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No. 27

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THE UNIVERSITY OF CHICAGO

THE INSTITUTE FOR COMPUTER RESEARCH

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The Institute for Computer Research

The University of Chicago

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THE COMPUTER INTERFACING LABORATORY

(This is a copy with minor revisions of an information memorandum circulated to the University community.)

Many experimental research efforts can advantageously incorporate real time processing with a stored program digital computer. By placing the logic of process control, data acquisition, and data storage in a modifiable program the experimenter can gain tremendous flexibility in the configuration of his experiment. The advent of the so-called "minicomputer" has made it possible to incorporate digital control economically into a wide variety of man-machine situations, appropriate to the experimental environment in the physical, biological, and social sciences. As the techniques get more sophisticated, however, the nature of the expertise required to implement such experimental setups becomes more specialized. The Institute for Computer Research has been involved in work of this kind for several years, and the Computer Interfacing Laboratory is intended to provide a means whereby its resources can be made available to the entire University community.

The components of the Maniac III System, described in detail on the attached list, include IBM compatible input/output equipment, a direct link to the Computation Center, analog input/output and computation facilities, and other special features making it suitable for experimental interaction. In particular, it may be used as a compiler, debugger and

maintenance system for a battery of minicomputers. The two IBM 1301 disk storage units permit Maniac III and connected minicomputers to be used in applications requiring modest-sized data bases. At present there is one satellite minicomputer system, based on a PDP-8. This is portable, and can be moved to an experimental site at the appropriate time in the process of checkout to perform on-site experimental control and data acquisition. When not resident at an experimental site, however, the PDP-8 has all the resources of the Maniac III system available through a direct link, which leads to greater efficiency in processes such as extensive input/output, program editing, and the like.

The support staff of engineers, programmers and technicians is thoroughly familiar with the available hardware and software. Maniac III was constructed in the Institute for Computer Research, and all subsequent expansion, maintenance and systems programming has been done by the Institute staff. The PDP-8 has been under the charge of the engineering staff for five years, and during this time several special purpose peripherals have been built. The programming staff has done extensive machine-language coding on large-scale IBM equipment, Maniac III, and on smaller machines such as the PDP-8 and PDP-11, the NCR 315, and the ASI 6000 series. Their work includes writing special programs for monitoring experimental equipment, as well as designing simulators, assemblers, compilers and utility programs.

The key to the power of this facility is the hardware/software interfacing capability provided by the programming and engineering staff. Communication and understanding between the hardware and software staffs is remarkably good and the resulting cooperation and coordination is especially useful in the design, testing and maintenance of special purpose systems. The concept of a central support system with a variety of peripherals and portable minicomputers, along with a flexible interfacing capability, is not only a unique resource for the University, but represents a mode of operation beginning to be exploited in the computer field as a whole.

The Computer Interfacing Laboratory can provide service along two lines:

1. Consultation and necessary hardware/software design support for investigators intending to purchase computer equipment. The staff keeps up-to-date information on commercially available computer equipment, which permits the matching of experimental requirements with hardware/software in such a way as to maximize performance and minimize cost.
2. Use of the ICR hardware/software facilities for prototypical or short-term experimental setups. This allows the investigator access to equipment too expensive to purchase or maintain on his own. It also allows him to test the feasibility of his experimental setup before undertaking purchase.

A schedule of charges for these services has been devised, and cost estimates will be made for specific proposed projects. The staff will be happy to explore the possibilities regarding such projects with potential users.

COMPUTER INTERFACING LABORATORY

HARDWARE SYSTEMS

The primary components and their interconnections are as follows (see also Figure 1):

A. PDP-8 System. This completely portable system may be transported anywhere for on-site support. It includes:

1. 2 Dec Tape Drives. These mini-tape drives store the PDP-8 operating system and user programs. They may also be used to collect experimental data.
2. Cipher Tape Drive. This tape drive, currently being interfaced, offers two advantages: a) the drive will allow data collection at 20 KHz; b) the drive produces 9-track IBM compatible tapes which may be processed on the 360/65.
3. A/D Converter. This unit provides up to 8 analog input channels at variable rates and precision.
4. D/A Converter. Applications include driving graphics output devices or control of experiments.

B. Extended Maniac III System. The devices specified below enhance the ability of the Maniac III complex to develop software, collect data and provide information in real time, and to transmit data to the appropriate data processor.

1. IBM Z61A Display Unit. This is a large CRT display with a full range of vector and character capabilities. This display as interfaced to Maniac III is suitable for projects requiring online graphic output and for many types of man-machine interaction.

2. Tektronix Storage Scope. This CRT, with long wire interface to Maniac III, provides a modest display capability that may be used at a remote location for monitoring an experiment connected to the Maniac III complex.

3. Maniac III-IBM 360 Link. The direct connection between Maniac III and the Computation Center computer system gives Maniac III access to the peripherals and computing power of the Computation Center. In particular, it makes it possible to use Maniac III as a remote job entry station for the 360/65 and 7094 and, in turn, to use 360 Asp to provide main processor support for Maniac III.

4. Maniac III-PDP-8 Link. One advantage of this connection is that an experimenter using the PDP-8 locally has flexible access to data storage, computing power, and a range of peripherals. An additional advantage for program development is that programs assembled on Maniac and stored on the 1301 disk can be easily loaded into the PDP-8. Similarly, core dumps may be printed by Maniac III.

5. PDP-8 Bus Simulator. This subcomputer, analogous to multiplexor channel, allows Maniac III to run directly any device that can be attached to the PDP-8. The utility of being able to plug in devices to Maniac III without building additional interfaces is obvious. For example, Maniac III, which has no magnetic tape units, can read and write IBM compatible tapes using the Cipher Drive. Another major advantage stems from the fact the subcomputer duplicates the input/output characteristics of the PDP-8 well enough to allow a PDP-8 simulator to run on Maniac III. Thus, if the PDP-8 is inoperative or at a remote site, Maniac III can actually fill in for it. In particular, program development for several projects can be done concurrent with the running of experiments.

6. IBM 360 Bus Simulator. The Maniac III interface to the IBM equipment is implemented as a main frame instruction and is consequently more limited than the PDP-8 Bus Simulator. However, it performs most of the functions of an IBM 360 selector channel, and nearly any IBM 360 compatible device can be interfaced to Maniac III.

C. EAI 680 System. This analog computer and plotter serves along with the PDP-8 to give a hybrid capability.

1. EAI 680. This analog unit can be used as a stand-alone unit, or be interfaced with the PDP-8 system as shown in the block diagram, Figure 1.

2. EAI 8850/680 Variplotter. This 11" by 17" X-Y plotter is driven by the EAI 680 Analog Computer, and can therefore also be used as an output device for Maniac III or the PDP-8. It can plot points or draw lines under computer control, although it is not equipped for symbol printing.

Fig.1

REALIZATION IS UNIVERSAL*

J.A. Goguen

Abstract

Behavior is left adjoint to minimal realization, as functors between certain categories of machines and behaviors. This gives a succinct characterization of minimal realization valid for discrete as well as linear machines, with or without finiteness condition. Realization theory is therefore expressed in contemporary algebra in a way which reveals its inner structure and suggests generalizations. An adjunction between regular sets and finite state acceptors follows as a corollary.

It is well-known that the Nerode construction gives a canonical realization, but the sense in which "canonical" is to be understood seems not to have been stated sufficiently strongly or generally. This paper gives conditions which characterize minimal realization uniquely up to isomorphism. The very same characterization statements are valid for all categories of machines one could expect, for example, discrete machines and linear machines, with or without finiteness condition. The characterizing conditions are universal, in the sense of categorical algebra.

If M is a machine, let EM denote its external behavior. Then the universal statement is that a machine N is a minimal realization of a behavior f iff N is reachable and there is a morphism of behaviors $e : EN \longrightarrow f$ such that every morphism

*Solei Dei Gloria.

$EM \longrightarrow f$ surjective on input alphabets, with M reachable, can be factored as $(Eg)e$ for a unique machine morphism $g:M \longrightarrow N$ which is surjective on input alphabets. An equivalent statement is that minimal realization is right adjoint to behavior, as functors between categories of machines and behaviors which will be defined later. In mathematics, adjoint situations have the connotation of fundamental structural relationships; see MacLane [8.]. A by-product of this is the fact that the minimal or reduced machines form a reflexive subcategory. These results embed realization theory in contemporary algebra in a way which reveals inner structure and suggests generalizations.

In system theory two common approaches are the state variable [1] and input-output relation [10]. We wish to suggest, as a philosophical principle arising from the present work, that these approaches are not opposed, but rather dual, or more precisely, adjoint. This principle will appear as theorems in various specific situations more complex than those treated in this paper, and will be explored in subsequent papers. However, we do treat the adjointness of regular sets and finite state acceptors in the last section.

The reader is assumed sufficiently familiar with automaton and control theories not to require much by way of motivation for our definitions, which are most often equivalent to the standard ones. In case of need, Arbib [1], Arbib and Zeiger [2] and Kalman [6], can be consulted. The reader who is only somewhat familiar with categorical algebra may consult the Appendix to this paper, which contains all the

definitions and results which we use. Since limitations on space preclude our giving very many examples or proofs, a reader with no categorical background may wish to consult some textbook before reading this paper. MacLane [8] is especially recommended and MacLane and Birkhoff [9] Chapter XV contains a fairly elementary exposition of the basic concepts.

The following is written explicitly for arbitrary discrete machines. The small changes that have to be made to treat finite discrete machines and linear machines (also called sampled data systems) are indicated in Section (O.); these results are an improvement of Arbib and Zeiger's [2] simplification of Kalman's [6] work.

(A.) A machine M is a sextuple $\langle X, S, Y, \delta, \lambda, \sigma \rangle$, where X, S, Y are sets (called the input, state, and output sets or alphabets, respectively), $\delta: X \times S \longrightarrow S$, $\lambda: S \longrightarrow Y$, and $\sigma \in S$ (called the transition and output functions, and initial state, respectively). A morphism $M \longrightarrow M'$ of machines is a triple $\langle a, b, c \rangle$ of maps, $a: X \longrightarrow X'$, $b: S \longrightarrow S'$, and $c: Y \longrightarrow Y'$ such that¹

¹For consistency, one might wish to define the initial state by a map $\sigma: 1 \longrightarrow S$, where 1 is a one point set, and if $*$ is that point, $\sigma(*) \in S$ is the initial state. For then the condition $b(\sigma) = \sigma'$ can be expressed by commutativity of the diagram

$$\begin{array}{ccc} & \sigma & \nearrow \\ 1 & & S \\ & \sigma' & \searrow \\ & & S' \end{array} \quad \begin{array}{c} \downarrow b \\ \end{array}$$

$b(\sigma) = \sigma'$, and the diagrams

$$\begin{array}{ccc} X \times S & \xrightarrow{\delta} & S \\ \downarrow a \times b & & \downarrow a \\ X' \times S' & \xrightarrow{\delta'} & S' \end{array} \quad \text{and} \quad \begin{array}{ccc} S & \xrightarrow{\lambda} & Y \\ \downarrow b & & \downarrow c \\ S' & \xrightarrow{\lambda'} & Y' \end{array}$$

commute. We shall call a, b, c respectively the first, second, and third, or input, state, and output, components of the morphism $\langle a, b, c \rangle$. It is easy to verify that machines and their morphisms constitute a category² under component-wise composition, which we denote $\underline{\text{Mach}}$.

(B.) Given M , define $\delta^+: X^* \rightarrow S$ recursively by $\delta^+(\Lambda) = \sigma$ and $\delta^+(xw) = \delta(x, \delta^+(w))$, for $x \in X$ and $w \in X^*$, where X^* is the free monoid generated by X and Λ is the empty word. If $\langle a, b, c \rangle: M \rightarrow M'$, then

$$\begin{array}{ccc} X^* & \xrightarrow{\delta^+} & S \\ \downarrow a^* & & \downarrow c \\ X'^* & \xrightarrow{\delta'^+} & S' \end{array}$$

commutes, where $a^*: X^* \rightarrow X'^*$ is the letter-wise extension of $a: X \rightarrow X'$ to words, defined by $a^*(\Lambda) = \Lambda$ and $a^*(xw) = a(x)a^*(w)$. For $c\delta^+(\Lambda) = c(\sigma) = \sigma'$, while $\delta'^+a^*(\Lambda) = \delta'^+(\Lambda)$; now assuming $c\delta^+(w) = \delta'^+a^*(w)$, it follows that $c\delta^+(xw) = \delta'^+a^*(xw)$, since $c\delta^+(xw) = c\delta(x, \delta^+(w)) = \delta'(a(x), c\delta^+(w))$ by the first diagram of (A.), and $\delta'^+a^*(xw) = \delta'^+(a(x)a^*(w)) = \delta'(a(x), \delta'^+(a^*(w)))$.

² This category is complete (or in the finite case, finitely complete) as it is easily seen to have equalizers and products (or finite products).

We say M is reachable iff δ^+ is surjective, and we denote the full subcategory of reachable machines by $\underline{\text{Machr}}$. Denote the subcategory³ of $\underline{\text{Machr}}$ all of whose morphisms have surjective input component by just $\underline{M}$. This is the category of machines we will find most useful.

(C.) A behavior is a function $f: X^* \longrightarrow Y$, where X and Y are sets; a morphism $f \longrightarrow f'$ of behaviors is a pair $\langle a, c \rangle$, where $a: X \longrightarrow X'$ and $c: Y \longrightarrow Y'$, such that

$$\begin{array}{ccc} X^* & \xrightarrow{f} & Y \\ \downarrow a & & \downarrow c \\ X'^* & \xrightarrow{f'} & Y' \end{array}$$

commutes. Behaviors and their morphisms constitute a category under component-wise composition, denoted $\underline{\text{Beh}}$. Let $\underline{B}$ denote the subcategory⁴ whose morphisms have surjective input component.

(D.) Given M , call $EM = \lambda \delta^+: X^* \longrightarrow Y$ the external behavior⁵

³It can be shown that Machr is complete (or finitely complete in the finite case), but that $\underline{M}$ has neither products nor equalizers. These facts are not used in this paper.

⁴It can be shown that $\underline{\text{Beh}}$ is complete (or finitely complete with the finiteness condition), but $\underline{B}$ has neither equalizers nor products.

⁵In some ways it would be more satisfying to define the external behavior to be the input sequence to output sequence function $\bar{f}: X^* \longrightarrow Y^*$, defined by the concatenation $\bar{f}(x_0 \dots x_n) = f(x_0)f(x_0 x_1) \dots f(x_0 \dots x_n)$. For this would be the input-output component of the behavior of the system diagram for the machine, in the general sense of Goguen [4], [5]. However it is easy to show that f and $\bar{f}$ determine each other uniquely, and f will be technically more convenient.

of M , and given $\langle a, b, c \rangle : M \longrightarrow M'$, let $E\langle a, b, c \rangle = \langle a, c \rangle$. Then $E\langle a, b, c \rangle$ is a morphism of behaviors; for commutativity of the diagram in (C.) follows from concatenation of the square in (B.) with the second square in (A.). Moreover, E is a functor $\underline{\text{Mach}} \longrightarrow \underline{\text{Beh}}$, since preservation of composition follows from concatenation of squares in $\underline{\text{Beh}}$. Because a is surjective in $\langle a, c \rangle$ if it is surjective in $\langle a, b, c \rangle$, E is also a functor $\underline{M} \longrightarrow \underline{B}$. We say M realizes f iff $EM = f$.

(E.) Given $f: X^* \longrightarrow Y$, let Nf denote its Nerode realization $\langle X, S_f, Y, \delta_f, \lambda_f, \sigma_f \rangle$, where $S_f = X^* / \equiv_f$, with $w \equiv_f y$ iff $f(uw) = f(uy)$ for all $u \in X^*$; $\delta_f(x, [w]) = [xw]$, where $[w]$ denotes the $\equiv_f$ -equivalence class of $w \in X^*$; $\lambda_f([w]) = f(w)$; and $\sigma_f = [\Lambda]$. The Nerode relation $\equiv_f$ is easily seen to be reflexive, transitive, and symmetric; write $[w]_f$ for the $\equiv_f$ -class of w if there is a danger of confusion, and $[w]$ otherwise. Let us check λ_f is well-defined: $w \equiv_f y$ implies $f(w) = f(y)$ follows from setting $u = \Lambda$ in the definition of $\equiv_f$.

Given $\langle a, c \rangle : f \longrightarrow f'$, define $N\langle a, c \rangle = \langle a, b, c \rangle : Nf \longrightarrow Nf'$ by $b[w]_f = [a^*(w)]_{f'}$. This is well-defined provided a is surjective: for $w \equiv_f y$ iff $f(uw) = f(uy)$ for all $u \in X^*$, and $a^*(w) \equiv_{f'} a^*(y)$ iff $u'f'(a^*(w)) = u'f'(a^*(y))$ for all $u' \in X'^*$, and since a is surjective, any $u' \in X'^*$ can be written $a^*(u)$ for some $u \in S^*$, and then we are reduced to showing $f'(a^*(u)a^*(w)) = f'(a^*(u)a^*(y))$, which follows from the square in (C.).

We now show $N\langle a, c \rangle = \langle a, b, c \rangle$ is a morphism of machines; that is, we show $b(\sigma_f) = \sigma_{f'}$, and

$$\begin{array}{ccc} X \times S_f & \xrightarrow{\delta_f} & S_f \\ \downarrow a \times b & \delta_{f'} & \downarrow b \\ X' \times S_{f'} & \xrightarrow{\delta_{f'}} & S_{f'} \end{array} \quad \text{and} \quad \begin{array}{ccc} S_f & \xrightarrow{\lambda_f} & Y \\ \downarrow b & \lambda_{f'} & \downarrow c \\ S_{f'} & \xrightarrow{\lambda_{f'}} & Y' \end{array}$$

commute. First, $b(\sigma_f) = [a^*(\Lambda)]_{f'} = [\Lambda]_{f'} = \sigma_{f'}$. Next, $b \delta_f(x, [w]_f) = b([xw]_f) = [a^*(xw)]_{f'}$, while $\delta_{f'}(a(x), c([w]_f)) = \delta_{f'}(a(x), [a^*(w)]_{f'}) = [a(x)a^*(w)]_{f'} = [a^*(xw)]_{f'}$. Finally, $c \lambda_f([w]_f) = cf(w)$, while $\lambda_{f'} b([w]_f) = \lambda_{f'}([a^*(w)]_{f'}) = f'a^*(w)$, and $cf(w) = f'a^*(w)$ by the square of (C.).

It is now easy to show that $N: \underline{B} \longrightarrow \underline{M}$ is a functor, by verifying that composition is preserved and noting that Nf is always reachable (but first see the next section for the notion of reachability).

(F.) Given $f: X^* \longrightarrow Y$, we show that $\delta_f^+(w) = [w]_f$ in Nf . First, $\delta_f^+(\Lambda) = \sigma_f = [\Lambda]_f$. Next, under the inductive assumption that $\delta_f^+(w) = [w]_f$ for all $w \in X^*$ of length $n \geq 0$, we show the same for length $n+1$. For $\delta_f^+(xw) = \delta_f(x, \delta_f^+(w)) = \delta_f(x, [w]_f) = [xw]_f$ for $x \in X$, $w \in X^*$ of length n ; see (B.).

