PARAMETERS OF THE RF SYSTEM
FOR THE "WEAK-FOCUSING" LATTICES

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(BNL January 20, 1984)
RF REQUIREMENTS
(Parabolic Distribution)

Bunch half length \[ = \sqrt{5\sigma_L} \]

Bunch phase half width \[ \phi = \sqrt{5} \sigma_L h/R \]

Bunch half height \[ \Delta E = \sqrt{5} \delta_E \]

Bunch area/amu \[ S = 5\pi \sigma_L \delta_E \gamma \frac{E_o}{c} = \frac{\gamma E_o}{2h f_o} \Delta E \phi \]

In the small-amplitude approximation and stationary

\[ \phi = \left( \frac{8\pi |\eta| h^3 f_o^2 A}{\gamma E_o e V} \right)^{1/4} \sqrt{S} \]

\[ \Delta E = \frac{2h f_o}{\gamma E_o} \frac{S}{\phi} \]

Bucket half height required, stationary

\[ \Delta_B = \frac{\Delta_E}{\sin \phi/2} \]

Bucket Area/amu required

\[ A_B = \frac{4 \gamma E_o}{\pi h f_o} \Delta_B \]

Voltage required

\[ V = \frac{\pi h |\eta| \gamma E_o A}{2e \Delta B} \]

\[ V \approx 8\pi \frac{|\eta| h^3 f_o^2 A S^2}{e \gamma E_o} \frac{\Delta E}{\phi} \quad (\phi \ll \pi) \]
Parameter Variations with $h_{rf}$
(Equipartition)
$\gamma = 100, \text{ Au}$

<table>
<thead>
<tr>
<th>Lattice</th>
<th>$N_B$</th>
<th>$h_{rf}$</th>
<th>$\delta E$</th>
<th>$S$</th>
<th>$L$</th>
<th>Diamond</th>
<th>$V$</th>
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<tbody>
<tr>
<td></td>
<td>$\times 10^9$</td>
<td>$\times 57$</td>
<td>$\times 10^{-4}$</td>
<td>$\times 10^{26}$</td>
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<tr>
<td>$\alpha = 0 \text{ mrad}$</td>
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<tr>
<td>$15/120^\circ$</td>
<td>1</td>
<td>12</td>
<td>11.3</td>
<td>3.8</td>
<td>10.5*</td>
<td>29</td>
<td>2.2</td>
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<td>6</td>
<td>9.8</td>
<td>6.6</td>
<td>14.*</td>
<td>57</td>
<td>0.85</td>
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<td>4.0</td>
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<td>0.81</td>
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<tr>
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<td>1</td>
<td>1</td>
<td>4.2</td>
<td>16.9</td>
<td>15.1*</td>
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<td>4.0</td>
<td>8.4</td>
<td>57</td>
<td>0.95</td>
</tr>
</tbody>
</table>

$\alpha = 2 \text{ mrad}$

| $15/120^\circ$ | 2     | 12       | 12.9      | 4.4 | 11.1  | 12      | 3.0 |
|                | 2     | 6        | 11.3      | 7.6 | 6.7   | 14      | 1.1 |
|                | 1     | 6        | 9.8       | 6.6 | 1.9   | 13      | 0.85 |
| $12/90^\circ$  | 2     | 12       | 7.9       | 2.7 | 7.6   | 14      | 2.8 |
|                | 2     | 6        | 6.9       | 4.6 | 4.8   | 18      | 1.1 |
|                | 2     | 1        | 4.8       | 19.4 | 1.2 | 17 | 0.087 |
|                | 1     | 6        | 6.0       | 4.0 | 1.4   | 16      | 0.81 |
| $9/120^\circ$  | 2     | 6        | 6.7       | 4.6 | 5.1   | 17      | 1.2 |

* $\Delta v > 0.003$

BB
MOMENTUM SPREAD AT TRANSITION

The momentum spread at transition scales like

$$\delta_E \propto \left( \frac{h^2}{V_{tr}^2} \frac{V^2}{B} \cos^2 \phi_s \right)^{1/6}$$

with $V \sin \phi_s = 2\pi R \rho B$

Assuming the same rf system, the lattices with $\gamma_{tr} = 25$ require at transition about 15% more momentum aperture than one with $\gamma_{tr} = 38$.

An acceptable rf system for the $\gamma_{tr} = 25$ lattices is obtained by using (primed quantities):

$$h' = \frac{1}{2} h; \quad V' = \frac{1}{5} V; \quad B' = \frac{1}{4} B; \quad \phi'_s = \frac{5}{4} \phi_s$$

leading to

$$\delta'_E = 0.7 \delta_E$$

The resulting physical aperture requirement due to momentum spread is

$$(X_p^{\text{max}} = 1.57 \text{ m}, \quad X_p^{\text{max}} = 0.7 \text{ m})$$

$$\sigma'_H = 1.57 \sigma_H$$
SUGGESTED rf PARAMETERS

\[ f_{\text{rf}} = 6 \times 57 \times f_o = 26.7 \text{ MHz} \]

\[ V_{\text{max}} = 1 \text{ MV} \]

\[ V_{\text{acceleration}} = 200 \text{ kV} \]

\[ \text{Acceleration time} = 2 \text{ min.} \]

Questions:

- What is dynamics of intrabeam scattering at operating point.

- Parzen will calculate \( L = L(t) \)
  \[ \sigma = \sigma_L(t) \]

- Slowest beam growth is expected, if full voltage is reached at the end of the acceleration cycle, since
  \[ \tau^{-1}_E \propto \frac{N_B}{\varepsilon S \delta \varepsilon}; \quad \tau^{-1}_H \propto \frac{N_B}{\varepsilon^2 S} \]
CHOICE OF TRANSITION ENERGY

Due to intrabeam scattering the momentum spread of the bunch increases until 
\( \Delta_E = \Delta_B \). If this limit is exceeded, the particles are lost.

At constant voltage

\[
\Delta_B^2 = \frac{1}{h \gamma |n| \Delta_E} = \frac{\gamma_{tr}}{h |\gamma/\gamma_{tr} - \gamma_{tr}/\gamma|}
\]

The bucket height requirements vary with energy according to

\[
\Delta_E = \frac{1}{\sqrt{\gamma}} \quad \text{(equipartition)}
\]

\[
\Delta_E(\gamma=12) > 1.4 \Delta_E(\gamma=100) \quad \text{(Parzen)}
\]

Equivalent performance over energy range, (i.e. \( L = \gamma \)) requires

\[
\gamma_{tr}^2 = \frac{(\Delta_1/\Delta_2)^2 \gamma_1 + \gamma_2}{\gamma_1 + (\Delta_1/\Delta_2)^2 \gamma_2}
\]

For \( \gamma_1=12 \) and \( \gamma_2=108 \) follows the optimized transition energy \( (\Delta_1/\Delta_2 = 1.4) \):

\[
\gamma_{tr} = 36 \times 0.6 = 22
\]