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PHYSICAL LIMITATIONS ON THE MEASUREMENT
OF TRANSIENT FIELDS IN AIR
AND IN DISSIPATIVE MEDIA
USING ELECTRIC AND MAGNETIC PROBES

by

C W Harrison, Jr

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SUMMARY

The properties of electric and magnetic probes for the measurement of transient electromagnetic fields in air and in dissipative media are discussed briefly.

It is shown that the effective height of an electrically small loop is independent of the ambient medium. This is also virtually true for a thin electrically short dipole (or monopole).

If the open-circuit voltage of a magnetic probe can be measured accurately, it is possible (in principle) to reconstruct the time history of the incident magnetic field, even if the loop is immersed in dissipative media of unknown characteristics. The time function of the open-circuit voltage of an electric probe is essentially a replica of the time history of the incident electric field.

In some schemes, the probes are lumped impedance loaded, and the voltage drop across the load impedances is measured. The source impedances of the probes are then involved in the equivalent circuits of the receiving antennas, and the leading terms in the expressions for these impedances depend on the properties of the environment.

If an electrically short monopole is base-loaded by a capacitor divider, the voltage wave appearing across any capacitor is a faithful reproduction of the time sequence of the incident electric field provided the measurement is made in air or other dielectric.

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PHYSICAL LIMITATIONS ON THE MEASUREMENT OF TRANSIENT FIELDS IN AIR AND IN DISSIPATIVE MEDIA USING ELECTRIC AND MAGNETIC PROBES

Introduction

By definition, a magnetic probe is a loop antenna, and an electric probe is a dipole antenna (or monopole erected vertically with respect to a highly conducting ground screen). Both are dimensionally small in terms of the wavelength of the highest significant frequency contained in the incident electromagnetic pulse field. This study was undertaken to determine the elementary properties of these devices as related to their use in the measurement of transient electromagnetic fields in air and in conducting media.

The Effective Height of a Magnetic Probe in Air and in Dissipative Media

The voltage induced in a single-turn, perfectly conducting loop that is dimensionally small in terms of the wavelength of the highest significant frequency contained in the electromagnetic field pulse is readily found using Faraday's Law

$$\oint \mathbf{e}(t) \cdot d\mathbf{l} = - \frac{\partial}{\partial t} \int_A \mathbf{b}(t) \cdot d\mathbf{a} \quad (1)$$

where $\mathbf{e}(t)$ and $\mathbf{b}(t)$ are the vector time functions of the electric and magnetic fields, respectively. For time dependence $\exp(j\omega t)$, the steady-state induced voltage is

$$V_{oc}(f) = -j\omega AB(f). \quad (2)$$

Here,

$B(f)$ is the magnetic field in an isotropic medium characterized by μ , ϵ , and σ , the absolute permeability, dielectric constant, and conductivity, respectively.

A is the area of the loop (whether it be an isolated loop or a half-loop over a perfectly conducting ground screen).

The effective height of the loop is the ratio $V_{oc}(f)/B(f)$. Thus,

$$h_e(f) [\text{magnetic probe}] = -j\omega A. \quad (3)$$

Note that Equation (3) is independent of the medium in which the loop is immersed. For example, the environment might consist of a plasma which behaves like a conducting dielectric. Also, Equation (3) is valid whether the loop consists of bare or insulated conductor.

The Effective Height of an Electric Probe in Air and in Dissipative Media

When the incident electric field is directed parallel to the axis of a dipole of half-length h and radius a , the effective height, $2h_e(f)$, of the receiving antenna may be calculated from the relation

$$2h_e(f) = \frac{1}{I_z(0)} \int_{-h}^h I_z(z') dz'. \quad (4)$$

In Equation (4), $I_z(z)$ is the current along the structure when it is center driven. It is given by the relation¹

$$I_z(z) = \frac{j2\pi khV}{\xi\Psi_{dl}} \left\{ \left(1 + \frac{k^2 h^2}{3}\right) \left(1 - \frac{|z|}{h}\right) + \frac{(3 \ln 2 - 1)k^2 h^2 - jk^3 h^3}{3(\Omega - 3)} \left(1 - \frac{z^2}{h^2}\right) \right\}, \quad (5)$$

