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A FAST REACTOR EXCURSION SIMULATOR

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ABSTRACT

This paper presents a technique for the study of fast reactor accidents by use of an analog computer. The technique is applied to reactor problems involving pump failure, orifice block, and fast excursion.

This method allows a good flux plot or temperature-distribution analysis to be made by the use of only a 5 x 5 mesh of an average channel in the core.

A sample problem is worked by this method and compared with a digital computer solution. All computations, circuits, and operating information necessary to duplicate the solutions are given. Results are included and compared with the digital solution.

Table 1

NOMENCLATURE

$A_{i,i+1}$	effective area between the i-th and i + 1-st cells
C_p	specific heat
$d_{i,i+1}$	distance between i-th and i + 1-st node points
h	heat transfer coefficient
i	radial coordinate of the mesh
j	axial coordinate of the mesh
$K_{i,i+1}$	conductivity between i-th and i + 1-st node points
K_S	surface conductivity between two solid regions
ρ	density
$q(v,t)$	power generated

Table 1 (Contd.)

T	temperature
V	volume
v	coolant flow rate
Δz	axial mesh size
t	time
n	prompt-neutron flux density
C_i	flux density of i-th delayed-neutron group
K_{ex}	excess reactivity
β_i	the fraction of prompt neutrons entering the i-th delayed-neutron group
$\beta = \sum \beta_i$	the fraction of prompt neutrons entering all delayed groups
ℓ	prompt-neutron lifetime
λ_i	decay constant of neutrons in the i-th delayed group
$n_1 = \frac{n(t)}{n(0)}$	the ratio of the prompt-neutron density at time t to the equilibrium value, n(0)
$C_{1i} = \frac{C_i(t)}{C_i(0)}$	the ratio of the density of neutrons in the i-th delayed group to the equilibrium value, $C_i(0)$
ΔK	feedback reactivity (in dollars)
K_{eff}	$K_{lex} - \Delta K$
$K_{lex} = \frac{K_{ex}}{\beta}$	excess reactivity (in dollars)
n'_1 C'_{1i} t' K'_{lex} η' ψ'_i	} primes denote the computer voltages which represent the unprimed variables

I. INTRODUCTION

A priori analyses of hypothetical fast reactor meltdown accidents are of interest because of their possible "autocatalytic" nature. The fuel

occupies approximately 30% of the core volume, and the remaining 70% is made up of structural materials and coolant. The fuel configuration is not optimum with respect to reactivity.

If the fuel were to melt and become rearranged into a denser configuration, large amounts of excess reactivity would become available. Thus, failure of a fuel element in a fast reactor might lead to large releases of nuclear energy. In such cases it is not the total excess reactivity available, but the rate at which the excess reactivity is added, that determines the ultimate severity of an accident.¹

The rate of addition of reactivity in a meltdown accident is determined by the time sequence of fuel element failure, which is dependent upon the temperature transients of the fuel elements.

In order to study the dynamics of a fast reactor accident, a detailed knowledge of spatial and time distributions of temperature in the fuel element is necessary. This is necessary to calculate the reactivity feedback effects, which determine the pre-meltdown trend of the accident. It also determines the coherence of pin failure, which determines the meltdown effects of the accident.

In view of the importance of temperature distribution in a fuel element in fast reactor accident analysis, an IBM 704 FORTRAN code "Argus"² was prepared at the Argonne National Laboratory to calculate the time- and space-dependent temperature distribution in multi-region cylindrical bodies. The code accommodates twenty-five concentric cylindrical regions (solid or liquid) with or without heat generation. The coolant can be static or flowing, and there is a provision for variable flow rate.

"Argus" is a temperature-transient code and does not calculate the reactivity feedback and subsequent change in reactor power.

Greebler et al.³ developed a digital program, "Fore," for Transac-2000, which takes into account the reactivity feedback effects on reactor

¹W. J. McCarthy, R. B. Nicholson, D. Okrent, and V. Z. Jankus, Studies of Nuclear Accidents in Fast Reactors, Proc. 2nd UN Intn'l. Conf. on the Peaceful Uses of Atomic Energy, Geneva, Switzerland, 12, 242(1958).

²D. F. Schoeberle, L. B. Miller, and J. Heestand, A Method of Calculating Transient Temperatures in a Multi-region, Axisymmetric Cylindrical Configuration. The Argus Program 1089/RP248, written in FORTRAN II, ANL-6654 (1963).

³P. Greebler, D. B. Sherer, and N. H. Walton, "Fore," A Computational Programme for the Analysis of Reactor Excursion, GEAP-4090.

power. This program is not as versatile as "Argus" in temperature calculations because of the restrictions on mesh size and because coolant flow rate must be constant.

Since such programs consume considerable time on large digital computers and due to the amenability of the entire problem to analog simulation, this paper presents an analog computer study of a fast reactor excursion.

The simulator takes into account the temperature dependence of the coolant properties, phase change of the fuel, feedback effects on reactor power, and variation of coolant flow.

