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A PHENOMENOLOGICAL RELATIONSHIP FOR PREDICTING THE SURFACE AREAS OF THERMAL PLUMES IN LAKES

J. G. Asbury and A. A. Frigo

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by

J. G. Asbury and A. A. Frigo

Center for Environmental Studies

April 1971

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TABLE OF CONTENTS

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A PHENOMENOLOGICAL RELATIONSHIP FOR PREDICTING THE SURFACE AREAS OF THERMAL PLUMES IN LAKES

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ABSTRACT

A phenomenological relationship for surface areas within isotherms has been developed for thermal plumes in large lakes. The relationship, based upon the field data of other investigators, represents a useful rule of thumb for predicting surface areas of buoyant thermal plumes.

1. INTRODUCTION

The problem of predicting the dispersive behavior of heated effluents in natural water bodies has received the attention of many investigators in recent years. From a practical point of view, a certain degree of success has been achieved with regard to describing the certain types of discharges. For example, the behavior of heated effluents in uniform river-type crossflows, can be reasonably well predicted. That this has been achieved without the emergence of a universally accepted theory of plume dynamics indicates the large role phenomenology currently plays in plume analysis.

The state of the art for predicting lake plumes is less satisfactory, even from a phenomenological point of view. Although the dynamics governing the zone of flow establishment are presumably the same for lake and river discharges, the dynamics in the region of established flow are quite different. The lake situation is considerably more complicated, due to much larger dynamic variations in the structure of the receiving body of water. Variations in the direction and magnitude of near-shore currents and in the ambient diffusivity greatly complicate the problem of modeling the dispersive processes.

One of the simplest methods of parameterizing plume dispersion is in terms of surface areas within isotherms. In the present study we have adopted a completely phenomenological approach in attempting to find a relationship that could be used to predict plume surface areas. The approach was based upon two considerations: 1) The behavior of lake plumes beyond the zone of flow establishment is governed by lake processes for which adequate models do not exist, and 2) there does exist a published set of lake-plume temperature measurements which can be examined for relationships among the plume variables.

2. STUDY PHILOSOPHY

Edinger and Polk, in an analysis of thermal-plume dispersion, derived a functional relationship for plume surface areas contained within excess temperature contours.[1] The authors assumed Fickian-type diffusion and considered conservative and nonconservative two-dimensional dispersion and conservative three-dimensional dispersion. For the simplest case--conservative, two-dimensional dispersion--they found a relationship of the form $A/A_n = f(\theta/\theta_0)$, where $f(\theta/\theta_0)$ is a function of the fractional excess temperature θ/θ_0, and A/A_n is the nondimensional plume area contained within the isotherm at excess temperature θ. For a two-dimensional, conservative plume, the scaling area A_n was found equal to $4/\pi^{3/2} \cdot Q^3/Du^2d^3$, where Q is the volumetric discharge flow rate, D is the ambient diffusivity, u is the ambient velocity, and d is the plume depth, assumed constant. After numerically integrating $f(\theta/\theta_0)$, the authors graphed the relationship θ/θ_0 versus A/A_n on log-log paper.

Perhaps more important than giving exact analytical solutions, the work of Edinger and Polk provides an elegant method of data display which can be used to group and compare data. In particular, it suggested to us that an empirical relation for plume spreading might be discovered by reducing existing field data and presenting it on plots of θ/θ_0 versus A/A_n. The parameterization of A_n could be adjusted to provide the best agreement among existing lake plume data. Such an empirical, "plume-area" approach would greatly simplify some of the problems normally encountered in developing predictive formulas for thermal plumes and at the same time would consider one of the most important features of the thermal plume, namely, the areal extent of its surface water.

The parameterization of A_n will be limited by the type, quality, and quantity of the existing lake plume data. The data are discussed in the next two sections.

3. SURVEY OF THE DATA

We identified seven useful sets of published lake plume data during the course of a literature survey. The sources of data are listed in Table I.

TABLE I. Lake Plume Data

Site	Number of Plumes Analyzed	Source
Waukegan, Lake Michigan	5	Ref. 2
Big Rock, Lake Michigan	1	Ref. 3
Milliken, Cayuga Lake	3	Ref. 4
Waukegan, Lake Michigan	1	Ref. 5
Michigan City, Lake Michigan	2	Ref. 5
Allen S. King, Lake St. Croix	9	Ref. 6
Douglas Point, Lake Huron	2	Ref. 7

With one exception, Table I lists all the lake plume data which we could identify and which we judged to be useful for the type of analysis outlined in Section 2. The one exception is the rather extensive temperature data collected at Waukegan by Biotest Laboratories under contract with Commonwealth Edison Company. The sole reason for not including the Biotest data, which were readily available to us, was our inclination to not overly emphasize the Waukegan site in the analysis.

