_{-\infty}^{\infty} K_2^2(x) dx \\ &= \mu_0 a_4 / \alpha \int_{-\infty}^{\infty} K_2^2(x) dx. \end{aligned} \quad (41)$$

Here α is τ_2/τ_1 . The sum of these energies can be interpreted as the total axial force impressed on the moving E-layer by the resistors.

Equations (35)-(37) are simultaneously solved for the currents K_1 and K_2 assuming the sine squared E-layer distribution of Eq. (34). These are used to evaluate the integrals of Eqs. (40) and (41). In Appendix A, the formulas for E_{d1} and E_{d2} are given. Some results of these calculations plotted with the aid of the "on line" University of California, Santa Barbara

Computer facility are given in Figs. 25 to 27. For these figures $r_1 = 0.5$ M, $r_2 = 0.3$ M, and $r_e = 0.4$ M.

Figure 25 shows the energy dissipated per meter traveled by the E-layer in the wall and cantilever resistors as well as the total energy dissipated as a function of the normalized parameter τ_p/τ_1 for a peak E-layer strength of 1 A/m. The pulse width τ_p is the total width given $\tau_p = L/v_z$ and is not the FWHM value, which for a sine squared distribution would be smaller by one-half. This curve shows that the maximum interaction occurs when the FWHM value of the pulse width is equal to the outer wall time constant. These curves were calculated for a ratio of

Fig. 25. Wall and cantilever resistors. Variation of loss with normalized pulse width τ_p/τ_1 for $\alpha = \tau_2/\tau_1 = 0.3$. For these data $r_1 = 0.5$ m, $r_2 = 0.3$ m, and $r_e = 0.4$ m.

cantilever resistor time constant to outer wall time constant of 0.3. Figure 26 shows the distribution

Fig. 26. Wall and cantilever resistors. Variation of loss with normalized cantilever resistor time constant α for $\tau_p/\tau_1 = 2$. For these data $r_1 = 0.5$ m, $r_2 = 0.3$ m, and $r_e = 0.4$ m.

of energy loss per meter in the inner and outer resistors as a function of the ratio of cantilever to wall resistor time constant for τ_p/τ_1 equal to two. Here a maximum interaction is observed at $\tau_2/\tau_1 = 0.3$. Figure 27 shows the energy dissipated in the outer resistors per meter traveled as a function of the normalized pulse width for various outer resistor radii. The results of Fig. 27 were obtained assuming no resistor structures on the cantilever. As can be seen, the coupling of the layer to the resistors is a strong function of the resistor radius.

EXPERIMENT

Measurements of the injection and trapping processes for an E-layer were made with fast-response magnetic probes which have a rise-time of 10 nsec in response to a step magnetic field.¹¹ The probes consist of four turns of wire having an area of about 10 cm^2 which are enclosed in a shield made of 0.005-cm brass. The shield serves two purposes: it eliminates pickup of strong rf fields that occur during injection, and four times less than the shield time constant of 6 μsec , it integrates the field inside the loop so that the output voltage is proportional to the local magnetic field. The probes are shown in Fig. 22 and were located behind the resistor wires and spaced at 24.4-cm or

Fig. 27. Wall resistors only. Total energy loss as function of normalized pulse width for E-layer at 0.40 m.

48.8-cm intervals along the axis of the experimental chamber.

During the injection and trapping of a pulse of electrons, probe measurements of the magnetic field of the moving layer were recorded simultaneously at up to 20 axial locations on photographic film. The recorded signals were then digitized at 10-nsec intervals for computer analysis. The result is an experimental determination of $H_1(t, z)$.

Measurements of E-layer trapping in several resistor geometries are reported here. The resistor geometry was changed by varying the resistor radii while keeping the cantilever and wall radii fixed. For each resistor geometry the surface resistance was chosen to maximize energy loss at the pulse width of the injected beam. The experiments were conducted under vacuum conditions ($P \approx 10^{-6}$ Torr) and in hydrogen gas. For vacuum conditions the beam was unneutralized during trapping. Most of the experiments in gas were at pressures of 15 mTorr or greater. At these pressures plasma densities greater than the beam density are created during trapping.

The measurements were analyzed to determine if resistor damping plays a significant role in the trapping process. The results of one trapping experiment at relatively low injected beam current are presented in detail to illustrate the trapping phenomena. For this experiment the injected elec-

trons behave as a single bunch. This is not always the case, particularly at higher injected currents where the loops indicate the presence of more than one bunch. Multiple bunches are probably due to E-layer tearing from self-forces.¹² This occurs when the axial pressure of the layer and the self-forces are not in balance.

SINGLE BUNCH TRAPPING

For detailed comparison of the measured loss with theory, the results are presented of an experiment with a 200-A beam of 5.8-MeV energy injected into 17-mTorr hydrogen. The E-layer is trapped in a 2% magnetic well. At 17 mTorr, neutralization of the E-layer occurs in about 0.25 μ sec. Since the trapping transient lasts for more than 1 μ sec, the E-layer is neutral during most of the trapping process. For this experiment there were no resistors on the cantilever so that the magnetic probe measurements yielded the total energy loss.

A digital computer was used to solve Eqs. (35)-(37) with $K_2 = 0$ for the fields and currents as a function of time and position using the experimental values of $H_1(t, z)$ as input. The computer was used to construct plots of layer current as a function of axial length at time increments of 10 nsec. By viewing the time sequence of plots as a motion picture, the motion of the layer as it moves into the trap can be carefully studied.

The computer was also used to calculate the total power dissipated in the resistors as a function of time. This was done by calculating $K_1(t, N)$ from measurements of $H_1(t, N)$ using the techniques described above and summing Eq. (38) over all N probe locations. The computer numerically evaluates

$$P_{d1}(t) = 2\pi r_1 R_1 D \sum_{n=1}^N K_1^2(t, N), \quad (42)$$

where D is the probe spacing. Thus, the total power dissipated in the resistors as a function of time is experimentally determined.

Figure 28 shows the position of peak E-layer current density as a function of time as the E-layer oscillates in the magnetic well. The damped motion shows the loss of axial energy.

Figure 29 shows E-layer charge as a function of time showing that about one-half of the charge entering the trap region is trapped. The high frequency fluctuations in the curve are not real since the total charge in the machine cannot increase after injection has ceased. The dotted curve is probably a reasonable approximation of the true charge variation. It is drawn to take into account the 6- μ sec droop of the loop signals.

Figure 30 shows the energy dissipated in the resistors calculated from the measured fields as a percent of total injected beam energy (168 J). The "knees" in the curve correspond to the turn-

around points of the layer in the mirrors when the rate of dissipation becomes very small.

Figure 31 shows the measured E-layer and resistor current as the layer passes the bottom of the magnetic well at time $t = 270$ nsec. It is seen that a sine-squared distribution of current is a reasonable approximation to the experiment.

Fig. 28. Position of peak E-layer current density versus time.

Fig. 29. Total E-layer charge as a function of time during trapping.

The small "negative" E-layer current reflects the crudity of the model.

Fig. 30. Energy dissipation in the resistors calculated from the measured magnetic fields.

Fig. 31. E-layer and resistor current density calculated from measured magnetic fields at time $t = 270$ nsec.

COMPARISON BETWEEN THEORY AND EXPERIMENT FOR A SINGLE BUNCH

Because of the large number of processes occurring at injection and the simplification inherent in the model, only an approximate agreement between theory and experiment can be expected. As shown in Fig. 29, nearly one-half the stored charge in the machine is lost in the period of interest. Moreover, at least two distinct instabilities are also occurring during this time. The two instabilities are the negative mass mode for relativistic electrons¹³ and a beam-plasma instability occurring at frequencies near the cold electron upper-hybrid frequency.¹ The negative mass mode occurs both in vacuum and gas for these experiments. The hybrid mode requires the presence of gas for creation of a plasma. The various processes occurring at injection and their effect on the system are not fully understood. Thus, we have chosen the somewhat empirical procedure described below to obtain some comparison of experiment with theory.

A comparison of the experiment and theory is made at the times the layer passes the equilibrium position. At these instants the axial velocity v_z and power dissipation P_{E1} are well defined from the data in Figs. 28 and 30. From the experimental data gathered as previously described, the computer is used to determine the normalized axial velocity β_z , the total charge in the layer Q_T , the peak layer

Table 5. Comparison of resistor loss theory with experiment.

Time (nsec)	β_z	Q_T (μC)	K_e (A/cm)	P_T (MW)	P_{E1} (MW)	P_{E2} (MW)	W_z (J)
275	0.1	14.8	16.0	1.8	1.6		0.47
430	0.057	10.0	9.0	0.32	0.49	1.6	0.1
650	0.065	7.9	7.3	0.25	0.17	-0.13	0.1
780	0.047	7.4	7.8	0.21	0.13	0.37	0.05

current per unit length K_e , and the instantaneous power dissipation P_{E1} . From these values of K_e and β_z a "theoretical" power dissipation is obtained using the curves of Fig. 27 and the simple relation $P_T = c\beta_z E_{d1}$. For the case under study the results of these calculations are given in Table 5. It is not surprising that good agreement is found between P_T and P_{E1} .

A more realistic comparison of theory and experiment was obtained by noting that the damped motion of the E-layer apparent in Fig. 28 can be used to estimate the energy dissipated by the layer resulting from all effects. We assume that the layer can be considered as a super-particle having a mass $\gamma m Q_T/e$. Thus, when the layer passes the equilibrium position it has an energy

$$W_z = \gamma m c^2 Q_T \beta_z^2 / 2e. \quad (43)$$

The last column (W_z) in Table 5 shows the number of joules present in the axial motion of the E-layer at the indicated times as given by Eq. (43). From these numbers the average power dissipated in these time intervals was calculated after

first subtracting the energy convected out of the layer by the escaping electrons.

The convection loss in each time increment was estimated by assuming each escaping particle carried with it the mean drift energy of the layer in the time period. The total energy lost by convection in the time increment is then

$$W_c = (W_{z1} + W_{z1+1})(Q_{T1} - Q_{T1+1}) / (Q_{T1} + Q_{T1+1}). \quad (44)$$

Here the convection loss W_c is obtained between the i th and $i+1$ th crossing of the equilibrium position. The mean power dissipation in the time period is then

$$P_{E2} = (W_{z1} - W_{z1+1} - W_c) / \Delta t. \quad (45)$$

The results of these calculations are given in the P_{E1} column of Table 5. Comparison of P_{E1} and P_{E2} in Table 5 show agreement to within a factor of two.

EQUILIBRIUM TIME FOR E-LAYER TRAPPING IN SEVERAL RESISTOR GEOMETRIES

This section presents measurements of the damping time for 11

E-layer trapping experiments carried out in several resistor geometries. It is meaningful to compare the time to reach equilibrium (damping time) with predictions of the resistor loss theory. For several of the experiments there was evidence of more than one electron bunch.

The equation of motion for a super-particle of total charge Q_T in a parabolic potential well is

$$\frac{d^2 z}{dt^2} + \omega^2 z = eF/\gamma m Q_T \equiv f. \quad (46)$$

The quantity F is the total resistor drag force, $\gamma m f$ is the force per electron, and ω is the oscillation frequency for the super-particle in the well.

An upper and lower bound for the damping time is obtained by solving Eq. (46) for several assumed functional forms for f .

