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PHYSICAL (HYDRAULIC) MODELING OF HEAT DISPERSION IN LARGE LAKES:

A Review of the State of the Art

by

Edward Silberman and Heinz Stefan

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PHYSICAL (HYDRAULIC) MODELING OF HEAT DISPERSION IN LARGE LAKES: A Review of the State of the Art

by

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PREFACE

The St. Anthony Falls Hydraulic Laboratory has undertaken to collaborate with the Argonne National Laboratory in its survey of the state of the art of heat dispersion from steam generating plants into large lakes. The phase of the work to be covered by the St. Anthony Falls Hydraulic Laboratory relates to the prospects and problems associated with physical or hydraulic modeling of thermal plumes and heat dispersion. The study is supported under Contract No. 31-109-38-2404 with Argonne National Laboratory, which became effective on February 16, 1970.

This part of the work has been subdivided into several topics which can be classified broadly as

1. Physical concepts of thermal plumes derived from fundamental experimental studies and criteria for hydraulic modeling of dispersion;
2. Adaptability of hydraulic models to providing parameters for and checking of mathematical models; and
3. Case histories of hydraulic models applied to predicting thermal plumes at actual power plants.

This report treats the above topics in some detail, although not in the order cited. It is based both on the experience of the authors in their own research and on their contacts with others in the field through the published and unpublished literature and through direct interviews, mostly by long-distance telephone.

Recently several reviews and summaries on thermal pollution larger in scope or emphasizing different aspects than the present one have been published. They are:

Engineering Aspects of Thermal Pollution, Proc. of the National Symposium on Thermal Pollution, 1968, ed. by F.L. Parker and P.A. Krenkel, Vanderbilt University Press, Nashville, 1969.

Bibliography on Thermal Pollution by the Committee on Thermal Pollution, Proc. ASCE, San. Eng. Div., June 1967.

Biological Aspects of Thermal Pollution, Proc. of the National Symposium on Thermal Pollution, 1968, ed. by F.L. Parker and P.A. Krenkel, Vanderbilt University Press, Nashville, 1969.

Parker, F.L. and Krenkel, P.A., Thermal Pollution: Status of the Art, Report No. 3, Dept. of Environmental and Water Resources Engineering, Vanderbilt University, Nashville, 1969.

I. PRINCIPLES OF THERMAL PLUME MODELING

A. Hydraulic Modeling

Hydraulic modeling of engineering works is a long-established practice which serves several purposes. These include the following:

1. Qualitative visualization of the behavior of the prototype is readily obtained. Gross omissions in design of the work and over-refinement of details become apparent in the model, where changes can be made at a small fraction of the cost of corresponding changes in the prototype.

2. Quantitative estimates of some of the flow quantities, such as velocities, depths, and pressures, can be obtained for use in designing the prototype. If these estimates are carried on simultaneously with the design process, the feedback between model and calculation can produce a nearly optimum design. (Not all parameters lend themselves equally well to quantitative evaluation in a model, however.)

3. Observation in the model leads to better understanding of the physical phenomena influencing the behavior of the prototype.

4. Visualization in a model leads to better understanding of a proposed project by laymen.

A single model of a given engineering work may not serve all these purposes simultaneously. Sometimes two or more models are used to emphasize different aspects of a project and different purposes, usually at different scales.

The dispersion of heated effluent from a steam generating plant is an engineering work in the sense used above, and it seemed natural to consider studying the phenomena involved using hydraulic models. In fact, such modeling has been under way since the 1950's. The modeling of thermal plumes posed several new problems in hydraulic modeling, and some old ones had to be reconsidered as well. One of the latter was the problem of distortion of horizontal relative to vertical scales. Some new ones involved the effect of stratification on model-prototype relations and the modeling of the air-water interface. These and other problems were recognized when model studies were undertaken, and they have been solved to a certain extent.

Given the present state of knowledge, it can be stated that hydraulic models of thermal plumes are capable of serving the same broad purposes as outlined above for models of other engineering projects. Unfortunately, experience in developing models of thermal plumes has been much more limited, and not all the rules for modeling have been clearly established. For example, thermal plumes involve a near field (adjacent to the outlet) and a far field, and how these should be joined in a model is still open to some question.

Just as in the case of models of other hydraulic phenomena, observation, visualization, and comprehension are facilitated by the small scale of the model as compared to the prototype. The time scale, being related to the geometrical scale, is compressed, and the effects of changes in meteorological or other input conditions can be recognized easily in the model, though they are difficult to identify in the prototype. Furthermore, changes in these input conditions can be applied at will in the model, whereas days, weeks, or months of waiting may be required for experiencing a given set of input conditions in the prototype. Also, the cost of collecting data in a model is modest compared to the cost of obtaining field data. As for other engineering works, different models can be used for different aspects of a thermal plume. For example, the outlet, whether on the surface or submerged, can be modeled without distortion, whereas the spread of the plume over a lake surface requires a model of much larger surface area which normally has to be heavily distorted.

It might be noted that many of the advantages claimed for a hydraulic model of thermal plume dispersion could also be claimed for a mathematical model augmented by a suitable three-dimensional display device. In fact, the physical geometry is usually simpler in the thermal plume case, especially over a lake surface, than it is for most other kinds of engineering problems, and this tends to facilitate mathematical modeling. The fact remains, however, that mathematical modeling yields little physical insight; also, there are certain fundamental physical parameters which must be obtained either from field measurements or from a physical model representation of the prototype phenomena if mathematical models are to be used. These parameters include mixing coefficients both at the outlet and between layers, interfacial friction between stratified layers, wind stress on the water surface, and other factors related to speed of advance and spreading of a thermal plume. They may vary not only with site geometry, but also from point to point in a given realization. A physical model offers great advantages in both the obtaining of these parameters for specific sites and the understanding of their fundamental physical significance, as already outlined. Once a physical model has been constructed, it can work in harmony with a mathematical model through feedback to predict thermal plume behavior much as more conventional engineering design problems are solved through the use of hydraulic models.

Meaningful hydraulic modeling requires that relations be established between model and prototype. This is necessary even if the model is being used only for visualization, although the exact quantitative relationships may not need to be so well defined in the visualization case. It is generally impossible to reproduce exactly in a single model all the phenomena which occur in a given prototype, simply because of the great difference in scale. It is therefore necessary to determine which quantities need to be faithfully modeled in a given situation and which can be neglected. Consideration needs also to be given to whether a single model or several different models should be used. These decisions can best be made using dimensionless ratios wherein lengths, velocities, temperatures, and other parameters are measured with respect to fixed reference quantities. These ratios and the relationships between them can be obtained in various ways--from the equations of motion in exact or approximate form, from ad hoc experiments, by testing intuitive hypotheses, or by combinations of these. The methods of obtaining these ratios will not be discussed in detail herein, but the remainder of this chapter will be devoted to a discussion of the important dimensionless numbers and their application to hydraulic modeling of thermal plumes.