Several sources of data are not included in Table I because they failed to satisfy the "usefulness" criteria eventually imposed on the data base. (See below.) There are doubtless other sources of published data, which would have satisfied these criteria, but which simply did not come to our attention.

The type and quality of the plume data referenced in Table I vary considerably; however, a few generalizations about the data are possible. (The individual measurements are summarized more carefully in Section 4.)

A "typical" plume measurement consisted of temperature readings collected over a preestablished spatial grid. Additional data usually included intake and outfall temperatures as well as the volumetric discharge flow rate. Where discharge flow rates were not reported, we were able to obtain these through personal communication, either with the investigator or with the utility personnel.

Ambient-current measurements were not always performed and reported. In general, the paucity of current data prevented a more extensive analysis and comparison of data from the various sites.

Constant-temperature contours (isotherms) were usually constructed from the raw data by the investigators. The isotherm plots presented the opportunity for determining plume area within isotherms. Most of the studies listed in Table I included temperature measurements at several depths. There is, however, the problem of assigning a unique value to plume depth which is characteristic of a given plume. For this reason, a comparison of plumes on the basis of depth is very difficult.

Two conclusions can be drawn from a survey of the data:

(1) The analysis is limited, essentially, by the quality of the weakest data sets. The scaling area, A_n, can be easily related only to the volumetric discharge flow rate. Lack of data or the difficulty of assigning plume-wide values to other variables prevents the parameterization of A_n in terms of other variables such as ambient velocity, ambient diffusivity, and plume depth. The most straightforward parameterization of A_n, therefore, is of the form $A_n = Q^a$, where the exponent a is to be determined.

(2) Not all the data of Refs. 2-7 are "useful" for the kind of analysis outlined in Section 2. The plume measurements from any given report are not of equal quality. Temperature data for some plumes are very sparse. In other cases, a strong temperature gradient, usually due to upwelling, makes it impossible to assign a unique value to the temperature of the ambient lake water. We therefore found it necessary to establish a set of criteria, which could be used to define an acceptable ("useful") plume measurement. An acceptable measurement included:

(a) Sufficient temperature data to permit the drawing of at least three closed isotherms.

(b) Measurements of the intake and outfall temperatures.

(c) Measurement of the volumetric discharge flow rate.

(d) Measurement of the ambient lake temperature, with no indication of large thermal gradients in the ambient lake water.

4. SOURCES OF DATA

Plume data from six publications were used. The data were collected at six different sites. (At Waukegan, data were gathered by two authors.) The following is a brief description of the individual measurements. Numerical values of plume variables which were used in the analyses are given in the appendix. For more complete descriptions of the individual measurements, see the original publications.

(1) Romberg et al.[2] conducted 17 surveys of thermal plumes near the outfall of Commonwealth Edison's fossil-fuel power plant at Waukegan, Illinois, during the summer of 1970. Data collected included surface and subsurface temperatures in the plume, as well as in the ambient lake and in the discharge canal. Plant operating data, including generating load and discharge flow rate, were reported for each survey. Meteorological data and current data were collected during most of the measurements. Diffusivities were not measured. The investigators constructed constant-temperature contours for 16 of the surveys.

We took areas within isotherms directly from Ref. 2. Of the 16 plumes, only five satisfied the "usefulness" criteria of Section 3. Eight plumes were rejected due to the presence of upwelling, which made it impossible to assign a unique value to the ambient lake temperature.

(2) As part of a study of biological effects of heated discharges, Krezoski[3] surveyed the plume at Consumer Power Company's Big Rock Point Nuclear Plant near Charlevoix, Michigan. Isotherms were drawn

for the plume measured on June 18, 1968. We obtained the discharge flow rate through personal communication with Mr. C. Axtell of Consumer Power Company.

(3) Sundaram et al.[4] conducted an extensive investigation of the physical effects of thermal discharges on Cayuga Lake. This work included a study of the thermal plume from the Milliken Generating Station. Six isotherm maps, which were developed from infrared overflight data, are presented in Ref. 4. We did not examine three of these plumes because they did not include at least three closed isotherms. The discharge flow rates were furnished to us by Mr. S. A. Lyon of New York State Electric and Gas Corporation.

Reference 4 contains considerable plume, lake, and meteorological data which were not used in the present study.