Since each element of S_f is of the form $[w]_f$, it follows that Nf is reachable.

(G.) $EN = \underline{B}$, the identity functor⁶ on $\underline{B}$.

Proof: Let $f: X \xrightarrow{*} Y$. Then $Nf = \langle X, S_f, Y, \delta_f, \lambda_f, \sigma_f \rangle$, and by

(F.), $(ENf)(w) = \delta_f^+([w]) = f(w)$; i. e., $ENf = f$, proving (G.) for

objects. But $EN\langle a, c \rangle = E\langle a, b, c \rangle = \langle a, c \rangle$, so (G.) also holds for

morphisms, provided the domain and codomain behaviors of $EN\langle a, c \rangle$

are the same as for $\langle a, c \rangle$. But that follows from $ENf = f$, proved

above.

(H.) Given M , let $\gamma_M: M \rightarrow NEM$ denote $\langle X, q_M, Y \rangle$, in which X

and Y are used to indicate the identity functions, and where $q_M(s) = [w]_f$

for $s \in S$, where $w \in X^*$ is chosen such that $\delta^+(w) = s$; this is

possible because of reachability. First, q_M is well-defined. For if

$\delta^+(w) = \delta^+(v) = s$, then $w \equiv_f v$, since $f(w) = \lambda \delta^+(w) = \lambda \delta^+(v) = f(v)$.

Denote q_M by just q if confusion is unlikely.

We now show that $\langle X, q, Y \rangle$ is a machine morphism. First,

$q(\sigma) = [\Delta]_f = \sigma_f$; it remains to show that

$$\begin{array}{ccc} X \times S & \xrightarrow{\delta} & S \\ \downarrow X \times q & \searrow \delta_f & \downarrow q \\ X \times S_f & \xrightarrow{\delta_f} & S_f \end{array} \quad \text{and} \quad \begin{array}{ccc} S & \xrightarrow{\lambda} & Y \\ \downarrow q & \searrow \lambda_f & \downarrow Y \\ S_f & \xrightarrow{\lambda_f} & Y \end{array}$$

commute. Let $s \in S$ and assume $\delta^+(w) = s$. Then for $x \in X$,

$q(\delta(x, s)) = [xw]$ since $\delta^+(xw) = \delta(x, s)$, while $\delta_f(x, q(s)) =$

$\delta_f(x, [w]) = [xw]$ also. Finally, $\lambda_f q(s) = \lambda_f([w]) = f(w)$, while

$\lambda(s) = \lambda \delta^+(w) = f(w)$.

⁶In this paper we often identify objects with their identity morphisms. Extended to the category of categories, in which functors are morphisms and categories objects, this gives the notation used in (G.); it is convenient in later calculations. See also the Appendix.

(I.) η is a natural transformation⁷ $\eta: \underline{M} \Rightarrow \underline{NE}$, where $\underline{M}$ denotes the identity functor.

Proof: Let $\langle a, b, c \rangle: M \longrightarrow M'$. We show that

$$\begin{array}{ccc} M & \xrightarrow{\eta_M} & NEM \\ \downarrow \langle a, b, c \rangle & & \downarrow NE\langle a, b, c \rangle \\ M' & \xrightarrow{\eta_{M'}} & NEM' \end{array}$$

commutes, by showing commutativity for each component. Since N and E do not change the input and output components, it suffices to check the state component. Letting $N\langle a, c \rangle = \langle a, b', c \rangle$, we must show

$$\begin{array}{ccc} S & \xrightarrow{q_M} & S_f \\ \downarrow b & & \downarrow b' \\ S' & \xrightarrow{q_{M'}} & S_{f'} \end{array}$$

commutes. Let $s \in S$, and assume $\delta^+(w) = s$. Let $s' = b(s)$. Then $\delta^+(a^*(w)) = s'$, by the square in (C.), so that $q_M(s) = [w]_f$ and $q_{M'}(s') = [a^*(w)]_{f'}$. We therefore must show that $b'([w]_f) = [a^*(w)]_{f'}$; but this is already the definition of the state component b' of $N\langle a, c \rangle$ given in (E.).

(J.) $E\eta_M = EM$, the identity morphism in $\underline{B}$ on EM .

Proof: Since $\eta_M: M \longrightarrow NEM$ in $\underline{M}$, $E\eta_M: EM \longrightarrow ENEM$ in $\underline{B}$. But by (G.) $EN(EM) = EM$, so $E\eta_E: EM \longrightarrow EM$. Since

⁷ The morphism $\eta_M: M \longrightarrow NEM$ is the mapping of the machine M onto its minimal realization. Thus NEM should be isomorphic to the usual quotient machine constructed from M . This will follow from later results.

$\eta_M = \langle X, q, Y \rangle$, $E\eta_M = \langle X, Y \rangle$, the identity morphism on $EM: X^* \rightarrow Y$.

This result says the same thing (G.) does, but in a different way, that NM has the same behavior as M , for M reachable.

(K.) $\eta_{Nf} = Nf$, the identity morphism on Nf in $\underline{M}$.

Proof: Given $f: X^* \rightarrow Y$ in $\underline{B}$, $Nf = \langle X, S_f, Y, \delta_f^+, \lambda_f, \sigma_f \rangle$,

and $\eta_{Nf}: Nf \rightarrow NENf = Nf$, again by (G.). Since $\eta_{Nf} = \langle X, q_{Nf}, Y \rangle$ by (H.), we need only show that $q_{Nf} = S_f$, the identity on S_f . Every state in S_f is of the form $[w]_f$. By (F.) $\delta_f^+(w) = [w]_f$, so $q_{Nf}([w]_f) = [w]_f$, by its definition the $\equiv_f$ -class of some $w \in X^*$ which δ_f^+ moves to $[w]_f$. But this just says $q_{Nf} = S_f$, as desired.

This result says that applying NE to a Nerode machine has no effect; that is, they are already reduced (this is an intuitive statement; the term "reduced" is not defined until Section (M.)).

(L.) E is left adjoint to N , as functors between $\underline{M}$ and $\underline{B}$ (this is written $E \dashv N$).

Proof: It is known (see Theorem 4 of the Appendix) that it suffices to give natural transformations $\eta: M \rightarrow NE$ and $\epsilon: EN \rightarrow B$ such that the triangular identity diagrams

$$\begin{array}{c} \text{ENE} \\ \nearrow \quad \searrow \\ \text{E} \quad \quad \text{E} \\ \text{E} \quad \quad \text{E} \end{array} \quad \text{and} \quad \begin{array}{c} \text{NEN} \\ \nearrow \quad \searrow \\ \text{N} \quad \quad \text{N} \\ \text{N} \quad \quad \text{N} \end{array}$$

commute. We take η as defined in (H.), and let ξ be the identity; recall again that $EN = \underline{B}$ by (G.).

To verify the first triangle on a reachable machine M , we must have commutativity of

$$\begin{array}{c} \text{ENEM} \\ \nearrow \quad \searrow \\ (E\eta)_M \quad \quad (\xi E)_M \\ \text{EM} \quad \quad \text{EM} \\ \text{EM} \quad \quad \text{EM} \end{array}$$

where $(E\eta)_M = E\eta = EM$ by (J.), $ENEM = EM$ by (G.), and $(\xi E)_M = \xi_{EM} = EM$ since ξ_f is always the identity. Thus the condition is just $(EM)(EM) = EM$, that the composite of identities is the identity.

For a behavior f , the second triangular identity becomes commutativity of

$$\begin{array}{c} \text{NENf} \\ \nearrow \quad \searrow \\ \eta_{Nf} \quad \quad N(\xi_f) \\ \text{Nf} \quad \quad \text{Nf} \\ \text{Nf} \quad \quad \text{Nf} \end{array}$$

where $\eta_{Nf} = Nf$ by (K.), $NENf = Nf$ by (G.), and $(N\xi)_f = N(\xi_f) = Nf$ since ξ_f is the identity on f . We are therefore reduced to the identity $(Nf)(Nf) = Nf$ on the identity Nf .

Thus (J.) and (K.) in effect are the triangular identities.

(M.) A machine M is reduced⁸ iff all its states are inequivalent under the relation $s \approx s'$ iff $f(w, s) = f(w, s')$ for all $w \in X^*$, where $f(w, s) = \lambda \delta^+(w, s)$, with $\delta^+(w, s)$ defined by $\delta^+(\Lambda, s) = s$ and $\delta^+(xw, s) = \delta(x, \delta^+(w, s))$.

In this section we show that the full subcategory $\underline{R}$ of $\underline{M}$ whose objects are the reduced machines is a reflexive subcategory (this definition is reviewed in the Appendix) of $\underline{M}$: that is, we show that for each reachable machine M there is a reduced reachable machine RM and a morphism $\eta_M: M \rightarrow RM$ in $\underline{M}$ such that for any $g: M \rightarrow M'$ with M' reachable and reduced factors as $h\eta_M$ for some unique $h: RM \rightarrow M'$. A reflection RM of M is called a reduction of M .

We show first that the image subcategory $\underline{NB}$ of $\underline{N}$ in $\underline{M}$ is reflexive. This follows from the fact that for any M in $\underline{M}$

⁸In control and linear system theories, the term observable is more likely to be used than the automaton theoretic synonym reduced. Moreover, observable is more likely to be defined by the condition that the function $F: S \rightarrow Y^{X^*}$ be injective, where Y^{X^*} is the set of all functions $f: X^* \rightarrow Y$ (i.e., behaviors) and $F(s) = \lambda \delta^+(-, s): X^* \rightarrow Y$, with $\delta^+(w, s)$ as defined just a little later in the text (for example, [2] and [6] proceed this way). The above definition of observable is easily seen to be equivalent to our definition of reduced.

$\eta_M: M \rightarrow NEM$ is a universal arrow from M to N (this fact follows from the adjointness $E \dashv N$ by Theorem 2 of the Appendix).

In fact, the adjunction gives us that any $M \xrightarrow{g} Nf$ with f in $\underline{B}$ factors as $\eta_M Nh$ for a unique $h: EM \rightarrow f$ in $\underline{B}$; i.e., any $M \xrightarrow{g} M'$ with M' in $N\underline{B}$ factors as $\eta_M Nh$ for a unique $Nh: NEM \rightarrow M'$ in $N\underline{B}$.

$$\begin{array}{ccc}
 M & \xrightarrow{\eta_M} & NEM \\
 & \searrow g & \downarrow \exists! Nh \\
 & & Nf
 \end{array}$$

We next show that $N\underline{B}$ is an equivalent subcategory of $\underline{R}$. Since both $\underline{R}$ and $N\underline{B}$ are full subcategories of $\underline{M}$, and since each Nerode machine is reduced, $N\underline{B}$ is a full subcategory of $\underline{R}$; moreover, an inclusion functor is certainly faithful, so it remains to check that each M in $\underline{R}$ is isomorphic to some machine in $N\underline{B}$. We show that if M is reduced, then $\eta_M: M \rightarrow NEM$ is an isomorphism; for this it suffices to show that $q_M: S \rightarrow S_f$ is an isomorphism, because if a and d are isomorphisms, then the left diagram commutes iff the right diagram does,

$$\begin{array}{ccc}
 A & \xrightarrow{c} & B \\
 \downarrow a & & \downarrow d \\
 C & \xrightarrow{b} & D
 \end{array}
 \qquad
 \begin{array}{ccc}
 A & \xrightarrow{c} & B \\
 \downarrow a^{-1} & & \downarrow d^{-1} \\
 C & \xrightarrow{b} & D
 \end{array}$$

proving that $\langle X, q_m^{-1}, Y \rangle$, the inverse to η_M , is a machine morphism.

To show q_M is bijective, we show it is injective and surjective (this special trick will work in the categories of finite sets, sets, linear spaces, and finite dimensional linear spaces). First surjectivity. Recall $q_M(s) = [w]_f$ where $w \in X^*$ is such that $\delta^+(w) = s$,

and letting $\{w\}_f$ be an arbitrary element of S_f , we see that $q_M(\delta^+(w)) = \{w\}_f$. Now let $s \neq_f s'$ and suppose that $\delta^+(w) = s$ and $\delta^+(w') = s'$; to prove q_M injective we show that $\{w\}_f \neq \{w'\}_f$. Because M is reduced, $s \neq s'$, which means that $\lambda \delta^+(v, s) \neq \lambda \delta^+(v, s')$ for some $v \in X^*$. But $\delta^+(v, s) = \delta^+(vw)$ and $\delta^+(v, s') = \delta^+(vw')$, so that $f(vw) = \lambda \delta^+(v, s) \neq \lambda \delta^+(v, s') = f(vw')$, so that $w \neq_f w'$.

Thus we have shown that $\underline{NB} \subseteq \underline{R} \subseteq \underline{M}$, and that $\underline{NB}$ is an equivalent subcategory of $\underline{R}$. But $\underline{NB}$ is isomorphic to $\underline{B}$, so $E \dashv N$ implies that $\underline{NB}$ is a reflexive subcategory of $\underline{M}$. Therefore, by the Lemma in the Appendix, $\underline{R}$ is a reflexive subcategory of $\underline{M}$, as claimed.

From this we obtain the usual result that a machine M is a minimal realization of its behavior iff it is reachable and reduced (or observable, in the control theoretic language).

(N.) We are now prepared to take a somewhat more general point of view. The functor N gives a particular reduced realization for each behavior, and its adjointness to E expresses its minimality. But there are other constructions for minimal realization (e.g., via the Myhill semigroup of the machine) and so we assert that a minimal realization functor is exactly a right adjoint to the behavior functor E . We first give a universal arrow style characterization of the minimal realization of a single behavior.

In fact, Theorem 2 of the Appendix allows us to assert from the adjunction $E \dashv N$, that $M = Nf$ is characterized up to isomorphism by the condition: there is a morphism $e: EM \rightarrow f$ such that every $EM' \rightarrow f$ which is surjective on inputs and has M' reachable, can be factored as $(Eg)e$ for a unique $g: M' \rightarrow M$, surjective on inputs. Notice that e is not required to be an isomorphism; but we know that if

$$\begin{array}{ccc} & M & \\ \exists! g \uparrow & \vdots & \\ & M' & \end{array} \quad \begin{array}{ccc} EM & \xrightarrow{e} & f \\ \uparrow Eg & \nearrow & \\ EM' & & \end{array}$$

e satisfies this condition, it will be an isomorphism. Moreover, the condition need be satisfied only for a rather small class of M' and $EM' \rightarrow f$. In this sense, the condition is rather weak, and the characterization result rather strong. In case we want not isomorphism but actual equality on the input and output alphabets, we require e to be an equality, and get the following simple criterion: M is a realization of f isomorphic to Nf , i.e., M is a minimal realization of f , iff every morphism $EM' \rightarrow f$ surjective on inputs with M' reachable is of the form Eg for a unique $g: M' \rightarrow M$ surjective on inputs.

Now suppose $L: \underline{B} \rightarrow \underline{M}$ is a functor such that Lf is reduced and $ELf = f$. We will show that Lf satisfies the condition of the previous paragraph; then by Theorem 2 of the Appendix again,

$E \dashv L$; this is our assertion that minimal realization functors are right adjoints of E . First, $\gamma_{Lf}: Lf \longrightarrow NELf$ is an isomorphism since Lf is reduced, and its codomain is Nf , since $ELf = f$. But then, the universal property for Nf given in the previous paragraph also holds for Lf : every morphism $EM \longrightarrow f$ surjective on inputs with M reachable is of the form Eg for a unique $g: M \longrightarrow Lf$ surjective on inputs. For if also $g': M \longrightarrow Lf$ in $\underline{M}$ such that $g \neq g'$ but $Eg = Eg'$, then $\gamma_{Lf}g \neq \gamma_{Lf}g': M \longrightarrow Nf$ in $\underline{M}$, but $E(\gamma_{Lf}g) = E(\gamma_{Lf}g'): EM \longrightarrow F$, contradicting the condition for Nf ; thus we have uniqueness of g . For existence, we compose the unique morphism arising from Nf with the inverse of γ_{Lf} .

(O.) As already mentioned, the extension from arbitrary discrete machines to finite machines is immediate. If $\underline{M}^F$ denotes the full subcategory of $\underline{M}$ whose objects are machines with finite input, state, and output sets, then we take $\underline{B}^F = \underline{E}\underline{M}^F$, which is the full subcategory of $\underline{B}$ whose objects are behaviors of finite machines. We then have an adjunction between E and N as functors between $\underline{B}^F$ and $\underline{M}^F$ just as before. Of course, it is of interest to characterize $\underline{B}^F$ independently of the notion of machine, but this is already well-known (see Arbib [1]).

Certain special subcategories are of particular interest in the finite discrete case. Let $\underline{2}$ denote the set $\{0, 1\}$. Then a function $f: X^* \longrightarrow \underline{2}$ determines a subset $f^{-1}(1)$ of X^* , and a subset L

of X^* determines a function $f: X^* \longrightarrow \underline{2}$ by $f(w) = 1$ iff $w \in X^*$; thus such behaviors are bijective with subsets L of X^* , which are also called languages over the alphabet X . Languages arising from finite machines are called regular sets. Let $\underline{M}_2^F$ be the subcategory of $\underline{M}^F$ whose objects all have output set $Y = \underline{2}$ and morphisms $\langle a, b, c \rangle$ such that $c = \underline{2}$, the identity map of $\underline{2}$. This is the category of finite acceptors. Let $\underline{B}_2^F = \underline{EM}_2^F$, the category of regular sets. Then, very much as before, E and N are adjoint functors between $\underline{M}_2^F$ and $\underline{B}_2^F$; that is, finite acceptors and regular sets are in an adjoint relationship. The characterizations of regular sets independently of machines are too well-known to bear repetition here (see Arbib [1]).

This section is primarily concerned with the case of linear machines -- more specifically, the case where X, S, Y are left k -modules, where k is an arbitrary ring with unit. Although things are quite analogous to the discrete case, it seems best to be generous with details, in part because Giv'on and Zalkstein [3] have claimed that the simple dynamic treatment in Arbib and Zeiger [2] of the linear case (k a field) cannot be made rigorous. We show that a dynamic treatment of the linear case can be obtained from the preceding discrete treatment with quite modest changes, yielding the same concise and sharp results, without the awkward machinery (k -linear bundle monoids) or restrictions (k is commutative) of [3]. Rings of matrices

and of polynomials in non-commuting variables are important and non-commutative.

Let k be a ring with unit. A k -linear machine M is a quintuple $\langle X, S, Y, \delta, \lambda \rangle$ where X, S, Y are left k -modules (k -linear spaces, if k is a field), and $\delta: X \times S \rightarrow S$ and $\lambda: S \rightarrow Y$ are k -linear maps. We can omit specification of an initial state because we will always take it to be the zero in S . One obtains a category $\underline{\text{Mach}}_k$ of k -linear machines, with the machines as objects and morphisms $\langle a, b, c \rangle$ exactly as in the discrete case (i.e., such that the same diagrams commute), except that each a, b, c are required to be k -linear. Hereafter we shall frequently fail to mention k explicitly. Let $\underline{\text{Mach}}^F$ be the full subcategory with objects machines such that X, S , and Y are finitely generated.

