STEAM GENERATOR AND CIRCULATOR MODEL FOR
THE HELAP CODE*

Hans Ludewig

Fast Reactor Safety Division
Department of Applied Science
BROOKHAVEN NATIONAL LABORATORY
Upton, New York

July 1975

*Work carried out under the auspices of the United States Nuclear Regulatory Commission.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED
NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the U.S. Nuclear Regulatory Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

Printed in the United States of America
Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road
Springfield, VA 22161
Price: Printed Copy $4.00; Microfiche $2.25

October 1975
131 copies
STEAM GENERATOR AND CIRCULATOR MODEL FOR
THE HELAP CODE*

Hans Ludewig

Fast Reactor Safety Division
Department of Applied Science
BROOKHAVEN NATIONAL LABORATORY
Upton, New York

July 1975

*Work carried out under the auspices of the United States Nuclear Regulatory Commission.
TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table of Contents</td>
<td>i</td>
</tr>
<tr>
<td>List of Figures</td>
<td>ii</td>
</tr>
<tr>
<td>Nomenclature</td>
<td>iii</td>
</tr>
<tr>
<td>Acknowledgments</td>
<td>v</td>
</tr>
<tr>
<td>Summary</td>
<td>1</td>
</tr>
<tr>
<td>Introduction</td>
<td>2</td>
</tr>
<tr>
<td>I Steam Generator</td>
<td>3</td>
</tr>
<tr>
<td>A. Heat Exchanger Equations</td>
<td>4</td>
</tr>
<tr>
<td>B. Heat Transfer Correlation for the Primary Side</td>
<td>5</td>
</tr>
<tr>
<td>C. Heat Transfer Correlation for the Secondary Side</td>
<td>6</td>
</tr>
<tr>
<td>II Circulator Model</td>
<td>9</td>
</tr>
<tr>
<td>A. Compressor Work</td>
<td>11</td>
</tr>
<tr>
<td>B. Turbine Work</td>
<td>12</td>
</tr>
<tr>
<td>III Steady State Determination</td>
<td>13</td>
</tr>
<tr>
<td>IV Conclusion</td>
<td>17</td>
</tr>
<tr>
<td>Appendix</td>
<td>18</td>
</tr>
<tr>
<td>References</td>
<td>24</td>
</tr>
</tbody>
</table>
List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure I</td>
<td>Schematic Layout of Steam Generator and Circulator</td>
</tr>
<tr>
<td>Figure II</td>
<td>Gas Cooled Reactor Steam Generator</td>
</tr>
<tr>
<td>Figure III</td>
<td>Gas Cooled Reactor Circulator</td>
</tr>
<tr>
<td>Figure IV</td>
<td>Schematic Operating Map for a Helium Compressor</td>
</tr>
<tr>
<td>Figure V</td>
<td>Schematic Operating Map for a Steam Turbine</td>
</tr>
</tbody>
</table>
Nomenclature

B = Coefficient in heat transfer correlation (primary side)
C = Heat capacity of steam generator tube walls
D = Hydraulic diameter
G = Mass flux of coolant
H = Enthalpy
h = Film heat transfer coefficient
I = Moment of inertia
K = Thermal conductivity
M = Mass flow rate
N = Rotational speed
n = Exponent in heat transfer correlation (primary side)
Nu = Nusselt number
P = Pressure
Pr = Prandtl number
Q = Heat flux
R = Wall radius
r = Radius
R_f = Fouling factor
Re = Reynolds number
T = Temperature
t = time
U = Overall heat transfer coefficient
x = Steam quality
ΔP = Pressure differential
Δt = Time step size
ρ = Density
w = Rotational speed
μ = Viscosity

Superscripts

m = m^{th} time step (in Appendix only)
### Subscripts

<table>
<thead>
<tr>
<th>Subscript</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Overall</td>
</tr>
<tr>
<td>c</td>
<td>Compressor</td>
</tr>
<tr>
<td>f</td>
<td>Fluid</td>
</tr>
<tr>
<td>g</td>
<td>Gas</td>
</tr>
<tr>
<td>m</td>
<td>Time step</td>
</tr>
<tr>
<td>NB</td>
<td>Nucleate boiling</td>
</tr>
<tr>
<td>Sat</td>
<td>Saturation</td>
</tr>
<tr>
<td>t</td>
<td>Turbine</td>
</tr>
<tr>
<td>w1</td>
<td>Inner wall</td>
</tr>
<tr>
<td>w2</td>
<td>Outer wall</td>
</tr>
<tr>
<td>1</td>
<td>Compressor inlet</td>
</tr>
<tr>
<td>2</td>
<td>Compressor outlet</td>
</tr>
<tr>
<td>3</td>
<td>Turbine inlet</td>
</tr>
<tr>
<td>4</td>
<td>Turbine outlet</td>
</tr>
</tbody>
</table>
The Author would like to express his appreciation to M. M. Levine and C. J. Hsu for many helpful discussions in formulating the steam generator model.
SUMMARY

This document outlines the work carried out in the past fiscal year on the GCFBR safety research project. It was a low level effort budgeted at .5 MY, and consists of the development of improved steam generator and circulator (steam turbine driven helium compressor) models which will eventually be inserted in the HELAP (1) code. Furthermore, a code was developed which will be used to generate steady state input for the primary and secondary sides of the steam generator.