(4) Ayers et al.[5] surveyed plumes at the Michigan City Generating Station on June 26 and 28, 1969, and at the Waukegan Generating Station on June 30, 1969. These surveys were made in support of biochemical investigations in the vicinity of the two outfalls. Subsurface as well as surface temperature contours were developed for two of the three surveys. We used the areas within surface isotherms from all three surveys.

Intake and outfall temperatures were reported by Ayers et al. for all three plumes.

(5) Fitch[6] conducted 10 temperature surveys in Lake St. Croix near Northern States Power Company's Allen S. King Generating Plant. The measurements, performed during the summers of 1969 and 1970, were made using the Minnesota State coordinate grid system.

Intake, outfall, and ambient lake temperatures were measured at the beginning and end of each survey.

We obtained details concerning the measurement technique and the presentation of the data through personal communication with Mr. D. Bohn and Mr. J. Bechthold of Northern States Power Company. We learned that some of the temperature data were corrected in order to account for observed changes in the ambient water temperature during the measurements. This change in ambient water temperature no doubt also accounts for the difference between outfall temperatures which were reported for the beginnings and ends of some of the surveys. To eliminate this ambiguity, we set the discharge temperature T_D equal to T_{200} + 1°F, where T_{200}, the temperature of the plume water 200 ft from the outfall along the plume axis, was determined from the isotherm plots. This simple algorithm for outfall temperature is based upon the observed rates of temperature decay in the near-field region of the Allen S. King Plant and should be accurate to ±0.5°F.

Using a planimeter, we measured areas within isotherms for the nine isotherm plots that satisfied the criteria given in Section 3. Areas for the three surveys conducted during 1969 were measured from large engineering drawings supplied by Northern States Power Company.

(6) Csanady et al.[7] performed rather detailed measurements of several shore-parallel plumes at the Douglas Point Nuclear Power Plant on Lake Huron during August 1970. Sufficient vertical temperature data and current data were collected during these studies so that the authors were able to determine plume heat fluxes across vertical transects.

Of the four horizontal isotherm plots presented in Ref. 7, two (those for August 24 and 25) include three or more closed isotherms. Although these isotherms refer to a depth of 1.5 ft, we considered them to be representative of the surface-temperature distributions. That this is a good approximation can be seen from an inspection of Table IV of Ref. 7.

5. ANALYSIS

All the plume data identified in Section 4 as useful were accompanied by isotherm plots of the individual plumes. Data reduction, therefore, essentially consisted of planimeter measurements of areas within isotherms for those cases for which the areas were not reported by the field investigators.

The data were displayed on log-log paper with θ/θ_0 plotted against A/Q^a. The most consistent grouping of the buoyant-plume data occurred for $a = 1$, that is, on a plot of the form θ/θ_0 versus A/Q. Figure 1 shows the data so displayed. (The data key for Fig. 1 is shown in Table II.) The curve drawn through the buoyant-plume data is an eyeball fit to these data. The "sinking"-plume curve may not be representative, since it is based on data from only two plume measurements.

