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ENHANCED INDUCED EMISSION FOR
STIMULATED RAMAN SCATTERING IN
PLASMAS

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Enhanced Induced Emission for Stimulated Raman
Scattering in Plasmas

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ABSTRACT

It is shown how longitudinal bremsstrahlung can be included in the formalism for calculating the enhanced induced emission of high-frequency waves in a plasma influenced by a pump field.

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Recently, Oberman and Auer¹ presented a simple formalism for calculating the enhanced fluctuations of high-frequency waves in a plasma influenced by a pump which is even below threshold for the excitation of parametric instabilities. These enhanced fluctuations are due to the scattering of the pump from low-frequency density fluctuations which arise from the discrete particles and their clouds. The discrete particle effects are taken into account by using the Rostoker superposition principle² of dressed, uncorrelated particles. In Ref. 1, straight line orbits were used in the bare particle microdensities, and hence the electrostatic emission was Cerenkov radiation.

For sufficiently small wave numbers, however, electron waves are damped by collisions rather than by Landau damping, and their emission is essentially due to longitudinal bremsstrahlung.³ It is the purpose of this note to show how longitudinal bremsstrahlung can be included in the formalism. These effects are of higher order in the plasma parameter $1/n\lambda_D^3$.

In the following, we use the notation of Ref. 1. Since we are dealing with electron waves here, we can neglect the ion susceptibility χ_i and the ion source term S_i . According to Birmingham, Dawson, and Oberman³ the emission of radiation due to particle encounters can be calculated by using the following Fourier-transformed electron source term

$$S_e(\vec{k}, \omega) = \frac{ie}{m\omega} \sum_{i,e} (\vec{k} \cdot \vec{E}_{ie}) \exp(-i\vec{k} \cdot \vec{r}_i) . \quad (1)$$

Here, $\vec{E}_{ie}$ is the shielded electric field of the e-th electron at the site of the i-th ion:

$$E_{ie}(\vec{r}_i, \vec{r}_e, \vec{v}_e, \omega) = \frac{ie}{\pi} \int d^3k \frac{\vec{k}}{k^2 \epsilon(\vec{k}, \omega)} \exp[i\vec{k} \cdot (\vec{r}_i - \vec{r}_e)] \delta(\omega - \vec{k} \cdot \vec{v}_e) \quad (2)$$

where $\epsilon(\vec{k}, \omega)$ is the usual dielectric function.

For computing the average of the square of the scattered field, we must calculate

$$\langle \sum_{i, i', e, e'} S_e(\vec{r}_i, \vec{r}_e, \vec{v}_e, \vec{k}, \omega) S_e^*(\vec{r}_{i'}, \vec{r}_{e'}, \vec{v}_{e'}, \vec{k}', \omega') \rangle = X. \quad (3)$$

In accord with the superposition principle of dressed particles, the electrons are taken uncorrelated. When ion correlations are neglected, only terms with $i = i'$ and $e = e'$ remain and we obtain

$$X = (2\pi)^6 \frac{n_o^2 e^4}{m^2 \omega^4 \pi^2} \delta(\vec{k} - \vec{k}') \delta(\omega - \omega') \int d^3k'' \frac{(\vec{k} \cdot \vec{k}'')}{k''^4 |\epsilon(\vec{k}'', \omega)|^2} \frac{1}{k''} \tilde{f}_e\left(\frac{\omega}{k''}\right) \quad (4)$$

where $\tilde{f}_e\left(\frac{\omega}{k}\right) = \int d^3v f_e(\vec{v}) \delta(\omega/k - \hat{k} \cdot \vec{v})$.

Using the following relation for a Maxwellian distribution function

$$\text{Im } \epsilon(\vec{k}, \omega) = \pi \frac{\omega}{k} (k\lambda_D)^{-2} \tilde{f}_e\left(\frac{\omega}{k}\right), \quad (5)$$

and integrating over all directions of $\vec{k}''$ in Eq. (4), we find

$$X = (2\pi)^6 \frac{n_o^2 e^4}{m^2 \omega^5 \pi^2} (k\lambda_D)^2 \operatorname{Im} \int_0^{k_{\max}} dk'' k''^2 \frac{1}{\epsilon(k'', \omega)} \quad (6)$$

The dispersion relation for longitudinal electron waves damped by collisions is given by³

$$\epsilon^c(\vec{k}, \omega) = 1 - \frac{\omega_p^2}{\omega^2} (1 + \sigma_1) \quad (7)$$

with

$$\sigma_1 = \frac{2e^2}{3\pi m \omega^2} \int_0^{k_{\max}} dk k^2 \left(\frac{1}{\epsilon(\vec{k}, \omega)} - \frac{1}{\epsilon(\vec{k}, 0)} \right) \quad (8)$$

