75
3/20/70

WAPD-TM-652

AEC RESEARCH AND
DEVELOPMENT REPORT

MASTER

**AN ANALYSIS OF TRANSIENT CLAD
STRAINS IN CYLINDRICAL FUEL ELEMENTS
INCLUDING THE EFFECTS OF OXIDE PELLET
CRACKING (STRIPE)**

(LWBR Development Program)

FEBRUARY 1970

CONTRACT AT-11-1-GEN-14

BETTIS ATOMIC POWER LABORATORY, PITTSBURGH, PA.,
OPERATED FOR THE U. S. ATOMIC ENERGY COMMISSION
BY WESTINGHOUSE ELECTRIC CORPORATION

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

UC-80: Reactor Technology
Special Distribution

AN ANALYSIS OF TRANSIENT CLAD STRAINS IN CYLINDRICAL FUEL ELEMENTS
INCLUDING THE EFFECTS OF OXIDE PELLET CRACKING (STRIPE)
(LWBR Development Program)

W. J. O'Donnell*
W. G. Clarke
W. R. Campbell

February 1970

Contract AT-11-1-GEN-14

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States nor the Commission, nor any person acting on behalf of the Commission, makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe upon any privately owned rights or constitute an advertisement or product endorsement.

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or any employee or contractor of such contractor, to the extent that such employee or contractor of the Commission, or any employee or contractor of such contractor, provides access to, or information present to the employment or contract with the Commission, or his employment with such contractor.

Printed in the United States of America
Available from

Clearinghouse for Federal Scientific and Technical Information
National Bureau of Standards, U. S. Department of Commerce
Springfield, Virginia 22151

Price: Printed Copy \$3.00; Microfiche \$0.65

NOTE

This document is an interim memorandum prepared primarily for internal reference and does not represent a final expression of the opinion of Westinghouse. When this memorandum is distributed externally, it is with the express understanding that Westinghouse makes no representation as to completeness, accuracy, or usability of information contained therein.

**BETTIS ATOMIC POWER LABORATORY, PITTSBURGH, PA.,
OPERATED FOR THE U. S. ATOMIC ENERGY COMMISSION
BY WESTINGHOUSE ELECTRIC CORPORATION**

*Dr. O'Donnell is presently Vice-President, KPA Computer Techniques, Inc.

SPECIAL EXTERNAL DISTRIBUTION (UC-80)

	<u>No. Copies</u>
AEC Library, Washington for J. M. Simmons	1
Ames Laboratory (AEC)	1
Argonne National Laboratory (AEC)	10
Atomics International (AEC)	4
Babcock and Wilcox Company (AEC)	2
Battelle Memorial Institute (AEC)	2
Battelle-Northwest (AEC)	9
Brookhaven National Laboratory (AEC)	4
Combustion Engineering, Inc. (AEC)	1
Combustion Engineering, Inc. (NRD) (AEC)	1
Du Pont Company, Aiken (AEC)	4
General Electric Company, Cincinnati (AEC)	2
General Electric Company, San Jose (AEC)	4
Gulf General Atomic Incorporated (AEC)	2
Idaho Nuclear Corporation (AEC)	6
IIT Research Institute (AEC)	1
Knolls Atomic Power Laboratory (AEC)	2
Lawrence Radiation Laboratory, Livermore (AEC)	2
Los Alamos Scientific Laboratory (AEC)	2
Mound Laboratory (AEC)	1
Nuclear Materials and Equipment Corporation (AEC)	1
Oak Ridge National Laboratory (AEC)	8
Westinghouse Electric Corporation (AEC)	2
DTIE	3
PNR, Manager	3
Total	<u>78</u>

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

CONTENTS

	<u>Page</u>
I. INTRODUCTION	1
II. MECHANICS MODEL	3
A. General Solution for Effect of Pellet Cracking on Clad Strains	5
1. Region (1): Uncracked Pellet (or Pellet with Compression across Existing Cracks)	6
2. Region (2): Cracked Portion of Pellet	7
3. Region (3): Cladding	8
4. General Method of Solution	9
B. Plastic Flow Treatment in Inner Core of Fuel Pellet	10
C. Calculational Model for Including Clad Yielding	12
III. CALCULATIONAL PROCEDURES IN STRIPE (MO650)	12
A. Preliminary Calculations	12
1. Plane Stress Option	13
2. User-Supplied Temperature Profile Option	13
3. Internally Computed Temperature Profile Option	14
4. Temperature-Dependent Material Properties	14
B. The Numerical Method of Solution	15
1. Governing Equations	15
2. Solution by Method of Bisection	16
3. Tensile Yielding in the Clad	17
4. Compressive Flow in Pellet	18
C. Auxiliary Calculations Performed by STRIPE	19
1. Radial Displacements	19
2. Stresses	20
D. Finite Difference Approximations	21
IV. INPUT REQUIREMENTS FOR STRIPE	22
A. Control Information	23
B. Initial Conditions	24
C. System Parameters	24
D. Final Conditions - User Supplied	26
E. Material Properties, Initial and Final Conditions by Formula	26
F. Termination Card	28

CONTENTS (Cont)

	<u>Page</u>
V. DISCUSSION OF STRIPE INPUT PREPARATION	28
A. Options	28
B. Temperature-Independent Parameters	30
C. Temperature Profiles	32
D. Temperature-Dependent Fuel Properties	34
E. Temperature-Dependent Clad Properties	38
VI. STRIPE OUTPUT	41
VII. COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS	42
A. Irradiation Experiments at Chalk River	42
B. Material Properties Used in Calculations	43
C. Comparison of Calculated and Measured Results	46
VIII. CONCLUSIONS	48
APPENDIX A - INPUT AND OUTPUT FOR A SAMPLE PROBLEM	49
APPENDIX B - COMPUTER PROGRAM ABSTRACT	55
REFERENCES	57

FIGURES

<u>Figure</u>		<u>Page</u>
1	Effective Elastic Properties of Porous Pellet Material	4
2	Geometry of Cracked Pellet	4
3	Cross Section of Fuel Element	5
4	Detailed Geometry of the Cracked Region of a Fuel Pellet	7
5	Geometry of the Clad Region of a Fuel Rod	8
6	Comparison of Yield Stress Determinations for 97.5-Percent TD ThO ₂	11
7	Plastic Flow-Stress as a Function of Fuel Temperature and Creep Rate for UO ₂ Fuel	37
8	Thermal Expansion of UO ₂ Cylindrical Fuel Elements - Comparison of Calculated and Measured Results	39
A-1	Input Data for the First Fuel Element in Chalk River Tests	50
A-2	Input Edit for Cross-Checking Input Data	51
A-3	Output Edit for Calculated Circumferential Stresses (SIGMA THETA) and Radial Stresses	53
A-4	Output Edit for Radial Displacement	54

TABLES

<u>Table</u>		<u>Page</u>
1	Test Element Parameters	43
2	Summary of Fuel and Clad Properties Used in STRIPE Calculations of Thermal Expansion of Chalk River Fuel Elements	44
3	Comparison of Calculated and Measured Fuel Expansion	47

This report contains an analysis of the deformations imposed on the cladding of cylindrical fuel elements by differential thermal expansions during an abrupt reactor power increase. The effects of pellet cracking are included in the analysis and are found to be very significant. The analytical model also includes the effects of temperature-dependent material properties, possible pellet-to-cladding gaps, compressive plastic flow in the hot inner core of the pellet, and tensile yielding of the cladding. The solution was programmed in FORTRAN IV for the CDC-6600 digital computer as STRIPE (M0650). Calculated results are compared with experimental results obtained on UO₂ fuel rods irradiated at Chalk River. Good agreement was found over a very wide temperature range.

AN ANALYSIS OF TRANSIENT CLAD STRAINS IN CYLINDRICAL FUEL ELEMENTS
INCLUDING THE EFFECTS OF OXIDE PELLET CRACKING (STRIPE)
(LWBR Development Program)

W. J. O'Donnell
W. G. Clarke
W. R. Campbell

I. INTRODUCTION

Clad stresses caused by differential thermal expansion during abrupt reactor-power increases appear to play a major role in the cracking of fuel elements during irradiation tests. This report describes a calculational model for predicting the relevant circumferential clad stresses and strains, including the effects of radial and circumferential pellet cracking. The resulting equations are solved by means of a FORTRAN IV digital computer program designated STRIPE† (M0650). The solution is non-linear (i.e., doubling the temperature change more than doubles the clad strain) because of the effects of cracking and the temperature dependence of material properties. Clad strains are needed to evaluate susceptibility to cracking resulting from plastic instability (Reference 1) or fatigue. Up-power transients tend to strain the cladding of a cylindrical fuel element into the plastic range due to differential thermal expansion resulting

†STRIPE is a digital program for determining STress In PEllets.

from the tendency of the fuel pellet to expand much more than the cladding. The fuel pellet expansion must be accurately known since it affects the structural integrity of the cladding in three major ways:

1. The expansion may fracture the cladding due to the concentration of strain which occurs once the cladding becomes plastically unstable (Reference 1).
2. The expansion imposes a severe fatigue loading on the cladding.
3. The expansion tends to increase the gap that occurs between the pellet and cladding when the power is reduced because the clad diameter has been permanently increased by plastic flow. This increase in gap is significant because it aggravates the axial ridging caused by the external system pressure. The strains associated with axial ridging may approach the ductility limits of the material and otherwise tend to impose severe fatigue loads.

Since the temperatures near the center of the pellet are much higher than those at the periphery, the center has a tendency to expand much more than the periphery, thus inducing tensile stresses near the periphery of the pellet and compressive stresses near the center. If it is assumed that the pellet does not crack radially, the colder portion of the pellet near its periphery then tends to impose a hoop restraint on the diametral expansion which might otherwise occur due to the much higher temperatures near the center of the pellet. In spite of system pressure and the restraint of the cladding, both of which tend to produce compression in the pellet, high circumferential tensile stresses exist at the periphery of the pellet due to the very high temperature gradient and the non-linear thermal expansion coefficient.

In the event of a reduction in power, the temperature near the center of the pellet drops much more than the temperature near the periphery. Since the center material tends to contract more than the peripheral material, tensile stresses are induced near the center of the pellet.[†] Radial and circumferential cracks should appear at about the same time in a solid pellet. Annular pellets are less likely to crack circumferentially. The existence of these cracks is significant in the present analysis only if they do not heal before the next heatup. In that case, the effect of the radial cracks can be included in the

[†]The assumption is that the stresses in the pellet are fairly uniform when the power drop begins. This would be the case if the pellet had been at power with neutron softening for an appreciable period of time. Cooldowns which occur only a short time after heatups are more complicated since residual stresses play a more complex role.

analysis given in Section II simply by setting the crack propagation strength of the pellet material equal to zero†. Circumferential cracks are important because they inhibit heat transfer, even when they are subjected to compressive loads. This effect can be included in the analysis simply by using the appropriately higher radial temperature profile in the vicinity of the crack.

II. MECHANICS MODEL

During a rapid reactor-power increase, the temperature near the center of the fuel pellet increases much more than the temperature near the periphery. Since the center attempts to expand more than the periphery, compressive hoop stresses are induced near the center of the pellet and tensile hoop stresses are induced near the periphery. The latter are much higher than the compressive hoop stresses due to the clad restraint. Since the pellet cannot sustain much tensile stress, that is, approximately 15,000 psi (Reference 3), radial cracks will be initiated at the periphery of the pellet. These cracks will propagate inward until the hoop stress drops below the crack propagation stress of the material at the local temperature.

The effect of porosity on the elastic properties of the pellet can be approximated using the perforated plate results of Reference 16. The effective elastic modulus E_f^* and Poisson's ratio ν_f^* of the porous oxide is shown in Figure 1.

Pellet cracking influences clad strains in two ways:

1. The unrestrained thermal expansion of the outside periphery of the pellet (u_b) increases, tending to increase the cladding stresses. For an uncracked pellet, this expansion is given by the plane strain axisymmetric solution:‡

$$u_b = \frac{2(1 + \nu_f^*)b}{b^2 - a^2} \int_a^b \alpha(r) \Delta T(r) r dr \quad (1)$$

where

a = inside radius of pellet ($a = 0$ for solid pellet)

b = outside radius of pellet

†This ignores the Hertzian compression at the cracks which can be included by using a suitable reduction in E_f^* in the cracked area. However, the magnitude of this reduction depends on the number of cracks and this cannot be accurately predicted. Clad stresses calculated by neglecting this effect will be slightly high.

‡Neglecting the variation of the effective elastic modulus of the pellet with temperature and porosity.

ν_f^* = effective Poisson's ratio of porous pellet material (Figure 1)
 $\Delta T(r)$ = temperature increase at radius r
 $\alpha(r)$ = mean coefficient of thermal expansion for the temperature range $\Delta T(r)$.

Figure 1. Effective Elastic Properties of Porous Pellet Material

A pellet which has radial cracks (Figure 2) from the periphery into a radius η expands by

$$u_b = \frac{2(1 + \nu_f^*)\eta}{\eta^2 - a^2} \int_a^\eta \alpha(r)\Delta T(r)rdr + (1 + \nu_f^*) \int_\eta^b \alpha(r)\Delta T(r)dr \quad (2)$$

Figure 2. Geometry of Cracked Pellet

As an example, the unrestrained expansion of a solid pellet ($a = 0$) due to a parabolic temperature increase is 1/3 greater if the cracks extend into the center of the pellet ($\eta = 0$).

