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IONIZATION BY NUCLEAR TRANSITIONS

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I. INTRODUCTION

Ionization and excitation in an atom induced by nuclear processes [1,2] within that atom exemplify the so-called single step and two-step mechanisms. The former is characterized as a sudden shaking transition of the composite nucleus-atom system, analogous to shaking in photoemission. In the latter process the charged particles emitted in a nuclear decay transfer energy by a relatively slow "direct collision" final state interaction with atomic electrons. These are not truly distinct mechanisms; rather they are labels for the extremes of the time spans involved in different types of nuclear decays. In some theories they are even coherent interactions. For either the shaking or direct collision processes, we can give a phenomenological description of an ionization event that will apply to both types.

In any given nuclear transition, alpha, beta or electron capture decay, or internal conversion of the decay of a nuclear excited state, a definite energy release occurs, and nuclear radiation is emitted. These decays are each associated with some change in the charge or charge distribution in the central core of the atom. If an orbital electron is simultaneously ionized, the same total energy release is shared by the orbital electron and the nuclear particle or particles in a statistical way, but the energy division occurs with the orbital electron most probably taking only a small fraction of the total as its kinetic energy, usually an amount of the order of magnitude of its initial shell binding energy. The nuclear particle(s) take most of the energy in the usual energy partition. Thus the energy distributions of emitted electron and nuclear particle are continuous spectra, which in principal overlap over the entire

energy range from zero to maximum, but are largely concentrated at the extremes. Figure 1 shows an idealization of the electron spectra emitted from the K and L shells associated with a nuclear decay in which a single particle is emitted, e.g., an alpha particle or the neutrino in K electron capture or an inner shell internal conversion electron. Note that the K electron spectrum extends beyond the L spectrum, as is expected from its larger binding energy, although its intensity is very much weaker; ejection probability increases rapidly with shell number. Complementary to each is the mirror image spectrum of the alpha or neutrino or conversion electron. Each of these has its maximum energy displaced below the normal nuclear transition energy by the shell binding energy. Just below the strong line spectrum of the normal nuclear radiation one sees the weak lines of the nuclear radiation corresponding to excitation of electrons to unoccupied bound states. The composite spectrum of line and continua satellites is the analogue of shaking processes seen in the photoelectron spectrum.

Orbital electron emission can occur from any shell. It's always a weak process in inner shells for any type of nuclear decay; typically K ejection occurs with a probability per nuclear decay in the range 10^{-6} - 10^{-3} at medium Z values, depending on the transition energy and type of decay. For outer shells the probability is much higher, the total emission being about 20-30% for all shells, with a major contribution from valence shell ionization for all Z values for beta decay or internal conversion: for alpha decay the probability is nearly unity since it varies as $(\Delta Z)^2$. In contrast, the total ejection probability is negligibly small for nuclear electron capture, because in this case the effective change in central atomic charge is nearly zero.

Fig. 1. Schematic energy spectra of electrons emitted from K and L shells, and complementary "nuclear" particle satellite continua. Satellite lines corresponding to excitation of K or L electrons and K and L binding energies B_K and B_L shown.

Most of the experimental studies have been done on inner electron ionization despite its relative rarity. Inner shell electrons usually have energies that can be detected whereas the energies of outer shell electrons will mostly be too low to observe. Similarly, the complementary spectra of the nuclear particle or internal conversion electron can be resolved from the normal transition line only if it is displaced by a sufficient (inner shell) binding energy. The specific shell ionized is characterized by the shell x-ray; this is difficult for outer shells.

II. SHAKING IN NUCLEAR DECAYS

The principal mechanism contributing to ionization in beta and electron capture decay and in internal conversion is the shaking process. In fact, there is no evidence of any direct collision component in any of these decays, at a level much below its expected intensity. The familiar description of any nuclear decay process is that of a bare nuclear event. If the process involves emission or redistribution of electric charge on a fast time scale within the nucleus or the inner core of the atom, then this description of nuclear events in real atoms is incomplete; the only valid description is that of a one-step transformation of the system nucleus + atom as an entity, including both atomic and nuclear variables in initial and final states. The usual picture is that the shaking process is induced by the sudden change in central atomic charge, which leads to a conceptualization which seems rather to imply a sequence of events; first the nuclear event emits a charged particle with a certain initial energy, then an interaction results in the excitation or ionization of atomic electrons, which derive their energy from the emitted charged particle. Most of the errors of interpretation that one finds in the literature arise in this misconception of the process, which is actually a valid description only of the usually much weaker direct collision process.

