MEASUREMENT OF THE $\pi^-p \rightarrow \pi^0 n$ and $\pi^-p \rightarrow \eta n$ DIFFERENTIAL
CROSS SECTIONS AT BEAM MOMENTA FROM 20 TO 200 GeV/c

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Abstract

This dissertation presents the results of a measurement of the $\pi^-p \rightarrow \pi^0 n$ and $\pi^-p \rightarrow \eta n$ differential cross sections at six pion beam momenta from 20 to 200 GeV/c. The data for these cross sections were collected at the Fermi National Accelerator Laboratory in Batavia, Illinois. A 73.5 x 73.5 cm lead-scintillator hodoscope was used to detect the two photons from the decay of the π^0 and the η ; all other reactions were eliminated by detection of their charged particles and additional photons in an array of veto counters. The zero-degree charge exchange cross sections determined from these measurements are shown to be consistent with the measured π^+p and π^-p total cross section differences. Effective Regge trajectories for the ρ and the A_2 are also extracted from the data.

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I. Introduction

Ever since pions were artificially produced at the Berkeley cyclotron in 1948, their scattering distributions from nucleon targets have provided important information about strong interactions. Despite the large amount of data accumulated, no exact quantum mechanical theory of the strong interactions has yet been formulated, and only a few phenomenological principles have been discovered. One of these principles is that the strong interactions of similar hadrons are independent of charge (isospin invariance). This and the Wigner-Eckhart theorem for spins make it possible to relate all pion-nucleon elastic and charge exchange scattering parameters to four independent amplitudes. These complex amplitudes, which represent the maximum amount of available information about these elastic and charge exchange reactions*, can be determined (up to an overall phase) from seven measurements: differential cross sections for $\pi^+p \rightarrow \pi^+p$, $\pi^-p \rightarrow \pi^-p$, and $\pi^-p \rightarrow \pi^0n$, polarization measurements for the same reactions, and any spin-rotation measurement for an elastic scattering. Unfortunately, the polarization and spin-rotation measurements are so complex that very few charge exchange polarization measurements have been made, and spin-rotation parameters have only been measured

* These amplitudes also give some information about inelastic processes. The imaginary parts of the forward π^+p and π^-p amplitudes are related to the total π^+p and π^-p cross sections, and the zero-degree $\pi^-p \rightarrow \pi^0n$ differential cross section is proportional to the difference between these two total cross sections (for a complete discussion, see Section IV.A). At high energies, the measurement of the forward charge exchange cross section provides a very sensitive determination of that difference.

at 6 GeV/c.¹ Consequently, only at that one energy can the exact amplitudes be derived from the data.²

Eventually, enough parameters will be measured at other energies so that additional amplitude determinations will be possible. However, until those measurements are completed, various approximations must be made to extract the amplitudes from a less than complete set of data.

Pion-nucleon amplitudes can be approximated if the very stringent assumption is made that only a few (usually less than six) angular momentum states contribute to each amplitude. This assumption is reasonable at low energies because of the short-range nature of strong interactions. If there are no interactions past a radius, r_0 , then the maximum angular momentum state that can affect an amplitude is

$$l_{\max} = p_{\text{beam}} r_0 / \hbar.$$

For an r_0 of 1 Fermi and an $l_{\max}$ of 5, p_{beam} is approximately 1 GeV/c. Once the cutoff in angular momentum states is assumed, the amplitude is expanded in terms of partial waves and a best fit to the data can be made by varying the coefficients of each partial wave. Such analyses have quite a few inherent problems and ambiguities, but nevertheless they have revealed many properties of low energy pion-nucleon amplitudes.³

Two difficulties arise when this method is extended to higher beam momenta; first, the number of partial waves needed in the expansion implies a complexity that cannot be handled by available computers; and second, there have as yet been no

high energy measurements of the full 180° differential cross sections and polarization parameters needed to constrain the higher partial waves. However, the formal existence of the expansion is useful at these energies. By analytical continuation of the angular momentum variable of the partial waves into the complex plane, the expansion can be expressed as a sum over a set of analytical singularities (the Regge expansion). Each residue of a singularity adds together the effects of many partial waves. For most reactions, many singularities must be included to fit the data. However, for the two reactions

$$\pi^- p \rightarrow \pi^0 n$$

and

$$\pi^- p \rightarrow \eta n$$

(hereafter referred to as charge exchange and eta production), a single pole-type singularity is thought to dominate each expansion, and the high energy differential cross sections should behave like a power law of center-of-mass energy

$$d\sigma/dt = A(t) E_{cm}^{-b(t)}.$$

For charge exchange, the quantum numbers transferred in the t -channel are those of the ρ meson (see figure I.1); therefore the pole term in the Regge expansion is called the ρ -pole. Likewise, the pole for eta production is called the A_2 -pole. Although a second singularity is needed to explain polarization measurements of the two reactions, very simple Regge-type expansions fit all of the available data from 4 to 50 GeV/c.⁴

To find out if this power law also applies to higher energies, the measurement of the charge exchange and eta production

CHARGE EXCHANGE

ETA PRODUCTION

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Figure I.1

t-Channel Exchanges in Charge exchange
and Eta Production

differential cross sections up to 200 GeV/c was undertaken at Fermi National Accelerator Laboratory. That investigation is the subject of this dissertation.

II. The Apparatus

A. General Description of the Experiment

If the low energy cross sections are extrapolated to 100 GeV/c, the π^-p total cross section should be almost 8000 times larger than the $\pi^-p \rightarrow \pi^0 n$ cross section. To measure the latter, an event trigger must be designed that will accept only a small percentage of all possible reactions. When all events are rejected in which charged particles emerge from a target, both the non-interacting beam particles and most of the other interactions are eliminated (total neutrals cross section/total cross section = .1% at 100 GeV/c). In this experiment, the target was completely surrounded with charged particle veto counters ("anticounters"). The combination of a signal from the beam telescope and no signal from the anticounters indicated that a neutral final state reaction had occurred and triggered the readout of the apparatus.

Charge exchange events have only two photons and a neutron in the final state; 38% of eta production events have the same signature*. At high energies, the two photons go forward from

*The only eta production events analyzed in this experiment were those in which the eta decayed into two photons; hereafter, "eta production" will refer only to those events.

the target and no gammas emerge in any other direction. The energies and positions of the two photons were measured by a 73.5 x 73.5 cm lead-scintillator hodoscope. Unfortunately, the energy resolution of this detector was not sufficient to distinguish charge exchange π^0 's from the π^0 's in reactions such as

$$\pi^- p \rightarrow \pi^0 N^* \rightarrow \pi^0 (\pi^0 n)$$

To identify these and other competing neutral final state reactions, lead-scintillator and lead-Cerenkov plastic photon detectors ("gamma vetoes") were placed in all directions not covered by the detector. For each trigger, information from the detector, from the gamma vetoes, and from various beam definition counters and scalers was read out and recorded on magnetic tape.

In order to maintain constant transverse momentum acceptance at all energies, the distance between the target and the detector was set proportional to the beam momentum. The gamma vetoes were moved along the beam line and their apertures adjusted so that they detected all photons which missed the detector.

The neutrons emerged from the target essentially perpendicular to the beam line. Since the neutron kinematic parameters were not needed to identify either charge exchange or eta production, it was desirable to let the neutrons escape undetected. In fact, great care was taken to make the veto system as transparent as possible to neutrons.

In the following sections, details of the construction and operation for each system are discussed.

B. The Beam

The experiment was done in the Meson Laboratory of Fermi National Accelerator Laboratory. During the data-taking periods (several time intervals between December, 1973, and September, 1974) the main accelerator was operated at 300 GeV/c. Protons of that momentum were extracted from the main ring and focused on a 2/3 interaction-length Be target in the Meson Laboratory target hall. The M2 beam line⁵ was used to transport secondary pions to the experiment's target area. This beam was a two-stage system with each stage comprised of two quadrupole magnets and a string of dipole magnets. The first stage formed a momentum disperse focus at a set of horizontal and vertical collimators; the second stage recombined the beam and focused it on the target. Beam particle momentum was selected by setting the field of the first dipole bend-string; momentum spread was determined by the horizontal collimator width ($\Delta p/p$ was kept at $\pm 1\%$). The beam spot size varied from a little over 1 cm at 20 GeV/c to less than 5 mm at 200 GeV/c. Beam intensity was typically 500,000 particles per beam spill.

Differential cross section data were taken at six nominal beam momenta: 20, 40, 65, 100, 150, and 200 GeV/c. To determine the bend string currents needed for each momentum, the magnetization curve of a beam-line dipole was measured.⁶ The beam line

focal properties were studied to find the excitation functions of the quadrupoles.⁷ These studies also showed that the program TRANSPORT⁸ accurately predicted the magnet field strengths needed for proper focusing. All of the beam line measurements were done with the M2 line in a slightly different configuration than actually used during data collection (fewer magnets in each bend string and slightly different quadrupole placements). When the line was upgraded to the new configuration in summer 1973, the magnetic field strengths predicted by TRANSPORT were used without extensive retesting. Although all of the beam studies were internally consistent, a normalization error could have occurred in the dipole field calibrations. For this reason, the absolute beam energy was determined from the charge exchange and eta production data (as described in Appendix 1).

The particle composition of the negative beam was determined from Cerenkov counter pressure curves. The electron flux was time dependent and had to be monitored throughout the experiment.

C. The Hydrogen Target

The 24" of liquid hydrogen used as the target in this experiment was divided into two flasks, each of which could be filled independently of the other. When warm and pressurized to 15 psi, the upstream flask was 15.8" long and the downstream one was 8.0" (when cold they contracted by .3%). Both flasks were filled with liquid hydrogen for the measurements at 60 GeV/c and above. At 20 and 40 GeV/c, where the uncertainty in the interaction position dominates the t-resolution, only the upstream flask was filled.

Other pertinent target dimensions were the following: the upstream flask was 2-1/2" in diameter, the downstream flask was 3"; the walls of the flasks were made of 5 mil mylar and covered with 20 layers of 1/4 mil aluminized mylar super-insulation; the vacuum housing which surrounded the flasks was 5-1/2" in diameter and made of 25 mil aluminum; the beam entry and exit ports were made of 5 mil and 14 mil mylar, respectively.

Two 12 watt helium refrigerators cooled and condensed the hydrogen in the target. When three liters of liquid were condensed, the H_2 gas intake line was closed. The hydrogen was then contained in a closed system; any hydrogen which boiled in the flasks was recondensed on the refrigeration plates in the target's reservoir. The target's temperature was regulated by a power feed-back loop between a sensing resistor and two heating resistors in the reservoir. The reservoir pressure was recorded regularly throughout the experiment; the hydrogen density was calculated from this information.⁹ Under normal conditions, the reservoir pressure was held at 5 psia and the hydrogen density was .0694 g/cc.