Lower Bound

A lower bound is obtained if f is assumed constant and equal to the maximum value of the total energy dissipation E_d . This is shown as curve A in Fig. 32 for a typical resistor loss curve. For this case, $f = eE_d K_e^2/\gamma m Q_T$. For weak damping, zero initial velocity, and an initial displacement z_0 from equilibrium, the damping time is

$$\tau_{Dm} = \frac{\pi}{2} \frac{z_0 \omega}{f}. \quad (47)$$

Fig. 32. Typical resistor energy loss curve. Curves A and B are the assumed forms for calculation of an upper and lower bound for E-layer damping time.

Upper Bound

The true resistor drag force must approach zero as $v_z \rightarrow 0$. It is easily found from the expression for E_d (see Appendix A) that in the limit $v_z \rightarrow 0$, $E_d \propto 1/\tau_p \sim v_z/L$, where L is the layer length. Consequently, an upper bound is obtained by replacing f by $f \hat{z}/v_0$, where v_0 is the initial velocity of the super-particle. This assumes that at the initial velocity of the bunch, the energy loss rate is maximum. Curve B in Fig. 32 shows the assumed functional form.

The solution of Eq. (46) for this case is exponentially damped motion. Assuming a damping time equal to two e-folding times, the result is

$$\tau_{Dm} = \frac{4v_0}{f} \approx \frac{4z_0 \omega}{f}, \quad (48)$$

where the relation $v_0 = z_0 \omega$ holds for weak damping.

Figure 33 shows a plot of measured damping times and also the

theoretical upper and lower bounds plotted as a function of the parameter $z_0\omega/\langle f \rangle$. All quantities except E_d are measured. E_d is determined from the theory and varies by more than a factor of three for these experiments. The brackets $\langle \rangle$ refer to an average over the time interval from injection to equilibrium. In general, f is not constant during trapping since the peak current density and charge of the layer change because of losses and changes in the length of the layer. The points labeled G were at gas pressures of 15-17 mTorr hydrogen. Point M was at 2.5 mTorr and data point G1 is

Fig. 33. Measured damping times versus the quantity $z_0\omega/\langle f \rangle$ for several resistor geometries.

the experiment described under single bunch trapping. All other data is in vacuum ($<10^{-6}$ Torr).

Most data points fall within the limits predicted by the theory including those experiments with evidence of more than one electron bunch. The resistor drag force is proportional to the current density of an electron bunch. The average for the force γmf takes into account the lower current density resulting from multiple bunches and it is perhaps not surprising that the theory and experiment are in reasonable agreement.

Although the theory for resistor loss presented here assumes a charge neutralized layer, the experimental points in vacuum are in good agreement with theory. This suggests that the effects of the electrostatic field of the layer on the resistors is not an important loss mechanism.

The greatest departure from the theoretical curves is for the experiments at high gas pressure. As mentioned earlier, plasma is created during trapping and one or more instabilities are present which, in some unknown way, might contribute to more rapid damping. Two other effects are present because of the creation of plasma. The electron of an electron-ion pair created by ionization is in the potential (tens of kV) of the negative space charge of the E-layer. It can therefore gain energy from the field and carry away momentum from the layer. The

ion, because of its greater mass, is left behind the layer as it moves along the axis and can exert a drag force. These effects are complicated and have not been evaluated. For the E-layers achieved in the experiments, the electric field potential of the unneutralized layer is comparable to its initial directed axial kinetic energy. Hence, these effects might be significant for trapping.

DISCUSSION AND CONCLUSIONS

In this chapter, we have attempted to give an explanation of how the Astron resistors work along with quantitative measurements of the energy dissipated in the resistors and the resulting behavior of the layer. Toward this end we have only partially succeeded because of the complex phenomena occurring in the first microsecond after injection.

The theoretical calculation of resistor loss agrees well with loop measurements. Moreover, the damping time for E-layer axial motion due to all effects agrees with estimates of the theory within a factor of two. Therefore, we conclude that the damping of the axial motion and trapping of the E-layer in vacuum results from the interaction of the moving layer with the resistor structures. In the presence of gas, additional phenomena take place which may contribute to trapping.

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5. The Electrostatic Precession of Electron Layers

Part 1: Infinitely Long E-Layers*

W. C. Condit, Jr. and R. V. Lovelace

The precessional motion of an unneutralized E-layer resulting from image charges on the walls of the Astron tank is computed. The layer is treated as an infinitely

long, rigid cylinder. Its axis is assumed to be displaced an amount ϵ from the axis of the conducting tank, and the parameter ϵ/R , where R is the E-layer equilibrium radius,

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is assumed to be small. Under the assumption of rigid or coherent precessions (justified by experimental results), only the motion of a single typical particle need be computed, and the full relativistic equations for such a particle are solved to first order in ϵ/R . The results indicate that the electrostatic precessions of the E-layer occur at a much higher frequency than the magnetic precessions that have been observed after the E-layer is neutralized.

Precessional motions of relativistic electron rings (E-layers) in an external magnetic field were predicted¹⁴ and observed¹⁵ several years ago. There are several contributing effects determining the frequency and stability of these motions. The most basic is the single-particle precession in the external magnetic field, which must have a "mirror" shape if the layer is to be confined axially. This leads to a precession frequency in the forward (positive) direction, $\omega_m = -n_x \omega_c / 2$, where $n_x = R[d(\ln B_z)/dr]$ is the field index at the orbit radius, R . This motion is unstable in the presence of dissipation.¹⁴ However, image currents in conducting walls¹⁵ or the addition of a small toroidal field¹ will stabilize the mode for a neutralized E-layer. In the image-current case, there is a contribution to the precession frequency in the negative (stabilizing) sense which is proportional to the E-layer strength parameters, $\zeta = B_{\text{layer}}/B_{\text{mirror}}$.

In this chapter, we compute the precessional motion due to the image charges in conducting walls when the layer is unneutralized, or in the presence of background ions of density n_E . As in the image-current case, a frequency contribution proportional to the strength parameter ζ will be found. The electrostatic term, however, is always positive (destabilizing).

The precessional motion due to the image electric field of the E layer can be calculated in much the same way that the magnetic precessions have been calculated.^{14,15} There is a significant difference in the results. In the magnetic case, $\omega_p = \omega_m - (\zeta/25)(\omega_c)$ to $\omega_m - (\zeta/50)(\omega_c)$ for typical E-layer parameters, whereas in the electric case the coefficient of ζ is both positive and much larger. This arises because the experiments are fairly close to the "long E-layer" limit, where, magnetically, the wall images do not "know" that the layer is off center. In the electrostatic case, however, the outer wall images change when the layer is shifted off center, and while the cantilever does not see a change in the field directly from the E-layer, it does see the image field change, and thus it has image charges induced in it. The net result is that there is a precessional effect for an infinitely long unneutralized E layer in which both outer and inner walls are important, and that is what is computed here.

The equations of motion are:

$$d/dt(\tilde{m}\dot{\phi}) = \tilde{m}r\dot{\phi}^2 - \frac{e}{c} r\dot{\phi}B_z + F_r \quad (49)$$

and

$$d/dt(\tilde{m}r^2\dot{\phi}) = + \frac{e}{c} r\dot{r}B_z + rF_\phi. \quad (50)$$

We have used $q = -e$ for the electron charge.

We shall calculate a nongrowing precession; growth rates will be discussed later. We postulate that the precession is coherent, and we therefore follow a single particle with no z velocity (Berk and Sudan¹⁶ have shown that if the z -motion is fast, one can average over it. During injection, the z -motion is uniform, and it can be ignored). Since particles of all phases perform the same motion, we choose one for which $\phi = 0$ at $t = 0$. Thus, our trial solutions for Eqs. (49) and (50) are, following Furth¹⁴:

$$r = R + \epsilon e^{i(\omega_p - \omega_c)t + i\delta} \\ = R + \epsilon e^{-i\omega_c t} \quad \text{for zero growth, (51)}$$

$$\phi = \omega_c t - i(1 + a)(\epsilon/R)e^{i(\omega_p - \omega_c)t}, \quad (52)$$

and

$$\tilde{m} = \tilde{m}_0(1 + \epsilon b e^{-i\omega_c t} + i\epsilon d e^{-i\omega_c t}). \quad (53)$$

Here $\tilde{m}(t) = \gamma(t)m_e$, where m_e is the rest mass, and $\tilde{m}_0 = \gamma_0 m_e$ is the zero-order (time-averaged) relativistic mass. Also, $\omega = \omega_c - \omega_p$. For zero growth, we find that $d = 0$

and $\delta = 0$. We note that one can write an energy conservation equation:

$$c^2(d\tilde{m}/dt) = R\omega_c F_\phi.$$

The zero-order equation of motion is

$$\omega_c = \frac{e[B_M + B_O(\tau)]}{\tilde{m}_0 c} \\ \times \left(1 + \frac{E_O(\tau)}{B_M + B_O(\tau)} \frac{\gamma_0}{\sqrt{\gamma_0^2 - 1}} \right),$$

where B_M is the externally applied mirror field and B_O , E_O are the axisymmetric image fields. Thus, the zero-order effect of the image fields is to change the cyclotron frequency, and, since $R = \frac{c}{\omega} \sqrt{\gamma^2 - 1}/\gamma$, the radius also changes.

In Appendix B, the off-axis image forces are computed to first order in ϵ in the form $F = \epsilon K\omega_c e^{-i\omega_c t}$, $F_\phi = i\epsilon(\epsilon K\omega_c) e^{-i\omega_c t}$. Since the layer is assumed long, no magnetic image forces are included. The magnetic forces and frequency contributions can be added linearly to the electrostatic and mirror terms.* Inserting Eqs. (51)-(53) in (49) and (50) and keeping terms of order ϵ , we get

$$-\tilde{m}_0(\omega_p - \omega_c)^2 = \tilde{m}_0\omega_c^2(1 + bR) \\ + \tilde{m}_0\omega_c(1 + a)(\omega_p - \omega_c) \\ - \tilde{m}_0\omega_c^2(1 + n_x) + (K\omega_c) \quad (55)$$

*Refer to Ch. 6.

and

or

$$R(\omega_p - \omega_c)\tilde{m}_0\omega_c + \tilde{m}_0R(1 + a)(\omega_p - \omega_c)^2 \\ = - \frac{\alpha(K\omega_c)R}{\gamma^2} \quad (56)$$

$$2\omega_p\omega_c = \frac{\beta^2\alpha K}{\tilde{m}_0}\omega_c - n_x\omega_c^2 + \frac{K\omega_c}{\tilde{m}_0},$$

or

$$\omega_p = \frac{\omega_c}{2} \left[-n_x + \frac{K}{\tilde{m}_0\omega_c} (1 + \alpha) \right]. \quad (59)$$

Energy conservation has been used to eliminate b from Eq. (56). Up to this point we have made no assumptions about the magnitude of ω_p/ω_c . We will now assume $(\omega_p/\omega_c \leq 0(\zeta))$.

