Tune Spreads and Tune Splitting due to Random Field Errors in RHIC

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(1)

Desired V -spread

Goal is to stay
within box shown —
and not cross
resonances below
10th order

$$\Delta V (\text{beam-beam}) \approx 6 \times .004 = .024$$

What is ΔV due to random b_k, a_k ?

Low Amplitude Non-resonance Result

For small enough ϵ_x, ϵ_y , $\delta = \Delta p/p$
and not near a resonance

$$\Delta V = \Delta V(\delta, \epsilon_x, \epsilon_y)$$

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Example, ΔV due to Sextupoles

$$\Delta V_x = C_x \delta + a E_x + b E_y$$

$$\Delta V_y = C_y \delta + c E_x + d E_y$$

To find ΔV -spread, compute $\Delta V_x, \Delta V_y$ along the line $E_x + E_y = E_t$, $E_t = 6.5$ for RHIC at $\delta = 30$ for several values of δ .

See Fig 1 for Sextupole Result

For Sextupoles, $\Delta V \approx 5 \times 10^{-3}$ for
 $\beta^* = 3$ lattice.

Sextupole γ -s pread

$\beta^* = 3$ lattice

$$\epsilon_x + \epsilon_y \leq 6.5, \quad C_x = C_y = 0$$

$$\Delta \gamma \leq 5 \times 10^{-3}$$

Fig 1

ν -spread near Coupling Resonances

Tracking results, ^{point to} something going on not indicated by low amplitude non-resonance result.

~~Linear~~ Linear Coupling Resonance (Review)

Both x and y motion contain two ν -values ν_x

$$x \cong A_1 e^{i\nu_1\theta} + A_2 e^{i\nu_2\theta} + \text{c.c.}$$

$$y = B_1 e^{i\nu_1\theta} + B_2 e^{i\nu_2\theta} + \text{c.c.}$$

Both x and y have the same ν -values ν_1 and ν_2

Splitting $\nu_1 - \nu_2 \cong 2 |\Delta\nu_{11}|$
 $\Delta\nu_{11}$ is coupling stop-band

For RHIC, without Magnet Shuffling, $\Delta\nu_{11, \text{max}} \cong .03$
 and $\nu_1 - \nu_2 \lesssim .06$

Linear coupling gives a ν -splitting, but not a ν -spread.

Non-Linear Coupling Resonance ν -shifts

ν -splitting depends on ϵ_x, ϵ_y

Fig 2 shows Tracking Result for ν_y versus χ_0 , initial x -betatron oscillation, when $y_0 = 0$ or $\delta = .005$.

ν_y splitting increases with χ_0 .
At largest χ_0 , $\chi_0 = 18 \text{ mm}$, $\epsilon_x = 6.5$,
 ν -splitting is $\Delta\nu = .034$

This ν -splitting is a ν -spread; all intermediate ν -values reached at lower values of χ_0 .

V-Spread Study for 10 Sets of Random Field Errors

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$$\epsilon_x + \epsilon_y = 6.5$$

Seed		$\epsilon_x = \epsilon_y = 3.25$	$\epsilon_x = 0, \epsilon_y = 6.5$	$\epsilon_y = 0, \epsilon_x = 6.5$	ΔV_{max}
Seed 3	$\delta = .005$	V_x	.828, .817, .823, .832	.827, .818, .823, .831	.830
		V_y	.822, .818, .828, .833	.822, .819, .828, .831	.830, .817
	$\delta = -.005$	V_x	.818	.823, .833	.839
		V_y	.818	.822, .833	.810, .839
Seed 1	$\delta = .005$	V_x	.818	.827, .821, .815	.834
		V_y	.819	.821, .815, .827	.834, .814
	$\delta = -.005$	V_x	.821, .825	.828, .820	.818
		V_y	.823, .818, .820	.820	.824, .819
Seed 2	$\delta = .005$	V_x	.830	.828, .825	.841
		V_y	.820	.825, .822, .829	.841, .807
	$\delta = -.005$	V_x	.827	.820, .830	.831
		V_y	.821	.820	.831, .816
Seed 4	$\delta = .005$	V_x			.834
		V_y			.816, .834
	$\delta = -.005$	V_x			.842
		V_y			.809, .842
Seed 5	$\delta = .005$	V_x			.833, .837
		V_y			.817, .833
	$\delta = -.005$	V_x			.834
		V_y			.834, .815
Seed 6	$\delta = .005$	V_x			.829
		V_y			.829, .819
	$\delta = -.005$	V_x			.835
		V_y			.811, .835

V-splitting Versus N_{seed}

$\epsilon_y = 0$, $\mu_0 = 18 \text{ mm}$

Scenario for γ -spread

at the start

$$\Delta V_{\text{beam-beam}} \sim .024$$

$$\Delta V (\text{betatron osc.}) \sim \text{small}$$

after many hours

Beam grows due to intrabeam scattering

$$\Delta V_{\text{beam-beam}} \sim .008$$

$$\Delta V (\text{betatron osc.}) \sim .02$$

Also, these ΔV are to some extent
not additive

$\Delta V_{\text{beam-beam}} \rightarrow$ Large at small betatron osc.
Small at large betatron osc

$\Delta V (\text{betatron osc.}) \rightarrow$ Small at small betatron osc.
Large at large betatron osc.

Multipole Break down - Tune Splitting

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Initial $\epsilon_y = 0$, $\epsilon_x = 6.5$, $u = 18\text{mm}$

Kmax		Seed 3	Seed 8
no	v_x	.829	.829
b_k, a_k	v_y	.819	.819
2	v_x	.827	.842
	v_y	.818, .827	.807, .841
3	v_x	.825	.846
	v_y	.817, .825	.846, .801
4	v_x	.839	.841
	v_y	.801, .839	.841, .806
5	v_x	.838	.840
	v_y	.803, .838	.840, .807
6	v_x	.837	.839
	v_y	.837, .804	.839, .807
7	v_x	.836	.839
	v_y	.836, .804	.839, .807
8	v_x	.836	.840
	v_y	.836, .805	.840, .807
9	v_x	.837	.840
	v_y	.837, .805	.840, .806
10	v_x	.836	.840
	v_y	.836, .806	.840, .807
$\Delta v_{\max}$		.030	.033
δ		-.005	-.005

Which multipoles b_k, a_k are responsible for the v -splitting? In above, $k_{\max}$ is the highest multipole present.

Answer not clear.