A better picture of the process follows the recognition that the atomic wave functions of the initial atom are not eigenfunctions of the final state Hamiltonian, since this corresponds to a different effective central atomic charge. The atom must relax into one of the infinite number of available final eigenstates. The initial eigenfunctions cannot persist for a finite time as non-stationary states; the fast charge change does not permit a slow adiabatic adjustment of the wave function. The nuclear and atomic transitions are simultaneous processes in the composite nucleus-atom system. The overall transition rate is given by Fermi's Golden Rule No. 2 for the sudden approximation of time-dependent perturbation theory by

$$\lambda = 2\pi |M|^2_{\rho} \quad ; \quad (1)$$

ρ is the density-of-final-states factor expressing the conservation of energy and the statistical sharing of momentum among all final state emitted particles, and the matrix element of the composite system can be written

$$|M|^2 = |\langle \phi_{fin} \psi_{fin} U_{fin} | H'_{NUC} | \phi_{in} \psi_{in} U_{in} \rangle|^2 ; \quad (2)$$

ϕ denotes the nuclear wave function, ψ is the atomic wave function, and U is the wave function of any particles incident onto or emitted by the nuclear process. H'_{NUC} is the perturbation term in the nuclear Hamiltonian that generates the overall process. The familiar "bare nucleus" transition matrix element which appears in the normal nuclear transition rate without shaking is:

$$|M_{NUC}|^2 = |\langle \phi_{fin} U_{fin} | H'_{NUC} | \phi_{in} U_{in} \rangle|^2 . \quad (3)$$

If $|M_{NUC}|$ is independent of the energy of the emitted particles, which is the case in allowed beta decay and in K electron capture, then equation (2) can be factored into a product of nuclear and atomic factors

$$|M|^2 = |M_{NUC}|^2 \cdot |M_{AT}|^2 . \quad (4)$$

Here the atomic matrix element

$$|M_{AT}| = \int \psi_{fin}(Z') \cdot \psi_{in}(Z) d\tau \quad (5)$$

is the wave function overlap integral of initial and final atomic states. This procedure is a whole-atom-decay description, and is applicable if the charge perturbation is fast enough; this leads to the criterion that the speed of the emerging charged particle should be large compared to the orbital speeds of the bound electrons, so that they cannot rearrange adiabatically as the charge changes. Expressed in terms of energies, this is equivalent to the rule that the transition energy E_0 , which comes mainly from the nucleus, must be large compared to the binding energy B_i of the electron under consideration; $E_0 \gg B_i$. The suddenness condition is fulfilled for all measured nuclear decay cases except only in the ejection of K, L or M electrons in alpha decay; in this case the alpha velocity is less than the bound orbital electron velocity, and only the direct collision mechanism operates.

The atomic matrix element M_{AT} has the simple form of an overlap integral, with an atomic operator of unity, as a result of the (infinitely) sudden approximation involved in the derivation. As in the case of photoionization, it represents a monopole electronic transition, for which the selection rule is $\Delta L = \Delta J = 0$. In a higher

approximation when the finite rate of charge shift is taken into account, the contribution of the monopole shaking transition decreases, and added terms appear representing dipole or higher multipole transition operators; such terms contribute in inner shell ionization in alpha decay.

The transition rate for orbital electron ionization depends on the ejected particle energies in the statistical factor ρ , and also in the overlap integral; this integral decreases very rapidly with increasing emitted orbital electron energy, about as $(E_e/B_i)^{-7/2}$. The latter dependence explains the concentration at low energy, near B_i , for the shakeoff electrons, and correspondingly at high energy for the complementary satellite spectra of the nuclear particles.