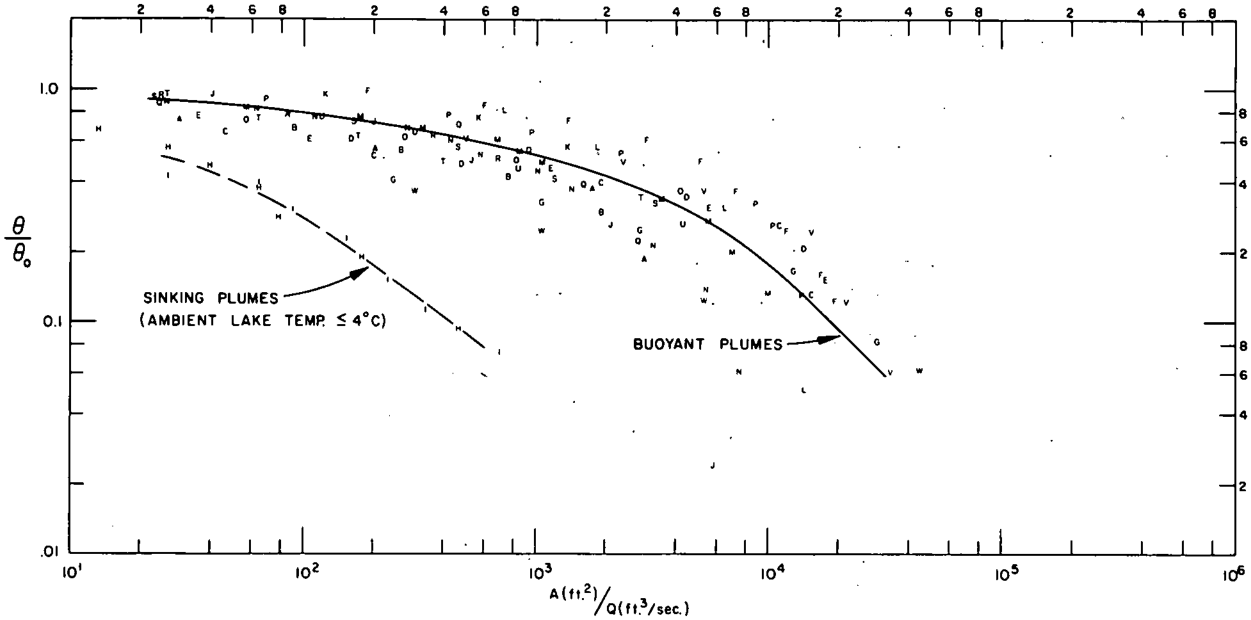

Fig. 1. Fractional Excess Temperature $\left(\frac{\theta}{\theta_0}\right)$ vs the Ratio $\frac{\text{Surface Area (A)}}{\text{Discharge Flow Rate (Q)}}$

TABLE II. Data Key for Fig. 1

A. Waukegan[2] (7/14/70, 12:30-13:37)	M. Allen S. King[6] (8/20/69)
B. Waukegan[2] (7/14/70, 14:50-16:10)	N. Allen S. King[6] (9/4/69)
C. Waukegan[2] (8/12/70, 12:00-13:57)	O. Allen S. King[6] (7/30/69)
D. Waukegan[2] (8/12/70, 16:22-17:53)	P. Allen S. King[6] (6/5/70)
E. Waukegan[2] (8/13/70, 12:12-13:26)	Q. Allen S. King[6] (6/12/70)
F. Big Rock Point[3] (6/18/68)	R. Allen S. King[6] (6/29/70)
G. Milliken[4] (9/17/68)	S. Allen S. King[6] (7/9/70)
H. Milliken[4] (12/10/68)	T. Allen S. King[6] (7/17/70)
I. Milliken[4] (1/8/69)	U. Allen S. King[6] (8/13/70)
J. Michigan City[5] (6/26/69)	V. Douglas Point[7] (8/24/70)
K. Michigan City[5] (6/28/69)	W. Douglas Point[7] (8/25/70)
L. Waukegan[5] (6/30/69)	

6. DISCUSSION

When considered against the various causes of scatter, the clustering of the buoyant-plume data about the central curve in Fig. 1 is remarkable. Sources of scatter among the data points include:

(1) Large plume-to-plume variation in ambient diffusivity, ambient velocity, and plume depth. Although the magnitude of the scaling area should be strongly dependent upon them, these variables have not been included in the parameterization of A_n.

(2) Inaccurate plume areas, especially in the far-field region, where ambient "noise" and plume meandering can produce significant errors in the mapping of thermal plumes. Nearly synoptic, infrared measurements of the type performed by Sundaram et al. should be less susceptible to this type of error.

(3) Different outfall geometries. Although all the outfalls are of the channel type, orifice dimensions and bottom topographies differ considerably. For example, at Waukegan the mouth of the outfall is approximately 6 ft deep by 60 ft wide and the bottom slope is at most 1:100, whereas at the Milliken Station the outfall mouth is approximately 10 ft square and the bottom slope is about 1:10.

(4) The parameterization $A_n \approx Q$ may not be optimum. There may exist a more suitable parameterization, which would lead to a more consistent grouping of the data.

All the data shown on Fig. 1 refer to channel outfall geometries. Other outfall geometries, such as submerged diffuser systems, could produce temperature decays that are inconsistent with those in Fig. 1. It may turn out, however, that the main difference between channel outfalls and other systems is the point at which the curve shown in Fig. 1 is initially intercepted. Consider, for example, a system that uses rapid, subsurface dilution and produces a maximum surface temperature of, say, $\theta/\theta_0 = 0.7$. Since the initial temperature reduction is achieved simply by dilution, the initial surface area within the $\theta/\theta_0 = 0.7$ isotherm may be consistent with that predicted by Fig. 1. The subsequent temperature decay may also follow the curve in Fig. 1.