This report describes the simulator and provides all computations, circuitry, and operating instructions for duplication of an EBR-II-type accident. Other data, for example, for a large ceramic reactor, can be used with appropriate input changes.

II. SIMULATOR

The simulator consists of three parts, as shown in Figure 1.

Fig. 1. Simulator Block Diagram

The system is designed such that the following studies can be made:

- 1) Obtain the detailed temperature distribution in an average fuel element due to a change in any of the operating conditions of the reactor.
- 2) Calculate the feedback reactivity due to temperature distribution in the fuel element.
- 3) Determine reactor power change due to feedback reactivity and externally inserted reactivity, if any.

A. Temperature-transient Equations

1. Assumptions

In the temperature-transient calculations, a fuel element is taken as the representative unit of the reactor. The fuel element is simulated by a mesh of point integrators for which the following assumptions are made:

- 1) The fuel is cylindrical with clad and coolant as concentric cylindrical shells.
- 2) Axial heat conduction in the fuel is negligible.
- 3) Temperature gradient in the radial direction in the coolant is zero, and only one radial mesh point need be considered in the coolant region.
- 4) There is axial heat transfer in the coolant region due to coolant movement.

For each cell in the fuel element mesh we have a heat balance expression:

$$\text{Rate of change of heat content} = (\text{heat flowing in} - \text{heat flowing out} + \text{heat generated}) \text{ per unit time.} \quad (1)$$

2. Equations

Referring to Figure 2 and considering the assumptions made above, the heat balance equation (1) for solid regions can be written as

$$(V\rho C_p)_i \frac{dT}{dt} = \left(\frac{KA}{d}\right)_{i,i-1} (T_{i-1} - T_i) - \left(\frac{KA}{d}\right)_{i,i+1} (T_i - T_{i+1}) + q(i,j,t) \quad (2)$$

for all i and $j \neq 0$. Rearranging the terms and integrating, we obtain

$$T_i(t) = T_i(0) + \int \{C_{i,i-1}T_{i-1} + C_{i,i+1}T_{i+1} - C_iT_i + q(i,j,t)\}dt, \quad (3)$$

where

$$\left. \begin{aligned} C_{i,i-1} &= \left(\frac{KA}{d} \right)_{i,i-1} \frac{1}{(V \rho C_p)_i} \\ C_{i,i+1} &= \left(\frac{KA}{d} \right)_{i,i+1} \left(\frac{1}{V \rho C_p} \right)_i \\ C_i &= C_{i,i+1} + C_{i,i-1} \end{aligned} \right\}, \quad (4)$$

with the following remarks:

$$(1) \quad C_{i,i+1} \neq C_{i,i-1}.$$

(2) If C_i and C_{i+1} are in two different regions, $K_{i,i+1}$ also takes into account the intersurface resistance and is given by the relation

$$\frac{d_{i,i+1}}{K_{i,i+1}} = \frac{1}{K_S} + \frac{d_i}{K_i} + \frac{d_{i+1}}{K_{i+1}}, \quad (5)$$

where $d_{i,i+1}$ and $K_{i,i+1}$ are the distance and effective conductivity between C_i and C_{i+1} . Here d_{i+1} and K_{i+1} are distance and conductivity, respectively, in the region of the $(i+1)$ point.

Fig. 2. Radial Node Points

For $i = 0$, there is no heat flowing in; therefore,

$$(v \rho C_p)_0 \frac{dT}{dt} = - \left(\frac{KA}{d} \right)_{0,1} (T_0 - T_1) + q(o,j,t), \quad (6)$$

or

$$T_0(t) = T_0(0) + \int \{C_{0,1}T_0 - C_{0,1}T_1 + q(o,j,t)\} dt. \quad (7)$$

The heat balance equation for the coolant region is

$$(v \rho C_p) \frac{dT}{dt} = - v \frac{dT}{dz} + q_2(v,z,t), \quad (8)$$

where $q_2(v,z,t)$ is heat flowing into the coolant from the solid region.

In the finite difference approximation, equation (6) can be written as

$$\frac{dT_{ij}}{dt} = -\frac{v}{\Delta z} (T_{i,j} - T_{i,j-1}) + \frac{hA_{i,i-1}}{(\rho C_p V)_{i,j}} (T_{i-1,j} - T_{i,j}), \quad (9)$$

where the i -th point is in the coolant and the $(i-1)$ -st point is the boundary point of the solid region, Therefore,

$$T_{i,j}(t) = T_{i,j}(0) + \int (C^* \Delta T_{j-1,j} + H_{i,j} \Delta T_{i-1,i}) dt, \quad (10)$$

where

$$C^* = v/\Delta z;$$

$$H_{i,j} = hA_{i,i-1}/(\rho C_p V)_{i,j}$$

$$\Delta T_{j-1,j} = T_{ij}(t) - T_{i,j-1}(t);$$

$$\Delta T_{i,i-1} = T_{i-1,j} - T_{i,j}.$$

Figure 3 shows a typical analog circuit diagram for the solution of a partial differential equation by finite difference approximation. The output voltage e_i of the integrator i is given by

$$e_i = e_i(0) + \int (C_{i-1}e_{i-1} + C_{i+1}e_{i+1} - C_i e_i + q_i) dt. \quad (11)$$

Fig. 3. Finite Difference Approximation Circuit

Clearly, equations (3), (10), and (11) are equivalent. Hence, an extended circuit of the type shown in Figure 3 can be used to simulate the time and spatial variation of temperature in the fuel element. The accuracy of simulation will depend on the fineness of the mesh, i.e., the number of integrators used.