To check the target length and the hydrogen density used in the cross section determinations, the 40 GeV/c π^-p total cross sections was measured. This measurement, described in appendix 4, verified that the length and the density to be correct to within 1%.

D. Beam Definition Counters and Electronics

Of the counters shown in figure II.1, those needed for beam

Figure II.1

Beam Definition and Charged Particle Veto Counters

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definition were two sets of hodoscopes (UX, UY and DX, DY), a threshold Cerenkov counter, a beam halo counter (AO), and a three counter beam telescope (M1, M2, and M3). M1, a $7/16"$ x $7/16"$ x $1/4"$ thick scintillation counter designed to have good pulse height and time resolution, set the timing for all coincidences in the experiment; M2, a $5/16"$ x $5/16"$ x $1/8"$ thick counter, limited the aperture of the telescope; and M3, a $1/16"$ thick scintillation counter (circular, 1" diameter) with an air light pipe placed within 2" of the hydrogen target, made certain that the particles entered the target region.

Figure II.2 is a schematic of the electronics used to define acceptable beam particles. A particle passing through the beam telescope would trigger the triple coincidence, BEAM. The rest of the beam definition circuits assured that each particle signal was separated from all others by at least 50 nsec. When two particles entered M1 within the time resolution of its phototube, the counter would give a double-sized pulse. The 2xMIN discriminator threshold was set to trigger on such pulses and veto the accompanying BEAM pulse at the circuit ONE PI. AO, a counter with a $3/8"$ hole at its center, detected particles in the beam halo; its signal would veto beam particles at the circuit ONE PI x AO. Finally, in the circuit DT PI, the signals EARLY PI and LATE PI vetoed BEAM signals that were within 50 nsec of each other. A pulse from DT PI indicated that an acceptable beam particle had entered the system.

The other two beam definition elements, the hodoscopes and the threshold Cerenkov counter, were not in the trigger logic.

BEAM DEFINITION ELECTRONICS

Figure II.2

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The hodoscopes, UX, UY, DX, and DY, gave the position of the incident particle at the target (1/16" resolution) and its direction (.2 mrad resolution). The hodoscopes were read by the computer through a string of data bits set with each event trigger. The Cerenkov counter distinguished pions from kaons and antiprotons (except at 20 GeV/c where it distinguished electrons from hadrons). An RCA 8850 photomultiplier in this counter insured a stable one photoelectron pulse. A discriminator with its threshold set below the one photoelectron level was used to set a data bit. The counter's pulse height was also integrated and digitized with each event; this information was used to check counter performance and discriminator threshold.

E. The Charged Particle Anticounters

The elimination of events with charged particles in the final state involved two steps: first, the non-interacting particles were vetoed by a small but highly efficient beam anticounter, A5 (4" x 4" x 1/4" thick), and by a forward wide-angle counter, A3 (7" x 7" x 1/4" thick); then the rest of the charged particle interactions were eliminated by two anticounters (A1A and A1B) before, and by four (A2A-A2D) alongside the target. Each A2 counter was 52" long, 7" wide, and 3/8" thick; the scintillation light was viewed by an RCA 8575 photomultiplier at the downstream end of the counter. The arrangement of the anticounters is shown in figure II.1

The discriminator thresholds for the anticounters A2 and A3 were set unusually high, about 2/3 of the straight-through

minimum ionizing peak for A3, and at the peak for A2 (the standard threshold for other counters was 1/3 of the peak), and the pulse heights of the counters were recorded with each trigger. It was therefore possible to study the effects of various threshold levels on the final result and to verify the calculations of the delta π^+ , noise, and neutron backgrounds in these counters after data collection was completed.

Figure II.3 is a schematic of the trigger electronics associated with these counters. All of the anticounters were susceptible to large light pulses that would saturate the final dynode stages and extend the pulse. To assure full efficiency for all counters, a big pulse veto (BIG VETO) was put in anti-coincidence with the DT PI signal at the circuit FLUX. After an interaction, BIG VETO would inhibit all subsequent DT PI signals until the anticounters had recovered. A similar veto for the big pulses from the gamma veto counters (BIG V-VETO) was also in this circuit. Each FLUX pulse indicated that an acceptable particle had entered the apparatus and that the apparatus was ready to accept an event. In order to derive the number of acceptable pions, two more coincidences were needed: (1) coincidences between FLUX and the Cerenkov counter were counted to determine the number of incident particles with masses less than the kaon mass, and (2) coincidences between FLUX and a full energy signal from the gamma detector were scaled to monitor the electron flux. The number of incident pions was then

$$N_{\pi^-} = N_{C.FLUX} - N_{E.FLUX}(1 - P_E^{\pi^-})$$

TRIGGER ELECTRONICS

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Figure II.3

where N_{π^-} = number of incident π^-

$N_{C.FLUX}$ = number of Cerenkov counter-FLUX coincidences

$N_{E.FLUX}$ = number of FLUX-full detector energy coincidences

and $P_E^{\pi^-}$ = correction term for π^- 's giving a full energy pulse in the detector (determined from Cerenkov counter pressure curves).

To veto forward charged particles, the sum of the two discriminated signals from A3 and A5 was put in anticoincidence with FLUX in the circuit INTERACT. The neutral final state identification was completed at the circuit NFS where the logic sum of A1 and A2 was used to veto INTERACT. Finally, the signal NFS was put in coincidence with the computer ready signal to trigger the computer read-out cycle.

F. The Gamma Veto System

The gamma veto system consisted of 5 sets of 4 counters each. Those surrounding the hydrogen target were called the veto house counters, and the others were called baffle counters (V1 upstream of the target, V2, V3, and V4 downstream). The arrangement of these counters is illustrated in figure II.4.

The veto house counters were the most critical in the gamma veto system (rejecting 84% of the inelastic reactions which fired only one gamma veto). Since the recoil neutrons from both eta production and charge exchange passed through the veto house counters, their neutron detection efficiency had to be as low as possible; in addition, they had to have high efficiency for

Figure II.4

65 GeV/c Gamma Veto Counter Arrangement

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Apertures and positions of V2, V3 and V4 were changed for each beam momentum. V4 was always 2 m in front of the detector so that photons from the target could be distinguished from back scattering from the detector.

detecting low energy gammas from recoil N*s (photons in the 100 MeV range) at all angles of incidence. The counters were nine-layer lead-Cerenkov plastic sandwiches five radiation lengths deep. Curved light pipes totaled the light pulses from all nine layers of each counter and channeled them to four RCA4522 photomultipliers (two at each end of the counter). Details of the veto house counter construction and their placement around the target are shown in figure II.5. To detect photons entering at oblique angles, the first radiation length of material had to be broken into five layers with plastic between each layer (the first 1/5 radiation-length was the counter's 1/8" steel case and the next 4 layers were each 50 mils of lead). To increase the detection efficiency for straight-through photons, each of the final four plastic layers had 1/4" of lead in front of it. Scintillation counters detect neutrons primarily by sensing the light from the low energy recoil products of neutron-proton and neutron-carbon collisions. Plastic Cerenkov counters, on the other hand, do not detect these products since their velocities are below Cerenkov light threshold. For this reason Pilot 425, a Cerenkov plastic doped with wave shifter, was used for the veto house counters. However, the amount of light produced by electron-positron pairs is small in Pilot 425 plastic; therefore, the photomultiplier amplification had to be augmented by 20 X amplifiers to detect the light of one particle transversing one layer. To reduce the effects of electronic noise at these amplification levels, a majority logic coincidence of pulses

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Figure 11.4

from at least two of the four photomultipliers was required. Such a coincidence would set a bit which was subsequently read by the computer.

The baffle counters which were needed to detect photons in the remaining solid angle were made of five layers of Pilot F scintillator interleaved with lead. V1 and V2 had 1/4" lead sheets between the layers while V3 and V4 had 1/2" sheets. Again, the light pipes added the signals from all layers of each counter. Each V1 and V2 counter had two side-by-side RCA 4522 photomultipliers, each V3 counter had one 4522, and each V4 counter had two RCA 8575 photomultipliers, one at each end of the scintillator. The discriminator thresholds for the baffle counters were set to trigger when one charged particle transversed one layer. For V1, V2 and V4, the analog signals from both phototubes were added before entering the discriminators. With each trigger, the discriminator output signals set data bits. Because the counters were made of scintillator instead of Cerenkov plastic, the tube voltages were lower than those of the veto house and noise was not a problem.

To check the photomultiplier gains and the discriminator thresholds, the pulse heights of each of the veto house photomultipliers and each baffle counter were digitized with each event. In addition, the absolute gains of the veto house, V2 and V3 counters were monitored frequently with LED light pulsers placed inside each counter.

G. The Gamma Detector

The gamma detector (illustrated in figure II.6) consisted of 16 planes of hodoscope counters interspersed among 19 sheets of lead. Each finger was 73.5 cm long, 1.05 cm wide, and .7 cm thick; one plane consisted of 70 adjacent fingers. The lead plates were 75 cm square and 6.4 mm thick. Planes of X and Y fingers alternated with each other and were separated by single lead sheets. Two additional lead sheets were placed in front of the first set of fingers, and two more behind the last set.

The light pipes from the eight fingers with the same X or Y coordinate went to one photomultiplier, thereby optically integrating the energy deposition of a photon shower along the beam direction. The transverse RMS spread of a typical shower was 40% larger than a finger, allowing the centroid of the shower to be measured to a fraction of a counter width (2 mm).

With each trigger, the output pulses from the photomultipliers of the detector, the charged-particle and gamma-veto counters and the Cerenkov counter were integrated in charge-to-time converters, digitized in 11 bit scalars and read out with a SAC data collection system by an ADS Sigma II computer. The computer's readout and record time limited the data rate to 80 events per beam spill. To monitor the zero points of all pulse heights, pedestal runs were taken every few hours throughout the experiment.

