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ABSTRACT

Considerable success has been achieved this year in research aimed at extending the asymptotic threshold nonlinear theory to plasmas governed by the Vlasov equation and in a group of linear studies concerned with drift instabilities in sheared magnetic fields. The nonlinear results are general and have been applied to the bump-on-tail instability and to the collisionless drift and the dissipative trapped electron instability in unsheared fields. The linear studies have dealt with the effects of resonant electrons, finite collisionality, finite-beta, neo-classical modifications and ion gyro-motion on trapped electron instabilities in sheared magnetic fields.

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Introduction

The past year has been a fruitful one for this project. Considerable success has been achieved in research problems which were undertaken and the scope of the project has broadened.

Part of the nonlinear work was carried out by the principal investigator while on sabbatical at the Institute for Advanced Study (Sept. 1974 - Feb. 1975) and then at Oxford University (March - Aug. 1975). Previously, a number of key difficulties had been encountered in attempting to extend the method of asymptotic threshold analysis to the collisionless "Vlasov plasma" regime. These difficulties were clearly identified by considering what is perhaps the simplest collisionless instability, the bump-on-tail. In collaboration with M. N. Rosenbluth, all of the special problems which were encountered were resolved and an unambiguous (and new) answer for the saturation of the bump-on-tail instability was obtained. Two papers are in preparation. These same techniques allowed Mr. Leonard Gross, who was studying the nonlinear behavior of the collisionless drift instability, to clarify several points in this calculation as well, and obtain an unambiguous result. This too is being prepared for publication and an abstract has been submitted to the St. Petersburg meeting of the A.P.S. Continuing further application of these techniques, A. Simon at Oxford (and Culham) has calculated the nonlinear saturation of the trapped electron instability in simple mirror geometry. This work is still in progress.

Meanwhile, P. J. Catto has initiated a broad program of studies on linear instabilities in collaboration with plasma physicists at Oak Ridge and Princeton. An improved calculation of critical magnetic shear in an inhomogeneous plasma has been carried out in collaboration with M. N. Rosenbluth

and has been accepted for publication in Nuclear Fusion. Finite-beta and resonant electron effects on the trapped-electron instabilities have been considered by P. J. Catto, W. M. Tang, C. S. Liu, M. N. Rosenbluth (Princeton) and J. D. Callen (Oak Ridge). An article has been submitted to Nuclear Fusion and will also appear as a Princeton Plasma Physics Laboratory Report, MATT-1153. In collaboration with K. T. Tsang and J. D. Callen (Oak Ridge) and W. M. Tang (Princeton), Catto has studied the importance of distinguishing between perpendicular and parallel components of electron velocity when considering the finite collisionality of the various regions (banana, plateau, Pfirsch-Schlüter) of electron velocity space for trapped-electron instabilities. This work will be submitted to Physics of Fluids and will also appear as an Oak Ridge Report. Finally, a new ordering of the Fokker-Planck equation was devised which is valid in sheared magnetic fields for arbitrary ratio of the ion gyroradius to the poloidal wavelength. Finite-beta, drift and neoclassical effects are retained. An abstract by P. J. Catto and K. T. Tsang (Oak Ridge) has been submitted to the St. Petersburg meeting of the A.P.S.

Details of all of these activities are given below.

1. Single-Mode Nonlinear Threshold Saturation of the Bump-on-Tail Instability.

In order to extend the asymptotic nonlinear threshold saturation method⁽¹⁾ to plasma governed by the Vlasov Equation, we considered perhaps the simplest case, a single mode unstable on a small bump on the tail of a Maxwellian-like distribution. Three new difficulties were encountered which did not appear previously for plasmas governed by fluid equations. One is the necessity of dealing with Van Kampen modes⁽²⁾ in an asymptotic analysis, the second is the

occurrence of indeterminate equations in solving for harmonics created by beating of the linear mode with itself, and the third is the appearance of highly singular terms (products of generalized functions) in the saturation condition. It was shown that one must use the Case type 1-c mode only⁽³⁾, and that the indeterminacies are resolved by considering the approach to the steadily oscillating saturated state. Finally, the highly singular terms are shown to cancel implicitly and indeed explicitly in the final expression for the saturation condition. All remaining singularities are identified as the usual generalized functions, with finite and known integrals. A unique expression was obtained for the saturation amplitude (totally unlike quasilinear theory) and the frequency shift. This solution is shown to be stable and to conserve energy and momentum. Two papers are in preparation by M. N. Rosenbluth and A. Simon. One considers immobile ions, while the other deals with the case when both species are allowed to respond. A numerical check of the results, by computer simulation, is under development.