B. Dimensionless Numbers

The study of the physical behavior of heated water discharged into a lake involves both hydrodynamic and thermodynamic considerations. There is frequently significant feedback from one to the other. A list of the many variables which must be considered in general is given in Ref. [1].* A modified and supplemented version of this list is reproduced in Table I-B-1.

*Numbers in brackets refer to List of References at end of report.

TABLE I-B-1 - Parameters Influencing Thermal Plumes

Discharge

I. Effluent characteristics

a. Flow rate

b. Density difference (relative to reference density in receiving water)

c. Velocity at outlet

II. Outlet characteristics

a. Location

b. Orientation

c. Submergence

d. Shape

e. Size (depth, width)

Receiving Water

I. Flow dynamics

a. Pre-existing velocity field (magnitudes and directions of local velocities)

b. Tidal currents

c. Wind-induced and other currents

d. Surface waves

e. Free turbulence

II. Stratification

a. Pre-existing stratification due to temperature, solids, and solvents

b. Wind and tidal effects on stratification

III. Geometrical characteristics

a. Shape

b. Size (widths and depths)

c. Bottom configuration and roughness near outlet

Atmosphere

I. Wind

a. Velocities (magnitude and direction)

b. Shear stresses at water surface

II. Air

a. Temperature

b. Relative humidity

III. Solar radiation

To facilitate determination of modeling criteria the variables are grouped into dimensionless numbers. The resultant set of dimensionless numbers must describe (a) the geometry, (b) the kinematics, (c) the dynamics, and (d) the thermodynamics of the plume and of the receiving water. The relationship between flow and heat transfer in the water and in the air must also be considered.

The flow geometry can be expressed by a humber of length, depth, and width ratios--e.g., the relative width of the outlet channel, the relative distance along a jet axis, and the relative depth of the impoundment using the outlet channel depth d_o as the reference length. One or more angles may be required to define the orientation of the outlet. Other possible geometrical ratios include relative roughness of the outlet channel and relative thickness of the air boundary layer for wind. There are usually a great many more of these geometrical parameters.

Kinematical ratios involve flow patterns and ratios of current velocities, exit velocities, and wind velocities relative to a reference velocity which is usually the plume velocity at the outlet, U_o. Kinematical considerations include local flow directions as well as magnitudes. It should be recognized that the form of the velocity distributions is important as well and requires identical representation in model and prototype.

The hydrodynamics of the flow are represented by ratios measuring the relative importance of the various forces. For example, the ratio of inertial to gravitational forces is given by a Froude number, $Fr_o = U_o/\sqrt{gd_o}$; of inertial to buoyancy forces by a densimetric Froude number $Fr_o' = U_o\Big/\sqrt{\dfrac{\Delta\rho_o}{\rho_o}\, gd_o}$; and of inertial to viscous forces by a Reynolds number, $Re_o = U_o d_o/\nu_o$. Herein the subscript "o" refers to reference conditions, which in the case of density ρ_o and viscosity ν_o are usually taken at lake ambient temperature T_o. Also important, and related to the densimetric Froude number, is the local Richardson number, the ratio of buoyant to inertial force gradient, $Ri = -\dfrac{g}{\rho}\dfrac{d\rho/dz}{(du/dz)^2}$, where z is the vertical coordinate. Forces due to the Earth's rotation can become important when large areas of the Earth's surface are being modeled. The ratio of rotational to inertial forces is given by a Rossby number, $Ro_o = \frac{2\Omega L_o}{U_o}$, where Ω_o measures the angular velocity of the Earth at the given latitude and L_o is a reference length comparable to the entire length of the heat dispersion area. Rotational forces can generally be ignored in a thermal plume model unless the Rossby number is considerably greater than unity.

The thermodynamic factors can be analyzed by lumping their effects into an equilibrium temperature T_E of the water. The equilibrium temperature is that water temperature at which the net heat flux through the water surface is zero; its value depends on atmospheric conditions. The difference between T_E and the lake surface temperature at any point T_ℓ, $(T_\ell - T_E)/T_o$, is then a measure of the removable portion of the excess temperature at any place in a plume, and the consequent heat transfer rate per unit area is measured by a coefficient of surface heat transfer, K_s. It is useful to display the surface heat exchange coefficient in dimensionless form as a Nusselt number, $K_s d_o/k$, where k is the thermal conductivity of the water. The relative density difference $\Delta\rho/\rho_o$, which is a reference for the buoyancy forces, measures the stratification; it is determined by the temperature ratio $(T_\ell - T_o)/T_o$ (although it may also be influenced by differences in salinity, in suspended load, or in other matter present). It is also useful to introduce the Prandtl number, Pr_o, the ratio of the viscosity coefficient to the product of thermal conductivity and specific heat of the water. Within a turbulent water body the horizontal and vertical coefficients of eddy viscosity, ν_v/ν_o and ν_h/ν_o, and of turbulent heat diffusion or mixing, D_h/ν_o and D_v/ν_o, respectively, are functions of the hydrodynamical ratios and of $\Delta\rho/\rho_o$; the ratio of the former to the latter forms a turbulent Prandtl number Pr_t.

Another important ratio which also depends on the previously listed dimensionless numbers is the entrainment coefficient. It is useful in measuring the cooling due to mixing with ambient waters as opposed to cooling to the atmosphere; it can be defined as $E = \frac{d(Q/Q_o)}{d(s/d_o)}$, the dimensionless rate of increase of discharge, Q, with distance due to entrainment of adjacent fluid.

C. Basic Modeling Requirements for Thermal Plumes

Exact modeling would require identical values for all dimensionless numbers in model and prototype, and in practice it is virtually impossible to achieve this. Only a selected number of ratios can be matched, Which numbers are selected for matching depends on the problem being modeled. When the ratios that are deemed important have been equated in model and prototype, similarity is said to have been attained.

In designing a hydraulic model, only overall dimensionless numbers can be specified in advance. These include the given geometrical ratios and the Froude number, densimetric Froude number, Reynolds number, Rossby number, Prandtl number, and overall removable heat $(T_{\ell o} - T_E)/T_o$. The other numbers, both overall and local, are consequences of the choices that have been made and must be determined in the model. Ackers [2, Ch. 6] distinguishes between the effects of jet diffusion, buoyant spread, convective spread, mass transport by ambient currents, ambient turbulent mixing, and surface cooling and gives scaling requirements for each of these. Unfortunately, the scaling requirements for these phenomena are not all compatible for a given plume; relationships among the various scaling requirements are discussed in Ref. [2]. For example, if the same density difference at the outfall is used in model and in prototype, it is possible to model jet diffusion and buoyant plume behavior near the outlet in a geometrically similar model of large size, and mass transport by ambient currents can also be modeled correctly. However, other properties will not be scaled correctly, and only if these other properties are unimportant in the given problem can it be said that the model is adequate.

