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NEUTRON - MODERATION MOISTURE METERS
ANALYSIS OF APPLICATION TO COAL AND SOIL

Final Report

By
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September 1963

Laboratories for Applied Sciences
University of Chicago
Chicago, Illinois

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**NEUTRON - MODERATION MOISTURE METERS:
ANALYSIS OF APPLICATION TO COAL AND SOIL**

R.A.Semmler

FINAL REPORT

Prepared for

**The Division of Isotopes Development
United States Atomic Energy Commission**

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Report Written by

R. A. Semmler

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**The University of Chicago
LABORATORIES FOR APPLIED SCIENCES**

Chicago, Illinois 60637

FOREWORD

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ABSTRACT

The influence of elements other than hydrogen on neutron-moderation moisture gauges using a Ra-Be neutron source is examined. A point source in a spherical cavity is used as a model of the gauge. Formulas are given to compute the change in gauge response caused by a change in composition using both two-group theory and age-diffusion theory. Moisture gauge response in coal is found to be significantly affected by the carbon density because of significant changes in the fast neutron transport cross section. The constants required for the calculations are tabulated for Al, B, C, Ca, Cl, Fe, H, K, N, Na, O, Si, and H_2O so that gauge response or parameters such as the Fermi age can be calculated for other media and other applications. The terminology and theory of neutron moderation is briefly reviewed. Thermal neutron flux equations are given for several geometries including a point source in a spherical cavity which is gray to both fast and slow neutrons.

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1. INTRODUCTION

The moderation (slowing down) of fast neutrons is the basic phenomena utilized in the moisture measuring instruments discussed here. The slowing down is accomplished by collisions between the neutrons and atoms in the medium. A fast neutron loses part of its energy and changes direction each time it collides with an atom of the medium until the average neutron energy is the same as the average atom energy (thermal energy). The neutron will then continue to be scattered with no change in energy (diffuse) until it is absorbed. This process is sensitive to the presence water because of the hydrogen in water. A neutron colliding with a light atom--hydrogen in particular--loses much more of its energy than in a collision with a heavy atom. The water thus greatly reduces the number of collisions required to reach thermal energy and as a result the neutron reaches thermal energy relatively close to the source of fast neutrons. The density of thermal neutrons produced around the source is used as a measure of moisture content. It should be noted that any phenomena which influences the density of thermal neutrons in the vicinity of the source will influence the response of the moisture probe.

These gauges do not measure water, as such, but respond to the hydrogen in the water and any other hydrogen in the medium. Coal for example is difficult to work with because of the potentially high concentrations of bound hydrogen which the gauge cannot distinguish from hydrogen in water. The water content must be inferred from data on the total hydrogen density and the bulk density. Figure 1 shows this problem graphically. The variation of hydrogen density with bulk density is shown for pure water and a medium with 5 w/o (weight percent) hydrogen. Basically the problem is to determine some point in between

Fig. 1. Hydrogen density as a function of bulk density for media with 5% hydrogen and various amounts of moisture.

the line drawn for the dry medium and the line for pure water. (For a non-hydrogenous medium such as sand, the line for the dry medium would be horizontal.) As the bound hydrogen content of the medium increases, the line for the dry medium approaches the line for pure water. The two lines are superimposed for a medium with a hydrogen content the same as in water (11 w/o). In such a situation, moisture determination by this method is inherently impossible since the medium behaves the same as water. Intermediate cases become progressively more difficult as this indeterminate situation is approached, particularly since the lines of constant moisture content have a finite width such as illustrated on the figure.

In general, the thermal neutron density is determined by three factors: (1) the distance traveled by the neutron during the slowing down process, (2) the distance traveled during the diffusion process, and (3) the geometrical arrangement of source, detector and medium. These factors are discussed briefly in the following paragraphs.

The slowing down length is of course strongly affected by the hydrogen concentration because of its moderating ability. Non-hydrogen elements can also influence this length merely because they scatter the neutron even if no moderation takes place. There are several theories available for calculating the slowing down length but in all of these, terms such as $(\xi \Sigma_s) (\Sigma_{tr})$ appear. ξ = average logarithmic energy decrement, Σ_s and Σ_{tr} are macroscopic scattering and transport cross sections. (See Section 6 for definitions of these terms.) The $\xi \Sigma_s$ term measures the moderating properties of the medium and Σ_{tr} measures the scattering properties. The moisture concentration is the dominant quantity which affects the $\xi \Sigma_s$ term because of the large ξ value for hydrogen. This is not true for the Σ_{tr} term, however.

Carbon in coal and silicon in soil contribute a large part of the total Σ_{tr} value. As a result, the probe response can be influenced by composition and density changes. Particularly in a medium such as coal with a high concentration of carbon, the neutron is scattered many times by the carbon nuclei and this significantly affects the number of slow neutrons around the gauge. In such situations, one cannot look only at the hydrogen content but must also look at the non-hydrogen elements as well in order to understand the variations in gauge response.

The following very relevant quotations from Ref. 1 and 2 point out the significance of non-hydrogen elements - in the presence of hydrogen - on neutron mean free paths:

From Ref. 1:

Furfural ($C_5H_4O_2$, density 1.16 gm/cm^3) in effect contains 0.435 gm/cm^3 of "water" and 0.725 gm/cm^3 of carbon. Dilution of water to this density without addition of other atoms would lead to mean square distances $5.3 (= (1/0.435)^2)$ times as large as those for water. Since the values of r^2 for furfural shown in Table 1 are only about 3 times as large as those for water, it is apparent that through their scattering properties, the carbon atoms reduce the mean square distance considerably.

From Ref. 2:

Of greater interest are the ages of monoenergetic neutrons in hydrogen of water density, shown in Table 9 and Fig. 8. These values are considerably higher than the corresponding ages in H_2O . For example, the age of 2-Mev neutrons in water is 24.6 cm^2 ; while in hydrogen

it is 40.8 cm^2 . This indicates that the oxygen in water is quite effective in reducing the age, which at first might be thought surprising considering how heavy the oxygen nucleus is. But suppose that the oxygen in water were replaced by a fictitious hydrogen similar in all respects to real hydrogen except that its cross section at 2 Mev is 1.6 barns (the same as that for oxygen) rather than 2.9 barns. Since the energy dependence of σ_T for the fictitious hydrogen has the same shape as that for real hydrogen, the change in cross section merely has the effect of reducing the density of the added hydrogen to a value $1.6/2.9$ of what it would be for actual hydrogen. The "hydrogenated" water is thus equivalent to pure hydrogen of a density 1.30 times greater than that of hydrogen in actual water. Hence, the age for a 2-Mev neutron in such a medium is $40.8/(1.30)^2 = 24.0 \text{ cm}^2$ - practically the same as for water. It therefore appears that the oxygen in water is as effective in slowing down as hydrogen of the same total cross section.

In truth, of course, the age is determined not only by the slowing-down kernel of the collisions but also by the properties of the angular distributions. Hydrogen is the most efficient nucleus for slowing down, but the scattering is all in the forward hemisphere. Oxygen is very poor in slowing down - witness the huge ages in pure oxygen. But it scatters nearly isotropically and its "function" in water is to assist

in keeping the neutrons in the vicinity of the source until enough collisions with hydrogen have taken the neutrons down to low energies. In synergistic action with hydrogen, the oxygen is thus as effective as hydrogen.

That it is the scattering properties of oxygen which are important and not the slowing-down behavior may be seen from some results on carbon and C_4H_7 presented below. Carbon is, of course, a significantly better slowing-down material than oxygen. For example, it may be inferred from Tables 8 and 13 that for 2-Mev neutrons the age in carbon is only 60% of the age in oxygen having the same atomic density. At this energy the cross section for carbon has about the same value as for oxygen, and increases even more rapidly with decreasing energy. Yet from Table 15 it is seen that the age for 2.0-Mev neutrons in C_4H_7 , 24.0 cm^2 , is identical with the age in H_2O when the atomic densities are adjusted. Thus, when heavy materials of comparable cross section and atomic density are mixed with hydrogen they will have equal effects on the age.

..... Calculations were also made for the slowing-down age at 1.44 ev for monoenergetic neutrons in oil of approximate composition C_4H_7 It will be noted from Table 15 that the ages in oil are nearly the same as in light water for source energies less than 3 Mev. This is not surprising since the densities for hydrogen and carbon in oil are practically the same as for hydrogen and oxygen, respectively, in water and the

carbon and oxygen cross sections are not greatly dissimilar between the narrow resonances. As discussed above, it appears that in mixtures of hydrogen with a heavy nuclide the heavy partner produces negligible slowing down by itself, but through its isotropic scattering events serves to keep the neutron in the vicinity of the source. This is not to say that the effect of the heavy nucleus on the age is small; indeed, the heavy component produces as much change in the age as would additional hydrogen having the same macroscopic cross section at the source energy. But this effect is produced through scattering and not from slowing down.

.... The slowing-down fission age for diphenyl was computed both with and without anisotropy in carbon scattering, the results being $46.39 \pm 0.5 \text{ cm}^2$ and $45.53 \pm 0.5 \text{ cm}^2$, respectively. Since the carbon atomic density in diphenyl is larger than the hydrogen density, it might be expected that the previous comments on the role of a heavy material in a hydrogen mixture do not apply here. It is therefore noteworthy that the carbon nucleus still remains almost as effective as hydrogen, barn for barn, in reducing the age. With a 2-Mev source, the age in diphenyl is equal to that obtained by replacing the carbon with hydrogen having a macroscopic cross section equal to about 90% of the carbon cross section at 2 Mev. Of course, as the hydrogen content is further reduced, the effectiveness

of the carbon decreases. But it appears that the hydrogen atomic density would have to be reduced below 5 to 10% before hydrogen would cease to be the prime slowing-down agent.

The diffusion length depends on the scattering and absorption characteristics of the medium at thermal energies. Hydrogen normally is the primary factor in determining these characteristics. However, the contribution of the non-hydrogen elements is usually not so small as to be completely negligible at typical moisture contents either in coal or soil.

The effects of geometry can only be partly evaluated on the basis of simplified ideal models. The evidence indicates that the geometry is significant in determining the shape of the calibration curve and the sensitivity to neutron absorbers.

Reports on this type of moisture probe have been largely empirical correlations of moisture content versus probe response. There is no doubt that the probe response will change with water content, but the influence of the factors mentioned above has not been analyzed. Such factors can alter the calibration curve and lead to erroneous moisture predictions, if ignored. The existence of conflicting calibration curves for moisture in coal was initially the impetus for this investigation.

It was felt that the contribution of non-hydrogen elements must be evaluated before this technique can be applied with accuracy to different materials or different densities without repeating extensive calibration data. Information on the allowable variations in the density and composition of the water bearing medium should be available to guide future applications of this type of moisture measuring device.

Of the several ways to improve the accuracy of these gauges, we have concentrated on understanding the influence of the non-hydrogen elements on the response. Although improvements in the determination of bound hydrogen content and moisture content are essential in order to accurately calibrate the gauge in difficult situations with large amounts of bound hydrogen, the additional information supplied by even a rough chemical analysis can be useful and is essential to an understanding of gauge operation.

With a knowledge of how various elements can influence the response, one can assess the possible influence of changes in chemical composition and density of the dry medium. Also, if a calibration curve is evaluated for one medium, a correction factor to give a calibration curve for another different medium can be obtained from the known differences in composition rather than by a redetermination of the calibration curve. Possibly the correction will be negligible, but when there is much spread in the calibration data, this approach is easier than the preparation of a large number of calibration standards which would be required to firmly establish the same information on an empirical basis. Data taken at random values of moisture and dry density can be adjusted to give the response at constant dry density or if field measurements are taken under conditions which do not match the calibration conditions, the significance of the differences can be evaluated without recourse to extensive recalibration.

2. COMPOSITION

2.1 Composition of coal

The primary elements in coal are carbon, hydrogen, and oxygen. Varying amounts of other elements are always present and are discussed below. The exact chemical composition of coal naturally depends on the source and will even vary for samples taken from the same

source. Kirner³ refers to data on the deviations of carbon, hydrogen, and nitrogen content of coals even taken from the same seam and shaft. The deviations are listed in Table 1.

Table 1

Deviations in carbon, hydrogen, and nitrogen content of coals taken from same seam and shaft⁴ expressed as percentages of the total amount of each element present.

	<u>Carbon</u>	<u>Hydrogen</u>	<u>Nitrogen</u>
Minimum	0.5%	7.0%	21.0%
Maximum	6.0	24.0	60.0
Average	3.7	13.0	40.5

An approximate representation of the major elements in coal is given in Fig. 2 which is taken from Ref. 5. The shaded region represents the approximate limits found from many ultimate analyses (dry basis) for carbon, hydrogen, and oxygen on a triaxial diagram. Carbon + hydrogen + oxygen equals 100 percent. As seen in Fig. 2, anthracite coal has the smallest bound hydrogen content. The volatile matter in anthracite is usually less than 8% whereas bituminous coal may contain 25 - 30%.

The nitrogen content of coal is usually of the order of 1 to 2%. Kirner³ cites references giving the nitrogen content in anthracite as 0.2 to 1.5% and in bituminous as 0.6 to 2.8% on an ash-and moisture-free basis. The content can vary considerably, however, as indicated earlier.

At least 95% of the mineral content of coal consists of silica, alumina, iron oxide, lime, and magnesia. The amounts usually vary within the limits given in Table 2.

Fig. 2. Triaxial diagram of coal composition. (From Ref. 5).

Table 2
Composition of coal ash

	<u>% of total ash</u>	
	max.	min.
SiO_2	70	15
Al_2O_3	40	0
CaO	55	0
Fe_2O_3	60	0

Other elements are also present in trace quantities; local variations again can be large. Thiessen⁶ gives a tabulation of the elements found in coal ashes which exhibit "enrichment" of many trace elements compared to the concentration found in the earth's crust. For example, "rich" ashes might contain an average of 0.06% boron (and a maximum of 0.3%) compared to the earth crust average value of 0.0003%.

The moisture content of coal cannot be easily determined with absolute accuracy. This is a major problem in the preparation of accurate calibration curves. The conventional technique is to dry the sample in an oven at 105°C and determine the moisture content by weight loss. However, some coals may not lose all of their water unless heated to 275°C and of course volatile substances other than water may be released. The permissible difference in moisture given for ASTM methods⁷ (0.5%) measures the reproducibility of analysis in the laboratory and is not an absolute error. They are purely analysis errors and do not include sampling errors. The permissible difference specified for hydrogen in an ultimate analysis is 0.07%.

The coals used by the American Electric Power Corporation were of primary interest in this investigation. Samples of coal from the

Glen Lyn, Kanawha, Kyger, and Clifty Creek Stations were spectrographically analyzed for trace-element concentration - particularly boron and cadmium.

The samples were taken from coal "on hand" and do not necessarily give an accurate sampling of the actual coal. Also, the analysis is only semi-quantitative and the data indicate only the order of magnitude involved. No large difference between the coals is apparent, however.

Table 3
Trace elements in several coal samples

Element	Kyger Creek (Consolidated Coal)	Clifty Creek	Glen Lyn (Pocahontas Coal)	Kanawha River (Morris Creek Coal)
Silicon	2%	1.5%	1.0%	2%
Aluminum	1	0.6	0.5	1
Iron	0.3	1	1	0.4
Calcium	0.04	0.2	0.15	0.04
Magnesium	0.1	0.09	0.07	0.1
Titanium	0.08	0.05	0.04	0.1
Barium	0.05	0.05	0.03	0.05
Strontium	0.02	0.03	0.005	0.01
Boron	0.003	0.01	0.02	Not detected < 0.002%
Manganese	0.004	0.008	0.007	0.003
Copper	0.0015	0.001	0.0006	0.002
Lithium	0.1	0.07	0.07	0.2
Potassium	Not detected < 0.2%	Not detected < 0.2%	Not detected < 0.2%	Not detected < 0.2%
Cadmium	Not detected < 0.01%	Not detected < 0.01%	Not detected < 0.01%	Not detected < 0.01%

The absorption cross section contributed by the hydrogen, iron, and nitrogen for a typical coal sample (Clifty coal with $\rho_{\text{dry}} = 64 \text{ lb/ft}^3$, and $\rho_w = 11 \text{ lb/ft}^3$) is 0.014 cm^{-1} . Boron at a concentration of 0.0064 lb/ft^3 (0.01%) would have an absorption cross section of 0.0040 cm^{-1} . Cadmium at a concentration of 0.0064 lb/ft^3 (0.01%) would have an absorption cross section of 0.0012 cm^{-1} . The cadmium cross section is well below the contribution of the hydrogen, iron, and nitrogen by a factor of about 10. The boron cross section at this concentration is about 1/3 of the hydrogen, iron, and nitrogen contribution. Although boron will have a negligible effect on the moderating properties, the samples analyzed here indicate that boron could significantly affect the thermal absorption cross section of the medium and as a result influence the gauge response.

2.2 Composition of soil

The composition of soil can likewise vary considerably but the principal constituents are the oxides of silicon, aluminum, and iron in approximately the same amounts as in the average composition of the earth's crust.

2.3 Relative Significance of the Elements

To establish the relative importance of the various elements in coal and soil in terms of neutron diffusion, the thermal neutron cross sections were calculated for a typical composition.

The coal analysis which was used and the resulting absorption and scattering cross sections are tabulated in Table 4. The most significant elements and their percentage contribution to the total cross section are also indicated in Table 4.

Table 4

Thermal neutron absorption and scattering cross sections in a dry Peabody coal sample with 5.06 w/o hydrogen and 12.6 w/o ash.

Component	w/o	A	σ_a (barn)	Contribution to Σ_a ($\rho = 1 \text{ gm/cm}^3$)	Percent- age of Σ_a	σ_s (barn)	Contri- bution to Σ_s ($\rho = 1 \text{ gm/cm}^3$)	Percent- age of Σ_s
C	66.4	12.0	0.003	0.0001 1	80%	4.8	0.16	12%
H	5.1	1.0	0.33	0.0101		38.	1.17	85%
O	10.4	16.0	0.0002	0.0000		4.2	0.02	1%
N	1.4	14.0	1.88	0.0011	9%	10.0	0.01	
S	4.2	32.1	0.49	0.0004	3%	1.1	0.00	
SiO ₂	5.1	60.1	0.13	0.0001		10.1	0.01	
TiO ₂	0.1	127.8	5.8	0.0000		12.4	0.00	
MnO ₂	0.01	86.9	13.2	0.0000		10.7	0.00	
Al ₂ O ₃	2.46	102.0	0.46	0.0001		15.4	0.00	
Fe ₂ O ₃	3.6	159.8	5.06	0.0007	6%	34.6	0.00	
CaO	0.8	56.1	0.43	0.0000		7.2	0.00	
MgO	0.1	40.3	0.063	0.0000		7.8	0.00	
Na	0.03	23.9	0.515	0.0000		4.0	0.00	
SO ₃	0.2	80.1	0.49	0.0000		13.7	0.00	
P ₂ O ₅	0.04	142.0	0.40	0.0000		31.0	0.00	
	99.94			.0126	98%		1.37	98%

Hydrogen is the most significant element in the determination of both the absorption and scattering thermal neutron cross sections. One could hardly say that all other elements are negligible, however, since the non-hydrogen elements contribute about 15% of the total thermal cross section.