2. Nonlinear Saturation of the Collisionless Drift Instability

The technique developed above seems to be sufficient to handle similar problems in much more complicated geometries. This has proven to be so in the collisionless drift instability in a uniform magnetic field. Although there is now a magnetic field, a nonuniform density distribution in equilibrium, finite boundaries in the radial and z-direction, and considerably more complex Van Kampen modes, nevertheless all indeterminacies which appear are analogous to those which were found in the bump-on-tail problem and can be resolved in like manner. Similarly, the highly singular terms can be forced

to cancel by appropriate free-parameter choice and the resulting saturation condition displayed in finite integral form. An unambiguous answer was obtained for the mode saturation amplitude and frequency shift. The mechanism of nonlinear saturation appears to be heating of the ions and subsequent increased ion Landau damping. A numerical solution for the saturation amplitude was obtained for a particular choice of parameters. This solution itself was shown to be unstable to small perturbations. Thus, in this case, explosive growth would occur at the stability threshold. Some details are given in an abstract submitted to the St. Petersburg meeting of the A.P.S. and attached to this report. A more complete paper is in preparation by A. Simon and L. Gross. This work also forms part of a Ph.D. thesis by L. Gross.

3. Nonlinear Saturation of the Dissipative Trapped-Electron Instability (Mirror Geometry).

In this work, which is still in progress, the technique described above was applied to the dissipative trapped-electron instability in mirror geometry; such as in the Columbia University experiments of Prager, Marshall and Sen⁽⁴⁾. While the trapped electrons are collisional, the circulating electrons and ions are collisionless and it is in their response that one encounters the Van Kampen modes, indeterminacies and highly singular functions similar to those of the bump-on-tail calculation. The same technique appears to resolve all these difficulties and predict a unique saturation limit. The details of the saturation mechanism are under study.

4. An Improved Calculation of Critical Magnetic Shear in an Inhomogeneous Plasma

For drift instabilities localized about isolated rational surfaces in sheared magnetic fields, resonant and non-resonant electron effects are treated via a perturbation theory which neglects resonant effects to lowest order and retains only the low phase velocity limit of the non-resonant electron terms. These inconsistencies are removed and a self-consistent solution and stability condition obtained by the present treatment. These improvements are accomplished by solving the appropriate differential equation in two overlapping regions. The resulting stability criterion and the perturbative technique result are shown to agree to within a multiplicative factor very nearly equal to one for the density and shear lengths of interest for Tokamaks. This calculation has taken on additional significance because of recent work [see item 5 below] which has shown the importance of carefully retaining resonant electron corrections to trapped electron instabilities. The work was completed by P. J. Catto at the University of Rochester in collaboration with M. N. Rosenbluth at the Institute for Advanced Study. This research, which was nearly complete at the time of our last annual report, has been accepted for publication in Nuclear Fusion⁽⁵⁾ and is available as Report No. PPG-4 of the Plasma Physics and Nuclear Energy Group of the College of Engineering and Applied Science, January 1975. The work was reported on at the Albuquerque meeting in November of 1974⁽⁶⁾.