It is generally agreed that buoyancy is one dominant aspect of the flow in modeling thermal plumes in lakes, and thus the attempt must be made to match densimetric Froude numbers. Near an outlet in a flowing river or tidal estuary, gravity is very important in controlling mixing, and the Froude numbers have to be matched. Geometrical similarity is also required in all modeling. However, distortion may be necessary because the horizontal extent of a plume is large compared to its depth, and if the depth were correctly modeled, surface tension and contaminants on the surface would completely destroy kinematical similarity; it would also be difficult to measure depths correctly.

Reynolds numbers are rarely matched, but this is generally not considered a serious handicap as long as the Reynolds number in the model is sufficiently large that the flow is turbulent. If the flow is turbulent in the prototype, as it usually is, and laminar in the model, shear on the solid boundaries will play too important a role in the model and mixing may be inhibited. Once the flow becomes turbulent, eddy viscosities and mixing coefficients are usually considered to be sufficiently defined by the dynamics at the boundaries of the flow, but the adequacy of this assumption is one of the unsolved problems in thermal plume modeling. Thus, another reason for distorting the model is this requirement that flow in the model be turbulent. In a model whose horizontal scale is fixed by other considerations, turbulence can usually be obtained only by artificially increasing the depth in the model. Barr [3] and others have proposed methods for calculating the required distortion, but this is another problem requiring further study.

Still another reason for distortion is the relative magnitude of the rate of heat loss at the water surface compared to the heat content of the plume. In models which do not control the atmosphere above the water surface, the heat loss may be too large in a geometrically scaled model; an increase in depth will adjust the relative heat loss rate to a more accurate value.

Model distortion, though a violation of the requirement for geometrical similarity, thus often facilitates thermodynamic and hydrodynamic modeling and has become rather common practice. All the models discussed in Chapter III resorted to geometrical distortion; this involved increasing the depth by factors in the range of 5 to 16, the larger factors accompanying the smaller horizontal scales.

The Richardson number is a direct measure of the stability of a density stratified flow. If it is of the order of close to unity or larger, large-scale mixing across an interface is inhibited. In turbulent flow small or negative values are associated with considerable mixing and entrainment. Hence, heat spread in a model will represent that in a prototype only if the Richardson number at various points in the model is near unity when prototype values are near unity or larger, and vice versa.

Thermodynamic parameters have rarely been modeled accurately, although they have been given some attention in many of the models to be discussed in

Chapter III. Attempts at accurate modeling including atmospheric effects are described in Refs. [4] and [5]. Modeling of these parameters appears to be more important for the far field than for the near field, because mixing is probably the principal mode of cooling in the latter case. In any event, it is probably not necessary to model all atmospheric parameters as such as long as the equilibrium temperature and the corresponding surface heat transfer coefficients (stratification and removable excess heat, as defined previously) are modeled.

In addition to cooling effects which may be included in equilibrium temperature, wind also causes a surface shear stress which produces currents in the receiving body of water and curvature of at least the upper part of the main trajectory of the plume. Experimental field studies of wind effects on the thermal plume have been carried out at existing thermal plant sites in Holland [6] and in Lake Ontario [7]; they show the importance of wind in plume migration. However, little fundamental work has been done on modeling wind-produced currents. Attempts to simultaneously model both the cooling and the wind drift effects are usually doomed to failure, because it is generally impossible to produce sufficient wind stress without producing too low an equilibrium temperature over the model.

D. Visual Aids

It might be pointed out that a number of films are available which can be useful in familiarizing completely inexperienced persons with some physical aspects of thermal plumes. On some occasions experiments have been undertaken for no other purpose than to show dynamical aspects of physical mechanisms. Unfortunately, laboratory film clips are seldom edited or available outside a specific laboratory. However, the following motion pictures dealing with stratified flows and buoyant outfalls have been located:

1. "Stratified Flow," by R. R. Long, ESI Educational Fluid Mechanics film.
2. "Experiments on Turbidity Currents," by G. V. Middleton, 16 mm color film, approximately 700 ft, 20 min, filmed at the W. M. Keck Laboratory of Hydraulics and Water Resources, California Institute of Technology, available from the California Institute of Technology Bookstore, Pasadena, California.

3. "Warm Water Flow into Impoundments," by H. Stefan, 16 mm color sound film, approximately 550 ft, 16 min, filmed at the St. Anthony Falls Hydraulic Laboratory, University of Minnesota.

The second of these films shows the head of a turbidity current (or density current) formed by sudden release of sediment-laden water into a still reservoir in the laboratory. A variety of conditions are demonstrated in the film.

In the third film the behavior of an interface between heated and cold water, made visible by dye, is shown. The interface was photographed near the outlet and also at some distance downstream. Surface spreading patterns (timelines) of a thermal plume in a wide tank are also shown. Some effects of turbulence and buoyancy on the shape and smoothness of the contours of a thermal plume are visible.

Some vertical motion pictures were also made during a study reported in Ref. [8]. Three types of flow were observed:

> For low initial jet velocities the flow appeared to be either laminar flow or simply a spreading of the warm water over the surface. For higher jet velocities a meandering occurred which was of appreciable magnitude. For still higher velocities the meandering became much smaller and turbulent mixing occurred, with relatively large eddies swirling throughout the mixing jet. Large eddies formed at the edges of the mixing jet, entrapping water from the surrounding receiving water.

Another series of film clips was produced at the Engineering Experiment Station, Oregon State University, Corvallis, Oregon, by L. S. Slotta and others during a study on stratified reservoir currents. These clips show the evolution of originally vertical timelines through a reservoir, represented by a tilted glass flume, which is stratified throughout its depth as the degree of salt concentration in the water is varied. The flow pattern is caused by surface discharge of fresh water from a channel at one end of the reservoir and simultaneous selective withdrawal from various layers at the other end.

II. FUNDAMENTAL LABORATORY STUDIES

A. Types of Problems

A thermal plume discharging into a body of water is influenced by the many factors listed in Table I-B-1. As was pointed out in Chapter I, not all of these factors are equally important in every case, and even those that are of considerable importance cannot all be studied in the laboratory in the same model at the same time because of the large geographical area that a plume covers. To facilitate laboratory study of thermal plumes it is convenient to consider a plume as consisting of three sequential elements.