To the first order in ω_p/ω_c and $1/\gamma^2$, Eq. (56) yields

$$a = \omega_p/\omega_c - \alpha K/\gamma_0^2\tilde{m}_0\omega_c. \quad (57)$$

Since $\beta = 1$ to a very good approximation, and since Appendix B shows that $K(1 + \alpha)/\tilde{m}_0\omega_c$ is approximately equal to 2.44ζ for the case where the resistors are at 30 cm and 50 cm, we get

$$\frac{\omega_p}{\omega_c} = 1.22\zeta + 0.5n_x. \quad (60)$$

From

$$c^2 \frac{d\tilde{m}}{dt} = R\omega_c F_\phi,$$

we find

$$b = \frac{R(\alpha K\omega_c)}{\tilde{m}_0 c^2}. \quad (58)$$

Then, neglecting terms of order ω_p^2/ω_c^2 , we get from Eq. (55)

$$-\omega_c^2 + 2\omega_p\omega_c = \omega_c^2 + \frac{\omega_c^2 R^2 \alpha K}{\tilde{m}_0 c^2} \omega_c$$

$$- \omega_c^2 - \tilde{m}_0 \omega_c^2 (1 + n_x) + \frac{K}{\tilde{m}_0} \omega_c,$$

For fractional neutralization f , the first term is reduced by $(1 - F)$. This is a rather high frequency for typical ζ in Astron; for $f = 0 - 0.9$, it is in the megacycle region, which is considerably higher than the magnetic precession frequency for long layers.

We have not yet computed growth rates or taken the finite length of the E-layer into account. The growth rates are probably considerably faster than the 50-200 μsec observed for magnetic precessions. The finite-length effects would not change the coefficients in Eq. (60) drastically.

6. The Electrostatic Precession of Electron Layers

Part 2: Finite-Length, Unneutralized E-Layers*

G. D. Porter and B. W. Stallard

The precessional stability of unneutralized, finite-length E-layers is examined both theoretically and experimentally. The theoretical analysis assumes the layer is a rigid cylindrical thin shell. Experimental measurements are consistent with the assumption of rigidity. It is shown that in the absence of a toroidal field, an unneutralized layer is unstable for all amplitudes. This is in contrast with a neutralized layer, which is stable above a critical amplitude. However, the precessional mode is stabilized by the application of a toroidal field. Experimental measurements of the precession frequency show agreement with theory within a factor of two. The discrepancy is attributed to the finite thickness of the layer.

INTRODUCTION

The stability of the Astron E-layer against the precessional mode was first considered by Furth¹⁴ who showed that the presence of resistive walls will drive this mode unstable. Christofilos¹⁵ later showed that, for an electrically neutral layer, the mode is stabilized by the image currents flowing

in the walls of the Astron tank. Thus, he found that there is a minimum E-layer amplitude above which the neutral layer is stable against precessions. The requirement that the E-layer amplitude exceed a certain minimum for stability was removed later, when it was found that the precession mode is stabilized by a small toroidal field.¹ This mode has been examined experimentally on the Astron machine^{1,15} and found to be in good agreement with the theory. The addition of a background plasma was carried out by Berk and Sudan¹⁶ who found that the layer is stable with large plasma densities. Finally, Lovelace and Sudan¹⁷ considered the precession of a strong E-layer and found that the precession frequency is reduced by self-field effects.

With the exception of Ref. 14, each of these calculations assumed that the E-layer is neutralized and hence, there are no self-electric fields. Furth¹⁴ considered the effects of an electrostatic drag force, but did not include the self-electric fields. However, recent experiments on the Astron have been performed at gas pressures which were sufficiently low that neutralization did not occur

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for several milliseconds. Thus, the question of the effect of self-electric fields on the precession mode is very pertinent. This effect was first considered by Condit and Lovelace* who assumed an infinitely long E-layer. In this chapter, we extend the calculations to E-layer profiles that closely approximate those measured on the experiment and present the results of measurements of the E-layer precession in the presence of self-electric fields.

THEORETICAL MODEL

The E-layer model used here is essentially the same as that in Ref. 14. The E-layer is assumed to be a rigid cylinder whose axis is slightly displaced from the center of the Astron tank (Fig. 34). The layer then precesses; this precession may be either damped or growing. No attempt is made to determine the growth rates, rather the result used is that an E-layer precessing forward (in the direction of the rotating electrons) is unstable; backward precession is stable.¹⁴ The problem is to determine the precession frequency and direction for motion in a field composed of external and self-fields. Since the layer is assumed to be a "superparticle," only the self-field contribution from image currents and charges is important. The equations of motion are:

$$\frac{1}{\gamma} \frac{d}{dt} \left(\gamma \frac{dr}{dt} \right) = r \left(\frac{d\theta}{dt} \right)^2 + \frac{e}{\gamma m} \left(E_r + r \frac{d\theta}{dt} B_z - \frac{dz}{dt} B_\theta \right), \quad (61)$$

$$\frac{1}{\gamma} \frac{d}{dt} \left(\gamma r^2 \frac{d\theta}{dt} \right) = \frac{e}{\gamma m} \left(E_\theta + \frac{dz}{dt} B_r - \frac{dr}{dt} B_z \right), \quad (62)$$

and

$$\frac{1}{\gamma} \frac{d}{dt} \left(\gamma \frac{dz}{dt} \right) = \frac{e}{\gamma m} \left(E_z + \frac{dr}{dt} B_\theta - r \frac{d\theta}{dt} B_r \right), \quad (63)$$

Fig. 34. Schematic cross-section of the Astron geometry. The E-layer is displaced off-axis a distance ϵ . The resistor layers are located inside and outside the layer at radii R_{cr} and R_{wr} , respectively.

*Refer to Ch. 5.

where e is the electronic charge, m is the rest mass, γ is the relativistic energy in rest mass units, the electric field $\vec{E}$ is the field from the image charges, and the fields B_r and B_z arise from both the image currents and the external magnetic well. The toroidal field arises from a current on the axis of the Astron tank. Hence, it can be written as

$$B_\theta(r) = B_\theta \frac{R}{r}. \quad (64)$$

Equations (61)-(63) are first linearized by assuming the E-layer is a rotating rigid cylinder, which is slightly off axis, i.e.,

$$\begin{aligned} \gamma(t) &= \gamma_0 + \Gamma \exp(i\omega t), \\ r(t) &= R + \epsilon \exp[i(\theta + \omega t)], \\ \theta(t) &= \omega_c t + \xi \exp(i\omega t), \end{aligned}$$

where

$$\frac{\Gamma}{\gamma_0}, \frac{\epsilon}{R} \ll 1,$$

and ω_c is the cyclotron frequency, which is determined from the zeroth order equation. The self-fields are also linearized for small displacements from $r = R$, i.e.,

$$\begin{aligned} \psi(r) &= \psi(R) + (r - R) \frac{\partial \psi}{\partial r} \Big|_{r=R} \\ &+ (z - z_0) \frac{\partial \psi}{\partial z} \Big|_{z=z_0}, \end{aligned} \quad (65)$$

where ψ is a general term for the fields.

The image field seen by the superparticle layer is, in fact, an

average over the z -dependence of the fields. By symmetry, it is assumed that

$$\left\langle \frac{\partial \psi}{\partial z} \right\rangle = \langle E_z \rangle = \langle B_r \rangle = 0,$$

for the motion of the superparticle. Finally, the fields can be expressed in the form:

$$E_\theta = i g_\theta \epsilon j_e \exp[i(\theta + \omega t)], \quad (66)$$

$$\begin{aligned} E_r &= j_e E_{r0} \\ &+ \epsilon j_e g_r \exp[i(\theta + \omega t)], \end{aligned} \quad (67)$$

$$\begin{aligned} B_z &= B_x + j_e B_{z0} \\ &+ \epsilon j_e M_z \exp[i(\theta + \omega t)], \end{aligned} \quad (68)$$

where j_e is the E-layer linear current density (A/m) and B_x is the external magnetic field. The linearized form of Eqs. (61)-(63) can then be reduced to a dispersion relation for the precession frequency ω ,

$$\begin{aligned} \left(1 + \frac{\omega}{\omega_c}\right)^2 - \left(\epsilon + \frac{\epsilon'}{1 + \frac{\omega}{\omega_c}} - M\right) \\ \times \zeta - n_x \left(\frac{1}{1 + \frac{j_e B_{z0}}{B_x}}\right)^{-1} - \left(\frac{\omega_\theta}{\omega_c}\right)^2 = 0, \end{aligned} \quad (69)$$

where

$$\epsilon = \frac{|e| B_x}{\mu_0 \gamma_0 m \omega_c^2} \left(\frac{\partial E_{r0}}{\partial r} + g_r \right),$$

$$\epsilon' = \frac{R^2 \omega_c B_x}{\mu_0 (B_x + j_e B_{z0}) c^2} g_\theta,$$

$$M = - \frac{RB_x}{\mu_0(B_x + j_e B_{zo})} \frac{\partial B_{zo}}{\partial r} - \frac{|e| RB_x}{\mu_0 \gamma_0 m \omega_c} M_z,$$

$$n_x = \frac{R}{B_x} \frac{\partial B_x}{\partial r},$$

$$\omega_c = \frac{|e|(B_x + j_e B_{zo})}{2\gamma_0 m} \times \left\{ 1 + \left[1 + 4 \frac{\gamma_0 m j_e E_{ro}}{e(B_x + j_e B_{zo})^2 R} \right]^{1/2} \right\},$$

$$\omega_\theta = \frac{|e| B_\theta}{\gamma_0 m},$$

and

$$\zeta = \frac{\mu_0 j_e}{B_x}.$$

The precession frequency and direction can be determined from Eq. (69) by calculating each field. As demonstrated by Furth,¹⁴ $\omega/\omega_c < 0$ implies damping, while $\omega/\omega_c > 0$ implies a growing mode. To obtain a simple expression, note that for all the Astron experiments reported here,

$$j_e B_{zo} \ll B_x,$$

$$4 \frac{\gamma_0 m j_e E_{ro}}{eR(B_x + j_e B_{zo})^2} \ll 1,$$

and

$$\frac{\omega}{\omega_c} \ll 1.$$

Under these conditions, Eq. (69) reduces to

$$\frac{\omega}{\omega_c} = (E - M)\zeta - \frac{1}{2} n_x - \frac{1}{2} \left(\frac{\omega_\theta}{\omega_c} \right)^2, \quad (70)$$

where

$$E = \frac{R}{2} \left[\frac{\epsilon_0 \beta c}{\beta^2} \left(\frac{\partial E_{ro}}{\partial r} + g_r \right) + \epsilon_0 \beta c g_\theta \right]$$

and

$$M \approx - \frac{R}{2\mu_0} \left(\frac{\partial B_{zo}}{\partial r} + M_z \right). \quad (71)$$

Note that for a mirror field, $n_x < 0$, hence, as shown in Ref. 14, the external field is destabilizing for the precession. Furthermore, if $E - M > 0$ (as will be shown in the next section), this term is also destabilizing. Thus, without a toroidal field, the E-layer is unstable to the precession for all values of ζ , i.e., for all E-layer amplitudes. This is in sharp contrast to the results for a neutral layer.¹⁵ In this case, $E = 0$ and $M > 0$ so that the mode is stable for all amplitudes such that

$$\zeta > \left| \frac{n_x}{2M} \right|.$$

Thus, the existence of the self-electric fields dramatically changes the character of the precessional mode in Astron. However, the presence of the toroidal field will, in principle, permit the stabilization of this mode for all amplitudes. The amplitude required for stabilization is calculated in the next section.