5. Finite- β and Resonant Electron Effects on Trapped-Electron Instabilities

Finite β and resonant electron corrections to the temperature gradient

and finite ion gyroradius driven dissipative trapped electron instabilities are evaluated. The resonant electrons introduce two new and important modifications to existing trapped electron results. The resonant electrons (I) can have a strong stabilizing influence on the electron temperature gradient driven term, and (II) result in a further destabilizing of the ion gyroradius driven term. Prior to this calculation the contributions of the resonant electrons were believed to unimportant for trapped electron instabilities. This work has shown that in a sheared magnetic field the resonant electron corrections are not smaller than the slab results (no trapped electrons) by the square of the wave over the electron bounce frequencies, as previously believed. In addition, there are three principal results due to the finite- β corrections. (I) A strong stabilizing modification of the ion gyroradius coupling to the shear Alfvén corrections. (II) No additional temperature gradient terms for β finite. (III) Stabilization of finite ion gyroradius driven terms for very small poloidal wavenumbers. This work was carried out by P. J. Catto in collaboration with W. M. Tang, C. S. Liu, M. N. Rosenbluth (Princeton) and J. D. Callen (Oak Ridge). The research has been submitted to Nuclear Fusion⁽⁷⁾ and will appear as Princeton Plasma Physics Laboratory Report⁽⁸⁾.

6. Resonant Electron Effects on Trapped Electron Instabilities

The importance of retaining resonant electron contributions when considering trapped electron instabilities has previously been noted [see item 5]. In this paper the importance of distinguishing between perpendicular and parallel components (or energy and magnetic moment) in electron velocity space, rather than simply considering energy, is discussed. A simple model is

presented which indicates how the various regions of finite collisionality (banana, plateau, and Pfirsch-Schlüter) in velocity space depend on both the perpendicular and parallel components of electron velocity. In addition, for much of the banana regime the resonant electron contributions are shown to be due to the circulating electrons with parallel phase velocities equal to and slightly below the electron thermal speed. This work has just been completed and was carried out by P. J. Catto in collaboration with K. T. Tsang and J. D. Callen (Oak Ridge) and W. M. Tang (Princeton). The work will appear as an Oak Ridge report and will be submitted to Physics of Fluids⁽⁹⁾.

7. Linearized Gyro-Kinetic Equation

Familiar drift kinetic orderings of the Fokker-Planck equation result in finite gyroradius terms only in higher order, and these corrections are only obtained in the limit in which the perpendicular wavelengths are large compared to the gyroradius. In order to investigate finite ion gyroradius effects for arbitrary perpendicular wavelengths, an ordering which retains the dominant gyroradius terms in the lowest order equation is required. In this work such a description is developed which also retains the minor-radial dependence required in treating problems in sheared magnetic fields, without resorting to a WKB technique⁽¹⁰⁾. Furthermore drift and neoclassical modifications are retained and a full electromagnetic description employed. The minor-radial dependence must be carefully treated in order to obtain the proper behavior when the poloidal wavenumber times the gyroradius is large ($k_{\perp}\rho \gg 1$). Finite- β effects are retained because previous work has only considered $k_{\perp}\rho \ll 1$. In addition, for large $k_{\perp}\rho$ a pitch angle scattering

form for the collision operator is no longer valid. As a result, a new collision operator valid for $k_{\rho} \gg 1$ is derived. This work is still in progress and is being performed by P. J. Catto in collaboration with K. T. Tsang (Oak Ridge). An abstract by Catto and Tsang has been submitted to the St. Petersburg meeting and is attached to this report.

8. Personnel:

FACULTY --

(a) A. Simon -- The principal investigator was on sabbatical leave during the 1974-1975 academic year at the Institute for Advanced Study, Princeton (Sept. 1974 - Feb. 1975) and Oxford University, Oxford, England (March - Aug. 1975), and continued to work on problems related to plasma instability. He supervised one graduate student during this time.

(b) Peter J. Catto -- Dr. Catto has devoted 20% of his time to this project during the academic year, and full time for two summer months. He is supervising one graduate student.

GRADUATE RESEARCH ASSISTANTS --

(a) Leonard S. Gross (MAS Dept.) -- Leonard S. Gross worked on a problem in nonlinear theory of the collisionless drift instability during this academic year under the direction of Dr. Simon. He has completed all degree requirements and passed his final thesis defense on Sept. 12, 1975 and has accepted employment at General Atomic, San Diego, Calif.

(b) Tom Speziale (MAS Dept.) -- Mr. Speziale has begun thesis research under the direction of Dr. Catto. His thesis topic is of interest to

both laser fusion research and magnetic confinement studies and is being supported at present by the Laboratory for Laser Energetics, Univ. of Rochester.