As will be shown in appendix 1, the data analysis is greatly simplified if the detector's energy response is uniform over its entire active surface. Three techniques¹⁰ were used to

Figure II.6

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The Gamma Detector

achieve this flat response over the full length of the fingers. First, Wratten 2E filters were placed in front of the photomultipliers to eliminate the effects of the short absorption length of blue light in scintillator. Second, the hodoscope fingers were very carefully polished and a wide angle filter was painted on each light pipe to reduce variation due to surface irregularities. Finally, to compensate for other attenuation effects, the amount of scintillation light reflected back into the fingers by the wrapping was controlled by a graded black pattern painted on the aluminized mylar. With these techniques, the pulse height deviated by less than 2% from the mean value over the length of the counter. To balance the finger-to-finger response, each detector finger was centered on the beam, its pulse height response to electrons measured, and its gain changed to equalize this response. This technique resulted in counter-to-counter variations in pulse height of less than 2%. In the process of a cross section measurement, the photomultiplier gains of the fingers in the beam changed by up to 10% due to photocathode aging. Fortunately, the majority of charge exchange events were detected in these fingers, and could be used to recalculate the gains.

III. Analysis of the Data

A. Identification of Charge Exchange and Eta Production

The first step in the analysis of the data was to simulate a more restrictive trigger than was originally required by the

electronics. To be included in the data set for further analysis, an event had to meet three requirements; the Cerenkov counter had to fire; the digitized pulse height of the A_2 and A_3 counters had to be less than $1/3$ of the minimum ionizing peak; and no gamma veto counters could have fired. The first requirement eliminated kaon- and antiproton-induced events; the second reduced the high discriminator biases on A_2 and A_3 and tightened the selection of neutral final state events; and the final requirement discarded events in which final state photons missed the detector. At 20 GeV/c, the Cerenkov counter was operated below pion threshold to veto beam electrons and to reduce the pion multiple scattering; therefore, at that energy, the Cerenkov counter was required to have no signal.

Next, the detector information was analyzed. The algorithms used to determine the pertinent parameters (production angle, mass and energy of the parent particle, number of photons hitting the detector, and the cosine of the decay angle in the parent particle rest frame) are described in appendix 1. These routines used energy and momentum conservation plus the known shower characteristics to determine the parameters directly from the energy distribution in the detector. Since the routines never needed to find the energies and positions of the individual photons, the computations were extremely fast. They allowed about 90% of the data to be analyzed on-line during data collection, which was a great help in debugging and monitoring the apparatus.

The lack of any substantial amount of background events is illustrated in the histograms of the 150 GeV/c two photon events, figures III.1 through III.4 (each figure is a histogram of events meeting all charge exchange conditions except the one for the abscissa variable). In figures III.3, the only events not found in the η mass² peak come either from η' mesons (the two-photon decay mode) or from ω mesons (feed-down from the three-photon decay mode; these events represent less than 5% of the ω signal that was seen in the three-gamma spectrum).

An event which met all veto system requirements was considered charge exchange if (1) two and only two distinct showers were found in the detector, (2) the integrated detector energy was within the full energy peak, (3) the calculated mass was within the pion mass peak, and (4) the calculated cosine of the decay angle was less than .7. The eta production events met the same requirements except the calculated mass had to fall within the eta mass peak. The position of each cut is shown in figures III.1 through III.4.

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Figure III.1

150 GeV/c Pion Energy Spectrum

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Figure III.2
150 GeV/c Pion $Mass^2$ Peak

XBL 759-8348

Figure III.3
150 GeV/c Eta Mass² Peak

Figure III.4
150 GeV/c Pion C&S Distribution

XBL 759-8331

B. Determination of the Total Cross Sections

At each beam momentum a group of runs with the best normalization information was selected for the computation of the total cross sections. Each group consisted of enough triggers to give approximately 10,000 charge exchange events. For each run, charge exchange and eta production events were selected in the manner described in the previous section and the two total cross sections were determined. The spread of the individual run cross sections was always consistent with the statistical accuracy of the measurements and the uncertainty of the hydrogen density. For each momentum, the individual run cross sections were averaged to give raw cross sections for charge exchange and eta production. The raw cross sections had to be corrected for beam contamination and attenuation, for background events mistakenly included by the electronic vetoes and the analysis cuts, and for true events lost; the magnitude of each correction is given in tables III.1 and III.2. In these tables, each correction is defined as

$$\text{Correction} = 1. - (\text{Fractional loss of events}).$$

The total correction is the product of the individual corrections, and the total cross section is the raw cross section divided by the total correction. An error was estimated for each correction, and all individual errors were added together in quadrature to find the total error. Where possible, these corrections were measured directly from the data or from special runs taken between data accumulation runs. Otherwise, they were calculated from known processes or previous calibration measurements.

Table III.1

Normalization Correction to the Charge Exchange Total Cross Sections

Correction *	Beam Momentum (GeV/c)					
	20.7	40.9	65.3	100.6	151.	199.
t independent corrections						
1. Target empty	1.047	1.047	1.032	1.027	1.044	1.035
2. Cos θ cut	.703	.703	.698	.703	.699	.703
3. One gamma conversion	.910	.910	.894	.921	.894	.894
4. Dalitz pair production	.988	.988	.988	.988	.988	.988
5. Delta ray production	.920	.920	.897	.897	.897	.897
6. N^* production	1.010	1.010	1.010	1.010	1.010	1.010
7. π^- absorbed by H_2	.980	.980	.970	.970	.970	.970
8. Target contraction	.997	.997	.997	.997	.997	.997
9. Mass ² cut	1.000	.996	.997	.996	.995	.990
10. Energy cut	.991	.979	.992	.997	.993	.974
11. Beam contamination	.944	.988	.991	.970	.998	.992
12. Feed-up to 3 gammas	.908	.907	.991	.971	.991	.996
t - dependent corrections (averaged over t)						
1. A_2 neutron detection	.972	.971	.971	.970	.970	.970
2. Veto house neutron detection	.951	.952	.953	.954	.955	.956
Total correction	.471	.485	.500	.483	.511	.497
Error	.028	.028	.026	.027	.026	.029

* Corrections are such that

$$\sigma_{\text{measured}} = (\text{correction}) \times \sigma_{\text{actual}}.$$

Table III.2

Normalization Corrections to the Eta Production Total Cross Sections

Correction*	Beam Momentum (GeV/c)					
	20.7	40.9	65.3	100.6	151.	199.
t independent corrections						
1. Target empty	1.064	1.043	1.043	1.044	1.012	1.042
2. Cos θ cut	.676	.676	.669	.676	.671	.676
3. One gamma conversion	.922	.910	.894	.921	.894	.894
4. Delta ray production	.920	.920	.897	.897	.897	.897
5. N^* production	1.010	1.010	1.010	1.010	1.010	1.010
6. π^- absorbed by H_2	.980	.980	.970	.970	.970	.970
7. Target contraction	.997	.997	.997	.997	.997	.997
8. Mass ² cut	1.000	1.000	1.000	1.000	.995	.990
9. Energy cut	.991	.979	.992	.967	.993	.974
10. Beam contamination	.944	.988	.991	.970	.998	.992
11. Feed-up to 3 gammas	.882	.882	.991	.971	.991	.996
t dependent corrections (averaged over t)						
1. A_2 neutron detection	.971	.971	.970	.970	.969	.969
2. Veto house neutron detection	.938	.940	.941	.943	.944	.946
Total correction	.452	.453	.486	.474	.475	.481
Error	.029	.029	.028	.027	.027	.032

* Corrections are such that

$$\sigma_{\text{measured}} = (\text{correction}) \times \sigma_{\text{actual}}$$

Most of the non-pion beam contamination was either eliminated by the Cerenkov counter cut or discounted by the electron correction. The only remaining particles were muons and, at 20 GeV/c, kaons and antiprotons. The flux of these particles was determined from the Cerenkov counter pressure curves and was subtracted from the pion flux. The pion beam attenuation was calculated from measured pion-proton total cross sections and from the target density.

Two types of events were mistakenly included in the raw total cross sections; "empty target events" in which charge exchange or eta production occurred in the beam counters and flask walls rather than in the hydrogen, and pion-proton inelastic reactions that were mistakenly identified as either charge exchange or eta production.

At each beam momentum, interspersed among the target full runs, target empty data were taken. These events were analyzed in the same manner as the target full events. No statistically significant difference between the target full and target empty distributions was found, so only a normalization correction was applied to the cross sections.

Two inelastic reactions that could be mistaken for charge exchange reactions are N^* production and diffractive pion resonance production. In all cases, there are additional hadrons and/or photons in the final state which might escape detection in the veto system.

The N^* that could most easily escape detection was the

$\Delta^0(1236)$, which could decay into a neutron and two low energy photons. The detection efficiencies of the veto house and a V2 counter for such photons were measured at the MARK III Linear Electron Accelerator at Stanford University. These tests are described in appendix 2. If all the solid angle except that covered by the detector had the measured detection efficiency, then only 1.6% of the neutral $\Delta^0(1236)$ decays would go undetected.

The Δ^0 production cross section was estimated from the low energy data. At 8 GeV/c¹¹

$$\frac{\sigma_t(\pi^+p + \Delta^{++}\pi^0)}{\sigma_t(\pi^-p + n\pi^0)} = \frac{.11 \text{ mb}}{.062 \text{ mb}} = 1.8. \quad \text{III.1}$$

Since both charge exchange and Δ^{++} production have approximately the same energy dependence, the ratio of their cross sections should be the same at high energies. The ratio of the Δ^0 cross section to the Δ^{++} cross section is 1/3 (from isospin symmetry and t-channel ρ -exchange). When equation III.1 is multiplied by this ratio and the $\Delta^0 \rightarrow \pi^0 n$ partial decay width, the result is

$$\frac{\sigma_t(\pi^-p + \Delta^0\pi^0 + (n\pi^0)\pi^0)}{\sigma_t(\pi^-p + \pi^0 n)} = .4. \quad \text{III.2}$$

A lower limit for the N^* background was obtained by assuming that the only N^* missed was the $\Delta^0(1236)$ produced at the rate given in equation III.2. If the photons from 1.6% of the $\Delta^0(1236)$ were missed (as indicated by the Monte Carlo calculations), then a .6% of the accepted events were actually N^* events.

An upper limit on the total number of N^* s produced was

obtained from the data. If all neutral final state events with a full energy π^0 in the detector, no other gammas in either the detector or the downstream gamma vetoes, and at least one gamma in the veto house were N^* s, then

$$\frac{\sigma_t(\pi^-p \rightarrow N^*\pi^0 + (\text{neutrals})\pi^0)}{\sigma_t(\pi^-p \rightarrow \pi^0n)} \lesssim 1.0.$$

The probability for missing all of the gammas from any one of these events is no more than the probability of missing a $\Delta^0(1236)$ (1.6%). As a result, an upper limit to the N^* background is 1.6%. The approximate N^* background was assumed to be the average of the two limits: 1%.