FIELD CALCULATIONS

The basic technique used to calculate the fields from a centered E-layer is found in Ref. 2 and Ch. 3. Briefly, the Green's function for the Astron geometry is evaluated and used to determine the field from an E-layer with current expanded in a Fourier series, i.e.,

$$j_e(z) = \sum_{n=0}^{\infty} j_n \cos k_n z.$$

(For convenience the E-layer is assumed symmetric about the origin, so it is unnecessary to include the $\sin k_n z$ terms.) The fields from a circular E-layer with an axis displaced from the tank axis can be treated in the same manner. In this case the fields vary as $\exp(10)$ rather than being azimuthally symmetric.

The Astron geometry (Fig. 35) consists of four concentric cylindrical surfaces. The surfaces of radii R_c and R_w are assumed to be perfect conductors and to terminate the magnetic field from the E-layer. The resistive surfaces at R_{cr} and R_{wr} allow the magnetic field to penetrate rapidly (on the order of 30 nsec) but to terminate the electric field of the E-layer. The E-layer is assumed to be infinitesimally thin in the radial direction, hence, there is a step discontinuity in B_z and E_r at $r = R + \epsilon \exp(10)$ (Fig. 34). The self-field solutions that satisfy these boundary conditions for $r \geq R$ are:

$$\begin{aligned} \phi(r, z) = & \frac{R}{\epsilon_0 \beta c} \sum_{n=0}^{\infty} j_n \left\{ \frac{I_0(k_n R, k_n R_{cr}) I_0(k_n r, k_n R_w)}{I_0(k_n R_{wr}, k_n R_{cr})} \right. \\ & - k_n \epsilon \exp(10) \\ & \left. + \left[\frac{I_0(k_n R) K_1(k_n R_{cr}) + I_1(k_n R_{cr}) K_0(k_n R)}{I_1(k_n R_{cr}, k_n R_{wr})} \right. \right. \\ & \left. \left. + I_1(k_n r, k_n R_w) \right] \right\} \cos k_n z, \end{aligned} \quad (73)$$

Section A-A

Fig. 35. Cross-section of the Astron showing location of the magnetic probes and resistor layers. Resistor and wall radii are $R_{cr} = 28$ cm, $R_w = 55$ cm, $R_c = 20.7$ cm and $R_w = 70$ cm.

and

$$\phi_s(r, z) = - \frac{R}{\epsilon_0 \beta c} \sum_{n=0}^{\infty} J_n \left\{ I_0(k_n R) K_0(k_n r) + k_n \epsilon \exp(i\theta) I_0(k_n R) K_1(k_n r) \right\} \cos k_n z, \quad (74)$$

where

$$f_n(a, b) = I_n(a) K_n(b) - I_n(b) K_n(a), \quad (75)$$

and ϕ_s is the electrostatic potential that would exist if there were no wall present, i.e., excluding the image charges. The fields seen by the superparticle are then

$$\vec{E} = - \nabla[\phi(r, z) - \phi_s(r, z)]. \quad (76)$$

Since only the axial component of the magnetic field is needed to evaluate Eq. (70), it is not necessary to derive the other components. The solution for $r \geq R$ is

$$B_z(r, z) = - \nu_0 R \sum_{n=0}^{\infty} k_n J_n \cos k_n z \left\{ \frac{f_1(k_n R, k_n R_c)}{f_1(k_n R_w, k_n R_c)} + [I_0(k_n R) K_1(k_n R_w) + I_1(k_n R_w) K_0(k_n R)] - k_n \epsilon \exp(i\theta) \left[\frac{I_1(k_n R) K_1^*(k_n R_c) - I_1^*(k_n R_c) K_1(k_n R)}{I_1^*(k_n R_w) K_1^*(k_n R_c) - K_1^*(k_n R_w) I_1^*(k_n R_c)} \right] + [I_1(k_n R) K_1^*(k_n R_w) - K_1(k_n R_w) I_1^*(k_n R)] \right\}, \quad (77)$$

and

$$B_{zs}(r, z) = - \nu_0 R \sum_{n=0}^{\infty} k_n J_n \cos k_n z + [I_1(k_n R) K_0(k_n r) + k_n \epsilon \exp(i\theta) I_1(k_n R) K_1(k_n r)]. \quad (78)$$

As before, the field seen by the superparticle is

$$B_z = B_z(r, z) - B_{zs}(r, z). \quad (79)$$

To evaluate the self-field terms in Eq. (70), the coefficients J_n in Eqs. (73) through (79) must be determined by assuming a current variation for the E-layer. It has been found on the Astron that the field from a current variation such as

$$J_e(z) = \frac{J_0}{2} (1 + \cos k_0 z) \quad |k_0 z| < \pi$$

$$= 0 \quad |k_0 z| > \pi \quad (80)$$

fits all data within experimental accuracy. Since this is not a periodic function, it cannot be rigorously expanded as in Eq. (72). However, if the periodicity is made large relative to E-layer length, the field profile in the region of the E-layer is not altered significantly. Thus, the following current variation is assumed:

$$J_e(z) = \frac{J_0}{2} \left\{ \frac{1}{2} + \frac{8}{3\pi} \cos \frac{\pi z}{2} \cos \frac{2\pi}{2} z + \frac{2}{\pi} \sum_{n=3}^{\infty} \frac{1}{n \left[1 - \left(\frac{n}{2} \right)^2 \right]^2} \sin \frac{n\pi}{2} \cos \frac{n\pi}{2} z \right\}. \quad (81)$$

This function is sketched in Fig. 36. It has the same functional form as Eq. (80) for $|z| < \ell/2$, but is periodic with period 2ℓ . In essence, this periodicity is long enough that field from the "side lobes" is negligible in the region $|z| \leq \ell/2$.

The self-field terms in Eq. (70) can be evaluated from Eqs. (76) and (79), by using the coefficients in

Fig. 36. Current variation assumed for all field calculations.

Eq. (81). The fields must be evaluated at the E-layer radius ($r = R$) and an average over z must be performed. In general, the fields can be written as

$$\psi(z) = \sum_n \psi_n \cos \frac{n\pi}{l} z.$$

The average field is then

$$\langle \psi \rangle = \frac{\int_{-l/2}^{l/2} \psi(z) j_e(z) dz}{\int_{-l/2}^{l/2} j_e(z) dz} = \sum_n \psi_n A_n, \quad \text{and} \quad (82)$$

where

$$A_n = \frac{2}{n\pi} \sin \frac{n\pi}{2} + \frac{\sin \frac{n\pi}{2} (n-2)}{(n-2)\pi} + \frac{\sin \frac{n\pi}{2} (n+2)}{(n+2)\pi}, \quad (83)$$

$$A_0 = 1,$$

and

$$A_2 = 0.5.$$

Recall that the E-layer is actually composed of particles that oscillate in z . This suggests a second definition for the average field, i.e.,

$$\langle \psi \rangle = \frac{1}{T} \int_0^{z_{\max}} \frac{\psi(z)}{v_z(z)} dz, \quad (84)$$

where

$$T = \int_0^{z_{\max}} \frac{dz}{v_z(z)},$$

and $z_{\max}$ is the turning point for an "average" particle. A reasonable assumption is that the average particle is one that oscillates to the midpoint of the distribution given by Eq. (81), i.e., $z_{\max} = l/4$. It can be shown that the motion of particles in the wells produced by the current variation given by Eq. (81) is very nearly simple harmonic. In this case,

$$v_z(z) = v_{z0} \left[1 - \left(\frac{4z}{l} \right)^2 \right]^{1/2}$$

$$\langle \psi \rangle = \sum_n \psi_n J_0 \frac{n\pi}{4}.$$

The self-field terms in Eq. (70) can now be evaluated. The expression for the self-electric and magnetic field is obtained from Eqs. (76) and (79), respectively, with the z -dependent term $\cos k_n z$ replaced by either $J_0(n\pi/4)$ or A_n and the coefficients $\{j_n\}$ given by Eq. (81). The two methods of averaging [Eqs. (82) and (84)] produce results that differ by less than 5%. The results of the evaluation of these expressions are presented in the next section with the results of experimental measurements of the precessions.

Fig. 37. Comparison of the superparticle theory and experiment for the amplitude modulation of the pickup loops as a function of axial distance. ΔB_z and B_{z0} are defined as shown in the figure. Best fit of experiment and theory occurs for a radial displacement of 4.5 cm. E-layer parameters are: $\zeta = 0.081$, $L = 62$ cm, and $B_0/B_z = 0.17$.

The magnetic field profile at any position (r, z) can be obtained by inserting the expansion coefficients given in Eq. (81) into Eq. (77). In particular, the z variation of the field at the radius of the loops used in the experiment can be determined. If the layer is off-axis and precessing, the magnetic field measured at a fixed point (r, θ, z) will show an oscillation at the precession frequency. The amplitude and phase of

this oscillation can be determined from Eq. (77) for a rigid layer. The result of this calculation is plotted in Fig. 37, where the ordinate is the fractional modulation of the field at a fixed instant of time. The amplitude of the displacement, ϵ , is chosen to obtain the best fit (in the least squares sense) to the experimental data shown in Fig. 37. The rather good agreement shows that the experimental results are consistent with the assumption of a rigid E-layer.

EXPERIMENTAL MEASUREMENTS OF PRECESSION

In this section we present the results of precession measurements for E-layers in the Astron experimental facility. Figure 35 shows a cross-section of the experimental tank and the location of the field-terminating resistors described previously. The toroidal field is created by an axial current carried by the cantilever. The diagnostic for the experiment is a number of magnetic pickup loops that are used to detect the self-magnetic field (B_z) of the E-layer. These loops are located behind the resistor layers inside and outside the E-layer and spaced at 24.4-cm intervals along the (z) axis.

The E-layer current amplitude, length, and mean radius are determined from the measured fields. Precession of the E-layer causes a modulation of the loop signal at the precession frequency.

The experiments consist of simultaneously recording oscilloscope traces of the loop output voltages for a large number of E-layers. The injected beam current and the toroidal field were varied to trap E-layers of various lengths and amplitudes. In this manner the precession frequency was measured as a function of E-layer amplitude and length and for several values of toroidal field.

Figure 38a shows a typical example of the time behavior of a precessing E-layer where the field reversal parameter ζ is greater than the threshold for stability.

Fig. 38. Typical experimental data for a toroidal field amplitude below the threshold for stability. The E-layer electrons circulate from 18 B to 18 N. The direction of precession changes from forward (unstable) to backward (stable) after reduction of E-layer amplitude $B_0/B_z \approx 0.2$.

Fig. 39. Comparison of theory and experimental measurements of the shift in precession frequency as a function of toroidal field. The mean E-layer radius is 38.3 cm. Experimental ranges of E-layer length and ζ are: $46 < L(\text{cm}) \leq 63$, $0.047 \leq \zeta \leq 0.095$.

In the figure, the precession amplitude grows until there has been sufficient current loss for stability. Thereafter, precession is damped. Figure 38b is an expanded scale comparison of two pickup loops located $\pi/2$ different in azimuth. The loops show a forward (backward) precession in the unstable (stable) regimes as predicted by theory.

Figure 39 shows the experimentally measured shift of precession frequency as the toroidal field is increased from zero. The data points are an average for a range of E-layer lengths and ζ values. The curve verifies the frequency shift predicted by the theory.