The following conclusions and suggestions for further work are made.

1) The steam-generator and circulator model are consistent with the volume and junction layout used in HELAP.

2) With minor changes these models, when incorporated in HELAP, could be used to simulate a direct cycle plant.

3) An explicit control valve model is still to be developed and would be very desirable to control the flow to the turbine during a transient. Initially this flow will be controlled by using the existing check valve model.

4) The friction factor in the laminar flow region is computed inaccurately, this might cause significant errors in loss of flow accidents.

5) It is felt that HELAP will still use a large amount of computer time and will thus be limited to design basis accidents without scram or loss of flow transients with and without scram. Finally it may also be used as a test bed for the development of prototype component models which would be incorporated in a more sophisticated system code, developed specifically for GCFBR's.
Introduction

In this report the progress made on the Gas Cooled Fast Breeder Reactor (GCFBR) program is outlined. This work consisted of improving the system code HELAP which was developed for a preliminary analysis of GCFBR transients. The preliminary analysis was carried out using a code based on RELAP 3 (MOD 36), which is based on an explicit algorithm. Improvements suggested by the preliminary work, which were considered during the reporting period are:

1) The adoption of an implicit algorithm, this was accomplished by changing to RELAP 3B.

2) Modification of the code to allow for two working fluids. The thermodynamic nature of the two fluids being left as arbitrary.

3) Development of realistic models for the steam generator and circulator (steam turbine driven helium compressor).

Items 1) and 2) were carried out under the auspices of a RSR funded LMFBR activity. No details of this work will be given here, since it is primarily a programming effort and involves no new component modeling.

Described in the next three sections is the work suggested under item 3) above. The first section describes the steam generator model, followed by a section discussing the circulator model. A final section is added in which a calculational technique is discussed which is used to determine steady state input for the steam generator. Finally an Appendix is added which outlines the numerical algorithm used in solving the steam generator equations.
I. STEAM GENERATOR

The current GCFR design has three steam generator modules, each of which is 10 ft. in diameter and approximately 57 ft. in length. These modules are axial flow heat exchanges made up of two separate helically wound tube bundles; these are the resuperheater and main steam generator bundles. The main steam generator bundle consists of the combined economizer, evaporator and superheater sections. Incoming feed water to the main bundle is converted to superheated steam which is used to drive the helium circulator turbine. The steam subsequently returns to the resuperheater before it flows to the main turbine. Water/steam flows upwards on the tube side of the heat exchanger and helium flows downwards across the tubes on the shell side. Figure I is a schematic depiction of this flow path.

Figure II shows the general arrangement of a steam generator inside the PCRV cavity. Helium carries the heat from the reactor down over the resuperheater, superheater, evaporator and economizer tube bundles. After its passage through the steam generator, it passes up through the annular passage between the outer shell of the steam generator and the thermal insulation to the main circulator. A more detailed description of the steam generator is available in the Preliminary Safety information document (PSID) and amendment #4 to the PSID. In the remainder of this section, the model used for simulating the steam generator in the HELAP code will be described.

The quantity of prime interest to be determined by the model is the amount of heat lost by the primary fluid (helium) and the amount of heat gained by the secondary fluid (water/steam). At steady state conditions, these two quantities are equal. However, during a transient, heat lost on the primary side is not equal to heat gained on the secondary side since the steam generator tube wall acts as a thermal inertia. The following description applies to an arbitrary primary volume, its corresponding secondary volume and the steam generator tube wall.

A description of the equations which determine the heat transfer rate from the primary to secondary follows in the next sub-section and the following two sub-sections deal with the heat transfer coefficients on the primary and secondary sides respectively.
A. Heat Exchanger Equations

The primary-to-secondary heat transfer rate and the temperature distribution within the tube wall, for times greater than zero, are determined by solving the following non-steady state heat conduction equation.

\[
\rho C \frac{\partial T}{\partial t} = K \left[ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right]
\]  

(1)

Boundary conditions at the inner and outer tube wall surfaces are given by

\[ r = R_{w1}; \quad -K \frac{\partial T}{\partial r} = Q_{w1} = U_{w1} \left[ T_{w1} - T_{sec} \right] \]  

(2)

and

\[ r = R_{w2}; \quad -K \frac{\partial T}{\partial r} = Q_{w2} = U_{w2} \left[ T_{pri} - T_{w2} \right] \]  

(3)

where

\[
\frac{1}{U_{w1}} = \frac{1}{h_{w1}} + R_{fw1}
\]

\[
\frac{1}{U_{w2}} = \frac{1}{h_{w2}} + R_{fw2}
\]

The initial condition \((t=0)\) is given by:

\[
T(r) = T_{w1} + \frac{T_{w2} - T_{w1}}{\ln \left( \frac{R_{w2}}{R_{w1}} \right)} \ln \left( \frac{r}{R_{w1}} \right)
\]

