NONLINEAR BEHAVIOR OF THE ONE-
DIMENSIONAL BEAM PLASMA SYSTEM*

by

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ABSTRACT

The effect of a beam of electrons on a Maxwellian plasma is studied by integrating numerically the Vlasov equation in one dimension. The solution is carried past the point of maximum electrostatic energy. After reaching its maximum value, the electrostatic energy fluctuates about its maximum. Some modes which are linearly stable grow due to mode-coupling with the unstable modes. The relative amount of energy in the fastest growing mode is shown to depend on the periodicity length of the system.

I. INTRODUCTION

The study of the effect of a small beam of electrons passing through a stationary Maxwellian plasma in one dimension has been the subject of numerous investigations. The resulting unstable electrostatic waves soon grow out of the linear regime and must be treated by non-linear techniques. Two numerical methods have been used to investigate this phenomenon, particle tracing codes^{1,2} and integration of the Vlasov equation.^{3,4,5}

The Vlasov equation is difficult to integrate in the (x, v) variables so generally the equation is Fourier transformed in space and either a Fourier transform or Hermite transform is made of the velocity variable. The Fourier transform in velocity space does not easily allow for more than one or two spatial modes to be considered, while the Hermite expansion leads to an infinite system of coupled equations. Because of computer time considerations, the Hermite transform has been so far restricted to symmetric functions in velocity. This means that for the beam-plasma system, two beams must be considered instead of one, and only standing waves are allowed.

We have used a hybrid expansion which is similar to the Hermite expansion, but allows an approximate closing of the system of coupled equations. We consider an asymmetric distribution in velocity which we hope permits closer comparison with the particle calculations and with the quasi-linear theory of the bump-on-the-tail distribution function.

II. METHOD OF CALCULATION

We consider the Vlasov equation in dimensionless units

$$\frac{\partial f(x, v, t)}{\partial t} + v \frac{\partial f(x, v, t)}{\partial x} - E \frac{\partial f(x, v, t)}{\partial v} = 0, \quad (1)$$

$$\frac{\partial E}{\partial x} = 1 - \int f(x, v, t) dv, \quad (2)$$

where the 1 in Eq. 2 represents the effect of a neutralizing background of positive charge. The method of transformation is discussed elsewhere⁶ so we will simply outline the procedure. We Fourier transform the functions in both the spatial and velocity variables. The resulting distribution function, $F_n(y, t)$ is a function of the spatial mode, n , the velocity transform variable, y , and the time. The macroscopic properties of the system are given by the derivatives of $F_n(y, t)$ evaluated at $y = 0$. For this reason we expand $F_n(y, t)$ in a power series in y obtaining

$$F_n(y, t) = \sum_{v=0}^{\infty} g_v \alpha_{n, v}(t) y^v e^{-y^2/2}, \quad (3)$$

where g_v and $e^{-y^2/2}$ have been introduced for convenience. Eq. 1 becomes a system of coupled equations

$$\dot{\alpha}_{n,v}(t) + k_0 n \gamma_v [(\nu + 1) \alpha_{n,\nu+1}(t) - \nu \alpha_{n,\nu-1}(t)] \\ + i \nu \gamma_r \sum_{m=-\infty}^{\infty} E_m(t) \alpha_{n-m,\nu-1}(t) = 0, \quad (4)$$

$$i E_m = - \frac{\alpha_{m,0}(t)}{m k_0}, \quad (5)$$

$$\text{where } \gamma_\nu = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{2} \Gamma(\frac{\nu+2}{2})}. \quad (6)$$

This system of equations is similar in form to the equations obtained from a Hermite expansion in velocity and, in fact, it can be shown that to within a complex constant the two expansion schemes are identical. The $\alpha_{n,\nu}$'s form a representation of $F_n(y,t)$ which is convenient for closing the system of Eq. 4. The essence of the closure method is that, given the coefficients $\alpha_{n,0}, \dots, \alpha_{n,\nu_{\max}}$ at any time, it is assumed that as a function of ν , these coefficients lie on a smooth curve. A 4th order polynomial fit to the last five of the coefficients is used to estimate the coefficient $\alpha_{n,\nu_{\max}+1}$, thus closing the system. The total number of coefficients used in this calculation has been on the order of 100.

The system of Eq. 4 has been numerically integrated on the IBM 360-65 computer at the University of Iowa using a modified Adams-Bashforth predictor-corrector integration algorithm. The results of this integration scheme are discussed in the next section.

III. NUMERICAL RESULTS

The initial space-averaged distribution function is shown in Figure 1. The main body of the distribution function contains 91 percent of the particles. The tail is peaked at $v = 4$ in units of the thermal velocity, $v_{th} = \sqrt{k T/m}$. The functional form of $f_o(v, 0)$ is

$$f_o(v, 0) = Ae^{-v^2/2} + Bv^{+16} e^{-v^2/2} \quad v > 0, \quad (7)$$

$$f_o(v, 0) = Ae^{-v^2/2} \quad v < 0. \quad (8)$$

In the vicinity of the peak, the function can be shown to be approximately a Maxwellian distribution centered at $v = 4$ with a temperature equal to the temperature of the main body of particles.