The coefficient E-M may be deduced from experimental data by evaluating the quantity

$$\left[\frac{\omega}{\omega_c} + 1/2 n_x + 1/2 (B_\theta/B_z)^2 \right] / \zeta.$$

The external radial gradient n_x is determined by the intercept at zero ζ of the quantity $\omega/\omega_c + 1/2 (B_\theta/B_z)^2$ plotted as a function of ζ , for a few E-layers of constant length. The value obtained is in reasonable agreement with a calculation based on a Hall probe measurement of the axial variation of the external field (B_x). The toroidal field B_θ is measured by an appropriately oriented pickup loop. Figure 40 shows a plot of the experimentally deduced values of E-M. The reduction in E-M as the length decreases is the result of geometry effects. As the length decreases, fringing of the electric field lines weakens the fields that drive the precession.

The ratio of the quantities E/M is not known accurately from the experiment. Estimates show that it is probably in the range $5 \leq E/M \leq 10$. This is consistent with the fact that the toroidal field required for stability is two to three times higher for an unneutralized layer compared to a fully neutralized layer.

Also shown in Fig. 40 is the theoretical curve for E-M which is calculated assuming harmonic oscillation of the E-layer electrons. The experimental values are about

Fig. 40. Experimentally deduced values of E-M as a function of E-layer length. The experimental range of ζ is $0.014 \leq \zeta \leq 0.10$. The solid curve is the theoretically calculated value assuming harmonic oscillation of the E-layer electrons. $R_w = 70$ cm, $R_c = 20.7$, $R_{wr} = 55$, $R_{cr} = 28$, and $\langle R \rangle = 38.2$ cm.

40% below the calculated values. The calculations are dependent on an average over the length of the layer. However, the difference between the experiment and theory probably cannot be accounted for by the averaging process.

The difference may be a result of assuming a zero thickness layer. The effects of finite thickness and self-field considered by Lovelace and Sudan¹⁷ lead to lower values of E-M. A more complete treatment for finite length should include these effects.

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7. High-Vacuum Trapping Experiments on the Astron*

R. E. Hester, G. D. Porter, B. W. Stallard,
J. Taska, C. W. Walker, and P. B. Weiss

Experimental results of high-vacuum (10^{-7} to 10^{-6} Torr) E-layer trapping experiments on the Astron are presented. Both single and multiple pulse experiments are reported. The precessional mode is stabilized with a toroidal field. At sufficiently low pressures ($\leq 5 \times 10^{-6}$ Torr), E-layers were observed to be stable for up to 200 msec. For greater times, the applied toroidal field had decayed sufficiently that the layer was unstable to precessions. Axial compression of the layer was observed as it was neutralized by ionization of the background gas. Compressions by a factor of two were observed. For layers whose self-well is the dominant focusing force, this compression agrees with adiabatic theory. A small ($\sim 20\%$) E-layer loss was observed during the compression. It is felt that this loss arises from scattering in electric field inhomogeneities. The effect of resistive walls on trapping efficiency was studied and it was shown that the efficiency scales with the theoretically calculated energy loss in the resistive

walls. Finally, the results of the multiple pulse experiments are presented. Failure to achieve buildup of the E-layer is attributed to a combination of losses during compression and poor trapping efficiency on later pulses.

INTRODUCTION

The central goal of the Astron approach to controlled thermonuclear research, as proposed by the late Christofilos,⁹ was to create an intense circulating ring of relativistic electrons known as the E layer. At a sufficiently large current, the ring creates a pattern of closed magnetic field lines (field reversal) with desirable minimum-B properties. In the summer of 1971, the Astron accelerator was improved to permit operation in the burst mode. In this mode, the accelerator is capable of firing a burst of up to 100 pulses at a maximum rate of 800 Hz. It was anticipated that this mode could be used to build the E-layer up to field reversal. Because of the requirement of rapid

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E-layer neutralization to stabilize the precessional mode, it was originally planned to "stack" the electron pulses in the presence of gas (10 to 20 mTorr).¹ However, early experiments indicated some severe difficulties would arise in this approach to E-layer buildup. First, trapping in 10 to 20 mTorr of H_2 produced plasma densities of the order of the beam density (10^8 to $10^9/cc$) in the transport region between the injection point and the trapping location (10 m apart in these experiments). In the presence of this plasma, additional pulses suffered large losses before reaching the trap. This loss is not well understood. By using the first pulse to create enough plasma to stabilize the precession mode for the second pulse, it was possible to lower the gas pressure to about 10^{-5} Torr. However, a second problem became apparent in the lower pressure regime. An abrupt E-layer loss occurred at a time that scaled as $P^{-4/3}$ over a pressure range from 5×10^{-6} to 10^{-4} Torr. This loss was great enough to prevent the buildup by multiple pulse injection into gas above 5×10^{-6} Torr. This problem is discussed in the following section. Finally, a "theoretical" problem began to emerge from simulation code studies¹⁸ and analytical work.⁵ This work showed that the interaction between an injected electron bunch and an existent neutralized E-layer created a saturation effect for the buildup.

Basically, the addition of a pulse causes the E-layer to lengthen while the peak amplitude remains the same.

Each of these problems is minimized by trapping in high vacuum. If the vacuum is adequate ($<10^{-7}$ Torr), the formation of plasma in the transport region is delayed until after the injection of several pulses. The abrupt loss observed in the experiment is delayed by 10 msec at 10^{-7} Torr. Finally, the effect of the lengthening that results from pulse interaction is overcome by the axial compression that accompanies slow neutralization of the layer.

Two discoveries made high vacuum trapping experiments feasible: First, it was found that the addition of a toroidal field, B_θ , stabilizes the precessional mode,* and second, the importance of the resistive walls for providing axial focusing of an unneutralized layer was found to be even greater in cylindrical geometry[†] than in plane geometry, as previously calculated by Christofilos. Each of these developments has been discussed in previous papers,*[†] hence, details of the calculations will not be presented here. The following section presents the results of a series of single-pulse trapping experiments in high vacuum. In general, these experiments contributed to a better understanding

*Refer to Ch. 6.

†Refer to Ch. 3.

of the properties of an unneutralized E-layer. We also present the results of multiple pulse experiments in high vacuum.

PROPERTIES OF UNNEUTRALIZED E LAYERS

Stability

Stabilization of the precessional mode by the application of a toroidal field was first discovered during trapping experiments in gas. In this case the layers are neutralized and it was found that $B_0/B_z > 0.1$ was adequate to stabilize all layers whose amplitude is greater than 1% of that required for field reversal. The requirements are somewhat different for an unneutralized layer. In this case the stability condition can be written*

$$\frac{1}{2} n_x + \frac{1}{2} \left(\frac{B_0}{B_z} \right)^2 - (E-M)\zeta > 0, \quad (86)$$

where n_x is the external field gradient

$$\left(n_x = \frac{B}{B_x} \frac{\partial B_x}{\partial R} \right)$$

and ζ is the field reversal parameter, i.e., it is the ratio of the change in the self-magnetic field across the layer to the external field. Since $E-M > 0$ and $n_x < 0$ for a mirror field, both of these terms are destabilizing. Thus, for an unneutralized layer, the toroidal field is the only stabilizing term. The amplitude of the toroidal field

must therefore be increased to maintain the stability of an E-layer as it is built up. If however, the layer is slowly neutralized as it builds up (as it must because of ionization of the background gas), the parameter E in inequality Eq. (86) will decrease and thus lower the required toroidal field. It was found theoretically that $B_0/B_z \approx 0.3$ would be adequate to form the reversed field layer. The precessional mode has been studied in detail experimentally and is reported by Porter and Stallard.*

The second instability that is a concern for high vacuum buildup is the abrupt loss discussed in the introduction. While this instability is not well understood, it appears to arise from the presence of a toroidal field in the plasma.¹⁹ It has been found that, at a fixed pressure, the onset of the loss can be delayed by lowering the toroidal field and can be eliminated with very low toroidal field. More importantly, it was found that, at sufficiently low pressures ($< 5 \times 10^{-7}$ Torr), the instability is eliminated for all amplitudes of toroidal field. This is evident in Fig. 41, which shows an E-layer that persists for 200 msec. During this time there is only about a 10% loss. It is felt that the disappearance of this instability is the result of insufficient accumulation of plasma for instability because

*Refer to Ch. 6.

*Refer to Ch. 6.

at these low pressures, the plasma diffusion rate is comparable to the generation rate. The plasma density was at the threshold of detection ($\approx 10^9 \text{ cm}^{-3}$) for the interferometers used and hence is not known accurately. The rapid "dump" at 200 msec in Fig. 41 occurs because the toroidal field has decayed to a low enough value that the layer is unstable to the precessional mode. Earlier experiments at higher pressures also produced stable layers.¹ An initial instability occurring near the cold electron, upper hybrid frequency created plasmas with densities much greater than the E-layer density in these experiments.¹ These results then indicate there are two plasma regimes in which the E-layer is stable:

Fig. 41. Long lifetime E layer achieved by trapping in vacuum at 1×10^{-7} Torr. The time scale is 50 msec/div. The upper trace is the signal from a diamagnetic probe located near the cantilever; the lower trace is from a probe located near the outer wall. The E-layer strength $\zeta \approx 10\%$.

first if the plasma density is low enough (probably no greater than two or three times the E-layer density) and second, when the plasma density is much greater than the E-layer density. Layers have been observed to be stable for many milliseconds in both regimes.

Equilibrium

The Astron geometry is shown in Fig. 42. The surfaces at R_{cr} and R_{wr} are composed of very fine resistive wires that terminate the self-electric field of a layer. The walls at R_c and R_w are made of aluminum and may be considered to be perfect conductors for E-layers that persist for less than 80 msec. Thus, after the initial image currents in the resistive walls have decayed (20-30 nsec), the E-layer reaches an equilibrium in which the electric field is terminated at R_{cr} and R_{wr} while the magnetic field is terminated at R_c and R_w . The resistive walls tend to short out the electric field, which is an axially defocusing field, thereby producing a net axial focusing force for an unneutralized layer. The details of the calculation of this focusing force are presented by Christofilos et al.* Briefly, for a layer whose length is greater than the spacing to the walls, the axial variation of the electric and magnetic fields are identical. In this case, the net axial force can be written as

$$F_z = ne v_{\theta} B_r, \quad (87)$$

*Refer to Ch. 3.

where

$$\eta = 1 - \frac{E_z}{v_{\theta} B_r} . \quad (88)$$

Fig. 42. Schematic diagram of the Astron geometry used in the experiments reported here. The middle figure shows a typical external field profile for these experiments.

Since the magnetic force $e v_{\theta} B_r$ is always focusing, the net axial force will be focusing for all geometries that yield $\eta > 0$. In cylindrical geometry, this is possible with the presence of resistive walls as shown in Fig. 42.

When a layer is slowly neutralized, the axial force increases and the layer shortens (compresses). If the neutralization process is slow relative to an axial bounce frequency, i.e., if the gas pressure is low, this compression is adiabatic. The adiabatic invariant can be expressed as the product of the axial temperature and the square of the E-layer length (TL^2) .⁴ For a long, thin layer with no r - z coupling, B_r is proportional to I/L , where I is the total circulating current. Thus, the temperature T is proportional to $\eta I/L$. Note that for a fully neutralized layer, $\eta = 1$, i.e., $E_z = 0$. Thus, the final length after complete neutralization will be given as,

$$L = \frac{\eta_0 I_0}{I} L_0 . \quad (89)$$

With no current loss during the compression, the final length is $\eta_0 L_0$. Since the value of η_0 was typically around 0.3 for many of the experiments reported here, we would expect a factor of three compression. However, this compression produces layers that are much too short (~ 30 cm) for the long layer limit to be applicable. A calculation of the adiabatic invariant for arbitrary length

layers has been done²⁰ and shows that the amount of compression is somewhat smaller than that indicated by Eq. (89). Furthermore, if there is current loss during the compression, as observed experimentally, the amount of compression will be further reduced. Such losses are usually assumed to be adiabatic.