(4)

In equation \((4)\) wall temperatures \(T_{w1}\) and \(T_{w2}\) are determined from equations \((2)\) and \((3)\) respectively. The solution of these equations require heat fluxes and temperatures. The heat fluxes \(Q_{w1}\) and \(Q_{w2}\) are computed from input quantities. Heat transfer coefficients \(U_{w2}\) and \(U_{w1}\) are in general also...
also functions of $T_{w1}$ and $T_{w2}$, since the transport properties (conductivity, viscosity, etc.) are evaluated at the mean film temperature which is defined as $(T_{wall} + T_{average Fluid})/2.0$. In the special case of boiling, the value of $U_{wl}$ is also a function of heat flux $Q_{w2}$. It is thus necessary that Equations (2) and (3) be solved iteratively when determining $T_{w1}$ and $T_{w2}$.

For times greater than zero, Equation (1) with boundary conditions (2) and (3) and initial condition (4) are solved numerically using an implicit scheme. This scheme is patterned after the one used in the HELAP code for determining the temperature distribution within the fuel pin. Six radial nodes within the tube wall are allowed for, resulting in six simultaneous equations. These six equations form a matrix equation which is solved for the temperature distribution within the tube wall. The desired heat fluxes $Q_{w1}$ and $Q_{w2}$ are determined from equations (2) and (3). In solving this equation density $\rho$, and the heat capacity $C$, are assumed to be constant over the temperature range of interest. However, the conductivity $K$ is allowed to be a function of temperature and its value is determined at the average temperature of the tube wall. Details of the numerical algorithm used are outlined in the Appendix.

B. Heat Transfer Correlation for the Primary Side

The heat transfer correlation used in computing the primary side heat transfer coefficient, $h_{w2}$ is based primarily on work carried out by Grimison in 1937. Grimison correlated data for straight tubes in both staggered and inline arrangements with gas flowing across the tube bundles. Subsequently experiments have been carried out using helically wound bundles which were either staggered or inline arrangements. These experiments confirmed the original work of Grimison and together they form an experimental base from which an acceptable heat transfer correlation could be extracted.

A correlation between the Nusselt number, $Nu$, Reynolds number, $Re$, and Prandtl number, $Pr$, of the following form is adopted.

$$Nu = B (Re)^n (Pr^{1/3})^{1/3}$$

$$Nu = B (Re)^{0.49} (Pr^{1/3})^{0.69}$$

(5)
The transport properties (Pr, viscosity, etc.) are determined at the volume pressure and mean film temperature. To determine the Reynolds number, a mass flow rate per unit area is required; this flow rate is based on the flow across the cross section of minimum area. The values of the coefficient B and the exponent n are a function of the bundle arrangement i.e. an inline or staggered arrangement; and on the values of the transverse and longitudinal pitch. These values are part of the heat exchanger input and may be different in each heat exchanger volume. It is thus possible to allow for different arrangements in the different components of the steam generator. Finally, the value of B is also used to reflect the change in heat transfer coefficient due to the presence of baffles. The determination of B, which depends on the parameters mentioned above and which reflects the enhancement of the heat transfer coefficient due to baffles, is described in a latter section where the method of obtaining the steady state input for the steam generator is outlined.

C. Heat Transfer Correlations for the Secondary Side

The heat transfer correlations used on the secondary side (steam/water) are based on correlations currently in use in RELAP 3B. Six modes of heat transfer are provided for, the choice of the correct mode is based on fluid quality and surface temperature. The correlations corresponding to these six different modes are given below.

Mode 1. - Subcooled Forced Convection \( x \leq 0.0 \ T_{\text{w1}} < T_{\text{NB}} \)

\[
h = \frac{0.023K}{D} \left( \frac{DG}{\mu} \right)^{0.8} \left( \frac{Pr}{\mu} \right)^{1/3} \left[ \frac{\mu \left( T_{\text{sec}} \right)}{\mu \left( T_{\text{w1}} \right)} \right]^{0.14}
\]

Mode 2. - Subcooled Nucleate Boiling \( x < 0.0 \ T_{\text{w1}} > T_{\text{NB}} \)

\[
T_{\text{NB}} = T_{\text{sat}} + 0.072 + 1260 \ Q_{\text{w1}}^{0.5}
\]

\[
h = \frac{Q_{\text{w1}}}{T_{\text{w1}} - T_{\text{sec}}}
\]
Mode 3. - Nucleate Boiling $0.0 < x < \cdot 1$

The heat transfer coefficient is calculated by interpolating with respect to quality between equation (8) and (9).

Mode 4. - Forced Convective Boiling $13) 0.1 \leq x \leq 0.6$

$$h = 6700 \left[ \frac{q_{w1}}{G \rho g} + 0.00035 \right] \left\{ \left( \frac{\rho f}{\rho g} \right)^{0.9} \left( \frac{\mu g}{\mu_f} \right)^{0.1} \right\} 0.66$$

$$+ \left[ \left( \frac{0.023 K}{D} \right)^{0.8} \left( \frac{D G (1-x)}{\mu_f} \right)^{0.8} \left( Pr \right)^{0.4} \right]$$  (9)

The physical properties are evaluated at saturation conditions.