The soil composition assumed here is the same as the average composition of the earth's crust (neglecting a few minor elements) such as in Refs. 8 and 9. Boron with a concentration of 30 ppm is included because of its large absorption cross section. The composition and cross sections are given in Table 5. The most significant elements and their percentage contribution to the total cross sections are also indicated.

Table 5
Thermal neutron absorption and scattering
cross section in soil

Component	w/o	A	$\sigma_a / 1.128$ (barn)	Contribution to Σ_a ($\rho = 1.5$)	Percent- age of Σ_a	σ_s (barn)	Contri- bution to Σ_s ($\rho = 1.5$)	Percent- age of Σ_s
Si	28.1	28.1	0.142	0.00129	18%	1.7	.015	10%
O	47.3	16.0	0.000	0.00000		4.2	.112	75%
Al	8.2	27.0	0.204	0.00056		1.4	.004	
Fe	5.1	55.8	2.24	0.00184	25%	11.	.009	
Ca	3.7	40.1	0.390	0.00033		3	.002	
Mg	2.1	24.3	0.056	0.00004		3.6	.003	
K	2.6	39.1	1.83	0.00110	15%	1.5	.001	
Na	2.8	23.0	0.448	0.00050		4.0	.004	
B	30 ppm	10.8	670.	0.00168	23%	4.	.000	
	99.9 w/o			0.00734	81%		.150	85%

The dominant absorbing element is iron although boron contributes almost as much. The trace elements have also been compared with the major elements using the average range of soil constituents given by Prince in Ref. 9. The range of Σ_a calculated for the major constituents is graphed in Fig. 3. The density was assumed to be 1 gm/cm^3 . Boron is potentially a disturbing element although it should be noted that the boron range indicated on the graph is equivalent to the range 15 - 100 ppm whereas the average boron content in the United States soils is 30 ppm. The Fe_2O_3 line corresponds to 0.5 - 17% whereas U. S. soils usually have 2 - 6%. Cadmium is not included in the tabulation by Prince but the average value for the earth's crust is 0.2 ppm giving $\Sigma_a = 2.62 \times 10^{-6} \text{ cm}^{-1}$ which is negligible.

The relative importance of the elements during the slowing down process depends both on the scattering cross section and the average energy loss per collision. A quantity called the slowing down power (defined as $\xi \Sigma_s$) is often used to measure moderating ability and is intimately associated with the calculation of the neutron age in various media. The scattering properties are best measured by the transport cross section (Σ_{tr}) which includes the effects of non isotropic scattering. Values for $\xi \Sigma_s$ and Σ_{tr} can be calculated or taken from tabulations.

These cross sections are energy dependent and if a single value is desired to represent the entire slowing down process, an average must be taken. The average cross sections used here are calculated in Appendix A and are tabulated in Section 8.9. Using these flux weighted averages the contribution of the various elements to $\overline{\xi \Sigma_t}$ and $\overline{\Sigma_{tr}}$ for soil have been calculated using the same analysis as before except 5% moisture was assumed. The results are given in Table 6 together with the percentage contribution of each.

Fig. 3. Average range of Σ_a in soil for various components.

Hydrogen dominates the slowing down cross section but not the transport cross section at this moisture concentration. At other moisture values and with other media the exact values differ but much the same results are obtained - hydrogen primarily determines the slowing down cross section but does not dominate the transport cross section and may not even be the major factor in the evaluation of the transport cross section. Because the terms $\xi \Sigma$ and Σ_{tr} appear as a product in calculations of the slowing down length, the assumption that slowing down is accomplished only by hydrogen does not remove a dependence of slowing down length upon non-hydrogenous materials.

Table 6
Fast neutron slowing down and transport
cross sections in soil

Component	w/o	$\xi \sigma$	Contribution to $\xi \Sigma$ ($\rho = 1.5$)	Percentage of $\xi \Sigma$	σ_{tr}	Contribution to Σ_{tr} ($\rho = 1.5$)	Percent- age of Σ_{tr}
Si	26.7 w/o	0.259	0.0022		4.34	0.0373	22%
O	44.9	0.140	0.0036	10%	3.19	0.0809	49%
Al	7.8	1.09	0.0028		3.03	0.0079	
Fe	4.8	2.43	0.0019		3.99	0.0031	
Ca	3.5	.271	0.0002		4.80	0.0038	
Mg	2.0	.268	0.0002		3.19	0.0024	
K	2.5	.237	0.0001		4.36	0.0025	
Na	2.7	.343	0.0004		4.64	0.0049	
H ₂ O	5.0	10.3	0.0258	69%	9.33	0.0234	14%
	99.9 w/o		0.0372	79%		0.1662	85%

3. LIMITATIONS IMPOSED BY BOUND HYDROGEN

The general problem of moisture determination is particularly complicated if bound hydrogen is present in appreciable quantities. Organic matter in soil and bound hydrogen in coal cause an instrument response which must be separated from the contribution due to moisture.

For example, a typical bound hydrogen content of bituminous coal is 5% on a dry basis. Assuming a typical moisture value of 10% and a wet density of 75 lb/ft³, the approximate hydrogen densities are

$$\rho_{\text{bound H}} = 3.4 \text{ lb/ft}^3$$

$$\rho_{\text{moisture H}} = 0.8 \text{ lb/ft}^3$$

The moisture hydrogen contributes only 20% to the total hydrogen content giving a very low "signal-to-noise" ratio. If air dried coal was being examined, the moisture content would be only 1 - 2%. To obtain quantitative moisture information in such a situation requires very accurate data even if there are no other complicating factors.

In the actual use of a moisture gauge, one would like to convert gauge response immediately to a moisture value either by an appropriately calibrated meter or by a calibration curve. For media with bound hydrogen there will not be a single curve relating the response, R, and the moisture content. This can be seen most simply in terms of an approximate equation for R. To a first approximation, the response is a linear function of the hydrogen density, i. e.,

$$R = a \rho_H. \quad (1)$$

Using f_m and f_H to represent the moisture fraction and bound hydrogen fraction,

$$\rho_H = \frac{\rho_{\text{wet}} f_m}{9} + \rho_{\text{wet}} (1 - f_m) f_H, \quad (2)$$

then

$$\left(\begin{array}{c} \text{moisture} \\ \text{fraction} \end{array} \right) = \frac{\rho_{H_2O}}{\rho_{wet}} = f_m = \left\{ \frac{1}{a \rho_{wet} \left(\frac{1}{9} - f_H \right)} \right\} R - \left\{ \frac{f_H}{\frac{1}{9} - f_H} \right\} \quad (3)$$

The relationship is linear between f_m and R but the slope depends on the bound hydrogen content f_H and the wet density.

If the dry rather than the wet density is used as a variable, a similar expression results:

$$\left(\begin{array}{c} \text{moisture} \\ \text{fraction} \end{array} \right) = f_m = \frac{\rho_{H_2O}}{\rho_{wet}} = \frac{R - a \rho_{dry} f_H}{R + a \rho_{dry} \left(\frac{1}{9} - f_H \right)} \quad (4)$$

If the fraction of hydrogen in the medium happens to be the same as in water, i. e., $1/9$, then the moisture content is indeterminant if only information about the gross (wet) density is available since both the medium and water are then equivalent. R and f_m will be uniquely related in such a situation only if ρ_{dry} and f_H are constant as seen in Eq. 4.

Even assuming the coefficient (a) were determined with negligible error, the determination of moisture content is sensitive to the accuracy of the bound hydrogen determination of the sample medium. Using Eq. 3, the change in predicted moisture, Δf_m , due to a change only in bound hydrogen; Δf_H , is given by:

$$\Delta f_m = \left(- \frac{1 - f_m}{\frac{1}{9} - f_H} \right) \Delta f_H \quad (5)$$

If e_m is the maximum desired moisture error than Δf_H must satisfy $|\Delta f_m| \leq e_m$, or

$$\frac{1 - f_m}{\frac{1}{9} - f_H} \cdot |\Delta f_H| \leq e_m \quad (6)$$

Solving for f_m ,

$$f_m \geq \frac{-e_m (\frac{1}{9} - f_H)}{|\Delta f_H|} + 1 \quad (7)$$

The need for an accurate hydrogen analysis can be seen from the following example. Using the ASTM limits of acceptable reproducibility for the same laboratory as the uncertainty in f_H and 1% as the maximum desired moisture error (e. g., percent moisture = 8.3 ± 1.0), then

$$f_m \geq 1 - \frac{1\%}{0.07\%} (\frac{1}{9} - f_H) = 14.29 f_H - 0.59 \quad (8)$$

For typical bituminous coal with 5% hydrogen, the moisture content must therefore be greater than 12.5% to achieve the desired accuracy even assuming R , ρ_{wet} , and "a" are known exactly.

The same type of analysis for the sensitivity to other specific errors can be made using Eq. 3. In order to compare the terms, relative changes can be used and Δf_m is then given by

$$\Delta f_m \text{ due to } \Delta f_H = \left\{ - \frac{(1 - f_m) f_H}{\frac{1}{9} - f_H} \right\} \left(\frac{\Delta f_H}{f_H} \right) \quad (9)$$

$$\Delta f_m \text{ due to } \Delta R = \frac{R}{a \rho_{wet} (\frac{1}{9} - f_H)} \left(\frac{\Delta R}{R} \right) = \left\{ \frac{\rho_H}{\rho_{wet} (\frac{1}{9} - f_H)} \right\} \left(\frac{\Delta R}{R} \right) \quad (10)$$

$$\Delta f_m \text{ due to } \Delta a = - \frac{R}{a \rho_{\text{wet}} \left(\frac{1}{9} - f_H \right)} \left(\frac{\Delta a}{a} \right) = \left\{ - \frac{\rho_H}{\rho_{\text{wet}} \left(\frac{1}{9} - f_H \right)} \right\} \left(\frac{\Delta a}{a} \right), \quad (11)$$

$$\Delta f_m \text{ due to } \Delta \rho_{\text{wet}} = - \frac{R}{a \rho_{\text{wet}} \left(\frac{1}{9} - f_H \right)} \left(\frac{\Delta \rho_{\text{wet}}}{\rho_{\text{wet}}} \right) = \left\{ - \frac{\rho_H}{\rho_{\text{wet}} \left(\frac{1}{9} - f_H \right)} \right\} \left(\frac{\Delta \rho_{\text{wet}}}{\rho_{\text{wet}}} \right). \quad (12)$$

Using typical values of $\rho_{\text{wet}} = 75 \text{ lb/ft}^3$, $f_H = 5\%$, $f_m = 10\%$, $\rho_H = 4.2 \text{ lb/ft}^3$, the bracketed quantities are $(-.74)$, $(+.92)$, $(-.92)$, and $(-.92)$. If f_H , R , a , and ρ_{wet} are known with accuracy p (e. g. $\Delta f_H/f_H = p$) then the total uncertainty in f_m is given by

$$\Delta f_m = \sqrt{(-.74p)^2 + (+.92p)^2 + (-.92p)^2 + (-.92p)^2} = 1.76p. \quad (13)$$

In order to achieve $\Delta f_m = 1\%$, (using the typical values previously) an accuracy of $p = .006 = 0.6\%$ is required, a very low level of error.

$\frac{\Delta R}{R}$ can be easily reduced by extending the counting time or increasing the source strength. In general, less than 1% (std. dev.) error can be obtained with $> 10,000$ total counts which is easily obtained. The other sources of error are not so readily controlled and require accurate information about the calibration curve.

4. INFLUENCE OF ACCESS TUBING

Besides the fluctuations produced by the medium itself, the access tubing used to introduce the gauge into the medium also can influence the response.

The effect of wall thickness and access tube diameter on the moisture probe for a water medium is given in Figs. 4 and 5. A Nuclear-Chicago P-19 gauge was used to obtain this data. The moisture probe readings decrease as the tube radius increases. In a medium with a smaller hydrogen content, the readings would also decrease but the effect would be smaller due to the greater spread in the thermal neutron density distribution, i. e., due to the larger sampling volume. The decrease in count rate as the wall thickness increases is due to greater absorption and scattering of thermal neutrons in the wall. The fast-neutron response (as measured with a cadmium shield over the probe) amounted to only 2.5 to 3.0% of the total counts and did not change significantly as the access tube wall thickness was increased.

The effect of collars used to couple lengths of access tubing has also been examined to determine the approximate range of influence. Figure 6 indicates how the count rate decreases as the probes pass through a 6-inch collar in a water medium. Both tube and collar have a 16-gauge wall. The access tube was 1-3/4 inch O. D. seamless steel tubing (MT-1015). The collar affects the moisture probe readings over a total length of about 13 inches. This would be increased somewhat in a medium with a smaller hydrogen content.

Fig. 4. Effect of access tube wall thickness on moisture gauge response.

Fig. 5. Effect of access tube diameter on moisture gauge response.

Fig. 6. Effect of access tube collar on moisture gauge response.

5. EMPIRICAL SIGNIFICANCE OF HYDROGEN AND NON-HYDROGEN COMPONENTS

Several empirical equations were first examined to determine whether the original data collected by the American Electric Power Corporation indicated any obvious perturbations caused by the non-hydrogen components. When the count ratio, R , is expressed as $R = a + b \rho_H + c \rho_{\text{non-H}}$ the values of the coefficients using the original calibration data are given in Table 7.

Table 7

Coefficients in $R = a + b \rho_H + c \rho_{\text{non-H}}$ using AEP calibration data

	<u>a</u>	<u>b</u>	<u>c</u>		<u>No. of Points</u>
Clifty	+ 0.00098	0.18	- 0.0021	+ 0.004	16
Kyger	- 0.15	0.11	+ 0.0043	+ 0.019	4
Glen Lyn	- 0.044	0.21	- 0.0053	+ 0.005	4
Kanawha	+ 0.073	0.081	+ 0.0037	+ 0.006	4

In Table 7, none of the data indicate a coefficient of the non-hydrogen components significantly different than zero. The coefficients are not well determined, however, because of the small number of points. The theoretical calculation of the influence of the non-hydrogen components is given in Section 7.

6. NEUTRON MODERATION THEORY

6.1 Introduction

A general discussion of the terminology and conventional notation used in a description of the neutron moderation process and the calculation of the various terms is given in the following sections. This review is included partly to explain the notation used here and partly to encourage others to attempt more analysis of moisture gauge data and not merely tabulate the response in media x , y , and z .

6.2 Calculation of Flux Equation Parameters

Essentially one set of variables appears in the neutron flux equations regardless of what particular theory is used. Values for some of these variables are tabulated; others must be calculated. The evaluation of these parameters for pure elements and mixtures is discussed below:

σ , Microscopic Cross Section. One way to indicate the probability for a particular neutron reaction is to specify the "target area" presented by an individual atom to the incoming neutron. This area is the microscopic cross section and is measured in cm^2 or in barns* ($1 \text{ barn} = 10^{-24} \text{ cm}^2$) and is designated by σ . Subscripts are used to indicate the particular reaction involved such as σ_a , σ_s , and σ_t for the absorption, scattering, and total cross section, respectively. Values for σ are tabulated or graphed in various references such as Refs. 11, 12, 13, 14, 15, 16, 17, and 18.

Σ , Macroscopic Cross Section. This cross section is very similar to σ except that instead of measuring the area of an individual atom, the total target area within 1 cubic centimeter is given. The dimensions are thus area per unit volume or $\text{cm}^2/\text{cm}^3 = \text{cm}^{-1}$. The same subscripts are used for Σ as for σ . The total target area in 1 cm^3 is just the number of atoms per cubic centimeter, N , times σ or

$$\Sigma = N\sigma = \left(\frac{\rho N_o}{A} \right) \sigma, \quad (14)$$

where ρ = density, N_o = Avogadro's number, A = atomic weight. For a

* In Ref. 10, the term "Fermi" is used instead of "barn". This terminology is neither conventional nor recommended.

mixture, the contributions to the area of each part are summed.

$$\begin{aligned}\Sigma &= N_1 \sigma_1 + N_2 \sigma_2 + \dots = \frac{\rho_1 N_o}{A_1} \sigma_1 + \frac{\rho_2 N_o}{A_2} \sigma_2 + \dots \\ &= \rho N_o \left(\frac{f_1 \sigma_1}{A_1} + \frac{f_2 \sigma_2}{A_2} + \dots \right) = \rho N_o \sum_i \frac{f_i \sigma_i}{A_i} ,\end{aligned}\quad (15)$$

where ρ_i is the effective density (actual mass per unit volume) of the i^{th} component and f_i is the weight fraction of the i^{th} component. Tabulated values of Σ for pure elements and a few compounds can be found in Refs. 16 and 18 or they may be calculated directly from σ using Eqs. 14 and 15.

λ , Mean Free Path. λ designates the average distance traveled by a neutron (total zig-zag path) before undergoing a particular reaction. λ is related to Σ by

$$\lambda = \frac{1}{\Sigma} . \quad (16)$$

(This equation and others merely stated here are derived in any standard text on neutron diffusion such as Refs. 10 and 19.) The dimensions are pure length — cm.

The difference between the scattering mean free path, λ_s , and the transport mean free path, λ_{tr} , should be noted. λ_s is the average distance traveled between scattering collisions and is also the average distance traveled before the neutron flight path is deflected 90° if the scattering is isotropic. If the neutrons are preferentially scattered in the forward direction (in a laboratory coordinate system) the neutron will travel farther than λ_s before being deflected 90° . This greater distance

is called λ_{tr} . λ_{tr} and λ_s are related through the average cosine of the scattering angle θ by

$$\lambda_{tr} = \frac{\lambda_s}{1 - \overline{\cos \theta}} \quad (17)$$

Values for $\overline{\cos \theta}$, which is often denoted as $\bar{\mu}$, are given in Ref. 16 for all the elements assuming that the scattering is isotropic in the center of mass system. With this assumption, $\overline{\cos \theta}$ can be calculated from

$$\overline{\cos \theta} = \frac{2}{3A} \quad (18)$$

The value of λ for a mixture is calculated by first finding the total cross section, Σ , for the process and then using $\lambda = 1/\Sigma$.