2. It is easier to restrain the growth of the cracked pellet. This tends to reduce the differential thermal expansion stresses in the cladding. The deflection of an uncracked pellet due to radial compression p_2 is given by

$$u_b = - \left[\frac{a^2 + b^2}{b^2 - a^2} - \frac{\nu_f^*}{(1 - \nu_f^*)} \right] (1 - \nu_f^{*2}) \frac{p_2 b}{E_f^*} \quad (3)$$

where

E_f^* = effective elastic modulus of porous pellet material
(Figure 1).

p_2 = compressive stress at $r = b$, psi.

The deflection of a pellet having radial cracks from the periphery into a radius η is given by

$$u_b = - \left[\ln \left(\frac{b}{\eta} \right) + \frac{a^2 + \eta^2}{\eta^2 - a^2} - \frac{\nu_f^*}{(1 - \nu_f^*)} \right] (1 - \nu_f^{*2}) \frac{p_2 b}{E_f^*} \quad (4)$$

A. General Solution for Effect of Pellet Cracking on Clad Strains

For a calculational model, the fuel element is visualized as being divided into three basic regions as shown in Figure 3:

Figure 3. Cross Section of Fuel Element

where

a = inside radius of pellet, inch

b = outside radius of pellet, inch

c = outside radius of cladding, inch

Note that the material properties are allowed to vary with temperature within each region.

1. Region (1): Uncracked Pellet (or Pellet with Compression across Existing Cracks)

In the uncracked region, let

p_1 = radial stress at $r = \eta$, psi

u_η = radial displacement of outside radius, η , inch

ν_f^* = effective Poisson's ratio of porous pellet material (Figure 1), dimensionless

E_f^* = effective elastic modulus of porous pellet material (Figure 1), psi

E_f = elastic modulus of 100-percent dense pellet material

$\Delta T(r)$ = temperature change at position r , $^{\circ}\text{F}$

$\alpha_f(r)$ = mean coefficient of thermal expansion of pellet material for temperature range $\Delta T(r)$, $(^{\circ}\text{F})^{-1}$

σ_c = crack propagation strength of the pellet material†, psi.

Figure 1 is the actual relative mass density of the material and not the "apparent" density based on the volume of fabricated voids remaining. The program equations describing the above properties are based on Figure 1.

The thick-walled plane strain solutions for this region are:

$$u_\eta = - \left[\frac{a^2 + \eta^2}{\eta^2 - a^2} - \frac{\nu_f^*}{(1 - \nu_f^*)} \right] \frac{p_1 \eta (1 - \nu_f^{*2})}{E_f^*} + \frac{2(1 + \nu_f^*)\eta}{\eta^2 - a^2} \int_a^\eta \alpha_f(r) \Delta T(r) r dr \quad (5)$$

Since the circumferential stress at η is equal to the crack propagation strength of the material (σ_c), Hook's Law at $r = \eta$ gives

$$u_\eta = \frac{\eta}{E_f^*} \left[(1 - \nu_f^{*2}) \sigma_c + \nu_f^* (1 + \nu_f^*) p_1 \right] + \eta (1 + \nu_f^*) \alpha(\eta) \Delta T(\eta) \quad (6)$$

If the pellet were cracked before the power increase of interest, the correct solution may be obtained by letting $\sigma_c = 0$ in Equations (6) and (7). Furthermore,

$$\sigma_c = \sigma_{ULT}(\eta) = - \frac{\eta^2 + a^2}{\eta^2 - a^2} p_1 + \frac{E_f^*}{(1 - \nu_f^*)} \left[\frac{2}{\eta^2 - a^2} \int_a^\eta \alpha_f(r) \Delta T(r) r dr - \alpha_f(\eta) \Delta T(\eta) \right] \quad (7)$$

†The fracture strength is given in Reference 3. The crack propagation strength would be lower.

Only two of the three equations, (5), (6), and (7), are independent. The hoop stress in the pellet may approach the crushing strength of the pellet material if the temperature is raised very suddenly. The maximum value of this hoop stress exists at the center and, for the case of no cracking, is given by

$$\sigma_{\theta}|_{r=a} = \frac{E_f^*}{(1 - \nu_f^*)} \left[\frac{2}{b^2 - a^2} \int_a^b \alpha_f(r) \Delta T(r) r dr - \alpha_f(a) \Delta T(a) \right] . \quad (8)$$

2. Region (2): Cracked Portion of Pellet

Figure 4. Detailed Geometry of the Cracked Region of a Fuel Pellet

For the cracked region (Figure 4), $\sigma_{\theta} = 0$. Let

p_2 = radial stress at $r = b$, psi

σ_r = radial stress at position r , psi

ϵ_r = radial strain at position r , inch/inch

u_b = radial displacement of radius b , inch.

Equilibrium of a radial element requires that

$$r d\theta p_1 = r d\theta \sigma_r$$

or

$$\sigma_r = \frac{\eta}{r} p_1 . \quad (9)$$

For plane strain, Hook's Law gives:

$$\epsilon_r = \frac{(1 - \nu_f^{*2}) \sigma_r}{E_f^*} + (1 + \nu_f^*) \alpha_f(r) \Delta T(r) . \quad (10)$$

Now

$$u_b = u_n + \int_n^b \frac{du}{dr} dr = u_n + \int_n^b \epsilon_r dr \quad . \quad (11)$$

Substituting from Equations (9) and (10) into Equation (11):

$$u_b = u_n + (1 - \nu_f^{*2}) \eta p_1 \int_n^b \frac{dr}{E_f^{*}(r)r} + (1 + \nu_f^{*}) \int_n^b \alpha_f(r) \Delta T(r) dr \quad . \quad (12)$$

3. Region (3): Cladding

Figure 5. Geometry of the Clad Region of a Fuel Rod

From Figure 5,

p_E = external system pressure, psi

ν_c = Poisson's ratio of clad material, dimensionless

E_c = elastic modulus of clad material, psi

$\sigma_c(r)$ = mean coefficient of thermal expansion of clad material for temperature range $\Delta T(r)$, $(^\circ F)^{-1}$

$\sigma_\theta|_r$ = hoop stress in clad at position r , psi.

From Equation (9)

$$p_2 = \frac{\eta}{b} p_1 \quad . \quad (13)$$

The plane strain solution is:

$$u_b = \frac{(1 - \nu_c^2)}{E_c} \left[\frac{c^2 + b^2}{c^2 - b^2} + \frac{\nu_c}{1 - \nu_c} \right] \eta p_1 + \frac{2(1 - \nu_c)}{c^2 - b^2} c \int_b^c \alpha_c(r) \Delta T(r) r dr - \frac{2(1 - \nu_c^2)}{E_c} \frac{bc^2}{(c^2 - b^2)} p_E \quad (14)$$

and

$$\sigma_{\theta}|_r = \frac{1}{(c^2 - b^2)} \left[b\eta p_1 + \frac{bc^2\eta}{r^2} p_1 - c^2 p_E - \frac{b^2 c^2}{r^2} p_E \right] + \frac{E_c}{(1 - \nu_c)} \frac{1}{r^2} \times \left[\frac{r^2 + b^2}{c^2 - b^2} \int_b^c \alpha_c(r) \Delta T(r) r dr + \int_b^r \alpha_c(r) \Delta T(r) r dr - \alpha_c(r) \Delta T(r) r^2 \right]. \quad (15)$$

If any initial gap (g) exists between the fuel and clad, the radial displacement of radius c would be

$$u_c = u_b - g. \quad (16)$$

4. General Method of Solution

Equations (5) and (6) give

$$p_1 = f_1(\eta). \quad (17)$$

Equating u_b from Equations (12) and (14), respectively, using u_{η} from either Equations (5) or (6), gives

$$p_1 = f_2(\eta). \quad (18)$$

Equations (17) and (18) contain only the unknowns p_1 and η . The formulas which define the above functions are given in Section III.

An important parameter in the general solution is the coefficient of thermal expansion. The fact that the coefficient of thermal expansion for UO_2 - ZrO_2 increases with temperature means that power increases which are initiated at higher power levels are more deleterious than equal power increases occurring at lower levels.

The treatment herein is axisymmetric, and no attempt has been made to treat the local stress conditions in the cladding in the vicinity of pellet cracks. Obviously, the stresses would be slightly higher at such points due to locally high contact forces, slight temperature perturbations, and dynamic loading conditions.[†] These effects are judged to be of minor importance provided that the cladding has not been embrittled by severe hydriding.

[†]Since the crack propagation strength of the pellet material is undoubtedly lower than the crack initiation stress, the crack growth will not be a gradual quasi-steady-state process. As the temperature increases, there will be a series of rapid crack growths interrupted by periods of no crack growth. This may induce some dynamic loading of the cladding at the open end of the crack.

B. Plastic Flow Treatment in Inner Core of Fuel Pellet

The high temperatures and stresses present in the inner core of the fuel pellet may cause the fuel material to exceed its plastic flow stress. Approximate flow properties for various oxide fuels are given in Reference 4. The values for ThO_2 are shown in Figure 6.

The modified calculational procedure which accounts for a plastic flow zone is as follows:

1. Solve by an iterative technique the non-linear algebraic-integral equations described in Section II.A, assuming no plastic zone.
2. Test if the circumferential stress at $r = a$ is greater than the plastic flow stress, σ_{FLOW} :

$$\text{Is } |\sigma_\theta| \Big|_{r=a} > \sigma_{\text{FLOW}} \Big|_{r=a} \quad ?$$

If no, the solution in Step 1 is appropriate, and the problem is terminated.

If yes, additional calculations must be performed as follows:

- a. Determine the value of r for which

$$|\sigma_\theta| \Big|_r < \sigma_{\text{FLOW}} \Big|_r$$

and define this radial position as $\zeta \equiv r$.

- b. Beginning with this first choice for ζ , solve the original problem but with the addition of an internal pressure p_ζ , defined by

$$p_\zeta = \frac{1}{\zeta} \int_a^\zeta \sigma_{\text{FLOW}}(r) dr \quad , \quad (19)$$

which acts on the inside of the uncracked fuel region (Region 1). The inside of this elastic ring will now have a radius ζ instead of a , and the internal pressure (p_ζ) term will result in an alteration of Equation (7).

$$\begin{aligned} \sigma_\theta \Big|_r = & \frac{E_f^*}{(1 - \nu_f^*)} \frac{1}{r^2} \left[\frac{r^2 + \zeta^2}{\eta^2 - \zeta^2} \int_\zeta^\eta \alpha_f(r) \Delta T(r) r dr + \int_\zeta^r \alpha_f(r) \Delta T(r) r dr \right. \\ & \left. - \alpha_f(r) \Delta T(r) r^2 \right] - \frac{\eta^2}{\eta^2 - \zeta^2} \left[p_1 + \frac{\zeta^2}{r^2} p_1 \right] + \frac{2\zeta}{(\eta^2 - \zeta^2)} p_\zeta \quad . \quad (20) \end{aligned}$$

Figure 6. Comparison of Yield Stress Determinations
for 97.5-Percent TD ThO_2

3. For each new solution of ζ , the standard non-linear equations must be solved again by iteration. The search is terminated when $|\sigma_\theta|_\zeta < \sigma_{\text{FLOW}_\zeta}$.

C. Calculational Model for Including Clad Yielding

The program calculational model also examines possible clad yielding for the fuel rod, based on the yield strength of the clad and final conditions in the reactor (e.g., final radial temperature profile in the clad). For this average yield strength in the clad, $(\bar{\sigma}_c^y)$, a corresponding radial stress at the surface of the pellet is determined:

$$p_2^y = \frac{\bar{\sigma}_c^y(c - b)}{b} + \frac{p_E c}{b} \quad (21)$$

This radial stress is then compared with the interference pressure between the cladding and fuel (p_2) for the final converged value of p_1 . From Equation (13),

$$p_2 = \frac{\eta}{b} p_1 \quad (22)$$

If $p_2 > p_2^y$, yielding of the clad has occurred and will result in further cracking (a reduction in the value of η). In this situation, the value of η is reduced until $p_2 < p_2^y$, then increased at a much smaller increment. The search is terminated when p_2 is only slightly greater than p_2^y .

III. CALCULATIONAL PROCEDURES IN STRIPE (M0650)

Solution of the non-linear algebraic-integral equations described in Section II is obtained by applying numerical differencing techniques to the model. The resulting set of finite difference equations is then solved on a digital computer. The development of these difference equations, along with other necessary program computations, are presented in this section.

A. Preliminary Calculations

Material properties used in the STRIPE program account for the porosity and relative density (ρ_f) of the fuel pellet as indicated in Figure 1.