Mode 5. - Forced Convective Boiling

The heat transfer coefficient is calculated by interpolating with respect to quality between equations (9) and (10).

Mode 6. - Single Phase Steam

$$h = 0.023 \frac{K_g}{D} \left( \frac{DG}{\mu_g} \right)^{0.6} \left( Pr_g \right)^{0.4}$$  (10)

Physical properties are evaluated at the mean film temperature

$$(T_{w1} + T_{sec}) / 2.0$$

The heat transfer coefficients computed using the above correlations would result in values applicable to straight tubes. The enhancement of the heat transfer coefficient due to secondary flows induced in the helically wound tubes of the steam generator is accounted for by means of a Nusselt number multiplier, as follows: $10,15)$
\[
\text{Nu (helical)} = \text{Nu (straight)} \left[ 0.562 + 0.09177 \left( \frac{\text{Re} \left( \frac{D}{D_{co}} \right)^2}{1/4} \right) \right] \quad (11)
\]

where \((D/D_{co})\) is the ratio of the hydraulic diameter of the tube to the mean bundle diameter. Finally, in those situations where the final heat transfer coefficient exceeds 10,000 it is limited to 10,000. \(\text{9)}\)
II. CIRCULATOR MODEL

Each of the three main coolant loops is equipped with one main circulator. Each circulator consists of a single stage axial flow compressor and a single stage steam turbine. The helium compressor is connected to the outlet of the steam generator and discharges the compressed helium to the reactor inlet plenum. The steam turbine is located between the superheater outlet and the resuperheater inlet. Thus, the full flow from the steam generators drives the circulator turbines before passing to the resuperheater and then the main steam turbine. This layout is shown schematically on Figure 1. Each circulator unit is mounted vertically in its respective steam-generator-cavity closure plug, as shown in Figure III.

The compressor and steam turbine discs are mounted on a single vertical shaft overhung at opposite ends of a central housing that contains the thrust and journal bearings. These bearings are of the water lubricated type. For more details on the circulator, the reader is referred to the PSID. Quantities of prime interest to be determined by the circulator model are the pressure increase on the primary side due to compression (ΔPc), the pressure drop on the secondary side due to expansion (ΔPt), the energy added to the primary fluid due to compression, and the energy lost by the secondary fluid due to expansion. These quantities can be determined by solving an energy balance equation for the circulator.

The energy balance of a turbo-compressor assembly is given by:

\[ \frac{1}{2} I \omega^2 = H_t - H_c \]  

(12)

It is seen that if the turbine produces more energy than the compressor needs the rotational speed of the shaft increases and vice versa.

In order to solve equation (12) at any instant in time during the transient it is written in digital form using the trapezoidal rule, with the new rotational speed \( N_{n+1} \) as the unknown.
\[ N_{m+1} = \left[ \frac{2\Delta t}{1} W_{ct} + N_m^2 \right]^{1/2} \]  \hfill (13)

where
\[ W_{ct} = H_{c(m+1)} - H_{c(m+1)} \]  \hfill (14)

In the above equation the subscripts \((m+1)\) and \(m\) refer to the current and the previous time steps respectively.

In attempting a solution of Equation (13) it is assumed that \(P_1, T_1, P_3, T_3, M_c, M_m\) and \(N_m\) are known. The desired parameter \(H_{c(m+1)}\), \(H_{t(m+1)}\), \(\Delta P_c\), \(\Delta P_t\) and \(N_{m+1}\) can thus be computed. The first two quantities are the energy added and subtracted from the primary and secondary fluids respectively, the next two quantities are the pressure increase and pressure decrease of the primary and secondary fluids respectively and the final quantity is the new shaft speed of the circulator which is required for the solution of the following time step.

Since both \(N_{m+1}\) and \(W_{ct}\) are unknown quantities in equation (13) it is solved iteratively. The solution follows the following procedure.

i) Assuming the shaft speed to be \(N_m\) and the expansion and compression in the circulator to isentropic a value for \(W_{ct}\) can be determined using the methods to be described in the next two sub-section.

ii) It is now possible to compute a new value of \(N_{m+1}\) using equation (13). In principle, the procedure could be repeated until a converged solution is achieved. However, it has been found that the rate of convergence is unacceptably slow and a Newton type acceleration scheme is used. In this scheme the function

\[ F = N_{m+1}^2 - N_m^2 - 2\Delta t W_{ct} \]  \hfill (15)

is forced to zero.
The improved prediction of \( N_{m+1} \) is given by

\[
N_{m+1}^{\text{(improved)}} = N_{m+1} - \frac{F'}{F},
\]

(16)

\[
F' = \frac{F_{m+1} - F_m}{N_{m+1} - N_m}
\]

(iii) A revised value of \( N_{m+1} \) is thus computed and used in step i) rather than the original value computed in step ii). Convergence is achieved when the fractional change in the shaft speed from iteration to iteration is within a prescribed error.

The terms \( H_g \) and \( H_c \) will be discussed in the following two sub-sections.