where

$$k = \beta - j\alpha = \omega\sqrt{\mu\epsilon} [f(p) - jg(p)], \quad (6)$$

$$\left. \begin{aligned} f(p) &= \cosh\left(\frac{1}{2} \sinh^{-1} p\right) \\ g(p) &= \sinh\left(\frac{1}{2} \sinh^{-1} p\right) \\ p &= \frac{\sigma}{\omega\epsilon} \end{aligned} \right\}, \quad (7)$$

$$\Omega = 2 \ln \frac{2h}{a}. \quad (8)$$

Equation (5) is valid provided $\beta h < 1$, $\alpha h < 1$. Accordingly,

$$\begin{aligned} 2h_e(f) &= \int_{-h}^h \left\{ \frac{3(\Omega - 3) \left(1 + \frac{k^2 h^2}{3}\right) \left(1 - \frac{|z'|}{h}\right) + [(3 \ln 2 - 1)k^2 h^2 - jk^3 h^3] \left(1 - \frac{z'^2}{h^2}\right)}{3(\Omega - 3) \left(1 + \frac{k^2 h^2}{3}\right) + (3 \ln 2 - 1)k^2 h^2 - jk^3 h^3} \right\} dz' \\ &= \frac{3(\Omega - 3) \left(1 + \frac{k^2 h^2}{3}\right) h + [(3 \ln 2 - 1)k^2 h^2 - jk^3 h^3] \frac{4}{3} h}{3(\Omega - 3) \left(1 + \frac{k^2 h^2}{3}\right) + (3 \ln 2 - 1)k^2 h^2 - jk^3 h^3} \approx h. \end{aligned} \quad (9)$$

The effective height of a monopole is, therefore,

$$h_e(f) [\text{monopole}] = \frac{h}{2}. \quad (10)$$

Thus, the effective height of the electric probe, i.e., the ratio $V_{oc}(f)/E(f)$, where $E(f)$ is the incident electric field, is virtually independent of the ambient medium, especially if the probe is thin. In this case, Ω is large. For a magnetic probe, the ratio $V_{oc}(f)/B(f)$ is completely independent of the environment, as explained earlier.

The Evaluation of Transient Magnetic and Electric Fields in Air and Conducting Media by Means of Open-Circuit Voltage Measurements on Magnetic and Electric Probes

Let the open-circuit voltage $v_{oc}(t)$ of a magnetic probe be measured, using a high-impedance cathode follower and oscilloscope. It should be possible to make this measurement with fair accuracy since the source impedance of the loop is low (in air). Then,

$$V_{oc}(f) = \int_{-\infty}^{\infty} v_{oc}(t) e^{-j2\pi ft} dt. \quad (11)$$

Since

$$B(f) = \frac{V_{oc}(f)}{-j\omega A}, \quad (12)$$

it follows that

$$b(t) = -\frac{1}{A} \int_{-\infty}^t v_{oc}(t) dt, \quad (13)$$

where $b(t)$ is the time sequence of the magnetic field. Alternatively,

$$b(t) = \int_{-\infty}^{\infty} \frac{jV_{oc}(f)}{\omega A} e^{j2\pi ft} df. \quad (14)$$

In principle, the Fourier transforms (11) and (14), or the integral (13), may be evaluated by use of a digital computer.

To obtain $b(t)$ --the time history of the magnetic field--from a knowledge of $v_{oc}(t)$, the writer favors following through with steps (11), (12), and (14) because the spectrum of the magnetic field is available from (12). Knowledge of the spectrum permits one to ascertain if the loop dimension was indeed small in terms of the wavelength of the highest significant frequency contained in the pulse.

It can be demonstrated that the measured value of $v_{oc}(t)$ is not independent of the dipole mode existing in the loop. This source of error can be minimized by keeping the loop small.

The accurate measurement of $v_{oc}(t)$ for a monopole is not a simple matter. This is because the monopole source impedance is high (in air) and, at high frequencies, cathode followers present significant loads.

Since the voltage induced in a monopole is

$$V_{oc}(f) = -h_e(f)E(f), \quad (15)$$

and $h_e(f) = h/2$, it follows that

$$v_{oc}(t) = -\frac{h}{2} \int_{-\infty}^{\infty} E(f)e^{j2\pi ft} df = -\frac{h}{2} e(t). \quad (16)$$

This relation shows that the time history of the open-circuit voltage of a short monopole is a replica of the time sequence of the incident electric field.