Instead⁹ of X^* , which does not have a natural k -linear structure, we use $X^+ = \coprod_{t=0}^{\infty} X$, the countable copower of X (i.e., the countable coproduct of X with itself). It is known that X^+ can be taken to be the space of all sequences $\underline{x} = \langle x_t \rangle$ such that $x_t \in X$ for $t \in \underline{N}$ and all but a finite number of x_t are zero, where $\underline{N}$ is the set of non-negative integers.

⁹Our X^+ is isomorphic to the X^*/P used by Arbib and Zeiger [2], but we are thinking of time as running from the past to now at $t=0$. Unfortunately, Arbib and Zeiger sometimes confusingly identify P -classes in X^* with their representatives. Kalman's [6] time runs into both the past and future, since he uses an Ω isomorphic to $(\coprod_{t=0}^{\infty} X) \times (\prod_{t=1}^{\infty} X)$ to replace X^* .

Define an operation $\cdot$ of X^* on X^+ by $w \cdot \underline{x} \in X^+$ for $w \in X^*$, $\underline{x} \in X^+$, has $(w \cdot \underline{x})_t = w_t$ for $0 \leq t < |w|$ and $(w \cdot \underline{x})_{t+|w|} = x_t$ for $t \in \mathbb{N}$; thus, $w \cdot \underline{x}$ is w concatenated onto the front of $\underline{x}$. In particular $\Lambda \cdot \underline{x} = \underline{x}$. This operation cannot be linear because X^* is not linear, but it is very convenient in giving a development of the linear case parallel to the discrete. Notice that each $\underline{x} \in X^+$ is of form¹⁰ $w \cdot \underline{0}$ for some (indeed, many) $w \in X^*$, because $\underline{x}$ has only finitely many initial non-zero components.

Define $\delta^+: X^+ \rightarrow S$ by $\delta^+(\underline{0}) = 0$ and $\delta^+(x \cdot \underline{x}) = \delta(x, \delta^+(\underline{x}))$, for $x \in X$, $\underline{x} \in X^+$; this gives an inductive definition since each $\underline{x} = w \cdot \underline{0}$, some $w \in X^*$. Then δ^+ is k -linear. For convenience of notation in this proof only, let us write $\underline{w}$ for $w \cdot \underline{0}$. We begin by proving homogeneity, that is, for $a \in k$, we show

$\delta^+(a\underline{w}) = a \delta^+(\underline{w})$. This is true for $w = \Lambda$, since then both sides are zero. Assume the equation for all $w \in X^*$ of some length $|w| = n \geq 0$, and let $|w| = m$, $x \in X$, and $a \in k$. Then $\delta^+(a(\underline{xw})) = \delta^+(ax \cdot a\underline{w}) = \delta(ax, \delta^+(a\underline{w})) = \delta(ax, a \delta^+(\underline{w})) = a \delta(x, \delta^+(\underline{w})) = a \delta^+(\underline{xw})$.

We now prove additivity, that is, for $w, w' \in X^*$, we show

$\delta^+(\underline{w+w'}) = \delta^+(\underline{w}) + \delta^+(\underline{w'})$. This is true if $|w| = |w'| = 0$, i.e., $w = w' = \Lambda$, since both sides are zero. Assume the equation for all

¹⁰ The equivalence relation generated by the surjection $X^* \rightarrow X^+$ given by $w \mapsto w \cdot \underline{0}$ is exactly the P mentioned in the footnote 8. Notice in particular that $w0^n \cdot \underline{0} = w \cdot \underline{0}$, for all $n \in \mathbb{N}$. In fact, wPw' iff $w0^n = w'$ or $w = w'0^n$, some $n \in \mathbb{N}$.

$w, w' \in X^*$ with $|w|, |w'| \leq n$, for some $n \geq 0$, and let $x, x' \in X$.

Then $\mathcal{S}^+(\underline{xw} + \underline{x'w'}) = \mathcal{S}^+((x+x') \cdot (\underline{w} + \underline{w'})) = \mathcal{S}(x+x', \mathcal{S}^+(\underline{w} + \underline{w'})) = \mathcal{S}(x+x', \mathcal{S}^+(\underline{w}) + \mathcal{S}^+(\underline{w'})) = \mathcal{S}(x, \mathcal{S}^+(\underline{w})) + \mathcal{S}(x', \mathcal{S}^+(\underline{w'}))$. This

works because each $\underline{x} \in X^+$ is of the form $w \cdot \underline{0}$, and w can be

padded out on the right with as many zeros as one wishes. The

commutative square condition in (B.) holds for k -linear machines (as usual $+$ replaces $*$).

We can now define a machine M to be reachable iff $\mathcal{S}^+$ is surjective, and define the category $\underline{M}_k$ as $\underline{M}$ before, with objects reachable machines and morphisms $\langle a, b, c \rangle$ such that a is surjective.

The external behavior of a machine M is $EM = \lambda \mathcal{S}^+: X^+ \longrightarrow Y$,

linear because both λ and $\mathcal{S}^+$ are. We have a category $\underline{\text{Beh}}$ of behaviors $f: X^+ \longrightarrow Y$, for X, Y left k -modules, and morphisms

$\langle a, c \rangle: f \longmapsto f'$ with $a: X \longrightarrow X'$, $c: Y \longrightarrow Y'$ such that the same diagram

as before commutes. We also have the subcategory $\underline{B}$ of $\underline{\text{Beh}}$ all

of whose morphisms have surjective input component. Then

$E: \underline{\text{Mach}} \longrightarrow \underline{\text{Beh}}$ is a functor as before, which gives $E: \underline{M} \longrightarrow \underline{B}$

when restricted, the behavior functor. We seek a right adjoint.

Given $f: X^+ \longrightarrow Y$ linear, define the equivalence relation

$\equiv_f$ on X^+ by $\underline{x} \equiv_f \underline{x'}$ iff $f(w \cdot \underline{x}) = f(w \cdot \underline{x'})$ for all $w \in X^*$. We show

this is a linear congruence relation. First homogeneity: $\underline{x} \equiv_f \underline{x'}$

implies $a\underline{x} \equiv_f a\underline{x'}$, for $a \in k$. For $f(w \cdot (a\underline{x})) = f(w \cdot \underline{0} + 0^{|w|} \cdot a\underline{x}) =$

$f(w \cdot \underline{0}) + af(0^{|w|} \cdot \underline{x})$, and similarly $f(w \cdot (a\underline{x'})) = f(w \cdot \underline{0}) + af(0^{|w|} \cdot \underline{x'})$,

but $x \equiv_f x'$ implies $af(0^{|w|}, \underline{x}) = af(0^{|w|}, \underline{x}')$, and therefore

$f(w \cdot (a\underline{x})) = f(w \cdot a\underline{x}')$. Now additivity: $\underline{x} \equiv_f \underline{x}'$ and $\underline{y} \equiv_f \underline{y}'$ implies $\underline{x} + \underline{y} \equiv_f \underline{x}' + \underline{y}'$. For $f(w \cdot (\underline{x} + \underline{y})) = f(w \cdot \underline{x} + 0^{|w|} \cdot \underline{y}) = f(w \cdot \underline{x}) + f(0^{|w|} \cdot \underline{y}) = f(w \cdot \underline{x}') + f(0^{|w|} \cdot \underline{y}') = f(w \cdot (\underline{x}' + \underline{y}'))$, as desired.

Given $f: X^+ \longrightarrow Y$, set $S_f = X^+ / \equiv_f$, another left k -module, and define $\delta_f(x, [\underline{x}]_f) = [x \cdot \underline{x}]_f$, $\lambda_f([\underline{x}]_f) = f(\underline{x})$, and verify that $Nf = \langle X, S_f, Y, \delta_f, \lambda_f \rangle$ is a linear machine. Well-definedness goes as in the discrete case, and linearity follows mainly from the linearity of $\equiv_f$. This is the pattern for all the rest: replace $*$ by $+$ and proceed as before, occasionally verifying linearity. For example, it must be checked that $\gamma_M: M \longrightarrow NEM$ is linear, i.e., that

$q_M: S \longrightarrow S_f$ is linear, where $f = EM$. This is straightforward. We define $\delta^+(w, s)$ for $w \in X^*$, $s \in S$ exactly as in (M.), $\delta^+(\Lambda, s) = s$ and $\delta^+(xw, s) = \delta(x, \delta^+(w, s))$, and we prove by induction that

$\delta^+(w \cdot \underline{x}) = \delta^+(w, \delta^+(\underline{x}))$. In particular, $\delta^+(w, \underline{0}) = \delta^+(w, \underline{0})$.

We then define for $s, s' \in S$, $s \equiv_f s'$ iff $\lambda \delta^+(w, s) = \lambda \delta^+(w, s')$

for all $w \in X^*$, and we call a machine reduced iff no two distinct states are equivalent. Everything goes as before, and we obtain not only adjointness, but reflectivity of reduced machines, etc.

The finiteness modification is similar to the discrete case.

Let $\underline{M}^F$ be the intersection of the subcategories $\underline{M}$ and $\underline{Mach}^F$ of $\underline{Mach}$, and let $\underline{B}^F = \underline{EM}^F$, the full subcategory of $\underline{B}$ whose objects are behaviors of finitely generated machines. The restrictions E and N are again adjoint. Once again, machine independent characterization

of $\underline{B}^F$ is desirable, but in this case it does not seem to be well-known. Indeed, the usual references (including Kalman [6] and Arbib and Zeiger [2]) seem not to realize (even assuming k is a field) that not every linear function $f: X^+ \longrightarrow Y$ can be realized with a finite dimensional state space. At this writing I am unsure whether the characterization needed for the case we are considering actually appears in print, but it turns out that Nerode, in the paper [12] from which the famous equivalence relation was taken, considered a closely related problem, and it is likely that a similar characterization covers the present case.

These results suggest additional research. The definition of machine given in Section (A.), as modified in the footnote 1, can easily be extended to give a notion of a discrete-time machine in a category with products and terminal object. For example, a strong argument can be made that affine maps are more natural than linear, and that machines in affine categories ought to be studied. The question of what additional assumptions must be made on the category for our constructions to be valid, and in particular, what to use for X^* , will be treated in a later paper with M.A. Arbib. Other work will consider continuous time, and other more general types of machine, as well as other results on languages.

APPENDIX

This appendix gives results and definitions from category theory in the form needed in the body of the paper. The reader who wants more detail or motivation should consult the standard references [7],[9],[11], and especially the excellent [8]. We prove two easy lemmas which do not seem to be in the literature.

Definition: A category $\underline{C}$ consists of a class $|C|$ whose elements are called objects, for each $A, B \in |C|$ a class $\underline{C}(A, B)$ whose elements are called morphisms from A to B , and for each $A, B, C \in |C|$, a mapping

$$\circ : \underline{C}(B, C) \times \underline{C}(A, B) \longrightarrow \underline{C}(A, C)$$

called composition, with $\circ(f, g)$ denoted fg , such that:

- (1.) For each $A \in |C|$ there is a morphism $A \in \underline{C}(A, A)$ called the identity at A such that $Ag = g$ and $fA = f$, whenever these compositions are defined;
- (2.) For each $A, B, C, D \in |C|$, $h \in \underline{C}(A, B)$, $g \in \underline{C}(B, C)$, $f \in \underline{C}(A, B)$, the associative law $f(gh) = (fg)h$ holds.

The following conventions are used: $f \in \underline{C}(A, B)$ is indicated by $f: A \longrightarrow B$ or $A \xrightarrow{f} B$; write $\underline{C} = \bigcup_{A, B \in |C|} \underline{C}(A, B)$; identify $A \in |C|$ with $A \in \underline{C}(A, A)$, so that $|C| \subseteq \underline{C}$; call A the domain and B the codomain of $f: A \longrightarrow B$. A morphism $f: A \longrightarrow B$ in $\underline{C}$ is called an isomorphism iff there is a morphism $g: B \longrightarrow A$ in $\underline{C}$ such that $gf = A$ and $fg = B$; write $g = f^{-1}$, as it is uniquely determined if it exists.

The standard examples are: $\underline{Sets}$, whose objects are sets and morphisms are set mappings; $\underline{Fin}$, whose objects are finite sets, and morphisms as in $\underline{Sets}$; $\underline{Lin}_k$, with objects k -linear spaces (k -modules), and morphisms k -linear maps, for k a field (or ring with unit); $\underline{Flin}_k$, whose objects are finitely generated k -linear spaces,

and morphisms as in $\underline{\text{Lin}}_k$. Other examples occur in the body of the paper.

A category $\underline{B}$ is a subcategory of $\underline{C}$ iff $\underline{B} \subseteq \underline{C}$, $|\underline{B}| \subseteq |\underline{C}|$, and composition in $\underline{B}$ agrees with composition in $\underline{C}$. A subcategory $\underline{B}$ of $\underline{C}$ is full iff for each $B, B' \in |\underline{B}|$, $\underline{B}(B, B') = \underline{C}(B, B')$. Thus to specify a full subcategory $\underline{B}$ of $\underline{C}$, one need only give the objects $|\underline{B}|$.

Each category $\underline{C}$ has an opposite category $\underline{C}^{\text{op}}$, defined by $|\underline{C}^{\text{op}}| = |\underline{C}|$, $\underline{C}^{\text{op}}(A, B) = \underline{C}(B, A)$, and for $f \in \underline{C}^{\text{op}}(B, C)$, $g \in \underline{C}^{\text{op}}(A, B)$, $fg \in \underline{C}^{\text{op}}(A, C)$ defined to $gf \in \underline{C}(C, A)$. Given $f \in \underline{C}(C, B)$, it is often convenient to write $f^{\text{op}}: B \longrightarrow C$ for the corresponding morphism in $\underline{C}^{\text{op}}(B, C)$.

$T \in |\underline{C}|$ is terminal iff for each $C \in |\underline{C}|$, $\underline{C}(C, T)$ has cardinality one; and $I \in |\underline{C}|$ is initial iff for each $C \in |\underline{C}|$, $\underline{C}(I, C)$ has cardinality one.

Definition: A function $F: \underline{B} \longrightarrow \underline{C}$ between categories is a functor iff $F(fg) = (Ff)(Fg)$ whenever fg is defined, and $F|\underline{B}| \subseteq |\underline{C}|$.

Notice we write Ff rather than $F(f)$, but $F(fg)$ for clarity.

In general, we try to avoid the proliferation of parentheses. An inclusion $\underline{B} \subseteq \underline{C}$ of a subcategory is a functor. A functor $F: \underline{B} \longrightarrow \underline{C}$ is said to be full iff $F\underline{B}(B, B') = \underline{C}(FB, FB')$ for all $B, B' \in |\underline{B}|$, and is said to be faithful iff F restricted to $\underline{B}(B, B')$ is injective for each pair $B, B' \in |\underline{B}|$. An inclusion functor is always faithful, and is full iff the subcategory is full.

Functors $F: \underline{B} \longrightarrow \underline{C}$ and $G: \underline{A} \longrightarrow \underline{B}$ can be composed to give a functor $FG: \underline{A} \longrightarrow \underline{C}$. There is an identity functor $\underline{A}: \underline{A} \longrightarrow \underline{A}$, defined by $\underline{A}f = f$ for all $f \in \underline{A}$. This gives rise to a "meta-category" $\underline{\text{Cat}}$ with categories as objects and functors as morphisms.

Definition: A natural transformation η from F to G , where $F, G: \underline{B} \longrightarrow \underline{C}$ are functors, is a collection $\{\eta_B \mid B \in |\underline{B}|\}$ of morphisms $\eta_B: FB \longrightarrow GB$ in $\underline{C}$, such that for each $f: B \longrightarrow B'$ in $\underline{B}$, the diagram

$$\begin{array}{ccc} FB & \xrightarrow{\eta_B} & GB \\ \downarrow Ff & & \downarrow Gf \\ FB' & \xrightarrow{\eta_{B'}} & GB' \end{array}$$

commutes. Write $\eta: F \Longrightarrow G$ or $\eta: F \Longrightarrow G: \underline{B} \longrightarrow \underline{C}$. If each η_B is an isomorphism, η is called a natural equivalence, or natural isomorphism, of functors F, G .

If $F, G, H: \underline{B} \longrightarrow \underline{C}$, $\eta: G \Longrightarrow H$, and $\phi: F \Longrightarrow G$, define the composite transformation $\eta\phi: F \Longrightarrow H$ to be the family $\{\eta_B \phi_B \mid B \in |\underline{B}|\}$. It can be directly verified that $\eta\phi$ is a natural transformation, that this notion of composition is associative, and that the identity transformations $F: F \Longrightarrow F$, defined by $F_B = FB: FB \longrightarrow FB$, function as identities. Therefore there is a category whose objects are functors $\underline{B} \longrightarrow \underline{C}$ and whose morphisms are transformations. It is denoted $[\underline{B}, \underline{C}]$ or $\underline{C}^{\underline{B}}$, and is called a functor category.

If $\gamma: G \Rightarrow H: \underline{B} \longrightarrow \underline{C}$ and $F: \underline{A} \longrightarrow \underline{B}$, then

$\gamma F: GF \Rightarrow HF: \underline{A} \longrightarrow \underline{C}$ is given by $(\gamma F)_A = \gamma_{FA}$. Similarly,

if $\gamma: G \Rightarrow H: \underline{B} \longrightarrow \underline{C}$ and $F: \underline{C} \longrightarrow \underline{D}$, then $F\gamma: FG \Rightarrow FH: \underline{A} \longrightarrow \underline{D}$

is given by $(F\gamma)_A = F\gamma_A$.

Definition: A functor $F: \underline{A} \longrightarrow \underline{B}$ is left adjoint to a functor

$G: \underline{B} \longrightarrow \underline{A}$ iff there exists a natural isomorphism

$$\phi_{AB}: \underline{B}(FA, B) \longrightarrow \underline{A}(A, GB).$$

One also says G is a right adjoint to F , and writes $F \dashv G$, or

$F \dashv G: \underline{A} \longrightarrow \underline{B}$, and one calls ϕ the adjunction.

In this definition $\underline{B}(-, -)$ and $\underline{A}(-, -)$ are discrete class valued functor with domains $\underline{B}^{\text{op}} \times \underline{B}$ and $\underline{A}^{\text{op}} \times \underline{A}$, respectively, so that $\underline{B}(F-, -)$ and $\underline{A}(-, G-)$ are both discrete class valued functors with domain $\underline{A}^{\text{op}} \times \underline{B}$, and ϕ is a transformation between them.

Theorem 1: Any two left adjoints to $G: \underline{B} \longrightarrow \underline{A}$ are naturally isomorphic.

Any two right adjoints to $F: \underline{A} \longrightarrow \underline{B}$ are naturally isomorphic.

Given $F: \underline{A} \longrightarrow \underline{B}$ and $B \in |\underline{B}|$, let F/B be the category with objects pairs $\langle A, b: FA \longrightarrow B \rangle$ where $A \in |\underline{A}|$ and $b \in \underline{B}$, and with morphisms $\langle A, b: FA \longrightarrow B \rangle \longrightarrow \langle A', b': FA' \longrightarrow B \rangle$ taken as morphisms $a \in \underline{A}(A, A')$ such that

$$\begin{array}{ccc} FA & \xrightarrow{Fa} & FA' \\ & \searrow b & \nearrow b' \\ & B & \end{array}$$

commutes in $\underline{B}$. A terminal object in F/B is called a universal arrow

from F to B . Dually, given $G: \underline{B} \longrightarrow \underline{A}$ and $A \in |\underline{A}|$, A/G has

objects $\langle a:A \longrightarrow GB, B \rangle$ and morphisms $b:B \longrightarrow B'$ such that

commutes; and an initial object in A/G is called a (co-) universal arrow from A to G . Categories such as F/B and A/G are called comma categories.