A. Compressor Work (\( H_g \))

It is assumed that the inlet temperature (\( T_1 \)), inlet pressure (\( P_1 \)) and the flow rate (\( M_c \)) to the compressor are known. Furthermore the operating map, which relates outlet pressure to flow rate is assumed given. The specific parameters involved in the map of the current model are

\[
\frac{\Delta P_c}{P_1} \text{ vs. } \frac{M_c \sqrt{T_1}}{P_1}
\]

for various rotational speed parameters \( \frac{N}{\sqrt{T_1}} \)

A family of similar curves results which are shown schematically on Figure IV. These were obtained from the FULTON plant PSAR.\(^{16}\). The values of temperature and speed are made dimensionless by their respective rated values. These curves are limited at one end by surging, (too low a flow rate for a given rotational speed) and at the other end by choked flow.\(^{17}\) It is now possible, knowing the values of \( P_1, T_1, P_c \) and \( N \) to determine the value of \( \Delta P_c \) and hence \( P_2 \) from this family of curves. Making the assumption that the compression is
isentropic, the outlet temperature can be computed from \(^{18}\)

\[
T_2 = T_1 R^{.4017} \left[ -1505 (\varphi(T_2) P_2 - \varphi(T_1) P_1) \right]
\]  

(18)

where \(R\) is the compression ratio \((P_2 / P_1)\), and

\[
\varphi(T) = \left( T^{-1/3} - 12580T^{-2} \right) \frac{1}{T}
\]

(19)

The work required to carry out this compression is given by the enthalpy change from the inlet condition \((T_1, P_1)\) to the outlet condition \((T_2, P_2)\) i.e.

\[
\Delta H = .43006 \left[ 5.196 (T_2 - T_1) + 3.13 \left( \left( T_2^{-1/3} - 4715T_2^{-2} \right) P_2 \right. \right.
\]

\[
\left. \left( T_1^{-1/3} - 4715T_1^{-2} \right) P_1 \right] \right]
\]

(20)

In the above expressions Equations 18 - 20 the temperatures are in \(^\circ\)K and the pressures in atmospheres.

The actual work required by the compressor is obtained by dividing the value of work determined above by the efficiency and multiplying it by the flow rate.

Compressor efficiency \(\eta_c\) is determined from input data, in which efficiency is read in as a function of \(\frac{M_c T_1}{P_1 N}\)

\[
\eta_c = \frac{\Delta H M_c}{\eta_c}
\]

(21)

B. Turbine Work \((H_t)\)

As in the case of the compressor, it is assumed that the inlet temperature \((T_3)\), inlet pressure \((P_3)\) and flow rate \((M_c)\) are known. Furthermore,
the operating characteristics for the turbine are assumed to be given. In the case of the turbine, they are a plot of $P_4/P_3$ against $M_c\sqrt{T_3/P_3}$ for various shaft speeds, $N/\sqrt{T_3}$. A schematic representation of such a family of curves is given in Figure V, it is seen that the curves show a choking limit i.e. regardless of increase in pressure ratio no increase in mass flow rate results. It is thus possible, with the available input data to obtain ($P_4$) the outlet pressure. In order to compute the final temperature ($T_4$) and work done under isentropic expansion from initial state ($P_3, T_3$) to final state ($P_4, T_4$) appropriate subroutines from the code system STEAM-67 are used. These subroutines form a module which is built into HELAP. Thus given $T_3, P_3$ and $P_4$ the output from the above module is $T_4, H_3$, and $H_4$ from which $\Delta H$ the work due to isentropic expansion can be computed.

$$\Delta H = H_3 - H_4$$ (22)

The actual turbine work done is obtained by multiplying $\Delta H$ by the turbine efficiency $\eta_c$ and the mass flow rate $M_c$. The turbine efficiency is obtained from input data which relates efficiency to $N/\Delta H$.

Thus the actual turbine work is given by

$$H_t = \Delta H \eta_c M_c$$ (23)

III. STEADY STATE DETERMINATION

As part of the input to any RELAP, or RELAP based calculation i.e. HELAP or NALAP it is required to start with a steady state solution. This implies that the enthalpy changes in the fluid for any volume in the circuit has to be consistent with the energy added to or subtracted from that volume. In the context of the steam generator this involves balancing the heat transfer rate determined by a product of the overall heat transfer coefficient and the mean fluid temperature difference, and the heat transfer rate implied by the
power fraction deposited in or subtracted from a volume. A steady state of the turbine-compressor module implies that the work required by the compressor is exactly balanced by the work supplied by the turbine. Furthermore this balance is required to occur at the steady state shaft speed of the unit, implying a definite position on the operating maps of the turbine and the compressor. These positions on the respective operating maps imply pressure differentials, which also have to be consistent with the steady state condition of the reactor plant.

The balance outlined above is not easily obtained and the only systematic manner of achieving it is by means of a numerical procedure. A program to determine the above balance was constructed and is described below.