The first of these is the near field or outlet region, the region surrounding the point of emission of the plume. The emitted flow can be generally classified into submerged buoyant jets and semi-submerged or surface buoyant jets. The former are typically represented by a submerged pipe outlet and the latter by a surface canal. The emitted flow could be further classified by other geometrical properties, such as angle of discharge, but it is not necessary to do so at this time. The advantage of separating the near field or outlet region from the remainder of the plume in a laboratory model is that this region can then be modeled on a reasonably large and undistorted scale which facilitates measurements and observations of those factors which are most important at the outlet. In fact, several such model studies have been made. The disadvantage is that the feedback from the remainder of the plume is lost and has to be represented by artificial boundary conditions. Sometimes, such as when considerable mixing occurs at the outlet, the required boundary conditions are obvious; but under other conditions several possibilities are open, and it is not easy to choose the correct one. Some of the completed studies have not recognized this problem. Several different boundary conditions may have to be used in a given outlet study to represent a whole range of far field conditions produced by wind, currents, and lake thermodynamics.

Regardless of the geometry of the near field problem, the plume eventually becomes nearly horizontal and spreads in such a fashion that the influence of the near field has completely disappeared--it makes no difference, for example, whether the jet was initially submerged or semi-submerged. This region can be called the far field. The important considerations here are rate of cooling to the atmosphere, convection by mean flow, tides, wind currents and other currents, and boundary configuration of the water body. The far field is much larger than the near field and in laboratory studies generally has to be represented on a distorted scale. Therefore, it cannot be modeled according to the same laws as the near field. By definition, however, the near field is unimportant, and it makes little difference for the far field studies how the plume is produced. One problem is that of recognizing where the far field begins and reproducing the initial conditions at that point. Far field problems in lakes are generally more difficult than in rivers or estuaries because in the lake case the several forces creating motion and turbulence--buoyancy, wind stress, natural currents--are all of the same order of magnitude and have to be found as part of the solution. In rivers and estuaries, on the other hand, the driving force due to gravity far outweighs the others and establishes the flow pattern almost independently of buoyancy, temperature becoming merely a marker of fluid particles.

The third element of a plume is the region joining the near field and the far field. Its length may vary considerably from one physical realization to another. In the submerged jet case it includes the region of the vertically rising plume. The laws of modeling involve both mixing and entrainment problems associated with the near field and surface cooling and convection problems associated with the far field. An undistorted laboratory model is desirable in this joining region, but the laws of modeling are considerably more complicated than in the outlet region because of the increased number of important factors. The true joining region has, in fact, been little studied in the laboratory because of the limitations imposed by tank walls and by the difficulty of properly representing the feedback from the far field.

Several processes are at work in spreading heat from a warm water outlet through a body of water. In one group are those processes associated with the spread of jet momentum--convection and momentum transfer as measured by eddy viscosity. If these were the only processes, heat would act as a marker of fluid particles and heat spread could be calculated once the velocity pattern was known (allowing for surface cooling). In another group are processes associated with the large-scale eddy structure of the flow, which mixes masses of warmer and cooler fluid almost independently of momentum transfer (although momentum transfer is partly responsible for the large-scale eddy structure). These large-scale mixing processes are inhibited when the densimetric Froude number is small or, more properly, when the local Richardson numbers are large--small velocity gradients accompanied by large, stable density gradients. This last situation usually prevails at the thermocline in the far field, and there is little mixing across it. Hence in the far field, if it were not for horizontal mixing, the heat spread could be calculated if the velocity field were known, and vice versa. Since the horizontal mixing is frequently small, it is not usually necessary to measure both streamlines and dispersion patterns in model experiments in the far field.

Nearer an outlet, however, the large-scale mixing processes are usually much more important, because the velocity gradients are large where the density gradients are large--near the edge of the jet--and the local Richardson numbers may become considerably less than unity (or even negative, as on the upper side of a warm, submerged jet). Hence in the near field and in the joining region it is essential to study momentum spread and heat spread together if full understanding is to be attained. One cannot usually be calculated if only the other is known. Early investigations have sometimes ignored this fact in model studies of outlets where the local Richardson numbers were small in the model and large in the prototype or vice versa.

Small Richardson numbers are, of course, characteristic of non-buoyant jets ($Ri \equiv 0$). Hence it can be expected that near an outlet, for large enough densimetric Froude numbers (so that inertial forces are large compared to buoyant forces), experimental results and analysis for non-buoyant jets will be applicable to buoyant jets. For smaller densimetric Froude numbers where buoyant forces are important this cannot be the case, and independent experiments on the effect of buoyancy are necessary. In any event, what is usually required is the entrainment coefficient E, which will measure the rate of dilution of the emitted warm water. The entrainment coefficient is a function of Richardson number, as well as of geometry, and the relationship has been discussed at some length by Ellison and Turner [9].

In the following sections the work on non-buoyant jets will be reviewed first because of its applicability to the near field and the joining region of buoyant jets at large densimetric Froude numbers. The research conducted on the two basic outlet geometries, submerged and semi-submerged (or surface buoyant) jets, at smaller densimetric Froude numbers will then be reviewed in separate sections. In each case the near field and much of the joining region will be treated together. (Chapter VII of Parker and Krenkel [1] should also be referred to in connection with this research.) A section on the far field will follow. Final sections will summarize the numerical coefficients available from laboratory experiments and describe laboratory instrumentation.

B. Non-Buoyant Jets

Only a brief review of experimental work on non-buoyant jets will be undertaken in view of the vast literature which exists. Simple geometries have been used for the most part in these experiments: two-dimensional jets discharged from slots or circular jets discharged from circular nozzles. Experiments have frequently been carried out in air, but the results are the same for water as long as the fluid is homogeneous.

The earliest measurements were for time-smoothed velocities in cross sections perpendicular to the axis of a jet issuing into fluid at rest; typical papers which have now become classics are those by Albertson, et al. [10] and by Forstall and Gaylord [11]. With the advent of newer instrumentation, such as the hot-wire anemometer, and better analytical methods, turbulent velocity fluctuations could also be measured and momentum and energy transport evaluated; the recent work of Wygnanski and Fiedler [12] is typical. Jet experiments show a developing region of the jet about 5 to 10 slot widths or nozzle diameters long in which there is a linearly narrowing region of constant velocity across each cross section as measurements proceed downstream; this region can be equated with the near field of buoyant jets. Further along, the centerline velocity decreases and turbulence from the mixing at the jet edge builds up in the core until, 60 or so slot widths or diameters downstream, complete similarity prevails; this last region is equivalent to the far field, and the region between to the joining region, of buoyant jets. Ambient fluid is entrained into the jet in both developing and fully developed regions.