Theorem 2: $F:\underline{A} \longrightarrow \underline{B}$ has a right adjoint iff each $B \in |\underline{B}|$ has a universal arrow from F ; and then the value GB of the adjoint G at B is the object A occurring in the universal arrow $\langle A, AF \longrightarrow B \rangle$ from F to B . Dually, $G:\underline{B} \longrightarrow \underline{A}$ has a left adjoint iff each $A \in |\underline{A}|$ has a universal arrow to G ; and the value FA of the adjoint F at A is the object B in the couniversal arrow $\langle A \longrightarrow GB, B \rangle$.

Actually, Theorem 1 follows from Theorem 2 because of the uniqueness up to isomorphism of terminal and initial objects, and Theorem 2 can be proved fairly directly.

Theorem 3: Given $F \dashv G: \underline{A} \longrightarrow \underline{B}$, the universal arrows $\eta_A: A \longrightarrow GFA$ constitute a natural transformation $\eta: \underline{A} \Longrightarrow GF$ called the unit of the adjunction, and the universal arrows $\epsilon_B: FGB \longrightarrow B$ constitute a natural transformation $\epsilon: FG \Longrightarrow \underline{B}$ called the counit of the adjunction. Moreover, F, G, η, ϵ satisfy the triangular identities given by commutativity of the diagrams

and

Theorem 4: If $F: \underline{A} \longrightarrow \underline{B}$, $G: \underline{B} \longrightarrow \underline{A}$, $\eta: \underline{A} \xrightarrow{\sim} GF$, and $\varepsilon: FG \xrightarrow{\sim} \underline{B}$ satisfy the triangular identities, then $F \dashv G$, and η, ε are the unit and co-unit of the adjunction.

Categories $\underline{A}$ and $\underline{B}$ are equivalent iff there is a functor $F: \underline{A} \longrightarrow \underline{B}$ which is full, faithful, and such that each $B \in |\underline{B}|$ is isomorphic to FA for some $A \in |\underline{A}|$. $\underline{A}$ is an equivalent subcategory of $\underline{B}$ iff the inclusion functor satisfies these conditions. Equivalently, $\underline{A}$ and $\underline{B}$ are equivalent iff there are functors $F: \underline{A} \longrightarrow \underline{B}$ and $G: \underline{B} \longrightarrow \underline{A}$ such that FG and GF are both naturally isomorphic to the identity. Yet another equivalent condition is that there is an adjunction between $\underline{A}$ and $\underline{B}$ whose unit and co-unit are both isomorphisms.

A subcategory $\underline{A}$ of $\underline{C}$ is reflective in $\underline{C}$ iff the inclusion functor has a left adjoint, called a reflector. Equivalently, $\underline{A} \subseteq \underline{C}$ is reflective iff for each $C \in |\underline{C}|$ there is an object RC in $\underline{A}$ and a morphism $\eta_C: C \longrightarrow RC$ in $\underline{C}$ such that every morphism $g: C \longrightarrow A$ in $\underline{A}$ has the form $g = f \eta_C$ for a unique $f: RC \longrightarrow A$ in $\underline{A}$. This equivalence follows from Theorem 2. The following trivial lemma is used in Section (M.).

Lemma 1: Suppose $\underline{A}$ is an equivalent subcategory of a subcategory of $\underline{B}$ of $\underline{C}$, and that $\underline{A}$ is a reflective subcategory of $\underline{C}$. Then $\underline{B}$ is a reflective subcategory of $\underline{C}$.

Proof: Let $C \in |\underline{C}|$. Then we have $\eta_C: C \longrightarrow RC$, with $RC \in |\underline{A}|$, universal from C to the inclusion. Since $\underline{A} \subseteq \underline{B}$, of course $RC \in |\underline{B}|$, and we have to show η_C is still universal from C to the inclusion of $\underline{B}$ in $\underline{C}$. So let $g: C \longrightarrow B$, $B \in |\underline{B}|$, and let A be an object of $\underline{A}$ isomorphic to B , $j: B \xrightarrow{\cong} A$, using equivalence. Then there is a unique $f: RC \longrightarrow A$ such that $f\eta_C = jg$. We conclude that there is a unique $f': RC \longrightarrow B$ in $\underline{B}$ such that $f'\eta_C = g$, namely $f' = j^{-1}f$, using fullness. For if also $f'': RC \longrightarrow B$ satisfies $f''\eta_C = g$, then $(jf'')\eta_C = jg$, so that $jf'' = jf'$, or $f'' = f'$, since j is an isomorphism.

A much simpler but more sophisticated proof uses the composition of adjoint situations (see [8.]).

The following fact is useful in considering various restrictions of the main adjunctions in the body of this paper.

Lemma 2: If $F: \underline{A} \longrightarrow \underline{B}$, $G: \underline{B} \longrightarrow \underline{A}$, $F \dashv G$, and if $\underline{A}_0$ and $\underline{B}_0$ are full subcategories of $\underline{A}$ and $\underline{B}$ (respectively) such that the restriction of F to $\underline{A}_0$ factors through $\underline{B}_0$ and the restriction of G to $\underline{B}_0$ factors through $\underline{A}_0$, yielding $F_0: \underline{A}_0 \longrightarrow \underline{B}_0$ and $G_0: \underline{B}_0 \longrightarrow \underline{A}_0$ (respectively), then $F_0 \dashv G_0$.

Proof: The restriction of the natural isomorphism φ on $\underline{A}^{op} \times \underline{B}$ to $\underline{A}_0^{op} \times \underline{B}_0$ yields the desired natural isomorphism φ_0 , noting

that $B(Fa, b) = B_0(F_0 a, b)$ and $A(A, Gb) = A_0(a, G_0 b)$, for $a \in A_0$ and $b \in B_0$.

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SOME COMMENTS ON DENNING'S PRINCIPLE OF OPTIMALITY⁺

J. Pomeranz

Denning and others have proposed a page replacement strategy and asked whether it is generally optimal in the sense of minimizing the expected number of faults [1,3,4]. One statement of this so-called "principle of optimality" is*

Let $S = \{1', 2', \dots, m'\}$ be the memory state at time t , the moment of a page fault, and let $t(i') \geq t$ be the earliest moment at which page i' is next referenced. Define $\tau(i') = t(i') - t$. Replace that page i' for which $\tau(i')$ is maximum. If the future is not precisely known, replace that page i' for which the expected time $E[\tau(i')]$ is maximum [3, p. 178].

When the future is precisely known, this algorithm is essentially Belady's MIN algorithm [2]. However, because time until next reference is not the key factor in the MIN algorithm, the extension to expectation fails. Consider a two page memory containing pages a and b and suppose that there are two possible future reference sequences:

⁺Prepared for SIGOPS Review.

*In this paper time is measured in "virtual time units" (vtu) and one page reference is assumed to require one vtu.

$cacb^n$ with probability p

ccb^na with probability $1 - p$

(n positive but fixed). Note that (1) n is a factor in the expected time until page a is again referenced but not a factor for pages b or c , (2) each reference sequence requires only two replacements, and (3) for the first sequence, b must be replaced to achieve this minimum, while for the second sequence either a or b may be replaced. Letting $T(x)$ be the expected time until next reference to page x and assuming equal time intervals and $T(c) = 0$, then

$$T(a) = 1(p) + (n + 2)(1 - p) = 1 + (n + 1)(1 - p)$$

$$T(b) = 3(p) + 2(1 - p) = 2 + p$$

Letting $R(x)$ be the expected minimum number of replacements if page x is now replaced,

$$R(a) = 3(p) + 2(1 - p) = 2 + p$$

$$R(b) = 2(p) + 2(1 - p) = 2.$$

So, for $p > 0$, page a should not be replaced. If $n > (2p)/(1 - p)$ then $T(a) > T(b)$ and using the above algorithm, page a is replaced. Thus for example if $p = .5$ and $n = 3$ then $T(a) = 3$, $T(b) = 2.5$ and page a would be replaced even though the corresponding replacement rate is almost 25% above the achievable minimum.

I considered an interesting alternative that appeared to overcome this time-related difficulty. When only one reference sequence is involved, time until next reference ranks the pages in the same order that they reappear in the sequence and optimality is achieved. With

many alternative sequences, the ordering can be incorrect as above.

Thus one might propose:

Calculate the expected rank by next appearance.

Replace that page with expected last position.

However, the previous counterexample with $n = 1$ yields possible future reference sequences:

cacb with probability p

ccba with probability $1 - p$.

Then as before,

$$R(a) = 2 + p$$

$$R(b) = 2$$

and b is the better choice for replacement. But, if $1 - p > .5 > p$ then a is expected to reappear after b , a would be replaced, and suboptimal replacement occurs.

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MINIMIZING EXPECTED COST WITH PREEMPTION
ALLOWED AND NON-DETERMINISTIC SERVICE TIMES

K. Sevcik

ABSTRACT

A selection rule is derived for the optimal scheduling of a finite set jobs on a single processor where the performance measure is expected total cost, and the cost of each job is a linear nondecreasing function of its waiting time to completion. A full proof of the optimality of the selection rule is deferred to a later paper.

I. Introduction

Consider a finite set of jobs which are to be executed on a single processor. Each job has an associated constant which is the rate at which the value of its results depreciates until the job is completed (i.e. linear loss functions). We wish to determine a selection rule for picking the next job to be run at any time in order to minimize expected total cost over all jobs. If service times are known precisely in advance, the optimal selection rule is well known: select the job whose ratio of service time to loss constant is the smallest. When service times are not known, but the expected service time of jobs of any type is known statistically, and preemption is not allowed, the optimal selection rule is to select the job whose ratio of expected service time to loss constant is the smallest. However, when preemption is allowed and service times are nondeterministic, no optimal scheduling strategy is known according to past literature.

Derivation of an optimal selection rule for this situation is the major goal of this paper.

II. The Wrong Way

When service times are known precisely in advance and preemption is allowed, an optimal selection rule is known even when we allow arbitrary arrivals of new jobs. In this case, the selection rule must pick the job whose ratio of remaining service time to loss constant is smallest. The pertinence of this information to the problem at hand is seen in Figure 1.

Preemption Service Times	Deterministic	Nondeterministic
	SPT/C Shortest Processing Time	SEPT/C Shortest Expected Processing Times
Not Allowed		
Allowed	SRPT/C Shortest Remaining Processing Times	? ? ?

Fig. 1. Optimal Selection Disciplines¹

The intuitive consequence of an examination of this table is the presumption that the open quadrant may be filled in with the selection rule which picks the job whose expected remaining processing time to loss constant ratio is smallest (SERPT/C). We may generate a counterexample to this presumption as follows²:

¹See [1].

²The possibility of deriving a counterexample to SERPT's optimality was suggested to me by Linus Schrage in a private communication.

Let the job set consist of one job from each of two job classes. Class 1 jobs are either short (service time ϵ) with probability p , or they are long (service time β) with probability $(1-p)$. Class 2 jobs all have service time α , where $\epsilon < \alpha < \beta$, and $\frac{\epsilon}{p} < \alpha < \beta - \alpha$. Also we require that β be sufficiently large that $\alpha < p\epsilon + (1-p)\beta$. Assume that jobs from both classes have unit loss constants. Then the expected remaining service times for the two jobs are:

$$\text{Class 1 job: } p\epsilon + (1-p)\beta$$

$$\text{Class 2 job: } \alpha$$

We selected β so that $\alpha < p\epsilon + (1-p)\beta$, so that SERPT strategy indicates that the class 2 job should be run first, then the class 1 job.

But consider an alternative strategy: run the class 1 job for ϵ seconds. If it completes (as it will with probability p) simply run the class 2 job. If it fails to complete, preempt it, and run the class 2 job before finishing it.

Under the first strategy, one job finishes after α , and the other after $\alpha + \epsilon$ with probability p , and after $\alpha + \beta$ with probability $1-p$. Thus total expected cost is $2\alpha + p\epsilon + (1-p)\beta$. Is total expected cost lower under the second strategy?

$$\begin{aligned} 2\alpha + p\epsilon + (1-p)\beta &\stackrel{?}{>} \epsilon + \alpha + p\epsilon + (1-p)(\alpha + \beta) \\ \alpha &\stackrel{?}{>} \epsilon + (1-p)\alpha \\ p\alpha &\stackrel{?}{>} \epsilon \\ \alpha &\stackrel{?}{>} \epsilon/p \end{aligned}$$

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We picked α , ϵ and p so that this last inequality holds. Thus a strategy other than SERPT yields lower expected total cost in this situation.

One particular set of parameter values which lead to a counterexample to the optimality of SERPT are:

$$\begin{aligned} p &= .5 & \alpha &= 5 \\ \epsilon &= 1 & \beta &= 10. \end{aligned}$$

III. The Right Way

In generating a counterexample to the optimality of SERPT in the preceding sections the scheduling strategy which proved superior to SERPT employed preemption of a variety which has gone untreated in previous literature in the field; namely, preemption due to dynamic prediction of the job's characteristics. Preemption at the arrival of a new job in the system has been treated extensively, as has regular (quantum-controlled) preemption due to continuing identical ignorance of service time (i.e. the assumption of exponentially distributed service times), such as is employed in time-slicing systems. Here, we wish to investigate how empirically gathered estimates of service time distributions by job class may be fully exploited in scheduling for minimum expected total cost.

Introducing some notation:

Let $f_k(t)$ be the probability density of service times of class k jobs.

Let $g_j(t, x)$ be the marginal probability density function of job j of class k which has already executed for x time units without completing. Then $g_j(t, x) = \frac{f_k(t+x)}{\int_x^\infty f_k(s) ds}$

Consider scheduling two jobs, j_1 and j_2 , which are respectively from job classes having service time distributions f_1 and f_2 . Assume that expected service time of class 2 jobs is less than that of class 1 jobs:

$$\int_0^\infty t f_2(t) dt < \int_0^\infty t f_1(t) dt$$

Then, if preemption is disallowed, the optimality of the SEPT selection rule dictates that j_2 should be run first. But, with preemption allowed, are there conditions under which j_1 should be selected initially? Let us compare the two following particular strategies:

- S1) Run j_2 to completion, then run j_1 to completion.
- S2) Run j_1 for some time ϵ . If it completes, run j_2 to completion, otherwise preempt j_1 and run j_2 to completion before finishing j_1 .

Then the expected total costs under the two strategies are:

$$EC(S1) = \int_0^\infty \int_0^\infty f_1(y) f_2(x) [2x + y] dy dx$$

$$EC(S2) = \int_0^\infty \int_0^\epsilon f_1(y) f_2(x) [2y + x] dy dx +$$

$$\int_0^\infty \int_\epsilon^\infty f_1(y) f_2(x) [2(x + \epsilon) + (y - \epsilon)] dy dx$$

Under what conditions will $EC(S2) < EC(S1)$?

$$\int_0^{\infty} \int_0^{\infty} f_1(y)f_2(x)[2x+y]dydx \stackrel{?}{>} \int_0^{\infty} \int_0^{\epsilon} f_1(y)f_2(x)[2y+x]dydx + \int_0^{\infty} \int_{\epsilon}^{\infty} f_1(y)f_2(x)[2x+y+\epsilon]dydx$$

$$\int_0^{\infty} \int_0^{\infty} f_1(y)f_2(x)[x]dydx \stackrel{?}{>} \int_0^{\infty} \int_0^{\epsilon} f_1(y)f_2(x)[y]dydx + \int_0^{\infty} \int_{\epsilon}^{\infty} f_1(y)f_2(x)[x+\epsilon]dydx$$

$$\int_0^{\infty} xf_2(x)dx \stackrel{?}{>} \int_0^{\epsilon} yf_1(y)dy + \left[\int_{\epsilon}^{\infty} f_1(y)dy \right] \left[\int_0^{\infty} xf_2(x)dx \right] + \epsilon \int_{\epsilon}^{\infty} f_1(y)dy$$

$$\int_0^{\epsilon} f_1(y)dy \int_0^{\infty} xf_2(x)dx \stackrel{?}{>} \int_0^{\epsilon} yf_1(y)dy + \epsilon \int_{\epsilon}^{\infty} f_1(y)dy$$

$$\int_0^{\infty} xf_2(x)dx \stackrel{?}{>} \frac{\int_0^{\epsilon} yf_1(y)dy + \epsilon \int_{\epsilon}^{\infty} f_1(y)dy}{\int_0^{\epsilon} f_1(y)dy} \quad (EQI)$$

Thus S2 is better than S1 whenever we can find a value for ϵ such that the inequality above holds. An examination of EQI for intuitive meaning reveals that the numerator of the righthand side is precisely the expected amount of time j_1 would run if it were allowed to run as much as ϵ .

$$\int_0^{\epsilon} y f_1(y) dy = \int_0^{\epsilon} f_1(y) dy \left[\int_0^{\epsilon} y f_1(y) dy / \int_0^{\epsilon} f_1(y) dy \right]$$

is (the probability that j_1 will complete by ϵ)
times (the expected length of a job of length less
than or equal to ϵ)

and

$$\epsilon \int_{\epsilon}^{\infty} f_1(y) dy \quad \text{is (the probability that } j_1 \text{ will not complete} \\ \text{by } \epsilon \text{) times (} \epsilon \text{)}$$

The denominator of the righthand side is the probability that j_1 will complete by ϵ . Thus, the righthand side is the ratio of the expected time invested to the probability of completion in scheduling j_1 for ϵ time units. Note that similarly, the lefthand side is the ratio of expected time invested to probability of completion in scheduling j_2 to run to completion. Note that we are allowed to choose ϵ in such a way as to minimize the ratio of expected time invested to probability of completion.

These observations provide a basis for a preliminary conjecture that an optimal selection rule is one which selects the job of smallest rank where the rank of a job is defined as:

$$\text{Rank of } j_i = R(j_i) = \min_{\epsilon > 0} \left[\left(\int_0^{\epsilon} y f_i(y) dy + \epsilon \int_{\epsilon}^{\infty} f_i(y) dy \right) / \left(\int_0^{\epsilon} f_i(y) dy \right) \right]$$

Further investigation has indeed shown that SR (Smallest Rank) scheduling minimizes expected total cost. A full proof of its optimality will follow in another paper [2].

IV. Conclusions

In this paper, we have described the steps which led to the development of the SR selection rule for scheduling. This selection rule minimizes expected total cost under non-deterministic service times when preemption is allowed, and jobs are assumed to have linear loss functions (i.e. the cost of the job increases linearly with time spent in system).

It is useful to note that SR selection reduces to SRPT when service times are, in fact, deterministic. Also, it reduces to SEPT when preemption is disallowed (i.e. the only value ϵ may take on is ∞).

Forthcoming results include

- 1) A full proof of SR's optimality under even non-identical linear loss functions;
- 2) A proof that SR's optimality holds up even under poisson arrivals;
- 3) A generalization of SR to provide optimal selection under general arrival patterns;
- 4) A generalization of SR to provide optimal selection when the time lost in a preemption is significant.