Input to this program are the necessary dimension of the heat exchanger i.e. area, tube length and size, secondary side fouling factor, primary side baffle factor etc., the primary and secondary volume pressures and flow rates. The computation proceeds sequentially, starting at the cold end i.e. primary side outlet and secondary side inlet, and proceeds to the hot end. A final step is to balance the circulator module. Values of temperature and enthalpy for the secondary inlet are known from the feed pump outlet conditions, and the primary side discharge conditions are equal to the compressor inlet, which is also known. With the above input, the computation proceeds along the following steps:

I) A power fraction, $P_2$, to be transferred from the primary side to the secondary side is assumed. It is thus possible to determine the corresponding volume inlet enthalpy on the primary side and the outlet enthalpy on the secondary side. On the primary side the inlet temperature can now be determined, knowing the heat capacity, $C_p$, which in the case of helium is a function of temperature and pressure only. Thus

$$ T \text{ (Primary inlet)} = \frac{H_{in} + \Delta H}{C_p} \quad (24) $$

where

$$ C_p = .2389 \left[ 5.196 - 1.043 \left( T^{-1/3} - 28300 T^{-2} \right) \frac{P}{T} \right] \quad (25) $$

-14-
T and P are the temperature and pressure in °K and atmospheres respectively, $H_{in}$ is the inlet enthalpy and $\Delta H$ the enthalpy change corresponding to $P_c$.

On the secondary side the determination of outlet temperature could, in principle, be carried out in a similar manner. However, the heat capacity is not known, with great enough precision. Instead, the steam tables as used in the HELAP code, (same temperature and pressure mesh) are used to determine the temperature corresponding the volume pressure and outlet enthalpy. Finally, it is possible to compute a heat flux $Q_p$ which corresponds to the assumed power fraction $P_f$ transferred from the primary to the secondary and steam generator dimensions.

II) It is now possible to compute the primary side heat transfer coefficient, as outlined in Section I. (A function of flow rate, pressure, and mean film temperature) and thus the primary side wall temperature.

$$T_w2 = T_{pri} - \frac{Q_p}{w2 \left( T_{w2}, T_{pri}, P_{pri}, G_{pri} \right)}$$

This step is carried out iteratively since the mean film temperature is a function of the wall temperature, which in turn depends on the heat transfer coefficient $U_w2$.

III) Knowing the primary wall temperature and the heat flux, it is now possible to compute the secondary side wall temperature. This is possible since it is assumed that the steam generator is operating at steady state and thus the heat flux per unit area on the primary side equals that on the secondary side.

$$T_{w1} = T_w2 - \frac{Q_{sec} R_{w2}}{K \left( T_{w1}, T_w2 \right)} ln \left( \frac{R_{w2}}{R_{w1}} \right)$$

This calculation is carried out iteratively since the wall heat conduction, $K$, is a function of the mean wall temperature.

IV) The secondary side heat transfer coefficient can now be determined. This determination is based on the correlations outlined in Section I.
In general this coefficient is a function of flow rate, mean film temperature, pressure, and in the case of boiling also the heat flux. The secondary side enthalpy is also required to determine the secondary side quality.

V) Using the results of steps II, III and IV the overall heat transfer coefficient can be determined

\[
U_A = \frac{1}{\frac{1}{U_2} + \frac{R_{w2}}{R_{w1}w_1} + \frac{R_{w2}}{K} \ln \frac{R_{w2}}{R_{w1}}} \quad (28)
\]

Furthermore a heat flux \( Q_U \) can be determined based on the product of \( U_A \) and the temperature difference between the mean fluid temperature on the primary and secondary sides.

\[
Q_U = U_A \left[ \left( \frac{T_{pri (in)} - T_{pri (out)}}{2} \right) - \left( \frac{T_{sec (in)} - T_{sec (out)}}{2} \right) \right] \quad (29)
\]

VI) At this point the heat flux \( Q_U \) computed from step V and the heat flux \( Q_p \) computed from step I are compared. If they are in acceptable agreement the desired balance is achieved for the volumes under consideration. If this agreement is not sufficiently precise the power fraction \( P_f \) is suitably modified and steps I thru V are repeated.

The above procedure is repeated for each of the steam generator volume pairs, starting at the secondary side inlet and the primary side outlet. The full flow of the steam generator flows through the steam turbine which drives the helium compressor. Helium from the outlet of the steam generator flows into the inlet plenum of the compressor where it is compressed. A balance of the steam turbine-helium compressor module is achieved when the work done by the turbine exactly balances the work required by the compressor. This desired balance is required to be achieved at the design speed of the circulator, which together with the performance maps gives the pressure changes across the turbine and compressor, which must also equal the designed operating condition. A correct choice of characteristics (operating map) and inlet conditions results in a balanced circulator module.
IV. CONCLUSION

The following conclusion can be made regarding the preceding work:

1) The steam generator and circulator (steam turbine driven helium compressor) models are consistent with the volume and junction formulation used in the HELAP code. No attempt at analyzing hydrodynamic flow instabilities and their consequences, or details of compressor stall are made. Such computing algorithms exist (21, 10, 17, 22) however, their inclusion in HELAP would be inconsistent with the remainder of the code.

