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COMPUTATION OF OPTIMAL CONTROLS
FOR A NUCLEAR ROCKET REACTOR

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By

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ABSTRACT

Various optimal reactor control problems with complicated constraints are solved for a nuclear rocket engine. The technique utilizes a digital computer with the numerical algorithm formed by a succession of linear programming problems. Each programming problem has a solution which is used to relinearize the system's dynamical equations.

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Introduction

This paper considers the computation of the optimal controls for a nuclear rocket reactor. Optimal controls have been calculated for a nuclear rocket reactor in reference 1 by means of the maximum principle and its extensions for state constraints. In that study, it was necessary to keep the constraints fairly simple. Utilizing a programming technique the present study considers more complicated constraints.

Most numerical procedures for the calculation of optimal control fall into the general categories of either gradient methods, generalized Newton methods or methods based on the maximum principle. In almost all cases, Lagrange multiplier functions are used to addend the constraint differential equations to the optimality criterion. In many cases, as for the neutron kinetics, the natural instability of the system makes a practical application of the preceding techniques very difficult.

The numerical procedure used here, however, is based on an iterative sequence of linear programming problems (reference 2). In this context the distinction between state and control loses much of its significance, and inequality constraints, which form an integral part of linear programming, are as easily applied to the state as to the control. Additionally, no Lagrange Multipliers

or penalty functions are usually needed, since each iterate can easily be forced to satisfy all inequality constraints and boundary conditions. The fact that each iterate satisfies all inequality constraints eliminates the instability problems associated with the nuclear reactor.

Nuclear Rocket Reactor

The nuclear rocket reactor to be considered is one which makes use of hydrogen as the propelling medium. Figure 1 is a schematic illustration of a nuclear rocket propulsion system. Here, hydrogen is pumped over the reactor core, where heat is transferred to the propellant. The hot hydrogen gas then exits through a nozzle to furnish the thrust. It is assumed that the only variables available to control the nuclear rocket reactor are the hydrogen flow-rate, u_2 , and the reactivity u_1 .

The differential equations which approximate the nuclear reactor behavior are

$$\dot{Q} = \left(\frac{u_1 - \beta}{\ell} \right) Q + \lambda C, \quad (1)$$

$$\dot{C} = \frac{\beta}{\ell} Q - \lambda C; \quad (2)$$

where: Q is the reactor power level,
 C is the neutron precursor level,
 u_1 is the reactivity,
 u_2 is the hydrogen flow rate

β is the proportion of neutrons which are delayed due to the precursors,
 ℓ is the mean generation time of neutrons, and
 λ is the average decay constant of the neutron precursors.

Equations (1) and (2) are the classical one-lump, spatially-independent neutron kinetics, which are discussed in reference 3.

The heat transfer between the reactor core and the hydrogen is approximated by

$$\dot{T} = \frac{Q}{M_c} - au_2 T; \quad (3)$$

where: T is the average core temperature,
 M_c is the effective core mass heat capacity,
 a is a heat-transfer coefficient.

Equation (3) is a one-lump heat transfer process, which is discussed in reference 1.

Optimal Control Problem

The specific problem considered is that of reactor startup. It is assumed that the reactor is operating at a relatively low power level -- sufficient to supply power to various auxiliary systems. It is desired to transfer the reactor from this low power level to a power level and temperature which corresponds to maximum thrust. During this transfer, the state and control variables are subject to inequality magnitude and rate constraints for mechanical

and safety reasons. The totality of constraints which are considered are

$$\begin{aligned}
 0 &\leq Q \leq Q_{\max} , \\
 0 &\leq C \\
 0 &\leq T \leq T_{\max} , \\
 \dot{T} &\leq \dot{T}_{\max} , \\
 -0.9\beta &\leq u_1 \leq 0.9\beta , \\
 \dot{u}_1 &\leq \dot{u}_1 \max , \\
 u_{2\min} &\leq u_2 \leq u_{2\max} .
 \end{aligned}
 \tag{4}$$

The initial boundary conditions ($Q(0)$, $C(0)$, $T(0)$) are specified by the steady-state low power level. The terminal values of power and temperature ($Q(t_1)$, $T(t_1)$) are fixed by the maximum thrust criteria. The terminal value of precursor level ($C(t_1)$), as well as the terminal time (t_1) itself, are allowed to be free, subject of course to the restrictions

$$\begin{aligned}
 0 &\leq t_1 , \\
 0 &\leq C(t_1) .
 \end{aligned}$$

The criterion of most interest in the operation of the nuclear rocket reactor is the quantity of hydrogen consumed in the process of transferring from one operating condition to another. A small percentage saving in hydrogen consumed can result in a large percentage increase in payload. This is particularly true since not only is a saving affected in the hydrogen proper, but also a saving

is affected in the size of the on-board storage tanks. For this reason, the optimality criteria is that the quantity J , defined by

$$J = \int_{t_0}^{t_1} u_2(t) dt, \quad (5)$$

have minimum value.