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FEWER RETRIEVALS IN THE HSIAO-HARARY FILE STRUCTURE

J. Pomeranz

The algorithm proposed by Hsiao and Harary for retrieving records by parallel processing of directory lists [1] can be improved. Their implied objective is minimizing the number of records that must be examined. They assume that the number of records containing each keyword is known and that nothing is known about interactions among keywords.

In their notation, for each term of the description they choose a keyword contained in the fewest records and call it prime. Further processing is only on lists of prime keywords. This is the algorithm's shortcoming. When a record is retrieved, it may contain non-prime keywords. If those occurrences were counted, then in a conjunct, a non-prime keyword might be found to occur less frequently in the remaining records than the prime keyword. Dynamically determining prime keywords would be an improvement. For example, applying the Hsiao-Harary algorithm to the description and file in Figure 1 yields four retrievals, the last of which is unnecessary.

Consider the following algorithm (for simplicity the algorithm operates on the directory in place).

0. E and F are empty sets. E will ultimately be the set of records satisfying the description. F is the set of addresses of records retrieved.

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1. If any term has a keyword for which no records remain unexamined in the file, delete the term. If no terms remain, stop. Otherwise, select a keyword which has a minimum number of records remaining in the file.
2. From the selected keyword's beginning addresses, choose an address not in F. Fetch the record with that address.
3. For each distinct keyword in the description determine whether it occurs in the record. If it does, reduce by one the number in the directory of records remaining in the file with that keyword and add the keyword pointer in the record to the beginning address sequence for the keyword.
4. Determine whether the record satisfies the description and, if so, include the record in E. Always put its address in F. Go to step 1.

Note that in step 1 any keyword could have been chosen with the limitation that within some term of the description, it was a keyword occurring least frequently in records remaining in the file. Thus, this algorithm will always do as well or better than the Hsiao-Harary algorithm. It uses more of the information.

A worthwhile extension of this model would be the inclusion of a function $n(D)$ that for any description D gave the expected number of records satisfying D . In this way, interactions among keywords could be assessed.

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Figure 1

description: $(K_1 \wedge K_3) \vee (K_2 \wedge K_4)$

directory		
K_1	2	001
K_2	2	004
K_3	3	002
K_4	3	003

file		
001	K_1	003
	K_4	002
002	K_2	000
	K_3	004
	K_4	000
003	K_1	000
	K_3	000
	K_4	001
004	K_2	002
	K_3	003

a'	σ_1	σ
		(001, 004)
001	003	(<u>001</u> , 003, 004)
003		(<u>001</u> , <u>003</u> , 004)
004	002	(<u>001</u> , 002, <u>003</u> , <u>004</u>)
002		(<u>001</u> , <u>002</u> , <u>003</u> , <u>004</u>)

A SYSTEM FOR CONTROLLING NON-BOOK MATERIALS
IN A SMALL TECHNICAL LIBRARY

C. Weil

Scientific and technical libraries receive large quantities of non-book materials -- principally preprints of articles, items from series of university departments or corporations, and government studies. These are generally paper-bound, too small to stand independently on the shelves, and not worth binding. If the organization, or some of its members, has a defense contract, the library may receive quantities of papers through the Defense Documentation Center. Most of these papers are in microfiche or microfilm form.

More critical than the storage of the items is the problem of access to the information in them. Collecting and storing the papers is useful only if good indices are provided. These materials are of shorter-term interest than books. That is, most worthwhile reports will be published in a more permanent form or manuals will be updated, so that periodic discarding of them is appropriate. The method of indexing reports and papers ought to facilitate identification of the papers to be discarded.

What to do with these materials is most acute in small departmental libraries with one professional and maybe a clerical worker and with such limited resources that only part of what would be "nice" to have can be bought. In a large, well-staffed, generously

financed institution these materials can be treated like books -- catalogued and even bound -- though it is probably not good library technique nor sound economics to do so. Therefore a good system would be useful for a large library as well.

This article describes a method of handling non-book materials in a small library, the library of the Institute for Computer Research of the University of Chicago. Books and journals are handled in this small library just as in the regular University library. The Library of Congress call numbers are the same as in the main collection and an author-title-subject catalogue is maintained. The major effort in the departmental library, however, goes into handling non-book materials; an average of 250 articles, papers, preprints, etc., arrive monthly. Many of these are unique copies on campus and most are the only copies not belonging to individuals. The staff of the library is one half-time professional librarian.

Storage of Non-Book Materials

Storage is simple. The papers are kept in 5"-wide boxes in chronological order with others from the same institution. For example, boxes 1-3 contain University of Illinois Computer Science Department papers; boxes 107-108 contain IBM manuals. If box 3 is filled when another University of Illinois paper comes, box 4 (or 3A if 4 is assigned to another institution) is inserted. There are also boxes labelled "Miscellaneous schools," "Miscellaneous corporations,"

etc. for papers from sources whose papers do not fill a box. There are 130 boxes at present and, although the number has been growing, it appears that discarding unused and out-of-date papers will lead to a steady state of about 150 boxes.

Microfiche are stored in special file drawers by AD, TIP or ATI numbers, accession numbers assigned by the Defense Documentation Center.

Indexing System

Formerly there were three card catalogues for information about these papers: each paper was indexed by its sponsoring organization(s), its author(s) and its subject(s). Monthly acquisition lists citing papers by sponsoring organization only were typed and disseminated. This system became unsatisfactory because typing all the cards and lists was so time-consuming that the librarian could not prepare a useful subject index.

Since the library is in a computation center and the librarian has convenient access to the IBM 7090/7094 system, an automated scheme seemed sensible. However, limited funds precluded any elaborate schemes involving machine-searchable records or even large blocks of processing time devoted to sorting and printing. The card sorter and lister -- a machine which provides printed copy called a listing of information on punched cards -- were free except for labor costs.

It was decided to produce a keyword-in-context (KWIC) index, although in this application the correct term would be KWOC index since the keywords are aligned at the left side of the line "out of context." Several modifications as described below were made on the classical KWIC indexing concept to correct some of its weaknesses.

Implementation of the System

First, when a document comes in, one 3"x5" card is typed containing the eventual physical location for the document (box number for a paper or AD number for microfiche or microfilm), the originating institution, the number given the article by the originating institution, the authors, title and date. Subject words derived from author-supplied abstracts or indexing terms and from the library's subject catalogue (compiled from Library of Congress terms and terms in general usage in the department) are often added in parentheses following the title. This procedure corrects for ambiguous titles and assures that all titles on a single subject will appear together under one term. For example, all papers on automata have that word added if it is not in the title, so they all appear under automata in the keyword list. Another example -- papers on characteristic vectors, latent vectors and eigenvectors have the term matrices added so that they all appear together at least once in the list. This card file is not necessary but is insurance in case punched cards, produced next, are lost or are being processed when a user wants information. The 3"x5" card

is prepared in the library when the paper is shelved and then is later used at another location as a source for keypunching.

At the time the card is typed, a code is assigned to each paper. The code for a paper includes three letters for the sponsoring organization, two digits for the year, two letters plus a digit for the author -- e.g. SM1 for the first article by an author whose name begins Sm -- and three digits for the box number, followed by a period. For example MIT70J02061. might represent a second paper in box 61 by Jones or Johnson of MIT written in 1970. If the document is microfiche, the code has a similar first five symbols for organization and year followed by a period and the AD number; that is MIT70.651089 might represent a paper from MIT written in 1970 and filed by its AD number 651089 in the microfiche file. Microfilms have "f" instead of a period before the AD number.

Each document is given a unique code and the codes provide useful information that need not be spelled out in full (organization, date and location). Specialists will recognize that the code is an elementary form of hash coding. More information such as report number, month as well as year of publication, specific department within the organization could have been provided, but these were deemed unnecessary for the intended users or for efficient retrieval.

A pack of cards is punched for each paper as follows. Each author's name is punched on a new card beginning in column 1. The

title begins on a new card and is punched between a "T" and a "=",

Any added subject terms are enclosed in parentheses before the "=",

The code is repeated on every card beginning in column 68. A pack of cards for an article might look like this:

(COMPUTERS-APPLICATIONS-EDUCATION) =	/ILL69RI1004.
PHYSICALLY HANDICAPPED (COMPUTER-ASSISTED-INSTRUCTION)	/ILL69RI1004.
T THE IMPLEMENTATION OF CAI SYSTEMS IN THE EDUCATION OF THE	/ILL69RI1004.
CARLTON, GEORGE S.	/ILL69RI1004.
RICE, ALBERT	/ILL69RI1004.

The parenthetical phrases are the added subject terms.

Next the punched cards, each pack representing an article, are hand-sorted by code -- alphabetized by organization code, arranged chronologically by date within the organization, alphabetized by author within each year, etc. A blank card is inserted between packs. This ordered set of packs is called the deck. The listed form of this deck is easy to read as it contains few distracting symbols.

Then the deck is submitted to the IBM 7090/7094 with a COMIT program that duplicates the author and title cards and sorts them into two stacks. The author cards are readily alphabetized on a sorter.

Next the titles with subject words -- any symbols appearing between a "T" and a "=" -- are submitted with another COMIT program that produces the KWIC index. The program's input is the titles as originally punched. The output is a card for each word not in the blocked word list. (See Appendix A.)

The blocked word list, or stop list, forms the bulk of the program. The list contains about 650 words, numbers and symbols which in the context of computer science literature are not suitable as index terms. That is, no researcher would look up words like "the," "theory," "2," or "September" in an index. Therefore, the program skips over these words. Words that are so common or uninformative or both that they would ordinarily be blocked individually -- like information and theory -- but have special meaning as a phrase are hyphenated and treated as one term.

Each output card has a keyword (non-blocked word) beginning in column 1, followed by three blank spaces and the part of the title surrounding the word with the keyword replaced by an asterisk, and the code at the right beginning in column 68. For example:

MAPPINGS	ON N-DIMENSIONAL NONLINEAR * GENERALIZING SEVERAL TYPES/MAR70RHL374
MAPS	SLOPE MEASUREMENT FROM CONTOUR * = /MAR70PAL374

This output is also in a form that is easily alphabetized on the sorter.

The keyword deck is then edited manually. Cards for words which the editor expects not to be helpful, such as the same word or root of a word appearing twice in a title, are removed. Cards starting with words that are not useful as index terms are also removed, and, if their recurrence seems probable, they are added to the stop list. The stop list has been expanding, but at a decreasing rate; each month fewer useless words show up in the keyword sort.

To compensate for the occurrence of synonyms, homographs, neologisms, etc. a deck of punched cards containing cross-indexed terms taken from the subject catalogue is sorted with the keyword deck. These cards are in the form

EDUCATION. SEE ALSO COMPUTERS-APPLICATIONS-EDUCATION.

The keyword deck is next compared with keyword lists submitted by the library's users and a list of matches is produced. Then the three indexed lists (author-title by code, authors and keywords) are copied and addressed to the users; keywords are starred if they match those on the addressee's keyword list. The lists are distributed to users with a sheet of directions for using the index. The directions remind users what the codes represent and therefore how to locate documents. The card decks are cumulated semi-annually, listed, and the lists retained in a notebook. The cards are stored for future uses.

The system has been in operation since May of 1968 so by now most of the articles in the library are included in the indices. The unindexed papers are either being discarded as current value decreases or are added to the indices.

Possible Extensions of the Concept

The punched cards have several potential uses. Because the codes are aligned it is easy to a) sort the cards by box number for a "shelf list" or by year to assist in weeding out old items, b) separate

microfiche articles and sort them by AD number, and c) resort by whole code numbers in case of a mixup. There is enough information on the cards so that if it were ever decided to set up a machine-searchable file the information could be converted automatically to tapes or whatever method would be desired.

The system could be used for journal articles. A pack of cards would be made for every article in journals taken by the library, using the journal title, volume and page as a code. These cards could be used to produce a complete periodical index, similar in form to the Computing Reviews indices though tailormade to the particular library. It would involve a large key-punching effort, however, and the gains do not seem sufficient to warrant it for a computer library because Computing Reviews indices cover most of the same literature.

The system could be adapted for books in at least two different ways. Either the library could maintain the card catalogue, which users seem to prefer, and use a more complicated program to print the information on 3"x5" cards in a form similar to usual catalogue cards. Or a new file could be started using a different form, such as punched cards or listings of cards. This would require either conversion of the old card file or maintenance of two files. Printing on 3"x5" cards seems most practical. Such a system has been employed in at least one small library [1]. It is particularly useful for collections of papers that should be catalogued separately but for which librarians are reluctant to make such an effort since it requires typing so many cards.

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A	ASSOCIATED	COMMODITY
ABOUT	AT	COMMON
ABSTRACT	ATTRIBUTES	COMMUNICATIONS
ABSTRACTS	AUGUST	COMPANY
ACADEMY	AUTHORS	COMPARABILITY
ACCEPTED	AUTOMATIC	COMPARE
ACCURATE	AUTOMATICALLY	COMPLETE
ACHIEVING	AVAILABLE	COMPILATION
ACTION	BACKGROUND	COMPUTATIONAL
ADAPTATION	BASE	COMPUTER
ADDITIONAL	BASED	COMPUTERS
ADVANCE	BASIC	COMPUTING
ADVANCED	BASIS	CONCEPT
ADVANCES	BE	CONCEPTS
AGREEING	BECAUSE	CONCEPTION
AID	BECOME	CONCEPTUAL
AIDS	BEEN	CONCERNED
ALL	BEHIND	CONCERNING
ALLOW	BEST	CONDITION
ALLOWED	BETTER	CONDITIONS
ALLOWS	BETWEEN	CONFERENCE
ALWAYS	BEYOND	CONSIDER
ALTERNATIVE	BODY	CONSIDERATION
ALTERNATIVES	BOOK	CONSIDERATIONS
AM	BOTH	CONSTRUCTING
AMONG	BREAKTHROUGH	CONSTRUCTION
AN	BUILDING	CONTACT
ANALYSES	BUT	CONTAIN
ANALYSIS	BY	CONTAINED
ANALYZE	CALCULATION	CONTAINS
ANALYZED	CAPACITY	CONTAINING
ANALYZING	CASE	CONTRIBUTION
AND	CASES	COUNCIL
ANNOTATED	CENTER	CREATE
ANNUAL	CENTERS	CREATED
ANOTHER	CERTAIN	CRITERIA
ANY	CHANGE	CRITIQUE
APPENDIX	CHANGED	CURRENT
APPLICABLE	CHANGES	DE
APPLICATION	CHAPTER	DECEMBER
APPLICATIONS	CHAPTERS	DEFINED
APPLIED	CHARACTERISTICS	DEFINITION
APPROACH	CHARACTERIZATION	DEFINITIVE
APPROACHES	CHARACTERIZATIONS	DEGREE
APPROXIMATE	CHARTS	DEL
APRIL	CHECK	DEPARTMENT
ARE	CHECKING	DESCRIBE
AREA	CITED	DESCRIBED
AREAS	CLASS	DESCRIBING
ARISE	CLASSES	DESCRIPTION
ARISING	CLASSICAL	DESIGNATION
ARITHMETIC	CODE	DETERMINATION
ARRANGEMENT	COLLECTED	DETERMINED
ARTICLES	COLLECTION	DETERMINING
AS	COME	DEVELOP
ASPECT	COMMENT	DEVELOPING
ASPECTS	COMMENTS	DEVELOPMENT
ASSIGNMENT	COMMITTEE	DEVELOPMENTS

DEVICE
 DIFFERENT
 DIGITAL
 DIMENSION
 DIMENSIONS
 DIRECT
 DIRECTED
 DIRECTORY
 DISCUSSION
 DIVISION
 DOCUMENT
 DOCUMENTS
 DOMAIN
 DUE
 EASILY
 DURING
 EASTERN
 EFFECT
 EFFECTIVE
 EFFECTS
 EFFICIENT
 EIGHT
 ELECTRONIC
 ELEMENT
 ELEMENTARY
 ELEMENTS
 EMPLOY
 END
 ENDING
 ENVIRONMENT
 EQUATION
 EQUIPMENT
 EQUIVALENCE
 ESTABLISHING
 ESTIMATE
 ESTIMATES
 ESTIMATION
 ET
 EVALUATING
 EVALUATION
 EVENTS
 EVIDENCE
 EVOKED
 EXACT
 EXAMPLE
 EXAMPLES
 EXPECTED
 EXPERIENCE
 EXPERIMENT
 EXPERIMENTAL
 EXPERIMENTATION
 EXPERIMENTS
 EXPLOITABLE
 EXPLORATORY
 EXPLORE
 EXPRESSED
 EXPRESSIONS
 EXTENDED

EXTENDING
 EXTENSION
 EXTENSIONS
 EXTRACTING
 FACILITY
 FACTOR
 FACTORS
 FAMILIES
 FAMILY
 FAST
 FEATURES
 FEBRUARY
 FIELD
 FINAL
 FIND
 FINDING
 FIRST
 FIT
 FIVE
 FLEXIBLE
 FOR
 FORM
 FORMAL
 FORMAT
 FORMULATION
 FOUNDATION
 FOUNDATIONS
 FOUR
 FOURTH
 FROM
 FULL
 FULLY
 FUNDAMENTAL
 FUNDAMENTALS
 FURTHER
 FUTURE
 GENERAL
 GENERALIZATION
 GENERALIZE
 GENERALIZED
 GENERALLY
 GENERATING
 GENERATION
 GIVEN
 GOVERNING
 GROUND
 GUIDE
 HANDLE
 HANDLING
 HAS
 HAVE
 HAVING
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 HELD
 HER
 HIGH
 HIGHLY
 HIS

HOLD
 HUMAN
 I
 IDENTITY
 IF
 II
 III
 ILLUSTRATING
 IMPACT
 IMPLEMENTATION
 IMPLICATIONS
 IMPROVE
 IMPROVED
 IMPROVING
 IN
 INCLUSION
 INCOMPLETE
 INCORPORATION
 INCREASING
 INDENTATION
 INDEX
 INDIVIDUAL
 INEQUALITIES
 INFORMATION
 INSTITUTE
 INSTRUCTION
 INSTRUCTIONS
 INTEREST
 INTERNATIONAL
 INTO
 INTRODUCING
 INTRODUCTION
 INTRODUCTORY
 INVESTIGATE
 INVESTIGATION
 INVESTIGATIONS
 INVOLVE
 INVOLVED
 INVOLVING
 IRREGULAR
 IS
 ISSUE
 ISSUES
 IT
 ITEMS
 ITS
 IV
 JANUARY
 JOINT
 JULY
 JUNE
 KEEP
 KEY
 KIND
 KINDS
 KNOW
 LA
 LABORATORY