2) When the models described above are incorporated into HELAP, it would also be possible to simulate a direct cycle gas turbine plant (23). This would be done by simulating the recuperator by the steam generator model. In this case the secondary side heat transfer coefficient, $h_{w2}$, will be computed using a different correlation, which would be consistent with the high pressure side of the recuperator. The steam turbine can be replaced by a helium gas turbine and the pre-coolers can be represented by dump heat exchangers.

Listed below are further improvements which would make this code more accurate in its representation of the plant during a transient.

3) To control the steam turbine speed, which determines the compressor outlet pressure, it is necessary to control the steam flow rate into the turbine. This requirement implies the ability to control the flow rate between two volumes in HELAP by means of a valve model. At the present time no realistic control valve model exists in the HELAP code, and the steam flow control will be simulated by using multiple check valves scheduled to close sequentially. This results in a pre-determined step wise reduction in flow rather than a continuous variation of the flow which could be pre-determined, or be a function of an internally computed quantity i.e. clad temperature.

4) The shortcomings in the code due to inaccurate determination of the friction factor in the laminar flow range still exist. This might lead to significant error in loss of flow transients.
Finally a comment will be made about the types of problems the HELAP code will eventually be able to deal with.

5) Despite the fact that the models outlined in this report will be incorporated in the version of RELAP which uses an implicit time step algorithm (implying larger time steps than the previous code) a large increase in time step size is not anticipated. The largest time step used, with confidence, in water reactor transients is approximately .002 sec., which is a four fold increase over corresponding calculations using the explicit time step algorithm. Unless the time step size in gas cooled systems is very much larger than .002 secs transients lasting longer than 50 - 100 secs cannot be expected to be analyzed on a routine basis. This limitation would limit the code to the analysis of design basis accidents, without scram and loss of flow accidents with or without scram. In all three of these cases partial clad failure is expected within 100 sec. One further use of this code would be its usefulness as a test bed for the development of new component models to be used in a more advanced, yet to be developed, system code.
In this appendix, the details of the finite difference equations used in solving the heat exchanger transient response is given. The method is patterned after the method used in HELAP to compute the temperature distribution within the fuel pin.

First, the inner mesh points will be treated to be followed by consideration of the boundary points. Within the tube Equation (1) applies, which may be written as:

$$\rho C_p \frac{\partial T}{\partial t} = \nabla \cdot K \nabla T$$  \hspace{1cm} (A1)

In arbitrary mesh space within the steam generator tube wall is shown in Figure 1A.

Figure 1A. Mesh Point Layout

The finite difference equation is obtained by integrating equation (A1) over the volume indicated by the dotted lines. Since only cylindrical geometry is of interest, i.e. no axial gradients, the volume to be considered is that which results when rotating the figure about \( r = 0 \). Finally, the factor \( 2\pi \) common to all terms will be omitted.
Using forward differences, the first term becomes:

\[ \int \rho C \frac{dT}{\Delta t} dv = \rho C \frac{(T_{n+1}^m - T_n^m)}{\Delta t} \left[ \frac{\Delta s}{2} \left( r_n - \frac{\Delta r}{4} \right) + \frac{\Delta s}{2} \left( r_n + \frac{\Delta r}{4} \right) \right] \]  

Equation (A2)

The subscript \( n \) refers to the \( n \)th mesh point and the superscripts \( m \) and \( m + 1 \) refer to the time \( t_m \) and \( t_{m+1} \) respectively.

The second term of equation (A1) can be written:

\[ \int_v \cdot \nabla T dv = \int_s \nabla T ds \]  

Equation (A3)

For cylindrical geometry this is approximated by

\[ \int_s \nabla T ds = \frac{K(T_n - T_{n-1})}{\Delta r} \left( r_n - \frac{\Delta r}{2} \right) + \frac{K(T_{n+1} - T_n)}{\Delta r} \left( r_n + \frac{\Delta r}{2} \right) \]  

Equation (A4)

Now defining

\[ \alpha_n^v = \frac{\Delta s}{2} \left( r_n - \frac{\Delta r}{4} \right) \quad \alpha_n^v = \frac{\Delta s}{2} \left( r_n + \frac{\Delta r}{4} \right) \]

\[ \alpha_n^s = \frac{1}{\Delta r} \left( r_n - \frac{\Delta r}{2} \right) \quad \alpha_n^s = \frac{1}{\Delta r} \left( r_n + \frac{\Delta r}{2} \right) \]

Then collecting terms, the finite difference from the equation (A1) can be written

\[ \frac{T_{n+1}^m - T_n^m}{\Delta t} \delta_n = \delta_n \]  

Equation (A5)
The superscript on $\delta_n$ has been omitted thus far. If a purely explicit formulation is desired, the following relationship results

$$\frac{T_n^{n+1} - T_n^{n}}{\Delta t} \delta_n = \delta_n^m$$  \hspace{1cm} (A6)

However, a more desirable implicit formulation will be used, where the $\delta_n$ will be a mixture of the $n^{th}$ and $(n+1)^{th}$ time steps, i.e.

