LUMINOSITY LOSS RATE DUE TO INTRABEAM SCATTERING

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In the limit of short interaction region the luminosity can be very well represented by

\[ L = \frac{N^2 B \text{ freq}}{4\pi \sigma_v^* \sigma_H^* \gamma} \]

where, for identical colliding beams,

- \( N \): number of particles per bunch,
- \( B \): number of bunches per beam,
- \( \text{freq} \): revolution frequency,
- \( \sigma_v^* \): vertical rms beam size at the collision point,
- \( \sigma_H^* \): horizontal rms beam size at the collision point,

assuming crossing at an angle on the horizontal plane

\[ \gamma = \sqrt{1 + p^2} \]

and

\[ p = \frac{\alpha e}{2 \sigma_H^*} \]
\[ \alpha, \text{ total crossing angle} \]

\[ \sigma_e, \text{ rms bunch length} \]

Intrabeam scattering has the effect of having \( \sigma_v^*, \sigma_h^* \) and \( \sigma_e \), the r.m.s. energy spread to vary, generally by growing.

Define the luminosity loss rate

\[ \chi_l^{-1} = - \frac{1}{L} \frac{dL}{dt} \]

Obviously:

\[ \chi_l^{-1} = \frac{1}{\sigma_v^*} \frac{d\sigma_v^*}{dt} + \frac{1}{\sigma_h^*} \frac{d\sigma_h^*}{dt} + \frac{1}{f} \frac{df}{dt} \]

One can work out that

\[ \frac{1}{f} \frac{df}{dt} = \frac{f^2}{\sigma_e^2} \left[ \frac{1}{\sigma_e} \frac{d\sigma_e}{dt} - \frac{1}{\sigma_h^*} \frac{d\sigma_h^*}{dt} \right] \]
Assumptions:

1. The bunch length increases (or decreases) at the same rate the energy spread does, that is

\[ \frac{1}{\sigma_e} \frac{d\sigma_e}{dt} = \tau_e^{-1} \]

2. We assume that the two modes of oscillations (H and V) are fully coupled to each other on a time shorter than the intrabeam scattering diffusion times, that is

\[ \frac{1}{\sigma_v^+} \frac{d\sigma_v^+}{dt} = \frac{1}{\sigma_H^+} \frac{d\sigma_H^+}{dt} = \tau_H^{-1} + \tau_V^{-1} \]

Here, \( \tau_e^{-1} \), \( \tau_H^{-1} \), and \( \tau_V^{-1} \) are the diffusion (or damping rates) from intrabeam scattering respectively in energy, horizontal (betaatron) side and vertical (betaatron) side.
In conclusion

$$\tau_{L}^{-1} = \frac{p^2}{p^2} \tau_{E}^{-1} + \left(2 - \frac{p^2}{p^2}\right) \left(\tau_{H}^{-1} + \tau_{V}^{-1}\right)$$

For head-on collision, $p=0$, $f=1$ and

$$\tau_{L}^{-1} = 2 \left(\tau_{H}^{-1} + \tau_{V}^{-1}\right)$$