LAG	OBJECTIVE	PREPARATION
LANGAGE	OBJECTS	PREREQUISITE
LARGE	OBSERVATION	PRESENT
LE	OBSERVATIONS	PRESENTATION
LES	OBTAINING	PRESENTATIONS
LITERATURE	OCCUR	PRESUPPOSITIONS
LOCATION	OCCURRED	PRIMER
LOCATIONS	OCCURRING	PRINCIPAL
LONG	OCCURS	PRINCIPLE
LOOK	OCTOBER	PRINCIPLES
LOW	OF	PROBLEM
MACHINE	OFFICE	PROBLEMS
MACHINES	OFTEN	PROCEDURE
MADE	ON	PROCEDURES
MAIN	ONCE	PROCEEDINGS
MAKE	ONE	PROCESS
MAKING	ONES	PROCESSES
MANUAL	ONLY	PROCESSING
MANY	OPERATION	PRODUCED
MARCH	OPERATIONS	PRODUCING
MATERIAL	OPTIMUM	PROGRAM
MAY	OR	PROGRAMMING
MEANING	ORDER	PROGRAMS
MEANS	ORGANIZING	PROGRESS
MECHANIZED	OTHER	PROJECT
MEETING	OUR	PROJECTION
METHOD	OUT	PROJECTS
METHODOLOGY	OUTLINE	PROPERTIES
METHODS	OUTLOOK	PROPOSAL
MIGHT	OVER	PROPOSED
MODERN	PACKAGE	PROVE
MODIFICATIONS	PAIR	PROVES
MOMENT	PAIRS	PUBLICATION
MOMENTS	PAPERS	PUBLICATIONS
MOTION	PART	PUBLISHED
MUCH	PARTS	PURE
MUTUAL	PAST	QUALITY
MY	PERFORM	QUANTITY
N	PERFORMANCE	QUARTERLY
NATIONAL	PERIOD	QUESTION
NATURAL	PERMITS	READILY
NEAR	PERSPECTIVE	REAL
NECESSARY	PERTAINING	REALIZING
NEED	PHILOSOPHICAL	RECENT
NEVER	PHILOSOPHY	RECEIVED
NEW	PLACE	RECOGNITION
NEWS	PLAN	REFERENCE
NEWSLETTER	PLANNING	REFERENCES
NEXT	PLAY	REGULAR
NINE	POINT	RELATE
NJ	POLICIES	RELATED
NORM	POLICY	RELATING
NOTE	POUR	RELATION
NOTES	POSSIBLE	RELATIONS
NOVEMBER	PRACTICAL	RELATIONSHIP
NOW	PRACTICE	REMARK
NUMBER	PRELIMINARIES	REMARKS
NUMBERS	PRELIMINARY	REPLY

REPORT	SOCIETY	THROUGH	WHETHER
REPORTS	SOLUTION	TO	WHICH
REPRESENTATION	SOLUTIONS	TODAY	WHILE
REPRESENTATIONS	SOLVE	TOMORROW	WHOSE
REPRESENTED	SOLVING	TOOL	WILL
REPRESENTING	SOME	TOPIC	WITH
REQUIRED	SPECIAL	TOPICS	WITHOUT
REQUIREMENT	SPECIFIC	TOWARD	WORK
REQUIREMENTS	SPECIFIED	TOWARDS	WORKING
REQUIRING	STAGE	TRANSACTIONS	WOULD
RESEARCH	STANDARD	TRANSLATED	WRITTEN
RESULT	STANDPOINT	TREATMENT	X
RESULTS	START	TREND	Y
REVIEW	STATE	TRENDS	YEAR
REVISED	STATEMENT	TWO	YEARLY
ROLE	STRUCTURE	TYPE	YEARS
ROLES	STUDENT	TYPES	YOU
ROUTINE	STUDENTS	UNDER	ZERO
ROUTINES	STUDIES	UNDERSTAND	ZEROS
ROW	STUDY	UNDERSTANDING	*
ROWS	SUBPROGRAM	UNIFIED	*+
RULE	SUBROUTINE	UNION	*
RULES	SUCCESS	UNIT	*{
SATISFYING	SUGGESTED	UNITS	*}
SCHEME	SUM	UNIVERSITY	
SCHEMES	SUMMARY	UPDATED	
SCIENCE	SUPPLEMENT	UPDATING	
SCIENCES	SURVEY	UPON	
SCIENTIFIC	SURVEYS	USAGE	
SECOND	SYSTEM	USE	
SELECT	SYSTEMATIC	USED	
SELECTED	SYSTEMS	USEFUL	
SELECTING	TASK	USER	
SELECTION	TASKS	USERS	
SEMI-ANNUAL	TEACHER	USER**S	
SEMINAR	TECHNICAL	USER-S	
SEMINARS	TECHNIQUE	USES	
SEPTEMBER	TECHNIQUES	USING	
SERIES	TECHNOLOGIES	UTILIZATION	
SERVICE	TECHNOLOGY	UTILIZE	
SETS	TEN	UTILIZING	
SEVEN	TEND	VARIATION	
SEVERAL	TENTATIVE	VERSION	
SHALL	TEST	VERSUS	
SHE	TESTING	VERY	
SHOULD	THAT	VIA	
SIMILARITY	THE	VOLUME	
SIMPLE	THEIR	VOLUMES	
SIMPLIFIED	THEIRS	VS	
SIMPLIFYING	THEN	VS.	
SIMULTANEOUS	THEOREM	WAS	
SINCE	THEORETIC	WAY	
SINGULAR	THEORETICAL	WAYS	
SITUATION	THEORY	WE	
SIX	THERE	WERE	
SIZE	THEY	WESTERN	
SMALL	THIRD	WHAT	
SO	THREE	WHEN	

THE USE OF I.C.R. FACILITIES FOR LINGUISTICS -- I

W. Fisher

During the past year the author has been investigating the application of computers, in particular the equipment of the I.C.R., to problems in linguistics. This article is a first report on some of the results; it is intended primarily as a record and guide for others who may want to use or build on the programs written.

In an earlier article [3] Yngve wrote:

"Computer applications to traditional or usual linguistic methods are the most straightforward. These are methods that have been useful in an un-automated form for many years. Concordance making, text searching, and the handling and sorting of linguistic data lend themselves to easy automation. The use of the computer can bring speed, convenience, accuracy, and relief from a certain amount of drudgery."

Historical linguistics was the first area chosen for examination, and the first programs written were intended to automate some of the simpler traditional or usual methods, as suggested by Yngve. It was felt that these preliminary data manipulating programs would be of the most value for the practical linguist and that they would be a base for the development of more advanced or theoretically controversial work.

Historical, or diachronic, linguistics attempts to find the history of a language -- how it has changed through the years. The basic assumption usually used is that if two languages are quite

similar now, then at some time in the past, they were the same language. This postulated ancestor language is referred to as the Proto-language. Language change is seen as change in the language faculty, or grammar, of succeeding generations of speakers: rules will be added, changed, lost, and lexical representations of items such as morphemes or words will change. Most of the work done so far in historical linguistics has concentrated on the phonological part of grammar -- the sound system of the language -- and the work reported on here is involved with sound change, not syntax change.

How does one proceed in historical linguistics? Ideally, the linguist will have before him complete grammars of the two contemporary languages being compared, and will then construct a single grammar assumed to represent the ancestor, or Proto-grammar, along with postulated changes in the Proto-grammar which will produce the two grammars in evidence. The changes postulated will be in accord with universal notions of how languages can change, and will be maximally simple in some sense.

Unfortunately, the practising linguist usually does not have before him completely adequate grammars of the languages. The minimum data with which historical linguistics can proceed are vocabulary lists for the two languages, and the first programs written were designed to collect, sort, and print out vocabulary lists.

The sounds of the words must be represented in a phonological orthography, and program INPHON lets the user define to the computer his code for representing the phonemes -- or phonological segments -- of a language. This coding is stored on disc; the order in which the phoneme orthographies is read in is taken as the order for alphabetic sorting purposes.

The program used to input a lexicon (vocabulary list) and store it on disc is called LEXIN. With this program, the user may create or update a lexicon of up to 2000 items, each item consisting of five fields of data: phonological data, prosodic data, grammatical classification, definition or gloss, and source citation. Input is from punched cards.

The program which sorts a lexicon, FASTSORT, may be called as an option in program LEXIN or run separately. It uses a modified version of Algorithm 347 of the Collected Algorithms of the A.C.M., operating on sort keys derived from the information in the phonological field of the lexicon. The major modification consisted of changing the algorithm so that it would not actually move the lexicon items, but rather sort a list of pointers, or order vector, which would then be stored in the data base on disc, reflecting the sorted order of items in the lexicon. By avoiding moving the actual items, considerable savings in running time is effected; in addition, the items in the lexicon remain available to other routines in both their original and sorted orders.

Program LEXOUT provides the user with several options for printing out and examining the lexicon he has created. He may specify the order: as read in, reverse as read in, alphabetic, or reverse alphabetic. In the phonological orthography, digraphs are conventionally enclosed in parentheses; the user may have the print-out with these parentheses removed. The user may also use the console to specify a pattern which will be used in a selective print-out of the lexicon.

Another program, STAT1, can be used to compute and print out the percentages of occurrence in the lexicon of each phonological segment.

The programs described above [1] are the major ones debugged and running now, allowing the user to build and examine a data base consisting of two lexicons. The next step for the historical linguist working with vocabulary lists is to locate cognate items in the two lexicons, and the automation of this task promises to be challenging.

Cognates are key evidence for historical reconstruction; they are items in the two lexicons which are similar in sound and meaning and therefore presumably are descendants of a single item in the Proto-language.

In finding cognates, we would presumably take one item in lexicon A and search lexicon B for an item close in sound and meaning. This is essentially the classic problem in pattern recognition: given

an input pattern, and a list of patterns in memory, to find the match. We need a distance function defined between two items, and we need a decision function to tell when the distance is small enough to qualify as a match.

The distance function should be a composite of phonological and semantic distance. The decision function may be non-linear and complex, and should be changeable at least parametrically by the user in an interactive scheme.

Since semantics is not well understood at present, there is little to be said about what a "semantic distance" should be. As for "phonological distance," we might propose first a simple scheme to calculate the percentage of phonemes differing in two items, perhaps weighting some phonemic differences more than others. And here we come across one minor difficulty: the patterns we are matching (say "words") are composed of ordered sub-patterns ("phonemes"), and we must find the right correspondence between the sub-patterns, or phonemes. For example, here are two ways to put the phonemes of the words "skas" and "kas" in correspondence:

A:	s	k	a	s
	ϕ	k	a	s

B:	s	k	a	s
	k	a	s	ϕ

Obviously (to a human) correspondence A is correct, and the two words are probably close enough to qualify as cognates; A involves a simple difference of a consonant prefix, whereas B involves

a complicated difference: $s : k$, $k : a$, $a : s$, and $s : \emptyset$. One way to get around this problem would be to use the correspondence which results in the minimum of phonological distance.

Some work has been done in computing phonological distance [2], and in closing, I will briefly describe a cognate finding program now being developed, which builds on Ladefoged's previous work.

The routine to find a match will have available a set of transformations to apply to the input, and it will look for an exact match in the lexicon. The transformations will be in the form of standard phonological rules, with an associated probability. In trying to find a match, the routine will heuristically determine the subset of transformations of maximum probability which finds matches. Initially the transformation probabilities will be a priori, derived from language universals and the linguist's experience; final decisions on cognatehood will be in the hands of the user, and after they are made, the probabilities of the transformations yielding those cognates will be increased, probably to 1.00. It is felt that such a program will be reasonably efficient and will allow the straightforward incorporation of most of the intuitions used by linguists in finding cognates.

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A FRINGE DETECTOR FOR USE WITH MICHELSON STELLAR INTERFEROMETERS

R. H. Miller

ABSTRACT

Fringe patterns are recognizable through their continuity properties; these imply that the "power" of fringe patterns mostly appears in a restricted portion of the Fourier transform space. This fact can be utilized to measure the fringe visibility. This report is a preliminary design study of a detector based on this principle. Signal-to-noise ratio estimates look promising and have been confirmed by computer simulation. Two possible ways of building a detector utilizing these principles are discussed. "Seeing" does not limit the aperture size that may be used.

1. THE MICHELSON STELLAR INTERFEROMETER

The Michelson Stellar Interferometer is a device that samples the light coming from a stellar source at two widely separated points, then combines the two samples to form an interference pattern, which can be interpreted to give a measurement of the mutual coherence function (Born and Wolf, 1959, Chapter 10), as shown in Figure 1. The diameter of the star emitting the light can be inferred from measurements of this function at various separations of the sampling points. In a practical case, the incoming wavefront is distorted by atmospheric effects into

some kind of irregular surface. In the literature on seeing, the surface is said to be "corrugated." In the absence of the atmosphere, the surface would be nice and flat. The average height of the surface is the same at both apertures, the interferometer having been arranged to assure this.

If it were not the same (to within the coherence length of the light; effectively determined by the spectral passband through the relation

$L \approx \lambda^2 / (\delta\lambda)$, the interference pattern could not be found.

One way of forming the interference pattern is to combine the two beams with the help of a beamsplitter, as shown in Figure 1. Then, with a certain phase difference between the two beams, all the light might come out on one side of the beamsplitter. With the relative phase differing by π , all the light would come out the other side, and at intermediate phase differences, the light would divide between the two sides. Many other arrangements might be considered to display the interference patterns; because they are all mathematically equivalent, this particular arrangement will be considered in this report to avoid enumerating many possibilities each time a detector is mentioned. The two light beams will be assumed collimated so they are parallel (in the absence of seeing effects). There is then a 1:1 mapping of points in the cross-section of the combined beams (in the fringe pattern) to points in each sampling aperture. This mapping is determined in a ray-optics sense and is as valid as diffraction effects permit.

In the absence of atmosphere the incoming wavefront would be smooth and flat, and the light pattern emerging from the half-silvered surface might be arranged to be uniform or barred (if the two beams form a small angle with each other). Suppose it is arranged to be uniform (this condition will be called a "flat field"): with the real atmosphere present, the actual pattern would be nonuniform. It would have a mottled appearance, with light and dark streaks running irregularly through it, but moving about because of the seeing. At one instant, the pattern might look like the upper portion of Figure 2, and a fraction of a second later it might look like the lower portion of Figure 2. The regions in Figure 2 are shown cross-hatched to suggest that the "dark" regions in the pattern need not be completely dark, but are merely less bright than the bright regions. A pattern like this is called a fringe pattern, and the amplitude of the normalized mutual coherence function is determined from the "fringe visibility," as Michelson called it, which is given by the maximum and minimum intensities in the fringe pattern:

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \quad (1)$$

The object of the fringe detector is to measure this quantity in the practical case where the field is irregular and constantly changing. In particular,

the fringes will not be evenly spaced and can be curved and have forks in them, as shown in Figure 2; these properties complicate matters with most automatic detection and measuring methods.

The fringe pattern will be random and can be described by a function like

$$B = B_0 (1 + V \cos \phi) \quad (2)$$

where B is the brightness observed at a point (x, y) at a time t , B_0 is the mean brightness averaged over the pattern, and ϕ represents the phase difference between the two beams: ϕ is a function of x , y , and t with certain continuity properties. The randomness enters through $\phi(x, y, t)$; but ϕ is smooth and can change by 2π (the fringe pattern goes from a bright fringe to the next bright fringe across a dark fringe) in a distance that is typically about 10 cm in 1-arcsecond seeing. (Distances are referred to the incoming wavefront throughout this report.) Seeing affects the Michelson Stellar Interferometer through a structure function that governs the behavior of the optical phase (ϕ). There is very little in the literature on seeing to allow valid inferences about this function--other properties of the seeing phenomenon have dominated most of the investigations.

The essential point to the proposed fringe detection scheme is that a measurement of V is wanted, but one really does not care (or even want to know) what the function ϕ is doing, or what the precise shape of the fringe pattern is.

II. WHY AUTOMATIC FRINGE VISIBILITY MEASUREMENTS?

Michelson and Pease estimated fringe visibilities visually, and in so doing, demonstrated that stellar interferometers could be made to work. There are two principal reasons for wanting to use automatic fringe detection schemes in preference to visual estimates: (1) the ability to work at lower light levels, and thus to work on fainter objects, and (2) to achieve more precise measurements of the fringe visibility than visual estimates can provide. Precise measurements are necessary in some of the more refined applications of the interferometric method--some of those that are always mentioned, such as limb-darkening measurements and so on. Limb darkening measurements are probably so difficult that they cannot practically be made; but it is not the object of this report to discuss those problems.

III. PREVIOUS METHODS OF AUTOMATIC FRINGE VISIBILITY MEASUREMENT

Several attempts have been made, in recent years, to measure fringes in Michelson Stellar Interferometers by automatic methods. I know of four, which I shall attempt to describe briefly in this section.

A. Beavers' work:

W. Beavers has made the only measurements of fringe visibility on single stars that I know of in recent years. This work is mostly described in Beavers' thesis (Beavers, 1965); he has continued to work on the problem and now has a telescope especially adapted to interferometric

applications nearly ready to operate. Beavers and Swift (1968) have reported laboratory tests of equipment that should be an improvement over the earlier methods. The new method consists of forming the fringes into a barred pattern which is scanned across a "picket fence" by a rotating mirror. The light so modulated is detected by a photocell whose output is analyzed for the presence of fringes.

B. Elliot's method:

J. Elliot and F. Scherb (Elliot, 1965, Elliot and Scherb, 1967, Scherb and Elliot 1967) proposed a method whereby the fringes were formed into a barred pattern which was passed through a "picket-fence" of the same spatial frequency into a photocell. The "picket-fence" was moved back and forth at a fairly high frequency to modulate the light beam. This is a pure modulation method, and is based on stationary uniform patterns, although the method can tolerate departures from the ideal pattern. Elliot's thesis consists of an analysis of the expected signal-to-noise ratio of the method.

C. Currie's scheme:

D. Currie (1967) proposed a method of detecting fringes in which the two light beams are arranged to produce a flat field. The region sampled is kept so small that the entire field can be assumed to have uniform brightness (there is no fringe pattern inside

the accepted field). This restricts the available field to 2-3 inches in the incoming wavefront. The uniform field has randomly fluctuating brightness as a function of time only, because of the seeing. In effect, the seeing provides a random modulator in this scheme. Currie pointed out that the characteristic of a fringe pattern in this detector would be that a photoelectron registered in one of two detectors (like D_1 and D_2 of Figure 1) would with higher probability appear in the same detector as its predecessor. In the absence of fringes, successive photoelectrons would with equal probability be recorded in either detector. Thus, if the sequence of arrivals is noted, counting one in a "direct" channel if two successive photoelectrons are recorded in the same detector, or counting one in a "switched" channel if two successive photoelectrons are recorded in opposite detectors, the fringe visibility can be worked out from the ratio $(n_D - n_S)/(n_D + n_S)$. The signal-to-noise ratio for this scheme has also been worked out (Miller, 1967a, b). I have used this method in the laboratory, but have not succeeded in making it work at the telescope. I understand that Currie has undertaken some experiments with it.

D. Twiss' method:

Since returning from Narrabri, R. Q. Twiss has been working on a Michelson Stellar Interferometer. Twiss has stressed need for high precision measurements of fringe visibility (on the order of 1%), and has pointed out that fringe curvature places a serious limitation

to the accuracy of any automatic fringe measuring method. Evidently, Twiss' method consists of using small (2-inch diameter) flats to redirect the light beams before forming fringe patterns; he then hopes to servo-control the orientation of these flats to produce a "flat field" from the fringe pattern. Approximately half the starlight is to be used to provide the control information for the servos to orient the flats. A two-meter instrument has been built in England, and installed in Italy; the instrument is to be used for instrumental studies only, using Polaris as the object. Twiss hopes to get it into operation this year.