For the system, the fundamental length was chosen to be 84 Debye lengths ($k_o = 0.075$). Eight modes were included of which four were linearly unstable. The fastest growing mode was the fourth with $k = 0.3(L = 21\lambda_D)$ and a phase velocity $\omega/k = 3.16$. Of the remaining modes, modes 3, 5, 6 were linearly unstable, mode 7 was marginally stable, mode 8 was stable and modes 1 and 2 had phase velocities so large that they were not of interest.

The initial perturbation was a perturbation of the density of the beam.

$$f_n(v,0) = \begin{cases} \epsilon_n B v^{16} e^{-v^2/2} & v > 0 \\ 0 & v < 0 \end{cases}, \quad (9)$$

where ϵ_n was selected so that all of the modes had the same electric field at $t = 0$.

Figure 2 shows the total electrostatic energy density. The units are chosen so that the ordinate represents the ratio of the electrostatic energy to the initial kinetic energy of the particles. As can be seen, the energy grows exponentially and then levels off at a value on the order of 1.2 percent of the particle kinetic energy and fluctuates about this value. The time behavior of the electric fields of four of the eight modes of the system is displayed in Figure 3. These are the three most unstable modes 3,4,5 and mode 8 which was initially stable. After the initial conditions die out, modes 3,4,5 behave as one would expect according to the quasi-linear theory, eventually leveling off when the energy of the fields becomes large enough to distort the distribution of particles. Mode 8, however, which is initially damped, couples through the mode-mode terms and begins to grow. The growth rate is about twice the growth rate of mode 4. This type of mode coupling was noted earlier by Knorr.⁷ We have run the system leaving out the mode coupling terms

in the equations and this behavior disappears. The differences obtained from leaving out the mode coupling are seen from comparing Figures 4a and 4b for modes 4, 6, and 7.

When the mode coupling terms are omitted, the rapid fluctuations of the electrostatic energy seen in Figure 2 disappear. These fluctuations also disappear when mode coupling is included, but modes 6, 7, and 8 are omitted. The rapid fluctuations are apparently due to mode coupling of modes 3, 4, and 5 with modes 6, 7, and 8.

Figure 5 shows the space averaged distribution function at $t = 36 \omega_p^{-1}$ as contrasted with the value at $t = 0$. By this time the bump has almost disappeared. There is a larger density of particles with velocities greater than $v = 4$ at $t = 36$ than at $t = 0$. This may be the beginning of a "heating" of the beam particles as discussed by Dawson and Shanney¹. They found the time scale for this effect $\sim 100 \omega_p^{-1}$ so that it is not fully developed by the time we are forced to stop the calculation.

IV. DISCUSSION

The features of this calculation are, in many respects, similar to the other calculations using particle models and symmetric beam-plasma distributions. There are some results which differ from the other models. For example, Morse and Nielson² find that after the initial growth of the electrostatic fields, there is a fall off of the electrostatic energy to about one half of its maximum value with variations about this value. Although we are limited in the maximum time of the calculation to values far below those of the particle model, we see in Figure 2 no tendency of the energy to "relax" to an energy below the maximum value. We have, unfortunately, no explanation for this difference.

Armstrong and Montgomery⁴ found that the fastest growing mode contained about 80 percent of the total electrostatic energy of the system. We find (see Figure 6) that mode 4 contains typically 55 percent of the total energy. This is apparently a function of the "density" of allowed modes in the vicinity of the fastest growing mode. To check this assumption we ran a similar calculation with a periodicity length of $64 \lambda_D (k_0 = 0.1)$. The fastest growing mode is again $k = 0.3$, but now the modes on either side are more widely

separated in phase velocity. The results of this run are also shown in Figure 6. Now the energy in the fastest growing mode is about 80 percent of the total energy. This seems to be a property of the enforced periodicity of the system, and should always be taken into consideration in numerical calculations.

FIGURE CAPTIONS

- Figure 1 Initial space averaged distribution function, $f_0(v,0)$.
The velocity units are in terms of the thermal velocity,
$$v_{th} = \frac{kT}{m}^{1/2}.$$
- Figure 2 Total electrostatic energy as a function of time. The dimensionless energy is in terms of the initial particle kinetic energy.
- Figure 3 Magnitude of the electrostatic field as a function of time for modes 3(— - —), 4(———), 5(----), and 8(— -- — --).
- Figure 4a Magnitude of the electrostatic field as a function of time for modes 4(—), 6(— - —), and 7(---).
- Figure 4b Magnitude of the electrostatic field as a function of time for modes 4(—), 6(— - —), and 7(---), when the mode coupling is neglected.
- Figure 5 Comparison of the initial space averaged distribution function and the distribution function at $t = 36(\omega_p^{-1})$.
- Figure 6 Percentage of the electrostatic energy in the fastest growing mode for systems of different length. — is the system 84 Debye lengths long. ---- is the system 64 Debye lengths long.

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Figure 1

Figure 2

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Figure 3

Figure 4a

Figure 4b

Figure 5

Figure 6