(c) Stephano Migliuolo (Physics Dept.) -- Mr. Migliuolo has just begun his thesis under Dr. Simon's direction.

Other Federal Support -- None of the personnel working on this contract have other federal support. The university recovers 10% of the academic salary of both Profs. Simon and Catto from the Laboratory for Laser Energetics.

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4. S. C. Prager, T. C. Marshall and A. K. Sen, "Dissipative Trapped Electron Instability in a Linear Machine", Phys. Rev. Lett. 33, 692 (1974).
5. M. N. Rosenbluth and P. J. Catto, "An Improved Calculation of Critical Magnetic Shear in an Inhomogeneous Plasma", Nuc. Fusion 15, (1975).
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7. W. M. Tang et al, "Finite-beta and Resonant Electron Effects on Trapped Electron Instabilities", submitted to Nuc. Fusion.
8. W. M. Tang et al, Princeton Plasma Physics Laboratory Report, MATT-1153, to be issued.
9. P. J. Catto et al, "Resonant Electron Effects on Trapped Electron Instabilities", to be submitted to Phys. Fluids.
10. P. H. Rutherford and E. A. Frieman, "Drift Instabilities in General Magnetic Field Configurations", Phys. Fluids 11, 569 (1968).

PREVIOUS PUBLICATIONS UNDER THIS CONTRACT

1. A. Simon, "Nonlinear Transport in Weakly Ionized Plasma", Phys. Fluids 11, 1181 (1968), UR-3942-1.
2. A. Simon, "Convection of Weakly Ionized Plasma in a Non-uniform Magnetic Field", Phys. Fluids 11, 1186 (1968), UR-3942-2.
3. J. J. Kim and A. Simon, "Nonlinear Theory of the ExB Instability", Phys. Fluids 12, 895 (1969), UR-3942-3.
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8. V. G. Abhyankar and A. Simon, "Linear Threshold of the Buneman-Farley Instability", Phys. Fluids 17, 261 (1974), C00-3497-3.
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11. A. Simon, L. Gross and P. J. Catto, "Shear Stability Boundaries for Collisionless Drift Modes in Finite Geometry", Phys. Fluids 18, 231 (1975), C00-3497-6.

THESES COMPLETED UNDER THIS CONTRACT

1. John J. Kim, "The Second Order Shifts in Frequency and Amplitude in the Non-Linear Theory of ExB Instability", Physics Dept., 1969.
2. Jeng-Nan Shiau, "Small Amplitude Nonlinear Theory of the Positive Column Spiral Instability", MAS Dept., 1969.
3. Arthur M. Sleeper, "The Drift Dissipative Instability", Physics Dept., 1970.
4. Cheng-I Weng, "Nonlinear Theory of the Mirror Flute Instability", MAS Dept., 1972.
5. Donald A. Monticello, "Nonlinear Theory of the Collisional Drift Wave Instability", Phys. Dept., 1973.
6. Vishwas Govind Abhyankar, "Density and Temperature Gradient Flute-Mode Instabilities", MAS Dept., 1974.
7. Leonard Gross, "Nonlinear Theory of the Collisionless Drift Instability", Applied Physics Program, 1975.

List of Publications

<u>Title</u>	<u>Authors</u>	<u>Journal/Status</u>	<u>Percentage AEC Support</u>
Shear Stability Boundaries for Collisionless Drift Modes in Finite Geometry	A. Simon, L. Gross and P. J. Catto	Physics of Fluids <u>18</u> , 231 (1975)	100%
Single-Mode Saturation of the Bump-on-Tail Instability, Part I and Part II	M. N. Rosenbluth and A. Simon	to be submitted for publication	75%
An Improved Calculation of Critical Magnetic Shear in an Inhomogeneous Plasma	M. N. Rosenbluth and P. J. Catto	Nuclear Fusion (accepted for publication) To appear in 1975	100%
Finite-Beta and Resonant Electron Effects on Trapped Electron Instabilities	W. M. Tang, C. S. Liu, M. N. Rosenbluth, P. J. Catto and J. D. Callen	submitted to Nuclear Fusion	100%

Table I