A typical set of compression data is shown in Fig. 43. In this case, the pressure was rather high ($\sim 3 \times 10^{-6}$ Torr), so the neutralization time was 110 μ sec. There was about a 16% loss of current, and the layer was compressed by slightly more than a factor of two.

Fig. 43. E-layer compression as a result of the neutralization of the self-electric fields. The compression time is approximately 110 μ sec. The E-layer strength for the unneutralized layer is $\zeta = 11.5\%$ and is 20% for the neutralized layer. The initial trapped current is 3100 A and there is a 16% loss during the compression. The injected current was 585 A.

Fig. 44. Comparison of the experimentally observed compressed lengths with the theoretically predicted lengths.

This is good agreement with theoretical calculations. In general, however, the agreement was not this good. Figure 44 shows the variation of the difference between the experimentally observed and theoretically predicted compressed length with the initial amplitude of the E-layer. The product $n_0 j_0$ ($j_0 = I/L$) is roughly proportional to the initial depth of the self-well, and $n_0 j_0 = 5$ corresponds to a well which is comparable to the magnitude of the external mirror depth ($\sim 1\%$). Therefore, the effect of the external mirror cannot be neglected for these layers. Since the external well depth is not known accurately, this effect

is difficult to assess. However, the agreement between theory and experiment is within about 15% for layers with self-well depths greater than twice the external well.

The E-layer loss that occurs during the compression was the subject of a long period of experimental effort. It will be shown in the next section that this loss is very detrimental to buildup by multiple pulse injection. Several characteristic features of the loss were found:

- This loss ceased at the time the E-layer became neutralized.
- The loss rate was roughly independent of the neutralization time, hence compression experiments done with long neutralization times were plagued by larger losses.
- Observations of the x rays produced in the walls by these losses indicated that electrons were lost both radially (centered in the trap) and axially (on the walls far from the ends of the trap). The axial loss appeared somewhat greater.
- Finally, it was observed that larger amplitude layers had a larger loss rate.

Each of these observed phenomenon is consistent with a loss that arises because of scattering from inhomogeneities in the electric field produced by the unneutralized E-layer. This is analogous to

previously observed losses resulting from magnetic field inhomogeneities. It is also consistent with a random walk calculation.²¹ Field anomalies can arise from imperfections in the resistive walls. For example, missing resistor wires can create a perturbation in the electric field.

Two experiments were done to study the effect of removing these perturbations. In the first, the geometry was identical to previous geometries except that the resistive walls were made as uniform as possible. In this experiment, the initial loss rate was reduced slightly. However, a rather large (~60%) radial loss occurred late in the neutralization phase. Furthermore, the compression was greatly enhanced by this radial loss. This additional compression was much greater than that predicted by adiabatic theory. This indicates a nonadiabatic loss such as one would expect from the layer scraping the walls. Thus, it appears that this experiment was dominated by a radial thickening of the layer during compression. In the final Astron experiments a geometry was chosen which increased the radial room by 40% while maintaining the uniformity of the resistive walls ($R_c = 5$ cm, $R_{cr} = 21$ cm, $R_{wr} = 60$ cm, $R_w = 70$ cm). In the experiments the loss rates were reduced by a factor of two to three. However, the value of n_0 was so low (~0.15) that there was very little

compression, i.e., the external well dominated. Thus, this is not felt to be a definitive test of the loss mechanism.

Resistive Energy Losses

The resistive walls were first proposed as a trapping mechanism by Christofilos. He originally assumed plane geometry to calculate the axial energy loss due to the current induced in the resistors. This calculation is extended to cylindrical geometry by Stallard and Fessenden.* The energy loss per meter of travel past the resistors (ΔE) is a function of the radii of the resistive walls. In an effort to increase the available radial room, these radii were varied during the vacuum experiments. Hence it was possible to study the effect of the resistor energy loss on the trapping efficiency. The result is shown in Fig. 45. The vertical bars indicate the range in trapping efficiency that was observed. Since very little experimental time was spent in the geometry with $\Delta E = 3$, the vertical bar there is small. It should be emphasized that the resistor geometry was changed (and hence ΔE) only in a 1-m-long region centered about the trap location. The resistor geometry (≈ 4 m long) was not altered between the injector and the trap location. Thus, these results indicate that only the resistor

loss in the trap region is important for E-layer trapping. This result can be explained in part by the shape of the external well shown in Fig. 42. In general, there was a small magnetic "hill" located approximately 1 m from the center of the trap location. This hill separates the transport region from the trapping region. If ΔE in the trap region is too low, a significant fraction of the electrons return from the trap region into the transport region and lose axial energy before they arrive back at this hill. Thus, the probability of getting back into the trap region is low. They will be trapped in the transport region instead. This effect is evident in Fig. 46, which shows the fraction of electron current in the trap region and the transport region during the trapping phase. While there is some transfer

Fig. 45. Variation of the trapping efficiency with the energy loss to the resistive walls.

*Refer to Ch. 4.

of charge across the magnetic hill, it is rapidly damped, making the resistor loss in the trapping region very important.

To create the layer by multiple pulse injection, it is desirable to reduce η_o . (See the next section.) It is evident that this must be done without reducing ΔE . Since both of these parameters depend on the geometry, it is not possible to independently adjust them by only varying the radii of the resistive walls. Figure 47 shows the range of values possible for two different values of the inner conductor radius (R_c). There is a great deal of flexibility made possible by the variation of R_c . It has been demonstrated experimentally that the trapping efficiency is roughly the same for

different geometries with identical values of ΔE .

MULTIPLE PULSE INJECTION

As pointed out in the Introduction, one of the main problems anticipated for multiple pulse buildup of the layer was an expansion of the layer that occurs when a pulse is injected into an existing layer. This effect is discussed by Christofilos and Fowler.* It is shown that the pulse interaction can be characterized by a parameter f , which is a measure of the change in the adiabatic invariant that occurs upon the injection of a pulse. Using the definitions of Christofilos and Fowler,* the adiabatic invariant after injection can be written as

$$TL^2 = T_o L_o^2 \left(\frac{I}{I_o} \right)^{2f}, \quad (90)$$

where the subscript (o) refers to E-layer parameters just before injection of another pulse. While it only approximates the experiment, the long layer limit is helpful in understanding the magnitude of this expansion problem. In this case, with the assumption that the self-well is much greater than the external well, $T = \eta I/L$ and we find

$$\frac{I}{L} = \frac{\eta}{\eta_o} \frac{I}{I_o}^{2(1-f)} \frac{I_o}{L_o}. \quad (91)$$

If experiments are done in gas so that prompt neutralization occurs,

*Refer to Ch. 2.

Fig. 46. Experimentally observed E-layer oscillations during the trapping phase. The circles (o) show the total current located between the injector and the magnetic "hill"; the triangles (Δ) show the current beyond the "hill" (see Fig. 42).

Fig. 47. Variation of resistive energy loss (ΔE) and focusing (η_0) parameters as a function of the inner resistor radius (R_{cr}) for different values of the outer resistor radius (R_{wr}): (a) Inner conductor radius of 21 cm; (b) inner conductor radius of 5 cm. The outer conductor is at 70 cm.

$\eta = \eta_0$. Furthermore, if $f = 1$ (this is felt to be the upper limit of f),* the layer amplitude, I/L , will not be changed by the addition of another pulse. The length of the layer is then proportional to the current, and the E-layer amplitude saturates. However, if the buildup is done in high vacuum so

that neutralization is slow, analysis of the model shows that the number of pulses that can be injected before saturation is about $1/\eta_0$ even if $f = 1$. Hence, it is desirable to do trapping experiments with η_0 as small as possible.

In computer simulation studies,¹⁸ the parameter f is found to vary from pulse to pulse. It is less than unity for the first two or

*Refer to Ch. 2.

three pulses, but then tends to approach unity. Experimentally, the value of f has been determined from Eq. (90), using calculations of the adiabatic invariant (TL^2) found in Ch. 4. It was found that f does not vary significantly from pulse to pulse. In general, it was found to be nearly unity. Therefore, the expansion is not felt to be a deterrent to multiple pulse buildup.

One major problem encountered in the multiple pulse experiments

Fig. 48. Theoretical predictions of the effect of the losses during compression on E-layer buildup. Curve a shows the buildup for three-pulse injection with no losses. Curve b shows the buildup with the current variation assumed to be $I = I_0 \exp(-0.2 t/\tau_n)$. The current variation is assumed to be $I_0 \exp(-0.5 t/\tau_n)$ in curve c. The parameter τ_n is the neutralization time, and ζ is the initial E-layer strength.

Fig. 49. Experimentally observed E-layer buildup for three-pulse injection.

was the losses that occurred during neutralization. These losses were described in the previous section. To analyze the effect these losses have on the buildup, scaling calculations were done using Eq. (90) with $f = 1$. It was assumed that the current decayed exponentially between pulses. The results are shown in Fig. 48 for no losses and for a decay rate such that 20 and 40% of the current is lost in one neutralization time. A minimum experimental loss rate was near 20%. It was also assumed that the trapping efficiency of later pulses was only 70% of that on the first. This is more optimistic than observed experimentally. A typical set of data taken for three-pulse injection is shown in Fig. 49. It

is clear that the observed losses largely account for the poor results of the multiple-pulse injection experiments. The decrease in trapping efficiency on later pulses also accounts for part of the poor results. As discussed in the previous section, losses were substantially reduced in the final Astron experiments. However, in this geometry the value of $n_0 j_0$ was small. Therefore, the external well was dominant, and the compression was inadequate for successful stacking.

SUMMARY AND CONCLUSIONS

In summary, the behavior of an E-layer formed by single-pulse injection into high vacuum is well understood. The trapping efficiency scales with geometry as expected. At sufficiently low pressures, the E-layer is stable against all modes except the pre-compression mode, which is easily stabilized by a toroidal field. The compression that accompanies neutralization of the layer is consistent with adiabatic theory.

However, the results of multiple-

pulse experiments are not well understood. The experiments reported here were plagued by losses during compression. It is felt that this problem has been identified and is soluble. The elimination of this loss would have a dramatic effect on the multiple-pulse experiments. Successful stacking would then depend on the trapping efficiency of the later pulses. In short, buildup by multiple-pulse injection still appears to be a difficult task.

With these problems in mind, a new approach to the formation of reversed field E-layers was tried during the final Astron experiments.* In these experiments, the electrons were injected into gas (≈ 0.1 Torr) with large toroidal field ($B_\theta/B_z \approx 1$). With the exception of the toroidal field, these experiments are similar to those of Refs. 22 and 23. The results of these experiments are very encouraging. E-layer amplitudes of $\zeta = 0.75$ were achieved. Clearly, at the present time, this would be a more viable approach to E-layer formation.

*Refer to Ch. 8.