Figure 4 shows the complete network which simulates the temperature transients in a fuel element by means of a 5×5 mesh. All

Fig. 4. Analog Circuit Diagram for Temperature Transients

of the coefficients by which the temperatures are to be multiplied, except in the clad and coolant regions, are accomplished by linear potentiometers.

The "h" which enters into the coefficients of variation of coolant and clad temperatures is coolant-velocity dependent. Accommodation for coolant-velocity variation in these parameters is accomplished by the use of servo-mechanical multipliers for coefficient multiplication in these regions.

In order to decrease the effect of inherent error of the servo-mechanical multipliers, temperature differences are taken in the clad and coolant regions before multiplying by the coolant velocity variable coefficient.

Fig. 5. Phase-transformation Network

Fig. 6. Heat Content of Uranium-5% Fissium Alloy

3. Special Circuits

a. Phase Transformation

Simulation of phase transformation in the fuel is accomplished by the use of high-speed differential relays in the fuel integrator networks. The relay circuit is shown in Figure 5. The relay is set to operate at 1010°C , the phase-change temperature of the fuel-region node. At this temperature there is a sudden change in the enthalpy vs. temperature curve (see Figure 6) for, say, the uranium-5% fissium alloy. This is equivalent to a change in the specific heat of the alloy, which changes the values of the coefficients in the fuel equations. Upon operation of the relays all inputs to the node integrator are multiplied by C_{p1}/C_{p2} , where C_{p1} is the specific heat of the fissium alloy prior to phase change and C_{p2} is the effective specific heat of the alloy after phase change.

b. Coolant Limiter

A series output limiter is used in the coolant integrator networks to limit the particular coolant node to its boiling point. The analog circuitry used for this purpose is shown in Figure 7.

Fig. 7. Coolant Output Limiter Circuit

The performance equation for the limiter is given by

$$\left(\frac{e_0}{A} - e_i \right) \frac{1}{R_i} + \left(\frac{e_0}{A} - e_0 \right) \frac{1}{R_f} = 0 \quad (12)$$

when the diode is not conducting. Since A is of the order of 10^8 , e_0/A can be neglected. Thus,

$$e_0 = - \frac{R_f}{R_i} e_i. \quad (13)$$

When the diode conducts

$$e_0 = \frac{R_2 R_f}{(R_2 + R_f) R_i} E - \frac{R_2 R_f}{(R_2 + R_f) R_i} e_i, \quad (14)$$

where $R_2 R_f / (R_2 + R_f) R_i$ is the final slope of the limiter characteristics. The value of R_2 is chosen such that it is much less than R_i and the final slope is approximately zero, thus limiting the coolant temperature to the boiling point.

B. Reactivity Feedback

Generally, temperature coefficients of reactivity are available for the reactor as a whole rather than for any particular fuel element. In the reactor accident analysis it sometimes becomes necessary to study other than the average fuel element. In such an event the average fuel pin is simulated to obtain the reactor power transient. The power transient thus obtained, with the proper multiplication factor, is used as a source term in simulating the particular fuel element of interest.

The reactivity feedback simulation is such that the temperature at each point is multiplied by the respective temperature coefficient of

reactivity and summed. This sum along with any externally inserted reactivity goes into the reactor kinetics simulator as K_{eff} . Figure 8 shows the circuitry for K_{eff} .

Fig. 8. Reactivity Feedback Circuit

C. The Reactor Kinetics Equation

Reference is made to ANL-6027, Analogue Computer Solution of the Nonlinear Reactor Kinetics Equation, by L. T. Bryant and N. Frank Morehouse, Jr. The equations and diagrams used are repeated here for completeness.

The reactor kinetics equations are

$$\frac{\ell}{\beta} \frac{dn_1}{dt} = (1 - \beta) K_1 \text{ex} n_1 - n_1 + \sum_{i=1}^6 \frac{\beta_i}{\beta} C_{1i} \quad (15)$$

and

$$\frac{dC_{1i}}{dt} = \lambda_i \beta K_1 \text{ex} n_1 + \lambda_i n_1 - \lambda_i C_{1i} \quad (i=1,2,\dots,6). \quad (16)$$

Let

$$\left. \begin{aligned} \eta &= \ell \ln n_1 \\ \epsilon_i &= e^{-\eta} C_{1i} \\ \epsilon_i - 1 &= \psi_i \end{aligned} \right\}. \quad (17)$$

Substituting equations (17) into equations (15) and (16), we obtain

$$\frac{d\eta}{dt} = \frac{\beta}{\ell} \left[(1 - \beta) K_1 \text{ex} + \sum_{i=1}^6 \frac{\beta_i}{\beta} \psi_i \right] \quad (18)$$

and

$$\frac{d\psi_i}{dt} = \lambda_i \beta K_1 \exp - \frac{d\eta}{dt} - \psi_i \left(\lambda_i + \frac{d\eta}{dt} \right), (i=1,2,\dots,6). \quad (19)$$

It is noted that in order to study the system for a large excursion, it is necessary to solve the kinetics equations in terms of $\ln n_1$. The antilog is taken by utilization of a diode function generator for insertion of n_1 as power into the temperature equations.