A meson resonance which could be mistaken for charge exchange would have to decay into a $K^0\bar{K}^0\pi^0$ state. Although the veto system surrounded the target with many radiation lengths of lead, it only comprised a fraction of an interaction length. A small percentage of the kaons could escape from the system without being detected. In figure III.5, the π^0 energy spectrum from an isotropic decay of a 1.4 GeV/c² meson into $K^0\bar{K}^0\pi^0$ is compared with the energy spectrum of 200 GeV/c charge-exchange-like events (events which satisfied the veto system requirements, had two gammas hit the detector, passed the $\cos \theta$ cut, and, for the detected energy, had a mass² within the pion peak). Essentially all of the low energy pion production can be explained by this process. Since phase space for the π^0 falls so quickly toward the full energy peak, no π^0 s produced in this manner would satisfy the energy cut and no correction to the

Figure III.5

XBL 759-8345

200 GeV/c Pion Energy Spectrum

Heavy line is the expected pion energy spectrum from the isotropic decay of a $K_0^* K_0^* \pi^0$ resonance normalized to the number of events below $.6E_{\text{beam}}$.

total cross section needed to be made.

There is very little evidence of the existence of other inelastic events, but what there is is disturbing. Figure III.6 shows the energy versus transverse-momentum² (p_T^2) plot for all 150 GeV/c charge-exchange-like events. The high energy band in this scatter plot represents the true charge exchange events (here, p_T^2 is equal to $-t$); the clumping at low energies represents meson resonance production. The unexplained events are those above $.4(\text{GeV}/c)^2$ in p_T^2 and at approximately three-quarters of the beam energy. No mechanism has been invented to explain this distribution which appears predominately at 150 and 200 GeV/c. In order to eliminate the events which fell below the break in the event distribution at 90% of the beam energy, the energy requirement was changed for the high t events. If that break is only a statistical aberration and if the anomalous events continue into the full energy region, then the high t cross sections given in section III.C are too large. An understanding of the nature and source of the "unexplained" events will be needed to determine in the future if or to what extent they contaminate the high t signal.

The veto system could reject good events by detecting a recoil neutron, an electron or positron from a gamma which converted prior to leaving the target region, or a delta-ray produced in a π^-e collision upstream of the interaction vertex.

The kinematics of charge exchange and eta production required the escape of final state neutrons through the veto

XBL 759-8329

Figure III.6

Pion Transverse Momentum Distribution as a Function of Pion Energy
 (150 GeV/c data; shaded area represents 20,000 events)

house and A_2 counters. To determine the frequency of neutron-vetoed events, the neutron detection efficiencies (as a function of neutron kinetic energy) were measured at the Lawrence Berkeley Laboratory's 184" cyclotron. The details of those measurements are given in appendix 3. For each beam momentum, the efficiency was weighted by the differential cross section to determine the average neutron detection efficiency. The total cross sections were increased by this amount.

The number of gamma conversions was calculated from the quantity of material downstream of the interaction point (approximately 7% of a radiation length). At 20 GeV/c, gammas from some high t events could miss the A_5 counter; therefore, a small t -dependent correction had to be made at that momentum.

To monitor the delta-ray spectrum in the A_2 counters, runs triggered on FLUX (instead of NFS, q.v. figure II.3) were taken every few hours throughout the experiment. For two groups of these runs (40 GeV/c target full, and 65 GeV/c target 2/3 full), the A_2 pulse heights for straight-through beam particles* were histogrammed. The spectra of delta-rays produced in the hydrogen and detected in the A_2 counters were assumed to be the measured spectra corrected for the target empty spectra. Theoretical delta-ray spectra were calculated from the π^-e differential

* A flux event was considered a straight-through beam particle if a minimum ionizing particle appeared in the detector on the trajectory of the incident particle, and if no other particles were found in either the detector or the downstream veto system.

cross section. These calculations agreed with the measured spectra as shown in figure III.7. Similar calculations were made for the delta-ray spectrum in A_3 . The total correction was the sum of the calculated charge exchange corrections for A_2 and A_3 (thresholds at .7 MeV) and the measured target empty correction.

The biggest loss from the detector analysis cuts was from the $\cos \theta$ cut. Since both the pion and the eta decay isotropically in their rest frame, the $\cos \theta$ cut eliminated approximately 30% of the good events. This cut was necessary to limit the maximum separation between the two gammas (so that the detector was fully efficient for detecting etas from $t=0$ to $t=-2$), and to limit the minimum energy of the smallest gamma (so that the photon counting algorithm was always fully efficient). The algorithm used to compute the cosine of the decay angle had some inherent inaccuracies caused by the zero point bias (see appendix 1). Monte Carlo calculations using the measured shower shape determined the effective cut position at each detector gain setting; these values are given in tables III.1 and III.2.

Three types of problems occurred with the two-gamma cut: two-gamma events could be mistakenly classified as one-gamma events (two-gamma feed-down), two-gamma events as three (two-gamma feed-up), and three-gamma events as two (three-gamma feed-down). The loss of events due to two-gamma feed-down accounts for the depletion of events at high $\cos \theta$ in figure III.4. This loss was already included in the $\cos \theta$ correction,

Figure III.7a

NBL 759-8093

Measured and Calculated Delta-Ray Spectra
(target full)

NBL 759-8094

Figure III.7b

Measured and Calculated Delta-Ray Spectra
(target $2/3$ full)

so no additional correction was necessary.

Two-gamma feed-up occurred when the pulse height in one channel of the detector fluctuated enough to be counted as a photon (q.v. photon counting algorithm in appendix 1). Figure III.8 illustrates the magnitude of this effect in the 20 and 150 GeV/c data. The number of such events was assumed to be the difference between the number of events in the π^0 peak in the three gamma spectrum and the number of events in the same mass² interval immediately above the peak. At 20 and 40 GeV/c, where this correction was large (about 10%), Monte Carlo calculations agreed with the measured corrections.

Evidence for three-gamma feed-down is found in the long non-Gaussian tail on the high side of the π^0 mass² peak (figure III.2). When events in this tail were examined individually, three types were found: events with an additional pulse height bump in the detector that was too small to be detected by the gamma counting algorithm; events which showed only two showers in each plane, but which had a mismatch in the photon energies between the planes; and a small number of events which looked like good two-shower events. The first two categories indicated the presence of additional gammas in the final state; these events had to be eliminated by making the mass² cut as close as possible to the π^0 peak. Only a small correction had to be made for the good events excluded by this cut. These corrections are listed in tables III.1 and III.2.

Figure III.8

XBL 7511-9037

Comparison of the Two and Three Photon Mass² Spectra. Only events in the full energy peak which satisfied the veto system requirements are included. No $\cos \theta$ cut was made on the two photon events.

The only remaining correction is for the good events eliminated by the energy cut. If one or both showers of a good event developed late, the detected energy would be less than expected. Fortunately, as noted in appendix 1, the calculated mass² was essentially unaffected by this fluctuation. Such events could be separated from the inelastic events as shown in figure III.9 (the mass² is calculated using the beam energy instead of the detected energy in this figure; it is for this reason that the inelastic events form a parabola). The total cross section was increased to include the good events eliminated by the energy cut. These good events were not included in the differential cross sections because of their positional uncertainty.

The total cross sections which result from the corrections applied to the raw cross sections are given in table III.3. (for eta production, only the cross section for the two-gamma decay mode is listed). They are compared with similar measurements of other groups¹²⁻¹⁵ in figures III.10 and III.11. The cross sections agree very well with the lower energy measurements, but have a normalization offset from the numbers of Bolotov, et al.^{13,15}

XBL 7510-8802

Figure III.9

Energy versus Mass^2 for Two Photon Events (with $\cos\theta = 0.7$ and with no signal from any veto counter) from the 150 GeV/c Data

Total Cross Sections

Nominal Beam Momentum (GeV/c)	Actual Beam Momentum ** (GeV/c)	$\sigma_{TOT}(\pi^-p + \pi^0n)^*$ (μb)	$\sigma_{2\gamma}(\pi^-p + \eta n)^*$ (μb)
20	20.7	22.2 ± 1.3	$3.64 \pm .25$
40	40.9	$9.8 \pm .6$	$1.39 \pm .10$
65	65.3	$5.6 \pm .3$	$.65 \pm .05$
100	100.6	$3.2 \pm .2$	$.33 \pm .04$
150	151.	$1.96 \pm .10$	$.184 \pm .014$
200	199.	$1.40 \pm .08$	$.127 \pm .009$

*

Error includes normalization error

** Error in momentum determination is estimated at $\pm .5\%$.

Table III.3

NBL 7510-8-463

Figure III.10
Charge Exchange Total Cross Sections

In addition to the points from this experiment, data from references 12 and 13 are plotted. The line is the best power law fit to the points from this experiment:

$$\sigma_{\text{CEX}} = (920^{+120}) p_{\text{lab}}^{-1.23 \pm 0.03} \mu\text{b}$$

Figure III.11
Eta Production Total Cross Sections

In addition to the points from this experiment, data from references 14 through 16 are plotted. The solid line is the best power law fit to the points from this experiment:

$$\sigma_{\text{ETA}} = (350 \pm 60) p_{\text{lab}}^{-1.50 \pm 0.04} \mu\text{b}$$

C. The Differential Cross Section

To measure any differential cross section, a scattering parameter must be found for each event. In this case, the parameter was the deflection angle between the incident π^- direction and the π^0 or η direction. The Lorentz-invariant scattering variable t (the four-momentum transfer squared) was determined from the deflection angle and the kinematics of the assumed reaction.

The accuracy with which t could be measured for each event was related to four parameters; beam momentum spread (kept at $\pm 1\%$), uncertainty in the interaction point (the target length: 40 cm at 20 and 40 GeV/c, and 60 cm at all other momenta), accuracy in determining the incident π^- direction (limited by the hodoscope resolution), and the accuracy of the positional measurement of the π^0 or η in the detector (found from Monte Carlo studies to be less than $\pm .1$ cm for π^0 and $\pm .3$ cm for η). The t resolution for the two limiting cases (20 and 200 GeV/c), along with the differential cross section bin sizes which were used at all momenta, is shown in figures III.12 and III.13.

Finally, events which met the veto system and detector requirements and which fired each plane of the hodoscopes were grouped into appropriate t bins. Each t bin was corrected for the neutron detection efficiency of the A_2 and veto house counters (and, in the case of 20 GeV/c, the gamma conversion efficiency) and was renormalized to the total cross sections computed in section III.B. This procedure gave the final

Figure III.12

Charge Exchange t Resolution Functions and Differential Cross Section Bin Sizes

NBL 759-8095

Figure III.13

NBL 759-8096

Eta Production t Resolution Functions and Differential Cross Section Bin Sizes

differential cross sections presented in tables III.4 and III.5 and figures III.14 and III.15 for charge exchange and table III.6 and figure III.16 for eta production.