D, Diffusion Coefficient. The diffusion coefficient is a characteristic of the medium which relates the movement of neutrons to the spatial distribution of the neutron density, n . D is defined by $\underline{J} = -D\nabla\phi$ where $\underline{J}$ = net neutron current, ϕ = neutron flux = nv , and v = neutron velocity. When the scattering is isotropic (laboratory coordinate system), and the absorption is weak ($\Sigma_a \ll \Sigma_T$) D is given by

$$D = \frac{\lambda_s}{3} = \frac{1}{3 \Sigma_s} \quad (19)$$

When the scattering is not isotropic in the laboratory system, D can be calculated from

$$D = \frac{\lambda_{tr}}{3} = \frac{1}{3 \Sigma_{tr}} = \frac{1}{3 \Sigma_s (1 - \overline{\cos \theta})} \quad (20)$$

although this still assumes isotropic scattering in the center of the mass system and that the absorption is weak.

If the absorption cannot be entirely neglected, the theory of neutron diffusion indicates that D is given by

$$D = \frac{\Sigma_s}{3 \Sigma_T \Sigma_{tr}}, \quad (21)$$

where Σ_{tr} is defined as $\Sigma_T - \bar{\mu} \Sigma_s$. This reduces to equation (20) for $\Sigma_a \ll \Sigma_s$.

L, Diffusion Length. L is a measure of the average "crowflight" distance traveled by a thermal neutron before capture. Specifically, the root-mean-square distance traveled is given by

$$\sqrt{r^2} = r_{rms} = \sqrt{6} L. \quad (22)$$

L is calculated from two previously mentioned parameters.

$$L = \sqrt{\frac{D}{\Sigma_a}}. \quad (23)$$

Using Eq. (20)

$$L = \sqrt{\frac{1}{3 \Sigma_a \Sigma_{tr}}}. \quad (24)$$

Using Eq. (21)

$$L = \sqrt{\frac{\Sigma_s}{3 \Sigma_T \Sigma_{tr} \Sigma_a}}. \quad (25)$$

For mixtures, the procedure is the same as for D , i. e., the correct macroscopic cross sections, Σ , are calculated for each component, then these are added. For example, using Eq. 20 the diffusion coefficient for 3 component mixture is given by

$$D = \frac{1}{3 (\Sigma_{tr1} + \Sigma_{tr2} + \Sigma_{tr3})} \quad (26)$$

and not by $1/3 \Sigma_{tr_1} + 1/3 \Sigma_{tr_2} + 1/3 \Sigma_{tr_3}$.

ξ , Logarithmic Energy Decrement. ξ is a measure of the average amount of energy lost by a neutron in a collision during the slowing down process and is defined by

$$\xi = \frac{E_i - E_f}{\ln E_i - \ln E_f} = \ln \frac{E_i}{E_f}, \quad (27)$$

where E_i and E_f are the energy before and after collision. Assuming again that the scattering is isotropic in the center of mass system, Eq. 27 can be evaluated as:

$$\xi = 1 + \frac{(A-1)^2}{2A} \ln \left(\frac{A-1}{A+1} \right) = \frac{2}{A + 2/3} \quad (28)$$

Values of ξ for all elements are given in Ref. 16.

u , Lethargy. The lethargy of a neutron is merely the energy expressed on a compressed scale. The energy range from fast to thermal neutrons is about 5×10^6 ev to 0.025 ev. This large energy range corresponds to a lethargy range of 0.7 to 19.8 (lethargy increases as the neutron slows down). Lethargy is defined using an arbitrary reference level of 10^7 ev by

$$u = \ln \frac{E_0}{E} = \ln \frac{10^7}{(\text{energy in ev units})} \quad (29)$$

and is a pure number.

τ , Age. The "age" of a neutron can be related to the true chronological age of the neutron but the definition is more directly related to the average "crowflight" distance traveled by the neutron during the slowing down process. The root-mean-square distance from the

source to the point where the neutron has an age τ is given by

$$\sqrt{r^2} = r_{\text{rms}} = \sqrt{6\tau} . \quad (30)$$

One can talk about the age of a neutron with any energy but almost all tabulated values are given for thermal neutrons. Also, most of the tabulated information is for a source energy of 2 Mev which corresponds to the average energy of neutrons emitted in the fission process.

Artificial neutron sources such as Ra-Be give neutrons with a higher average energy and a greater age (greater distance traveled) at thermal energy.^{17, 18}

The calculation of age is by far the most difficult parameter to evaluate since not only the distribution in space but also the distribution in energy is changing with time. There are two basic ways to simplify the description of the slowing down process. Both involve replacing the random changes in energy by a systematic behavior which is more amenable to analysis. (Monte Carlo or random walk calculations do not use this smoothing technique and as a result, give excellent results but the computations are so lengthy that electronic computers must in general be used.)

One approach (Fermi age theory) assumes that the neutron loses energy continuously rather than in many discrete steps. The other (group theory) assumes that the neutron loses energy in a step-wise fashion but the steps are few in number and regularly spaced.

The simplest form of age theory assumes no absorption during moderation and results in the following well known expression

$$\tau = \int_E^{E_0} \frac{D}{\xi \Sigma_s} \frac{dE}{E} = \int_E^{E_0} \frac{1}{3 \xi \Sigma_s \Sigma_{tr}} \frac{dE}{E} . \quad (31)$$

There are various modifications which allow for absorption during the slowing down process. If the $(\xi \Sigma_s)$ term in Eq. 31 is replaced by the more general term $(\xi \Sigma_s + a \Sigma_a)$, the various forms can be characterized by the value of (a) . $a = 0$, ξ , γ respectively in the "Fermi," "Wigner," and "Goertzel-Greuling" approximations. Values of γ are given in Ref. 19. The different forms mentioned here plus a so-called "Hurwitz" approximation are compared with an exact solution in Ref. 20. In general, the Fermi approximation is poorest. The Wigner, Goertzel-Greuling and Hurwitz forms are progressively better if there is absorption during the slowing down process. The simplest form which includes any absorption effects is the Wigner form given by

$$\tau = \int_E^{E_0} \frac{D}{\xi \Sigma_T} \frac{dE}{E} \quad (32)$$

Of more importance than capture during the slowing down process, which is usually very small, is the particular problem of slowing down in a hydrogenous medium. If the moderation is accomplished primarily by hydrogen, the slowing down process is relatively rapid and involves few collisions (about 18 for a 2 Mev source). The assumption that the typical energy variation is composed of many discrete steps and can be replaced by a continuous curve begins to break down. For sources of neutrons with high energy, the problem is particularly complicated since the hydrogen cross section begins to decrease at about 1 Mev causing a low reaction probability and a very long path to the first collision. This is sometimes allowed for using a "first-flight" correction^{21, 22} which is added to the conventional age. (An analysis of the transport problem which includes the angular dependence of the scattering can be found in Refs. 10, 19, or 22). Hellens²² derives an expression

for the age in a hydrogenous medium using the Selengut-Goertzel approximation and finds a first flight correction of

$$\text{ffc} = \frac{1}{3\Sigma_{\text{tr}}\Sigma_0} \quad \left| \quad E = E_0 \right. \quad (33)$$

where Σ_{tr} is defined as before, i.e., $\Sigma_{\text{tr}} = \Sigma_T - \bar{\mu}\Sigma_s$, and $\Sigma_0 = \Sigma_a + \Sigma_H$, Σ_H = hydrogen scattering cross section.

If there is no absorption $\Sigma_a = 0$ and Eq. 33 reduces to

$$\text{ffc} = \frac{1}{3\Sigma_{\text{tr}}\Sigma_H} \quad \left| \quad E = E_0 \right. \quad (34)$$

This is not the only form for the first flight correction that one will see, however. Wigner¹⁰ gives the following simple form which contains no corrections for anisotropic scattering:

$$\text{ffc} = \frac{1}{3\Sigma_T^2} \quad (35)$$

6.3 Possible models

Various models of differing complexity can be used to approximate the geometry of a neutron moisture gauge. Some of these are tabulated below. The numbers refer to the following sections where the thermal neutron flux equation is generally given for that particular geometry. A collection of solutions for many other simple geometries using monoenergetic diffusion and continuous slowing down theory can be found in Ref. 23.

Models and Theories Discussed in Section 6

	1 Group Theory	2 Group Theory	Continuous Slowing Down Plus Diffusion
Point Source in Homogeneous Medium	6.3.1	6.3.2	6.3.3
Point Source in Spherical Cavity	6.3.4	6.3.5	6.3.6
Point Source in Black Spherical Cavity	-----	6.3.7	-----
Point Source in Cylindrical Cavity	6.3.8	6.3.8	-----

6.3.1 The solution of the diffusion equation for a point source of thermal neutrons in a homogeneous medium gives

$$\phi(r) = \frac{Q}{4\pi D} \frac{e^{-r/L}}{r}, \quad (36)$$

where ϕ = thermal neutron flux, Q = source strength, r = radial distance, L = diffusion length = $\sqrt{D/\Sigma_a}$, D = diffusion coefficient. All constants are evaluated at thermal energy.

The diffusion equation is also used to describe the neutron distribution at each discrete energy in group theory. True fast neutron absorption is usually neglected but the absorption term, $\Sigma_a \phi$, is not zero since neutrons are removed (slowed down) from the fast group to a lower energy group. In two group theory, the fast group is usually called the first group and the removal cross section is often denoted Σ_1 .

Since Eq. 36 has the characteristic that $\overline{r^2} = 6 D/\Sigma_a$ and for fast neutrons $\overline{r^2} = 6\tau$, the removal cross section can be denoted by

$$\Sigma_1 = \frac{D_1}{\tau} \quad (37)$$

With this notation, the flux of neutrons with age τ using group theory is given by

$$\phi = \frac{Q}{4\pi D_1} \frac{e^{-r/\sqrt{\tau}}}{r} \quad (38)$$

D_1 is the average diffusion coefficient over the energy range E_o to E . Using $D_1 = \Sigma_1 \tau$ this can be written as

$$\text{removal rate} = \phi \Sigma_1 = \frac{Q}{4\pi \tau} \frac{e^{-r/\sqrt{\tau}}}{r} \equiv q \quad (39)$$

q is called the slowing down density and gives the rate of removal from the fast group for the case of no fast absorption. This equation from group theory is comparable to $q = e^{-r^2/4\tau} / (4\pi\tau)^{3/2}$ from age theory. In both, $\overline{r^2} = 6\tau$.

The flux per unit lethargy, $\phi(u)$, is related to the slowing down density by

$$q = \bar{\xi} \Sigma_s \phi(u), \quad (40)$$

or

$$\begin{aligned} \phi(u) &= \frac{q}{\bar{\xi} \Sigma_s} = \frac{Q}{\bar{\xi} \Sigma_s 4\pi \tau} \frac{e^{-r/\sqrt{\tau}}}{r} \\ &= \frac{Q}{4\pi D'} \frac{e^{-r/\sqrt{\tau}}}{r} \end{aligned} \quad (41)$$

where $D' = \xi \Sigma_s \tau$, $\phi(u) = E \phi(E)$ = neutron flux per unit lethargy.

6.3.2 The derivation of the neutron flux around a point source of fast neutrons using two group theory is given in Appendix B. The result is merely stated here:

$$\phi_1 = \frac{Q}{4\pi D_1} \frac{e^{-k_1 r}}{r}, \quad (42)$$

$$\phi_2 = \frac{Q k_1^2}{4\pi D_2 (k_2^2 - k_1^2)} \left[\frac{e^{-k_1 r}}{r} - \frac{e^{-k_2 r}}{r} \right], \quad (43)$$

where ϕ_1 and ϕ_2 are the fast and slow neutron fluxes, $k_1^2 = 1/\tau$, $k_2^2 = 1/L^2$, D_1 and D_2 are the fast and slow diffusion coefficients. At the origin, the thermal flux is given by:

$$\phi_2(0) = \frac{Q k_1^2}{4\pi D_2 (k_2^2 + k_1^2)} = \frac{Q L \sqrt{\tau}}{4\pi D_2 \tau (L + \sqrt{\tau})} = \frac{Q}{4\pi \Sigma_a L \sqrt{\tau} (L + \sqrt{\tau})}. \quad (44)$$

6.3.3 If one uses the continuous slowing down model (age theory) to determine the distribution of neutrons arriving at thermal energy and uses this result as the source term in the diffusion equation, the thermal neutron flux is given²³ by

$$\phi_{th} = \frac{Q e^{-\tau/L^2}}{8\pi D r} \left\{ e^{-r/L} \left[1 - \operatorname{erf} \left(\frac{\sqrt{\tau}}{L} - \frac{r}{2\sqrt{\tau}} \right) \right] - e^{+r/L} \left[1 - \operatorname{erf} \left(\frac{\sqrt{\tau}}{L} + \frac{r}{2\sqrt{\tau}} \right) \right] \right\}. \quad (45)$$

if the $e^{\pm r/L}$ terms and error functions are expanded in series form

and the limit $r \rightarrow 0$ is taken, the result is

$$\phi_{th}(0) = \frac{Q}{4\pi D} \left\{ \frac{1}{\sqrt{\pi \tau}} - \frac{e^{-\tau/L^2}}{L} \left[1 - \operatorname{erf} \frac{\sqrt{\tau}}{L} \right] \right\} \quad (46)$$

6.3.4 The one group treatment of a point thermal neutron source in a spherical cavity with radius R_o gives²³

$$\phi(r) = \frac{\frac{Q}{4\pi D} e^{-\frac{r-R_o}{L}}}{\left(1 + \frac{R_o}{L}\right)r} \quad (47)$$

This one group equation can be treated in the same manner as equation 36 to describe the neutron flux due to any one group in a multigroup theory.

6.3.5 The derivation of the flux around a spherical cavity using two group theory is given in Appendix C. The thermal flux is given by

$$\phi_2(r) = D \frac{e^{-k_2 r}}{r} + E \frac{e^{-k_1 r}}{r}, \quad (48)$$

where

$$D = \frac{-k_1^2 Q e^{k_2 R_o}}{4\pi D_2 (k_2^2 - k_1^2) (1 + k_2 R_o)}$$

$$E = \frac{k_1^2 Q e^{k_1 R_o}}{4\pi D_2 (k_2^2 - k_1^2) (1 + k_1 R_o)}$$

At the surface of the sphere, this reduces to

$$\begin{aligned}\phi_2(R_o) &= \frac{k_1^2 Q}{4\pi D_2 (k_2 + k_1) (1 + k_2 R_o) (1 + k_1 R_o)} \\ &= \frac{Q}{4\pi \Sigma_a L^2 \sqrt{\tau} \left(1 + \frac{\sqrt{\tau}}{L}\right) \left(1 + \frac{R_o}{L}\right) \left(1 + \frac{R_o}{\sqrt{\tau}}\right)}\end{aligned}\quad (49)$$

6.3.6 If a continuous slowing down model is used to describe the fast flux rather than a group description, the thermal flux around a point source in a spherical cavity is given in Ref. 23. The general expression is rather long and is not repeated here.

The value at the surface of the sphere is

$$\phi_{th} = \frac{Q}{4\pi D_2 \left(1 - \frac{R_o^2}{L^2}\right)} \left[\frac{e^{\tau/R_o^2}}{R_o} \left(1 - \operatorname{erf} \frac{\sqrt{\tau}}{R_o}\right) - \frac{e^{\tau/L^2}}{L} \left(1 - \operatorname{erf} \frac{\sqrt{\tau}}{L}\right) \right] \quad (50)$$

(The term $\exp(\tau/R_o^2)$ is missing in Ref. 23.)

6.3.7 If all thermal neutrons incident on a spherical cavity are assumed to be absorbed, the resulting flux equation is given by the following equation derived in Appendix E as a special case of a gray cavity:

$$\phi_{th} = D \frac{e^{-k_2 r}}{r} + E \frac{e^{-k_1 r}}{r}, \quad (51)$$

where

$$D = - \frac{k_1^2 Q (1 + k_1 R_o + \frac{R_o}{2D_2}) e^{k_2 R_o}}{4\pi D_2 (k_2^2 - k_1^2) (k_1 R_o + 1) (1 + k_2 R_o + \frac{R_o}{2D_2})}$$

$$E = \frac{k_1^2 Q e^{k_1 R_o}}{4\pi D_2 (k_2^2 - k_1^2) (k_1 R_o + 1)}$$

At the surface of the sphere, the flux is

$$\phi_{th}(R_o) = \frac{k_1^2 Q}{4\pi D_2 (k_2 + k_1) (1 + k_1 R_o) (1 + k_2 R_o + \frac{R_o}{2D_2})} \quad (52)$$

The rate of absorption in the sphere is given by

$$\left(\begin{array}{c} \text{absorption} \\ \text{rate} \end{array} \right) = 4\pi R_o^2 J_{in} = 2\pi R_o^2 \quad (53)$$

where J_{in} is the incident thermal neutron current. This particular model has been used by Nuclear-Chicago²⁵ to estimate the dependence of neutron moisture gauges on composition.

6.3.8 If one uses a model consisting of a point source in a cylindrical geometry, the mathematical description becomes more complex. Tittle²⁴ has examined one group theory for this situation and gives the epithermal flux in a series form for a point source on the axis of a cylindrical medium and for two coaxial cylindrical media. Glauberman²⁶ has used 2 group theory for the general case of a point source on the axis of two coaxial cylindrical media with different properties. The results are given in integral form, however.

Several of the response equations given in this Section and in Appendix F are collected together in Table 8 for comparison.

7. PREDICTED VARIATION OF THE NEUTRON FLUX IN COAL AND SOIL

As a first trial model for an immersion type moisture gauge, a simple point source in an infinite medium was used. A 2 Mev source of neutrons was assumed since this corresponds to the average energy of fission neutrons and much of the data is readily available for this energy. The response was assumed to be proportional to the thermal flux in the vicinity of a fast neutron source assuming continuous slowing down followed by diffusion. Carbon, hydrogen, and water were the only constituents included in this calculation. Figure 7 shows the results plotted along with the experimental data reported in Ref. 27.

The general agreement between the experimental data and the simple model encouraged further examination. In particular, the following changes were made: 1) the source energy was changed to 5 Mev to approximate artificial neutron sources, 2) more elements were taken into account in the constants calculations, and, 3) the equations were simplified to facilitate calculations. Simplified equations for the thermal neutron flux which were considered are derived in Appendix F.