One is usually interested in the electron shaking rate relative to the rate of normal nuclear decay without shaking, rather than in the absolute rate. Since the nuclear perturbation operator appears only in $|M_{\text{NUC}}|$ in equation (4), and since this factor appears also in the normal nuclear decay rate, it cancels in the relative shaking rate. Thus this normalized rate is approximately independent of the detailed nature of the nuclear operators causing the decay, their multipolarity or forbidden character. This does not, of course, imply that shakeoff probability is the same for all types of nuclear decay; the nuclear transition characteristics also determine the effective charge change ΔZ_{eff} , and shaking varies as $(\Delta Z_{\text{eff}})^2$. Moreover the shaking rate still depends on the transition energy and final energies of the nuclear particles.

II A. Shakeoff in Internal Conversion

Shakeoff in internal conversion is an almost perfect analogue of shakeoff-multiple-ionization in photoemission; it similarly produces two inner shell vacancies and either an electronic shakeup excitation or a shakeoff continuous spectrum plus a matching internal-conversion-electron continuum. But there is a competitive process in the decay of a nuclear excited state that can also eject two orbital electrons with continuous energy distributions, namely, the rare second order double conversion process. Thus all measurements which determine the probability of producing a pair of inner shell vacancies per normal internal conversion necessarily measure the summed contributions of the double conversion and shakeoff processes and one must resort to rather unreliable theory to separate them. Theory indicates that the shakeoff process usually dominates but by only a small factor. Such measurements consist of double K x-ray coincidence or K x-ray hypersatellite experiments.

One can distinguish the separate contributions because they yield very different shapes for the electron spectra; the shakeoff and complementary satellite conversion electron continua are

concentrated at the low and high energy extremes of the range, whereas double conversion gives a continuous two electron spectrum that is in general broadly distributed between these limits. Figure 2 shows the first observation of a satellite continuous spectrum below the K-conversion line of the 122 keV transition in ^{57}Fe as observed in the Argonne high resolution beta spectrometer. It rides on the tail of the K line, with a tiny fraction of the line intensity and its high energy threshold is displaced from the conversion line centroid by just the calculated L_3 subshell binding energy in an Fe ion with one K vacancy. It represents the K conversion electron continuous spectrum associated with the shakeoff of an L electron. It is a continuous spectrum extending down about 1% in momentum, in comparison with the narrow K line of .05% width shown fitted to the upper limit of the continuum. The L shakeoff electron spectrum lies within a kilovolt of zero; this satellite spectrum is at ~ 114 keV. The intensity of the double conversion spectrum in this region would be negligibly small. The 136 keV transition in ^{57}Fe gives an almost identical result, with exactly the same relative intensity per normal internal conversion. This transition is electric quadrupole whereas the 122 keV transition is magnetic dipole, demonstrating that shaking is independent of the intrinsic character of the nuclear transition; in contrast, their internal conversion probabilities differ by tenfold.

The spectrum area corresponds to an L shakeoff probability of 0.9% per K conversion. The self-consistent-field (SCF) wave function overlap calculations of Dr. Carlson's Oak Ridge group

FIG. 2. Complementary L-shakeoff satellite continuum on tail of K conversion line of 122 keV transition in ^{57}Fe .

corrected by a Slater screening factor to take account of the smaller effective change in central charge for K conversion compared to beta decay, gives 0.95%. Figure 2 shows the excellent agreement of the continuum shape with the theoretical shape for the satellite. A few similar cases now establish the phenomenon; in each, the shakeoff electron is from a shell outside the conversion shell. Even the satellite due to valence electron shakeoff has been seen. For shakeoff outside the conversion shell the measured shakeoff intensity lies within a factor of 2 of the Oak Ridge predictions.