Except for the forwardmost bin ($0. \leq -t \leq .02$ for charge exchange; $0. \leq -t \leq .1$ for eta production) the 20 and 40 GeV/c shapes agree extremely well with the previous experiments.¹²⁻¹⁵ The difference in the forward direction may be due to the vastly improved t-resolution in this experiment. Recent eta production data from Serpukhov¹⁶ with equivalent t-resolution shows the strong forward dip in the eta cross section in agreement with this experiment.

Table III.4

 $\pi^- p + \pi^0 n$ Differential Cross Sections ($\mu\text{b}/\text{GeV}^2$)
 (Only statistical errors are given)
Regge fit parameters
(from equation IV.6)

- t bin	Beam momentum						Regge fit parameters (from equation IV.6)		
	20.7	40.9	65.3	100.6	151	199	N	α_{eff}	χ^2 *
0 - .004	104. $\pm$ 5.	49.0 $\pm$ 2.3	30.4 $\pm$ 1.2	19.6 $\pm$.8	12.7 $\pm$.5	9.7 $\pm$.4	4730 $\pm$ 580	.475 $\pm$.012	1.4
.004 - .008	102. $\pm$ 5.	58.0 $\pm$ 2.6	32.9 $\pm$ 1.3	19.4 $\pm$.8	12.8 $\pm$.5	9.7 $\pm$.4	5500 $\pm$ 640	.465 $\pm$.012	7.6
.008 - .016	117. $\pm$ 4.	55.9 $\pm$ 1.8	33.9 $\pm$.9	20.3 $\pm$.6	13.7 $\pm$.4	10.5 $\pm$.3	5890 $\pm$ 490	.463 $\pm$.008	3.4
.016 - .024	118. $\pm$ 4.	60.9 $\pm$ 1.9	34.9 $\pm$ 1.0	21.5 $\pm$.6	13.9 $\pm$.4	10.5 $\pm$.3	6370 $\pm$ 520	.459 $\pm$.008	3.8
.024 - .032	134. $\pm$ 4.	60.8 $\pm$ 1.9	36.2 $\pm$ 1.0	22.3 $\pm$.6	14.4 $\pm$.4	10.5 $\pm$.3	7910 $\pm$ 640	.441 $\pm$.008	.6
.032 - .048	131. $\pm$ 3.	62.5 $\pm$ 1.4	36.3 $\pm$.7	21.8 $\pm$.4	13.3 $\pm$.3	10.0 $\pm$.2	8850 $\pm$ 500	.426 $\pm$.006	4.4
.048 - .064	125. $\pm$ 3.	58.2 $\pm$ 1.3	35.8 $\pm$.7	20.9 $\pm$.4	12.8 $\pm$.3	9.0 $\pm$.2	8890 $\pm$ 510	.421 $\pm$.006	12.3
.064 - .08	115. $\pm$ 3.	55.1 $\pm$ 1.3	30.3 $\pm$.6	18.2 $\pm$.4	11.4 $\pm$.3	8.6 $\pm$.2	8130 $\pm$ 500	.420 $\pm$.006	6.2
.08 - .1	106. $\pm$ 2.	48.1 $\pm$ 1.1	28.3 $\pm$.6	16.3 $\pm$.3	9.66 $\pm$.21	6.7 $\pm$.2	9420 $\pm$ 530	.391 $\pm$.006	9.9
.1 - .12	92.3 $\pm$ 2.2	38.9 $\pm$ 1.0	23.3 $\pm$.5	13.1 $\pm$.3	7.90 $\pm$.19	5.52 $\pm$.14	8610 $\pm$ 540	.381 $\pm$.006	5.1
.12 - .16	72.9 $\pm$ 1.4	32.1 $\pm$.6	17.4 $\pm$.3	9.94 $\pm$.18	5.60 $\pm$.11	3.93 $\pm$.09	8560 $\pm$ 430	.353 $\pm$.005	11.2
.16 - .20	50.4 $\pm$ 1.1	19.9 $\pm$.5	11.5 $\pm$.3	5.90 $\pm$.14	3.65 $\pm$.09	2.54 $\pm$.07	6320 $\pm$ 400	.339 $\pm$.006	10.4
Fitted parameters from Eq. IV.5	Zero Degree Differential Cross Section						Eq. IV.4 C	Parameters α	χ^2 *
$\frac{d\sigma}{dt} _{t=0}$	96.4 $\pm$ 3.9	49.0 $\pm$ 1.1	29.5 $\pm$ 1.0	17.7 $\pm$.6	12.3 $\pm$.4	9.4 $\pm$.3	2360 $\pm$ 180	.476 $\pm$.009	4.23
F	3560 $\pm$ 350	1690 $\pm$ 100	985 $\pm$ 90	640 $\pm$ 45	378 $\pm$ 39	290 $\pm$ 31	* Each fit has 4 degrees of freedom		
γ	15.3 $\pm$.9	15.9 $\pm$.5	16.0 $\pm$.8	17.1 $\pm$.5	17.4 $\pm$.9	18.2 $\pm$.9	** Each fit has 6 degrees of freedom		
χ^2 **	7.1	5.1	9.4	2.7	3.1	8.8			

Table III.5
 $\pi^- p + \pi^0 n$ Differential Cross Section ($\mu\text{b}/\text{GeV}^2$)
 (Only statistical errors are given)

Regge Fit Parameters
 (from Equation IV.6)

- t bin	Beam Momentum						Regge Fit Parameters		
	20.7	40.9	65.3	100.6	151.	199.			
							N	α_{eff}	χ^2 *
0. - .02	112. $\pm$ 2.	55.4 $\pm$ 1.1	33.0 $\pm$.6	20.1 $\pm$.4	13.4 $\pm$.2	10.1 $\pm$.2	5700 $\pm$ 300	.463 $\pm$.005	4.4
.02 - .04	128. $\pm$ 3.	62.0 $\pm$ 1.2	36.5 $\pm$.6	21.8 $\pm$.4	13.8 $\pm$.3	10.5 $\pm$.2	7800 $\pm$ 400	.440 $\pm$.005	5.5
.04 - .06	130 $\pm$ 3.	60.9 $\pm$ 1.2	35.4 $\pm$.6	21.5 $\pm$.4	13.0 $\pm$.2	9.3 $\pm$.2	9400 $\pm$ 500	.417 $\pm$.005	6.6
.06 - .08	114. $\pm$ 2.	54.4 $\pm$ 1.1	31.6 $\pm$.6	18.6 $\pm$.4	11.7 $\pm$.2	8.7 $\pm$.2	7800 $\pm$ 300	.425 $\pm$.003	4.5
.08 - .10	106. $\pm$ 2.	48.1 $\pm$ 1.1	28.3 $\pm$.6	16.3 $\pm$.3	9.7 $\pm$.2	6.7 $\pm$.2	9400 $\pm$ 500	.391 $\pm$.006	9.9
.10 - .12	92. $\pm$ 2.	38.9 $\pm$.9	23.3 $\pm$.5	13.1 $\pm$.3	7.9 $\pm$.2	5.52 $\pm$.14	8600 $\pm$ 500	.381 $\pm$.006	5.1
.12 - .16	72.8 $\pm$ 1.4	32.1 $\pm$.6	17.4 $\pm$.3	9.9 $\pm$.2	5.60 $\pm$.11	3.93 $\pm$.09	8600 $\pm$ 400	.353 $\pm$.005	1.2
.16 - .20	50.4 $\pm$ 1.1	19.9 $\pm$.5	11.5 $\pm$.3	5.90 $\pm$.14	3.65 $\pm$.09	2.54 $\pm$.07	6300 $\pm$ 400	.339 $\pm$.006	10.3
.20 - .24	33.7 $\pm$.9	13.1 $\pm$.4	7.2 $\pm$.2	3.80 $\pm$.11	2.27 $\pm$.07	1.41 $\pm$.05	5300 $\pm$ 400	.310 $\pm$.008	5.9
.24 - .28	21.3 $\pm$.7	8.4 $\pm$.3	4.55 $\pm$.16	2.11 $\pm$.09	1.28 $\pm$.06	.79 $\pm$.04	4400 $\pm$ 400	.277 $\pm$.010	10.9
.28 - .34	11.4 $\pm$.4	4.1 $\pm$.2	2.43 $\pm$.09	.99 $\pm$.05	.59 $\pm$.03	.35 $\pm$.02	3030 $\pm$ 120	.241 $\pm$.003	21.3
.34 - .40	4.5 $\pm$.3	1.78 $\pm$.12	.960 $\pm$.060	.39 $\pm$.03	.22 $\pm$.02	.137 $\pm$.013	1300 $\pm$ 200	.233 $\pm$.018	9.7
.40 - .50	1.71 $\pm$.13	.54 $\pm$.05	.260 $\pm$.024	.107 $\pm$.012	.073 $\pm$.008	.037 $\pm$.005	740 $\pm$ 190	.168 $\pm$.025	3.5
.5 - .6	.44 $\pm$.07	.10 $\pm$.02	.065 $\pm$.012	.028 $\pm$.006	.011 $\pm$.003	.008 $\pm$.003	260 $\pm$ 160	.12 $\pm$.06	1.8
.6 - .7	.34 $\pm$.06	.13 $\pm$.03	.045 $\pm$.010	.013 $\pm$.004	.008 $\pm$.003	.009 $\pm$.003	340 $\pm$ 200	.06 $\pm$.07	5.2
.7 - .8	.51 $\pm$.07	.07 $\pm$.02	.038 $\pm$.009	.017 $\pm$.005	.002 $\pm$.002	.005 $\pm$.002	3200 $\pm$ 2200	-.20 $\pm$.06	8.3
.8 - 1.0	.49 $\pm$.05	.080 $\pm$.014	.038 $\pm$.007	.012 $\pm$.003	.006 $\pm$.002	.0039 $\pm$.0013	1400 $\pm$ 600	-.10 $\pm$.05	3.1
1.0 - 1.2	.24 $\pm$.03	.041 $\pm$.010	.015 $\pm$.004	.006 $\pm$.002	.002 $\pm$.001	.0007 $\pm$.0006	1600 $\pm$ 1000	-.20 $\pm$.07	.7
1.2 - 1.4	.15 $\pm$.03	.023 $\pm$.008	.008 $\pm$.003	.001 $\pm$.001	.002 $\pm$.001	.0004 $\pm$.0004	2400 $\pm$ 1600	-.33 $\pm$.04	2.0
Total number of events	22154	20395	26175	23720	21055	18832	* Each fit has 4 degrees of freedom