Computational method

The numerical procedure used to obtain solutions to the optimal nuclear rocket reactor problem makes use of an iterative sequence of linear programming problems. Figure 2 is a schematic illustration of the algorithm. A convergence proof for a very restricted class of problems is presented in reference 2. The first step is to approximate the continuous differential equations and the integral performance criterion by a set of discrete algebraic equations. There are of course many methods to obtain these approximations, most of which are patterned after numerical integration procedures. The technique used here for the process differential equations is the second-order approximation

$$Q_{k+1} - Q_k = \frac{\Delta t}{2} \left\{ \left(\frac{u_{1k}^{-\beta}}{\ell} \right) (Q_{k+1} + Q_k) + \lambda (C_{k+1} + C_k) \right\} \quad (6)$$

$$C_{k+1} - C_k = \frac{\Delta t}{2} \left\{ \frac{\beta}{\ell} (Q_{k+1} + Q_k) - \lambda (C_{k+1} + C_k) \right\} \quad (7)$$

$$T_{k+1} - T_k = \frac{\Delta t}{2} \left\{ \frac{1}{M_c} (Q_{k+1} + Q_k) - a u_{2k} (T_{k+1} + T_k) \right\} \quad (8)$$

In obtaining equations 6, 7 and 8, it is assumed that the controls are stepwise approximations; that is, the controls u_1 and u_2 are constant over each time interval Δt . Since the approximations are second-order, the truncation error of the Taylor series, over any interval t to $t+\Delta t$ (or k to $k+1$) is proportional to Δt^3 . The interval Δt , while variable, is determined by

$$\Delta t = \frac{(t_1 - t_0)}{N} ; \quad (9)$$

where: N is the number of time increments in the interval $(t_1 - t_0)$.

Due to the fact that the controls are piecewise constant over each Δt , the integral performance criterion can be represented algebraically by

$$J = \Delta t \sum_{k=1}^N u_{2k} \quad (10)$$

Equations 6, 7, 8 and 10 are nonlinear, and thus are not directly suitable for a linear programming problem. The general linear programming problem is to determine a vector $\underline{x}$ such that the linear objective function

$$\phi(\underline{x}) = \underline{c}^T \underline{x} ; \quad (11)$$

where: $\underline{c}^T$ is a constant row vector, has minimum (maximum) value. The vector $\underline{x}$ must also satisfy the constraints

$$A \underline{x} (\leq) \text{ or } (=) \text{ or } (\geq) \underline{b} ; \quad (12)$$

where: A is a constant matrix,

$\underline{b}$ is a constant vector,

and

$$\alpha_i \leq x_i \leq \beta_i ;$$

where: α_i and β_i are constants, and

i ranges over all components of $\underline{x}$.

A complete discussion of linear programming can be found in many sources; see for example reference 4.

In order to execute linear programming procedures, the nonlinear equations 6, 7, 8 and 10 must be linearized to conform to equations 11 and 12. The linearization is accomplished by retaining the first two terms (the constant and linear terms) of the Taylor series expansion for the nonlinear equations. Since the terminal time is free, Δt is treated as a variable in the Taylor series and in the ensuing linear programming problems.

Except for the rate constraints, linearization completes the formulation of the linear programming problem. From a practical standpoint, the rate constraints are easily formulated as linear inequality constraints of the form shown in equation 12. That is, the inequality constraints

$$u_{l_{k+1}} - u_{l_k} \leq \Delta t \dot{u}_l \text{ max} \quad (14)$$

and

$$T_{k+1} - T_k \leq \Delta t \dot{T}_{\max} \quad (15)$$

will prevent the reactivity and temperature from increasing too rapidly.

As can be seen from figure 2, the solution to the non-linear problem is obtained as the limit of a sequence of linear programming problems with each new problem in the sequence resulting from a linearization about the previous solution. Because of the generalized Newtonian character of this process, convergence, if it occurs, will be quadratic.