The improved cross sections were used first to calculate the response of the moisture gauge in soil. This calculation used a point source of fast neutrons in an infinite medium. The gauge response was assumed to be proportional to the thermal flux at the origin. This model was later modified by putting the source in a spherical cavity to approximate the actual geometry of a source within a cylindrical pipe. The thermal flux at the surface of the cavity was then used as an indication of gauge response.

TABLE 8. Approximate forms for the moisture gauge response in terms of constants for the medium.

<u>Model</u>	<u>Theory</u>	<u>Response</u>	<u>Comment</u>
1) Point source, infinite medium	Continuous slowing down followed by diffusion	$R = \frac{k_1}{\Sigma_a \tau^{3/2}}$	Approximate form using 2 terms of asymptotic expansion
2) "	"	$R = \frac{k_2}{\Sigma_a \tau^{3/2} \left\{ 1 - \frac{3L^2}{2\tau} \right\}^{-1}}$	Approximate form using 3 terms of asymptotic expansion
3) Point source in hollow spherical cavity with radius a	"	$R = \frac{k_3}{\Sigma_a \tau^{3/2}}$	Approximate form using 2 terms of asymptotic expansion
4) "	"	$R = \frac{k_4}{\Sigma_a \tau^{3/2} \left\{ 1 - \frac{3(L^2 + a^2)}{2\tau} \right\}^{-1}}$	Approximate form using 3 terms of asymptotic expansion
5) Point source, infinite medium	2 group diffusion	$R = \frac{k_5}{\Sigma_a L \sqrt{\tau} (L + \sqrt{\tau})}$	
6) Point source in hollow spherical cavity with radius a	"	$R = \frac{k_6}{\Sigma_a (L + a) (\sqrt{\tau} + a) (L - \sqrt{\tau})}$	
7) Point source in "black" spherical cavity with radius a	"	$R = \frac{k_7}{\Sigma_a \left(L + a + \frac{a}{2 \Sigma_a L} \right) (\sqrt{\tau} + a) (L + \sqrt{\tau})}$	(This form has been used by Nuclear Chicago Corporation)

Fig. 7. Original coal calibration data and calculated calibration curves assuming fission energy neutrons.

The early efforts again used age-diffusion theory to describe the slowing down process. This theory for an infinite medium gives

$$R = \frac{\Sigma_a \tau^{3/2} \Big|_{\text{H}_2\text{O}}}{\Sigma_a \tau^{3/2} \Big|_{\text{medium}}} \quad (54)$$

The assumed composition of the soil is the same as previously given in Table 5. Σ_a is given by $\Sigma_a = \rho (.00377) \text{cm}^{-1}$ when ρ is in gm/cm^3 . For packed soil with a density of 90 lb/ft^3 (1.44 gm/cm^3), this is $\Sigma_a = .00544 \text{ cm}^{-1}$. Iron contributes the major part (about 1/3) of this value.

Table 9

Fast and thermal neutron cross sections for use in Eqs. 54 and 58.

Fast neutron cross sections for use in Eq. 57

	Weight Percent	$\sigma_a/1.128$	$\xi \sigma_1$	$\xi \sigma_2$	$\sigma_{tr,1}$	$\sigma_{tr,2}$	$\sigma_{tr,thermal}$
Si	28.1	0.142	0.216	0.16	3.01	2.26	1.66
O	47.3	0.0	0.357	0.48	2.27	3.64	4.02
Al	8.2	0.204	0.246	0.13	2.47	1.78	1.36
Fe	5.1	2.24	0.103	0.32	2.76	8.93	10.9
Ca	3.7	0.390	0.121	0.12	2.37	2.40	2.95
Mg	2.1	0.056	0.277	0.27	2.26	3.31	3.50
K	2.6	1.83	0.136	0.11	2.65	2.10	1.47
Na	2.8	0.448	0.275	0.28	3.02	3.40	3.88
Average			0.304	0.365	2.48	3.27	3.41
H			3.55	19.2	1.18	6.4	

Taking Σ_a for water as 0.0197 cm^{-1} the absorption cross section for the soil-water mixture is:

$$\Sigma_a = 0.0197 \rho_w + 0.00544. \quad (55)$$

τ was calculated using a form similar to the Deutsch formula (Refs. 17, 28).

$$\tau = \frac{\tau_1}{\left[\rho_w + \rho_x \alpha_1 \right] \left[\rho_w + \rho_x \beta_1 \right]} + \frac{\tau_2}{\left[\rho_w + \rho_x \alpha_2 \right] \left[\rho_w + \rho_x \beta_2 \right]}, \quad (56)$$

where τ_1 and τ_2 are the ages over the lethargy intervals 0 to 4 and 4 to thermal lethargy. (A lethargy of 4 corresponds to $E = 0.183$ Mev.)

These were taken as $\tau_1 = 43.0 \text{ cm}^2$ and $\tau_2 = 6.5 \text{ cm}^2$ to give a total age in water of 49.5 cm^2 . The coefficients are given by

$$\alpha_1 = \frac{A_w \xi \sigma_{x,1}}{A_x \xi \sigma_{w,1}}, \quad \beta_1 = \frac{A_w \sigma_{tr,x,1}}{A_x \sigma_{tr,w,1}}, \quad (57)$$

$$\alpha_2 = \frac{A_w \xi \sigma_{x,2}}{A_x \xi \sigma_{w,2}}, \quad \beta_2 = \frac{A_w \sigma_{tr,x,2}}{A_x \sigma_{tr,w,2}}.$$

The slowing down and transport cross sections used for this calculation are slightly different than those calculated later with more care but were calculated using a flux weighting scheme similar to Appendix A.

The cross sections which were used are tabulated in Table 9. The average transport cross sections for soil are $\sigma_{tr,x,1} = 2.48$ barn and $\sigma_{tr,x,2} = 3.27$ barn. The resulting coefficients are $\alpha_1 = 0.346$, $\alpha_2 = 0.00796$, $\beta_1 = 0.455$, and $\beta_2 = 0.169$. Several computed values of the count ratio are given in Table 10.

Table 10

Calculated count ratios in soil using (1) a point source in a homogeneous medium and (2) a point source in a cavity.

ρ_w	$R = 6.85 / \Sigma_a \tau^{3/2}$	$R = \frac{6.85}{\Sigma_a \tau^{3/2}} \cdot \left(\frac{\frac{2}{3} - \frac{L^2 + a^2}{\tau}}{0.311} \right)$
0.0	0.00572	0.010
0.1	0.0722	0.077
0.2	0.165	0.219
0.3	0.279	0.355
0.4	0.416	0.504
0.5	0.575	0.645

The introduction of a cavity alters the formula for the count ratio to

$$R = \frac{\left[\Sigma_a \tau^{3/2} \right]_{H_2O}}{\left[\Sigma_a \tau^{3/2} \right]_x} \cdot \frac{\left[\frac{2}{3} - \frac{L^2 + a^2}{\tau} \right]_x}{\left[\frac{2}{3} - \frac{L^2 + a^2}{\tau} \right]_{H_2O}} \quad (58)$$

L^2 was calculated from $L^2 = 1/3 \Sigma_a \Sigma_{tr, \text{thermal}}$ and τ was calculated as before. The cavity radius, $a = 3.2$ cm, was chosen to normalize the calculated values to an experimentally observed count ratio of $R = 0.645$ at $\rho_w = 0.5$ gm/cm³. The resulting values are also given in Table 10 and are plotted in Fig. 8. Calibration points for a Nuclear-Chicago Model P-19 moisture gauge as given in the operating manual are also included in Fig. 8.

The addition of boron at a concentration of 57 ppm (about twice the content of average United States soils) would shift the curve

Fig. 8. Theoretical moisture gauge calibration curves showing the effects of geometry and soil scattering.

to lower values of R as indicated in Fig. 9. In general, the model which includes a cavity around the neutron source is less sensitive to change in Σ_a than the infinite medium model.

A change in soil density to 78 lb/ft^3 (1.25 g/cm^3) corresponding to a loosely packed soil would shift the curve to lower values of R as indicated in Fig. 9.

8. CALCULATED CORRECTIONS TO ALLOW FOR COMPOSITION CHANGES

Since even the absolute value of the predicted count ratio is close to the observed value, it is believed that the influence of composition changes on the calibration curve can probably be inferred with good accuracy from the theoretical equations. An initial calibration curve can then be changed to correspond to a new medium, or, limits of composition change can be specified which will keep the error within given bounds.

Multiplicative type correction factors are used in the discussion below since they are relatively easy to calculate and do not depend on the reference medium used to calibrate the instrument. For example, given a calibration curve of count ratio, R , versus water density, ρ_w , for medium I:

$$R_I = R_I(\rho_w). \quad (59)$$

The calibration curve at the same water density for medium II is given by

$$R_{II} = CR_I, \quad (60)$$

where C is a function of the water density and the changes in composition between medium I and II. A table or chart of C versus ρ is required to make the correction.

Fig. 9. Theoretical moisture gauge calibration curves showing the effects of soil density.

C is merely the ratio of two R functions, and R is related to the composition through τ , L, and Σ_a by one of the appendix equations F-2, F-3, F-6, B-19, C-10, or E-1. C therefore can be expressed directly as a function of the changes in these parameters.

In order to relate C to basic cross section data, L and τ can be put in the form (see Section 6):

$$L^2 = \frac{1}{3 \Sigma_a \Sigma_{tr2}}, \quad (61)$$

$$\tau = \frac{1}{3 \Sigma_1 \Sigma_{tr1}}.$$

In the following sections, correction factors are calculated as indicated above for 3 of the gauge models.

8.1 Notation

In the following sections, subscripts 1 and 2 will refer to epithermal and thermal constants, subscripts I and II will refer to media I and media II. A cross section without an explicit I or II is always to be evaluated for medium I. The following notation will be used for the more common cross sections:

Σ_1 = epithermal "absorption" cross section (removal cross section)

Σ_a = thermal absorption cross section

Σ_{tr1} = epithermal transport cross section

Σ_{tr2} = thermal transport cross section

The macroscopic cross section, Σ , is given in terms of the microscopic cross section σ by

$$\Sigma_x = \sum_j \frac{\rho_j N_o}{A_j} \sigma_{x,j}, \quad (62)$$

where N_o and A are Avogadro's number and atomic weight respectively. The derivative with respect to ρ_i is

$$\frac{\partial \Sigma_x}{\partial \rho_i} = \sum_j \frac{\partial \rho_j}{\partial \rho_i} \frac{N_o}{A_j} \sigma_{x,j} = \frac{N_o \sigma_{x,i}}{A_i}. \quad (63)$$

Defining $\alpha_{x,i}$ as $\sigma_{x,i} A_w / \sigma_{x,w} A_i$, terms with the general form $(1/\Sigma) \partial \Sigma / \partial \rho_i$ can be written as

$$\frac{1}{\Sigma_x} \frac{\partial \Sigma_x}{\partial \rho_i} = \frac{\frac{N_o \sigma_{x,i}}{A_i}}{\sum_j \frac{\rho_j N_o}{A_j} \sigma_{x,j}} = \frac{\alpha_{x,i}}{\sum_j \rho_j \alpha_{x,j}}, \quad (64)$$

8.2 Equivalence Factors

The values of α defined above can also be used to convert any medium to an equivalent amount of water using any one of the 4 characteristic cross sections as a basis for comparison. From Eq. 62, Σ can be written as

$$\Sigma_x = \sum_j \frac{\rho_j N_o}{A_j} \sigma_{x,j} = \frac{\sigma_{x,w} N_o}{A_w} \sum_j \rho_j \alpha_{x,j} \quad (65)$$

$$= \frac{\sigma_{x,w} N_o}{A_w} \left[\rho_{\text{dry}} \sum_{\substack{j=\text{dry} \\ \text{medium}}} (\rho_j \alpha_{x,j}) + \rho_w \right]$$

An equivalence factor K_j can now be defined as

$$K_j = \sum_{\substack{j=\text{dry} \\ \text{medium}}} \rho_j \alpha_{x,j} \quad (66)$$

Using only the percentage composition and values of α , one can obtain an equivalence factor to convert the dry medium density into an equivalent moisture density for cross section calculations, or for making comparisons, from

$$\begin{array}{l} \text{Equivalent moisture density on the} \\ \text{basis of the } j\text{-th type interaction} \end{array} = K_j \rho_{\text{dry}} \quad (67)$$

8.3 Correction factor based on appendix equation F-2 (age-diffusion theory for infinite medium)

From Eq. 60, the correction factor is given by

$$C = \frac{R_{II}}{R_I} \quad (68)$$

The ratio R_{II}/R_I is evaluated by calculating $\Delta \ln R = \ln R_{II} - \ln R_I = \ln(R_{II}/R_I)$ and assuming the count ratio R is proportional to the thermal neutron flux. As noted before, the ratio R_{II}/R_I is used rather than the change $R_{II} - R_I$ so that the correction term is independent of the reference medium.

The total change in $\ln R$ is given in a linear approximation

by

$$\Delta \ln R = \sum_i \frac{\partial \ln R}{\partial \rho_i} \Delta \rho_i, \quad (69)$$

where $\Delta \rho_i$ is the change in the mass density of the i^{th} component.

Using Eq. F-2 for the neutron flux and noting that differentiation of the proportionality constant makes no contribution, one has

$$\begin{aligned} \frac{\partial \ln R}{\partial \rho_i} &= - \frac{\partial \ln \Sigma_a \tau^{3/2}}{\partial \rho_i} = - \frac{\partial \ln \Sigma_a}{\partial \rho_i} - \frac{\partial \ln (\Sigma_1 \Sigma_{tr1})^{-3/2}}{\partial \rho_i} \\ &= \frac{3}{2} \frac{1}{\Sigma_1} \frac{\partial \Sigma_1}{\partial \rho_i} + \frac{3}{2} \frac{1}{\Sigma_{tr1}} \frac{\partial \Sigma_{tr1}}{\partial \rho_i} - \frac{1}{\Sigma_a} \frac{\partial \Sigma_a}{\partial \rho_i} \end{aligned} \quad (70)$$

Using Eq. 64 gives

$$\Delta \ln R = \frac{3}{2} \frac{\sum_i \Delta \rho_i \alpha_{1,i}}{\sum_j \rho_j \alpha_{1,j}} + \frac{3}{2} \frac{\sum_i \Delta \rho_i \alpha_{tr1,i}}{\sum_j \rho_j \alpha_{tr1,j}} - \frac{\sum_i \Delta \rho_i \alpha_{a,i}}{\sum_j \rho_j \alpha_{a,j}} \quad (71)$$

Defining f_1 as

$$f_1 = \frac{\sum_i \Delta \rho_i \alpha_{1,i}}{\sum_j \rho_j \alpha_{1,j}}, \quad (72)$$

and using similar definitions for f_{tr1} and f_a , one obtains

$$\Delta \ln R = \frac{3}{2} f_1 + \frac{3}{2} f_{tr1} - f_a. \quad (73)$$

The above definition of f_1 is equivalent to using $f_1 = \Delta \Sigma_1 / \Sigma_1$.

The correction factor for the calibration curve is obtained directly from Eq. (73) since

$$\Delta \ln R = \ln R_{II} - \ln R_I = \ln \left(\frac{R_{II}}{R_I} \right) = \frac{3}{2} f_1 + \frac{3}{2} f_{tr1} - f_a.$$

Therefore

$$C = \frac{R_{II}}{R_I} = e^{\frac{3}{2} f_1 + \frac{3}{2} f_{tr1} - f_a}, \quad (74)$$

or

$$R_{II} = C R_I = R_I e^{\frac{3}{2} f_1 + \frac{3}{2} f_{tr1} - f_a}.$$

In a non-hydrogenous medium, one needs the correction factor at constant ρ_w as a function of water density. This is obtained by explicitly separating the ρ_w term in the expression for f and letting $\Delta \rho_w = 0$. It may be noted that f will be a small quantity since it is merely the fractional change in cross section $\Delta \Sigma / \Sigma$. An approximate form* for C , valid for small changes in composition and density is

* As noted later in section 8.7, this approximation of Eq. 74 is actually just as accurate as equation 74 itself when non-linear terms become significant.

given by

$$C = e^{\frac{3}{2} f_1 + \frac{3}{2} f_{tr1} - f_a} = 1 + \frac{3}{2} f_1 + \frac{3}{2} f_{tr1} - f_a \quad (75)$$

The f terms can also be separated into the contributions from density and composition changes by noting that - at constant ρ_w -

$$\Delta \rho_i = \Delta(p_i \rho) = p_i \Delta \rho + \rho \Delta p_i, \quad (76)$$

where p_i is the dry weight fraction of the i^{th} component and ρ is the total dry density.

A typical equation for f is

$$f_1 = \frac{\sum_i \Delta \rho_i \alpha_{1,i}}{\sum_j \rho_j \alpha_{1,j}} = \frac{\Delta \rho_{\text{dry}} \sum_{i=\text{medium}} p_i \alpha_{1,i} + \rho_{\text{dry}} \sum_{i=\text{medium}} \Delta p_i \alpha_{1,i}}{\rho_{\text{dry}} \sum_{i=\text{medium}} p_i \alpha_{1,i} + \rho_w} \quad (77)$$

For a medium which contains significant hydrogenous material, a calibration curve usually is a plot of count ratio versus total hydrogen density. In this case one needs a correction factor at constant ρ_H (ρ_H = total hydrogen density) as a function of ρ_H . This is obtained by explicitly separating ρ_H rather than ρ_w in the expression for f and taking $\Delta \rho_H = 0$. The oxygen in water now must be included as a separate term and the expression for f is altered in this case to (using f_1 as a typical example):

$$f_1 = \frac{\Delta \rho_{\text{dry}} \sum_{i=\text{non-H medium}} p_i \alpha_{1,i} + \rho_{\text{dry}} \sum_{i=\text{non-H medium}} \Delta p_i \alpha_{1,i} + \Delta \rho_o \alpha_{1,o}}{\rho_{\text{dry}} \sum_{i=\text{non-H medium}} p_i \alpha_{1,i} + \rho_o \alpha_{1,o} + \rho_H \alpha_{1,H}} \quad (78)$$

where the "o" subscript refers to the oxygen in the water. Dividing by α_H to obtain ρ_H explicitly and expressing the oxygen density in terms of water density gives

$$f_1 = \frac{\frac{\Delta\rho_{\text{dry}}}{\alpha_{1,H}} \sum_{\substack{i=\text{non-H} \\ \text{medium}}} p_i \alpha_{1,i} + \frac{\rho_{\text{dry}}}{\alpha_{1,H}} \sum_{\substack{i=\text{non-H} \\ \text{medium}}} \Delta\rho_i \alpha_{1,i} + \frac{8}{9} \Delta\rho_w \frac{\alpha_{1,o}}{\alpha_{1,w}}}{\frac{\rho_{\text{dry}}}{\alpha_{1,H}} \sum_{\substack{i=\text{non-H} \\ \text{medium}}} p_i \alpha_{1,i} + \frac{8}{9} \rho_w \frac{\alpha_{1,o}}{\alpha_{1,w}} + \rho_H} \quad (79)$$

In this case, one is presented with a need for $\Delta\rho_w$ when ρ_w is not known in the new medium. As an approximation at constant ρ_H , it is assumed that ρ_w in the new medium is about the same as in the old medium so that the $\Delta\rho_w$ term can be neglected.