1. Plane Stress Option

In order to obtain the plane stress solution ($\sigma_z = 0$) using the plane strain expressions contained herein, the following transformations are performed in the fuel and clad material properties:

$$\begin{array}{ll}
 \text{FUEL} & \text{CLAD} \\
 E'_f = \frac{(1 + 2\nu_f^*)}{(1 + \nu_f^*)^2} E_f^* & E'_c = \frac{(1 + 2\nu_c)}{(1 + \nu_c)^2} E_c \\
 \nu'_f = \frac{\nu_f^*}{(1 + \nu_f^*)} & \nu'_c = \frac{\nu_c}{(1 + \nu_c)} \\
 \alpha'_f = \frac{(1 + \nu_f^*)}{(1 + 2\nu_f^*)} \alpha_f & \alpha'_c = \frac{(1 + \nu_c)}{(1 + 2\nu_c)} \alpha_c
 \end{array} \quad (23)$$

Other preliminary calculations include the establishment of an equally spaced fuel and clad computing grid. For the fuel,

$$\Delta r_f = \frac{b - a}{\text{NFUEL}} \quad (24)$$

where a and b are the inside radius and outside radius of the fuel pellet and NFUEL is the user specified number of radial sections in the fuel region. For the clad,

$$\Delta r_c = \frac{c - b}{\text{NCLAD}} \quad (25)$$

where c is the outside radius of the clad and NCLAD is the number of radial sections in the clad.

Essential initial and final temperature profiles may be specified by one of the two methods available as discussed in Section II.C.2 and II.C.3.

2. User-Supplied Temperature Profile Option

For this option, temperature distributions may be inputted on a uniform temperature grid ΔT independent of the equally spaced computing grids Δr defined in Equations (24) and (25). These fuel and clad temperature profiles $T(r)$,

which may be of coarser or finer detail than the region computing grids, are then overlaid on to the basic computing grids. For each computing mesh point j ,

$$T_j = \frac{1}{\Delta r} \int_{r_j - \frac{1}{2} \Delta r}^{r_j + \frac{1}{2} \Delta r} T(r) dr \approx \frac{1}{\Delta r} \sum_{j - \frac{1}{2}}^{j + \frac{1}{2}} T(r) \Delta r \quad (26)$$

3. Internally Computed Temperature Profile Option

For this option, the STRIPE program computes temperature profiles in the pellet and clad at each computing mesh point by the following formulas:

$$\text{Pellet: } T(r) = C_b \left[\frac{b^2 - r^2}{b^2 - a^2} - \frac{2a^2}{b^2 - a^2} \ln\left(\frac{b}{r}\right) \right] + T_b \quad ; \quad (27)$$

$$\text{Clad: } T(r) = C_c \ln\left(\frac{c}{r}\right) + T_c \quad . \quad (28)$$

It is suggested that the user-supplied coefficients, C_b and C_c , be calculated as follows:

$$C_b = \frac{c \phi_s}{24 K_f} \quad ; \quad (29)$$

$$C_c = \frac{c \phi_s}{12 K_c} \quad (30)$$

where

ϕ_s = surface heat flux, Btu/hr-ft²

K_f = thermal conductivity of the fuel, Btu/hr-ft-°F

K_c = thermal conductivity of the clad, Btu/hr-ft-°F.

4. Temperature-Dependent Material Properties

Temperature-dependent material properties may be computed in either of two ways: (1) in a user-specified table of material property versus temperature, or (2) by a second-order polynomial equation. For the user-supplied tables, fuel properties must be provided over the expected range of fuel temperatures; up to 50 entries are permitted. Clad properties are likewise supplied, but in a

separate table for the range of expected clad temperatures. Linear interpolation is employed in calculating these material properties, which include:

Fuel:

$\alpha_f(T)$ (coefficient of thermal expansion, $1/^\circ\text{F}$)

$E_f(T)$ (Young's modulus for the fuel, psi)

$\sigma_{ULT}(T)$ (ultimate tensile strength of the fuel, psi)

$\sigma_{FLOW}(T)$ (compressive plastic flow stress of the fuel, psi).

For $\sigma_{FLOW}(T)$, the linear interpolation is performed on a semilog plane.

Clad:

$\alpha_c(T)$ (coefficient of thermal expansion, $1/^\circ\text{F}$)

$E_c(T)$ (Young's modulus for the clad, psi)

$\sigma_{YIELD}(T)$ (yield strength of the clad, psi).

For the second-order polynomial fit method, the following calculation illustrates the procedure. At each station j in the fuel region computing grid,

$$\alpha_{f,j} = AF_0 + AF_1 T_{f,j} + AF_2 (T_{f,j})^2 ; j = 1, \dots, (NFUEL + 1) \quad (31)$$

Similar polynomial fitting is used for the other temperature-dependent material parameters.

B. The Numerical Method of Solution

The basic mathematical problem in STRIPE involves the simultaneous solution of a pair of non-linear algebraic-integral equations for their roots. Physically, the model must determine the depth of radial cracking in an annular (or solid) fuel pellet from the periphery to a radius $r = \eta$ (Figures 2 and 3) and the radial compressive stress (p_1) at the base of the crack.

1. Governing Equations

The governing equations for each region of the model are given herein. Setting the circumferential stress at the periphery of the uncracked portion

of the pellet ($r = \eta$) equal to the fracture stress $\sigma_{ULT}(\eta)$, and solving for the radial compressive stress at η :

$$p_1^U = - \frac{(\eta^2 - a^2)}{(\eta^2 + a^2)} \left\{ \left\{ \sigma_{ULT}(\eta) - \frac{\bar{E}_f^*}{(1 - \nu_f^*)} \left\{ \frac{2}{(\eta^2 - a^2)} \int_a^\eta \left[\alpha_f^F(r) T_f^F(r) - \alpha_f^I(r) T_f^I(r) \right] r dr \right. \right. \right. \\ \left. \left. \left. - \left[\alpha_f^F(\eta) T_f^F(\eta) - \alpha_f^I(\eta) T_f^I(\eta) \right] \right\} - \frac{2a^2 p_c}{(\eta^2 - a^2)} \right\} \right\} . \quad (32)$$

Equating the radial displacements of the cracked and uncracked portions of the pellet at their interface, and solving for the radial compressive stress at η :

$$p_1^c = \frac{2 \left\{ \frac{(1 + \nu_f^*)\eta}{(\eta^2 - a^2)} \int_a^\eta \left[\alpha_f^F(r) T_f^F(r) - \alpha_f^I(r) T_f^I(r) \right] r dr \right. \\ \left. - \frac{(1 + \nu_c)c}{c^2 - b^2} \int_b^c \left[\alpha_c^F(r) T_c^F(r) - \alpha_c^I(r) T_c^I(r) \right] r dr + \frac{bc^2 p_E (1 - \nu_c^2)}{\bar{E}_c (c^2 - b^2)} \right\}}{\eta \left\{ \left[\frac{(c^2 + b^2)}{(c^2 - b^2)} + \frac{\nu_c}{(1 - \nu_c)} \right] \frac{(1 - \nu_c^2)}{\bar{E}_c} + \left[\frac{(\eta^2 + a^2)}{(\eta^2 - a^2)} - \frac{\nu_f^*}{(1 - \nu_f^*)} \right] \frac{(1 - \nu_f^{*2})}{\bar{E}_f^*} \right\}} . \quad (33)$$

In Equations (32) and (33), the superscripts "I" and "F" denote initial and final temperature conditions, respectively.

2. Solution by Method of Bisection

The STRIPE computer program solves the problem using the method of bisection iteration scheme as follows:

- (1) Define $\Delta p = p_1^c - p_1^u$; initialize a counter (ICOUNT = 0).
- (2) Compute Δp at $\eta_1 = a$ and $\eta_2 = b$, utilizing Equations (32) and (33).
- (3) Test: Is $\Delta p_1(\eta_1 = a) > 0$? If yes, proceed to Step (4); if no, there is complete cracking of the pellet. Terminate the calculations by setting $p_1 = p_1^c$.
- (4) Test: Is $\Delta p_2(\eta = b) < 0$? If yes, proceed to Step (5); if no, there is no cracking of the pellet. Terminate the calculations by setting $p_1 = p_1^u$.
- (5) Modify the counter (ICOUNT = ICOUNT + 1); compute an average $\eta_3 = \frac{1}{2}(\eta_1 + \eta_2)$.

- (6) Compute Δp_3 ($\eta = \eta_3$).
- (7) Test: Is $\Delta p(\eta = \eta_3) \geq 0$? If yes: Set $\eta_1 = \eta_3$ and $\Delta p_1 = \Delta p_3$, and proceed to Step (8). If no: Set $\eta_2 = \eta_3$ and $\Delta p_2 = \Delta p_3$; and proceed to Step (8).
- (8) Test: Is the counter ICOUNT ≥ 20 ? If no, return to Step (5). If yes, proceed to auxiliary calculations and a final edit.

Step (7) is the key step in the method of bisection; here the plane of possible solutions is separated into two halves. Only that half in which the solution lies is retained; the other half is discarded from consideration. If this process is performed 20 times, the final error in η , the radial distance to the base of the crack will be

$$\epsilon = \frac{1}{2^{20}} (b - a) . \quad (34)$$

3. Tensile Yielding in the Clad

After the iterations are completed, the program tests whether yielding has occurred in the clad. This test is based on an average yield strength in the clad, $\bar{\sigma}_c^y$; for this yield point, a radial stress at the surface of the pellet (p_2^y) is determined from the relationship,

$$p_2^y = \frac{\bar{\sigma}_c^y (c - b)}{b} + \frac{p_E c}{b} . \quad (35)$$

The radial stress is then compared with an interference pressure (p_2) between the cladding and the fuel for the final converged value of p_1 as obtained from the simple relationship,

$$p_2 = \frac{\eta}{b} p_1 . \quad (36)$$

If $p_2 > p_2^y$, then yielding has occurred, and a special modification must be performed:

- (1) Since yielding results in further cracking, reduce η by the increment $\Delta\eta = 0.01(b - a)$.
- (2) Repeatedly solve Equations (32) and (33) for each new decreasing value of η from Step (1) until $p_2 < p_2^y$.
- (3) When Step (2) is satisfied, increase η by the increment $\Delta\eta = 0.001(b - a)$ and again repeatedly solve Equations (32) and (33) for each new increasing value of η until $p_2 \geq p_2^y$.
- (4) Adjust the p_1 dependent radial displacements (u_η, u_b, u_c which are subsequently defined) to match this final solution.

4. Compressive Flow in Pellet

A further test provided in the STRIPE program is for the presence of a plastic flow portion in the inner region of the fuel pellet where the temperatures are high (typically 3500°F). The logic commences by testing a counter n which was initialized to zero at the beginning of the program:

$$\text{If } n = 0, \text{ test: } \left| \sigma_{\theta} \right|_{r=a} > \sigma_{\text{FLOW}} \quad (37)$$

If $n \neq 0$, jump to Step (3) below.

If $|\sigma_{\theta}|$ as computed by Equation (48) is larger than σ_{FLOW} , the entire problem must be resolved using the following special procedure:

- (1) Compare $|\sigma_{\theta}|_r$ with σ_{FLOW_r} for $p_{\zeta} = 0.0$ at $r = a + 0.01m(\eta - a)$, $m = 1, 2, \dots, M$ until $|\sigma_{\theta}|_r \leq \sigma_{\text{FLOW}_r}$. The term $\sigma_{\theta}|_r$ for the fuel is obtained from

$$\begin{aligned} \sigma_{\theta}|_r = & \frac{\bar{E}_f^*}{(1 - \nu_f^*)r^2} \left\{ \frac{(r^2 + a^2)}{(\eta^2 - a^2)} \int_a^{\eta} \left[\alpha_f^F(r)T_f^F(r) - \alpha_f^I(r)T_f^I(r) \right] r dr \right. \\ & + \int_a^r \left[\alpha_f^F(r)T_f^F(r) - \alpha_f^I(r)T_f^I(r) \right] r dr \\ & \left. - \left[\alpha_f^F(r)T_f^F(r) - \alpha_f^I(r)T_f^I(r) \right] r^2 \right\} - \frac{\eta^2(r^2 + a^2)}{r^2(\eta^2 - a^2)} p_1 \quad (39) \end{aligned}$$

- (2) Define this value of $r = \zeta_1$, where ζ_1 is the initial choice of the outer radius ζ_n of the plastic region for $n = 1$.
- (3) For any iteration n , solve the original problem but with the addition of an internal pressure $p_{\zeta_n} > 0.0$:

$$p_{\zeta_n} = \frac{1}{\zeta_n} \int_a^{\zeta_n} \sigma_{\text{FLOW}}(T_f, r) dr \quad (40)$$

- (4) This internal pressure p_{ζ_n} acts on the inside of the elastic ring; the inside radius of this elastic region is now ζ_n instead of the inside radius of the pellet, a . Substitute this modified inside radius,

$$a' = \zeta_n \quad (41)$$

in all equations where "a" appears. Also, increase the iteration counter by unity ($n = n+1$).

(5) Solution of the original problem must be performed for this internal pressure p_{ζ_n} ; immediately return to the inner loop of Equations (32) and (33) and again solve this set by the method of bisection, assuming $\eta_1 = \text{converged } \eta_{20}$.