Most large scale digital computers have general linear programming systems available which considerably simplify the programming effort required for the solution of problems in this class. The calculations in this paper were performed on an IBM-360/40 digital computer located at the University of New Mexico Computing Center. The Mathematical Programming System/360 (MPS/360) is a general system available for solution of linear programming problems on the IBM-360 series of computers. Using the source language of MPS/360, only sixteen statements were required for the solution of the linear programming problems associated with the nuclear rocket reactor. This program includes the reinsertion of the previous optimal basis as the starting basis for each new iteration.

The matrix of coefficients and other data associated with the linearized algebraic equations are illustrated in figure 3. The recursive nature of the algebraic equations and the inequality rate constraints generates a blocked matrix. The non-zero coefficients of this matrix, together with the coefficients in the right-hand-side column and the data on bounds for each variable, form the input data for the MPS/360 program. On each iteration these coefficients must be generated in the proper format for MPS/360. Since the matrix is blocked, the data may be generated by a series of DO-loops in a Fortran program. The complete algorithm shown in figure 2 was formed by alternately running the Fortran and MPS/360 programs. Convergence was indicated when the input data to the MPS/360 program was optimal in accordance with the standard tolerances set in the MPS/ 360 program.

Computational Results for the Nuclear Rocket Reactor

Computational results for the nuclear rocket reactor have been obtained using the linear programming algorithm previously described. For numerical calculations, the parameter values, constraints and initial conditions were assumed to be those shown in table 1.

Table 1
Numerical Values

<u>Parameter</u>	<u>Value</u>
β	0.64×10^{-2}
ℓ	0.3×10^{-4} sec
λ	0.4 (sec) $^{-1}$
M_c	0.555×10^{-2} MJ/k $^{\circ}$
a	0.18×10^{-1} # $^{-1}$
u_{lmin}	-0.9β
u_{lmax}	0.9β
u_{2min}	2.0 #/sec
u_{2max}	400.0 #/sec
$\dot{u}_{lmax}$	0.15×10^{-1} /sec
$\dot{T}_{max}$	1600.0 K $^{\circ}$ /sec
Q_{max}	20.0 MW
T_{max}	2500.0 $^{\circ}$ K
$Q(0)$	0.01 MW
$c(0)$	5.332
$T(0)$	277.8 $^{\circ}$ K
$Q(t_1)$	20.0 MW
$c(t_1)$	free
$T(t_1)$	2500.0 $^{\circ}$ K

Figure 4 shows the trajectories obtained in the absence of the reactivity and temperature rate constraints. The data in this figure confirm the results of Mohler (reference 1) that the minimum propellant process is also one of minimum time. That is, the optimal policy is to insert maximal reactivity until the power level constraint is reached. The reactivity must then decrease in order to hold the power level on its constraint boundary. The constant power level condition continues until the terminal temperature boundary condition is satisfied. During the entire startup process, the propellant flow rate remains on its lower boundary. The process is certainly an absolute minimum, since the integral performance criteria assumes its absolute minimum value under these conditions. The computation time taken to obtain the data is shown in table 2.

The data illustrated in figure 5 is similar to that of figure 4; however, the effective, core, mass heat capacity was artificially lowered by a factor of ten so that the temperature constraint was reached prior to the power constraint. In this case, the optimal process is also one of minimal time; but, the propellant consumption is much higher than that of figure 4. The higher propellant consumption results from the fact that the hydrogen flow rate must increase in order to keep the temperature on its boundary, during the latter portion of the trajectory. The optimality of the trajectory shown in figure 5 might

be questioned, since there is an alternative policy which can keep the temperature on its boundary. That is, one could allow the propellant flow rate to remain on its lower bound and use reactivity variations to vary the power level, Q , so that the temperature differential equation is satisfied. The temperature rate on the temperature boundary is zero, however, so that the power level would have to be constant, i. e.,

$$\dot{T} \equiv 0 = \frac{Q}{M_C} - a u_{2\min} T_{\max}.$$

If the power level remains constant, the terminal power level boundary condition cannot be satisfied.