The experimentally much more difficult case of shakeoff in the same shell as that in which conversion occurs is especially interesting as an indication of the need for taking account of correlations between the two closely associated bound electrons. The very weak K shakeoff in K conversion was definitely seen for transitions in ^{57}Fe and ^{137}Ba , but the weak continua could not be as accurately delineated as for L shakeoff. The assumption of the theoretical continuum shape gave shakeoff yields per K conversion between 3 and 8 times higher than predictions from the same calculations. A similar result applies to the L shakeoff in L conversion of the 89 keV transition in ^{109}Ag . Perhaps this larger disparity is indicative of the need to use a many body calculation for such cases, although accounting for correlations in the similar case of K shakeoff in nuclear K capture actually lowers the predicted intensity. However, Professor Shimizu's Kyoto group [3] showed that relativistic screening constants applied to the SCF wave function overlap integrals of the Oak Ridge group brought much nearer agreement in these cases of the same shell shakeoff. The Kyoto groups also calculated the shakeoff probability in internal conversion using relativistic hydrogenic wave functions with screening constants obtained from SCF wave functions. For K shakeoff their predictions are somewhat lower, and hence in poorer agreement with the meager data, than the earlier SCF calculation; for shakeoff in a higher shell, the Kyoto predictions are about 1/3 of the SCF predictions, again in poor agreement with the data. In summary one can only conclude that the question of the adequacy of a single electron model is not clear at present; better data is needed, particularly for shakeoff in the same shell, and by the unambiguous but very difficult electron-spectroscopic technique.

The final state has two electrons in the continuum so we write a form differential in each energy, and keep only energy dependent factors. The relative probability is

$$N(W_S, W_K) dW_S dW_K = |\{F(Z_f, W_S) B(W_S)\}|^2 p_S W_S p_K W_K \delta(A - W_S - W_K) dW_S dW_K. \quad (6)$$

The subscript K denotes the K conversion electron and S, the shakeoff electron; p is momentum and W is energy. The curly bracket is

the atomic overlap integral of the continuum final state Coulomb-field Fermi function $F(W_K)$ with the initial bound state wave function, $B(W_K)$. The density of states factors follow, including the energy-conserving δ function, the constant A being the decay energy minus the sum of both electron binding energies. By integrating the formula over W_K one obtains the shakeoff spectrum and integration over W_K gives the mirror image conversion electron satellite spectrum of Fig. 2. Note that the nuclear matrix element which governs the internal conversion probability has been factored out of equation (6) in writing the shakeoff probability per internal conversion. This has removed the sensitive dependence on multipolarity that internal conversion exhibits. The bound electron wave functions $B(W_K)$ were evaluated with non-relativistic hydrogenic wave functions to produce the shapes shown in Fig. 2; the Kyoto group improved these with relativistic wave functions.

II B. Shakeoff in Electron Capture

Nuclear capture of an atomic electron is a beta-decay process alternate to positron emission. If the transition energy exceeds the K electron binding energy, K electron capture is most probable. The K electron charge is transferred into the nucleus from near the nuclear surface leaving a K vacancy ion, and a neutrino of unique energy is emitted. The effective change in central atomic charge is much smaller in K capture than the unit charge change in positron decay since no charge is removed from the atom; for the remaining K electron the decrease in nuclear charge is nearly compensated by the removal of the partial screening afforded by the captured K electron; for outer shells this compensation is almost complete since the screening constant of a K electron for outer shells is nearly unity. Thus K shakeoff is about an order of magnitude less probable than for positron decay, being typically 10^{-4} - 10^{-5} per decay; L and higher shell shaking is still much weaker. Recall that shakeoff probability varies as the square of the effective change in central charge.

When K shakeoff occurs, the energy spectrum resembles that of the shakeoff component in internal conversion, with its maximum at zero energy and rapidly decreasing in intensity with energy above zero. The complementary satellite continuous spectrum is that of the unobserved neutrino with which the emitted electron shares the available energy. In consideration of the low interaction cross-section of neutrinos, there is no direct collision process; this is the purest example of shaking. The residual atom is doubly ionized in the K shell, so the definitive evidence of the process is the emission of two K vacancy deexcitations in coincidence, either K x-rays or Auger electrons, or the identification of a K x-ray hyper-satellite as the first K vacancy is filled. K shakeoff has been

observed in six isotopes, ^7Be , ^{37}Ar , ^{55}Fe , ^{71}Ge , ^{131}Cs and ^{165}Er , and L shakeoff in none.

The atomic matrix element overlap integral in the shakeoff rate has two bound K electrons in the initial state, one of which is destroyed, and one free electron in the final state. The principal problem in its evaluation is how to account for the initial state interaction or correlation between the two electrons; all published theories do so, by widely varying means, and since they enjoy comparable agreement (or lack of it) with experimental results, selection among them is as present a matter of theoretical preference. Because of the very small change in effective central charge in K capture, the initial and final wave functions of the remaining core change only very slightly. This means that the overlap integral between initial bound state and final continuum state wave functions is between nearly orthogonal functions, and thus it is very sensitive to their small details. It seems certain that omitting the correlation interaction would yield a grossly different result.