Figure III.14

XBL 7510-8462

Small t Charge Exchange Differential Cross Sections
 (lines are solutions of fits to equation IV.5)

SBL 75108101

Figure III.15

Charge Exchange Differential Cross Sections

Table III.6

$\pi^- p \rightarrow \eta n; \eta \rightarrow 2\gamma$ Differential Cross Section ($\mu\text{b}/\text{GeV}^2$)
(Only statistical errors are given)

Regge Fit Parameters
(from equation IV.6)

- t bin	Beam Energy						N	α_{eff}	χ^2 *
	20.7	40.9	65.3	100.6	151.	199.			
0 - .02	$8.5 \pm .7$	$4.1 \pm .3$	$1.89 \pm .14$	$1.01 \pm .08$	$.58 \pm .05$	$.52 \pm .04$	1080 ± 230	$.34 \pm .02$	8.8
.02 - .06	$10.6 \pm .5$	$4.4 \pm .2$	$2.22 \pm .11$	$1.17 \pm .06$	$.76 \pm .04$	$.54 \pm .03$	1380 ± 120	$.33 \pm .01$	4.6
.06 - .10	$11.0 \pm .5$	$4.6 \pm .2$	$2.40 \pm .11$	$1.37 \pm .07$	$.84 \pm .04$	$.56 \pm .03$	1330 ± 180	$.35 \pm .01$	.8
.10 - .16	$11.1 \pm .4$	$4.6 \pm .2$	$2.08 \pm .09$	$1.09 \pm .05$	$.62 \pm .03$	$.46 \pm .02$	2180 ± 250	$.28 \pm .01$	7.1
.16 - .22	$8.7 \pm .4$	$3.4 \pm .2$	$1.56 \pm .07$	$.88 \pm .04$	$.43 \pm .03$	$.30 \pm .02$	2090 ± 270	$.25 \pm .01$	3.9
.22 - .30	$6.5 \pm .3$	$2.64 \pm .13$	$1.16 \pm .06$	$.50 \pm .03$	$.28 \pm .02$	$.164 \pm .013$	2570 ± 330	$.19 \pm .01$	13.9
.3 - .4	$4.2 \pm .2$	$1.35 \pm .08$	$.61 \pm .04$	$.26 \pm .02$	$.130 \pm .011$	$.083 \pm .008$	2540 ± 410	$.13 \pm .02$	1.7
.4 - .6	$1.64 \pm .09$	$.43 \pm .03$	$.22 \pm .02$	$.080 \pm .007$	$.039 \pm .004$	$.019 \pm .003$	1720 ± 330	$.05 \pm .02$	7.3
.6 - .8	$.52 \pm .05$	$.12 \pm .02$	$.045 \pm .007$	$.012 \pm .003$	$.005 \pm .002$	$.0024 \pm .0010$	2340 ± 920	$-.14 \pm .05$	2.2
.8 - 1.0	$.10 \pm .02$	$.019 \pm .007$	$.003 \pm .002$	$.0027 \pm .0014$	$.0020 \pm .0010$	$.0012 \pm .0008$	430 ± 650	$-.16 \pm .17$	4.1
1.0 - 1.2	$.043 \pm .016$	$.013 \pm .006$	$.002 \pm .002$	$.0014 \pm .0011$	$.0010 \pm .0008$	$.0008 \pm .0006$	80 ± 140	$-.04 \pm .19$	1.7
Total number of events	3480	2700	3042	2464	1926	1624			

* Fit has 4 degrees of freedom

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Figure III.16

Eta Production Differential Cross Sections

IV. Phenomenological Implications

A. The Zero Degree Charge Exchange Cross Section and the π^+p and π^-p Total Cross Sections

Isospin symmetry relates the charge exchange amplitude (A^{CEX}) to the amplitudes for π^+p and π^-p elastic scattering (A^+ and A^- respectively)

$$A^{CEX} = \frac{1}{\sqrt{2}} (A^+ - A^-).$$

Since the imaginary parts of the forward elastic scattering amplitudes are proportional to the total cross sections (by the optical theorem)

$$\text{Im } A_{t=0}^{CEX} = \frac{1}{\sqrt{2}} (\text{Im } A_{t=0}^+ - \text{Im } A_{t=0}^-)$$

$$\begin{aligned} \text{Im } A_{t=0}^{CEX} &= \frac{k}{4\pi} \frac{(\sigma^+ - \sigma^-)}{\sqrt{2}} \\ &= \frac{k}{4\pi} \frac{\Delta\sigma}{\sqrt{2}} \end{aligned}$$

where σ^+ and σ^- are the total cross sections for π^+p and π^-p scattering ($\Delta\sigma$ is their difference) and k is the momentum of the pion beam. The zero-degree charge exchange cross section is

$$\begin{aligned} \left. \frac{d\sigma}{dt} \right|_{t=0} &= \frac{\pi}{k^2} \left[(\text{Re } A_{t=0}^{CEX})^2 + (\text{Im } A_{t=0}^{CEX})^2 \right] \\ &= \frac{\pi}{k^2} \left[1 + \xi^2 \right] (\text{Im } A_{t=0}^{CEX})^2 \\ &= \frac{1}{32\pi} \left[1 + \xi^2 \right] (\Delta\sigma)^2 \end{aligned} \quad \text{IV.1}$$

where ξ is the ratio of the real to the imaginary part of the charge exchange amplitude. Since ξ^2 is always positive, the

zero-degree cross section places an upper limit on the difference between the two total cross sections. This limit is not very restrictive, however, since it is approximately twice the measured $(\Delta\sigma)^2$.

If it is further assumed that the imaginary part of the forward charge exchange amplitude behaves as a pure power law

$$\text{Im } A_{t=0}^{\text{CEX}} = B k^{-\alpha} \quad \text{IV.2}$$

then dispersion relations* give the value of

$$\xi = \tan \frac{\pi\alpha}{2} \quad \text{IV.3}$$

and the total cross section differences can be computed directly from the charge exchange cross sections. Equation IV.3 also implies that the zero-degree cross section should have the form

$$\left. \frac{d\sigma}{dt} \right|_{t=0} = C k^{2\alpha-2} \quad \text{IV.4}$$

where $C = B^2 / \cos^2 (\pi\alpha/2)$.

*The dispersion relationship¹⁷ for the charge exchange amplitude is

$$\text{Re } A^{\text{CEX}}(k) = \frac{1}{\pi} \int_0^\infty 2kdk' \frac{\text{Im } A^{\text{CEX}}(k')}{k'^2 - k^2}$$

If equation IV.2 is assumed, then

$$\text{Re } A^{\text{CEX}}(k) = \frac{\text{Im } A^{\text{CEX}}(k)}{\pi} \int_0^\infty \frac{dx}{(1-x) x^{\frac{\alpha+1}{2}}}$$

and $\text{Re } A^{\text{CEX}}(k) = \text{Im } A^{\text{CEX}}(k) \tan \frac{\pi\alpha}{2}$

It was assumed that the fluctuations in the resonance region made no contribution to the dispersion integral; to that extent, equation IV.3 is an approximation.

The first step in computing the total cross section differences from the charge exchange data is to find the $t=0$ differential cross sections. For each energy, the data from the t bins from 0. to $\pi,1$ were fit to a function of the form¹⁸

$$\frac{d\sigma}{dt} = \left[\left(\frac{d\sigma}{dt} \right)_{t=0} - F t \right] e^{-\gamma t} \quad \text{IV.5}$$

(the constants of the best fit were given in table III.4; these functions were compared with the data in figure III.14). The values for the best fit of the $\left. \frac{d\sigma}{dt} \right|_{t=0}$ to equation IV.4 is

$C = 2360 \pm 180$ and $\alpha = .476 \pm .009$ (this function is compared with the actual zero-degree cross sections in figure IV.1).

The two constants C and α determine the prediction for the total cross section difference (from equations IV.1 and IV.3). This prediction is compared with the measured differences¹⁹⁻²¹ in figure IV.2. Although the line is not the best possible fit to the total cross section differences, it shows remarkably good agreement between the forward cross sections and the measured differences. Any presumed discrepancy is probably no bigger than the systematic errors in the experiments and the fitting procedures.

B. Energy Dependence of the Differential Cross Sections; Effective ρ and A_2 Trajectories.

The effective ρ and A_2 trajectories (the complex angular momenta at which the ρ^- and A_2^- poles occur) are found by fitting the energy dependence of each bin of the differential

Figure IV.1

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Solution of Power Law Fit to Zero-Degree Charge Exchange Cross
Sections

Figure IV.2

XBL 7510-8389

π^+p and π^-p Total Cross Section Differences (references 19 - 21)
 Solid line is the prediction of the zero-degree charge exchange data;
 dashed line is one standard deviation from the prediction.

cross section to the functional form⁴

$$\frac{d\sigma}{dt} = N \frac{(\bar{v}^2)^{\alpha_{\text{eff}}(t)}}{k s} \quad \text{IV.6}$$

where

$$\bar{v}^2 = v^2 - v_0^2$$

$$v = (s - u)/2$$

$$v_0 = (s_{\text{min}} - u_{\text{min}})/2$$

and s , t , and u are the Mandelstam variables. The best values of N and $\alpha_{\text{eff}}(t)$ were listed in tables III.4 and III.5; comparisons of a few of the fitted functions with the differential cross sections are shown in figure IV.3. The values of $\alpha_{\text{eff}}(t)$ for the ρ and the A_2 trajectories are plotted in figures IV.4 and IV.5. Though no straight line represents the data well, the best are

$$\alpha_{\text{eff}}^{\rho}(t) = .46 + .71t$$

and
$$\alpha_{\text{eff}}^{A_2}(t) = .37 + .66t.$$

In the ρ trajectory, the points up to $t = -.15$ fall very close to the line through the masses² of the ρ and the g mesons, but after $t = -.15$, they tend to flatten out. This high- t tendency differs from previous Regge analyses,^{4,22} whose predictions tend to underestimate the actual high- t , high energy cross sections. In the A_2 trajectory, the points at low t fall below the line connecting the ρ and A_2 masses, and at higher t they tend to approach that line. This effect has already been observed at lower energies and has been included in Regge

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Figure IV.3

Solutions of Regge Fits to Charge Exchange and Eta Production Differential Cross Sections

Figure IV.4

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Effective ρ Trajectory
 (straight line is line through ρ and g poles)

Figure IV.5

XBL 7510-8391

Effective A_2 Trajectory(straight line is line through ρ and A_2 poles)

fits.²³ As a result, the fits tend to resemble the qualitative features of these data.