One could postulate an optimal policy which allowed the temperature to remain interior to its boundary. It is shown in Appendix A that interior to state constraints the optimal control policy must be bang bang; that is, a singular condition is not feasible for a startup trajectory. Using this fact, combined with the fact that temperature increases faster than power, (for this non-typical case) one can conclude that the trajectory shown is optimal. The computation time required to obtain the data of figure 5 is similar to that shown for figure 4 in table 2.

Figure 6 illustrates the results of calculations with all constraints imposed. It is seen that the optimal policy is to insert reactivity at maximum rate until the reactivity upper bound is attained. At this time the

reactivity rate drops to zero so that the reactivity remains on its upper bound until the temperature rate intersects its upper bound. Control reactivity is then decreased so that temperature increases at the maximum allowable rate. The constant rate of temperature increase requires an almost constant (slightly increasing ramp) power level. Sometime prior to the end of the trajectory, reactivity is again inserted at maximum rate to bring the power level to its terminal boundary value. The propellant flow rate remains on its lower bound until the final portion of the trajectory, where the power level is rapidly increasing. The flow rate must increase at this time so that the temperature rate constraint is not exceeded. It is obvious that this process is also one of minimum time.

An interesting feature of the trajectory is the apparent oscillatory behavior of power level during the latter portion. In an effort to more closely evaluate this characteristic, the trajectory of figure 7 was calculated with approximately twice the definition of figure 6; that is, the value for Δt is approximately halved. It can be seen that the frequency of this oscillatory behavior approximately doubled and its amplitude decreased slightly. It is concluded that the oscillatory behavior is due to the fact that the algebraic equations used in the calculations are only approximations to the continuous differential equations. In fact, in the limit as Δt approaches

zero, the oscillations must disappear, since the constraint on reactivity rate will eventually prohibit fast variations in power level. The oscillatory behavior does illustrate, however, that discrete approximations to differential equations must be used with some caution.

The propellant consumption for the smoothed versions of the trajectories shown in figures 6 and 7 is very close to the absolute minimum consumption possible. For this reason, it is concluded that the control policy shown is indeed optimal. The computation times required to obtain the data of figure 6 is shown in table 2.

Conclusions

The optimal control and state trajectories, in the presence of control, control-rate, state and state-rate constraints, have been calculated for a nuclear rocket reactor. Although the optimality criteria was assumed to be minimum propellant consumption, the process is also one of minimal time.

The linear programming algorithm is very powerful for problems of this nature, although it has received little attention in the optimal control literature. In addition, the computation time for problems of this size is not excessive, and the programming effort is minimal.

Table 2

Nuclear Rocket Engine MPS/360 Computation Times (1)

FIGURE	NO. OF ITER. TO FEASIBLE	NO. OF ITER. TO OPTIMAL (3)	MAJOR ITERATION	COMPUTING TIME (MIN) (2)
4	13	---	1	3.12
	---	11	1	2.69
	1	---	2	2.81
	---	2	2	2.80
	Feasible at start	---	3	2.65
	---	1	3	3.68
	Feasible and optimal at start			2.60
	TOTAL			20.35
6	21	---	1	5.63
	---	40	1	3.03
	34	---	2	10.42
	---	29	2	7.25
	16	---	3	4.25
	---	14	3	2.92
	19	---	4	5.05
	---	17	4	4.38
	Feasible and optimal at start			2.53
	TOTAL			45.46

- (1) The starting trajectories were obtained by integrating the differential equations with maximum reactivity and minimum propellant flow rate for 1.5 seconds.
- (2) These computation times do not include running of the Fortran program between each major iteration. One run of this program requires approximately 1.50 minutes.
- (3) Optimal here refers to optimal for the linear program.

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APPENDIX A

POTENTIALLY SINGULAR SUBARCS FOR THE NUCLEAR ROCKET REACTOR

A.1 Introduction

Potentially singular subarcs, interior to the state constraint boundaries, are investigated for the nuclear rocket engine. It is shown that a zero-power-level subarc along which the average core temperature is decreasing is the only singular condition permissible, if the problem is normal. From physical considerations it does not seem plausible that a zero-power-level subarc forms part of the optimal trajectory for the nuclear rocket reactor. However, if one investigates the problem of reactor shutdown, then this singular subarc might form a portion of the optimal trajectory.

A.2 Nuclear Rocket Reactor Singularity

Theorem 25 of reference 5 states that interior to the state constraint boundaries, the solution to an optimization problem must satisfy the maximum principle.