To gain some feeling for the magnitudes of the hydrogen and non-hydrogen terms in f for a soft coal medium, the values in the denominator of f in the form

$$t = \frac{(\Sigma_{\text{II}} - \Sigma_{\text{I}})_{\text{non-H}}}{\Sigma_{\text{non-H}} + \Sigma_{\text{H}}} \quad (80)$$

were evaluated for a Kanawha coal with a dry density of 62.4 lb/ft^3 ($\rho = 1 \text{ gm/cm}^3$) containing 4 lb/ft^3 of moisture. The $\Sigma_{\text{non-H}}$ term includes the oxygen in water and the Σ_{H} term includes the hydrogen both in the medium and in the water. The ratio of the terms is given in Table 11.

Table 11

Comparison of hydrogen and non-hydrogen cross sections
in Kanawha coal with 4 lb/ft³ moisture

	$\Sigma_H / \Sigma_{\text{non-H}}$	Elements included in the calculation of Σ
Σ_1	10.4	H, C, O
Σ_{tr1}	0.5	H, C, O
Σ_a	8.0	H, C, O, Fe, N
Σ_{tr2}	5.6	H, C, O, N

Hydrogen dominates all cross sections except for the fast transport cross section. It is largely because of the dependence of the fast transport cross section on non-hydrogen elements that a composition dependence exists.

8.4 Correction factor based on appendix equation B-19 (2 group theory for infinite medium)

Using Eq. B-19

$$\begin{aligned}
 \frac{\partial \ln R}{\partial \rho_i} &= - \frac{\partial}{\partial \rho_i} \ln \left[D_2 \sqrt{\tau} \left(1 + \frac{\sqrt{\tau}}{L} \right) \right] \\
 &= - \frac{\partial}{\partial \rho_i} \ln (3 \Sigma_{\text{tr2}})^{-1} - \frac{\partial}{\partial \rho_i} \ln (3 \Sigma_1 \Sigma_{\text{tr1}})^{-1/2} - \frac{\partial}{\partial \rho_i} \ln \left(1 + \frac{\sqrt{\tau}}{L} \right) \\
 &= \frac{1}{\Sigma_{\text{tr2}}} \frac{\partial \Sigma_{\text{tr2}}}{\partial \rho_i} + \frac{1}{2} \frac{1}{\Sigma_1} \frac{\partial \Sigma_1}{\partial \rho_i} + \frac{1}{2} \frac{\partial \Sigma_{\text{tr1}}}{\partial \rho_i} - \frac{1}{\left(1 + \frac{\sqrt{\tau}}{L} \right)} \frac{\partial \left(\frac{\sqrt{\tau}}{L} \right)}{\partial \rho_i} \quad (81)
 \end{aligned}$$

The last term in the equation can be altered as follows:

$$\begin{aligned}
& - \frac{1}{(1 + \frac{\sqrt{\tau}}{L})} \frac{\partial (\frac{\sqrt{\tau}}{L})}{\partial \rho_i} = - \frac{1}{1 + \frac{L}{\sqrt{\tau}}} \left(\frac{1}{\sqrt{\tau}} \frac{\partial \sqrt{\tau}}{\partial \rho_i} - \frac{1}{L} \frac{\partial L}{\partial \rho_i} \right) \\
& = - \frac{1}{1 + \frac{L}{\sqrt{\tau}}} \left[(\Sigma_1 \Sigma_{tr1})^{1/2} \frac{\partial (\Sigma_1 \Sigma_{tr1})^{-1/2}}{\partial \rho_i} - (\Sigma_a \Sigma_{tr2})^{1/2} \frac{\partial (\Sigma_a \Sigma_{tr2})^{-1/2}}{\partial \rho_i} \right] \\
& = \frac{1/2}{1 + \frac{L}{\sqrt{\tau}}} \left(\frac{1}{\Sigma_1} \frac{\partial \Sigma_1}{\partial \rho_i} + \frac{1}{\Sigma_{tr1}} \frac{\partial \Sigma_{tr1}}{\partial \rho_i} - \frac{1}{\Sigma_a} \frac{\partial \Sigma_a}{\partial \rho_i} - \frac{1}{\Sigma_{tr2}} \frac{\partial \Sigma_{tr2}}{\partial \rho_i} \right) \quad (82)
\end{aligned}$$

The value of $L/\sqrt{\tau}$ is nearly constant for a wide range of moisture values and the quantity $(1/2)/(1 + L/\sqrt{\tau})$ is represented by α in the following equations. Combining Eq. (81) and (82) gives:

$$\begin{aligned}
\frac{\partial \ln R}{\partial \rho_i} &= \frac{1}{2} \frac{1}{\Sigma_1} \frac{\partial \Sigma_1}{\partial \rho_i} + \frac{1}{2} \frac{1}{\Sigma_{tr1}} \frac{\partial \Sigma_{tr1}}{\partial \rho_i} + \frac{1}{\Sigma_{tr2}} \frac{\partial \Sigma_{tr2}}{\partial \rho_i} + \\
&\quad \alpha \left(\frac{1}{\Sigma_1} \frac{\partial \Sigma_1}{\partial \rho_i} + \frac{1}{\Sigma_{tr1}} \frac{\partial \Sigma_{tr1}}{\partial \rho_i} - \frac{1}{\Sigma_a} \frac{\partial \Sigma_a}{\partial \rho_i} - \frac{1}{\Sigma_{tr2}} \frac{\partial \Sigma_{tr2}}{\partial \rho_i} \right) \quad (83)
\end{aligned}$$

Therefore,

$$\Delta \ln R = \sum_i \frac{\partial \ln R}{\partial \rho_i} \Delta \rho_i = \frac{1}{2} f_1 + \frac{1}{2} f_{tr1} + f_{tr2} + \alpha (f_1 + f_{tr1} - f_a - f_{tr2}). \quad (84)$$

The correction factor, C, is then given by

$$C = \exp(\Delta \ln R) = \exp \left[\left(\frac{1}{2} + \alpha \right) f_1 + \left(\frac{1}{2} + \alpha \right) f_{tr1} - \alpha f_a + (1 - \alpha) f_{tr2} \right]$$

$$= 1 + \left(\frac{1}{2} + \alpha \right) f_1 + \left(\frac{1}{2} + \alpha \right) f_{tr1} - \alpha f_a + (1 - \alpha) f_{tr2} \quad (85)$$

8.5 Correction factor based on appendix equation C-10 (2 group theory for medium with a transparent spherical cavity)

Using Eq. C-10,

$$\frac{\partial \ln R}{\partial \rho_i} = - \frac{\partial}{\partial \rho_i} \ln \left[D_2 \sqrt{\tau} \left(1 + \frac{\sqrt{\tau}}{L} \right) \left(1 + \frac{r_o}{L} \right) \left(1 + \frac{r_o}{\sqrt{\tau}} \right) \right] \quad (86)$$

The first three terms in the brackets are the same as in Eq. 81. The last two terms can be rewritten as follows:

$$- \frac{\partial}{\partial \rho_i} \ln \left(1 + \frac{r_o}{L} \right) = \frac{1}{1 + \frac{L}{r_o}} \cdot \frac{1}{L} \frac{\partial L}{\partial \rho_i} = - \frac{1/2}{1 + \frac{L}{r_o}} \left(\frac{1}{\Sigma_a} \frac{\partial \Sigma_a}{\partial \rho_i} + \frac{1}{\Sigma_{tr2}} \frac{\partial \Sigma_{tr2}}{\partial \rho_i} \right) \quad (87)$$

$$- \frac{\partial}{\partial \rho_i} \ln \left(1 + \frac{r_o}{\sqrt{\tau}} \right) = - \frac{1/2}{1 + \frac{\sqrt{\tau}}{r_o}} \left(\frac{1}{\Sigma_1} \frac{\partial \Sigma_1}{\partial \rho_i} + \frac{1}{\Sigma_{tr1}} \frac{\partial \Sigma_{tr1}}{\partial \rho_i} \right) \quad (88)$$

Define γ and δ as

$$\gamma = \frac{1/2}{\left(1 + \frac{L}{r_o} \right)}, \quad \delta = \frac{1/2}{\left(1 + \frac{\sqrt{\tau}}{r_o} \right)} \quad (89)$$

Combining equations (86), (87), and (88) gives

$$\begin{aligned} \frac{\partial \ln R}{\partial \rho_i} = & \frac{1}{\Sigma_1} \frac{\partial \Sigma_1}{\partial \rho_i} \left(\frac{1}{2} + \alpha - \delta \right) + \frac{1}{\Sigma_{tr1}} \frac{\partial \Sigma_{tr1}}{\partial \rho_i} \left(\frac{1}{2} + \alpha - \delta \right) \\ & + \frac{1}{\Sigma_a} \frac{\partial \Sigma_a}{\partial \rho_i} (-\alpha - \gamma) + \frac{1}{\Sigma_{tr2}} \frac{\partial \Sigma_{tr2}}{\partial \rho_i} (1 - \alpha - \gamma), \end{aligned} \quad (90)$$

therefore

$$\Delta \ln R = \sum_i \frac{\partial \ln R}{\partial \rho_i} \Delta \rho_i = \left(\frac{1}{2} + \alpha - \delta \right) (f_1 + f_{tr1}) - (\alpha + \gamma) f_a + (1 - \alpha - \gamma) f_{tr2}. \quad (91)$$

C is then given by,

$$\begin{aligned} C = \exp & \left[\left(\frac{1}{2} + \alpha - \delta \right) (f_1 + f_{tr1}) - (\alpha + \gamma) f_a + (1 - \alpha - \gamma) f_{tr2} \right] \\ & = 1 + \left(\frac{1}{2} + \alpha - \delta \right) (f_1 + f_{tr1}) - (\alpha + \gamma) f_a + (1 - \alpha - \gamma) f_{tr2}. \end{aligned} \quad (92)$$

8.6 Correction factor based on D-9 (2 group theory for a medium with a gray spherical cavity)

Equation D-9 which includes an albedo parameter for the surface of the spherical cavity can be written as

$$R \propto \frac{1}{D_2 \sqrt{\tau} \left(1 + \frac{\sqrt{\tau}}{L} \right) \left(1 + \frac{r'_o}{L} \right) \left(1 + \frac{r_o}{\sqrt{\tau}} \right)}, \quad (93)$$

where

$$r'_o = r_o \left[1 + \frac{1 - \beta}{2 (1 + \beta) \Sigma_a L} \right]$$

$\Sigma_a L = (1/\sqrt{3}) (\Sigma_a / \Sigma_{tr2})^{1/2}$ will be a slowly varying quantity since both Σ_a and Σ_{tr2} are primarily determined by one element - hydrogen. The effect of introducing an albedo for the surface of the cavity can be viewed as an effective increase in the radius term associated with L or as a decrease in L. For a transparent cavity, $\beta = 1$ and $r'_o = r_o$. For a black cavity, $\beta = 0$ and $r'_o = r_o (1 + 1/2 \Sigma_a L)$. $\Sigma_a L$ will be of the order 0.05 so that r'_o is significantly different than r_o for a black cavity.

Taking Σ_a / Σ_{tr2} as a constant will make r'_o constant and Eq. 93 is the same form as used in Eq. 86 except that r'_o replaces r_o in the term containing L. The correction factor would thus have the same form as Eq. 92

$$C = \exp \left[\left(\frac{1}{2} + \alpha - \delta \right) (f_1 + f_{tr1}) - (\alpha + \gamma') f_a + (1 - \alpha - \gamma') f_{tr2} \right], \quad (94)$$

$$\text{where } \gamma' = \frac{1/2}{1 + \frac{L}{r'_o}}. \quad (95)$$

8.7 Simplified correction factors

The correction factors will have the simplest form in those cases where hydrogen is the dominant element in the determination of the various cross sections and the non-hydrogen elements introduce only second order effects. When the cross sections are primarily determined by one element, cross section ratios are nearly constant

and terms like Σ_a / Σ_{tr2} and $\Sigma_a \Sigma_{tr2} / \Sigma_1 \Sigma_{tr1}$ will be approximately constant. In this case the $\Sigma_a L$ and $\sqrt{\tau}/L$ terms can be taken as constants and in addition,

$$f_a = f_{tr2} = \frac{1}{2} f_1 + \frac{1}{2} f_{tr1} \quad (96)$$

The last equality follows from

$$D_2 = \Sigma_a L^2 = (\Sigma_a L) L = (\text{constant}) L = (\text{constant}) \sqrt{\tau} \quad (97)$$

With these simplifications, the correction factor for an infinite medium using two group theory can be written

$$\begin{aligned} C &= e^{\frac{1}{2} (f_1 + f_{tr1}) + f_{tr2} + \alpha (f_1 + f_{tr1} - f_a - f_{tr2})} \\ &= e^{2 f_{tr2}} \end{aligned} \quad (98)$$

The approximate age-diffusion equation (Eq. F-2) leads to the same result. The correction factor for a medium with a spherical cavity using two group theory can be written

$$\begin{aligned} \bar{C} &= e^{\left(\frac{1}{2} - \delta\right) (f_1 + f_{tr1}) + (1 - \gamma) f_{tr2} - \gamma f_a + \alpha (f_1 + f_{tr1} - f_a - f_{tr2})} \\ &= e^{\left(\frac{1}{2} - \delta\right) (2 f_{tr2}) + (1 - \gamma) f_{tr2} - \gamma f_{tr2}} \\ &= e^{2 f_{tr2} (1 - \gamma - \delta)} \end{aligned} \quad (99)$$

where γ and δ are defined the same as in Eq. 89. The smaller exponent in Eq. 99 as compared to Eq. 98 indicates a reduction in the sensitivity to changes in composition for a cavity geometry as compared to an infinite medium.

8.8. Higher order corrections

The equations derived in Sections 8.2 - 8.5 are based on a linear expansion for $\Delta\Sigma$. If the assumption of a linear variations is not made, one has (for Eq. F-2)

$$e^{\Delta \ln R} = \exp \left[\ln \frac{\left(1 + \frac{\Delta \Sigma_1}{\Sigma_1}\right)^{3/2} \left(1 + \frac{\Delta \Sigma_{tr1}}{\Sigma_{tr1}}\right)^{3/2}}{1 + \frac{\Delta \Sigma_a}{\Sigma_a}} \right] = \frac{\left(1 + \frac{\Delta \Sigma_1}{\Sigma_1}\right)^{3/2} \left(1 + \frac{\Delta \Sigma_{tr1}}{\Sigma_{tr1}}\right)^{3/2}}{1 + \frac{\Delta \Sigma_a}{\Sigma_a}} \quad (100)$$

Depending on whether one used $\ln(1 + \Delta) \doteq \Delta$, or $e^\Delta \doteq 1 + \Delta$, Eq. 100 can be written approximately as

$$e^{\Delta \ln R} \doteq e^{\frac{3}{2} f_1 + \frac{3}{2} f_{tr1} - f_a} \quad (101)$$

or

$$e^{\Delta \ln R} \doteq 1 + \frac{3}{2} f_1 + \frac{3}{2} f_{tr1} - f_a \quad (102)$$

Compared to the exact Δ equation, the first approximation which is the same as the previous result when only the linear terms in $\Delta \Sigma$ were included gives somewhat high results and the second gives somewhat low results. That is, the equation derived in Section 8.2 including only first order terms in the expansion of $\Delta \Sigma$ will tend to give high results when f^2 cannot be neglected compared to f .

Using a Δ treatment for Eq. B-19 gives exactly

$$e^{\Delta \ln R} = \frac{(1 + f_1)^{1/2} (1 + f_{tr1})^{1/2}}{(1 + f_{tr2})} \left(1 + \frac{\Delta \left(\frac{\sqrt{\tau}}{L} \right)}{1 + \frac{\sqrt{\tau}}{L}} \right)^{-1} \quad (103)$$

Since $\sqrt{\tau}/L$ is usually a very slowly varying term, a linear approximation is adequate for the $\Delta(\sqrt{\tau}/L)$ term. This gives

$$e^{\Delta \ln R} = \frac{(1 + f_1)^{1/2} (1 + f_{tr1})^{1/2}}{(1 + f_{tr2})} \left[1 + \alpha (f_1 + f_{tr1} - f_a - f_{tr2}) \right] \quad (104)$$

where

$$\alpha \equiv \frac{1/2}{1 + \frac{L}{\tau}} \quad (105)$$

Again depending on the approximations used, one has

$$e^{\Delta \ln R} = e^{\frac{1}{2} f_1 + \frac{1}{2} f_{tr1} + f_{tr2} + \alpha (f_1 + f_{tr1} - f_a - f_{tr2})} \quad (106)$$

or

$$e^{\Delta \ln R} = 1 + \frac{1}{2} f_1 + \frac{1}{2} f_{tr1} + f_{tr2} + \alpha (f_1 + f_{tr1} - f_a - f_{tr2}). \quad (107)$$

The first approximation tends to give high results and the second tends to give low results.