Figure 9 and Figure 10 show the analog circuit diagrams for the reactor kinetics equation.

Fig. 9. Nonlinear Reactor Kinetics Schematic for η'_1

III. CONSTANTS FOR EBR-II ACCIDENT ANALYSIS

The constants given below are based on the values used in the "Argus" calculation.

A. Core Data (see also Table 2)

Length of the fuel pin	36.12 cm;
Radius of the fuel region	0.183 cm;
External radius of the clad + bond	0.221 cm.

Two types of pins are considered for the coolant region:

- (1) internal pin;
- (2) peripheral pin.

Effective external radius of the internal pin coolant channel 0.2895 cm;
effective external radius of the peripheral pin coolant channel 0.326 cm.

Table 2

MATERIAL PROPERTIES OF THE CORE

	ρC_p (cal/cc-°C)	k(cal/cm ² -sec)	Remarks
Fuel	1.061	0.1118	(ρC _p) of fuel includes energy of transformation at 630°C. (ρC _p) of coolant is taken at 650°C.
Clad + Bond	0.633	0.065	
Coolant	0.239	0.157	

$$\Delta z = 7.22 \text{ cm}$$

$$(\Delta r)_f = 0.0915 \text{ cm}$$

$$(\Delta r)_{cl+bond} = 0.038 \text{ cm}$$

$$(\Delta r)_{coolant} = 0.0685 \text{ cm (for internal pin)}$$

$$= 0.105 \text{ cm (for peripheral pin)}$$

B. Source Term Data

Operating power = 62.5 MW

Average (axial average) power of the internal pin of the 1st row = 816.72 cal/cc-sec (see also Table 3).

Table 3

RELATIVE POWER IN THE AXIAL SECTIONS OF THE PIN

Section	1	2	3	4	5	Avg.
Relative power	0.92	1.15	1.20	1.15	0.92	1.068
Temp rise (°C/sec)	662.9	828.7	864.7	828.7	662.9	

The phase transformation for the fuel is set at 1010°C.

$$\frac{C_{p1}}{C_{p2}} = \frac{4.437}{19.41} = 0.2286 \quad .$$

Coolant vaporization is set at 1020°C.

$$C^* = B\phi(t)/\Delta z$$

$$H_{ij} = K_2 + K_3\phi(t)$$

$$K_2 = 148.8 \text{ for internal pin}$$

$$= 88.88 \text{ for peripheral pin}$$

$$K_3 = 0.219 B\phi(t) \quad .$$

C. Reactor Kinetics and Feedback Data

$$\ell = 8 \times 10^{-8} \text{ sec} \quad ; \quad \beta = 0.007347 \quad .$$

Table 4 shows the delayed-neutron data.

Table 4

DELAYED-NEUTRON DATA

i	λ_i	β_i
1	0.0127	0.000232
2	0.0318	0.001423
3	0.1153	0.001333
4	0.3110	0.002956
5	1.40	0.001121
6	3.870	0.000282

The feedback coefficients (from ANL-5719 Addendum, p. 85) for fuel temperature change:

$$= -0.50 \times 10^{-5}.$$

For coolant temperature change:

$$= -2.22 \times 10^{-5}.$$

Reactivity feedback due to clad and structural temperature changes is included in the coolant coefficients on the assumption that the coolant has the average temperature of structure and clad.

Table 5 gives the weight factors and the coefficients used for the reactivity feedback of the several radial regions.

Table 5

WEIGHT FACTORS AND REACTIVITY
FEEDBACK COEFFICIENTS

Radial Region	Weight Factor	Coefficient
1	1/16	3.1×10^{-7}
2	1/2	2.5×10^{-6}
3	7/16	2.2×10^{-6}
coolant	1.0	2.22×10^{-5}

As the reactivity coefficients are given for the reactor as a whole, they are applicable only for the average fuel pin. Therefore, the accident is simulated for a pin with average power and its power transient is obtained. For all other pins, this power transient multiplied by the ratio of their respective power to the average power was used as the input power.

The ratio of radial maximum to minimum power for EBR-II was found to be 1.6.