The differential equations which approximate the behavior of the nuclear rocket engine are repeated here for convenience

$$\dot{Q} = \left(\frac{u_1 - \beta}{\ell}\right) Q + \lambda C, \quad (A.1)$$

$$\dot{C} = \frac{\beta}{\ell} Q - \lambda C, \quad (A.2)$$

$$\dot{T} = \frac{Q}{M_C} - a u_2 T. \quad (A.3)$$

The criteria for optimization is minimal propellant consumption; thus, trajectories along which

$$\underline{J} = \int_{t_0}^{t_1} u_2(t) dt$$

has minimum value are required.

The Hamiltonian for this system is

$$\begin{aligned} H(\underline{p}, \underline{x}, \underline{u},) &= p_0 u_2 + p_1 \left(\left(\frac{u_1 - \beta}{\ell}\right) Q + \lambda C\right) + p_2 \left(\frac{\beta}{\ell} Q - \lambda C\right) + p_3 \left(\frac{Q}{M_C} - a u_2 T\right) \\ &= (p_0 - p_3 a T) u_2 + p_1 \frac{Q}{\ell} u_1 + p_1 \left(-\frac{\beta}{\ell} Q + \lambda C\right) + p_2 \left(\frac{\beta}{\ell} Q - \lambda C\right) + p_3 \frac{Q}{M_C}; \end{aligned} \quad (A.4)$$

where: $p_0, p_1, p_2, p_3, Q, C, T, u_1$ and u_2 represent values along the optimal trajectory.

The corresponding costate equations are

$$\dot{p}_0 = 0, \quad (A.5)$$

$$\dot{p}_1 = -\left(\frac{u_1 - \beta}{\ell}\right) p_1 - \frac{\beta}{\ell} p_2 - \frac{1}{M_C} p_3, \quad (A.6)$$

$$\dot{p}_2 = -\lambda p_1 + \lambda p_2 , \quad (A.7)$$

$$\dot{p}_3 = a u_2 p_3 . \quad (A.8)$$

The Hamiltonian, equation A.4, indicates that this problem may be u_2 singular (i.e., maximum H is independent of u_2) if

$$p_0 - p_3 aT \equiv 0 ;$$

u_1 singular, if

$$p_1 \frac{Q}{\ell} \equiv 0 ;$$

or both. The possibility that these conditions may exist is investigated in the following material.

Case I: Assume that the problem is u_2 singular; that is,

$$p_0 - p_3 aT \equiv 0 . \quad (A.9)$$

Equation A.9 implies that

$$\frac{d}{dT}(p_0 - p_3 aT) = \dot{p}_0 - p_3 a\dot{T} - \dot{p}_3 aT \equiv 0 . \quad (A.10)$$

Substituting the values for $\dot{p}_0$ and $\dot{p}_3$ from equations A.5 and A.8 respectively, it follows that

$$-p_3 a\dot{T} - p_3 a^2 u_2 T \equiv 0 . \quad (A.11)$$

It follows from equation A.11 that either

$$\dot{T} \equiv a u_2 T , \quad (A.12)$$

or

$$p_3 \equiv 0 .$$

Equation A.12 together with equation A.3 imply that

$$Q \equiv 0 , \quad (A.13)$$

which in turn implies that

$$\dot{Q} \equiv 0 .$$

It follows from equation A.1 that

$$C \equiv 0 .$$

Thus, it seems that a zero-power-level singular u_2 solution may exist. Equation A.12 requires that the temperature decrease along this trajectory, since both a and u_2 are greater than zero. The condition

$$T \equiv 0$$

is trivial, since it results in a time-stationary point.

Now consider

$$p_3 \equiv 0 . \quad (A.14)$$

Equation A.14 and A.9 imply that

$$p_0 \equiv 0 .$$

Therefore, the problem is non-normal (as defined in the calculus of variations).

Case II: Assume that the problem is u_1 singular; that is,

$$p_1 \equiv 0 \quad (A.15)$$

Equation A.7 indicates that if equation A.15 is to be true, then

$$\dot{p}_2 \equiv \lambda p_2 ,$$

which in turn implies that

$$\dot{p}_1 \equiv 0 ,$$

so that equation A.6 requires that

$$p_3 \equiv -\left(\frac{M_c \beta}{\ell}\right) p_2 = -\left(\frac{M_c \beta}{\ell}\right) A \exp(\lambda t) . \quad (A.16)$$

the problem has free terminal time; therefore, the Hamiltonian must be zero. It follows from this that the constant A in equation A.16 cannot be zero, since if A is zero equation A.16 requires that

$$p_2 \equiv p_3 \equiv 0 ,$$

and equation A.4 requires that

$$p_0 \equiv 0 .$$

All of the components of the costate are therefore identically zero which violates the maximum principle.