8. Efficient Trapping of High-Level E-Layers in a Strong Toroidal Field*

R. J. Briggs, G. D. Porter, B. W. Stallard,
J. Taska, and P. B. Weiss

Efficient trapping of high-level E-layers in the presence of a strong toroidal field ($B_\theta \sim B_z$) has been achieved in the Astron experiment at Lawrence Livermore Laboratory. The beam was injected into hydrogen gas at pressures around 0.1 mTorr resulting in the prompt formation of plasma with a density of $10^{12}/\text{cm}^3$. Single-pulse trapping efficiencies greater than 50%, circulating currents in excess of 13 kA with a typical half-life of 1 msec, and diamagnetic strengths (ζ) up to 75% were achieved. Using the multiple pulse capability of the Astron linac, "maintenance" of an E-layer at $\zeta \sim 30\%$ was demonstrated.

INTRODUCTION

The central goal of the Astron approach to controlled thermonuclear research, as proposed by the late N. C. Christofilos,⁹ is to create an intense circulating ring of relativistic electrons known as the "E-layer." At a sufficiently large current, the ring creates a pattern of closed magnetic field lines (field reversal) with de-

sirable minimum-B properties. Our previous attempts to reach the reversed field state have relied on buildup by injection of successive pulses into a magnetic trap.¹ Success with this "pulse stacking" approach has been limited by E-layer losses between pulses and by the tendency for the layer to stretch when a new pulse is added,* leading to lengthening of the layer and only small increases in current density.

An alternate approach is that of Fleischmann and his co-workers^{22,23} at Cornell University who have achieved the reversed field state in a single pulse by injecting a very intense (up to 30 kA, 0.5 MeV) electron beam into high pressure hydrogen gas (several hundred mTorr to 2 Torr). The low beam energy and high gas pressure of the Cornell experiment result in an E-layer whose lifetime is limited to several tens of microseconds. The reversed field state exists for several microseconds followed by a smooth decay as a result of gas scattering and an abrupt E-layer loss (or dump) at 15-20% of field reversal.

Recently we have performed trapping experiments at relatively high

*Originally published in Phys. Fluids 16 (11), 1934 (1973).

*Refer to Ch. 2.

gas pressures (up to 0.2 Torr hydrogen) using the 6 MeV beam of the Astron linac, which is capable of beam currents up to 650 A and pulse widths up to 340 nsec. A significant difference between our experiments and the Cornell experiments has been our addition of a large toroidal field $B_\theta \leq B_z$. With the toroidal field B_θ approaching B_z , trapping efficiencies greater than 50% have been achieved. In addition, the dump phenomenon is eliminated by the toroidal field.¹ The maximum achieved diamagnetic strength of the layer was $\zeta = 0.75$ (with the definition $\zeta = \mu_0 \kappa_0 / B_0$), where κ_0 is the peak E-layer current density per unit length and B_0 is the vacuum magnetic field). For typical E-layer lengths in the present experiments (40 cm), field reversal occurs for $\zeta \sim 1.6$.

SINGLE PULSE TRAPPING

Figure 50 shows the geometry of the experimental apparatus. The available radial space was increased from previous experiments²⁴ and sufficient resistor loss was maintained by replacing the 21-cm-radius cantilever with a 5-cm-radius conductor. The center conductor is used to support resistors for damping E-layer axial motion¹ and to carry the current that establishes the toroidal field. The relativistic electrons were injected through a 15-cm-diam soft iron injector. The injection angle is 4° from a plane perpendicular to

the tank axis. A 35-cm-radius, single-turn pulse coil was located about 15 cm upstream from the injector. This coil was used to produce a B_r field at the injector

Fig. 50. Schematic diagram of the Astron geometry. The probes are separated by 24.4 cm and the typical trap location was 2 m from the injector. A typical external field profile is also shown.

plane to adjust the axial velocity of the electrons after they left the injector. A pair of 25 GHz interferometer horns were used for measurements of plasma electron density. The E-layer current profile (amplitude, length, and mean radius) were inferred from measurements of the self-magnetic field of the layer. An array of magnetic probes located inside and outside the layer (shown in Fig. 50) were used to measure the self-fields. Two types of magnetic probes were used: the first had a risetime of 10 nsec and was used to record the behavior of the E-layer during the first few microseconds after injection; the second type had a risetime of several hundred nanoseconds and was used to study the behavior of the E-layer on longer time scales.

We determined the dependence of the trapping efficiency on three experimental parameters: toroidal field ($B_\theta/B_z \leq 1.0$ at the E-layer radius), gas pressure ($p \leq 0.2$ mTorr hydrogen), and beam pulse length (≤ 340 nsec). At hydrogen gas pressures greater than about 30 mTorr and $B_\theta \geq 0.4B_z$, the trapping efficiency was roughly independent of pulse length. Hence, maximum trapped current occurred for the full-beam pulse width of about 300 nsec. This behavior is very different from previous results obtained generally at pressures less than 30 mTorr and in a geometry with less radial space.²⁴ Trapping efficiencies in these earlier experiments decreased with

increasing pulse width and the maximum trapped current (~ 3000 A) occurred for pulse widths of about 120 nsec.

The trapping efficiency was optimized for hydrogen gas pressures of 100 mTorr or greater. The efficiency slowly decreased in going from 100 mTorr to 30-40 mTorr and then dropped off sharply. The best combination of initial trapped amplitude and E-layer lifetime was generally around 100 mTorr, and for this reason the most extensive data was taken in this regime.

It is significant to note that the time scale for space charge neutralization is less than the beam injection time at pressures greater than about 13 mTorr hydrogen. At the higher pressures there is rapid plasma generation during the beam injection time and plasma phenomena may become important in the trapping of the layer. Plasma density measurements at 100 mTorr show peak electron densities in the range $5-10 \times 10^{11}/\text{cm}^3$ created before beam injection is completed, and these densities are adequate for substantial plasma currents excited by the beam "tail."

Figure 51 shows the variation of total trapped current with toroidal field at several values of pressure. With the injection currents and pulse lengths used here, 12 kA of trapped current corresponds to a trapping efficiency of about 50%. The rapid increase in trapping efficiency as B_θ is raised suggests that even greater trapping efficiency

may be achieved at larger values of toroidal field.

The lifetime of the E-layer appears to be dominated by scattering with the background neutral gas. Figure 52 shows the lifetime as pressure is varied for a narrow range of toroidal field values. Lifetime is defined as the time at which the self-magnetic field of the layer, as measured by a magnetic probe at the center of the layer, decreases by one-half. The straight line drawn in Fig. 52 has the functional form $\tau_{1/2} = 62/p$.

Fig. 52. The dependence of the E-layer half-life on the neutral pressure with a toroidal field $0.66 \leq B_\theta/B_z \leq 0.71$.

Fig. 51. The dependence of the total trapped current on the amplitude of the toroidal field for a full-width beam pulse of 300 nsec.

The maximum initial value of ζ achieved in these experiments was $\zeta = 0.64$ with a total circulating current of 11,200 A. This is not the maximum circulating current achieved; more than 13 kA has been trapped in longer layers. Figure 53 shows the time behavior of the E-layer current density, total current, and mean radius as inferred from the magnetic probes. As can be seen from this data, a compression (shortening) of the E-layer occurs which increases ζ from its initial value to a peak value of 0.75 about 0.3 msec after injection. The physical mechanisms causing this "compression" have not been identified at the present time.

Also shown in Fig. 53 is the E-layer mean radius as a function of time. The shrinking radius is evidently due to Coulomb energy

Fig. 53. The temporal behavior of E-layer amplitude, trapped current, and mean radius. The neutral pressure is 120 mTorr hydrogen.

loss to the background neutral gas. The inferred rate of energy loss (1.0 MeV/msec) is within a factor of two agreement with the classical energy loss of 1.8 MeV/msec calculated from the Bethe formula.²⁵ Measurements of radial shrinkage over a range of pressure and toroidal field show that the decrease of radius with time is slower than the calculated value by a factor of two or less. The rate of radial shrinkage at fixed gauge pressures also appears to be a decreasing function of total trapped current or peak current density.

There are several factors that can account for the difference be-

tween theory and experiment. The most obvious is that the true pressure at the E-layer may differ from the gauge pressure. This is likely since the energy deposited in the gas during E-layer decay should be sufficient to heat the gas, reducing the local gas density by expansion. A second effect arises from the inductive magnetic field of the layer. As electrons are lost from the layer, an azimuthal electric field (E_θ) is induced which tends to increase the average energy of the remaining electrons. Hence, the radial shrinkage should be less than expected from Coulomb losses alone. The latter effect can account for perhaps half of the reduction in radial shrinkage. Both effects are consistent with the observation of reduced radial shrinkage as the total trapped current increases.

Measurements were made of the axial motion of the injected electron bunch during the first several microseconds after injection by use of the fast-response magnetic loops. The distribution of magnetic field of the incoming electron bunch is presented in Fig. 54 for several instants of time; the case illustrated was for a toroidal field $B_\theta/B_z = 0.77$ and gauge pressure of 120 mTorr. The total trapped current at equilibrium is about 7300 A. The magnetic loops measure the total field, i.e., the field of both the E-layer and plasma currents. If plasma currents comparable to the E-layer current are present, the

field profiles cannot be interpreted directly in terms of the E-layer current. However, both the data and rough calculations suggest that plasma current effects are minimal insofar as the magnetic field measurements are concerned, at least for times greater than 0.5-1 μsec after injection. In interpreting the data, we will assume this to be the case.

A surprising result of these fast-time-scale loop measurements is the very slow axial velocity of the E-layer ($v_z/c \sim 5 \times 10^{-4}$) as it enters the trap. This compares with a typical value of $v_z/c \sim 0.1$ at pressures below 30 mTorr and for

small values of B_θ . It also appears that the "E-layer" which has formed near the injection point just after injection has ceased ($t = 0.3 \mu\text{sec}$ in Fig. 54) reaches the trap with little loss. Measurements made under different conditions ($p \approx 50$ mTorr, $B_\theta/B_z \approx 0.58$) show similar behavior except that the typical axial velocity is $v_z/c \sim 0.02$.

MULTIPLE PULSE INJECTION

The Astron accelerator has the capability of operating at the rate of 800 pulses per second for a burst of 20-100 pulses. Previous attempts to use this capability to build up the E-layer have not been successful.* Bursts of up to five pulses were injected during the experiments described here. In general, the pressure was lowered for the burst experiments. This produced longer E-layer lifetimes and lower plasma densities. The results of an experiment at 10 mTorr are shown in Fig. 55. The pressure was so low that the first pulse did not trap. However, the trapping efficiency of the second pulse was reasonable ($\sim 33\%$) due to the presence of plasma produced by the first pulse. The trapping efficiency of later pulses (3-5) was only 15-20%.

Figure 55 also shows the time history of the E-layer length. The

*Refer to Ch. 7.

Fig. 54. The behavior of the injected electron bunch for the first 10 μsec after injection. The neutral pressure is 120 mTorr hydrogen and $B_\theta/B_z = 0.77$.

lengthening of the layer immediately after injection of the later pulses is evident. This effect has been observed in both high-vacuum stacking experiments and in computer code simulations.²⁶ However, in the present experiments, this expansion is overcome by the compression which occurs between pulses, and the layer actually becomes shorter while maintaining roughly constant total current.