IV. DEFINITION OF SCALE FACTORS AND MACHINE EQUATIONS

A. Scale Factors

$$\begin{aligned}
 a_t &= t'; & d_i C_{1i} &= C'_{1i}, \text{ or } d_i \psi_{1i} = \psi'_{1i}; \\
 b n_1 &= n'_1; & f T_{ij} &= T'_{ij}, \\
 e K_{1ex} &= K'_{1ex};
 \end{aligned}
 \tag{20}$$

where the primed variables represent machine voltages, and a, b, e, d_i , and f_i are constants.

B. Machine Equations

1. Temperature Transients

(a) Fuel Regions

$$\begin{aligned} \frac{dT'_{ij}}{dt'} = & \frac{1}{a} \left(\frac{KA}{V\rho C_{pd}} \right)_{i,i-1} (T'_{i-1} - T'_i) + \frac{1}{a} \left(\frac{KA}{V\rho C_{pd}} \right)_{i,i+1} (T'_i - T'_{i+1}) \\ & + \frac{f q'(i,j,t)}{abV\rho C_p}. \end{aligned} \quad (21)$$

(b) Clad + Bond

$$\frac{dT'_{ij}}{dt'} = \frac{1}{a} \left(\frac{KA}{V\rho C_{pd}} \right)_{i,i-1} (T'_{i-1} - T'_i) + \frac{1}{a} \left(\frac{KA}{V\rho C_{pd}} \right)_{i,i+1} (T'_i - T'_{i+1}). \quad (22)$$

(c) Coolant

$$\frac{dT'_{ij}}{dt'} = -\frac{v}{a\Delta z} (T'_{i,j} - T'_{i,j-1}) + \frac{hA_{i,i-1}}{a(\rho C_p V)_{i,j}} (T'_{i-1,j} - T'_{i,j}). \quad (23)$$

(d) Reactor Kinetics

$$\frac{dn'_l}{dt'} = \frac{\beta}{a\ell} \left[\frac{(1-\beta)}{e} K'_l \text{ex } n'_l - n'_l + \frac{1}{b} \sum_{i=1}^6 \frac{\beta_i}{d_i\beta} C'_{li} \right]. \quad (24)$$

$$\frac{dC'_{li}}{dt'} = \frac{d_i\lambda_i\beta K'_l \text{ex } n'_l}{aeb} + \frac{d_i\lambda_i n'_l}{ab} - \frac{\lambda_i}{a} C'_{li}. \quad (25)$$

$$\frac{d\eta'}{dt'} = \frac{\beta}{a\ell} \left[\frac{(1-\beta)}{be} K'_l \text{ex} + \frac{1}{b} \sum_{i=1}^6 \frac{\beta_i}{d_i\beta} \psi'_i \right]. \quad (26)$$

$$\frac{d\psi'_i}{dt'} = \frac{d_i\lambda_i\beta K'_l \text{ex}}{ae} - \frac{d_i}{b} \frac{d\eta'}{dt'} - \frac{\lambda_i}{a} \psi'_i - \frac{1\psi'_i}{b} \frac{d\eta'}{dt'}. \quad (27)$$

V. ANALOG COEFFICIENTS

The potentiometer settings are listed in Tables 6, 7, and 8.

Table 6

Argonne National Laboratory
APPLIED MATHEMATICS DIVISION
ANALOG COMPUTER
POTENTIOMETER SETTINGS

PROBLEM NO. _____

DRAWING NO. _____

DATE _____

"B" TEMPERATURE TRANSIENT, HIGH SPEED*

POTENTIOMETER NO.		MATHEMATICAL VALUE	VALUE	CORREC- TION	SETTING	SET	PARAMETERS
DRAWING	MACHINE						
J=1	21	$C_{00}/a \cdot 10^{-1}$	.5034		0.5034		for Power Transient
	22	$fk_1/ab \cdot 10^{-1}$	.3315		0.0663(5)		a = 10 f = $5 \cdot 10^{-2}$
	23	$C_{10}/a \cdot 10^{-1}$	.05968		0.0597		b = 1.0
	24	$C_{01}/a \cdot 10^{-1}$	.5034		0.5034		V = $B\phi(\tau)$
	25	$C_{11}/a \cdot 10^{-1}$	.2388		0.2388		
	26	$fk_1/ab \cdot 10^{-1}$	.3315		0.0663(5)		
	27	$C_{21}/a \cdot 10^{-1}$	.2251		0.2251		
	28	$C_{12}/a \cdot 10^{-1}$	.1791		0.1791		
	29	$C_{22}/a \cdot 10^{-1}$	.6293		0.6293		
	31	$C_{32}/a \cdot 10^{-1}$	.6285		0.6285		
	32	$C_{23}/a \cdot 10^{-1}$	.4041		0.4041		
	30	$C_{33}/a \cdot 10^{-1}$	.6285		0.6285		
	42	$k_4 \cdot 10^{-1}$	.1163		0.1163		Central Pin
J=2			.0709		0.0709		Peripheral Pin
	33	$C_{00}/a \cdot 10^{-1}$	.5034		0.5034		
	34	$fk_1/ab \cdot 10^{-1}$	.4144		0.0829(5)		.829
	35	$C_{10}/a \cdot 10^{-1}$	.05968		0.0597		
	36	$C_{01}/a \cdot 10^{-1}$	.5034		0.5034		
	37	$C_{11}/a \cdot 10^{-1}$	.2388		0.2388		
	38	$fk_1/ab \cdot 10^{-1}$	.4144		0.0829(5)		.829
	39	$C_{21}/a \cdot 10^{-1}$	.2251		0.2251		
	40	$C_{12}/a \cdot 10^{-1}$	.1791		0.1791		
	41	$C_{22}/a \cdot 10^{-1}$	.6293		0.6293		
	43	$C_{32}/a \cdot 10^{-1}$	.6285		0.6285		