The preceding discussion indicates that

$$p_3 \equiv -\left(\frac{M_c \beta}{\ell}\right) A \exp(\lambda t) , \quad (A.17)$$

so that

$$\dot{p}_3 \equiv -\left(\frac{M_c \beta}{\ell}\right) A \lambda \exp(\lambda t) . \quad (A.18)$$

If the identities of equations A.17 and A.18 are substituted into equation A.8, the result implies that

$$u_2 = \frac{\lambda}{a} . \quad (\text{A.19})$$

The problem must therefore also be u_2 singular, since in general equation A.19 is not a u_2 bound. Again, the Hamiltonian, equation A.4, requires

$$p_0 - p_3 aT = 0 \quad (\text{A.20})$$

for u_2 singularity. But

$$p_3 = -\left(\frac{M_c \beta}{\ell}\right) A \exp(\lambda t) ,$$

so that

$$p_0 - \left(\frac{M_c \beta}{\ell}\right) A \exp(\lambda t) aT = 0 . \quad (\text{A.21})$$

Now if $T \neq 0$, equation A.21 implies that

$$T = -\left(\frac{\ell p_0}{M_c \beta A a}\right) \exp(-\lambda t) ,$$

so that

$$\dot{T} = -\left(\frac{\ell p_0 \lambda}{M_c \beta A a}\right) \exp(-\lambda t) \quad (\text{A.22})$$

If equation A.22 is inserted into equation A.3, it is easily seen that

$$Q = 0 ,$$

which in turn implies that

$$\dot{Q} = 0 .$$

Equation A.1 thus requires

$$C \equiv 0.$$

Again a zero-power-level solution with decreasing temperature is obtained. In addition, if $T \neq 0$, equation A.20 requires

$$p_0 \equiv 0 ,$$

so that the problem must be non-normal in this case too.

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FIGURE 1 Schematic Illustration of Nuclear Rocket.

FIGURE 2 Schematic Illustration of the Linear - Programming Algorithm for Optimal Control Problems

NOTE: SUPERSCRIPT REFERS TO COMPONENT NUMBER

NOTE: SUPERSCRIPT REFERS TO COMPONENT NUMBER

(2)

(1)

X_1^1	X_1^2	X_1^3	U_1^1	U_1^2	X_2^1	X_2^2	X_2^3	U_2^1	U_2^2	X_3^1	X_3^2	X_3^3	U_3^1	U_3^2	X_4^1	X_4^2	X_4^3	U_4^1	U_4^2	X_5^1	X_5^2	X_5^3	U_5^1	U_5^2	Δt
X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	
X	X	X	X	X	X	X	X	X	X																X
X	X	X	X	X	X	X	X	X	X																X
X	X	X	X	X	X	X	X	X	X																X
					X	X	X	X	X	X	X	X	X	X											X
					X	X	X	X	X	X	X	X	X	X											X
					X	X	X	X	X	X	X	X	X	X											X
										X	X	X	X	X	X	X	X	X	X						X
										X	X	X	X	X	X	X	X	X	X						X
										X	X	X	X	X	X	X	X	X	X						X
															X	X	X	X	X	X	X	X	X	X	X
															X	X	X	X	X	X	X	X	X	X	X
															X	X	X	X	X	X	X	X	X	X	X

RHS

(1) OBJECTIVE FUNCTION ROW

(2) RIGHT HAND SIDE

FIGURE 3 Illustration of Block^d Character of Work Matrix for Maximal Entries with Second-Order Approximation_P

FIGURE 4 Nuclear Rocket Reactor Trajectories with 20 Megawatts Power Level Constraint

COMPUTATION TIME 76.63 MIN.

8 MAJOR ITERATIONS

COST = 80.61 #

NOTE:

UI = REACTIVITY IS AT
UPPER BOUND OF 0.9β

FIGURE 5 Nuclear Rocket Reactor Trajectories with Fast Temperature Response

FIGURE 6 Nuclear Rocket Reactor Trajectories with Temperature-Rate Constraint

FIGURE 7 Nuclear Rocket Reactor Trajectories with Increased Definition