A similar exact treatment for Eq. C-10 gives

$$e^{\Delta \ln R} = (1 + f_1)^{1/2} (1 + f_{tr1})^{1/2} (1 + f_{tr2}) \left[1 + \frac{\Delta \left(\frac{\sqrt{\tau}}{L} \right)}{1 + \frac{\sqrt{\tau}}{L}} \right]^{-1} \times \left[1 + \frac{\Delta \left(\frac{r_o}{L} \right)}{1 + \frac{r_o}{L}} \right]^{-1} \left[1 + \frac{\Delta \left(\frac{r_o}{\sqrt{\tau}} \right)}{1 + \frac{r_o}{\sqrt{\tau}}} \right]^{-1} \quad (108)$$

The first term in brackets can be again approximated as $1 + \alpha (f_1 + f_{tr1} + f_a - f_{tr2})$. The approximation for the second term in brackets through second order terms is given by

$$\left[1 + \frac{\Delta \left(\frac{r_o}{L} \right)}{1 + \frac{r_o}{L}} \right]^{-1} = \left[1 + \frac{\frac{1}{2} f_a + \frac{1}{2} f_{tr2} - \frac{1}{8} (f_a - f_{tr2})^2}{1 + \frac{L}{r_o}} \right]^{-1} \quad (109)$$

The term $(f_a - f_{tr2})$ will be very small since f_a and f_{tr2} are normally of the same magnitude. Neglecting $(f_a - f_{tr2})^2$, this term can be written

$$\left[1 + \frac{\Delta \left(\frac{r_o}{L} \right)}{1 + \frac{r_o}{L}} \right]^{-1} = \frac{1}{1 + \gamma (f_a + f_{tr2})} \quad (110)$$

where

$$\gamma \equiv \frac{1/2}{1 + \frac{L}{r_o}} \quad (111)$$

A similar approximation for the last term in brackets results in

$$\left[1 + \frac{\Delta \left(\frac{r_o}{\sqrt{\tau}} \right)}{1 + \frac{r_o}{\sqrt{\tau}}} \right]^{-1} = \frac{1}{1 + \delta (f_1 + f_{tr1})} \quad (112)$$

To a very good approximation then, the entire expression can be written as

$$e^{\Delta \ln R} = \frac{(1 + f_1)^{1/2} (1 + f_{tr1})^{1/2} (1 + f_{tr2}) \left[1 + \alpha (f_1 + f_{tr1} - f_a - f_{tr2}) \right]}{\left[1 + \gamma (f_a + f_{tr2}) \right] \left[1 + \delta (f_1 + f_{tr1}) \right]} \quad (113)$$

The two possible approximations are

$$e^{\Delta \ln R} = e^{(\frac{1}{2} + \alpha - \delta)(f_1 + f_{tr1}) + (1 - \alpha - \gamma)f_{tr2} - (\alpha + \gamma)f_a}, \quad (114)$$

or

$$e^{\Delta \ln R} = 1 + (\frac{1}{2} + \alpha - \delta)(f_1 + f_{tr1}) - (\alpha + \gamma)f_a + (1 - \alpha - \gamma)f_{tr2}. \quad (115)$$

The first of these two equations tends to give high results and the second low but the latter gives results that are generally closer to the value obtained from Eq. 113. Since $\sqrt{\tau}/L$ is nearly constant, the results are hardly affected by setting $\alpha = 0$. This would give

$$C = e^{\Delta \ln R} = 1 + (\frac{1}{2} - \delta)(f_1 + f_{tr1}) - \gamma f_a + (1 - \gamma)f_{tr2}. \quad (116)$$

as a reasonably simple form for calculation of most correction factors using 2 group theory and a medium with a spherical cavity.

8.9 Thermal cross sections and parameters

In order to evaluate the parameters appearing in the equation for the correction factors, average cross sections for the various elements are required. The thermal scattering and absorption cross sections given in Ref. 16 for several elements of interest are tabulated in Table 12. The transport cross section is taken as $\sigma_{tr2} = \sigma_s(1 - \mu_0)$ and the average absorption cross section is taken as $\sigma_a/1.128$. The values for α_1 defined in Eq. 64 are also given in the table.

8.10 Fast cross sections and parameters

The average fast cross sections were initially determined for three epithermal energy groups. Although no single constant can

TABLE 12
Thermal Cross Sections and α Factors

	<u>A</u>		<u>σ_a</u>	<u>σ_s</u>	<u>$\sigma_a/1.428$</u>	<u>$\sigma_s (1-\bar{\mu}_0)$</u>	<u>α_a</u>	<u>α_{tr2}</u>
Aluminum	26.98	Al	0.241 b	1.4 b	0.214 b	1.37 b	0.243	.0131
Boron	10.82	B	795	4	705	3.76	2000	0.0897
Calcium	40.08	Ca	0.44	3	0.39	2.95	0.298	0.0190
Carbon	12.01	C	0.00373	4.8	0.00331	4.53	0.00844	0.0974
Chlorine	35.46	Cl	33.8	16	29.97	15.7	25.9	0.114
Hydrogen	1.008	H	0.332		0.294	32.9*	8.94	8.42
Iron	55.85	Fe	2.62	11	2.32	10.9	1.27	0.0504
Magnesium	24.32	Mg	0.069	3.6	0.0612	3.50	0.0771	0.0371
Nitrogen	14.01	N	1.88	10	1.67	9.52	3.65	0.175
Oxygen	16.00	O	0.00020	4.2	0.00018	4.02	0.000345	0.0648
Potassium	39.10	K	2.07	1.5	1.84	1.47	1.44	0.00970
Silicon	28.09	Si	0.16	1.7	0.142	1.66	1.55	0.0153
Sodium	22.99	Na	0.525	4.0	0.466	3.88	0.621	0.0436
Water	18.02	H ₂ O			0.588	69.8	1.000	1.000

* σ_{tr} for H obtained by using experimental data for water ($L = 2.70$ cm, Ref. 18) and solving

$$L^2 = 1/3 \sum_a \sum_{tr} \text{for } (\sigma_{tr})_H$$

exactly reproduce the results of a three group calculation, in the interest of simplifying the subsequent calculation of correction factors, the three-group values were averaged to a single constant which gives approximately the same age as the three-group treatment.

The source energy was taken as 5.0 Mev on the basis that age calculations in water using this energy generally agreed with experimental data. The first group includes only the first flight correction and actually uses only the cross section at 5 Mev. The second group extends from 5 Mev to 0.183 Mev. The third group extends from 0.183 Mev to 1.4 ev. The energy breakdown within the various groups and further details of the calculation are given at the beginning of the tabulation of cross sections in Appendix A. The resulting average fast cross sections and α factors are tabulated in Table 13.

TABLE 13
Fast Neutron Cross Sections and α Factors

<u>Element</u>	<u>σ_1</u>	<u>σ_{tr1}</u>	<u>α_1</u>	<u>α_{tr1}</u>
Al	0.06762	3.031	0.07044	0.2169
B	-	-	-	-
Ca	0.01690	4.804	0.01185	0.2315
C	0.02625	3.038	0.06143	0.4885
Cl	-	-	-	-
H	0.3161	3.069	8.813	5.879
Fe	0.1514	3.994	0.07618	0.1381
Mg	0.01668	3.194	0.01928	0.2536
N	0.02509	3.200	0.05034	0.4412
O	0.008692	3.191	0.01527	0.3851
K	0.01474	4.364	0.01060	0.2155
Si	0.01613	4.341	0.01614	0.2984
Na	0.02136	4.642	0.02611	0.3899
H ₂ O	0.6410	9.329	1.000	1.000

9. COMPARISON WITH EXPERIMENTAL DATA

9.1 Comparison of C/H Correction Factors

As one means of evaluating the various models and the calculated cross sections, the predicted influence of the carbon-to-hydrogen atom ratio, $N(C)/N(H)$, using the various theories can be compared with the experimental data of Tittle¹ by computing what Tittle calls an "interpolation factor, f ". This is equivalent to computing the flux for different values of $N(C)/N(H)$.

The relationship between Tittle's f -factor (which he introduced to compare data from carbon-water lattices with data from furfural) and the flux can be seen in the following resume of the pertinent parts of Ref. 1: Indium resonance data for various carbon-water lattices were normalized to the same peak height and the source-to-foil distances were multiplied by the actual water density in the lattice to give "the distributions in media containing the same concentration of hydrogen and oxygen but various concentrations of carbon". f was then defined as the ratio of the distance from the origin for a given intensity on one of the distributions to the distance from the origin for the same intensity on a distribution C' which nearly represented furfural, i. e.,

$$f = \frac{\text{distance to obtain intensity } I}{\text{distance to obtain intensity } I \text{ on } C'} \quad (117)$$

A ratio R was defined as below and calculated for each lattice using a plot of the f values obtained in Eq. 117 versus $N(C)/N(H)$.

$$R = \frac{f(\text{furfural})}{f(\text{lattice})} = \frac{\text{distance to obtain intensity } I \text{ in furfural}}{\text{distance to obtain intensity } I \text{ in lattice}} \quad (118)$$

The data for various lattices was then adjusted to the value of $N(C)/N(H)$ in furfural by correcting the distance and activity scales used for plotting as:

$$r_1 = Rr_o \quad , \quad (119)$$

$$A = A_o/R^3 \quad . \quad (120)$$

These adjustments correct the data to an equivalent media with the same value of $N(C)/N(H)$ as in furfural but with different atom densities. At the origin, only Eq. 120 is of interest. Further corrections for the atom densities were used by Tittle but are not required here.

The above corrections are partly complicated by the fact that a comparison with furfural was ultimately desired and none of the experimental lattices had a value of $N(C)/N(H)$ or water density which was identical to that in furfural. (The value of $N(C)/N(H)$ for distribution C' was 1.173 compared to 1.25 for furfural.) For comparison with theoretical calculations, one can use the lattice with distribution C' as the "standard" rather than furfural. For this case, one has

$$R = \frac{f(\text{standard})}{f(\text{medium})} = \frac{1}{f(\text{medium})} \quad , \quad (121)$$

and

$$A_1 = A_o/R^3 = A_o f^3 (\text{medium}). \quad (122)$$

Again, this adjusts the activity to an equivalent media with the same value of $N(C)/N(H)$ as in lattice C' but makes no correction for a difference in water densities.

Equation 122 can be written as

$$f(\text{medium}) = \left(\frac{\text{Activity in medium with } N(C)/N(H) = 1.173}{\text{Activity in medium with } N(C)/N(H) = N(C)/N(H)} \right)^{1/3} \quad (123)$$

This equation will be used as a basis for comparing the various theories by calculating the activity ratios and obtaining a theoretical value for f which can be compared with the experimental values. Thermal flux ratios were computed using (a) age-diffusion theory, (b) approximate age-diffusion theory, and (c) two-group theory for values of $\rho(C)/\rho(W) = 0.0, 0.5, 1.0, 2.0, 2.5,$ and 3.0 . For carbon-water lattices, the atom densities and mass densities are related by

$$\frac{N(C)}{N(H)} = 0.750 \frac{\rho(C)}{\rho(W)} \quad (124)$$

In age-diffusion theory, the flux at the origin in an infinite homogeneous medium is given by Eq. 46 which can be slightly modified to give

$$\phi(0) = \frac{S}{4\pi \sum_a L^3} \left[\frac{1}{\gamma \sqrt{\pi}} - e^{\gamma^2} (1 - \text{erf } \gamma) \right] = \frac{S}{4\pi \sum_a L^3} f(\gamma) \quad (125)$$

where γ is defined as $\gamma = \sqrt{\tau}/L$.

Values for $f(\gamma)$ are given in Table 14 and are plotted in Fig. 10.

Table 14. Values of $f(\gamma)$ defined in Eq. 125.

γ	$f(\gamma)$	γ	$f(\gamma)$
1.0	0.13661	2.0	0.02670
1.1	0.11117	2.1	0.02354
1.2	0.09162	2.2	0.02086
1.3	0.07635	2.3	0.01856
1.4	0.06425	2.4	0.01658
1.5	0.05454	2.5	0.01487
1.6	0.04665	2.6	0.01339
1.7	0.04021	2.7	0.01209
1.8	0.03488	2.8	0.01095
1.9	0.03043	2.9	0.009946
2.0	0.02670	3.0	0.009066

Fig. 10. Plot of $f(\gamma) = (1/\gamma\sqrt{\pi}) - e^{\gamma^2} (1 - \text{erf } \gamma)$ versus γ .

The flux ratio to be evaluated is

$$\frac{\phi(0)|_{\rho(C)/\rho(W) = 1.564}}{\phi(0)|_{\rho(C)/\rho(W)}} = \frac{f(\gamma)}{\Sigma_a L^3} \bigg|_{1.564} \frac{\Sigma_a L^3}{f(\gamma)} \quad (126)$$

In terms of the α coefficients, γ can be written as

$$\begin{aligned} \gamma &= \frac{\sqrt{\tau}}{L} = \sqrt{\frac{\Sigma_a \Sigma_{tr2}}{\Sigma_1 \Sigma_{tr1}}} = \left[\frac{\sigma_{a,W} \sigma_{tr2,W}}{\sigma_{1,W} \sigma_{tr1,W}} \right]^{1/2} \left[\frac{\sum_i \rho_i \alpha_{a,i} \sum_i \rho_i \alpha_{tr2,i}}{\sum_i \rho_i \alpha_{1,i} \sum_i \rho_i \alpha_{tr1,i}} \right]^{1/2} \\ &= \left[\frac{\sigma_{a,W} \sigma_{tr2,W}}{\sigma_{1,W} \sigma_{tr1,W}} \right]^{1/2} \left[\frac{(1 + \frac{\rho(C)}{\rho(W)} \alpha_{a,C}) (1 + \frac{\rho(C)}{\rho(W)} \alpha_{tr2,C})}{(1 + \frac{\rho(C)}{\rho(W)} \alpha_{1,C}) (1 + \frac{\rho(C)}{\rho(W)} \alpha_{tr1,C})} \right]^{1/2} \end{aligned} \quad (127)$$

Table 15 gives the value of γ obtained by inserting the cross sections and α factors from Tables 12 and 13 into Eq. 127. The corresponding values for $f(\gamma)$ are also given.

Table 15. Values of $\gamma = \sqrt{\tau}/L$ from Eq. 127 for various carbon-to-water mass densities.

ρ_C/ρ_W	γ (Eq. 122)	$f(\gamma)$
0.0	2.620	0.0131
0.5	2.374	0.0172
1.0	2.192	0.0211
1.5	2.052	0.0248
1.564	2.036	0.0255
2.0	1.938	0.0290
2.5	1.844	0.0328
3.0	1.765	0.0369

Any Σ can be written as

$$\Sigma = \frac{\sigma_W^{N_O}}{A_W} (\sum_i \rho_i \alpha_i + \rho_W) = \left(\frac{\sigma_W^{N_O}}{A_W} \right) \rho_W \left(1 + \frac{\rho(C)}{\rho(W)} \alpha_C \right). \quad (128)$$

At constant water density

$$\frac{\Sigma_a}{\Sigma_a} \bigg|_{\rho(C)/\rho(W) = 1.564} = \frac{\left(1 + \frac{\rho(C)}{\rho(W)} \alpha_{a, C}\right)}{\left(1 + \frac{\rho(C)}{\rho(W)} \alpha_{a, C}\right)_{1.564}} \quad (129)$$

and

$$\begin{aligned} \frac{L^3}{L^3} \bigg|_{\rho(C)/\rho(W) = 1.564} &= \left[\frac{\Sigma_a \Sigma_{tr2} \big|_{1.564}}{\Sigma_a \Sigma_{tr2}} \right]^{3/2} \\ &= \left[\frac{\left(1 + \frac{\rho(C)}{\rho(W)} \alpha_{a, C}\right) \left(1 + \frac{\rho(C)}{\rho(W)} \alpha_{tr2, C}\right) \big|_{1.564}}{\left(1 + \frac{\rho(C)}{\rho(W)} \alpha_{a, C}\right) \left(1 + \frac{\rho(C)}{\rho(W)} \alpha_{tr2, C}\right)} \right]^{3/2} \end{aligned} \quad (130)$$

The product of Eqs. 129 and 130 is

$$\frac{\Sigma_a L^3}{\Sigma_a L^3} \bigg|_{1.564} = \frac{\left(1 + \frac{\rho(C)}{\rho(W)} \alpha_{a, C}\right)^{1/2} \left(1 + \frac{\rho(C)}{\rho(W)} \alpha_{tr2, C}\right)^{3/2} \big|_{1.564}}{\left(1 + \frac{\rho(C)}{\rho(W)} \alpha_{a, C}\right)^{1/2} \left(1 + \frac{\rho(C)}{\rho(W)} \alpha_{tr2, C}\right)^{3/2}} \quad (131)$$

Using this equation and the value of $f(\gamma)$ from Table 15, Eq. 126 can be evaluated. The results are tabulated below and f is plotted in Fig. 11 along with Tittle's experimental data.

Table 16. Values of the thermal flux using age-diffusion theory in media with various carbon-to-water mass densities compared with the thermal flux in a medium with a carbon-to-water mass density of 1.564 (Eq. 126)

ρ_C/ρ_W	$\frac{\phi 1.564}{\phi}$	$\left(\frac{\phi 1.564}{\phi}\right)^{1/3} = f$
0.0	2.423	1.343
0.5	1.715	1.197
1.0	1.303	1.092
1.5	1.037	1.0123
1.564	1.000	1.000
2.0	0.8313	0.9403
2.5	0.6908	0.8840
3.0	0.5785	0.8332

As a simple approximation to Eq. 125, one can expand the error function and retain only the first term in the expression for $f(\gamma)$. The result is

$$f(\gamma) \doteq \frac{1}{2\gamma^3\sqrt{\pi}} \quad (132)$$

Using this simple form for $f(\gamma)$ in the computation of the flux ratios gives the curve labeled as "approximate age-diffusion theory" in Fig. 11. It may be noted that this approximation is equivalent to Eq. F-2 which is repeated here:

$$\phi(0) \propto \frac{1}{\sum_a \tau_a^{3/2}} \quad (133)$$

The two-group equation for the flux at the origin in an infinite homogeneous medium is given by Eq. B-19 which can be modified to read

$$\phi(0) = \frac{S}{4\pi \Sigma_a L^3} \left[\frac{1}{\gamma(1+\gamma)} \right] \quad (134)$$

The value of f obtained from Eq. 134 by again computing the cube root of the flux ratio is also plotted in Fig. 11.

The values of f computed here are relatively insensitive to the source energy. Before the cross sections appropriate for a Ra-Be source were evaluated a similar calculation was made using fast cross sections for fission energy neutrons (2 Mev). The resulting curves were essentially identical to the curves in Fig. 11.