(6) After the 20 iterations by the method of bisection and the test for yielding of the clad, recompute $|\sigma_\theta|_{r=\zeta_n}$, that is, $|\sigma_{\zeta_n}|$, by Equation (48) but with $a' = \zeta_n$.

(7) Test: Is $n = 2$?

If yes, proceed to Step (8); otherwise omit Step (8) and proceed to Step (9).

(8) Test: Is $|\sigma_{\zeta_2}| < \sigma_{\text{FLOW}}|_{r=\zeta_2}$?

If yes: This solution is abnormal; terminate the problem and write an error comment.

If no: Proceed to Step (9).

(9) Increase ζ_n by the formula

$$\zeta_n = \zeta_1 + 0.01(n-1)(b - \zeta_1), \quad n = 2, 3, \dots, N. \quad (42)$$

(10) Test: Is $n = 2$?

If yes, return to the inner loop of Step (5); otherwise proceed to Step (11).

(11) Test: Is $|\sigma_{\zeta_n}| < \sigma_{\text{FLOW}}|_{r=\zeta_n}$?

If yes, a satisfactory solution has been obtained; terminate the problem with a final edition.

If no, return to Step (5) for another iteration.

C. Auxiliary Calculations Performed by STRIPE

Auxiliary calculations performed in the STRIPE program include the radial displacements (u) in each region caused by differential thermal expansion and the circumferential stresses edited at selected points in the pellet and clad.

1. Radial Displacements

Radial displacement calculations (optionally) include the effect of a gap between the pellet and clad, and are as follows:

Region 1 (Uncracked portion of pellet)

$$u_\eta = \frac{\eta}{\bar{E}_f^*} \left[(1 - \nu_f^{*2}) \sigma_{\text{ULT}}(\eta) + \nu_f^* (1 + \nu_f^*) p_1 \right] + \eta (1 + \nu_f^*) \left[\alpha_f^F(\eta) T_f^F(\eta) - \alpha_f^I(\eta) T_f^I(\eta) \right] + \frac{2a^2 \eta p_\zeta (1 - \nu_f^{*2})}{\bar{E}_f^* (\eta^2 - a^2)} \quad (43)$$

If there is no cracking of the pellet as indicated in Step (4) of the method of iterations, u_η is computed from the alternate formula

$$u_\eta = - \left[\frac{b^2 + a^2}{b^2 - a^2} - \frac{v_f^*}{(1 - v_f^*)} \right] \frac{p_1 b (1 - v_f^{*2})}{\bar{E}_f^*} + \frac{2(1 + v_f^*)b}{b^2 - a^2} \int_a^b \left[\alpha_f^F(b) T_f^F - \alpha_f^I(b) T_f^I(b) \right] r dr + \frac{2a^2 b p_\zeta (1 - v_f^{*2})}{\bar{E}_f^* (b^2 - a^2)} \quad (44)$$

Region 2 (Cracked portion of pellet)

$$u_b = u_\eta + (1 - v_f^{*2}) \eta p_1 \int_\eta^b \frac{1}{\bar{E}_f^*(r)} \frac{dr}{r} + (1 + v_f^*) \int_\eta^b \left[\alpha_f^F(r) T_f^F(r) - \alpha_f^I(r) T_f^I(r) \right] r dr \quad (45)$$

For no cracking,

$$u_b = u_\eta \quad (46)$$

Region 3 (Clad)

A basic assumption in the radial displacement of the clad is that at the termination of the power transient, there is no gap (g) remaining between pellet and clad, that is,

$$u_c = u_b - g \quad (47)$$

where u_c is the radial displacement of the clad. If the thermal expansion of the fuel does not close this gap (i.e., $u_b < g$), a flag will be written and the problem terminated.

2. Stresses

Circumferential stresses are calculated in the pellet at $r = a$ and η using the following solutions:

$$\text{Pellet: } \sigma_\theta|_{r=a} = \frac{\bar{E}_f^*}{(1 - v_f^*)} \left\{ \frac{2}{\eta^2 - a^2} \int_a^\eta \left[\alpha_f^F(r) T_f^F(r) - \alpha_f^I(r) T_f^I(r) \right] r dr - \left[\alpha_f^F(a) T_f^F(a) - \alpha_f^I(a) T_f^I(a) \right] \right\} - \frac{2\eta^2 p_1}{\eta^2 - a^2} + \frac{(\eta^2 + a^2)}{(\eta^2 - a^2)} p_\zeta \quad (48)$$

$$\begin{aligned}
\sigma_{\theta}|_{r=\eta} = & \frac{\bar{E}_f^*}{(1 - \nu_f^*)\eta^2} \left\{ \frac{(\eta^2 + a^2)}{(\eta^2 - a^2)} \int_a^{\eta} \left[\alpha_f^F(r) T_f^F(r) - \alpha_f^I(r) T_f^I(r) \right] r dr \right. \\
& + \int_a^{\eta} \left[\alpha_f^F(r) T_f^F(r) - \alpha_f^I(r) T_f^I(r) \right] r dr \\
& \left. - \left[\alpha_f^F(\eta) T_f^F(\eta) - \alpha_f^I(\eta) T_f^I(\eta) \right] \eta^2 \right\} - \frac{(\eta^2 + a^2)}{(\eta^2 - a^2)} p_1 + \frac{2a^2 p_c}{(\eta^2 - a^2)} . \quad (49)
\end{aligned}$$

For the clad, circumferential stresses are calculated at $r = b, \frac{1}{2}(b + c), c$ by the solution:

$$\begin{aligned}
\text{Clad: } \sigma_{\theta}|_r = & \frac{1}{c^2 - b^2} \left[b\eta p_1 + \frac{bc^2\eta}{r^2} p_1 - c^2 p_E - \frac{b^2 c^2}{r^2} p_E \right] \\
& + \frac{E_c}{(1 - \nu_c)r^2} \left\{ \frac{r^2 + b^2}{c^2 - b^2} \int_b^c \left[\alpha_c^F(r) T_c^F(r) - \alpha_c^I(r) T_c^I(r) \right] r dr \right. \\
& \left. - \int_b^r \left[\alpha_c^F(r) T_c^F(r) - \alpha_c^I(r) T_c^I(r) \right] r dr - \left[\alpha_c^F(r) T_c^F(r) - \alpha_c^I(r) T_c^I(r) \right] r^2 \right\} . \quad (50)
\end{aligned}$$

D. Finite Difference Approximations

Finite difference approximations to the integral terms appearing in Equations (32) through (50) are presented as follows:

$$\begin{aligned}
& \int_{r_1}^{r_2} \left[\alpha^F T^F - \alpha^I T^I \right] r dr \\
& \approx \sum_{J=1}^N \left[\frac{(\alpha_{j+1}^F T_{j+1}^F - \alpha_{j+1}^I T_{j+1}^I) + (\alpha_j^F T_j^F - \alpha_j^I T_j^I)}{2} \right] \cdot \left[\frac{r_{j+1} + r_j}{2} \right] \Delta r . \quad (51)
\end{aligned}$$

$$\int_{\eta}^b \frac{1}{E_f^*(r)} \frac{dr}{r} \approx \sum_{j(\eta)}^{j(b)} \frac{2}{(E_{f,j+1}^* + E_{f,j}^*)} \ln \left(\frac{r_{j+1}}{r_j} \right) . \quad (52)$$

Numerical integrations performed in the course of the iterations account for the likelihood that calculated values of η will lie between established computing

mesh points r_j and r_{j+1} . In this event an additional mesh point (j_n) is introduced. Linear interpolation is employed to compute temperatures and temperature-dependent material properties at the new mesh point, for example,

$$n > r_j, f = \frac{n - r_j}{r_{j+1} - r_j} ; \quad (53)$$

$$T_n^F = T_j^F + f(T_{j+1}^F - T_j^F) ; \quad (54)$$

$$\left[\alpha_f^F \right]_n = \left[\alpha_f^F \right]_j + f \left[(\alpha_f^F)_{j+1} - (\alpha_f^F)_j \right] . \quad (55)$$

IV. INPUT REQUIREMENTS FOR STRIPE

The purpose of this section is to provide program users with the input requirements and restrictions for the preparation of a STRIPE (MO650) job input deck. The input card preparation and processing employ the Bettis input package (Reference 5) for the CDC-6600 digital computer (operating system SCOPE 3.1). To briefly summarize this input processing, all problems run under a single accounting card are called a job. Within this job there may be any number of individual problems called cases; special termination cards with a slash (/) in column 1 separate the data deck into these cases. The first case must be complete and self-contained while succeeding cases need supply only the modifications to this original case. Each case should be headed by an appropriate comment (title) card, identified by an equal sign (=) in column 1 and any descriptive title in columns 2 through 80.

STRIPE was written in FORTRAN IV and compiled on the CDC-6600 computer. The specific MO650 job card arrangement for the CDC-6600 is shown in the following tabulation:

CDC-6600 JOB CARD FORMAT

Job Identification	1-7
,	8
Priority	9
,	10
Running Time Limit	11-13
,	14
045 (Program Length)	15-17
,	18
0000.	19-23
MO650	24-28

,	29
User code name	30-34
,	35
Classification	36
,	37
P. (Time-charge class)	38-39

A CDC-6600 control card of the form TASK(M0650) must follow the job card. In the description of the input data requirements that follows, the symbols (X) and (F) associated with the data definitions denote the decimal form of input, that is,

(X): Input data of integer form

(F): Input data of real (floating-point) form

while ND indicates No Dimensions. Data cards must contain a four-digit card number (usually) in columns 1 through 4 followed by a comma in column 5; columns 6 through 80 contain the decimal data in the above integer or real form.

A. Control Information

1001, NTOP, NFUEL, NCLAD, NAXOP

NTOP: Temperature specification option control, ND, (X).

NTOP

Option

- 1 User-supplied temperature profiles; table lookup for temperature-dependent material properties.
- 2 Temperature profiles computed by built-in formula; second-order polynomial fit of temperature-dependent material properties.

NFUEL: Number of radial sections in fuel region ($1 \leq \text{NFUEL} \leq 100$), ND, (X)

NCLAD: Number of radial sections in clad region ($1 \leq \text{NCLAD} \leq 50$), ND, (X)

NAXOP: Axial restraint option control, ND, (X).

NAXOP

Option

- 1 Plane strain solution (no axial strain, $\epsilon_z = 0$)
- 2 Plane stress solution (no axial stress, $\sigma_z = 0$, applicable to dished pellets)
- 3 Generalized plane strain solution (constant axial strain, zero axial external load, $F_z = 0$)

B. Initial Conditions

1. Essential initial conditions

2001, p_E

p_E : External system pressure, psia, (F).

2. Initial conditions - user supplied (NTOP = 1 on Card 1001)

2002, P_I , P_F

P_I : Initial fractional reactor power, ND, (F)

P_F : Final fractional reactor power, ND, (F)

2101, $(T_f^I)_1, \dots, (T_f^I)_j, \dots, (T_f^I)_{NFS}$ [$2 \leq NFS \leq 101$]

$(T_f^I)_j$: Initial radial temperature-profile in fuel, °F, (F)

Here (and for all subsequent user-supplied temperature distributions) the temperature profile may be inputted independent of the equally spaced fuel computing grid determined by NFUEL on Card 1001.

2201, $(T_c^I)_1, \dots, (T_c^I)_j, \dots, (T_c^I)_{NCS}$ [$2 \leq NCS \leq 51$]

$(T_c^I)_j$: Initial radial temperature profile in clad, °F, (F).

The temperature profile indexing proceeds from the inner radius to the outer radius of each region. These essential temperature profiles may be obtained from output of the digital computer program FIGRO (Reference 6), which solves the radial heat conduction problem for cylindrical fuel elements, using a temperature varying thermal conductivity for oxide fuels.

C. System Parameters

1. Geometry

3001, a , b , c , g

a : Inside radius of pellet (fuel), in, (F)

b : Outside radius of pellet, in, (F)

c : Outside radius of clad, in, (F)

g : Initial radial gap between pellet and clad, in, (F).

2. Constant material properties

3002, ν_f , ρ_f , ν_c

ν_f : Poisson's ratio for the fuel at 100-percent relative density, ND, (F)

ρ_f : Relative fuel density, ND, (F)

ν_c : Poisson's ratio for the clad, ND, (F).

3. Temperature-dependent material properties - Table Lookup
(NTOP = 1 on Card 1001).

a. Fuel material properties

3101, $\alpha_{f_1}, \dots, \alpha_{f_i}, \dots, \alpha_{f_{NFE}}$ ($2 \leq NFE \leq 50$)

α_{f_i} : Coefficient of thermal expansion for fuel, $1/^\circ F$, (F)

3201, $E_{f_1}, \dots, E_{f_i}, \dots, E_{f_N}$ ($N = NFE$ on Card 3101)

E_{f_i} : Young's modulus for fuel at 100-percent relative density, fuel
psi, (F).