Experiments have also been made on jets discharging into flowing coaxial or parallel streams [13] and on jets discharging normally into moving streams [14,15]. Abramovich [16] indicates that for coflowing axially symmetrical jets the spreading rate is dependent on the ratio of free-stream velocity U_s to jet mean velocity U_j. Spreading or entrainment decreases for $1 < U_j/U_s < \infty$ and increases for $U_j/U_s < 1$. When an axisymmetrical jet is injected at an angle into a crossflow $(U_j > U_s)$, not only is it deflected, but the rate of mixing or entrainment is increased. Concentration and velocity profiles are skewed, as are the pressure profiles, and maximum centerline velocity decreases more rapidly than in the parallel flow case. Figure II-1 illustrates the typical kidney shape of the jet cross sections. Entrainment of ambient fluid occurs largely in the wake of the jet if the jet velocity is not much larger than the current, but entrainment becomes more like that of a jet into ambient fluid as the jet velocity increases.

Tracers such as dye, smoke, or heat (with negligible change in density) have been used to study mass transport in jets [17]. It is found that these tracers spread more rapidly than does momentum in turbulent jets for the reason given in Section II-A.

Many more studies than are cited have been presented in the recent periodical literature, and most have been undertaken for fairly high Reynolds numbers. All but the most recent work is summarized in several books, of which Refs. [16] and [18] through [21] are particularly useful, and is applied to buoyant problems in Chapter VII of Parker and Krenkel [1].

C. Submerged Buoyant Jets

Experimental and theoretical studies of buoyant jets originated fairly recently and were initially concerned with industrial processes, hot-air plumes such as those arising from chimneys, and fresh-water flows into the ocean. A natural extension of the non-buoyant jet experiments was the conducting of experiments with jets of one density discharging into homogeneous fluid of another density; papers by Chriss [22] and by Uberoi and Corby [23] are typical. For jets discharged at large densimetric Froude numbers, papers such as these report that the general behavior in the developing region is not too different from that in the non-buoyant case. The outstanding difference for horizontal jets was found to be an inhibition of turbulent mixing or entrainment between the jet fluid and the surrounding fluid, a result of increasing Richardson number accompanying the density differences.

As the centerline velocities of a jet decrease with distance, buoyancy forces, if present, will also affect the main trajectory of a jet if it is discharged at an angle to the direction of the acceleration of gravity. The point from which such effects can be noted moves closer to the point of discharge if the densimetric outlet Froude number is decreased. For values of the densimetric Froude number of the order of 20 or less such noticeable effects have been shown to exist from the very point of discharge onward [29]. Furthermore, rather than flowing into homogeneous fluid (of a different density than the jet), the jet may discharge into fluid with a density gradient or into fluid with two or more layers of differing densities. A buoyant jet discharging into homogeneous (isothermal) water of lower temperature would normally be expected to rise to the surface. However, if the receiving water is stratified due to salinity, for example, mixing of warm water from the outlet may be sufficient to produce a density stratified layer seeking a level intermediate between the bottom fluid and the surface fluid. Similarly, a saline, heated jet discharged into less saline, cooler water may become a non-buoyant jet under the proper conditions. It is also necessary to consider whether the jet flows into fluid at rest, into fluid moving parallel to the jet axis, or into fluid moving at an angle to the jet axis. Near an outlet in a lake situation, as opposed to rivers or estuaries, the first case often prevails, although pre-existing currents may be of some importance.

The problems of mixing and entrainment of a buoyant jet at a submerged outlet are subject to experimental study in laboratory models much as in the case of a non-buoyant jet. However, in order to account for the additional phenomena just outlined, proper modeling laws must be followed. The transition from a rising plume to a horizontal plume can usually be studied in the same model, but the eventual transition to the far field is a more difficult problem not easily studied without going to different models and model laws.

The direction and magnitude of the momentum of a jet at its orifice are two of the jet's main features. According to the impulse-momentum principle, the initial momentum will be modified by external and internal forces. These are essentially of two kinds, frictional forces and buoyancy. Most jet studies make the assumption that frictional forces can be ignored, while buoyant forces cannot. Under this assumption the jet is considered to be independent of the Reynolds number at the outlet if the Reynolds number is large enough. Experiments largely support this assumption for non-buoyant jets. In buoyant jets only the horizontal component of the initial momentum can remain unaltered; the vertical component will be changed by buoyant forces. Velocity distributions and temperature distributions in both buoyant and non-buoyant jets are generally found experimentally to be quite similar to normal probability distributions, although different from each other.

While the theoretical treatment of mixing in the developing region of the flow from non-buoyant submerged jets, as summarized in such references as [16], [20], and [21], leads to quite similar results, it appears [24] that the constants used in the equations are influenced by turbulence variations associated with initial and boundary conditions. Buoyant effects can be expected to be important in this connection, also, and the constants are not directly transferrable from non-buoyant to buoyant jets. Abraham [25] gives numerical results on the length of the zone, the shape of the axis of the jet, the velocity along the centerline, and other parameters in the zone of flow establishment of a horizontal jet in still water. The length of the zone of flow establishment for concentration, for example, varies from zero to 5.65 times the diameter of the outlet pipe, depending on densimetric Froude number, according to Abraham, and this is considerably shorter than for the non-buoyant jet.

The vertical buoyant jet from a point source with zero initial momentum (hence no developing region) was investigated experimentally by Rouse, Yih and Humphreys [26]. In this case the vertical gradient of the momentum flux is equal to the buoyancy of a horizontal stratum of unit thickness. The experimental study showed that local density differential and velocity distribution in the jet followed normal probability distributions in a cross section and that the analysis could follow a pattern familiar from homogeneous jets. It was shown that the volumetric flux Q and a local specific weight increment $\Delta\gamma$ changed with vertical distance s from the source as follows:

$$\Delta\gamma = -11.0 \left[\frac{\rho(-W)^2}{s^5}\right]^{1/3} \exp\left(-71 \frac{r^2}{s^2}\right)$$

$$Q = 0.153 \left(\frac{-W}{\rho}\right)^{1/3} s^{5/3}$$

where W is the original flux of the incremental weight, ρ is the average density, and r is the horizontal distance from the center of the jet. Figures II-2 and II-3 show the distribution functions and convection patterns. (In the figures the distance s is designated by the symbol x.) The spread of the velocity profiles is less than for a homogeneous jet.