Three different approaches are currently competitive. All are two-initial-electron models; the remaining electrons are regarded as static during the transition. All theories antisymmetrize the initial wave function for exchange of the two K electrons and all include a density-of-states factor. The calculation of Primakoff and Porter used a variational product of two hydrogenic 1s wave functions, with extra factors adjusted to account for the effect of their mutual Coulomb interaction on their spatial correlation and for their mutual screening of the nuclear charge. The constants were evaluated by matching the Hylleraas wave function for helium. They calculated the overlap with a free electron Coulomb wave function as a function of its energy.

The Kyoto group also calculated an overlap integral, but of a relativistic hydrogenic single bound electron with a continuum electron. They accounted for the initial interelectron interaction in terms of screening constants σ for initial and final wave functions that determine the effective nuclear charge $Z_{\text{eff}} = Z - \sigma$ that each electron sees. They calculate σ from relativistic self-consistent field wave functions, and take account of the change of screening in the final K-vacancy state.

Intemann uses a very different idea. His initial wave function is without interelectron interaction; instead the K-K interaction is put into the Hamiltonian as a perturbation along with the beta decay operator, and thus the wave function can be evaluated with any desired accuracy by perturbation methods. The initial basis set without interaction is obtained as solutions of a "semi-relativistic" Hamiltonian. The result is obtained without having to calculate screening constants or an effective nuclear charge. For

this reason it is probably the preferred approach. Intemann's method also includes significant corrections for dipole contributions to the dominant monopole ejection; these arise from the relativistic mixing of orbital angular momentum.

In summary of the theoretical results, the effect of making the calculation relativistic is to reduce the shakeoff probability by 30-50%, and to change the shape of the shakeoff electron spectrum by a modest amount compared to the non-relativistic calculation. Figure 3 shows the only truly valid measurement of the K electron spectrum in K capture, done on ^{55}Fe by the Kyoto group [4] in coincidence with two K x-rays. The solid curve is their prediction of the shape, which is close to Intemann's calculation; the dashed curve is that of the non-relativistic theory of Primakoff and Porter. The fit to the relativistic form seems better, and constitutes the only clear basis for choice among the theories. However, the agreement with experimental shakeoff probabilities is better for the non-relativistic predictions in three low atomic number isotopes, while it is better with both relativistic predictions in two higher Z cases, as one might expect. The Kyoto group also predict a large contribution of K shakeup to unoccupied bound states, about equal to their shakeoff values; the ensemble of experimental values is indecisive evidence pro or con. Overall the average agreement

FIG. 3. K shakeoff energy spectrum in K capture in ^{55}Fe . Solid curve; relativistic theory of Kyoto group. Dashed curve; non-relativistic theory of Primakoff and Porter.

is poor, but this is at least in part owing to large variance among the few measurements in these difficult experiments.

II C. Shakeoff in Beta Decay

Here the situation is less ambiguous than for electron capture; only the theory of Law and Campbell for K shakeoff in beta transitions with allowed shapes appears to be very well supported by the ensemble of recent experiments of better quality. These are measurements of shakeoff probability done with high resolution x-ray detectors and measurements of the composite energy spectra of shakeoff-electron plus-nuclear-beta done by magnetic spectrometry in coincidence with K x-rays. All of the dozen or so published calculations of K electron emission in beta decay have used the sudden (shaking) approximation, but the complication of a three particle final state, two free electrons and an antineutrino, has been handled by way of various approximations that are often inadequate. Law and Campbell's treatment is the culmination of several attempts of increasing sophistication spanning 30 years; it is the first to use a fully antisymmetrised, relativistic form incorporating nuclear and atomic variables. Its approximations involve: a) the use of a one electron atom model, so the initial state is described with a Dirac wave function without screening and of course without correlations; and b) apart from the antisymmetrization of the two electron wave function, the final state interactions of the two free electrons with each other (or with other electrons) are not yet formally included, though their interaction with the residual nucleus is accounted for.