3301, $\sigma_{ULT_1}, \dots, \sigma_{ULT_i}, \dots, \sigma_{ULT_N}$ ($N = NFE$ on Card 3101)

σ_{ULT_i} : Ultimate tensile strength of fuel, psi, (F)

3401, $\sigma_{FLOW_1}, \dots, \sigma_{FLOW_i}, \dots, \sigma_{FLOW_N}$ ($N = NFE$ on Card 3101)

σ_{FLOW_i} : Plastic flow compressive stress of fuel, psi, (F)

3501, $T_{MF_1}, \dots, T_{MF_i}, \dots, T_{MF_N}$ ($N = NFE$ on Card 3101)

T_{MF_1} : Temperature entry in fuel material property versus
temperature table, $^\circ F$, (F)

$T_{MF_{i+1}} > T_{MF_i}$ for all i .

Entries in Card Series 3101 through 3401 must correspond to the fuel temperatures entered on Card 3501, and these fuel temperature entries must be monotonically increasing for proper table lookup of temperature-dependent fuel material properties.

b. Clad material properties

3601, $\alpha_{c_1}, \dots, \alpha_{c_i}, \dots, \alpha_{c_{NCE}}$ ($2 \leq NCE \leq 50$)

α_{c_i} : Coefficient of thermal expansion for clad, $1/^\circ F$, (F)

3701, $E_{c_1}, \dots, E_{c_i}, \dots, E_{c_N}$ ($N = NCE$ on Card 3601)

E_{c_i} : Young's modulus for clad, psi, (F)

3801, $\sigma_{YIELD_1}, \dots, \sigma_{YIELD_i}, \dots, \sigma_{YIELD_N}$ (N = NCE on Card 3601)

σ_{YIELD_i} : Yield strength of the clad, psi, (F)

3901, $T_{MC_1}, \dots, T_{MC_i}, \dots, T_{MC_N}$ (N = NCE on Card 3601)

T_{MC_i} : Temperature entry in clad material property versus temperature table, °F, (F)

$T_{MC_{i+1}} > T_{MC_i}$ for all i.

Entries in Card Series 3601 through 3801 must correspond to the temperatures entered on Card 3901 and these clad temperatures must be monotonically increasing for proper table lookup of temperature-dependent clad material properties.

D. Final Conditions - User Supplied

The following card series are required for NTOP = 1 on Card 1001:

4101, $(T_f^F)_1, \dots, (T_f^F)_j, \dots, (T_f^F)_M$ (M = NFS on Card 2101)

$(T_f^F)_j$: Final radial temperature profile in fuel, °F, (F)

4201, $(T_c^F)_1, \dots, (T_c^F)_j, \dots, (T_c^F)_N$ (N = NCS on Card 2201)

$(T_c^F)_i$: Final radial temperature profile in clad, °F, (F).

E. Material Properties, Initial and Final Conditions by Formula

1. Temperature-dependent material properties (NTOP = 2 on Card 1001)

5001, AF_0, AF_1, AF_2

$\left. \begin{array}{l} AF_0: \\ AF_1: \\ AF_2: \end{array} \right\} \begin{array}{l} \text{Coefficients for second-order polynomial fit, coefficient} \\ \text{of thermal expansion for fuel, mixed, (F).} \end{array}$

For this material property and for the other temperature-dependent material properties which follow, the coefficients must be selected according to the formula

$$\alpha_f = AF_0 + AF_1 * T + AF_2 * T^2 \quad .$$

5002, EF_0 , EF_1 , EF_2

EF_0 :
 EF_1 :
 EF_2 :
Coefficients for second-order polynomial fit, Young's modulus for fuel, mixed, (F).

5003, SU_1 , SU_2 , SU_3

SU_1 :
 SU_2 :
 SU_3 :
Coefficients for second-order polynomial fit, ultimate tensile strength in fuel, mixed, (F).

5004, SF_1 , SF_2 , SF_3

SF_1 :
 SF_2 :
 SF_3 :
Coefficients for second-order polynomial fit, plastic flow compressive stress in fuel, mixed, (F).

5005, AC_0 , AC_1 , AC_2

AC_0 :
 AC_1 :
 AC_2 :
Coefficients for second-order polynomial fit, coefficient of thermal expansion for clad, mixed, (F).

5006, EC_0 , EC_1 , EC_2

EC_0 :
 EC_1 :
 EC_2 :
Coefficients for second-order polynomial fit, Young's modulus for clad, mixed, (F).

5007, SY_0 , SY_1 , SY_2

SY_0 :
 SY_1 :
 SY_2 :
Coefficients for second-order polynomial fit, yield strength of the clad, mixed, (F).

2. Initial and final conditions by formula (NTOP = 2 on Card 1001)

5101, T_b^I , C_c^I , T_c^I , C_c^I , P_I .

T_b^I : Initial pellet temperature at $r = b$, °F, (F)

C_b^I : Coefficient for initial pellet temperatures by formula, °F, (F)

T_c^I : Initial clad surface temperature at $r = c$, °F, (F)

C_c^I : Coefficient for initial clad temperatures by formula, °F, (F)

P_I : Initial fractional reactor power, ND, (F).

5102, T_b^F , C_b^F , T_c^F , C_c^F , P_F

T_b^F : Final pellet temperature at $r = b$, °F, (F)

C_b^F : Coefficient for final pellet temperatures by formula, °F, (F)

T_c^F : Final clad surface temperature at $r = c$, °F, (F)

C_c^F : Coefficient for final clad temperatures by formula, °F, (F)

P_F : Final fractional reactor power, ND, (F).

F. Termination Card

To run a series of cases as one job, special separator cards must follow the data deck of the first case and each case that follows. The first case must be complete, while succeeding cases need only supply the modified card series. Separator cards between cases are identified by a slash (/) in column 1, while the termination card of the last case is designated by a period (.) punched in column 1. For the CDC-6600 computer, an "END FILE" card must follow the termination card of the last case.

V. DISCUSSION OF STRIPE INPUT PREPARATION

The detailed procedures used for preparing STRIPE input are described in this section. Material property values are also given for UO_2 , $ThO_2 + UO_2$, and ThO_2 fuel.

A. Options

All options for the STRIPE program are included in the 1001 card.

Card 1001	Symbol	NTOP	No units
-----------	--------	------	----------

Option for method of determining temperature-dependent properties.

Recommended

Option 1 (Table Lookup).

Comments

The method of Table Lookup is recommended for a detailed analysis of fuel pellet expansion. Option 2 restricts the user to expressing fuel temperature profiles and other fuel parameters as first- or second-order polynomial functions of temperature. Table Lookup allows the listing of nonparabolic fuel temperature profile, such as the profile obtained from a FIGRO (Reference 6) calculation using variable fuel conductivity. Of equal importance is the description of fuel properties. Although the temperature variations of the thermal expansion coefficient, Young's modulus, and ultimate stress can be accurately expressed as polynomials, plastic flow stress is an exponential function of temperature and is described more precisely in an input table.

<u>Card 1001</u>	<u>Symbol NFUEL</u>	<u>No units</u>
------------------	---------------------	-----------------

Number of fuel sections.

Recommended

50.

Comments

The lowest number which will produce results with the desired precision should be used here. Sample runs have shown that the use of 50 sections provides stress calculations within 1 percent and fuel surface displacement within 0.1 percent of the same runs using 100 fuel sections, and the running time is about one-half. The running time can be halved again by using 20 sections, but some stress results would be off by 15 percent and fuel displacement by 3 percent. The specification of NFUEL has no bearing on the number of points necessary to describe the temperature profiles on Cards 2101 and 4101.

<u>Card 1001</u>	<u>Symbol NCLAD</u>	<u>No units</u>
------------------	---------------------	-----------------

Number of clad sections.

Recommended

10.

Comments

Sample runs have shown that the use of 10 sections when compared with the use of 20 sections results in identical stress calculations and fuel surface displacements. The use of 2 sections causes errors of 3 percent in stress and 1 percent in displacement results. No significant running time advantage

is gained by reducing the number of clad sections. The specification of NCLAD has no bearing on the number of points necessary to describe the temperature profiles on Cards 2201 and 4201.

Card 1001	Symbol NAXOP	No units
-----------	--------------	----------

Option describing the form of axial restraint.

Recommended

2 for dished-end pellets, 1 for flat-end pellets.

Comments

The plane stress solutions were used for analysis of the dished-end Chalk River pellets. It was found in the comparison of STRIPE results with Chalk River test results on UO_2 fuel pellets that the consideration of compressive plastic flow is necessary in pellets that reach temperatures higher than half the melting temperature, since a significant reduction in diametral expansion is achieved.

B. Temperature-Independent Parameters

System and pellet parameters which are specified as constants are contained on the following cards.

Card 2001	Symbol p_E	psi
-----------	--------------	-----

External system pressure.

Recommended

Nominal value of system pressure.

Comments

The greatest pellet expansion will result when the lowest possible system pressure is used. A variation of ± 200 psi above and below 1400 psi for a sample pellet resulted in a maximum of 6-percent change in fuel stresses and a 3-percent difference in fuel surface displacement.

Card 2002	Symbols P_I, P_F	No units
-----------	--------------------	----------

Initial and final power levels, fractions of full power.

Recommended

0., 1. for increase in power; 1., 0. for decrease in power.

The two values are used by the program to determine whether the implied transient is increasing or decreasing in power.

Inside radius of fuel.

For a solid pellet, the value of a is zero.

Outside radius of fuel.

This is the fuel radius at the initial temperature conditions described on Card Series 2101 (or on Card 5101).

Outside radius of clad.

This is the clad radius measured at the initial temperature conditions described on Card Series 2201 (or on Card 5102).

Radial fuel-clad gap size.

Use the fuel-clad gap size existing at the same initial temperature conditions described on Card Series 2101 and 2201 (or 5101 and 5102).

Poisson's ratio for fuel material.

See Figure 1.

Variation of v_f in sample pellet calculations showed no change in results when the value of v_f is increased by 10 percent.

Fuel density, fraction of theoretical density.

Comments

This value is used in the program to modify the values of Young's modulus on the Card Series 3201 (or Card 5002) for fuel porosity as indicated in Figure 1.

Poisson's ratio for clad material.

Recommended

0.325 for Zircaloy (Reference 7).

Comments

Results are not sensitive to as much as 10-percent variation in this value. Sensitivity to greater variations was not investigated.

C. Temperature Profiles

The initial and final temperature distributions in the fuel and clad are contained on these cards.

Initial temperature profile at uniform radial points in the fuel.

Comments

The first value is the temperature at the inside fuel radius a ; the final value is the temperature at the outside fuel surface b . Linear interpolation is used by the program to overlay this temperature profile on the independent computing grid; however, Card Series 2101 must contain as many values as Card Series 4101 to follow.

Initial temperature profile in clad.

Comments

The first value is the temperature at the inside surface of the clad; the final value is the temperature at the outside surface. Linear interpolation is used by the program to overlay this temperature profile on the clad

computing grid. Therefore, if both initial and final temperature profiles vary linearly through the thin clad, as is usually a good approximation, only two values are necessary. However, Card Series 2201 must contain as many values as Card Series 4201.

Cards 4101-4109 (or 5102)	Symbol (T_f^F) _j	°F
---------------------------	---------------------------------	----

Final temperature profile in fuel.

Recommended

The most effective tool for calculation of temperature profiles is the FIGRO computer code (Reference 6). This program includes (1) the effects of fuel temperature, porosity, and depletion on thermal conductivity; (2) the effects of fission gas release, surface roughness, and diminishing gap volume on fuel-clad gap conductance; and (3) the effects of flux depression on power distribution within the pellet.

Comments

The variation of thermal conductivity with fuel temperature is not accurately known at elevated temperatures. Reference 8 presents experimental results which are in general agreement at low temperatures. At temperatures higher than 3000°F, some tests show that conductivity continues downward with increasing temperature and other tests show an upswing in conductivity. For the sample cases and evaluation of Chalk River test data contained herein, the conductivity curve presented in Reference 7 was used. This conductivity curve increases with temperatures above 3200°F, and can be approximated in FIGRO by using the following constants:

$$k_{\text{SOLID}} = \frac{1}{B+CT}, \text{ watts/cm } ^\circ\text{K}$$

	<u>Below 1313 °K</u>	<u>Above 1313 °K</u>
B	11.15 cm °K/watt	44.6 cm °K/watt
C	0.01778 cm/watt	-0.00765 cm/watt

A β -value of 0.9 is used for UO_2 porosity effects where

$$K_{\text{POROUS}} = \frac{1-P}{1+\beta P} K_{\text{SOLID}} \quad .$$

P is the porosity, that is, $P = 1 - \rho_f$, where ρ_f is the relative density. Fuel-clad gap conductances for use in FIGRO are available for both helium and argon mixtures with fission gases. The Chalk River test elements

analyzed herein were welded in argon, according to References 12 through 14, but since the gaps are small enough to close up during the initial rise in reactor power, a constant gap conductance of $1770 \text{ Btu/hr-ft}^2\text{-}^\circ\text{F}$ was used. This is the equivalent of $1.0 \text{ watt/cm}^2\text{-}^\circ\text{C}$ used in temperature calculations in Reference 2, 7, 9, and 10.

Cards 4201-4209 (or 5102)	Symbol $(T_c^F)_j$	$^\circ\text{F}$
---------------------------	--------------------	------------------

Final temperature profile in the clad.