7. SUMMARY OF RESULTS

Figure 1 summarizes the results of the investigation. All the buoyant-plume data are seen to be reasonably well fit by the curve drawn through the data points. The curve thus represents a phenomenological fit relating fractional excess temperature to the quotient of plume surface area and volumetric discharge flow rate.

The plume data shown in Fig. 1 refer to channel outfall geometries. The fit, therefore, may not be applicable to other outfall geometries, particularly in the region where θ/θ_0 is large. Allowing for this restriction, we believe that the curve represents a useful rule of thumb for predicting surface areas of buoyant thermal plumes.

APPENDIX

Plume Areas

Power plant: Waukegan
Body of water: Lake Michigan
Investigators: Romberg et al.[2]
Date: 7/14/70 (12:00-13:57)
Discharge flow rate: $Q = 1871\ \mathrm{ft}^3/\mathrm{sec}$
Outfall temperature: $T_D = 22.3°\mathrm{C}$
Ambient temperature: $T_A = 17.0°\mathrm{C}$
$\theta_0 = T_D - T_A = 5.3°\mathrm{C}$

Excess Temp, θ, °C	Fractional Excess Temp, θ/θ_0	Total Plume Area, A, within Isotherm at Excess Temp, ft^2
4.0	0.755	5.488×10^4
3.0	0.566	3.841×10^5
2.0	0.377	3.293×10^6
1.0	0.189	5.488×10^6

Power plant: Waukegan
Body of water: Lake Michigan
Investigators: Romberg et al.[2]
Date: 7/14/70 (14:50-16:10)
Discharge flow rate: $Q = 1871\ \mathrm{ft}^3/\mathrm{sec}$
Outfall temperature: $T_D = 24.4°\mathrm{C}$
Ambient temperature: $T_A = 16.7°\mathrm{C}$
$\theta_0 = T_D - T_A = 7.7°\mathrm{C}$

Excess Temp, θ, °C	Fractional Excess Temp, θ/θ_0	Total Plume Area, A, within Isotherm at Excess Temp, ft^2
5.3	0.688	1.722×10^5
4.3	0.558	4.950×10^5
3.3	0.429	1.431×10^6
2.3	0.299	3.615×10^6

Power plant: Waukegan
Body of water: Lake Michigan
Investigators: Romberg et al.[2]
Date: 8/12/70 (12:00-13:57)
Discharge flow rate: $Q = 1730\ \mathrm{ft}^3/\mathrm{sec}$
Outfall temperature: $T_D = 30.5°\mathrm{C}$
Ambient temperature: $T_A = 23.0°\mathrm{C}$
$\theta_0 = T_D - T_A = 7.5°\mathrm{C}$

Excess Temp, θ, °C	Fractional Excess Temp, θ/θ_0	Total Plume Area, A, within Isotherm at Excess Temp, ft^2
5.0	0.667	8.070×10^4
4.0	0.533	3.497×10^5
3.0	0.400	3.309×10^6
2.0	0.267	1.945×10^7
1.0	0.133	2.690×10^7

Power plant: Waukegan
Body of water: Lake Michigan
Investigators: Romberg et al.[2]
Date: 8/12/70 (16:22-17:53)
Discharge flow rate: $Q = 1730\ \mathrm{ft}^3/\mathrm{sec}$
Outfall temperature: $T_D = 30.6°\mathrm{C}$
Ambient temperature: $T_A = 23.5°\mathrm{C}$
$\theta_0 = T_D - T_A = 7.1°\mathrm{C}$

Excess Temp, θ, °C	Fractional Excess Temp, θ/θ_0	Total Plume Area, A, within Isotherm at Excess Temp, ft^2
4.5	0.634	2.798×10^5
3.5	0.493	8.393×10^5
2.5	0.352	7.833×10^6
1.5	0.211	2.462×10^7

Power plant: Waukegan
Body of water: Lake Michigan
Investigators: Romberg et al.[2]
Date: 8/13/70 (12:12-13:26)
Discharge flow rate: $Q = 1624\ ft^3/sec$
Outfall temperature: $T_D = 29.4°C$
Ambient temperature: $T_A = 23.0°C$
$\theta_0 = T_D - T_A = 6.4°C$

Excess Temp, θ, °C	Fractional Excess Temp, θ/θ_0	Total Plume Area, A, within Isotherm at Excess Temp, ft^2
5.0	0.781	5.810×10^4
4.0	0.625	1.743×10^5
3.0	0.469	1.917×10^6
2.0	0.313	9.180×10^6
1.0	0.156	2.905×10^7