From Eqs. (6), (7), and (8) we obtain

$$X = (2\pi)^6 \frac{1}{2\pi} \frac{1}{\omega} n_o (k\lambda_D)^2 \delta(\vec{k} - \vec{k}') \delta(\omega - \omega') \operatorname{Im} \epsilon^c(\vec{k}, \omega) \quad (9)$$

The average of the squared transverse high-frequency field is then (see Ref. 1) :

$$\langle E^2(x, t) \rangle = \frac{1}{(2\pi)^2} \iint d^3k d\omega \frac{n_o (k\lambda_D)^2}{2\pi^2 \omega} \left(\frac{\omega_p^2}{n_o} \right)^2 \frac{\operatorname{Im} \epsilon^c(\vec{k}, \omega)}{|\epsilon_{nl}^c(\vec{k}, \omega)|^2} \quad (10)$$

$$\times \left\{ \frac{|\hat{\vec{k}}_+ \times \vec{E}_o|^2}{|D_+|^2} + \frac{|\hat{\vec{k}}_- \times \vec{E}_o|^2}{|D_-|^2} \right\} ,$$

where the nonlinear dispersion relation $\epsilon_{nl}^C(\vec{k}, \omega)$ consists of the collisional dispersion relation $\epsilon^C(\vec{k}, \omega)$ plus additional nonlinear terms.

Let us consider the case where $D_-(\vec{k} - \vec{k}_0, \omega - \omega_0) \approx 2\omega_0\omega - 2c^2\vec{k} \cdot \vec{k}_0 + c^2k^2 + i2\omega_0\Gamma_- \approx 0$, whereas D_+ is nonresonant. For Raman scattering the electrostatic wave is an electron plasma wave, i.e., $\text{Re } \epsilon^C \approx 0$, $\epsilon^C = i\text{Im } \epsilon^C$, $\omega = \omega_e$. Hence,

$$\epsilon_{nl}^C D_- \approx \epsilon^C(\vec{k}, \omega) 2\omega_0(\omega - \omega_e + i\Gamma_{nl}) \quad (11)$$

$$\frac{\text{Im } \epsilon^C}{|\epsilon^C|^2} \approx \frac{1}{\text{Im } \epsilon^C} = \frac{\omega_e}{2\Gamma_e^C}, \quad (12)$$

where Γ_e^C is the collisional damping rate of the electron plasma wave, Γ_- that of the free electromagnetic side-band at frequency $\omega - \omega_0$, and the nonlinear damping rate Γ_{nl} is given by

$$\Gamma_{nl} = \Gamma_- - \frac{k^2}{2\omega_0} |\hat{\vec{k}}_- \times \hat{\vec{v}}_0|^2 \frac{\omega_e}{2\Gamma_e^C}. \quad (13)$$

Substituting Eqs. (11) and (12) in Eq. (10) and integrating over the dominant electromagnetic resonance, we obtain the following result for the square of the scattered high-frequency field (normalized to the thermal equilibrium level)

$$I_k = \frac{\langle E_k^2 \rangle}{4\pi T_e / (2\pi)^3} = |\hat{\vec{k}}_- \times \hat{\vec{E}}_0|^2 \frac{E_0^2}{4\pi n_0 T_e} \frac{1}{8} \left(\frac{\omega_p}{\omega_0} \right)^3 (k\lambda_D)^2 \frac{\omega_0 \omega_p}{\Gamma_{nl} \Gamma_e^C}. \quad (14)$$

Let us return now to the case of dominant Landau damping and Cerenkov emission, respectively. The average over the source term yields (see Ref. 1):

$$\langle \sum_{e,e'} s_e(\vec{k}, \omega) s_{e'}^*(\vec{k}', \omega') \rangle = (2\pi)^5 n_0 \delta(\vec{k} - \vec{k}') \delta(\omega - \omega') \frac{1}{k} \tilde{f}_e\left(\frac{\omega}{k}\right) \quad (15)$$

$$= (2\pi)^5 n_0 \delta(\vec{k} - \vec{k}') \delta(\omega - \omega') (k\lambda_D)^2 \text{Im } \epsilon(\vec{k}, \omega) / \pi \omega ,$$

where we have used Eq. (5). Comparing Eqs. (15) and (9), we see that the computation of the average of the squared field proceeds in a quite analogous manner.

Thus, Eq. (14) together with Eq. (13) can be used to calculate stimulated Raman scattering in both cases, when the emission of the electrostatic wave is due to Cerenkov radiation or due to longitudinal bremsstrahlung, if only the appropriate damping rate (Landau or collisional) is inserted in these equations.

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