Recommended

The clad temperature distribution is considered to be linear between the temperatures at the outer and inner clad surfaces. These two temperatures are printed out with the FIGRO calculation of fuel temperatures.

Comments

The following values are recommended† for irradiated Zircaloy-4:

<u>Temperature, $^\circ\text{F}$</u>	<u>Conductivity, Btu/hr-ft-$^\circ\text{F}$</u>
600	8.69
650	8.88
700	9.08
750	9.28
800	9.49

A thermal conductivity of $8.09 \text{ Btu/hr-ft-}^\circ\text{F}$ was used herein for determining the temperature profile for the Zircaloy-2 cladding in the Chalk River test evaluation.

D. Temperature-Dependent Fuel Properties

The following cards, 3101 through 3509, describe the temperature-dependent properties of the fuel material. The properties on Card Series 3101 through 3401 correspond, term by term, to the temperatures on Card Series 3501. Therefore, each card series in this group must contain the same number of values that Card Series 3501 contains.

Cards 3101-3109 (or 5001)	Symbol α_{fi}	$^\circ\text{F}^{-1}$
---------------------------	----------------------	-----------------------

Coefficient of thermal expansion of fuel material.

†From data obtained by C. J. Guare and E. R. Pallard of Knolls Atomic Power Laboratory in a program for calculating thermal conductivity.

Recommended

For UO_2 fuel, $\alpha_{f_i} = 3.72 \times 10^{-6} + 1.79 \times 10^{-9} T_i$

For $\text{UO}_2 + \text{ThO}_2$ fuel, $\alpha_{f_i} = 4.87 \times 10^{-6} + 0.82 \times 10^{-9} T_i$

For ThO_2 fuel, $\alpha_{f_i} = 4.52 \times 10^{-6} + 0.69 \times 10^{-9} T_i$.

Comments

The values for thermal expansion coefficient depend on the fuel material and vary with the temperature listing on Card 3501 according to the equations above. Explicit values at 200°F intervals from 400 to 5000°F for UO_2 fuel are shown in the input listing of the sample problem. The expression for α_f of UO_2 fuel is derived from data in Reference 11 for measurements from room temperature to 4080°F. The expression for α_f of $\text{UO}_2 + \text{ThO}_2$ fuel is derived from data in Reference 12 for measurements from room temperature to 1832°F. The expression for α_f of ThO_2 fuel is derived from data in Reference 12 up to 1832°F and in Reference 13 up to 4500°F.

Cards 3201-3209 (or 5002)

Symbol E_{f_i}

psi

Young's modulus for fuel material at 100-percent theoretical density.

Recommended

$$E_{f_i} = 30.43 \times 10^6 - 1.376 \times 10^2 T_i - 1.239 T_i^2$$

Comments

Young's modulus is independent of fuel material. The above equation is derived from the following values from Reference 3, adjusted to 100-percent by the relation $E_{100} = E/(1 - 1.9P + 0.9P^2)$, where P is the porosity of the fuel.

<u>Temperature</u>	<u>E_f, 93% T.D.</u>	<u>E_f, 100% T.D.</u>
Room	26.5×10^6 psi	30.4×10^6 psi
1472°F	24.0×10^6 psi	27.5×10^6 psi
4900°F	0	0

Explicit values for E_f at 200°F intervals from 400 to 5000°F are shown in the input listing of the sample problem in Appendix A.

Ultimate strength of fuel material.

Recommended

14,000 psi for all temperatures.

Comments

This parameter is also called modulus of rupture, and is the circumferential tensile stress required for radial cracking to occur. Table 5.11 in Reference 3 lists a scattering of data points between 12,250 and 17,000 psi, with variations in fuel temperature, grain size, and irradiation history. There is no consistent correlation between ultimate strength and fuel temperature, so an average value of 14,000 psi is recommended. The value of σ_{ULT} has a significant effect on the results in some cases. For example, in the sample runs an increase in σ_{ULT} from 14,000 to 17,000 psi reduced the calculated strain by 20 percent in some cases and had no effect in other cases. When the plastic flow-stress is high enough to have no effect on the results, an increase in σ_{ULT} to 17,000 did not show up significantly in the results for any of the sample cases. The effect of σ_{ULT} on a particular fuel pellet can only be determined by using several values, each in a different STRIPE run.

Plastic flow-stress of fuel material.

Recommended

See Figure 7.

Comments

Plastic flow-stress is a function of the creep rate and temperature for a given fuel material. Figure 7 presents the variation in σ_{FLOW} with temperature, at several creep rates between 0.02 and 0.15 hr⁻¹, for UO₂ fuel. These curves are based on test data shown in Figure 4 of Reference 14 for unirradiated UO₂ of 55 μ grain size. Those test data provide three points for a log₁₀ σ_{FLOW} versus temperature curve at any plastic creep rate between 2 x 10⁻³ and 2 x 10⁻¹ hr⁻¹. The three points are at temperatures of 2912, 3272, and 3632°F. These three points, although very close together and representing only a small part of the 5000°F temperature range of interest, form a straight line on the log₁₀ σ_{FLOW} versus temperature curve. That

Figure 7. Plastic Flow-Stress as a Function of Fuel Temperature and Creep Rate for UO_2 Fuel

straight line is then extrapolated to the melting temperature, on the basis of findings in Reference 15 for hardness of pure metals. Here, Westbrook presents evidence that the logarithm of hardness varies linearly with absolute temperature above and below the Tamman temperature, which for UO_2 with a melting temperature of 5500°R , would be 2750°R , or about 2300°F .

The curves in Figure 7 were extended in a straight line below 2300°F for use with the sample problems because it was found that the STRIPE results were essentially the same for any slope on the curve below 2300°F . The results of STRIPE calculations show that the plastic flow effect is significant only in pellets with maximum temperatures above 2500°F . This can be seen in Figure 8, where the difference between the expansion calculated by STRIPE and the unrestrained thermal expansion is largely due to plastic flow near the center of the pellet. Other curves of $\log \sigma_{\text{FLOW}}$ versus temperature for 10μ grain size are available in Reference 4 for temperatures of 2600 , 2800 , and 3030°F .

Cards 3501-3509

Symbol T_{mf_i}

$^\circ\text{F}$

Temperatures for table of fuel properties.

Recommended

The number of temperature points needed to describe material properties is usually determined by the need for an accurate input of the plastic flow stress, σ_{FLOW} , which varies exponentially with temperature.

Comments

The range of temperatures listed on these cards should include the range of final temperatures listed on Cards 4101-4109. In the sample problems it was necessary to list fuel properties at 200°F intervals, from 400 to 5000°F , to define accurately the plastic flow stress and Young's modulus.

E. Temperature-Dependent Clad Properties

Cards 3601 through 3909 describe the temperature-dependent properties of the clad material. The properties on Card Series 3601 through 3801 correspond, term by term, to the temperatures on Card Series 3901. Therefore, each of the three card series in this group must contain the same number of values as Card Series 3901.

Figure 8. Thermal Expansion of UO_2 Cylindrical Fuel Elements - Comparison of Calculated and Measured Results

Coefficient of thermal expansion of clad material.

Recommended

For Zircaloy-4, annealed or cold-worked, the following values are recommended in Reference 14.

<u>Temperature, $^{\circ}\text{F}$</u>	<u>Thermal Expansion Coefficient</u>
0	$3.10 \times 10^{-6} \text{ }^{\circ}\text{F}^{-1}$
500	$3.60 \times 10^{-6} \text{ }^{\circ}\text{F}^{-1}$
1000	$4.10 \times 10^{-6} \text{ }^{\circ}\text{F}^{-1}$

Comments

Although the thermal expansion coefficient varies with temperature, a constant value of $3.5 \times 10^{-6} \text{ }^{\circ}\text{F}^{-1}$ was used in the evaluation and sample runs herein to facilitate separation of clad expansion due to thermal expansion from that due to elastic and plastic strains.

Young's modulus for clad material.

Recommended

The following are obtained from Reference 14:

<u>Temperature, $^{\circ}\text{F}$</u>	<u>Young's Modulus, psi</u>
0	14.2×10^6
500	11.6×10^6
1000	9.1×10^6

Comments

A constant value of 11.5×10^6 psi was used for the evaluation and sample runs to facilitate separation of clad expansion due to elastic expansion from that due to thermal and plastic expansion.

Yield strength of clad material.

Recommended

The clad yield strength depends on the composition heat treatment and fabrication history of the material as well as the temperature, current fast-flux level, strain rate, and total integrated fast flux.

Comments

The sensitivity of fuel expansion to clad yield strength is dependent on many factors. In general, the effect of clad restraint and therefore the sensitivity-to-clad strength is greater at higher temperature operation where the pellet is made plastic. For the 13 test elements considered in the evaluation of the STRIPE code, there was no effect on fuel expansion when clad yield strength was raised from 23,000 to 50,000 psi if the maximum fuel temperature was below 3000°F. Among the higher-temperature test elements, the greatest reduction in fuel expansion was approximately 20 percent for the same increase in yield strength.

Cards 3901-3909

Symbol T_{MC_i}

°F

Temperatures for table of clad properties.

Recommended

At least two temperature values are necessary, and usually three are sufficient since clad temperatures cover only several hundred degrees and the clad properties can be approximated adequately by two straight lines.

Comments

The temperatures on these cards should cover the range of clad temperatures listed on Cards 4201-4209.

VI. STRIPE OUTPUT

The STRIPE output format first prints an input edit of the first case only as a check on the input data.

The output is arranged to print each case on one line so that multi-cases may be conveniently displayed. Output data are aligned under the following headings:

1. Case number (NCASE)
2. Initial reactor power (fraction)

3. Final reactor power (fraction)
4. Circumferential stresses (σ_θ), psi, at
 - a. $r = a$ or ζ
 - b. $r = \eta$
 - c. $r = b$
 - d. $r = (b + c)/2$
 - e. $r = c$
5. Radial stress (P1) at $r = \eta$, psi
6. Converged solution of η (ETA), inch
7. A clad yield warning flag (yes or no)
8. Number of iterations (IT)
9. Radial stress (P2) at $r = b$, psi
10. Computer running time, seconds.

Typical computer running time is approximately 1 to 4 seconds per case on the CDC-6600 computer. Further output information printed in a separate block includes

1. Radial displacements (u), inch, at
 - a. $r = \eta$, denoted U. ETA
 - b. $r = b$, denoted U.B
 - c. $r = c$, denoted U.C
2. Radial stress (P0) at ζ
3. Converged solution of ζ (ZETA), inch
4. Number of iterations required in solving for ζ
5. Generalized plane strain solution:
 - a. Axial stress (σ_z), psi
 - b. Axial strain (ϵ_z), psi
 - c. Axial force (F_z), lb_f.

VII. COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS

A. Irradiation Experiments at Chalk River

The theoretical diametral expansions of 13 UO_2 fuel elements irradiated in the water-cooled loops of the NRX reactor at Chalk River were calculated using STRIPE and the results were compared with measured values. These 13 elements were tested over a wide range of temperatures up to a 4900°F peak center temperature. They were inserted into regions having different flux levels. The rod diameters ranged between 0.812 and 0.852 inch with clad thicknesses between

25 and 30 mils. The initial gaps were intentionally varied between 1.5 and 5.7 mils. The fuel density was 93 to 95 percent of theoretical density and the tests were run with a system pressure of 1400 psi. Dimensions and other parameters are listed in Table 1.

The fuel rods were irradiated for various times ranging between 16.7 efpd* ($\sim 0.13 \times 10^{20}$ f/cc) and 120 efpd ($\sim 0.8 \times 10^{20}$ f/cc) before second measurements were taken. At these low depletions, little fission-induced swelling would be anticipated. Therefore, the measured diametral expansions were due to differential thermal effects alone.

TABLE 1. TEST ELEMENT PARAMETERS

NRX Test No.	Element	Rod OD, in.	Clad Thick- ness, mils	Avg Gap (Dia), mils	Fuel T.D., %	Total Irradiation, efpd	Clad Surface, Flux $\times 10^3$ Btu/hr-ft ²
X211†	DFA	0.812	30	4	95	16.7	283
X211†	DFB	0.812	30	4	95	16.7	386
X211†	DFJ	0.812	30	4	95	16.7	381
X211†	DFK	0.812	30	4	95	16.7	281
X225‡	DFN	0.812	30	4	95	26.8	436
X2D††	LI	0.8485	25	2	93.4	115	146.6
X2D††	LJ	0.8527	25	5.75	94.3	115	200.8
X2D††	LL	0.8487	25	1.5	93.3	115	258.4
X2D††	LM	0.8506	25	4	94.6	122	258.3
X2D††	LO	0.8507	25	4	95.2	115	181.2
X2D††	LK	0.8520	25	4	94.5	43	245.8
X2D††	LN	0.8480	25	2	94.1	70	230.0
X2D††	LP	0.8506	25	1.75	94.0	28	250.4

†Information from Reference 2.

‡Information from Reference 9.

††Information from Reference 10.

B. Material Properties Used in Calculations

The fuel and clad properties used for this comparison were discussed in Section V and are listed in Table 2. In addition, studies were made to determine the sensitivity of the results to various material properties and design parameters.

*efpd = effective full power days.