Jets which are discharged with a horizontal velocity component into a homogeneous denser ambient fluid at rest are bent upward by buoyant forces, as is shown in Fig. II-4. The axes of such jets are curved. It has been shown by Abraham [25] and by Fan and Brooks [28] that the local coordinates, the mean velocity, and the mean density of the jet depend on one dynamic dimensionless variable: the densimetric Froude number at the outlet, Fr_o. The similarity principle for mean velocity profiles, well known from non-buoyant jets, is frequently applied to buoyant jets. However, the effect of buoyant forces on the turbulence pattern is different at the upper side and the lower side of a horizontal jet, and the assumption of similarity of velocity and concentration profiles is not completely justified for small values of the angle between the main trajectory of the jet and a horizontal plane.

Experimental evidence of fairly good similarity between velocity and concentration profiles was given by Bosanquet, et al. [27]. It was suggested by Abraham [25], Fan and Brooks [28], and Anwar [29] that velocity and concentration in the fully developed buoyant jet be described using the Gaussian distributions. Abraham used the forms

$$\frac{u}{U_m} = e^{-k(r/s)^2}$$

and

$$\frac{c}{C_m} = e^{-\mu k(r/s)^2}$$

respectively, where the coefficients k and μ were found experimentally and by speculation to equal

$$k = -304\left(\frac{\beta}{\pi}\right)^3 + 228\left(\frac{\beta}{\pi}\right)^2 + 77$$

and

$$\mu = 0.96\left(\frac{\beta}{\pi}\right)^3 - 0.72\left(\frac{\beta}{\pi}\right)^2 + 0.80$$

and where u is the local flow velocity, U_m is the centerline velocity, r is the coordinate in the transverse direction, s is the distance from the orifice measured along the axis of the jet, β is the angle between the tangent to the axis of the jet and the horizontal plane, c is the local concentration, and C_m is the concentration on the centerline of the trajectory. This trajectory is derived from the local values of the slope β, which can be found from

$$\tan \beta = \frac{g \int_0^s ds \int_A (\rho_a - \rho) dA}{\int_{A,s=0} \rho u^2 \, dA}$$

A similar approach was used in other studies on the same subject, for example those by Fan and Brooks [28] and by Anwar [29]. However, there is disagreement on the rate of entrainment between Abraham [25] and Fan and Brooks [28]. The latter follow the concept proposed by Morton [30,31], which makes the rate of entrainment dQ/ds a function of the local maximum flow velocity U_m and the nominal radius $b = \sigma\sqrt{2}$ of the jet where σ is the variance of the Gaussian distribution of velocity.

$$\frac{dQ}{ds} = 2\pi b \, U_m \, \alpha$$

where α is an entrainment coefficient which can be obtained from experimental work such as that reported in [26] and [10], wherein $\alpha = 0.082$ has been given for a buoyant plume and $\alpha = 0.057$ for a non-buoyant jet.

The analysis by Fan and Brooks is carried out in dimensionless form with the possibility of substituting various values of the entrainment coefficient into the solution, which is obtained numerically from the dimensionless equations. It is recommended, however, that 0.082 be used as the entrainment coefficient in all cross sections; this hypothesis is supported by the experimental work of Morton [30] and of Lee and Emmons [32]. The velocity and buoyancy profiles are Gaussian, but have different spreads because the diffusion coefficients for mass and momentum are found experimentally to be different. The dimensionless spread ratio is less for velocity than for buoyancy profiles by a factor selected as 1.16. The theory is also readily applicable to jets discharged into a stratified environment. The theories of Abraham [25] and of Fan and Brooks [28] produce very similar results for main trajectories and dilutions along the centerline, both of which are functions of the outlet densimetric Froude number as the sole dynamic variable. Results in terms of these variables are expressed in graphical form in the original references. Figures II-5, II-6, and II-7 reproduce typical data wherein the densimetric Froude numbers are variously given as F or Fr.

Anwar [29], whose work has been cited, also used a constant coefficient of entrainment, though some reservations were expressed based on experimental observations. An elaborate discussion of the entrainment principle for buoyant jets and in support of a non-constant entrainment parameter was presented by Abraham in a later paper [34] based on experimental data from Ricou and Spalding [33]; Fig. II-5 shows the data. It is felt, however, that for practical application a constant coefficient may be acceptable for the low densimetric Froude number range.

A comparison presented by Anwar [29] of experimental data on plume trajectories is reproduced in Fig. II-6; it shows agreement among various experimenters' data [29, 35, 36] for trajectories to within a few percentage points and is probably satisfactory for most practical purposes. The densimetric Froude number range covers values from 4 to 20 at the outlet. Other studies on forced plumes [37, 38, 39, 43, 46] which have not been discussed in detail essentially support the above conclusions.

While the previously described studies of horizontal buoyant jets dealt with jets in fluids of infinite extent, at least two experiments by Anwar [29] were carried out to examine the effect of proximity of the bottom on the buoyant plume. The nozzle was placed to discharge horizontally 1/4 inch above the bottom of a tank. Concentration measurements showed no sizable floor effect for outlet densimetric Froude numbers from 4 through 16. Density differences in these experiments were 0.016 to 0.028 and nozzle diameters were 1/2 inch and 1.0 inch. It is reported that the plumes were deflected upward a short distance from the nozzle. The lack of information on bottom effects points toward a need for further research in this area.

Experimental data taken in a harbor at semi-industrial scale (nozzle diameters from 0.6 to 5 cm, depth of submergence 4 m) were provided by Racou and Palmer [40] and are reanalyzed in Ref. [41]. The dilution rate was evaluated specifically, and no appreciable influence of Reynolds number was found in the range between 5000 and 40,000. Experimental values of measured concentrations along the centerline were found to be larger than those predicted by Abraham's theory [25]. Laboratory experiments by Cederwall [36, 42], on the other hand, indicate that measured concentrations were below Abraham's predicted values. Values of $C_m \frac{y}{D}$ ranged from 1.5 to 2.0 for dimensionless distances $y/D\ Fr^{1/2}$ from 1.0 to 8.0 where y is the vertical coordinate of a point on the jet centerline measured vertically from the center of the orifice. Measurements by Frankel and Cumming [43] gave somewhat lower values of $C_m \frac{y}{D}$ ranging from 0.8 to 1.4 for values of $y/D\ Fr^{1/2}$ from 0.8 to 15.0. Any disagreement between Abraham's theory and the data of Frankel and Cumming and of Cederwall was attributed to "confusion regarding terminology" in a review by Burdick and Krenkel [44]. It is also suggested that the concentration data of Racou and Palmer should be corrected for surface effects and that the correction would bring the data into better agreement with Abraham's theory.