9.2 Comparison with Fermi Age

The neutron age, τ , in various materials was also calculated for comparison with experimental values. One possible means of calculating the neutron age is to convert the non-water part of the medium to an equivalent water density as indicated in section 8.2 and combine this with the true water density, ρ_w . Since the age is inversely proportional to the macroscopic transport and slowing down cross sections, the age can then be calculated as:

$$\tau = \frac{\tau_w}{(\rho_w + \rho_1)(\rho_w + \rho_2)} \quad (135)$$

where τ_w is the neutron age in pure water, ρ_1 is the water equivalent of the medium on the basis of transport properties and ρ_2 is the water equivalent of the medium on the basis of moderating properties. Alternately, the age can be calculated directly from the cross sections which are used in the computation of the equivalence factors using:

$$\tau = \frac{1}{3 \sum_{tr1} \Sigma_1} = \frac{1}{3 \sum_i N_i \sigma_{i,tr1} \sum_i N_i \sigma_{i,1}} \quad (136)$$

Fig. 11. Calculated and experimental values of an interpolation factor to allow for a change in the carbon to hydrogen ratio of the medium.

where N_i is the number of atoms per cm^3 of type i , σ is the microscopic cross section, Σ is the macroscopic cross section, and the subscripts tr1 and 1 refer to the epithermal transport and slowing down cross sections respectively.

The results of several age calculations are shown in Table 17. The difference between the experimental and calculated values is generally well under 10%. The data for the carbon and iron lattices is plotted in Fig. 12.

Table 17. Comparison of Calculated Neutron Age with Experimental Age

<u>Medium</u>	<u>P_w</u>	<u>Note</u>	<u>One-Group Age to In Resonance</u>	<u>Experimental Age</u>	<u>Ref.</u>
Water	1.00		50.1 cm^2	48.5 cm^2	29
Furfural ($\text{C}_5\text{H}_4\text{O}_2$)	0.435		135	134.5	1
Saturated sand-water mixture	0.388	1	127	131.4	1
Carbon-water lattice B	0.769	2	65.4	71.3	1
Carbon-water lattice C	0.526	3	95.5	99.5	1
Carbon-water lattice D	0.346	4	134	136.8	1
Iron-water lattice with 1:1 volume ratio	0.500	5	59.8	57.9	29
Iron-water lattice with 1:2 volume ratio	0.667	5	55.9	56.0	29
Iron-water lattice with 1:3 volume ratio	0.750	5	54.2	52.3	29

Notes

- (1) Atom densities are based on 38.8 v/o water and an assumed density of $\rho = 2.65$ for SiO_2 .
- (2) Atom densities are based on 76.9 v/o water, $N(\text{C})/N(\text{H}) = 0.401$.

- (3) Atom densities are based on 52.6 v/o water, $N(C)/N(H) = 1.173$.
- (4) Atom densities are based on 34.6 v/o water, $N(C)/N(H) = 2.599$.
- (5) Atom densities are based on an assumed density of 7.86 g/cm^3 for the iron plates (reported as in the range SAE 1010 to SAE 1020).

10. APPLICATION TO THE COAL INVENTORY PROBLEM

Using the correction factors derived in Section 8 and the α factors given in Section 8, either the water or coal contribution to the count ratio can be adjusted to any value. If only the dry coal contribution to the count ratio is adjusted to a constant value, the count ratio should then be a smooth function of water density.

Such corrections were examined using the 2 gp. theory for a medium with a spherical cavity and it was found that the value of the cavity radius, r_o , which produced the least scatter in the adjusted Clifty coal data was $r_o \doteq 2 \text{ cm}$ when water was adjusted to a constant value and $r_o \doteq 10 \text{ cm}$ when coal (including the bound hydrogen) was adjusted to a constant value. In terms of the resulting corrections, this meant that more weight was apparently required for the change in cross section caused by water variations than for the coal variations. This apparent difference in weights could be caused by errors in the water and/or hydrogen determination, errors in cross sections, or inadequacy of the model. Because of the general agreement between the calculated and measured values of τ , it does not appear that the cross sections can be seriously in error. The most likely source of error would seem to be in the determination of the hydrogen and moisture content of the medium or the choice of the theoretical model. The data on the influence of bound hydrogen is conflicting.

Using the value of $r = 10 \text{ cm}$, correction factors to adjust the calibration date from all four coals to the same composition and dry coal

Fig. 12. Calculated and experimental values of the Fermi age in carbon-water and iron-water lattices.

density can be calculated. Factors were computed to adjust all data to a composition and density the same as Clifty composition and average dry density; the resulting adjusted data are plotted in Fig. 13. Ideally, all the adjusted data should now fall on the same curve. It is obvious from the figure that the known differences in composition and water content cannot explain the different behavior of these coals.

If the Kyger calibration data is adjusted, using Eq. C-10 with $r_o = 10$, to the average density for the Kyger field measurements, one obtains the solid line in Fig. 14. Using this line with the observed value of R ($R = 0.641$) gives a water density of 7.8 lb/ft^3 or 10.4% moisture. The measured value from core samples was 8.7%. When an uncorrected curve is used, the neutron gauge indicates 16.2% moisture. Although known differences in composition can reduce the error in the moisture determination using the gauge, they cannot explain all of the differences observed with this set of samples. For example, if one had attempted to determine a calibration curve for Kyger coal on the basis of what was known about Clifty coal by adjusting the Clifty data in composition and density to what is believed to be the average composition and density for the Kyger field measurements, one obtains the dashed line in Fig. 14. Comparison of this curve with the actual data using Kyger coal (plotted on the same figure) indicates that the available information on composition is not accurate enough to establish a reliable calibration curve in this manner.

11. REPRODUCIBILITY

The day to day reproducibility of the data during field operations appears to be excellent. Figure 15 which shows typical moisture data collected on different days with a Nuclear Chicago moisture gauge during a field trip to the Clifty Creek Station of the Indiana-Kentucky Electric Corporation.

Fig. 13. Plot of calibration data from four different coals after adjustment to the same chemical composition and dry coal density.

Fig. 14. Plot of calibration data for Kyger coal adjusted to the average Kyger field density, and calibration data for Clifty coal adjusted to Kyger composition and average Kyger field density.

Fig. 15. Field data of moisture gauge response versus depth taken on two different days.

12. POSSIBLE CALIBRATION AND EXPERIMENTAL MEDIA

Various organic compounds were examined in an attempt to find media which would enable direct measurement of the effect on the moisture probe of different carbon-to-hydrogen ratios. Relatively inexpensive, miscible liquids of the type $C_x(H_2O)_y$ were looked for initially. The list of Industrial Organic Compounds in the Handbooks of Chemistry and Physics (40th Edition) and the Quarterly Report on Current Prices in Chemical and Engineering News (March 28, 1960) were checked to indicate availability in commercial quantities and relatively low cost. Figure 16 is a plot of hydrogen density vs. non-hydrogen-to-hydrogen atom ratio for $C_x(H_2O)_y$ compounds with known density and indicates that there are no groups of liquids or solids with the same hydrogen density in the region of interest.

All high purity liquids of the type $C_xH_yO_z$ which are found in the Industrial Organic list were then examined and are shown in Fig. 17. Here also no pairs or groups of liquids with the same hydrogen density were found in the region of interest. The complete list of organic compounds in the Handbook of Chemistry and Physics was searched for C, H, and O-compounds with high values for the non-hydrogen-to-hydrogen ratio. The few that were found are also shown in Fig. 17. Only one is a liquid. The only inexpensive, available liquids which have values for the hydrogen density and atom ratio in the region of interest are furfural, demethyl phthalate, and acrylic acid. Unfortunately, each has a different hydrogen density.

In short, it does not seem possible to find a liquid C, H, O-compound which duplicates both the hydrogen density and carbon-to-hydrogen ratios which are found in coal. (Colloidal suspensions of graphite were also examined.)

Fig. 16. Plot of hydrogen density versus non-hydrogen-to-hydrogen atom ratio for $C_x(H_2O)_y$ compounds.

Fig. 17. Plot of hydrogen density versus non-hydrogen-to-hydrogen atom ratio for $C_x H_y O_z$ compounds.

13. POSSIBLE METHODS OF DETECTING ANOMOLOUS MEDIA

In order to have an indication in the field of whether the contribution of the non-hydrogen elements is essentially the same as in the calibration samples, one could add a second thermal neutron detector at a distance r from the source. This second detector would provide information on the rate of attenuation of the neutron density in the medium. For "normal" media, the ratio of the counting rate at r compared with the counting rate near the source will have a definite value for each moisture content since the migration length changes with moisture content. The value of this ratio could be used as an indication of typical media. If there are extraneous neutron absorbers present, the migration length is reduced and the counting rate at r compared to the rate near the source will be less than normal. If there are extraneous non-absorbing moderators present (e. g., carbon) the counting rate at r will be greater than normal. The general variation of neutron density with distance is indicated in Fig. 18 for the several cases discussed above. The addition, however, of a non-absorbing moderator such as carbon plus an absorbing material such as iron could produce effects which cancel and which would appear as "fictitious" hydrogen. (One possible means of recognizing such a situation would be to measure the slowing down time using a pulsed neutron source.

The use of epithermal detectors would avoid errors caused by extraneous thermal neutron absorbers and would possibly give information on fast neutron scattering but the fast neutron density is very small and would lead to poor counting statistics unless the source strength or counting time is increased by a factor of about 100. A BF_3 tube (a $1/v$ detector) gives the same response to equal densities of either fast or slow neutrons since

$$R \propto \sigma_a \phi = \left(\frac{k}{v} \right) (nv) = kn \quad (137)$$

Covering the moisture gauge with a cadmium filter to absorb the thermal neutrons reduces the counting rate to only 2 - 3% of the original value.

Fig. 18. Schematic variation of neutron density and distance for three samples with different moisture contents but with the same neutron density at the origin.

1. Normal variation of neutron density with distance for moisture M.
2. Neutron density at a higher moisture content than M but with absorbers present.
3. Neutron density at a lower moisture content than M but with carbon present.

14. SUMMARY

The general tendency to ignore the influence of non-hydrogen elements on neutron-moderating moisture gauges is not justified in many cases since the scattering as well as the moderating properties of the medium are significant. The response of a neutron-moderating moisture gauge is sensitive to the slowing down length which depends on both the scattering and moderating properties of the medium. This means that

changes in the dry medium composition and density can influence the gauge response.

In particular, the use of neutron moderating moisture gauges in soft coal does not appear practical for two reasons: 1) The analysis of the calibration media for hydrogen and water content must be exceedingly accurate since the amount of bound hydrogen in the coal is 3 to 5 times as great as the hydrogen in the water. Conventional techniques for moisture analysis of coal do not provide the necessary accuracy. 2) The high carbon content significantly affects the neutron age and thus fluctuations in density as well as moisture will alter the gauge response.

Neither of the above points is as significant in soil as in coal but they still cannot be ignored. For example, calibration data using samples with a uniform dry density and various amounts of moisture will give a smooth calibration curve and a misleading indication of the gauge accuracy. The media used for calibration must reflect the random changes in density and composition found under field conditions as well as the change in moisture content.

The relative importance of hydrogen and the non-hydrogen elements is examined in Section 2. Examination of the neutron cross sections for coal and soil in Section 2 indicates that the non-hydrogen components contribute significantly to the total cross section. The fast transport cross section is particularly influenced by the non-hydrogen components. The relationship between these cross sections and the neutron density is discussed in Section 6 where a brief review of neutron moderation theory and the equations for the thermal neutron flux in several simple geometries are given. These thermal flux equations are used in Section 7 to give theoretical calibration curves for media such as coal

and soil. The same equations are used in Section 8 as a basis for calculating correction factors which give the change in response caused by changes in composition and density. The calculation of correction factors, cross sections, or the water equivalent of a dry medium is facilitated by the use of a set of α coefficients which are tabulated in Sections 8.9 and 8.10. The accuracy of the α coefficients is checked by comparison with experimental data in Section 9.

The tabulation of α coefficients can, of course, be used in the study of any media and are not restricted to coal and soil. Also they are not restricted to any particular model of a moisture gauge since they give cross section information and not gauge response.

The correction factor based on two group theory for the flux at the surface of a spherical cavity is applied to coal data in Section 10 in an attempt to explain differing calibration curves for different coals. Application of correction factors to reduce all data to the same composition and density reduces the scatter in the data but does not remove the discrepancy between the various coals. Errors in the assumed composition seems to be the most likely source of error since the theoretical equations give reasonably good agreement with other experimental data.

APPENDIX A

CALCULATION OF AVERAGE FAST NEUTRON CROSS SECTIONS

One method of calculating average cross section values is to use the neutron flux as the weighting factor^{28, 30}. The energy dependent flux is given by

$$\phi(E) = \frac{q}{\xi \Sigma_T E}, \quad (A-1)$$

where q is the slowing down density and is constant if there is no epithermal absorption. The flux due to neutrons in the interval ΔE is then

$$\phi(E) \Delta E = \frac{q \Delta E}{\xi \Sigma_T E} = \frac{q \Delta u}{\xi \Sigma_T} = \left(\frac{q}{N}\right) \left(\frac{\Delta u}{\xi \sigma_T}\right), \quad (A-2)$$

where lethargy $= u = \ln(E_0/E)$. q/N will be constant. If the flux is primarily determined by slowing down due to hydrogen, the weight factor, W , is

$$W = \frac{\Delta u}{(\xi \sigma_T)_H}. \quad (A-3)$$

Using this weight function, average values for $\xi \sigma$ and σ_{tr} have been calculated for various elements using the cross sections in Table A-1 for the energy range 5.0 Mev to 1.44 ev. In Table A-1, the cross sections were taken from Refs. 31, 32, 33, and 34, when possible, otherwise they were calculated from basic cross section data in Ref. 11. (In Ref. 17 σ_{tr} for hydrogen and $\xi \sigma$ for aluminum are in error by a factor of 3 and 10 respectively.)

Table A-1. Cross Sections for the Range 5.0 - 1.44 ev.

Group	Δu	$\Delta u/\xi\sigma$	u	E(Mev)	σ_{tr}										
					O	Na	Mg	Al	Fe	Si	Ca	K	H	C	N
1(ffc)	1.0	.6173	.693	5	0.676	2.259	1.057	1.356	1.904	2.29	3.45	2.949	1.620	1.005	1.524
2	0.307	.1696	.693-1.0	5-3.68	1.260	2.320	1.098	1.534	2.178	2.20	3.64	3.145	1.563	1.646	1.524
3	0.5	.2145	1.0-1.5	3.68-2.23	1.248	2.59	1.23	1.649	2.198	2.54	2.54	3.24	1.445	1.774	1.571
4	0.5	.1613	1.5-2.0	2.23-1.35	1.945	2.91	1.44	1.919	2.264	2.93	2.35	2.73	1.448	1.690	1.905
5	0.5	.1359	2.0-2.5	1.35-.821	4.036	3.70	2.36	2.448	2.038	2.93	2.46	2.11	1.456	2.029	1.571
6	0.75	.1282	2.5-3.25	.821-.387	5.124	4.23	4.00	3.247	2.771	3.42	2.46	1.97	2.085	3.091	2.162
7	0.75	.0882	3.25-4.0	.387-.183	3.609	3.67	6.76	3.568	3.012	5.85	2.46	2.21	2.887	3.778	3.210
8	11.75		4-15.75	.183-1.44 ev	3.64	3.40	3.31	1.78	8.93	2.20	2.4	2.1	6.4	4.37	7.82

Group	$\xi\sigma$										
	O	Na	Mg	Al	Fe	Si	Ca	K	H	C	N
1	0.079	0.2039	0.153	0.6680	1.318	0.164	0.1722	0.1512	1.62	0.2342	0.2176
2	0.150	0.2149	0.159	0.6737	1.334	0.157	0.1820	0.1613	1.81	0.2806	0.2176
3	0.149	0.262	0.178	0.6076	1.124	0.182	0.1711	0.166	2.33	0.3000	0.2244
4	0.232	0.245	0.209	0.3931	0.777	0.209	0.1427	0.140	3.10	0.2821	0.2720
5	0.485	0.397	0.278	0.2624	0.413	0.209	0.1230	0.108	3.68	0.3952	0.2244
6	0.613	0.338	0.474	0.2406	0.0977	0.244	0.1230	0.101	5.85	0.5172	0.3087
7	0.430	0.300	0.679	0.2644	0.1062	0.419	0.1230	0.113	8.50	0.6320	0.4583
8	0.48	0.28	0.27	0.13	0.32	0.157	0.12	0.106	19.2	0.77	1.12

Table A-2. Few Group Cross Sections

Element	Number of epithermal groups	σ_{tr}			$\xi\sigma$			$\sigma_1 = \frac{\xi\sigma}{16.06}$
O	3	.676	2.583	3.64	.079	.3088	.48	0.00869
	2	1.806	3.64		.079	.48		
	1	3.191			.1396			
Na	3	2.259	3.105	3.40	.2039	.2851	.28	0.0214
	2	2.760	3.40		.2039	.28		
	1	4.642			.343			
Mg	3	1.057	2.353	3.31	.153	.287	.27	0.0167
	2	1.824	3.31		.153	.27		
	1	3.194			.2679			
Al	3	1.356	2.213	1.78	.668	.4431	.13	0.0676
	2	1.864	1.78		.668	.13		
	1	3.031			1.086			
Fe	3	1.904	2.344	8.93	1.318	.7471	.32	0.151
	2	2.165	8.93		1.318	.32		
	1	3.994			2.432			
Si	3	2.29	3.056	2.20	.164	.2184	.157	0.01613
	2	2.744	2.20		.164	.157		
	1	4.341			.259			
Ca	3	3.45	2.772	2.40	.1722	.1492	.12	0.0169
	2	3.048	2.40		.1722	.12		
	1	4.804			.2714			
K	3	2.949	2.677	2.1	.1512	.1372	.106	0.0147
	2	2.787	2.1		.1512	.106		
	1	4.364			.2368			

Table A-2. Few Group Cross Sections (cont'd)

Element	Number of epithermal groups	σ_{tr}			$\xi\sigma$			$\sigma_1 = \frac{\xi\sigma}{16.06}$
H	3	1.62	1.703	6.4	1.62	3.683	19.2	0.3161
	2	1.669	6.4		2.761	19.2		
	1	3.069			5.077			
C	3	1.005	2.158	4.37	.2342	.3712	.77	0.0262
	2	1.688	4.37		.2342	.77		
	1	3.038			.4216			
N	3	1.524	1.868	7.82	.2176	.2667	.77	.02509
	2	1.728	7.82		.2176	1.12		
	1	3.200			.403			
H ₂ O	3	3.92	5.989	16.44	3.32	7.675	38.9	0.641
	2	5.144	16.44		5.601	38.9		
	1	9.329			10.29			

From Table A-1, few group (I, II, III) epithermal cross sections were calculated and are tabulated in Table A-2. Group I ($\Delta u = 1$) is actually a first flight correction and uses the cross sections from group No. 1 in Table A-1. The cross sections for group II (5 Mev to 0.183 Mev, $\Delta u = 3.307$) are the flux weighted average of groups 2 through 7 in Table A-1. Group III (0.183 Mev to 1.44 ev., $\Delta u = 11.75$) uses the cross sections from group No. 8 in Table A-1.