Finally, since the actual ρ and A_2 trajectories differ only by their signature, it is expected that their t dependence should be the same (exchange degeneracy). When figures IV.4 and IV.5 are compared, it becomes apparent that, for the effective ρ and A_2 trajectories, this is not true. To preserve exchange degeneracy between the actual ρ and A_2 poles, additional singularities must be included in the Regge expansion; these additional singularities would have to have approximately the same energy dependence as the bare poles.

Despite the small technical difficulties just mentioned, the most amazing overall feature of the data is that they so closely approximate simple Regge theory. If an effective ρ -trajectory of the straight line through the ρ and g masses² is used to extrapolate the 5 GeV/c, $t=-1$. differential cross sections to 200 GeV/c, the calculated value differs from the measured values by only 30% even though the actual cross sections differ by five orders of magnitude.

C. Summary

In the realm of high energy interactions, where most concepts are complicated, it is refreshing to find reactions like charge exchange and eta production which are so simple. This experiment has shown that a power law can be used to relate the total charge exchange and eta production cross sections from a few GeV/c to 200 GeV/c. Since the zero-degree charge exchange sections are consistent with the measured $\pi^+ p$

cross sections, this power law relationship seems to extend to the imaginary part of the forward amplitudes. The energy dependence of the differential cross sections can be fit with effective Regge trajectories, implying that the production dynamics and qualitative features of the cross sections are about the same over the energy range. Although a much deeper dip at $t=0$ is seen in this data than in previous measurements, this difference can be attributed to a much better t resolution, rather than to changes in interaction dynamics.

At present, no fundamental theory exists that predicts these phenomena. Hopefully, these simple relationships will lead to the development of such a strong interaction theory.

Appendix 1. The Analysis of the Detector Information: The Moments Method.²⁴

For each detector plane, the position and energy of a photon can be represented by

$$k_i = \int_{\text{detector plane}} f_i(x) dx \quad \text{and} \quad x_i = \frac{1}{k_i} \int_{\text{detector plane}} f_i(x) dx \quad \text{A.1}$$

where k_i is the energy of a photon, x_i is its position in the detector, and $f_i(x)$ is its lateral energy distribution in one plane ($f_i(x)$ is proportional to the pulse height in the hodoscope finger at position x ; the integrals are evaluated as sums over fingers). The energy distribution that is actually measured by the detector is the sum of the individual photon distributions

$$f(x) = \sum_{i=1}^{\text{number of photons}} f_i(x). \quad \text{A.2}$$

As illustrated in figure A.1, the individual shower distributions, f_i , can overlap. To eliminate the need for unfolding the individual showers, an algorithm (called the "moments method") was developed which determines the energy, production angle, and mass of the parent particle directly from the distribution $f(x)$.

The relationships in the moments algorithm are derived from energy and momentum conservation equations. The energy of the parent particle (E_p) is the sum of the energies of the individual photons (k_i)

$$\begin{aligned} E_p &= \sum_i k_i \\ &= \sum_i \int f_i(x) dx = \int \sum_i f_i(x) dx \\ &= \int f(x) dx. \end{aligned} \quad \text{A.3}$$

Figure A.1

Two photon showers, f_1 and f_2 , can overlap to give measured distribution f . Detector integrates energy deposited in each finger width.

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Figure A.2

Angles and Detector Positions Used in Moments Derivations

Through transverse momentum conservation, the production angle of the parent (θ_p) is related to the production angles of the individual photons (θ_i)

$$E_p \sin \theta_p = \sum_i k_i \sin \theta_i$$

(approximating the momentum of the parent particle with its energy; angles are defined in figure A.2). In the small angle approximation ($\sin \theta_i = x_i/L$, where L is the distance from the target to the detector)

$$\begin{aligned} E_p \sin \theta_p &= \sum_i \frac{k_i x_i}{L} = \frac{1}{L} \sum_i \int x f_i(x) dx \\ &= \frac{1}{L} \int x \sum_i f_i(x) dx \end{aligned}$$

$$\text{or} \quad \sin \theta_p = \frac{1}{E_p L} \int x f(x) dx. \quad A.4$$

Finally, the mass of the parent particle (m_p) is related to the difference of its energy and its momentum

$$\begin{aligned} \frac{1}{2} \frac{m_p^2}{E_p} &\approx E_p - p_p = \sum_i k_i (1 - \cos \theta_{ip}) \\ &\approx \frac{1}{2} \sum_i k_i \theta_{ip}^2 \end{aligned}$$

$$m_p^2 = \frac{E_p}{L^2} \sum_i k_i [(x_i - \bar{x})^2 + (y_i - \bar{y})^2] \quad A.5$$

(where θ_{ip} is the angle between the flight path of the parent and that of the photon, q.v. figure A.2). This is related to the second moment of the detector energy distribution. The RMS width of the energy distribution in the X-plane is given by

$$\begin{aligned} (\Delta x)^2 &= \langle (x - \bar{x})^2 \rangle = \frac{1}{E_p} \int (x - \bar{x})^2 f(x) dx \\ &= \frac{1}{E_p} \sum_i \int (x - \bar{x})^2 f_i(x) dx. \end{aligned}$$

For each photon

$$\begin{aligned}(x - \bar{x})^2 &= [(x - x_i) + (x_i - \bar{x})]^2 \\ &= (x - x_i)^2 + (x_i - \bar{x})^2 + 2(x_i - \bar{x})(x - x_i)\end{aligned}$$

and

$$(\Delta x)^2 = \frac{1}{E_p} \sum_i \int (x - x_i)^2 f_i(x) dx + \frac{1}{E_p} \sum_i (x_i - \bar{x})^2 \int f_i(x) dx + 0 \quad A.6$$

(the last term is zero from equation A.1). The first term in equation A.6 is a measure of the photon shower width. The shower shape is approximately independent of energy so that

$$\frac{1}{E_p} \sum_i \int (x - x_i)^2 f_i(x) dx = \frac{1}{E_p} \sum_i k_i \delta_i^2 = \frac{\delta^2 \sum k_i}{E_p} = \delta^2$$

$$\text{and} \quad (\Delta x)^2 = \delta^2 + \frac{1}{E_p} \sum_i (x_i - \bar{x})^2 k_i. \quad A.7$$

Likewise, for the Y-plane

$$(\Delta y)^2 = \delta^2 + \frac{1}{E_p} \sum_i (y_i - \bar{y})^2 k_i. \quad A.8$$

Adding A.7 and A.8 together gives

$$\sum_i k_i [(x_i - \bar{x})^2 + (y_i - \bar{y})^2] = E_p [(\Delta x)^2 + (\Delta y)^2 - 2\delta^2] \quad A.9$$

and substituting A.9 into A.5 gives the final equation for the mass of the parent particle

$$m_p^2 = \frac{E_p^2}{L^2} [(\Delta x)^2 + (\Delta y)^2 - 2\delta^2]. \quad A.10$$

Thus, equations A.3, A.4 and A.10 give the three needed parameters of the parent particle directly from $f(x)$ without explicit reference to the individual photon showers or even to number of photons hitting the detector. This rendered unnecessary the time-consuming job of determining

the individual $f_i(x)$ s.

Two other parameters were used in identification of charge exchange and eta production events: (1) the number of photons that hit the detector and (2) in two-photon events, the cosine of the decay angle (the center of mass angle between the flight path of the parent particle and the forwardmost gamma). The number of photons was found by a simple bump counting scheme: in each plane the number of local maxima above a given threshold were counted and the maximum number of bumps in either the X or Y plane was assumed to be the number of photons. This crude routine worked well for low multiplicity events, but tended to undercount higher multiplicity ones. Also, at 20 and 40 GeV/c, there was approximately 10% feedup of two photon events into the three-photon category. A correction for this loss was made in the calculation of the total cross sections.

The cosine of the decay angle was computed by finding the plane which had the greatest separation between the two photons, computing the left-right energy difference around the lowest point between the two showers, and dividing by the energy sum. This approximated the high energy relationship

$$\cos \theta_{\text{decay}} = \frac{|k_1 - k_2|}{k_1 + k_2}. \quad \text{A.11}$$

The underlying assumption behind the moments calculations is that the proportionality constant between the integrated pulse height from the detector fingers and the incident photon energy is independent of both detector position and photon energy. Non-uniform detector gains or light attenuation in the detector fingers would introduce a positional dependence in the constant. Flattening the detector response reduced this effect below the intrinsic energy resolution of the detector. Small fluctuations in the energy response affect both the energy and positional measurement of an event, but tend to

cancel out in the mass calculation. For this reason, charge exchange events could be identified in the central region as the phototubes aged, and could be used to recalibrate the central fingers during the course of a run.*

To minimize the effects of zero point fluctuations in the pulse height analyzers, a 5-count bias was subtracted from all channels before the moments calculations were made. As a result, the proportionality constant depended, to a small extent, on the photon energy (the bias cut a larger percentage from small showers than big ones). One effect of the $\cos \theta$ cut was to keep this to a minimum.

In addition to the detector pulse height information, two parameters are needed in equation A.10 to determine the mass of a particle: the ratio p_{beam}/L (p_{beam} is the incident pion beam momentum, and L is the distance from the target to the detector), and the width of a single photon shower in the detector. Since the difference between the calculated η and π^0 mass² eliminates the dependence upon the shower width, this difference was used to determine the p_{beam}/L ratio.

The computed mass² difference had to be corrected for the systematic error introduced by the 5-count bias used in the moments calculations. The actual photon shower shape was measured from data taken with 50 GeV/c pions with the detector in the 100 GeV/c position. Since this configuration had a greater than normal spread between the two charge exchange photons, it

* The procedure for recalibration was to find pions which were centered on a finger and decayed either vertically or horizontally. The average energy of all such events was calculated for a given finger, and the gain of that finger was changed to conform to the average (as determined by electron sweeps).

was possible to separate the two showers and to obtain an accurate measurement of the photon shower shape and shape fluctuations. Monte Carlo calculations using these shapes indicated that when a $\cos \theta \leq .7$ cut was made, the average value of the eta mass² was reduced by 2 to 2.5% (depending upon the detector gain setting) while the pion mass² was virtually unaffected. The value of p_{beam}/L was determined from the corrected mass² difference and the actual particle masses.