Surface effects were also cited by Frankel and Cumming [45] to explain the differences. Plume widths were said to be larger in the experiments than in the theory because of the limited depth. Consequently, it was proposed that values of $\mu = 0.80$ and $k = 77$ (such that $\mu k = 0.61$) be used in the theoretical calculations referring to Frankel's and Cumming's experiments.

Only a few investigations have dealt with the discharge of buoyant plumes into already stratified waters. Brooks and Koh studied the subject in connection with ocean outfalls [47]. The purpose of their theoretical and experimental investigation was to predict the maximum rise of the plume in the stratified environment and to prevent the buoyant jet from rising to the surface. It was found that the maximum rise y_{max} above the orifice could be calculated from the relationship

$$y_{max}^3 = \xi_{max}^3 \frac{q_o \sqrt{\rho_1}\ (\rho_1 - \rho_d)}{8\alpha \sqrt{g}\ (- \frac{d\rho_o}{dx})^{3/2}}$$

where q_o is the volume flux per unit length or rate of discharge, ρ_1 is the ambient water density at the level of discharge, ρ_d is the density at the point of discharge, $d\rho_o/dx$ is the vertical density gradient, and α is the constant mixing coefficient. It was found that for most practical purposes the coefficient ξ_{max} could be approximated by a value of 2.7, resulting in errors of no more than $\pm$ 10 per cent. Forced plumes in stably stratified water were also treated by Hino [48] and by Rahm and Cederwall [49].

When heated water is discharged as a free jet into colder water it will rarely become so well mixed that it does not upwell at the water surface, and one must be particularly concerned with the surface spread of the heated water at the surface of the lake. Some experimental work on a fairly large scale is described by Abraham and Brolsma [50], but it is difficult to generalize the results given. A more detailed experimental study of the surface spread subsequent to the rise of the jets has been reported by Sharp [51]. A spread diagram is shown in Fig. II-8 representing essentially the radius of spread L versus a reduced Reynolds number

$$Ro' = \frac{Q_i (g')^{1/3}}{\nu^{5/3}}$$

in which Q_i is the volumetric flow rate, g' is the reduced acceleration of gravity $\frac{\Delta\rho}{\rho} g$, and ν is the kinematic viscosity at the source of surface spread. Time of spread T appears as a parameter in the experimental results. Spreading patterns are circles when seen from above. The study presents unsteady flow results with no heat transfer at the water surface. Therefore the results must be viewed with caution when they are applied to the steady state spread of heated water on a lake. Surface effects on jet mixing were also discussed by Burdick and Krenkel [44] with the conclusion that dilution is inhibited near the surface; a similar comment occurs in Frankel and Cumming [45].

If heated water discharges into smaller lakes initially filled with water of constant temperature, a temperature gradient gradually builds up and a stratified flow problem develops. There have been no direct experiments on this phenomenon, but experiments on the inverse condition of convection from a small source of negative buoyancy into a tank have been carried out by Baines and Turner [52]. Salt water was released near the surface into a tank of fresh water at very small densimetric Froude number. The receiving water became stably stratified with a density profile fixed in shape, changing with time at a uniform rate at all levels as the salt water descended. The effects of initial momentum and buoyancy on circulation in reservoirs of limited depth and width were investigated by Iamandi and Rouse [53]. Mean streamline patterns and lines of equal turbulence intensity are given graphically in dimensionless form for various length-to-depth ratios of the tanks.

To increase the initial mixing of a turbulent buoyant jet, various types of deflectors placed at 0.5 to 1.0 D_o from the outlet nozzle were investigated by Hansen and Schroder [54]. A maximum dilution improvement factor of $f = 2.19$ was found for a 120^o bent sheet dividing the jet, f being the ratio of dilution obtained with the special device to dilution obtained without it. It is also noteworthy that the mixing without deflectors was observed to produce 1.45 times larger dilutions than are derived from Abraham's theory [25].

The effects of currents on buoyant plumes are covered in the literature concerned with waste discharges into rivers [44] and also that concerning smoke plumes [55]. Reference [55] reports on a field study, but contains also several references on plume models. Experiments on jet injection in a two-layered parallel flow were carried out by Burdick and Krenkel [44] for densimetric Froude numbers from 27.5 to 55.5 and ratios of jet velocities to river velocities from 20 to 85. The jet discharged horizontally below the interface parallel to and at angles with the centerline. Time-smoothed centerline dilutions ranged from 28 to 55 per cent and were also found to be 1.4 to 1.8 times larger than instantaneous minimum centerline dilutions. The latter result is in agreement with the data of Frankel and Cumming [43]. Concentration measurements were taken where the jet plume intersected a standard measurement elevation. This elevation y, was chosen arbitrarily as $y/d_o = 27.5$ or $y/d_o = 41.3$, depending on the nozzle diameter d_o. The effect of jet orientation upon average centerline dilution was also investigated experimentally. As an example, the increase in average centerline dilution was between 40 and 50 per cent when the jet axis at the point of discharge formed a 45° angle with the current, this angle being measured in a horizontal plane. The studies also showed clearly the skewness of the concentration profiles in oblique jets.

Some experimental results referring to a submerged manifold sewage disposal system in a river were also reported by Burdick and Krenkel. An experimental study of the multiple-hole diffuser system for the TVA Brown's Ferry Plant condenser cooling system was carried out by Harleman, Hall and Curtis [56]. Obtaining information about the dispersion of the heated water in the river was one of the main goals of the investigation. The diffuser design is described in greater detail by Vigander, et al. [57].

Very low cross-current velocities were employed in an experimental study by Bosanquet, Horn, and Thring [27], and the effects on the buoyant jet trajectory were evaluated. It is doubtful whether the results are also applicable to high current velocities. It is stated that cross currents increase the initial dilution of a buoyant jet, but the validity of such a statement, in general, must remain suspect, at least until more data are accumulated. It is more likely that different effects will appear depending on the combination of relative velocities of the jet and the current, densimetric outlet Froude number, and Reynolds number. Pearson [58] presents some schematic representations of this effect which are reproduced as Fig. II-9. Earlier, dispersion of a boundary source without initial momentum into a crossflow had been examined by Rouse [59]. Pearson [58], however, suggests that the results might not be applicable to outfalls. There is a need for more experiments on buoyant jets in cross flows, but this need has relatively low priority for the lake situation. The literature on non-buoyant jets in cross flow previously referred to [14, 15, 16] is also useful in this connection.

The laboratory work reviewed herein shows that for submerged buoyant jets at all but the smallest densimetric Froude numbers there is naturally considerable mixing and entrainment near the outlet whether the jet be horizontal or vertical. Mixing and dispersion can be increased by several means, such as constricting the outlet to increase velocity, providing mechanical deflectors at the jet exit [54], or using multiple jets [57]. The effect of cross currents is not yet well understood and requires further investigation.