$$\frac{T_n^{n+1} - T_n^{n}}{\Delta t} \delta_n = \frac{1}{2} (\delta_n^m + \delta_n^{m+1})$$  \hspace{1cm} (A7)

Writing out equation (A7) in full and collecting terms the following equation for the $n^{th}$ internal mesh point

$$a_n T_n^{n-1} + b_n T_n^{n} + c_n T_n^{n+1} = d_n \quad (n = 2, 5)$$  \hspace{1cm} (A8)

where

$$a_n = - \frac{K \Delta t}{2} \left( \frac{T_n}{\Delta r} - \frac{1}{2} \right)$$

$$b_n = e_n - a_n - c_n$$

$$c_n = - \frac{K \Delta t}{2} \left( \frac{T_n}{\Delta r} + \frac{1}{2} \right)$$

$$d_n = e_n \rho C_{\Delta \Delta r \tau}$$
At the inner wall surface, the boundary condition is given by

\[- k \frac{dT}{dr} = U_{w1} \left[ T_{w1} - T_{sec} \right] \quad (A9)\]

Using equation (A9) in evaluating the surface integral on the inner wall surface, an equation analogous to equation (A8) can be divided, which has the form

\[ b_o T_{w1}^{m+1} + c_o T_1^{m+1} = d_o \quad (A10) \]

where

\[ c_o = - \frac{k^2 \Delta r}{2} \left( \frac{R_{w1}}{\Delta r} + \frac{1}{2} \right) \]

\[ b_o = \rho C K \frac{\Delta r}{2} \left( R_{w1} + \frac{\Delta r}{4} \right) + \frac{K R_{w1}^m R_{w1} \Delta t}{2} - c_o \]

\[ d_o = - c_o T_1^m + T_{w1}^m \left[ - \frac{k^2 \Delta r}{2} \left( R_{w1} + \frac{\Delta r}{4} \right) + \frac{K R_{w1} \Delta t}{2} U_{w1}^m + c_o \right] \]

\[- \frac{1}{2} K R_{w1} \Delta t \left[ U_{w1} T_{sec}^{m+1} + U_{w1} T_{sec}^m \right] \]

Similarly at the outer wall the boundary condition is given by

\[- k \frac{dT}{dr} = U_{w2} \left[ T_{pri} - T_{w2} \right] \quad (A11) \]
The finite difference form at this surface is given by

\[ a_N T_{N-1}^{m+1} + b_N T_w^m = d_N \]  \hspace{1cm} (A12)

where

\[ a_N = -\frac{K_r \Delta r}{2} \left( \frac{R_w}{\Delta x} - \frac{1}{2} \right) \]

\[ b_N = \frac{K_r C_F}{2} \left( \frac{R_w}{\Delta x} - \frac{\Delta r}{4} \right) + \frac{K R_w U_{w2}^m \Delta t}{2} - a_N \]

\[ d_N = -a_N T_{N-1}^m + T_w^m \left[ \frac{K_r C_F}{2} \left( \frac{R_w}{\Delta x} - \frac{\Delta r}{4} \right) - \frac{K R_w U_{w2}^m \Delta t}{2} - a_N \right] + \frac{1}{2} K R_w \Delta t \left[ U_w^m T_{pri}^{m+1} + U_{w2}^m T_{pri}^m \right] \]

The equations (A8), (A10) and (A12) form a tri-diagonal matrix equation of the following form

\[
\begin{pmatrix}
  b_0 & c_0 & 0 & 0 & 0 \\
  a_1 & b_1 & c_1 & 0 & 0 \\
  0 & a_2 & b_2 & c_2 & 0 \\
  & & & & \\
  & & & & \\
  & & & & \\
  & & & & \\
  & & & & \\
  & & & & \\
  & & & & \\
  & & & & \\
  & & & & \\
  & & & & \\
  a_{N-1} & b_{N-1} & c_{N-1} & 0 & a_N & b_N \\
  & & & & & T_{tw2} \end{pmatrix}
\begin{pmatrix}
  T_{w1}^m \\
  T_1^{m+1} \\
  . \\
  . \\
  . \\
  . \\
  . \\
  . \\
  . \\
  . \\
  . \\
  T_{tw2}^{m+1} \\
  d_N \end{pmatrix}
= \begin{pmatrix}
  d_0 \\
  . \\
  . \\
  . \\
  . \\
  . \\
  . \\
  . \\
  . \\
  . \\
  . \\
  d_N \end{pmatrix} \hspace{1cm} (A13)
\]

Equation (A13) is solved by a system routine which employs the Gauss elimination scheme.
References


4) Martin, B. A. et. al. NALAP - A LMFBR System Transient Code, Brookhaven National Laboratory (To be published).


24) Hsu, C., Private Communication (1975).
Figure 1. Schematic Layout of Steam Generator and Circulator
Figure III. Gas Cooled Reactor Circulator
Figure IV. Schematic Operating Map for a Helium Compressor
Figure V. Schematic Operating Map for a Steam Turbine

\[ \nu = \frac{N}{N_0} \]

\[ \psi = \frac{P_1}{P_0} \]

\[ \theta = \frac{T_1}{T_0} \]