If the magnitude of the voltage $v_{oc}(t)$, as delivered by either an electric or magnetic probe, is large, the electronics measuring circuits may be overdriven. In practice, it may be difficult to attenuate the signal because most schemes to achieve this involve in one way or another the source impedance of the probe. The source impedance of either antenna is very much a function of the properties of the ambient medium, as will be discussed in the following section. The characteristics of the medium may never be known; indeed, these may be functions of time during the course of the measurement.

The Use of Impedance-Loaded Probes in the Measurement of Transient Electromagnetic Fields in Air and in Conducting Media

It might be suggested that the magnetic field $b(t)$ may be derived from a measurement of (a) the short-circuit current in the loop or (b) the voltage drop across a series load impedance. In the frequency domain (a) leads to the relation

$$I_{sc}(f) = \frac{V_{oc}(f)}{Z_o(f)}, \quad (17)$$

and (b) to the expression

$$V_L(f) = \frac{V_{oc}(f)Z_L(f)}{Z_o(f) + Z_L(f)}. \quad (18)$$

Here, $Z_o(f)$ is the source impedance, i. e., the driving point impedance of the loop if used for transmission, and $Z_L(f)$ is the load impedance. It has been shown by Chen² that even the leading

term in the formula for the self-impedance of a small loop in a dissipative medium depends on the properties of the medium. Since the degree of ionization may be difficult to ascertain at the time of measurement, it appears that determination of $b(t)$ by measuring the short-circuit current $i_{sc}(t)$ or voltage drop across a load impedance $v_L(t)$ in a loop is ruled out under this circumstance.

In measuring electric fields in air, it is customary to employ a monopole probe base-loaded by a capacitive attenuator or divider. The voltage developed across one of the capacitors is measured, using a cathode follower and oscilloscope. For simplicity, suppose the load is a single capacitor, C_L . The voltage across the capacitor is

$$V_L(f) = - \frac{h_e(f)E(f)Z_L(f)}{Z_L(f) + Z_o(f)} \quad (19)$$

King³ has shown that for an electrically short monopole in air normal to a perfectly conducting ground screen,

$$Z_o(f) \approx jX_o(f) = -j \frac{\xi_o}{4\pi\beta h} (\Omega - 2 - 2 \ln 2) = -j \frac{K_1}{\beta}, \quad (20)$$

and

$$h_e(f) = h_e = \frac{h}{2} \left(\frac{\Omega - 1}{\Omega - 2 + \ln 4} \right) = K_2. \quad (21)$$

Since

$$Z_L(f) = - \frac{j}{2\pi f C_L}, \quad (22)$$

it follows from (20)-(22) that

$$V_L(f) = - \frac{K_2}{c C_L K_1 + 1} E(f), \quad (23)$$

where c is the velocity of light.

Hence,

$$e(t) = - \int_{-\infty}^{\infty} \frac{(c C_L K_1 + 1)}{K_2} V_L(f) e^{j2\pi f t} df = - \frac{c C_L K_1 + 1}{K_2} v_L(t). \quad (24)$$

Thus, the time history of the voltage across the capacitor is a replica of the time history of the incident field $e(t)$, assuming the measurement is made in the air.

In dissipative media, the leading term in $Z_o(f)$, which appears in (19), depends on the properties of the medium, as can be seen by forming the ratio $V/I_z(0)$, using (5).

Concluding Remarks

In summary, it appears that the measurement of the open-circuit voltage of a magnetic probe would permit determination of $b(t)$, and of an electric probe $e(t)$. The loop suffers from error due to the possible presence of a dipole mode. The electric probe is at the disadvantage that a capacitive attenuator may not be used when pronounced ionization is present, and it is a high-impedance device. In any measurement scheme involving the source impedance of either a loop or monopole, the results will depend fundamentally on the nature of the medium in which the fields are measured.

Schmitt⁴ reports that he has consistently observed strong transient ringing in short monopoles and small loops terminated in a cathode follower. He feels that contributions from the frequency spectrum of the pulse near probe resonance significantly affect the response. For a thin monopole, ringing may be reduced by the addition of a base resistor of about 300 ohms in series with the cathode follower. But, apparently there exists no panacea for loops. If ringing is manifest in loops, no simple relation, as reported here between $v_{oc}(t)$ and $b(t)$, obtains. Moreover, since the ringing is a function of the transit time of the waves along the conductor, the observed transient signal will be dependent on the physical properties of the medium.

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