Law and Campbell use the formalism of second quantization to express the beta decay transition for an initial atom with nuclear charge Z and one K electron to a final atom with charge $Z \pm 1$, plus two free electrons and a neutrino. The perturbation operator in the transition matrix element which induces the transition is the V-A beta decay Hamiltonian H_β . In a standard notation for Fermi's golden rule the K shakeoff rate in $\beta^\pm$ decay is written:

$$\lambda_{\pm} = 2\pi |\langle f | H_\beta | i \rangle|^2 \delta(E_i - E_f) ; \quad (7)$$

where the wave functions f and i include nuclear and atomic variables, the delta function conserves energy and the density of states is implicitly included. This means that all three emitted particles including the neutrino share the energy statistically. If p is the momentum of an electron and q is that of the neutrino, the differential shakeoff rate dependence on p , which is the electron momentum spectrum is

$$\lambda_{\pm}(p) dp = A p^2 dp \int q^2 dq M ; \quad (8)$$

this resembles a standard allowed beta spectrum. Constant A includes the β decay coupling constants and 2π 's. The second quantized form of the overall transition matrix element M brings in the third momentum, s, of the ejected electron, and also the essential difference between the β^+ and β^- cases. In the β^+ decay the two electrons in the free state can be distinguished by their charges; but not in β^- decay. The necessary antisymmetrization of the final state wave function with respect to the two electrons in the β^- case results in a complete formal symmetry of the final expression for the electron momentum spectrum; this symmetry arises from the coherent addition of the amplitudes of the wave functions of the free electrons. The double differential electron momentum spectrum shows this symmetry in all functions of s and p:

$$\lambda_{\pm}(p,s)dpds = Ap^2s^2(E_0-E_p-E_s)^2 dsdp\{|\langle\psi_s|\psi_K\rangle|^2F(Z',E_p) + |\langle\psi_p|\psi_K\rangle|^2F(Z',E_s) - \langle\psi_s|\psi_K\rangle\langle\psi_K|\psi_p\rangle\sqrt{F(Z',E_s)F(Z',E_p)}} \quad (9)$$

E denotes energy and the factor $(E_0-E_s-E_p)$ is the neutrino energy; F is the Fermi function of beta decay theory that accounts for the interaction of each electron with the nuclear Coulomb field; the angular brackets are the wave function overlap integrals of the s and p free electrons with the initial bound K electron. Inside the curly brackets are three terms, two squared terms and a negative interference term. Only the first term appears in the β^+ shakeoff calculation; this leads to a reduction of the predicted shakeoff probability by $\sim 1/2$ compared to that in β^- decay at the same energy and atomic number. By integrating over either electron momentum the spectrum of the other electron is obtained, but the momentum dependence of the overlap integrals results in different spectrum shapes for the three terms. The two leading terms, which give equal total contributions to the intensity, are relatively emphasized at high and low energy regions of the spectrum, and in analogy to the idealized shakeoff spectrum (Fig. 1) these are called the nuclear and orbital components, respectively. The interference term is of equal intensity in low energy decays with a low ratio of transition energy to K binding energy, but it becomes relatively small for high decay-energy cases.

The inner orbitals of heavy atoms and high energies involved in beta decay both require the use of relativistic wave functions in the overlap integrals; for simplicity, Law and Campbell used hydrogenic type. The calculated shakeoff rate is doubled to account for two initial K electrons. The calculation was extended to include shakeup excitation to unoccupied bound states, which adds significantly to the total shaking, particularly in cases with low ratios of transition energy to K binding energy. From the nuclear standpoint the theory is appropriate only for allowed beta transitions or for those forbidden transitions with allowed shapes.

Experimental comparisons to the theory are most reliable with 33 recent improved measurements of the K shaking probability P_K , covering 15 β^- decaying isotopes from $Z = 20$ -83, and ranging in P_K values from 2×10^{-6} to 2×10^{-3} ; the measurements were made by a variety of independent techniques. Only two cases of highly forbidden-shape transitions were in slight disagreement with the predictions; the averaged agreement for all isotopes lies within a few percent with Law-Campbell theory, but tenfold or more worse with any other calculation.