*For Power Transient a = 10^2 , "B" Machine in Slow SpeedHigh Speed 0.1 μ fd Feedback CapacitorSlow Speed 1.0 μ fd Feedback Capacitor.

Table 6 Contd.

Argonne National Laboratory
APPLIED MATHEMATICS DIVISION
ANALOG COMPUTER
POTENTIOMETER SETTINGS

PROBLEM NO. _____

DRAWING NO. _____

DATE _____

POTENTIOMETER NO.		MATHEMATICAL VALUE	VALUE	CORREC- TION	SETTING	SET	PARAMETERS
DRAWING	MACHINE						
	44	$C_{23}/a \cdot 10^{-1}$	.4041		0.4041		for Power Transient
	54	$C_{33}/a \cdot 10^{-1}$	.6285		0.6285		
	58	$k_4 \cdot 10^{-1}$			0.1163		
J=3					0.0709		Central Pin
	45	$C_{00}/a \cdot 10^{-1}$	.5034		0.5034		Peripheral Pin
	46	$fk_1/ab \cdot 10^{-1}$	.4324		0.0865(5)		.865
	47	$C_{10}/a \cdot 10^{-1}$	.0597		0.0597		
	48	$C_{01}/a \cdot 10^{-1}$	.5034		0.5034		
	49	$C_{11}/a \cdot 10^{-1}$	.2388		0.2388		
	50	$fk_1/ab \cdot 10^{-1}$	.4324		0.0865(5)		.865
	51	$C_{21}/a \cdot 10^{-1}$	.2251		0.2251		
	52	$C_{12}/a \cdot 10^{-1}$	.1791		0.1791		
	53	$C_{22}/a \cdot 10^{-1}$	.6293		0.6293		
	55	$C_{32}/a \cdot 10^{-1}$	.6285		0.6285		
	56	$C_{23}/a \cdot 10^{-1}$	.4041		0.4041		
	62	$C_{33}/a \cdot 10^{-1}$	.6285		0.6285		
	66	$k_4 \cdot 10^{-1}$	.1163		0.1163		Central Pin
J=4			.0709		0.0709		Peripheral Pin
	57	$C_{00}/a \cdot 10^{-1}$	.5034		0.5034		
	59	$C_{10}/a \cdot 10^{-1}$	.0597		0.0597		
	60	$C_{01}/a \cdot 10^{-1}$	.5034		0.5034		
	61	$C_{11}/a \cdot 10^{-1}$	.2388		0.2388		
	63	$C_{21}/a \cdot 10^{-1}$	.2251		0.2251		
	64	$C_{12}/a \cdot 10^{-1}$	.1791		0.1791		

Table 6 Contd.

Argonne National Laboratory
APPLIED MATHEMATICS DIVISION
ANALOG COMPUTER
POTENTIOMETER SETTINGS

PROBLEM NO. _____
 DRAWING NO. _____
 DATE _____

POTENTIOMETER NO.		MATHEMATICAL VALUE	VALUE	CORREC- TION	SETTING	SET				PARAMETERS
DRAWING	MACHINE									
	8	$fT_3J(0)$	30.00	Volts	.3000					
	9	$fT_4J(0)$	27.50	Volts	.2750					
	10	$fT_5J(0)$	25.00	Volts	.2500					
	11	$(1/3.5) C_{p1}/C_{p2}$	.8001		.8001					
	12	$(1/3.5) C_{p1}/C_{p2}$	.8001		.8001					
	13	$(1/3.5) C_{p1}/C_{p2}$	.8001		.8001					
	14	$(1/3.5) C_{p1}/C_{p2}$	.8001		.8001					
	15	$(1/3.5) C_{p1}/C_{p2}$	.8001		.8001					
	16	$(1/3.5) C_{p1}/C_{p2}$	.8001		.8001					
	17	$(1/3.5) C_{p1}/C_{p2}$	.8001		.8001					
	18	$(1/3.5) C_{p1}/C_{p2}$	.8001		.8001					
	19	$(1/3.5) C_{p1}/C_{p2}$	.8001		.8001					
	20	$(1/3.5) C_{p1}/C_{p2}$	.8001		.8001					
	61A	$(1/3.5) C_{p1}/C_{p2}$	.8001		.8001					
	62A	$(1/3.5) C_{p1}/C_{p2}$	.8001		.8001					
	63A	$(1/3.5) C_{p1}/C_{p2}$	.8001		.8001					
	64A	$(1/3.5) C_{p1}/C_{p2}$	.8001		.8001					
	65A	$(1/3.5) C_{p1}/C_{p2}$	.8001		.8001					
Feedback										
	77A	C_{1e}/Bf	.0169		.0169					$C_1 = 3.1 \cdot 10^{-7}$
	78A	C_{2e}/Bf	.1361		.1361					$C_2 = 2.5 \cdot 10^{-6}$
	79A	C_{3e}/Bf	.1198		.1198					$C_3 = 2.2 \cdot 10^{-6}$
	80A	C_{4e}/Bf	1.209		.1209(10)					$C_4 = 2.22 \cdot 10^{-5}$