These multiple pulse experiments

Fig. 55. The temporal behavior of an E-layer maintained by multiple pulse injection. The neutral pressure is 10 mTorr hydrogen.

indicate that it is possible to use the burst capability of the Astron accelerator to maintain an E-layer. In these experiments the layer was maintained at an average ζ of 28% (100 A/cm). In fact, the third pulse increased the layer strength by 60% and the last two pulses maintained that level. This behavior was not observed at higher pressures where single pulse trapping was most efficient. In that case, subsequent pulses added very little to the existing layer when trapping of the first pulse was optimized.

DISCUSSION AND CONCLUSIONS

One important difference between the Astron experiments reported here and previous ones is the increased radial room made possible by the use of a smaller center rod. The achieved E-layer amplitudes were significantly better than previous results, even in similar regimes, and this would imply that previous performance was limited at least in part by insufficient radial room.

One of the most important discoveries in this experimental run, however, was the marked improvement in trapping efficiency and initial E-layer amplitude made possible by a relatively strong toroidal field (B_θ). With strong B_θ and relatively high gas pressure (50-100 mTorr), the motion of the layer down to the trap region was very sluggish. Plasma densities of the order of

10^{12} per cm^3 were generated promptly in this regime, and an appealing interpretation of the very slow axial velocity of the layer ($v_z/c < 10^{-3}$) can be given in terms of a "drag" on the E-layer resulting from azimuthal plasma conductivity that prevent its rapid motion through the plasma. Estimates of this drag force indicate that it is strong enough to explain the observed behavior.

Another observation, which is not fully understood at present, is the axial shortening (compression) of the E-layer with high B_θ . With this compression, it was possible to maintain an E-layer with multiple pulse injection at relatively high levels ($\zeta \sim 30\%$) for five

pulses with decreasing rather than increasing axial length.

In summary, the results presented here have demonstrated that efficient trapping of high-level E-layers ($\zeta \sim 75\%$) is possible, and that large toroidal fields are an important feature of this efficient trapping. Maintenance of high-level ($\zeta \sim 30\%$) E-layers with roughly constant (or decreasing) axial lengths by multiple pulse injection was also demonstrated. The regime of even larger toroidal fields ($B_r < B_z$) could not be studied in the present experiments; however, the results presented here suggest that this regime would be an interesting and fruitful area for future Astron studies.

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years. We also thank the crew who worked on the Astron and Astron accelerator. Their diligent efforts made these experiments possible.

Appendix A

The integrations of Eqs. (40) and (41) give the following formulas for $E_{d1,2}$ (MKS units):

$$E_{d1} = 10^{-7} \pi a_1 \left(\frac{a_3}{a_{23}} \right)^2 \left(\frac{2\pi}{T} \right)^4 \left\{ \left(1 - e^{-T/T_+} \right) \left[s_{11}^2 T_+ + 2s_{11}s_{12}Q_1 \right. \right. \\ \left. \left. + \frac{2s_{11}s_{13}Q_2}{T_+} + 2s_{11}s_{14}Q_2 \left(\frac{2\pi}{T} \right) \right] \right. \\ \left. + \left(1 - e^{-T/T_-} \right) \left[s_{12}^2 T_- + 2s_{11}s_{12}Q_1 + \frac{2s_{12}s_{13}Q_3}{T_-} \right. \right. \\ \left. \left. + 2s_{12}s_{14}Q_3 \left(\frac{2\pi}{T} \right) \right] + \left(s_{13}^2 + s_{14}^2 \right) \frac{T}{2} \right\}$$

$$E_{d2} = 10^{-7} \pi \left(\frac{a_4}{a} \right) \left(\frac{a_2}{a_{23}} \right)^2 \left(\frac{2\pi}{T} \right)^4 \left\{ \left(1 - e^{-T/T_+} \right) \left[s_{21}^2 T_+ + 2s_{21}s_{22}Q_1 \right. \right. \\ \left. \left. + \frac{2s_{21}s_{23}Q_2}{T_+} + 2s_{21}s_{24}Q_2 \left(\frac{2\pi}{T} \right) \right] + \left(1 - e^{-T/T_-} \right) \right. \\ \left. \times \left[s_{22}^2 T_- + 2s_{21}s_{22}Q_1 + \frac{2s_{22}s_{23}Q_3}{T_-} \right. \right. \\ \left. \left. + 2s_{22}s_{24}Q_3 \left(\frac{2\pi}{T} \right) \right] + \left(s_{23}^2 + s_{24}^2 \right) \frac{T}{2} \right\},$$

where the following definitions apply:

$$T = \frac{\tau_2}{\tau_1}, \quad \alpha = \frac{\tau_2}{\tau_1}$$

$$\frac{1}{Q_1} = \frac{1}{T_+} + \frac{1}{T_-}$$

$$\frac{1}{Q_2} = \frac{1}{T_+^2} + \left(\frac{2\pi}{T} \right)^2$$

$$\frac{1}{Q_3} = \frac{1}{T_-^2} + \left(\frac{2\pi}{T} \right)^2$$

$$s_{11} = \left(\frac{a_{34}}{a_3 a} - \frac{1}{T_+} \right) \frac{Q_2}{\left(\frac{1}{T_-} - \frac{1}{T_+} \right)}$$

$$s_{12} = -s_{11} \frac{Q_3}{Q_2}$$

$$s_{13} = \left[\frac{1}{T_+ T_-} - \left(\frac{2\pi}{T} \right)^2 - \frac{a_{34}}{a_3 a Q_1} \right] Q_2 Q_3$$

$$ss_{14} = \left[\left(\frac{a_{34}}{a_3 a} \right) \left(\frac{1}{T_+ T_-} - \left(\frac{2\pi}{T} \right)^2 \right) + \left(\frac{2\pi}{T} \right)^2 \frac{1}{Q_1} \right] Q_2 Q_3 \left(\frac{T}{2\pi} \right)$$

$$s_{21} = \left(\frac{a_{12}}{a_2} - \frac{1}{T_+} \right) \frac{Q_2}{\left(\frac{1}{T_-} - \frac{1}{T_+} \right)}$$

$$s_{22} = -s_{21} \frac{Q_3}{Q_2}$$

$$s_{23} = \left[\frac{1}{T_+ T_-} - \left(\frac{2\pi}{T} \right)^2 - \left(\frac{a_{12}}{a_2 Q_1} \right) \right] Q_2 Q_3$$

$$s_{24} = \left[\left(\frac{a_{12}}{a_2} \right) \left(\frac{1}{T_+ T_-} - \left(\frac{2\pi}{T} \right)^2 \right) + \left(\frac{2\pi}{T} \right)^2 \frac{1}{Q_1} \right] Q_2 Q_3 \left(\frac{T}{2\pi} \right)$$

$$T_{\pm} = 2 \left[\left(\frac{a_{13}}{a_{23}} + \frac{a_{24}}{a a_{23}} \right) \pm \sqrt{\left(\frac{a_{13}}{a_{23}} + \frac{a_{24}}{a a_{23}} \right)^2 - \frac{4a_{14}}{a a_{23}}} \right]^{-1}$$

$$a_{1j} = \sum_{k=1}^j a_k.$$

Appendix B

THE ELECTROSTATIC FORCE

The fields of an E-layer of I statamps/cm located at $r_E = R + \epsilon \cos \theta$ (refer to Fig. B-1) are as follows.

Outside:

$$E_r = -\frac{B^>}{r} - \frac{\epsilon A}{r^2} \cos \theta \left(r^2 + R_W^2 \right), \quad (B-1)$$

$$E_\theta = \epsilon A \sin \theta \left(r^2 - R_W^2 \right) / r^2. \quad (B-2)$$

In these equations $A = (4\pi IR) / (R_W^2 - R_C^2) 8c$; the layer is negative; $I > 0$; and $B^> = (4\pi IR / 8c) M$, where $M = \ln R / R_C / (\ln R_W / R_C)$. Also, $4\pi I / c = \epsilon B_0$.

Inside:

$$E_r = -\frac{B^<}{r} - \frac{\epsilon A}{r^2} \cos \theta \left(r^2 + R_C^2 \right), \quad (B-3)$$

$$E_\theta = \epsilon A \sin \theta \left(r^2 - R_C^2 \right) / r^2, \quad (B-4)$$

where A is as before, $B^< = -(4\pi IR / 8c) N$, and $N = (\ln R_W / R) / (\ln R_W / R_C)$. We can let $\theta = \theta_0 + \omega_p t$ if $(\omega_p / \omega_c) \ll 1$, and the above expressions remain valid.

The first-order force on a particle following the orbit,

$$\rho = R + \epsilon e^{-i\omega t},$$

$$\phi = \omega_c t,$$

can be computed by evaluating the above expressions for E_r and E_ϕ . The expressions for the fields inside the layer are the appropriate ones to use, since the circular E-layer produces no fields inside itself, and thus the self-fields are not present in the "inside" expressions; the image charge contributions are continuous at the layer.

We thus see that

$$\begin{aligned} (K\omega_c)\epsilon \cos(\omega_p - \omega_c)t &= -eE_r = +\frac{eB^<}{R} \left[-\frac{\epsilon}{R} \cos(\omega_p - \omega_c)t \right] \\ &+ \frac{eA\epsilon \left(R^2 + R_C^2 \right)}{R^2} \cos(\omega_p - \omega_c)t. \end{aligned}$$

Thus,

$$\frac{K}{\tilde{m}_0 \omega_c} = \left(\frac{e}{\tilde{m}_0 \omega_c^2} \right) \left(\frac{\zeta B_0}{\beta R} \right) \left(N + \frac{R^2 + R_c^2}{R_W^2 - R_c^2} \right).$$

Now,

$$\frac{e B_0}{\tilde{m}_0 \omega_c^2 R} = 1/\beta,$$

so

$$\frac{K}{\tilde{m}_0 \omega_c} = \zeta \left[\frac{N}{\beta^2} + \frac{(R^2 - R_c^2)}{\beta^2 (R_W^2 - R_c^2)} \right]. \quad (B-5)$$

For the Astron 30-40-50 cm region, $\beta = 1$, $N = 0.44$, and the term in brackets is 2.00. Thus,

$$\frac{K}{\tilde{m}_0 \omega_c} = 2\zeta. \quad (B-6)$$

Also,

$$-ac(K\omega_c) \sin(\omega_c - \omega_p)t = -eE_0 = -e\epsilon A \left(\frac{R^2 - R_c^2}{R^2} \right) \sin(\omega_c - \omega_p)t.$$

So,

$$\frac{\alpha K}{\tilde{m}_0 \omega_c} = \frac{e}{\tilde{m}_0 \omega_c^2} \left(\frac{\zeta B_0 R}{\beta R^2} \right) \frac{R^2 - R_c^2}{R_W^2 - R_c^2},$$

$$\frac{\alpha K}{\tilde{m}_0 \omega_c} = \frac{\zeta}{\beta^2} \frac{R^2 - R_c^2}{R_W^2 - R_c^2}. \quad (B-7)$$

For Astron,

$$\frac{\alpha K}{\tilde{m}_0 \omega_c} = +0.44\zeta. \quad (B-8)$$

Fig. B-1. Cross section of the Astron showing radii of resistor layers and walls. The electric fields are terminated at R_{cr} ($= R_c = 30$ cm) and R_{wr} ($= R_w = 50$ cm). For the E-layer, $R = 40$ cm.

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