Accurate measurement of spectrum shapes again turns out to be a more sensitive test of the theory. The Argonne group has measured five representative cases covering a wide range in transition energy and in isotopes with both allowed and forbidden shapes for the singles spectrum. The composite nuclear beta- K electron spectrum focused in a magnetic spectrometer was registered in coincidence with K x-rays observed in a GeLi detector. In Fig. 4 is the momentum

FIG. 4. Composite β^- + K-orbital shakeoff momentum spectrum in β^- decay of ^{143}Pr . "N + O + X" curve is theory of Law and Campbell, summing nuclear (N), orbital (O) and interference (X) terms. Direct collision shape of Feinberg theory shown.

spectrum of ^{143}Pr , a 930 keV allowed shape β^- transition. The separate nuclear, orbital and interference terms of Law-Campbell theory are shown, and their sum clearly fits the observed spectrum very well over the full energy range from 6 keV up; for comparison, the K binding energy is 44 keV. Similar excellent fits are seen in the 227 keV decay of ^{147}Pm and even in ^{151}Sm , which has a transition energy of 77 keV, only 1.5 times the K binding energy of 49 keV. In this case the low energy shakeoff spectrum which has a maximum energy of only 28 keV serves as a severe test of the applicability of the sudden approximation treatment. The good fit of the shape to the Law-Campbell theory and the exact agreement of the K emission probability (obtained from the spectrum area) at 2.3×10^{-6} per decay attests the validity of the sudden approximation in this limit.

The agreement of the shakeoff value also sets the lowest limits on the contribution of the direct collision mechanism, to $\sim 1/5$ of that predicted by the Feinberg theory. Feinberg analyzed direct collision as a final state interaction of the electrons which is coherent with the shaking interaction; he then dropped the interference term, retaining only the additive direct collision component. It is in this sense that direct collision is regarded as an independent additive mechanism. His prediction states that the relative intensity ratio of direct collision to shakeoff is given approximately by the ratio of K binding energy to available beta transition energy. For ^{151}Sm this ratio is 38%. Thus there is no affirmative evidence for this final state interaction component, even in this most favorable case.

Shakeoff spectrum shapes for two transitions with large deviations from the allowed shape, ^{210}Bi and ^{89}Sr , disagreed with L-C theory, but much less when theory was corrected with the measured singles shape correction factor, per L-C's ad-hoc prescription. This is the principal deficiency of the theory.

To sum up the case in beta decay, there seems little indication of need to improve the simple one-electron relativistic wave functions or to take account of initial state correlations in the K shell. Not enough data exists for L electron or outer shell shakeoff on which to judge this question. Nor, it appears, need final state energy exchange be considered.

III. INNER ELECTRON EJECTION IN ALPHA DECAY

Alpha particle velocities are of order 10^9 cm/sec, which is below mean orbital electron velocities in K, L or M shells in heavy elements. Thus the α passage is essentially adiabatic so electron shakeoff due to the relatively slow reduction of the central field should contribute little to the ejection probability. The electron

emission arises instead from the direct collision process. In the first stage the α is ejected by the nucleus through the potential barrier past the outer classical turning radius of about 4 Fermis with a definite total energy. This is transformed to over 99% of the final α kinetic energy inside the mean radius of the K orbit. The second stage energy exchange with the inner electron mainly occurs in the vicinity of the orbit, since the Coulomb interaction potential between α and electron is maximum as the α passes the shell.

All but one recent calculation use time dependent perturbation theory in a higher order approximation than the sudden approximation to treat this problem. Thus, in the original theory of Migdal, the probability for ejection of an electron from the n shell of the ground state configuration g , to a final state, j , is given by

$$P_n(j) = \frac{i}{h} \int_0^\infty e^{-i(E_j - E_g)t} |\langle \psi_j | H'(t) | \psi_g \rangle|^2 dt \quad (10)$$

Here E is energy, and $H'(t)$ is the time-varying Coulomb perturbation operator given by:

$$H'(t) = -2e^2/r_\alpha = -2e^2/v_\alpha t \quad (11)$$

where r_α and v_α are the alpha radial position and velocity. In a later version by Levinger, this interaction is greatly diminished by an opposing contribution from the potential of the slowly recoiling nucleus with velocity v_r . The integral in equation (10) was developed in a series resembling a multipolar expansion. The monopole term corresponding to shaking was shown to be negligible; the dipole term was evaluated with hydrogenic wave functions to give an ejection probability for the n shell

$$P_n = \frac{(v_r - 2v_\alpha/Z)^2}{Z^2} C_n \quad (12)$$

where the C_n are squares of hydrogenic dipole matrix elements for the n shell. In Migdal's version the recoil velocity term is omitted. Various refinements such as relativistic and nuclear screening corrections and excitations to unoccupied bound levels have been added.