The sensitivity of the results to the fuel temperature profile at full power was studied because the thermal conductivity of UO_2 is not accurately known at elevated temperatures. Some data show a continuing decrease in the conductivity of bulk oxide fuels up to melting temperature, whereas other data show an increase above 3200°F . The latter data were used in Reference 7 for Chalk River's analysis of their own test pellets, and were also used in the study report herein. The decision to use the Chalk River conductivity curve herein was based on comparative FIGRO results obtained using both conductivity curves. Elements DFB and DFJ had calculated peak center-line temperatures well above melting (4900°F) for UO_2 fuel when the continually decreasing conductivity was used. Since these pellets showed definite diameter increases, they must not have reached the melting temperature. Apparently, the Chalk River conductivity curve results in a closer representation of the actual temperature profile.

TABLE 2. SUMMARY OF FUEL AND CLAD PROPERTIES USED IN STRIPE CALCULATIONS OF THERMAL EXPANSION OF CHALK RIVER FUEL ELEMENTS

Fuel Properties

Card 3101, temperature coefficient of expansion:

$$\alpha_f = 3.72 \times 10^{-6} + 1.79 \times 10^{-9} T, \text{ in units of } ^\circ\text{F}.$$

Card 3201, Young's modulus:

$$E_f = 30.43 \times 10^6 - 1.376 \times 10^2 T - 1.239 T^2, T \text{ in } ^\circ\text{F}, E_f \text{ in psi}.$$

Card 3301, ultimate strength:

$$\sigma_{\text{ULT}} = 14,000 \text{ psi}.$$

Card 3401, plastic flow-stress:

$$\log_{10} \sigma_{\text{flow}} = \log \sigma_{\text{FLOW}}(0^\circ\text{F}) - mT, T \text{ in } ^\circ\text{F}, \text{ and values of } \log \sigma_{\text{FLOW}} \text{ are obtained from Figure 7.}$$

Clad Properties

Card 3601, temperature coefficient of expansion:

$$\alpha_c = 3.5 \times 10^{-6} ^\circ\text{F}^{-1}.$$

Card 3701, Young's modulus:

$$E_c = 11.5 \times 10^6 \text{ psi}.$$

TABLE 2. (Cont)

Card 3801, flow strength:

σ_{FLOW} = varied from 23,000 psi for the low irradiation of the X211 and X225 elements, to 50,000 psi for the moderately irradiated (0.4×10^{21} nvt) X2S elements.

Temperature Profiles

Card 4101, fuel temperature profile:

obtained from printout of FIGRO runs using fuel conductivity from Figure 4 in Reference 7.

Card 4201, clad temperature profile:

straight-line interpolation between the inner and outer clad surface temperatures printed in FIGRO output.

Since the plastic flow stress is a function of the plastic strain rate, an iterative process was used to estimate the plastic strain rate of each of the 13 Chalk River pellets. No more than 3 STRIPE runs were necessary for convergence on the proper plastic flow rate for any of the 13 pellets. Flow rate was determined using the calculated radial expansion and assuming that the plastic-flow occurred during a 30-minute rise in pellet flux from a no-load condition to full power. Values of plastic strain rate thus calculated for the 13 pellets ranged from 0.022 to 0.094 per hour. The appropriate flow-stress curve from Figure 7 was then used in the final STRIPE iteration for each pellet.

The curves of Figure 7 are plotted from several experimental sources. Reference 4 provides data for plastic creep of UO_2 fuel at low temperatures and Figure 4 of Reference 14 provides plastic creep for fuel temperatures between 2900 and 3600°F. In Reference 15, Westbrook presents evidence that the curve of plastic flow-stress versus temperature is made up of two straight lines joining at a dividing temperature equal to one-half the melting temperature. For UO_2 , the dividing temperature is 2400°F. Since a straight line through the three data points at low temperatures intersects at the dividing temperature with a straight line through the high-temperature data for a creep rate of 0.060 per hour, and since 0.060 per hour lies in the range of appropriate creep rates for the Chalk River pellets, these two lines are used as the representative flow-stress curve. As shown in Figure 7, the straight lines above and below the dividing temperature have identical slopes and therefore form a single straight line over the entire temperature range.

C. Comparison of Calculated and Measured Results

STRIPE calculates the total diametral expansion of the fuel element in-pile. This value was obtained by Notley, et al (Reference 7) from the Chalk River test measurements by adding:

1. the measured permanent deformation of the cladding obtained in post-irradiation examinations,
2. the original pellet-clad gap dimension,
3. the calculated elastic unloading strains in the cladding, and
4. the calculated thermal expansion of the cladding between the in-pile temperature and the temperature existing during the post-irradiation measurements.

The circumferential strain was obtained by dividing the radial pellet expansion by the radius of the pellet.

To evaluate the effects of pellet cracking, two sets of calculated results were obtained. One set was obtained by including the effects of pellet cracking as described previously herein. Another set of values was calculated by setting a very high value on the tensile fracture strength of the pellets so that effectively there was no cracking. These calculated results are compared with the measured results in Table 3 and are plotted as a function of the peak fuel temperature in Figure 8. The unrestrained thermal expansion of the fuel pellets,

$$\frac{1}{b} \int_0^b \left[\alpha_F^F(r) T_F^F(r) - \alpha_F^I(r) T_F^I(r) \right] dr,$$

is also included in Table 3 and Figure 8 for comparison. This is the expansion that would occur if the pellet were completely cracked radially, provided there was no clad restraint. The STRIPE calculations showed that pellet cracking occurred in all 13 fuel rods tested. The radial depth of this cracking varied from 29 to 177 mils according to these calculations.

The triangular values Δ in Figure 8 were calculated without considering pellet cracking. These values are much lower than the experimental values (indicated by x) over the entire temperature range. This is also evident in Table 3. It means that simplified analyses which ignore pellet cracking give unconservative results, that is, such analyses seriously underestimate the deformations imposed on the cladding and the rod support springs by the expansion of the fuel.

TABLE 3. COMPARISON OF CALCULATED AND
MEASURED FUEL EXPANSION*

Element	Peak Center-Line Temperature, °F [†]	FUEL EXPANSION, % OF CLAD DIAMETER			
		Measured, % [‡]	CALCULATED		
			STRIPE with Cracking, %	STRIPE without Cracking, %	Unrestrained, %**
DFA	3550	1.26	1.61	0.92	2.11
DFB	4490	1.81	1.76	0.97	3.05
DFJ	4410	1.77	1.77	0.93	2.94
DFK	3460	1.28	1.55	0.85	2.01
DFN	4880	2.39	2.10	1.05	3.54
LI	2020	0.97	0.94	0.66	0.93
LJ	2690	1.10	1.31	0.87	1.39
LL	3375	0.87	1.38	0.73	1.96
LM	3320	1.32	1.45	0.84	1.91
LO	2420	1.12	1.18	0.78	1.20
LK	3170	1.12	1.51	0.85	1.78
LN	3010	1.25	1.44	0.77	1.64
LP	3230	1.06	1.60	0.74	1.83

*The listed fuel expansions are plotted against the peak center-line temperature in Figure 8.

†Calculated by FIGRO program, Reference 6, using the properties listed in Tables 1 and 2.

‡As listed in Reference 7.

**"Unrestrained" means the thermal expansion that would occur if there were complete radial cracking of the pellet and no clad restraint.

According to the calculations made herein, pellet cracking resulted in an increase in the peripheral expansion of the pellet ranging from 42 percent at low temperature to 116 percent at high temperatures. Part of the pellet expansion is accommodated by the stress-free thermal expansion of the cladding; therefore, it does not impose a structural burden on the cladding. Subtracting the free thermal expansion of the cladding, the remaining expansion must be accommodated by the pellet-to-clad gap and by stretching the cladding.

This remaining portion of the pellet expansion was increased by amounts ranging from 61 to 156 percent due to cracking of the pellets in the 13 cases tested. If the elastic portions of the clad deformations are also omitted as being recoverable and producing little fatigue damage, the remaining calculated plastic clad burdens were 2 to 3.5 times the magnitudes calculated when pellet cracking was neglected.

The results of Table 3 and Figure 8 also show that the measured pellet expansions are very close to the calculated unrestrained values (denoted by the symbol + in Figure 8) at very low temperatures. Recall that the so-called unrestrained expansion is that which would occur if there were no hoop constraints from the cooler periphery of the pellet and the cladding. However, when the peak-pellet center-line temperature exceeds 2500°F, compressive plastic flow near the center of the pellet begins to reduce the peripheral expansion below that which would otherwise occur. This plastic flow is caused by the restraining effect of the cooler periphery of the pellet and clad, and by the system pressure.

A comparison of the calculated (STRIPE) values (indicated by the symbol 0 in Figure 8) which include all of these effects with the measured values shows reasonably good agreement over the entire temperature range. The calculated values appear to be somewhat conservatively high at intermediate temperatures. This appears to be due to the use of too high a strain-rate-dependent compressive flow stress for the fuel.

VIII. CONCLUSIONS

Pellet expansions during up-power transients are seriously underestimated when the radial cracking of the pellet is not included in the calculations. Cracking may more than double the expansion of the pellet. The effects of this additional swelling must be included in the analysis of the structural integrity of the fuel rod cladding and support springs in order to avoid premature failures.

The effect of plastic flow near the inner core of the fuel pellets and the constraining effect of the system pressure and cladding also become significant at elevated temperatures. These effects should be considered when the peak temperature at the center of the pellet exceeds 2500°F.

All of these effects are included in the method of analysis and STRIPE digital computer program described herein. Moreover, this analysis gives results which agree reasonably well with experimental results over the entire temperature range of interest as shown in Figure 8.

APPENDIX A

INPUT AND OUTPUT FOR A SAMPLE PROBLEM

The following pages illustrate the input recommended in Sections V and VII and the corresponding STRIPE output for the 13 UO_2 fuel elements examined from the Chalk River tests. The comparison of the STRIPE predictions with these test data was discussed in Section VII. Figure A-1 shows a listing of the input cards for the first fuel element. This is followed by an input edit for cross-checking the input data (Figure A-2). The output edit appears on Figures A-3 and A-4 with the converged solutions for all 13 cases (fuel elements) conveniently displayed in a compact format of one case per line. Calculated circumferential stresses (SIGMA THETA) and radial stresses appear on Figure A-3. Radial displacements are shown on Figure A-4. Total computer running time per case is approximately 20 to 30 seconds.

```

= SAMPLE PROBLEM FOR WAPD-TM-652 STRIPE INPUT CHALK RIVER TESTS
* CHALK RIVER TESTS X211, X225, X25
* X211 TEST ELEMENT DFA CREEP RATE 0.022 PER HR
0001 1001,1,100,20,2
0002 2001,1400.
0003 2002,0.,1.
0004 2101,70.,70.,70.,70.,70.,70.,70.,70.,70.,70.
0005 2201,70.,70.,70.,70.,70.,70.
0006 3001,0.,,374.,406.,002
0007 3002,.3.,.95.,.3
0008 3101,405-5,479-5,515-5,551-5,587-5,623-5,658-5,694-5
0009 3102,730-5,766-5,802-5,837-5,873-5,909-5,945-5,981-5
0010 3103,1012-4,1052-4,1088-4,1124-4,1160-4,1195-4,1231-4,1267-4
0011 3201,300+8,297+8,293+8,289+8,283+8,277+8,270+8,262+8
0012 3202,253+8,243+8,230+8,218+8,203+8,188+8,172+8,155+8
0013 3203,137+8,118+8,100+8,081+8,063+8,046+8,027+8,009+8
0014 3301,14+5,14+5,14+5,14+5,14+5,14+5,14+5,14+5
0015 3302,14+5,14+5,14+5,14+5,14+5,14+5,14+5,14+5
0016 3303,14+5,14+5,14+5,14+5,14+5,14+5,14+5,14+5
0017 3401,661+5,537+5,436+5,355+5,288+5,229+5,191+5,151+5
0018 3402,125+5,100+5,822+4,661+4,537+4,436+4,355+4,288+4
0019 3403,234+4,191+4,155+4,126+4,102+4,832+3,675+3,525+3
0020 3501,4+3,6+3,8+3,1+4,12+4,14+4,16+4,18+4
0021 3502,2+4,22+4,24+4,26+4,28+4,3+4,32+4,34+4
0022 3503,36+4,38+4,4+4,42+4,44+4,46+4,48+4,5+4
0023 3601,3.5-6,3.5-6
0024 3701,11.5+6,11.5+6
0025 3801,23.+3,23.+3
0026 3901,500.,1000.
0027 4101,3552.,3527.,3451.,3322.,3138.,2893.,2582.,2198.,1737.,1265.,816.
0028 4201,659.,641.,623.,605.,587.,568.