Power plant: Big Rock Point
Body of water: Lake Michigan
Investigator: Krezoski[3]
Date: 6/18/68
Discharge flow rate: $Q = 111.4\ ft^3/sec$
Outfall temperature: $T_D = 18.0°C$
Ambient temperature: $T_A = 10.0°C$
$\theta_0 = T_D - T_A = 8.0°C$

Excess Temp, θ, °C	Fractional Excess Temp, θ/θ_0	Total Plume Area, A, within Isotherm at Excess Temp, ft^2
8.0	1.000	2.150×10^4
7.0	0.875	6.780×10^4
6.0	0.750	1.552×10^5
5.0	0.625	3.398×10^5
4.0	0.500	5.707×10^5
3.0	0.375	8.153×10^5
2.0	0.250	1.347×10^6
1.3	0.163	1.901×10^6
1.0	0.125	2.181×10^6

Power plant: Milliken
Body of water: Cayuga Lake
Investigators: Sundaram et al.[4]
Date: 9/17/68
Discharge flow rate: $Q = 254\ ft^3/sec$
Outfall temperature: $T_D = 31.0°C$
Ambient temperature: $T_A = 19.0°C$
$\theta_0 = T_D - T_A = 12.0°C$

Excess Temp, θ, °C	Fractional Excess Temp, θ/θ_0	Total Plume Area, A, within Isotherm at Excess temp, ft^2
5.0	0.417	6.2×10^4
4.0	0.333	2.72×10^5
3.0	0.250	7.22×10^5
2.0	0.167	3.292×10^6
1.0	0.083	7.422×10^6

Power plant: Milliken
Body of water: Cayuga Lake
Investigators: Sundaram et al.[4]
Date: 12/10/68
Discharge flow rate: $Q = 377\ ft^3/sec$
Outfall temperature: $T_D = 14.5°C$
Ambient temperature: $T_A = 4.0°C$
$\theta_0 = T_D - T_A = 10.5°C$

Excess Temp, θ, °C	Fractional Excess Temp, θ/θ_0	Total Plume Area, A, within Isotherm at Excess Temp, ft^2
7.0	0.667	4.923×10^3
6.0	0.571	9.846×10^3
5.0	0.476	1.477×10^4
4.0	0.381	2.462×10^4
3.0	0.286	2.954×10^4
2.0	0.190	6.892×10^4
1.0	0.095	1.772×10^5

Power plant: Milliken
Body of water: Cayuga Lake
Investigators: Sundaram et al.[4]
Date: 1/8/69
Discharge flow rate: $Q = 377\ \text{ft}^3/\text{sec}$
Outfall temperature: $T_D = 15.5°C$
Ambient temperature: $T_A = 2.5°C$
$\theta_0 = T_D - T_A = 13.0°C$

Excess Temp, θ, °C	Fractional Excess Temp, θ/θ_0	Total Plume Area, A, within Isotherm at Excess Temp, ft^2
5.5	0.423	9.846×10^3
5.0	0.385	2.462×10^4
4.0	0.308	3.446×10^4
3.0	0.231	5.908×10^4
2.0	0.154	8.862×10^4
1.5	0.115	1.280×10^5
1.0	0.077	2.658×10^5

Power plant: Michigan City
Body of water: Lake Michigan
Investigators: Ayers et al.[5]
Date: 6/26/69
Discharge flow rate: $Q = 537\ \text{ft}^3/\text{sec}$
Outfall temperature: $T_D = 21.1°C$
Ambient temperature: $T_A = 16.9°C$
$\theta_0 = T_D - T_A = 4.2°C$

Excess Temp, θ, °C	Fractional Excess Temp, θ/θ_0	Total Plume Area, A, within Isotherm at Excess Temp, ft^2
4.1	0.976	2.222×10^4
3.1	0.738	1.111×10^5
2.1	0.500	2.889×10^5
1.1	0.262	1.133×10^6
0.1	0.024	3.133×10^6

Power plant: Michigan City
Body of water: Lake Michigan
Investigators: Ayers et al.[5]
Date: 6/28/69
Discharge flow rate: $Q = 178\ \text{ft}^3/\text{sec}$
Outfall temperature: $T_D = 25.2°C$
Ambient temperature: $T_A = 20.0°C$
$\theta_0 = T_D - T_A = 5.2°C$

Excess Temp, θ, °C	Fractional Excess Temp, θ/θ_0	Total Plume Area, A, within Isotherm at Excess Temp, ft^2
5.0	0.961	2.222×10^4
4.0	0.769	1.000×10^5
3.0	0.576	2.444×10^5