The only isotope with a simple ground-to-ground state α decay scheme that is feasible for measurement of the weak ejection probabilities is ^{210}Po , on which essentially all experiments have been made. The probability is obtained from the intensity of the shell x-rays, corrected for fluorescence yield, measured either in singles or in coincidence with alphas or with ejected electrons. For the inner shells the probabilities are given in Table I, averaged over nine K shell, six L shell and three M shell measurements. One sees

TABLE 1. EMISSION PROBABILITIES PER ALPHA DECAY IN ^{210}Po

	Exp't	Hansen	Migdal	Levinger	Rubinson
K($\times 10^{-6}$)	1.88 ± 0.15	2.0	2.5	0.10	0.14
L($\times 10^{-4}$)	7.9 ± 0.4	5.9	1.13	0.6	0.028
M($\times 10^{-2}$)	2.3 ± 0.3	1.90	0.16		0.0056

from the comparison to the Migdal predictions that the agreement with the K shell values is fair, but for the L shell theory is at least a factor of seven low, and for the M shell, a factor of twenty; even worse for the later improved versions of the theory.

A recent calculation by Hansen [5], has achieved fairly good agreement with the data in all three inner shells. This theory is an adaptation of the binary encounter approximation used in the evaluation of electron ejection in ionic bombardment; ionization occurs when the coulomb interaction transfers at least the electron binding energy from the incident ion to a single electron. The cross section for this transfer is integrated over the infinite range of impact parameters in the ion beam. For α decay only the outgoing wave is considered, and only zero impact parameter. The energy-transfer cross section is evaluated with an exact two-body treatment of a classical α trajectory and a particular electron treated quantum mechanically; the cross section is a function of the α and averaged electron velocities. The electron velocities are calculated using screened relativistic hydrogenic wave functions, and the α acceleration is taken into account. (However, there appears to be an inconsistency in the relativistic velocity transformation, which seriously affects the results, particularly for the K shell.)

Again the shapes of the spectra of α and electron turn out to be more severe tests of the theory than the integrated probabilities. The α complementary satellite spectrum shape associated with L electron ejection was measured [6] with high resolution in the Argonne magnetic α spectrometer, in coincidence with K or L x-rays. Figure 5 shows these satellites corresponding to K and L electron ejection, displaced below the α line by the K and L binding energies. Near zero energy is the electron spectrum which was measured in the Argonne magnetic electron spectrometer in coincidence with L x-rays; the mirror image of this spectrum, broadened with the line shape of the alpha spectrometer, is the dashed curve (b), which fits the L shell α complementary spectrum quite well, as expected. But great disparities with the predictions of any theory appear in the K and L electron spectra extended to 100 keV. The binary encounter theory gives no match whatever to the measured shapes, despite the fairly good agreement with the ejection probabilities, and Migdal's predicted shape for the K spectrum also fails below 30 keV.

FIG. 5. α complementary satellite continua associated with ejection of K and L electrons in ^{210}Po . L electron spectrum (a) shown at left, and fit of its complement, convoluted with α_0 normal line, to the " α -L" continuum is shown dashed (b). B_K and B_L binding energies at top.

An unanticipated result emerged that is a further challenge to ejection theory; namely, it appears that in $\sim 2/3$ of the interactions of the emerging α with an L electron, (but not with a K electron), the electron is captured into a bound state of the α particle, which must then emerge as a singly charged He^+ ion. It would seem that an attempt to account for this result must consider the interaction of the developing level structures of the α particle and the residual lead nucleus, as the α moves through the shells, and must also reproduce the measured energy spectra.

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