```

Figure A-1. Input Data for the First Fuel Element in Chalk River Tests

STRIPE - STRESS IN PELLETS (FUEL RODS) BETTIS 01/30/69

INPUT EDIT

1. CONTROL INFORMATION

NTOP = 1 NFUEL = 100 NCLAD = 20 NAXOP = 2

2. INITIAL CONDITIONS

P-EXT = 1400.00

INITIAL RADIAL TEMPERATURE DISTRIBUTION OF FUEL

70.00	70.00	70.00	70.00	70.00	70.00	70.00	70.00	70.00	70.00
70.00									

INITIAL RADIAL TEMPERATURE DISTRIBUTION OF CLAD

70.00	70.00	70.00	70.00	70.00	70.00
-------	-------	-------	-------	-------	-------

3. SYSTEM PARAMETERS

A. GEOMETRY

A = 0. B = .3740 C = .4060 GAP = .0020

B. CONSTANT MATERIAL PROPERTIES

NU.F = .300 RHO.F = .950 NU.C = .300

C. TEMPERATURE DEPENDENT MATERIAL PROPERTIES

(1) FUEL MATERIAL PROPERTIES

ALPHA.F	405000-05	479000-05	515000-05	551000-05	587000-05	623000-05	658000-05	694000-05	730000-05	766000-05
	802000-05	837000-05	873000-05	909000-05	945000-05	981000-05	101200-04	105200-04	108800-04	112400-04
	116000-04	119500-04	123100-04	126700-04						
E.FUEL	300000+08	297000+08	293000+08	289000+08	283000+08	277000+08	270000+08	262000+08	253000+08	243000+08
	230000+08	218000+08	203000+08	188000+08	172000+08	155000+08	137000+08	118000+08	100000+08	810000+07
	630000+07	460000+07	270000+07	900000+06						
S.ULT	140000+05	140000+05	140000+05	140000+05	140000+05	140000+05	140000+05	140000+05	140000+05	140000+05
	140000+05	140000+05	140000+05	140000+05	140000+05	140000+05	140000+05	140000+05	140000+05	140000+05
	140000+05	140000+05	140000+05	140000+05						
S.FLOW	661000+05	537000+05	436000+05	355000+05	288000+05	229000+05	191000+05	151000+05	125000+05	100000+05
	822000+04	661000+04	537000+04	436000+04	355000+04	288000+04	234000+04	191000+04	155000+04	126000+04
	102000+04	832000+03	675000+03	525000+03						
TEMP.F	400.00	600.00	800.00	1000.00	1200.00	1400.00	1600.00	1800.00	2000.00	2200.00
	2400.00	2600.00	2800.00	3000.00	3200.00	3400.00	3600.00	3800.00	4000.00	4200.00
	4400.00	4600.00	4800.00	5000.00						

Figure A-2. Input Edit for Cross-Checking Input Data (Sheet 1 of 2)

(2) CLAD MATERIAL PROPERTIES

ALPHA.C 350000-05 350000-05
 E.CLAD 115000+08 115000+08
 S.YIELD 230000+05 230000+05
 TEMP.C 500.00 1000.00

4. FINAL CONDITIONS

FINAL RADIAL TEMPERATURE DISTRIBUTION OF FUEL

3552.00 3527.00 3451.00 3322.00 3138.00 2893.00 2582.00 2198.00 1737.00 1265.00
 816.00

FINAL RADIAL TEMPERATURE DISTRIBUTION OF CLAD

659.00 641.00 623.00 605.00 587.00 568.00

Figure A-2. Input Edit for Cross-Checking Input Data (Sheet 2 of 2)

OUTPUT EDIT													
NCASE	--- POWER ---		-----		SIGMA.THETA	-----		P1	ETA	CLAD	IT	P2	COMPUTER
	INITIAL	FINAL	A(ZETA)	ETA	B	MID	C			YIELD			TIME
1	0.	1.000	-120176+05	140000+05	222959+05	228097+05	237776+05	4531.5	.2879	YES	20	3488.0	30.49 SEC
2	0.	1.000	-177847+05	140000+05	216143+05	227582+05	243935+05	4026.9	.3240	YES	20	3488.6	34.17 SEC
3	0.	1.000	-178763+05	140000+05	216745+05	227695+05	243753+05	4106.6	.3178	YES	20	3489.0	33.45 SEC
4	0.	1.000	-131620+05	140000+05	243961+05	248187+05	256684+05	4848.1	.2823	YES	20	3659.7	28.26 SEC
5	0.	1.000	-368950+04	140000+05	213035+05	227183+05	247097+05	3780.6	.3450	YES	20	3487.9	36.27 SEC
6	0.	1.000	-130458+05	140000+05	510888+05	500336+05	492544+05	10389.6	.1826	YES	20	4763.9	8.99 SEC
7	0.	1.000	-133125+05	140000+05	508027+05	498602+05	492875+05	7764.8	.2564	YES	20	4995.6	18.08 SEC
8	0.	1.000	-166093+05	140000+05	503687+05	498442+05	497167+05	5864.5	.3209	YES	20	4721.1	28.64 SEC
9	0.	1.000	-127631+05	140000+05	504497+05	498367+05	496342+05	6097.8	.3191	YES	20	4884.5	27.41 SEC
10	0.	1.000	-126752+05	140000+05	509121+05	499383+05	493059+05	8781.4	.2217	YES	20	4887.9	15.43 SEC
11	0.	1.000	-111034+05	140000+05	350170+05	348897+05	351024+05	5558.2	.2775	YES	20	3865.4	24.30 SEC
12	0.	1.000	-127548+05	140000+05	402433+05	399113+05	399301+05	6137.1	.2664	YES	20	4108.1	23.33 SEC
-13	0.	1.000	-879643+04	140000+05	297642+05	299040+05	303567+05	5099.9	.2691	YES	20	3436.3	24.30 SEC

Figure A-3. Output Edit for Calculated Circumferential Stresses (SIGMA THETA)
and Radial Stresses

NCASE	RADIAL DISPLACEMENT			P0	ZETA	IT.Z	SIGMA.Z	EPS.Z	F.Z
	U.ETA	U.B	U.C						
1	.00583	.00654	.00454	4790.1	.2821	70	000000+00	0.	000000+00
2	.00672	.00713	.00513	4201.1	.3197	82	000000+00	0.	000000+00
3	.00669	.00716	.00516	4280.2	.3134	80	000000+00	0.	000000+00
4	.00558	.00631	.00431	5115.6	.2760	68	000000+00	0.	000000+00
5	.00831	.00853	.00653	4032.3	.3437	90	000000+00	0.	000000+00
6	.00248	.00399	.00299	13241.0	.1477	22	000000+00	0.	000000+00
7	.00453	.00557	.00269	8393.9	.2434	51	000000+00	0.	000000+00
8	.00537	.00587	.00512	6172.2	.3114	73	000000+00	0.	000000+00
9	.00563	.00614	.00414	6458.2	.3111	73	000000+00	0.	000000+00
10	.00371	.00504	.00304	9824.4	.2045	39	000000+00	0.	000000+00
11	.00548	.00643	.00443	5940.2	.2693	60	000000+00	0.	000000+00
12	.00510	.00612	.00512	6569.4	.2565	56	000000+00	0.	000000+00
13	.00571	.00680	.00592	5475.0	.2622	58	000000+00	0.	000000+00

END OF OUTPUT

Figure A-4. Output Edit for Radial Displacement

APPENDIX B

COMPUTER PROGRAM ABSTRACT

1. Program Name (and Title): M0650 (STRIPE), Cracking in Pellets and Clad Strains.
2. Computer and Language(s): CDC-6600, FORTRAN IV.
3. Problem Solved: This program examines the deformations imposed on the cladding of cylindrical fuel elements due to differential thermal expansions during abrupt power increases. The title is taken from STresses and cracking In PEllets. The calculational model includes effects of pellet cracking, the constraint of system pressure and cladding, compressive plastic flow in the fuel and the temperature dependence of material properties. Special features of the program include allowance for clad yielding and the selection of several possible axial restraint conditions.
4. Approximations and Method of Solution: Thick walled integral solutions are used for various regions of the fuel pellets and cladding. Trapezoidal integration is used in finite difference equations to approximate these integrals that result in describing differential thermal expansion. The method of bisection iteration scheme is used to solve a pair of nonlinear algebraic-integral equations to determine the depth of cracking.
5. Restrictions on Complexity of Problem: The total number of radial sections permitted for the computing grid is 100 in the fuel and 50 in the clad. Elastic-perfectly plastic properties are used for both the pellet and clad.
6. Relationship to Other Programs: Essential temperature profiles may be obtained from output of the digital computer program FIGRO, which solves the radial heat conduction problem for cylindrical fuel elements, using a temperature varying thermal conductivity for oxide fuels.
7. Estimated Typical Running Time: The running time is typically about 5 seconds per case. If plastic flow or clad yielding calculations are required, the computer time increases to about 30 seconds per case.
8. References:
 - (a) W. J. O'Donnell, W. G. Clarke, and W. R. Campbell "An Analysis of Transient Clad Strains in Cylindrical Fuel Elements Including the Effects of Oxide Pellet Cracking (STRIPE)," WAPD-TM-652, February 1970.
 - (b) C. J. Pfeifer, "CDC-6600 FORTRAN Programming Bettis Environmental Report," WAPD-TM-668, January 1967.
9. Unusual Features of the Program: The program determines the depth of radial cracking and its effects on pellet expansion. It accounts for a plastic inner zone in the fuel pellet as well as yielding of the cladding. Several axial restraint options such as plane strain or plane stress are available.

10. Status: The program is in production.
11. Machine Requirements: A FORTRAN IV compiler and 45,000 octal locations of central memory.
12. Operating System or Monitor Under Which Program is Executed: Bettis SCOPE 3.1 Operating System.
13. Name and Establishment of Authors:

W. J. O'Donnell (Analyst)
KPA Computer Techniques, Inc.
Lincoln Bank Bldg.
Large, Pa. 15025

W. G. Clarke (Programmer)
W. R. Campbell (Analyst)
Westinghouse Electric Corp.
Bettis Atomic Power Laboratory
P. O. Box 79
West Mifflin, Pa. 15122

Originated: February 1970

REFERENCES

1. W. J. O'Donnell, "Fracture of Cylindrical Fuel Rod Cladding Due to Plastic Instability," WAPD-TM-651, April 1967.
2. M. J. F. Notley, A. S. Bain, S. Ananthakrishnan, and G. W. Parry, "Zircaloy Sheathed UO_2 Fuel Elements Irradiated at Values of $\int k d\theta$ Between 40 and 83 W/cm," CRFD-117, December 1962.
3. J. Belle, Ed., "Uranium Dioxide: Properties and Nuclear Applications," p. 201, U. S. Government Printing Office, Washington, D. C., (1961).
4. R. A. Wolfe and S. F. Kaufman, "Mechanical Properties of Oxide Fuels (LSBR/LWB Development Program)," WAPD-TM-587, October 1967.
5. C. J. Pfeifer, "CDC-6600 FORTRAN Programming-Bettis Environmental Report," WAPD-TM-668, January 1967.
6. I. Goldberg, L. L. Lynn, and C. D. Sphar, "FIGRO - A FORTRAN-IV Digital Computer Program for the Analysis of Fuel Swelling and Calculation of Temperature in Bulk-Oxide Cylindrical Fuel Elements," WAPD-TM-618, December 1966 and Addendum 1, October 1967.
7. M. J. F. Notley, A. S. Bain, and J. A. L. Robertson, "The Longitudinal and Diametral Expansions of UO_2 Fuel Elements," AECL-2143, November 1964.
8. J. Belle, R. M. Berman, W. F. Bourgeois, I. Cohen, and R. C. Daniel, "Thermal Conductivity of Bulk Oxide Fuels," WAPD-TM-586 (Revised), April 1967.
9. M. J. F. Notley, "The Thermal Conductivity of Columnar Grains in Irradiated UO_2 Fuel Elements (Irradiation Tests X-255 and X-227)," CRFD-1166, July 1963.
10. R. D. MacDonald and A. S. Bain, "Irradiated of Zircaloy-2 Clad UO_2 to Study Sheath Deformation," EXP-NRX-2807, December, 1962.
11. J. B. Conway, R. M. Fincel, Jr., and R. A. Hein, "The Thermal Expansion and Heat Capacity of UO_2 to 2200C," Trans. Am. Nucl. Soc. 6, 153 (1963).
12. C. P. Kempter and R. O. Elliot, "Thermal Expansion of (UN), (UO_2), ($\text{UO}_2 + \text{ThO}_2$), and (ThO_2)," J. Chem. Phys. 30, 1524-1526, (1959).
13. B. Ohnysty and F. K. Rose, "Thermal Expansion Measurements of Thoria and Hafnia to 4500°F," J. Am. Ceram. Soc. 47, 398-400, (1964).

14. E. Duncombe, J. E. Meyer, and W. A. Coffman, "Comparisons with Experiment of Calculated Dimensional Changes and Failure Analysis of Irradiated Bulk Oxide Fuel Test Rods Using the CYGRO-1 Computer Program," WAPD-TM-583, September 1966.
15. J. H. Westbrook, "Temperature Dependence of the Hardness of Pure Metals," Trans. Am. Soc. Metals 45, 221-243 (1953); Discussion, Trans. Am. Soc. Metals 45, 243-248 (1953).
16. W. J. O'Donnell and B. F. Langer, "Design of Perforated Plates," Trans. Am. Soc. Mech. Engrs. Series B 84, 307-320 (1962).