Power plant: Waukegan
Body of water: Lake Michigan
Investigators: Ayers et al.[5]
Date: 6/30/69
Discharge flow rate: $Q = 1872\ \text{ft}^3/\text{sec}$
Outfall temperature: $T_D = 16.6°C$
Ambient temperature: $T_A = 12.8°C$
$\theta_0 = T_D - T_A = 3.8°C$

Excess Temp, θ, °C	Fractional Excess Temp, θ/θ_0	Total Plume Area, A, within Isotherm at Excess Temp, ft^2
3.2	0.842	1.39×10^6
2.2	0.579	3.48×10^6
1.2	0.316	1.219×10^7
0.2	0.052	2.700×10^7

Power plant: Allen S. King
Body of water: Lake St. Croix
Investigator: Fitch[6]
Date: 8/20/69
Discharge flow rate: $Q = 660\ \mathrm{ft}^3/\mathrm{sec}$
Outfall temperature: $T_D = 93.25°F$
Ambient temperature: $T_A = 79.1°F$
$\theta_0 = T_D - T_A = 14.15°F$

Excess Temp, θ, °F	Fractional Excess Temp, θ/θ_0	Total Plume Area, A, within Isotherm at Excess Temp, ft²
11.9	0.841	3.824×10^4
10.9	0.770	1.166×10^5
9.9	0.700	2.144×10^5
8.9	0.629	4.545×10^5
7.9	0.558	5.624×10^5
6.9	0.488	7.041×10^5
4.9	0.346	2.339×10^6
3.9	0.276	3.669×10^6
2.9	0.205	4.729×10^6
1.9	0.134	6.708×10^6

Power plant: Allen S. King
Body of water: Lake St. Croix
Investigator: Fitch[6]
Date: 9/4/69
Discharge flow rate: $Q = 660\ \mathrm{ft}^3/\mathrm{sec}$
Outfall temperature: $T_D = 91.13°F$
Ambient temperature: $T_A = 78.2°F$
$\theta_0 = T_D - T_A = 12.93°F$

Excess Temp, θ, °F	Fractional Excess Temp, θ/θ_0	Total Plume Area, A, within Isotherm at Excess Temp, ft²
11.8	0.913	1.708×10^4
10.8	0.835	4.175×10^4
9.8	0.758	7.400×10^4
8.8	0.681	1.853×10^5
7.8	0.603	2.821×10^5
6.8	0.526	3.814×10^5
5.8	0.449	6.641×10^5
4.8	0.371	9.437×10^5
2.8	0.217	2.108×10^6
1.8	0.139	3.574×10^6
0.8	0.062	4.948×10^6

Power plant: Allen S. King
Body of water: Lake St. Croix
Investigator: Fitch[6]
Date: 7/30/69
Discharge flow rate: $Q = 660\ \mathrm{ft}^3/\mathrm{sec}$
Outfall temperature: $T_D = 86.0°F$
Ambient temperature: $T_A = 78.0°F$
$\theta_0 = T_D - T_A = 8°F$

Excess Temp, θ, °F	Fractional Excess Temp, θ/θ_0	Total Plume Area, A, within Isotherm at Excess Temp, ft²
6.0	0.750	3.715×10^4
5.0	0.625	1.825×10^5
4.0	0.500	5.500×10^5
3.0	0.375	2.784×10^6

Power plant: Allen S. King
Body of water: Lake St. Croix
Investigator: Fitch[6]
Date: 6/5/70
Discharge flow rate: $Q = 457\ \mathrm{ft}^3/\mathrm{sec}$
Outfall temperature: $T_D = 85.25°F$
Ambient temperature: $T_A = 70.0°F$
$\theta_0 = T_D - T_A = 15.25°F$

Excess Temp, θ, °F	Fractional Excess Temp, θ/θ_0	Total Plume Area, A, within Isotherm at Excess Temp, ft²
14.0	0.918	3.150×10^4
12.0	0.787	1.969×10^5
10.0	0.656	4.331×10^5
8.0	0.525	1.063×10^6
5.0	0.328	4.071×10^6
4.0	0.262	4.995×10^6
2.0	0.131	6.433×10^6

Power plant: Allen S. King
Body of water: Lake St. Croix
Investigator: Fitch[6]
Date: 6/12/70
Discharge flow rate: $Q = 638.6\ ft^3/sec$
Outfall temperature: $T_D = 91.35°F$
Ambient temperature: $T_A = 79.2°F$
$\theta_0 = T_D - T_A = 12.15°F$