Currie's method actually evolved from discussions between Currie and Peter Bender, trying to reconstruct how Twiss might be measuring fringe visibility.

E. Difficulties with these methods

Each of these methods is based on the establishment of an expected fringe pattern, whether it be a flat field or a barred pattern. The total amount of light in the field is then measured by a photocell. The methods will work well as long as the pattern conforms to the expected pattern. But, in general, it will not. For example, Currie's scheme requires a flat field; but then the aperture must be so restricted that the field is actually flat. Some tolerance is allowed, of course; the limitations on field size are

calculated from estimates of the size of field that will conform to the allowed tolerances. The limitations may be appreciated by the following argument: suppose the actual fringe pattern contains a dark fringe separating two bright fields. Then the photocell, integrating the total amount of light in the aperture, sees a greatly reduced fluctuation of total light over what it would see if the field were uniform. In particular if there were one cycle of a sine wave across the field, the photocell would not register any change of total light even though the relative phases of the two beams changed enough to make the sinewave pattern reverse (go from dark to bright at some location). Although this description is based on a spatial pattern that does not change with the time, the same thing can be seen to happen even if the pattern changes with time.

There is no advantage in this respect with the barred pattern: Suppose the barred pattern is designed to have N dark bars across the field. The photocell will get confused if the actual pattern contains $N+1$ dark bars, and the angular deviations that take a pattern of N to a pattern of $N+1$ are just the same as those that take a flat field to a field with 1 dark fringe. This is illustrated in Figure 3, where an undistorted barred pattern is shown in the upper portion; the lower portion shows what the pattern would look like if it were distorted by the same seeing effects that would carry a flat field into the fringe pattern shown in the lower part of Figure 2. The bars are curved and

slope in directions that a picket-fence could not follow. A single photodetector behind the picket fence would not be able to distinguish this pattern from a field that contained no fringes.

In effect, in each of these methods, one has tried to force the fringe pattern into a certain mold; but the fringe pattern won't let itself be forced into that mold. Note that the entire argument is based on the notion that the phase-differences, ϕ of Eq. (2) may be several times 2π -- the "equiphase surface" height varies by several wavelengths. Danjon (1955) claims that the variation is asymptotically limited to about 1 wavelength. I don't believe this result, although I must admit that Danjon's observations deserve more weight than my prejudices. I hope to get a definitive answer to this question in the near future. If Danjon's result is correct, it should be much easier to design a good automatic fringe detector.

Converted to practical units, with "reasonable" estimates about what the seeing is doing, with 3" diameter apertures, with $2\text{\AA}$ passband, there is a limit achieved with Currie's scheme at about 7th magnitude. The $2\text{\AA}^0$ passband is selected to yield a coherence length of about a millimeter, because the coherence length determines the mechanical tolerances in an interferometer. The brightness limitation comes about in Currie's scheme because several photoelectrons must be collected before the fringe phase (ϕ of Eq. (2))

can change very much. These numbers were reported previously, along with a brief survey of the kinds of astronomical problems that might be attacked with an instrument having these limitations (Miller, 1968).

IV. THE PROPOSED METHOD

There are several troublesome points in the detectors discussed so far that somehow pose limitations on the methods, but which look as if they should not. For example, one ought to be able to detect and measure the visibility of fringes in a field of essentially arbitrary shape without caring what that shape is. One ought to be able to combine measurements from several of the small apertures of Currie's scheme, knowing that, with neighboring apertures, knowledge of the seeing phase ϕ in one of them implies that you know something about the phase in the other, and vice versa.

When you look at fringes, you readily recognize them quite independently of the shape. You readily distinguish cases in which the visibility is large (distinct fringes) from cases in which it is low. What properties of fringe patterns permit you to make this ready identification? How might these properties be utilized in an automatic fringe measuring device?

The essential property of a fringe pattern seems to be its continuity. If one point is bright, a neighboring point will also be bright. You must look at some distance from the original bright place before you find a dark region. Similarly, with changing patterns, if a spot is bright, it

will stay bright a little while before it becomes dark. Currie's detection scheme is based on this property of the time-variation. The property applies in space as well.

A more formal statement is that, if you were to fit a set of polynomials to the brightness distribution, low-order polynomials would suffice. The formal statement is useful for calculations--some calculations are required to determine how good or how bad the method might be. There is, of course, nothing magic about polynomials. Other functions would do as well. Sines and cosines are a nice set for such purposes. So the formal statement becomes: If you Fourier-analyze the pattern, low frequencies are sufficient to describe it. The discussion in this report is based on Fourier analysis, although it should be recalled that Fourier analysis is not necessarily the best way to treat the patterns. This limitation is placed to keep the report manageable and because more readers may be familiar with Fourier analysis than with other decomposition schemes.

Imagine an ideal detector that somehow told you "where" and "when" each photoelectron was recorded over a 2-dimensional surface: the i^{th} photoelectron was recorded at x_i , y_i , and at time t_i . The discussion of this section and of the next will be based on such a detector; approximations to it will be discussed in the final section. If the illuminated field were bright, the combination of very many events would give a brightness

$B(x, y, t)$ that depended on position and time.

This brightness function requires a three-dimensional analysis. The calculations are messier, but there is nothing new in principle in going from one to three dimensions. So for the moment, think of the brightness in time alone, $B(t)$. If there were no fringes, $B(t)$ would not change, and would look like the "boxcar" in the upper half of Figure 4. The square modulus of its Fourier transform (its power spectrum) would look like the solid line in the lower part of Figure 4. With fringes present, $B(t)$ would vary somehow, possibly like the dashed line at the top of Figure 4. Its power spectrum might look like the dashed line at the bottom of Figure 4. If the output were filtered, a filter extending from zero frequency to the first light vertical line would pass almost all the "power" of the "boxcar" that results from no fringes (a statistician would call this the "null hypothesis"), but a relatively small portion of the power that results if there are fringes present. A second filter, passing the frequencies between the two vertical lines, would pass very little power with no fringes present, but a substantial amount of power with fringes present. The high-frequency cutoff is convenient for noise rejection, as will be shown in the next section.

Now, think of the full three-dimensional function, $B(x, y, t)$. It is some kind of a "density" in a three-dimensional space like that at the top of Figure 5. Its Fourier transform is a different three-dimensional

object--some kind of a density distribution $\hat{B}(\hat{x}, \hat{y}, \hat{t})$. It is built up of things like the power spectrum of Figure 4, but in three dimensions. The filter-function shown at the bottom of Figure 5 lives in the first octant, as is appropriate for a power spectrum. It looks something like a section of an avocado--the nut at the center is the three-dimensional analogue of the low-pass filter, but it passes low spatial frequencies as well as low temporal frequencies. Most of the power from the case of no fringes (the null hypothesis) is in this central core. Some power from the fringe pattern is also there, and that is an unavoidable inefficiency (although here, use of another set of functions for the transformation might reduce this inefficiency). Outside this nut is the good part of the avocado--the filter whose pass band contains most of the power from bonafide fringes. It also passes a little power from the no-fringe (null hypothesis) case. The region outside both shells--the high-frequency region--will mostly contain noise, and by judicious choice of the boundaries of these filter regions, one hopes to maximize his ability to measure the power in the fringe pattern.

The fringe visibility measurement is a measurement of the amount of power in the region between the two shells in the three-dimensional transform space. That power can be shown to contain a term proportional to V^2 if the outputs of the detectors D_1 and D_2 of Figure 1 are first differenced at each point (x, y) .

This method gets around the shortcomings of the earlier methods described in the previous section. The fringes may have any orientation or shape; they may change at almost any rate. The brightness pattern is modulated by the random modulator of the atmosphere, and this modulation causes the power spectrum of the brightness to fill the region described. In fact, all that has been done so far is to describe the properties of the Fourier transform of the brightness distribution, and to point out that the part due to fringes lies in a certain part of the transform space. If one measures the total power in that part of the transform space, he measures the fringe visibility.

V. SIGNAL-TO-NOISE RATIO

A. Amplitudes and Phases of Beams; Scintillation Noise

Let the amplitude of the beam approaching from the left side of the beamsplitter of Figure 1 be described by $\exp(\alpha_1 + i\varphi_1)$, where α_1, φ_1 are real, and in general are functions of position and time. Imagine a "narrow band" approximation with the center frequency term left out, as in Born and Wolf (1959), Chap. 10. The scalar field picture used here can easily be extended to include polarization-dependent effects by superposing two polarized waves. The corresponding quantities for the beam approaching from the right are α_2, φ_2 . The two beams are considered parallel, and the time and position dependences in $\alpha_1, \alpha_2, \varphi_1, \varphi_2$

are unimportant because the interference phenomenon takes place through point-by-point correspondence of the two beams at equal times. When a beam is transmitted through the beamsplitter, it emerges with a new amplitude and phase: for example the amplitude and phase of the piece of beam 1 that is transmitted is $\exp(\alpha_1 + \alpha_t + i\varphi_1 + i\varphi_t)$, and for the reflected portion is $\exp(\alpha_1 + \alpha_r + i\varphi_1 + i\varphi_r)$. The beam leaving the beamsplitter is a superposition of the two original beams: that leaving on the left side has its amplitude and phase given by $\exp(\alpha_1 + \alpha_r + i\varphi_1 + i\varphi_r) + \exp(\alpha_2 + \alpha_t + i\varphi_2 + i\varphi_t)$, that on the right has r and t indices interchanged. The intensities are to be obtained in the usual way from the square modulus, giving

$$\begin{aligned}
 I_L &= e^{2\alpha_1 + 2\alpha_r} + e^{2\alpha_2 + 2\alpha_t} + 2e^{\alpha_1 + \alpha_2 + \alpha_r + \alpha_t} \cos[(\varphi_2 - \varphi_1) - (\varphi_r - \varphi_t)], \\
 I_R &= e^{2\alpha_1 + 2\alpha_t} + e^{2\alpha_2 + 2\alpha_r} + 2e^{\alpha_1 + \alpha_2 + \alpha_r + \alpha_t} \cos[(\varphi_2 - \varphi_1) + (\varphi_r - \varphi_t)],
 \end{aligned}
 \tag{3}$$

again the r and t indices associated with 1 and 2 indices are interchanged in the two expressions. If the two beams are only partially coherent, the entire argument carries through with the inclusion of $|\gamma|$, the absolute value of the normalized mutual coherence function, multiplying the cosine terms.

If the "reasonable" condition is imposed that the sum of the two intensities should be independent of the phase difference, $\cos(\varphi_2 - \varphi_1) = 0$, or the two differ in phase by $\pi/2$. This rather surprising result seems to have been noticed by several people but has not, so far as I am aware, been published. The "reasonable" condition says that you should not be able to determine the relative phase of two beams by a single measurement of the total transmitted intensity. The condition will be assumed; then $\sin(\varphi_2 - \varphi_1) = \pm 1$, and the expressions for the phase-dependent terms in the two intensities have opposite signs:

$$I_L = e^{2\alpha_1 + 2\alpha_R} + e^{2\alpha_2 + 2\alpha_L} + 2|\gamma|e^{\alpha_1 + \alpha_2 + \alpha_R + \alpha_L} \sin(\varphi_2 - \varphi_1) \quad (4)$$

$$I_R = e^{2\alpha_1 + 2\alpha_L} + e^{2\alpha_2 + 2\alpha_R} - 2|\gamma|e^{\alpha_1 + \alpha_2 + \alpha_R + \alpha_L} \sin(\varphi_2 - \varphi_1).$$

The sum of $I_L + I_R$ is independent of $(\varphi_2 - \varphi_1)$; this is the condition imposed-- and the difference is

$$I_L - I_R = (e^{2\alpha_1} - e^{2\alpha_2})(e^{2\alpha_R} - e^{2\alpha_L}) + 4|\gamma|e^{\alpha_1 + \alpha_2 + \alpha_R + \alpha_L} \sin(\varphi_2 - \varphi_1). \quad (5)$$

With a "good" beamsplitter, $\alpha_L = \alpha_R$, and the first term vanishes. In any event α_L and α_R are time-independent (they will vary from place to place over the beamsplitter surface). With $\alpha_L = \alpha_R$, the $e^{2\alpha_R}$ term factors out

of I_L or I_R , and the $e^{2\alpha_1}$ term left is just I_1 , etc. so this reduces to the familiar form. Eq. (2) has been recovered, with $U = |\gamma|$, and $\phi = \varphi_2 - \varphi_1 + \pi/2$. The rest of this note will be based on the $\alpha_R = \alpha_L$ case; the remaining terms show how to get the correct expressions when things are not perfect, but carrying them along merely obscures the basic principles.

The feature that is wanted from all this discussion is the autocorrelation of the difference of intensities (the sum is straightforward), because that shows how scintillation noise enters. Recall that the Wiener-Khinchin theorem states that the Fourier transform of the autocorrelation is the power spectrum; thus the autocorrelation is the object that ties this discussion in with that of the previous section. Take $e^{\alpha_R + \alpha_L} = 1/2$, so

$$\begin{aligned} I_L - I_R &= 2|\gamma| e^{\alpha_1 + \alpha_L} \sin(\varphi_2 - \varphi_1) \\ &= 2|\gamma| \sqrt{I_1 I_2} \sin(\varphi_2 - \varphi_1) \end{aligned} \tag{6}$$

in terms of the intensities of the incident beams. The autocorrelation can be obtained from an ensemble-average over intensity-differences at different times. Let $\phi = \varphi_2 - \varphi_1$ (it will shortly become apparent that the $\pi/2$ difference between this definition and that of Eq. (2) is unimportant), then the autocorrelation is

$$\begin{aligned}
& \mathcal{E}[(I_1(t) - I_2(t))(I_1(t+\tau) - I_2(t+\tau))] \\
&= 4|\gamma|^2 \mathcal{E}[\sqrt{I_1(t)I_2(t)} \sin \phi(t) \sqrt{I_1(t+\tau)I_2(t+\tau)} \sin \phi(t+\tau)] \\
&= 4|\gamma|^2 \mathcal{E}[\sqrt{I_1(t)I_1(t+\tau)} \sqrt{I_2(t)I_2(t+\tau)} \times \\
&\quad \times \frac{1}{2}(-\cos(\phi(t) + \phi(t+\tau)) + \cos(\phi(t) - \phi(t+\tau)))] \\
&= 2|\gamma|^2 \mathcal{R}_S(\tau) \mathcal{E}[\cos(\phi(t) - \phi(t+\tau))] \\
&= 2|\gamma|^2 \mathcal{R}_S(\tau) \mathcal{R}_{\phi}(\tau).
\end{aligned} \tag{7}$$

Several notions have been used in these successive steps: that the cross-correlation $\mathcal{E}[I_1(t) I_2(t+\tau)]$ is just equal to $\mathcal{E}[I_1(t) I_2(t)]$ for all τ (if the intensities had mean zero, the cross-correlation would be zero), that ensemble-averaging permits the expectation of a product to be written as the product of the expectations (this is why the insistence on ensemble-averages), that the ensemble-averaged cosine of a sum of phase angles is zero, but that of the differences is not (consider the case $\tau = 0$, then the difference-angle is zero), and that cross correlations between the intensities and the phase-differences vanish.

The important feature is that the autocorrelation of the scintillation multiplies the other terms in the expression for $|Y|^2$, so in cases where $|Y|$ is difficult to measure ($|Y|$ small), the scintillation correction is not important. In particular, with a properly balanced system, scintillation noise will not produce a fake difference term. This statement is, of course, only approximately true, to the extent that $e^{\alpha_x} - e^{\alpha_y} = 0$; with an imperfectly balanced system, where $\alpha_x \neq \alpha_y$, (or with unequal detector efficiencies) there will in general be a term due to scintillation noise in the difference even when $|Y| = 0$. This will be an additive contribution that can be subtracted, but at some cost in signal-to-noise. The autocorrelation of the scintillation will enter the expression for the autocorrelation of the sum of intensities, so there is an experimental measurement of this function available. The autocorrelation functions are functions of space as well as of time; for the general case, treating the "time difference," τ as if it were a vector in the three-dimensional (x,y,t) space will give the proper generalization.

B. Shot Noise

The limitations on a detector scheme are expected to be reached with faint objects; and there the limitation is expected to be set by shot noise. Again, the analysis will be treated in the language of the time domain; the generalization to the three-dimensional space-and-time domain merely consists in interpreting certain variables as vectors in a 3-space rather than as scalars.

The analysis follows the usual procedures for shot-noise, based on Campbell's theorem, as described, for example, by Rice (1945).

The problem is discussed from a statistical point of view by Cox and Lewis (1966, Sec. 5.5); the comparison is interesting and informative.

Without going into details, the expected power spectrum is (equation 2.6-11 of Rice, 1945)

$$\mathcal{E}[S(\omega)] = |S(\omega)|^2 \bar{K} [1 + |\Pi(\omega)|^2], \quad (8)$$

where the notation $\mathcal{E}[\dots]$ is used for expectation-value, $|S(\omega)|^2$ is the power spectrum of the pulse-shape resulting from one event in the shot-noise process, $\bar{K}$ is the expected number of counts under the assumption that the counts arrive according to a Poisson distribution, and $\Pi(\omega)$ is the Fourier transform of the probability distribution governing the arrival of photoelectron counts:

$$\Pi(\omega) = \int p(t) e^{i\omega t} dt. \quad (9)$$

This function is the "characteristic function" of statistics, in the sense that $p(t)$ is the probability distribution for a photoelectron to be counted at time t . In the absence of a fringe pattern, $p(t)$ is uniform over $(0, T)$, the interval during which the counter is turned on. If there are fringes, $p(t)$ will vary in some strange way with the time, producing a modified

power spectrum, such as that in Figure 4. The total "power" in some interval of frequencies is

$$E[P] = \int_{\omega_1}^{\omega_2} d\omega E[S(\omega)] = \int_{\omega_1}^{\omega_2} d\omega |S(\omega)|^2 [\bar{K} + \bar{K}^2 / |\Pi(\omega)|^2]. \quad (10)$$

The variance is required for a signal-to-noise ratio estimate; it involves the double integral over the covariance of the power spectral estimate at two frequencies:

$$\text{Var}[P] = \int_{\omega_1}^{\omega_2} d\omega \int_{\omega_1}^{\omega_2} d\nu \text{Cov}[S(\omega), S(\nu)]. \quad (11)$$

Again, without going into details, the covariance is:

$$\begin{aligned} \text{Cov}[S(\omega), S(\nu)] = & \bar{K} + \\ & + \bar{K}^2 \{ |\Pi(\omega+\nu)|^2 + |\Pi(\omega-\nu)|^2 + 2 |\Pi(\omega)|^2 + 2 |\Pi(\nu)|^2 \} \\ & + \bar{K}^3 \{ [\Pi(\omega+\nu) \Pi^*(\omega) \Pi^*(\nu) + \text{c.c.}] + [\Pi(\omega-\nu) \Pi^*(\omega) \Pi(\nu) + \text{c.c.}] \}, \end{aligned} \quad (12)$$

where the notation, "c.c." means complex conjugate. This form for the covariance implies that there is no higher-order structure in the probability distributions than is expressed by the second-order joint probability distribution; otherwise there would be higher-order terms present. For example, the term

linear in $\bar{K}$ would be replaced by a suitable fourth-order cumulant or semi-invariant, and so on. These details will be omitted in this report.