Table 6 Contd.

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ANALOG COMPUTER
POTENTIOMETER SETTINGS

PROBLEM NO. _____
 DRAWING NO. _____
 DATE _____

POTENTIOMETER NO.		MATHEMATICAL VALUE	VALUE	CORREC- TION	SETTING	SET	PARAMETERS
DRAWING	MACHINE						
	65	$C_{22}/a \cdot 10^{-1}$	6293		0.6293		
	67	$C_{32}/a \cdot 10^{-1}$	.6285		0.6285		
	68	$C_{23}/a \cdot 10^{-1}$	.4041		0.4041		
	70	$C_{33}/a \cdot 10^{-1}$	6285		0.6285		
	74	$k_4 \cdot 10^{-1}$	.1163		0.1163		Central Pin
J=5			.0709		0.0709		Peripheral Pin
	69	$C_{00}/a \cdot 10^{-1}$	.5034		0.5034		
	71	$C_{10}/a \cdot 10^{-1}$	.0597		0.0597		
	72	$C_{01}/a \cdot 10^{-1}$	.5034		0.5034		
	73	$C_{11}/a \cdot 10^{-1}$	.2388		0.2388		
	75	$C_{21}/a \cdot 10^{-1}$	.2251		0.2251		
	76	$C_{12}/a \cdot 10^{-1}$	.1791		0.1791		
	77	$C_{22}/a \cdot 10^{-1}$	.6293		0.6293		
	79	$C_{32}/a \cdot 10^{-1}$	.6285		0.6285		
	80	$C_{23}/a \cdot 10^{-1}$	.4041		0.4041		
	71	$C_{33}/a \cdot 10^{-1}$	.6285		0.6285		
	1	$k_4 \cdot 10^{-1}$	.1163		0.1163		Central Pin
Other Parameters			.0709		0.0709		Peripheral Pin
	2	$T_c \ln \cdot f$	18.60	Volts	.1860		
	3	$B/10^2 \Delta Z a$	1000		.1000		
	4	$k_3 B \phi (\tau) \cdot 10^{-2}/a$	.2159		.2159		
	5	k_2/a	14.88	Volts	.1488		
	6	$fT_1 J(0)$	30.00	Volts	.3000		
	7	$fT_2 J(0)$	30.00	Volts	.3000		

Table 7

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APPLIED MATHEMATICS DIVISION
ANALOG COMPUTER
POTENTIOMETER SETTINGS

PROBLEM NO. _____

DRAWING NO. _____

DATE _____

"A" REACTOR KINETICS η_1

POTENTIOMETER NO.		MATHEMATICAL VALUE	VALUE	CORREC- TION	SETTING	SET	PARAMETERS														
DRAWING	MACHINE																				
	21	λ_1/a	.0013		.0013		$\ell = 8 \cdot 10^{-8}$ $\beta = 0.007347$														
	22	λ_2/a	.0032		.0032																
	23	λ_3/a	.0115		.0115		<table><tr><td>i</td><td>λ_i</td></tr><tr><td>1</td><td>.0127</td></tr><tr><td>2</td><td>.0318</td></tr><tr><td>3</td><td>.1153</td></tr><tr><td>4</td><td>.3110</td></tr><tr><td>5</td><td>1.4000</td></tr><tr><td>6</td><td>3.8700</td></tr></table>	i	λ_i	1	.0127	2	.0318	3	.1153	4	.3110	5	1.4000	6	3.8700
i	λ_i																				
1	.0127																				
2	.0318																				
3	.1153																				
4	.3110																				
5	1.4000																				
6	3.8700																				
	24	λ_4/a	.0311		.0311																
	25	λ_5/a	.1400		.1400																
	26	λ_6/a	.3870		.3870																
	27	$\beta_1 b/\beta d_1$	.0316		.0316																
	28	$\beta_2 b/\beta d_2$	.1937		.1937		$\beta_1/\beta = .0316$ $\beta_2/\beta = .1937$ $\beta_3/\beta = .1814$ $\beta_4/\beta = .4023$ $\beta_5/\beta = .1526$ $\beta_6/\beta = .0384$														
	53	$\beta_3 b/\beta d_3$	.1814		.1814																
	54	$\beta_4 b/\beta d_4$	.4023		.4023																
	55	$\beta_5 b/\beta d_5$	.1526		.1526																
	56	$\beta_6 b/\beta d_6$	.0384		.0038(10)																
	33	$\lambda_1 d_1/ab$	.0013		.0013			$1 - \beta = .99265$													
	34	$\lambda_2 d_2/ab$	.0032		.0032		$a = 10$														
	35	$\lambda_3 d_3/ab$	.0115		.0115																
	36	$\lambda_4 d_4/ab$	.0311		.0311		$b = d_i = 1.0$														
	37	$\lambda_5 d_5/ab$	.1400		.1400																
	38	$\lambda_6 d_6/ab \cdot 10^{-1}$	.0387		.0387		$e = 20$														
	39	$\lambda_1 d_1/ab$	.0013		.0013																
	40	$\lambda_2 d_2/ab$	.0032		.0032																
	41	$\lambda_3 d_3/ab$	.0115		.0115																
	42	$\lambda_4 d_4/ab$	.0311		.0311																
	43	$\lambda_5 d_5/ab$	.1400		.1400																
	44	$\lambda_6 d_6/ab \cdot 10^{-1}$	.0387		.0387																