Excess Temp, θ, °F	Fractional Excess Temp, θ/θ_0	Total Plume Area, A, within Isotherm at Excess Temp, ft^2
10.8	0.889	1.550×10^4
8.8	0.724	2.946×10^5
6.8	0.560	5.969×10^5
4.8	0.395	1.047×10^6
2.8	0.230	1.791×10^6

Power plant: Allen S. King
Body of water: Lake St. Croix
Investigator: Fitch[6]
Date: 6/29/70
Discharge flow rate: $Q = 627.1\ ft^3/sec$
Outfall temperature: $T_D = 93.0°F$
Ambient temperature: $T_A = 78.8°F$
$\theta_0 = T_D - T_A = 14.2°F$

Excess Temp, θ, °F	Fractional Excess Temp, θ/θ_0	Total Plume Area, A, within Isotherm at Excess Temp, ft^2
13.2	0.930	1.563×10^4
11.2	0.789	5.469×10^4
9.2	0.648	2.266×10^5
7.2	0.507	4.375×10^5

Power plant: Allen S. King
Body of water: Lake St. Croix
Investigator: Fitch[6]
Date: 7/9/70
Discharge flow rate: $Q = 614.8\ ft^3/sec$
Outfall temperature: $T_D = 93.1°F$
Ambient temperature: $T_A = 80.9°F$
$\theta_0 = T_D - T_A = 12.2°F$

Excess Temp, θ, °F	Fractional Excess Temp, θ/θ_0	Total Plume Area, A, within Isotherm at Excess Temp, ft^2
11.1	0.910	1.504×10^4
9.1	0.746	1.053×10^5
7.1	0.582	2.857×10^5
5.1	0.418	7.444×10^5
4.1	0.336	2.090×10^6

Power plant: Allen S. King
Body of water: Lake St. Croix
Investigator: Fitch[6]
Date: 7/17/70
Discharge flow rate: $Q = 591.1\ ft^3/sec$
Outfall temperature: $T_D = 95.13°F$
Ambient temperature: $T_A = 81.1°F$
$\theta_0 = T_D - T_A = 14.03°F$

Excess Temp, θ, °F	Fractional Excess Temp, θ/θ_0	Total Plume Area, A, within Isotherm at Excess Temp, ft^2
12.9	0.919	1.538×10^4
10.9	0.777	3.846×10^4
8.9	0.634	1.000×10^5
6.9	0.492	2.462×10^5
4.9	0.349	1.700×10^6

Power plant: Allen S. King
Body of water: Lake St. Croix
Investigator: Fitch[6]
Date: 8/13/70
Discharge flow rate: $Q = 623.7\ ft^3/sec$
Outfall temperature: $T_D = 91.25°F$
Ambient temperature: $T_A = 81.3°F$
$\theta_0 = T_D - T_A = 9.95°F$

Excess Temp, θ, °F	Fractional Excess Temp, θ/θ_0	Total Plume Area, A, within Isotherm at Excess Temp, ft^2
7.7	0.774	7.576×10^4
6.7	0.673	1.894×10^5
4.7	0.472	5.152×10^5
2.7	0.271	2.598×10^6

Power plant: Douglas Point
Body of water: Lake Huron
Investigators: Csanady *et al.*[7]
Date: 8/24/70
Discharge flow rate: $Q = 397\ ft^3/sec$
Outfall temperature: $T_D = 28.3°C$
Ambient temperature: $T_A = 20.3°C$
$\theta_0 = T_D - T_A = 8.0°C$

Excess Temp, θ, °C	Fractional Excess Temp, θ/θ_0	Total Plume Area, A, within Isotherm at Excess Temp, ft^2
5.0	0.625	2.013×10^5
4.0	0.500	9.515×10^5
3.0	0.375	2.123×10^6
2.0	0.250	6.130×10^6
1.0	0.125	8.637×10^6
0.5	0.063	1.350×10^7

Power plant: Douglas Point
Body of water: Lake Huron
Investigators: Csanady *et al.*[7]
Date: 8/25/70
Discharge flow rate: $Q = 397\ ft^3/sec$
Outfall temperature: $T_D = 28.1°C$
Ambient temperature, $T_A = 20.1°C$
$\theta_0 = T_D - T_A = 8.0°C$

Excess Temp, θ, °C	Fractional Excess Temp, θ/θ_0	Total Plume Area, A, within Isotherm at Excess Temp, ft^2
3.0	0.375	1.200×10^5
2.0	0.250	4.200×10^5
1.0	0.125	2.060×10^6
0.5	0.063	1.824×10^7

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