Since the distance from the target to the detector was directly measured, the beam momentum was found from that distance and the ratio p_{beam}/L ; the error in this determination was approximately .5%.

Appendix 2. The Stanford Gamma Calibrations

In the experiment, the inelastic reaction that could most easily be mistaken for charge exchange was $\pi^- p \rightarrow \Delta^0 \pi^0 \rightarrow (\pi^0 n) \pi^0$. The veto system was designed to have high efficiency for detection of photons from the Δ^0 decay. To test this design, the low energy gamma detection efficiencies were measured at Stanford's Mark III Linear Electron Accelerator during February, 1973.

The equipment used to define a tagged photon beam is illustrated in figure A.3. The electron beam was focused on a 32 mil aluminum radiator. The bunker magnet was set to select scattered electrons with three-quarters beam energy; these electrons were detected in two 2" x 8" x 1/8" counters, T1 and T2 (the energy acceptance for the scattered electrons was $\pm 2\%$). Each T1 x T2 coincidence was assumed to be accompanied by a bremsstrahlung photon with one-quarter beam energy.

The gains of the Pilot 425 veto house counters were set in unorthodox fashion. High detection efficiency was desired for very low light levels; uniform efficiency over the face of the counter was also required. Therefore, all photomultiplier gains were set to give the same counting rate when the counter was exposed to Th^{228} . Figure A.4 shows the measured gamma detection efficiency for veto house counter 2 at the gain setting eventually used at Fermilab. Detection efficiency was also studied for different positions and angles of incidence of the gamma; only minor variations were found.

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Figure A.3

Bremsstrahlung Beam Used for Veto House Counter
Photon Efficiency Measurements

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Figure A.4

Gamma Detection Efficiency for Veto House Counter 2

Solid curve is the function
 Efficiency = $1.0133 - 14.2/E_{\text{gamma}}$

The efficiencies for the veto house counters, a V2 counter and the Stanford NaI crystal are listed in table A.1. The crystal was expected to be 100% efficient; if its measured inefficiency can be attributed to beam impurities, then the gamma efficiencies for the other counters are actually higher than those given in table A.1.

Gamma Detection Efficiency for Various Counters
(as measured at the Mark III Linear Accelerator)

Counter	Gamma Energy			
	30 MeV	50 MeV	100 MeV	200 MeV
<u>Veto house counters</u>				
VH1		75.6 $\pm$.9%		
VH2	54 $\pm$ 3%	73.6 $\pm$ 1.1%	86.6 $\pm$.9%	95.2 $\pm$.9%
VH4		77.4 $\pm$ 1.4%		
<u>V2 counter</u>				
V2A				96.5 $\pm$ 1.5%
<u>NaI crystal</u>		92 $\pm$ 2%	94 $\pm$ 2%	

Table A.1

Appendix 3. Veto House and A₂ Neutron Detection Efficiencies.

The neutron efficiencies of the veto house counters were measured at the Berkeley 184" cyclotron. Stripping 450 MeV deuterons in an internal beryllium target⁺ yielded a 200 ± 4 MeV beam with full width at half-max of 45 MeV. Tagged neutrons were obtained by targeting the beam on CH₂, then deducing the energy and direction of the neutrons by measuring the angle and energy of the recoil protons. The protons were detected in a four-counter telescope with their energies determined from their ranges in copper. The arrangement of the target and the counters is illustrated in figure A.5. The trigger for a tagged neutron was P1xP2xP3xP4. Carbon background was measured with a carbon target in place of the CH₂. The target thicknesses were chosen so that the energy loss of the scattered protons would be equal in each target. The no-target background was negligible in all cases.

The efficiency of each veto house counter was measured for normally incident neutrons at several kinetic energies from 6 to 120 MeV. Table A.2 lists these efficiencies. The 40 MeV efficiency for veto house counter 1 was also studied as a function of incident neutron angle. The tagged neutrons were directed at a spot on the entrance face of the counter 3.5" from the thin side (the center of the hydrogen target during data collection). With the veto house in this configuration, efficiencies were measured for three neutron trajectories: perpendicular to the counter face, at a 45° angle of incidence, and at a 30° angle of incidence. The results of these tests, given in table A.3, show that the measured

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Figure A.5

Apparatus Used in Neutron Efficiency Measurements

Table A.2

Veto House and A₂ Neutron Efficiencies

Neutron Kinetic Energy ± Beam Spread (MeV)	Detection Efficiency (Percentage)				
	A ₂	Veto House Counters			
		VH1	VH2	VH3	VH4
7.6 ± 2.8	4.7 ± .4	1.1 ± .3	1.5 ± .4	1.9 ± .4	1.5 ± .3
10.4 ± 3.6	4.2 ± .4	3.4 ± .6	-	-	-
21 ± 5	4.4 ± .6	4.3 ± .8	4.5 ± .5	2.8 ± .4	4.3 ± .5
40 ± 7	3.3 ± .4	3.6 ± .5	5.7 ± .3	4.0 ± .6	4.9 ± .8
60 ± 8	2.4 ± .4	-	-	-	-
80 ± 9	2.3 ± .3	-	-	-	5.0 ± .7
97 ± 9	2.8 ± .7	-	-	-	-
118 ± 10	1.7 ± .5	7.1 ± .8	7.4 ± .9	6.4 ± 1.0	7.5 ± 1.0

Table A.3

Angular Neutron Efficiency for VH1 at 40 MeV

Position	Length of travel through counter	Efficiency (Per cent)	Efficiency Normalized to Straight through length
Straight through center of counter	6"	$3.6 \pm .5\%$	$3.6 \pm .5\%$
Straight through 3" from edge of counter	6"	$4.8 \pm .6$	$4.8 \pm .6$
45° angle of incidence	9.5"	$6.1 \pm .8$	$4.3 \pm .6$
30° angle of incidence (neutron exits out side)	3.0"	$1.7 \pm .3$	$3.4 \pm .6$
			average $4.0 \pm .3$

efficiencies are proportional to counter thickness in the neutron direction. The efficiencies used to correct the charge exchange and eta production cross sections were extrapolated from the measured efficiencies.

The A_2 neutron efficiencies were measured both at the cyclotron (in the same manner as the veto house) and at Fermilab directly from the 200 GeV/c charge exchange data. The cyclotron measurements are given in table A.2.

At FNAL, a large amount of 200 GeV/c charge exchange data was taken with a trigger which made no electronic cuts on the A_2 pulse height (an INTERACT trigger). From this data, only those events which had no more than one A_2 counter with a pulse height above 1/3 minimum ionizing, which satisfied all of the other charge exchange requirements, and whose neutrons were aimed directly at an A_2 counter were selected. These events were grouped by neutron kinetic energy, and the neutron efficiency was computed for each group as follows:

$$\epsilon(T_n) = \frac{N_n(T_n) - N_{\pi^0}(T_n)}{N_{\text{events}}(T_n)} \quad (\text{A.12})$$

where T_n = neutron kinetic energy

$N_n(T_n)$ = number of events above 1/3 minimum ionizing for the A_2 counter
in the neutron direction

$N_{\pi^0}(T_n)$ = number of events in the π^0 direction,

and $N_{\text{events}}(T_n)$ = total number of events accepted in the T_n energy bin.

The subtraction was necessary to eliminate the effects of N^* s in the sample.

As shown in figure A.6, these efficiencies agree very well with the cyclotron

Figure A.6

A_2 Neutron Detection Efficiency

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measurements.

The expected A_2 neutron efficiencies were calculated by the program TOTEFF.²⁵ As illustrated in figure A.6, these efficiencies are about .9% below the measured efficiencies. This is surprising because, although TOTEFF is known to calculate incorrect efficiencies for thin counters, it tends to overestimate the efficiencies rather than underestimate them. The discrepancy was probably caused by neutrons exciting high nuclear levels in the 1/8" of iron immediately behind the A2 counters (TOTEFF calculates only the neutron-scintillator interactions). These states decay by emitting photons which could be detected in the A2 counter. If there is a 1 barn cross section for the production of nuclear states which emit 2 MeV gammas, then the A2 neutron efficiency is increased by 1%. Cross sections of this size were measured for neutrons of kinetic energies from 2 to 13 MeV.²⁶

The efficiencies used to correct the charge exchange and eta production cross sections were the TOTEFF predictions increased by up to .9% to include the neutron-iron interactions.

Appendix 4. The Measurement of the π^-p Total Cross Section at 40 GeV/c.

The 40 GeV/c π^-p total cross section was measured to check the length and density of the hydrogen target. The experiment used the "good geometry" beam attenuation technique. The beam was defined by the beam telescope and electronics. The Cerenkov counter was run below pion threshold and put in anticoincidence with the beam counters to veto electrons and muons. Three circular transmission counters of diameters 2.625", 3.75", and 4.55" were placed 31' 2.8" downstream from the center of the 15.79" target flask; these counters covered the t intervals of .02, .04, and .06 (GeV)² respectively. A fourth counter of .375" diameter was placed immediately downstream of the three; it was used for centering the beam and for measuring the efficiencies of the larger counters. Each counter was more than 99.9% efficient and all efficiencies were the same to $\pm .02\%$.

The total cross section was computed from the difference between the target $2/3$ full and the target empty rates, extrapolating to t of 0 (GeV)². The average total cross section (measured over a variety of target pressures) was $24.1 \pm .1$ mb.

From Cerenkov pressure curves, a 6% beam contamination of kaons and antiprotons was found. Assuming that the kaon/antiproton ratio is the same as in the M1 line²⁷ ($K^-/\bar{p}=3$) and that the 40 GeV/c K^-p and $\bar{p}p$ cross sections are 20.5 and 44.0 mb respectively, then these hadrons will raise the measured cross section by .6%. The beam also contained .3% muons which did not fire the Cerenkov counter. The correction to the total

cross section due to both the hadrons and the muon beam contamination is $-.3 \pm 1\%$.

The target empty rate was measured with H_2 gas in the target. A comparison of the target empty rate (with H_2 gas) to the target full and target evacuated rates found the H_2 gas density to be $.5 \pm .2\%$ of the liquid density.

When corrections for both the beam contamination and the empty target H_2 gas density are made, 40 GeV/c π^-p total cross section is $24.1 \pm .3$ mb. This is in good agreement with the Serpukhov value²⁰ at 40 GeV/c of $24.36 \pm .07$ and the FNAL value²¹ of $24.01 \pm .06$. Therefore, within a maximum error of 1%, the target length and the hydrogen density are correct, and the beam definition electronics are unbiased.

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