These cross sections appear to give reasonable ages as seen in Section 9.2. When the three groups were averaged to one group for simplicity, the primary criteria was that the ages calculated using the one group cross sections approximate the ages obtained using all three groups. The values of σ_{tr} for groups I and II were combined using a flux weighted average. The values of $\xi\sigma$ for hydrogen were combined using a weighted average which resulted in the same age for water as a three group calculation. This was almost the same as a flux weighted average. Because of the relatively large effect of the first flight correction on age calculations, the first group (I) or the original three is particularly significant and it was found that age calculations using two groups will be much closer to the results using three groups if the group I values for $\xi\sigma$ continue as the values for the higher of the two groups.

The contribution to the age by group III of the original three groups is relatively small, say, 10 - 15%. The single group values were obtained basically by scaling up the cross sections to allow for this small increase. For each element a multiplier, C, was determined for the average cross for section groups I and II as follows:

$$\frac{4.307}{(\sum_1 \Sigma_{tr})_{I, II}} + \frac{11.75}{(\sum_1 \Sigma_{tr})_{III}} = \frac{16.06}{C(\sum_1 \Sigma_{tr})_{I, II}} \quad (A-4)$$

or

$$C = \frac{16.06}{4.307 + 11.75 \frac{(\Sigma_1 \Sigma_{tr})_{I, II}}{(\Sigma_1 \Sigma_{tr})_{III}}} \quad (A-5)$$

In the cross section ratio, the Σ_1 terms are almost completely determined by the hydrogen and were taken as such. The transport cross sections are affected by many elements but to estimate the significance of each, the ratio $(\Sigma_{tr})_{I, II} / (\Sigma_{tr})_{III}$ was actually evaluated using $(\sigma_{tr})_{I, II} / (\sigma_{tr})_{III}$. The results of this procedure are given in Table A-2 as single group ($\Delta u = 16.06$) cross sections.

APPENDIX B

TWO GROUP THEORY FOR A POINT SOURCE IN A HOMOGENEOUS INFINITE MEDIUM

For the group of fast neutrons, the diffusion equation is given (except at the source) by

$$D_1 \nabla^2 \phi_1 - \Sigma_1 \phi_1 = 0,$$

or

$$\nabla^2 \phi_1 - k_1^2 \phi_1 = 0, \quad (B-1)$$

where $\Sigma_1/D_1 = k_1^2$. Expressing $\nabla^2 \phi_1$ in spherical coordinates and taking the case of spherical symmetry gives the solution

$$\phi_1 = \frac{Ae^{-k_1 r}}{r} + \frac{Ce^{+k_1 r}}{r} \quad (B-2)$$

Since ϕ_1 must remain finite as r increases without limit, C must be zero, therefore ϕ_1 is given by

$$\phi_1 = \frac{Ae^{-k_1 r}}{r} \quad (B-3)$$

For the group of thermal neutrons, the diffusion equation is

$$D_2 \nabla^2 \phi_2 - \Sigma_a \phi_2 + \Sigma_1 \phi_1 = 0,$$

or

$$\nabla^2 \phi_2 - k_2^2 \phi_2 = -\frac{\Sigma_1}{D_2} \phi_1, \quad (B-4)$$

where $k_2^2 = \Sigma_a/D_2$. The homogeneous part of this equation, $\nabla^2 \phi_2 - k_2^2 \phi_2$,

has the solution

$$\phi_2 = \frac{De^{-k_2 r}}{r} + \frac{Ee^{k_2 r}}{r} \quad (B-5)$$

after expressing $\nabla^2 \phi_2$ in spherical coordinates and assuming spherical symmetry. ϕ_2 must also remain finite as r increases without limit so that $E = 0$. The solution to the homogeneous part is then

$$\phi_2 = \frac{De^{-k_2 r}}{r} \quad (B-6)$$

The particular solution will have the form $S\phi_1$ so that the general solution for the thermal flux can be written

$$\phi_2 = \frac{De^{-k_2 r}}{r} + S\phi_1 = \frac{De^{-k_2 r}}{r} + \frac{SAe^{-k_1 r}}{r} \quad (B-7)$$

If Eq. (B-7) is a solution of Eq. (B-4), S must take on some value which can be determined by substituting Eq. (B-7) into Eq. (B-4). This gives

$$\nabla^2 (S\phi_1) - k_2^2 (S\phi_1) = -\frac{\Sigma_1}{D_2} \phi_1 \quad (B-8)$$

Using $\nabla^2 \phi_1 = k_1^2 \phi_1$, cancelling the ϕ_1 terms, and solving for S gives

$$S = \frac{\Sigma_1}{D_2 (k_2^2 - k_1^2)} \quad (B-9)$$

The form of the flux equations thus far are

$$\phi_1 = \frac{Ae^{-k_1 r}}{r} \quad (B-10)$$

$$\phi_2 = \frac{De^{-k_2 r}}{r} + \frac{SAe^{-k_1 r}}{r}, \text{ where } S = \frac{\Sigma_1}{D_2 (k_2^2 - k_1^2)} \quad (B-11)$$

The two constants are determined from two boundary conditions:

1. There is no leakage therefore all neutrons are absorbed in the medium, 2. The net neutron current through a spherical surface surrounding the source must approach Q , the source strength, as the sphere shrinks around the source. The boundary conditions can be written as

$$\int_{\text{all space}} \Sigma_a \phi_2 dV = Q \quad (B-12)$$

$$\lim_{r \rightarrow 0} (4\pi r^2 J_1) = Q \quad (B-13)$$

Application of the first boundary condition gives the following equation

$$D = \frac{Qk_2^2}{4\pi \Sigma_a} - \frac{SAk_2^2}{k_1^2} \quad (B-14)$$

Using

$$J_1 = -D_1 \nabla \phi_1 = -D_1 \frac{d}{dr} \left[\frac{Ae^{-k_1 r}}{r} \right] \quad (B-15)$$

the second boundary condition gives

$$A = \frac{Q}{4\pi D_1} \quad (B-16)$$

Inserting these values into the flux equations results in

$$\phi_1 = \frac{Q}{4\pi D_1} \frac{e^{-k_1 r}}{r} \quad (B-17)$$

$$\phi_2 = \frac{Qk_1^2}{4\pi D_2(k_2^2 - k_1^2)} \left[\frac{e^{-k_1 r} - e^{-k_2 r}}{r} \right] \quad (B-18)$$

The thermal flux at the origin can be written as

$$\phi_2(0) = \frac{Q k_1^2}{4 \pi D_2 (k_2 + k_1)} = \frac{\Omega}{4 \pi D_2 \sqrt{\tau} \left(1 + \frac{\sqrt{\tau}}{L}\right)} \quad (\text{B-19})$$

APPENDIX C

TWO GROUP THEORY FOR A POINT SOURCE IN A SPHERICAL CAVITY

For a point source in a spherical cavity, the derivation is the same as in Appendix B up to the application of the boundary conditions. The source condition is now (for a unit source)

$$J_1(r_o) = \frac{S}{4\pi r_o^2} = \frac{1}{4\pi r_o^2} \quad (C-1)$$

Since there are no thermal neutron sources or absorbers within the cavity, the net thermal neutron current at $r = r_o$ must be zero, i.e.,

$$J_2(r_o) = 0 \quad (C-2)$$

The general solutions for ϕ_1 and ϕ_2 can be taken from Appendix B and are

$$\begin{aligned} \phi_1 &= \frac{Ae^{-k_1 r}}{r} \\ \phi_2 &= \frac{De^{-k_2 r}}{r} + \frac{SAe^{-k_1 r}}{r} \end{aligned} \quad (C-3)$$

where $S = \frac{\Sigma_1}{D_2(k_2^2 - k_1^2)}$

Applying the first boundary condition, Eq. (C-1) gives

$$J_1 \Big|_{r_o} = -D_1 \nabla \phi_1 \Big|_{r_o} = \frac{AD_1 k_1 r_o e^{-k_1 r_o} + AD_1 e^{-k_1 r_o}}{r_o^2} = \frac{1}{4\pi r_o^2} \quad (C-4)$$

Solving for A:

$$A = \frac{1}{4\pi D_1 e^{-k_1 r_o} (1 + k_1 r_o)} \quad (C-5)$$

The coefficient SA in ϕ_2 is now also determined as

$$SA = \frac{k_1^2}{4\pi D_2 (k_2^2 - k_1^2) (1 + k_1 r_o) e^{-k_1 r_o}} \quad (C-6)$$

where $k_1^2 = \Sigma_1/D_1$ has been used.

Applying the second boundary condition, Eq. (C-2) gives

$$J_2|_{r_o} = -D_2 \nabla \phi_2|_{r_o} = -D_2 \left[\frac{-k_2 r_o De^{-k_2 r_o} - De^{-k_2 r_o}}{r_o^2} - \frac{k_1 r_o SA e^{-k_1 r_o} + SA e^{-k_1 r_o}}{r_o^2} \right] = 0 \quad (C-7)$$

Solving for D and inserting the value of SAe^{-k₁r_o} from Eq. (C-6) gives

$$D = \frac{-(SA e^{-k_1 r_o}) (1 + k_1 r_o)}{e^{-k_2 r_o} (1 + k_2 r_o)} = - \frac{k_1^2}{4\pi D_2 (k_2^2 - k_1^2) (1 + k_2 r_o) e^{-k_2 r_o}} \quad (C-8)$$

Inserting the values for D and SA in Eq. (C-3) for the thermal flux and setting $r = r_o$ gives the thermal flux at the surface of the cavity as

$$\phi_2(r_o) = \frac{k_1^2}{4\pi D_2 (k_2 + k_1) (1 + k_1 r_o) (1 + k_2 r_o)} \quad (C-9)$$

By using $k_1 = 1/\sqrt{\tau}$, $k_2 = 1/L$ and $D_2 = \Sigma_a L^2$, this can be written in more conventional notation as

$$\phi_2(r_o) = \frac{1}{4\pi \Sigma_a L^2 \sqrt{\tau} (1 + \frac{\sqrt{\tau}}{L}) (1 + \frac{r_o}{\sqrt{\tau}}) (1 + \frac{r_o}{L})} \quad (C-10)$$

APPENDIX D

TWO GROUP THEORY FOR A POINT SOURCE IN A "GRAY" SPHERICAL CAVITY

If the spherical cavity is assumed to absorb part of the incident thermal neutrons, this can be included by modifying the boundary condition for the thermal neutron flux. An albedo, β , for the surface is defined in the usual way as

$$J_{\text{out}} = \beta J_{\text{in}} \quad (D-1)$$

The boundary condition on the net current is now

$$J_2(r_o) = J_{\text{out}} - J_{\text{in}} = J_{\text{in}} (\beta - 1) \quad (D-2)$$

The incident current is given by

$$J_{\text{in}} = \frac{\phi_2(r_o)}{4} + \frac{D_2}{2} \nabla \phi_2 \Big|_{r_o} \quad (D-3)$$

Applying the boundary condition, Eq. (D-2) gives

$$J_2(r_o) = -D_2 \nabla \phi_2 \Big|_{r_o} = \left(\frac{\phi_2(r_o)}{4} + \frac{D_2}{2} \nabla \phi_2 \Big|_{r_o} \right) (\beta - 1) \quad (D-4)$$

Upon rearrangement, this becomes

$$\frac{2(1 + \beta)}{(1 - \beta)} D_2 \nabla \phi_2 \Big|_{r_o} = \phi_2(r_o) \quad (D-5)$$

Inserting the values of $\nabla \phi_2 \Big|_{r_o}$ and $\phi_2(r_o)$ from Appendix C and solving for the coefficient D gives

$$D = \frac{SAe^{-k_1 r_o} \left(1 + k_1 r_o + \frac{r_o (1-\beta)}{2D_2 (1+\beta)} \right)}{e^{-k_2 r_o} \left(1 + k_2 r_o + \frac{r_o (1-\beta)}{2D_2 (1+\beta)} \right)} \quad (D-6)$$

The thermal flux at the surface of the gray cavity can now be obtained by substituting this expression for D and the expression for SA from Appendix C (Eq. C-6) into the thermal flux equation and setting $r = r_o$. After some manipulation this can be written

$$\phi_2(r_o) = \frac{k_1^2}{4\pi D_2 (k_1 + k_2) (1 + k_1 r_o) \left(1 + k_2 r_o + \frac{r_o (1+\beta)}{2D_2 (1+\beta)} \right)} \quad (D-7)$$

Eq. (D-7) can also be written as

$$\phi_2(r_o) = \frac{1}{4\pi \Sigma_a L^2 \sqrt{\tau} \left(1 + \frac{\sqrt{\tau}}{L} \right) \left(1 + \frac{r_o}{\sqrt{\tau}} \right) \left(1 + \frac{r_o}{L} + \frac{r_o (1-\beta)}{2D_2 (1+\beta)} \right)} \quad (D-8)$$

$$= \frac{1}{4\pi \Sigma_a L^2 \sqrt{\tau} \left(1 + \frac{\sqrt{\tau}}{L} \right) \left(1 + \frac{r_o}{\sqrt{\tau}} \right) \left(1 + \frac{r'_o}{L} \right)} \quad (D-9)$$

where

$$r'_o = r_o \left(1 + \frac{1-\beta}{2 \Sigma_a L (1+\beta)} \right) \quad (D-10)$$

The incident thermal neutron current, J_{in} , can be obtained by substituting the value of $D_2 \nabla \phi_2 \Big|_{r_o}$ from Eq. (D-5) into Eq. (D-3). The result is

$$J_{in} = \frac{\phi_2(r_o)}{2(1+\beta)} \quad (D-11)$$

The absorption rate in the gray surface is given by

$$(J_{\text{in}} - J_{\text{out}}) (4\pi r_o^2) = J_{\text{in}} (1 - \beta) 4\pi r_o^2 = \frac{\phi_2(r_o)(1 - \beta) 4\pi r_o^2}{2(1 + \beta)} \quad (\text{D-12})$$

APPENDIX E

TWO GROUP THEORY FOR A POINT SOURCE IN A BLACK SPHERE

The case of complete absorption of all thermal neutrons incident on the sphere can be treated by solving the thermal flux equation using

$$J_{\text{out}}(r_o) = 0 \quad (\text{E-1})$$

as a boundary condition or by merely setting $\beta = 0$ in the results of Appendix D. The result in either case is

$$\begin{aligned} \phi_2(r_o) &= \frac{k_1^2}{4\pi D_2(k_1 + k_2)(1 + k_1 r_o)(1 + k_2 r_o + \frac{r_o}{2D_2})} \\ &= \frac{1}{4\pi \Sigma_a L^2 \sqrt{\tau} (1 + \frac{\sqrt{\tau}}{L})(1 + \frac{r_o}{\sqrt{\tau}})(1 + \frac{r_o}{L} + \frac{r_o}{2\Sigma_a L^2})} \end{aligned} \quad (\text{E-2})$$

APPENDIX F

APPROXIMATE FORMS OF THE COUNT RATIO USING AGE-DIFFUSION THEORY

Equation 46 can be simplified by using an asymptotic expansion³⁵ for erf x given below

$$1 - \operatorname{erf} x = \frac{e^{-x^2}}{x \sqrt{\pi}} \left(1 - \frac{1}{(2x^2)} + \frac{1 \cdot 3}{(2x^2)^2} - \frac{1 \cdot 3 \cdot 5}{(2x^2)^3} + \dots \right) \quad (\text{F-1})$$

With $x = \sqrt{\tau}/L$, Eq. 46 becomes

$$\begin{aligned} \phi(o) &= \frac{1}{4\pi D_2} \frac{1}{\sqrt{\pi \tau}} \left[\frac{1}{(2x^2)} - \frac{1 \cdot 3}{(2x^2)^2} + \dots \right] \\ &= \frac{1}{8\pi^{3/2} \Sigma_a \tau^{3/2}} \left[1 - \frac{1 \cdot 3}{\left(\frac{2\tau}{L^2}\right)} + \frac{1 \cdot 3 \cdot 5}{\left(\frac{2\tau}{L^2}\right)^2} - \dots \right] \end{aligned}$$

Since $2\tau/L^2$ is not very large (about 14 for water), the series will begin to diverge after about 6 terms so that some care must be used. If only count ratios with respect to water are used, the errors will tend to cancel and the following approximations will result using one or two terms of the series:

$$R = \frac{\Sigma_a \tau^{3/2} \big|_{\text{H}_2\text{O}}}{\Sigma_a \tau^{3/2} \big|_{\text{medium}}} \quad (\text{F-2})$$

$$R = \frac{\Sigma_a \tau^{3/2} \Big|_{H_2O} \left(1 - \frac{3L^2}{2\tau}\right) \Big|_{\text{medium}}}{\Sigma_a \tau^{3/2} \Big|_{\text{medium}} \left(1 - \frac{3L^2}{2\tau}\right) \Big|_{H_2O}} \quad (F-3)$$

Equation 50 can be treated similarly. With $x = \sqrt{\tau}/L$, $y = \sqrt{\tau}/a$, Eq. 50 becomes

$$\begin{aligned} \phi(a) &= \frac{1}{4\pi(L^2 - a^2)\Sigma_a} \frac{1}{\sqrt{\pi\tau}} \left[-\frac{1}{(2y^2)} + \frac{1.3}{(2y^2)^2} - \dots + \frac{1}{(2x^2)} - \frac{1.3}{(2x^2)^2} + \dots \right] \\ &= \frac{1}{8\pi^{3/2}\Sigma_a\tau^{3/2}} \left[1 - \frac{3(L^2 + a^2)}{2\tau} + \dots \right] \quad (F-4) \end{aligned}$$

Again using only one or two terms of the series expansion, the first two approximate equations for R in a cavity geometry using age-diffusion theory are

$$R = \frac{\Sigma_a \tau^{3/2} \Big|_{H_2O}}{\Sigma_a \tau^{3/2} \Big|_{\text{medium}}} \quad (F-5)$$

$$R = \frac{\Sigma_a \tau^{3/2} \Big|_{H_2O} \left(1 - \frac{3(L^2 + a^2)}{2\tau}\right) \Big|_{\text{medium}}}{\Sigma_a \tau^{3/2} \Big|_{\text{medium}} \left(1 - \frac{3(L^2 + a^2)}{2\tau}\right) \Big|_{H_2O}} \quad (F-6)$$

The first of these is the same as for an infinite medium. In the second, the diffusion length is effectively increased by the cavity.

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