Even with fairly simple forms assumed for the power spectrum, the integrals obtained by substituting Eq. (12) into Eq. (11) are not very manageable; the essential point, however, is that Eq. (11) calls for an integration over the frequencies. The integration will not modify the structure of the covariance: there will be terms depending on $\bar{K}$, on $\bar{K}^2$, and on $\bar{K}^3$ -- each with some kind of parametric dependence on ω_1 and ω_2 . It may be expected that the $\bar{K}^3$ term might be small, since it is a product of $\Pi(\omega)$ -terms which can be negative as well as positive, but the precise structure of these terms is not too obvious so it is safer to carry this term along.

The signal-to-noise ratio is usually given as the estimated power (actually, the difference of the power measured in the presence of a signal and the power measured with signal absent) divided by the square root of its variance. It is noteworthy that there is no term in $\bar{K}^4$ in Eq. (12); thus the signal-to-noise ratio asymptotically, in the limit of large $\bar{K}$, improves at least as fast as $\bar{K}^{\frac{1}{2}}$. This is the usual kind of dependence on $\bar{K}$.

A priori, it might be expected that there are three time-scales of importance: (1) the time scale of T , the duration of a measurement; (2) the time scale of typical changes of the phase pattern due to seeing, which may be expected to be dominated by changes with time-scales of the order of seconds to tenths of seconds, and (3) the mean inter-arrival

times for logging photoelectrons. It is interesting that the photoelectron time-scale appears mainly through $\bar{K}$, which implies that fringe detection is not limited to a regime in which several photoelectrons must be logged in a time during which the optical phase difference changes very little due to seeing. This conclusion is quite different from that arising from a signal-to-noise ratio estimate for Currie's scheme (Miller, 1967), in which the leading $\bar{K}^{\frac{1}{2}}$ -dependence is retained at all counting rates, but in which the numerical coefficient multiplying $\bar{K}^{\frac{1}{2}}$ decreases sharply when few photoelectrons are logged during times in which the phase-difference can change appreciably due to seeing. The reason for this difference is that Currie's scheme makes use of sequential pairs of photoelectron pulses. It therefore has an effective response to individual pulses that is determined by the probability distribution of the interval between successive pulses. This distribution is exponential (in the absence of signal), and Fourier transforms to give the function $|S(\omega)|^2$ of Eq. (8) the usual Lorentzian shape; the overlap of this instrumental response function with the power spectrum due to seeing decreases as the counting rate becomes lower. (In this language, the integral of Eq. (10) extends from 0 to ∞ for Currie's scheme; it is not natural to think of that scheme in terms of its Fourier transform.)

Arbitrarily large apertures can be used, just as arbitrarily long integration times are used in other applications. The only requirement is that the number of "picture-elements" in the detectors increase with

increasing aperture so that the linear dimension of the area each element represents on the incoming wavefront is independent of aperture size. With large apertures and long integration times, the method can be used to arbitrarily low light levels--thus promising a method that can be used for astronomically interesting objects such as the quasi-stellars. Of course, if the light-levels were pushed sufficiently low, detector and night sky background would have to be subtracted. These can be removed by standard methods.

It is not a priori evident that fringes can be detected, and their visibility measured, in cases where the photoelectron arrival rate is much slower than the time-scale for the optical phase difference to change appreciably due to seeing. With large apertures, this consideration must be restricted to small areas within the aperture, such that the optical phase difference due to seeing is small across that area. The arguments given here indicate that, if you wait long enough, the fringe pattern will emerge even in these cases of slow photoelectron arrivals.

C. Simulation

No detector, operating along the lines proposed here, has yet been built. In the absence of experimental data on a real detector, a Monte Carlo simulation was run on the computer at Kitt Peak. A brief description of the expected results and their confirmation by the simulator is included here to illustrate some of the ideas.

The simulator generated a time-dependent amplitude such as would be seen with a small area detector in one side of Figure 1. No scintillation noise was included, but phase noise was. The phase noise shows up in the interferometer as an amplitude term; it was mocked up by building a signal whose amplitude was

$$R = R_0 (1 + V \cdot RTS), \quad (13)$$

where R is the actual signal, R_0 the average photoelectron rate, V the fringe visibility, and RTS a time-dependent signal modulating the average rate. The signal, RTS , was chosen to be a "random telegraph signal," (Rice, 1945): a signal that is either $+1$ or -1 with equal probability, the number of switchings in a unit time interval being Poisson distributed with some average switching rate, μ . Finally, the actual signal was a set of simulated "photoelectron arrival times," "random" times generated in such a way that the "photoelectrons" arrive according to a Poisson process with time-varying parameter, the parameter being given by R of Eq. (13). The end result is a double random process: the random telegraph signal modulator, and arrival times for the simulated photoelectron pulses. This particular choice for the modulating signal tests one feature of the detector that might otherwise not be tested: the Fourier transform of the random telegraph signal, $\overline{\Pi}(\omega)$, has expectation zero, but its square modulus, $|\overline{\Pi}(\omega)|^2$, has nonzero expectation.

Each of the "photoelectron" pulses was treated like a delta-function, so the term $s(\omega) = 1$ in Eq. (8). Then the power spectrum can be estimated from

$$E[S(\omega)] = \frac{1}{N} \left| \sum_{k=1}^N e^{i\omega t_k} \right|^2, \quad (14)$$

where t_k is the time at which the k^{th} pulse arrived. These sums can be worked out for several different frequencies, ω , and a final sum over various frequencies then evaluated. This is not quite equivalent to an integral over a band of frequencies (the distinction is important in attempts to understand the results of the simulation), but it is close in principle to a trapezoidal integration if the actual spectra contain no pathological features.

The expected power spectrum is

$$|T(\omega)|^2 = \left(\frac{S_{10} \frac{\omega T}{2}}{\frac{\omega T}{2}} \right)^2 + V^2 \frac{4\mu/\tau}{4\mu^2 + \omega^2}. \quad (15)$$

The duration of the count, T , enters, as does the test-frequency, ω .

Note the manner in which the fringe-visibility, V , enters. The contribution due to the random telegraph signal decouples from that of the "window-function," and appears as an additive term in the power spectrum. This is a consequence of the vanishing of the (nonsquared) Fourier transform of the random

telegraph signal--which in turn, results because the expected strength (R of Eq. (13) at any time is constant at R_0 .

Numerical results of the simulations need not be given in detail-- suffice it to say that the variances and expectations computed from the shot-noise picture for the model (Eq. 15) and those from the sequence of times generated by the simulator agreed as well as the statistics would lead you to expect; the simulation confirms the design expectations. Considerable care is required in making the comparisons; in particular, approximations in estimating the comparison are dangerous and can obscure the effects sought.

VI. POSSIBLE REALIZATIONS OF THE SCHEME--REAL DETECTORS

The discussion so far has been based on some kind of ideal detector that could tell "where" and "when" each photoelectron was born over two spatial dimensions (x,y) and the time domain, as if this could be done with arbitrary precision. It is clear, however, that the bandwidth of the detector need only comfortably contain all the expected bandwidths of the seeing phenomenon: in the time domain, this is expected to extend up to about 10-50 cycles per second (about the rate at which the relative phases of two well-separated beams might change) and spatial frequencies extending up to 0.1 cm^{-1} referred to the incoming wavefront in 1 arcsecond seeing (about $s/10 \text{ cm}^{-1}$ in s-arcsecond seeing). Thus, for any real detector, the ideal conditions can be compromised to this extent. The detector should work

down to the single photoelectron level, although a lot of interesting astronomy could be done with less sensitive detectors.

It would be very elegant to use optical Fourier analysis, using the incident starlight directly. Unfortunately, that cannot be done, as optical Fourier analysis methods are based on the use of an intermediate recording of the image to be Fourier-analyzed.

In any practical measurement, the power-spectrum of the seeing $|\mathcal{T}(\omega)|^2$ must be calibrated to yield the actual fringe visibility. The power spectrum of the phase fluctuations in the seeing, and that due to scintillation, may be expected to change with time; they will differ from hour to hour as seeing conditions vary. Thus, these power spectra must be determined as part of the measurement. It is not obvious what is the best strategy for doing this. However, if the outputs of the detectors D_1 and D_2 of Figure 1 are combined to form $(D_1 + D_2)$ and $(D_1 - D_2)$ and these are analyzed as well as the outputs from D_1 and D_2 themselves, then it should be possible to determine these power spectra from various combinations. Recall that ideally the combination $(D_1 + D_2)$ does not contain the power spectrum of the phase variations, and $(D_1 - D_2)$ contains that of the scintillation in a simple way. There, are, in addition, certain relations among the power spectra of scintillation and of phase imposed by the structure of seeing itself that may be invoked. Thus, at this writing, the problem of determining $|\mathcal{T}(\omega)|^2$ at the time of measurement seems quite manageable.

Several ways of realizing the detection scheme can be imagined; two ways that seem to require a minimum of development work are (A) two arrays of photocells, each consisting of about 100 photocells arranged 10×10 , each representing one of the two detectors D_1 and D_2 of Figure 1--this number of array elements would be usable with apertures of 20-30 inches, and (B) a pair of image tubes, each followed by some kind of TV raster scan pickup. These two possibilities will be discussed in detail. These kind of detectors seem to be best suited to working at the "photon-limited" low light levels of astronomy.

A. Array of Photomultipliers

An array of photomultipliers can give the required time and location information for registered photoelectrons-- the time information is much more precise than required, but the spatial information is limited to the picture-element size of a photomultiplier collecting area mapped onto the incoming wavefronts. Since large numbers of photomultipliers may be required, design considerations force selection of as large a picture-element size as can be tolerated. This is probably about 2 inches diameter, referred to the incoming wavefront. Then a 10×10 array of photomultipliers would be adequate for an aperture of 20-30 inch diameter, and should collect fringe visibility information about 10 times as fast as a single 2" aperture. But so coarse a picture-element limits the spatial frequencies that can be measured to rather low values.

Each photomultiplier should have its own amplifier and discriminator adequate to amplify single-photoelectron pulses enough to permit logical operations by standard integrated circuit transistor or diode logic. The outputs could be combined to form a convenient set of logical functions. The appropriate combinations might be formed directly without need for a separate output of each of the functions. Counting-rate limitations on bright objects might be overcome by narrow-band optical filtering of the starlight reaching the detector.

Additional information yield from such a detector-array would increase like the square-root of the number of photomultipliers. The detector costs would rise linearly with the number of detectors--so there would be an optimum number of photomultipliers. A reasonable guess (it is a pure guess--not based on any careful cost analysis or bids) of costs would be about \$200 per photomultiplier, so a pair of detectors arranged 10×10 would cost about \$40,000. Of course, adequate utilization of more photomultipliers would require a larger aperture matched to the number of photomultipliers, so a reasonable optimization of the detector involves a complete interferometer design.

An interesting variant of this design would be to have the array of photomultipliers in one envelope--a kind of coarse Channeltron, with each channel brought out to a separate anode line. A detector of this kind, with about 16×16 picture-elements properly matched to the optical input, would have many astronomical applications.

Proper combination of detector outputs to form the sums and differences, $(D_1 + D_2)$, $(D_1 - D_2)$, would require matching the boundaries of individual apertures. This should not be a difficult instrumental problem, particularly since the alignment need be made only once, when the detector is installed.

If the photomultipliers are refrigerated, detector noise should be no more of a problem than with spectrophotometers and other low-level astronomical applications. Detector noise is more severe than with photometry because the optical input to individual photomultipliers may be severely limited by filtering and other beam-handling methods such that individual photomultipliers may operate at quite low counting rates (1 per second and lower).

Some interesting feasibility studies might be run with the array detector using fewer photomultipliers than mentioned here--two arrays of 6 photomultipliers each, arranged in a 2×3 array, would be adequate for checking out the ideas. Of course, very simple checks might be run with just two photomultipliers. Astronomically, the two photomultiplier system might find application in double-star work.

The photomultiplier array has the advantage that it should be relatively easy to equalize the photocathode responses for corresponding elements in the detector arrays, D_1 and D_2 .

B. TV Scan

Because the TV raster scan spot moves according to a linear relationship between its spatial position (x, y) and the time, a simple temporal Fourier analysis of the output, properly interpreted, contains the spatial and temporal Fourier analysis of the input signal. A TV raster scan is appealing because, in a rather elegant way, it carries out all the transform analysis directly--all that would be required would be some filtering of the output from the tube, which would then be ready to integrate directly.

A convenient way to think of the Fourier analysis as handled by a TV raster scan is to think of the field actually scanned by the raster scan as being periodically replicated over the (x, y) plane, with empty regions in between to correspond to the beam blanking during flyback. The scanning spot moves continuously along a single line over this plane, sampling the brightness inside the scan area over and over--if the scan field pattern changes with the time, the periodically replicated patterns change in the same way, so temporal variations are correctly handled. Then the spatial Fourier Transform is discrete--the temporal variations yield a continuous Fourier spectrum. At the output of the raster scan, there will be a continuous spectrum arising from temporal variations modulated by each of the relevant spatial frequencies: the spatial frequencies will each be multiplied by appropriate factors before being added into the scan output. For example, if ω_x is the horizontal scan

frequency (angular frequency $= 2 \pi$ /horizontal scan time) and ω_y is the vertical scan frequency (again an angular frequency, $= 2 \pi$ /vertical scan time $= 2 \pi$ /field scan time), the spatial frequencies (in a cartesian coordinate system) map into temporal frequencies according to

$$\omega = k_x \omega_x + k_y \omega_y + \Omega. \quad (16)$$

Here, k_x and k_y are the horizontal and vertical spatial frequencies (in units of 1/field dimension), Ω is the temporal frequency, and ω is the frequency at the raster scan output. There are unimportant corrections because the raster scan line is oblique to both the horizontal and vertical axes, and for the empty bands that represent the flyback over the extended (x, y) plane. The regions near the origin of the (x, y, t) space of Figure 5 can be excluded by filtering out the frequencies corresponding to (k_x, k_y) pairs $(0, 0)$, $(1, 0)$, $(0, 1)$, possibly also $(1, 1)$, $(2, 0)$, $(0, 2)$, $(2, 1)$, and $(1, 2)$. This can be accomplished with a transmission filter that extends from $2\omega_y$ to about $\omega_x - \omega_y$, and another extending from $\omega_x + \omega_y$ up to some convenient cutoff selected to reject shot noise contributions. Note that, in rejecting all the power corresponding to $(0, 0)$, etc., a cylinder along the t -axis in Figure 5 has been excised from the Fourier transform space. This represents a small loss of signal power, and is quite tolerable. The idea of a TV raster scan, of course is that the picture-field does not change much during a field scan time, so the temporal part of the spectrum is all

concentrated to frequencies well below ω_y . Similarly, the spatial frequencies present in the y-direction are assumed low enough that their transforms do not spill over above ω_x (there is no valid signal power for $k_y > \omega_x / \omega_y$).

The power-spectrum, or squared amplitude of the output, can be constructed at the output of the TV raster scan by using an analogue squarer--a diode, for example. It is also fairly simple to take point-by-point sums and differences, such as implied in the terms $(D_1 \pm D_2)$, if two TV scan tubes are slaved to each other by using a single sweep generator for both. Gain and positioning controls of adequate precision may require some care to design and to build. The sums and differences should be taken before squaring in those terms, of course.

The image tube placed between the TV raster scan tube and the optical beam serves two functions: (1), it provides temporary storage of the information that a photoelectron was registered until the TV raster scan gets around to scan that spot, and (2) it should permit operation at single-photoelectron level without the adverse noise properties of the TV scan tubes being a limitation. The actual kind of TV scan tube used is not critical--an image orthicon or image dissector would be quite usable, as would a vidicon. The SEC vidicon might have some advantages in this service, but they are not as clear-cut as usual--memory times of about 0.1 second should be adequate, and these are conveniently supplied by

phosphors in image tubes. The spatial resolution afforded by such a system is substantially greater than needed--operation at 100 line pairs over the field would be adequate for apertures up to about 200 inches. But the temporal resolution is compromised somewhat by the storage time in a phosphor. It is not likely that seeing will introduce important changes whose frequency extends well above 10 cycles per second.

The main trouble to be expected from the image-tube to TV raster scan detector comes from the extra noise injected by the system. This arises from several sources: notably from the grainy structure of the phosphor response, from the decay of the phosphor between registering a photoelectron and the time when the scan gets around to read that spot, and the noise inherent in the image tubes and in the TV pickup. It is difficult to predict the importance of these terms, but the overall impression is that, while they will compromise system performance, they are not bad enough to keep it from working.

Costs for an image-tube TV raster scan detector are more difficult to estimate than for the photomultiplier array. A very crude guess is about \$100,000 for a complete pair of tubes with associated circuitry. They have the advantage that the outputs can be taken in a simple form, but may not have the flexibility for equalization of photocathode response, etc., afforded by the photomultiplier array.

VI. CONCLUSIONS

The fringe detector proposed in this report looks feasible at this stage of design study, and seems to merit further work that might take the form of construction of an actual detector for bench tests and for some elementary tests at the telescope. The signal-to-noise ratio estimates indicate that arbitrarily large apertures can be used just as can arbitrarily long integration times, and that there is no natural size limit to apertures set by the seeing (2-3 inches) any more than there is an effective limit to integration times determined by the time it takes the seeing to change the phase difference between the two beams by an appreciable part of a radian (probably about 0.1 second): The most serious conceptual problem remaining is the determination of the power spectrum due to seeing, $|\Pi(\omega)|^2$, at the time of observation. This is not trivial to obtain because it represents a combination of two apertures that does not seem to admit of honest determination within each of the two apertures independently--although such an approximation might be adequate. This point would be readily settled with an actual interferometer using the detection scheme.

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FIGURE CAPTIONS

- Figure 1: Incoming wavefront (curved line) as sampled at two apertures (between vertical lines) in a Michelson Stellar Interferometer. The two sampled light beams are brought together by some optical apparatus at a beam-splitter, the output on the two sides of the beamsplitter is viewed by two detectors, D_1 and D_2 .
- Figure 2. Sketch of a possible fringe pattern as seen when looking toward the beamsplitter of Figure 1. Partially darkened regions shown crosshatched. (upper figure). (Lower figure) The same field a short time later--perhaps 1/4 second.
- Figure 3. Regular barred pattern formed if the two beams of Figure 1 are not virtually parallel (upper figure). (Lower figure) The same field distorted by the phase pattern that would produce the lower part of Figure 2. Phase changes of several full cycles are assumed to be produced by the seeing. Note that the fringes curve and spread about horizontally by more than one fringe spacing.
- Figure 4. Time response of detector turned on and off with no fringes present (solid line), and with fringes present (dashed line). (upper figure). (Lower figure) Power spectrum of time Fourier transform of upper part of Figure 4. Solid line: boxcar or window function in absence of fringes. Dashed line: possible power spectrum in presence of fringes.

Figure 5. Three dimensional detector output fills region in (x, y, t) space with points in the shot-noise limit. (upper figure). (Lower figure): Portions of power spectrum space of Fourier transform of detector output. Region near the origin is filled if no fringes are present. Region outside outer shell is filled with shot noise. Most of power of fringe pattern falls in between (spherical shell in Figure). Frequencies and powers are real-positive because power spectra are being used.

Fig.1

Fig.2

Fig.3

Fig.4

Fig.5