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ANALOG COMPUTER
POTENTIOMETER SETTINGS

PROBLEM NO. _____

DRAWING NO. _____

DATE _____

[illegible]

Table 8

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ANALOG COMPUTER
POTENTIOMETER SETTINGS

PROBLEM NO. _____

DRAWING NO. _____

DATE _____

"A" REACTOR KINETICS ℓ_{n_1}

POTENTIOMETER NO.		MATHEMATICAL VALUE	VALUE	CORREC- TION	SETTING	SET	PARAMETERS
DRAWING	MACHINE						
	21	λ_1/a	.0000		.0000		$a = 10^3$ $b = d_1 = 1.0$ $e = 20$
	22	λ_2/a	.0000		.0000		
	23	λ_3/a	.0000		.0000		
	24	λ_4/a	.0003		.0003		
	25	λ_5/a	.0014		.0014		
	26	λ_6/a	.0039		.0039		
	3	$\beta_1 b / \beta d_1$	.0316		.0316		
	4	$\beta_2 b / \beta d_2$	.1937		.1937		
	5	$\beta_3 b / \beta d_3$	.1814		.1814		
	6	$\beta_4 b / \beta d_4$	.4023		.0402(10)		
	7	$\beta_5 b / \beta d_5$	.1526		.03052(5)		
	8	$\beta_6 b / \beta d_6$	.0384		.0038(10)		
		d_1/b	1.0				Multiplier
		d_2/b	1.0				Gains Take
		d_3/b	1.0				Care of
		d_4/b	1.0				These
		d_5/b	1.0				Coefficients
		d_6/b	1.0				
	1	$(\beta b(1-\beta)/a \ell e) \cdot 10^{-1}$	.4558		.4558		
	2	$(\beta/ad) \cdot 10^{-1}$	9.184		.9184(10)		

6

VI. COMPARISON OF ANALOG AND DIGITAL RESULTS

The accident to be studied for comparison is as follows: it is assumed that a complete blockage occurs instantaneously for the central core subassembly, reactor power remains constant, and hydrodynamic effects of the blockage can be ignored, so that the heat transfer problem is that for a fuel element in stagnant sodium.

Results from the simulator are compared with output from "Argus" in Figure 11. Temperature traces are shown, as a function of time after flow blockage, for the fuel surface at the axial midplane.

Fig. 11

Comparison of Simulator
and Digital Temperature
Calculations for Instant-
aneous Flow Blockage

The effect of material phase change (at 1010°C) is shown clearly at 0.8 sec. Coolant boiling first appears at 1.0 sec, at a time when all coolant node points are rising slowly, so that the outputs do not show any large effect from the limitation of coolant rise.

The relatively linear temperature rise between 0.1 and 0.8 sec displayed by the simulator output is due to the use of a single average value of ρC_p over this temperature range. "Argus" output in this region is nonlinear, because the latter calculation included two different values of fuel ρC_p between 600°C and 1010°C, and one fuel phase change with appreciable heat of transformation at 630°C (see Figure 6).

Agreement in Figure 11 is satisfactory for coherence of failure studies.

Agreement is shown in Figure 12 for an accident in which coolant flow is important. In this case pumping power is assumed to fail, but reactor power remains constant.

Fig. 12. Comparison of Simulator and Digital Temperature Calculations for Pump Failure

The data shown are fuel surface temperatures for the top (hottest) axial position of the central fuel element in the core. In this case, coolant flow decays exponentially with a time constant of the order of 10 sec.

Note that both sets of output show relatively little effect of the phase transformation at 1010°C. This is probably due to the relatively slow rate of our temperature excursion.

Figure 13 presents the prompt-neutron response of EBR-II due to a loading accident. The S-shaped rod worth curve peaks out at 2%.

Fig. 13
Power Excursion due to
Central Subassembly Drop

VII. ACKNOWLEDGEMENTS

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