Mixing can be reduced only by increasing the local Richardson numbers (decreasing the densimetric Froude number), which can be done by reducing jet velocity gradients or increasing density gradients.

D. Buoyant Surface Jets

At high enough densimetric Froude numbers the buoyant surface jet is much like the lower half of a non-buoyant jet, and to this extent the information reviewed in Section II-B is applicable. The influence of densimetric Froude number on the surface spread of dye is well illustrated by photographs in the paper by Kashiwamura and Yoshida [60], which show the narrow, jet-like nature of the spread for high Fr_o and the widening spread which occurs as Fr_o is decreased until at the smallest Fr_o the spread is radial from the outlet. No velocity measurements were recorded, so that the relation between spread of dye and spread of momentum cannot be obtained.

The experiments by Kashiwamura and Yoshida were conducted for another purpose using fresh water flowing into salt water. But such experiments are equally applicable to warm water discharged into cooler water as long as surface cooling is not important, as is the case close to the outlet. Thus other experiments on fresh water discharge into salt water can be drawn upon to gain information about surface buoyant jets near an outlet. A number of experiments applicable to surface discharge of warm water plumes are summarized in Table II-D-1.

The experiments reported by Jen, Wiegel and Mobarek [61] were conducted at large Fr_o and measured the spread of heat directly. As can be seen from Table II-D-1, the receiving experimental tank was rectangular in shape, with vertical sides, and deep compared to the thickness of the heated water layer. Heat loss through the water surface was ignored because the dominant effect near the outlet was one of turbulent jet mixing. These experiments took only a few minutes, and presumably a quasi-steady condition was reached for the outlet region. At the largest Fr_o the spreading pattern was very much like that of half a submerged jet. Some temperature patterns are illustrated in Fig. II-10 for lesser Fr_o. The vertical spread of the jet was measured by introducing dye into the jet and photographing the flow. It was found to have a slope varying from 5.5 to 8 horizontal to one vertical with only weak

TABLE II-D-1 - Some Experiments on Horizontal Surface Plumes

Ref., Year of Report	JET Dimensions	JET Velocity	RECIPIENT Length	RECIPIENT Width	RECIPIENT Depth	Fr_o	Re_o	Instrumentation and Remarks
Kashiwamura and Yoshida [60], 1967	Channel - <15 cm deep by 4 cm and 8 cm wide		2.05 m	3.10 m	0.15 m	0.05 to 1.60*	100 to 9000	Fresh water into salt; camera above water surface photographed dye streamlines.
Jen, et al. [61], 1966	Pipe - D_o = 5 mm = 8 mm = 11 mm	7.24 fps 3.52 fps 1.74 fps	26 ft	15 ft	1.5 ft	18 to 180	5300 to 21,000	Copper-constantan thermocouples; null-point recording potentiometer; no velocity measurements.
Wood and Wilkinson [62], 1967	Pipe - D_o = 0.125 in. = 0.25 in.		10 ft	10 ft	4 ft	10 to 60	?	Fresh water into salt; camera above water surface photographed dyed fresh water.
Tamai, et al. [8], 1969	Pipe - D_o = 0.58 in. = 1.0 in. = 0.45 in.		25 ft	3.5 ft	2.0 ft	3.0 to 11.0	6600 to 21,300	Copper-constantan thermocouples; no velocity measurements.
Hayashi and Shuto [63], 1967	Channel - 5.3 cm deep by 5.3 cm wide		12 m	5.6 m	45 cm	1.4 to 16.1	3100 to 5600	Thermistors; ΔT from 0.2°C to 28.0°C; velocity by intermittent dye injection.
Tamai [64], 1969	Channel - 4 cm deep by 10 cm wide		3.0 m	10 cm	30 cm	1.2 to 4.1	3350 to 8150	Fresh water into salt; conductivity probe and hydrogen bubbles.
Stefan and Schiebe [66] 1970, [67] 1968	Channel - 0.16 ft deep by 0.5 ft wide		40 ft	17 ft	1.6 ft	0.62 to 7.2	1600 to 9400	Thermistor and tethered sphere probe for temperature and velocity.

*Obtained from values of Re and a stability parameter θ.

dependence on $\Delta\rho$. This observation together with those of Kashiwamura and Yoshida [60] seems to indicate that because of the large magnitude of the densimetric outlet Froude number, buoyancy had only a small effect on this flow. The vertical spread was definitely less than the lateral, however. The authors compared their results with those of half a submerged circular jet.

Analysis of their measured surface temperature distributions led Jen, et al. to propose, for surface spread, the relation

$$\frac{T_\ell - T_w}{T_o - T_w} = 7.0 \frac{D_o}{x} \exp -[3 \; Fr_o^{1/2} \left(\frac{y}{x}\right)^2]$$

and for jet width,

$$\frac{y_c}{D_o} = 0.57 \frac{x}{D_o \; Fr_o^{1/4}}$$

for $7 < x/D_o < 100$ where T_ℓ is the local temperature at a point in the surface with coordinates x along the centerline and y normal to it, T_o is the heated water discharge temperature, T_w is the cold water temperature of the lake, D_o is the diameter of the outlet pipe, y_c is the value of y where $\frac{T_\ell - T_w}{T_o - T_w} = \frac{1}{2}$, and Fr_o is the outlet densimetric Froude number. The equations indicate that the surface heat spread was Gaussian in the lateral direction and that the developing region of the jet was 7 diameters in length with the centerline temperature thereafter falling with x, all much in agreement with the non-buoyant case. Their best fit line for axial temperature distribution, along with data from several other sources, is reproduced in Fig. II-11, taken from the literature review of Tamai, et al. [8]. (These data for axial temperature distribution should not be confused with axial momentum distribution, which was not measured.)

Wood and Wilkinson, in their discussion of the paper by Jen, et al. [62], cited their own experimental results using fresh water plumes in salt water at comparable values of the densimetric Froude number. They verified the equation of Jen, et al., for jet width, but found the coefficient 1.25

instead of 0.57, indicating more rapid lateral spreading. The Wood and Wilkinson measurements were all in the vicinity of $x/D_o = 50$, and their boundary conditions, too, probably permitted only a quasi-steady flow to be achieved.

Flows at intermediate values of the densimetric Froude number have been investigated by Tamai, Wiegel and Tornberg [8]. The surface temperature patterns found could not be described precisely using the equations given earlier by Jen, et al. [61], nor was the lateral distribution of the surface temperature Gaussian as in the latter work. The temperature profiles were only roughly similar.

A study at smaller densimetric Froude numbers was also made by Hayashi and Shuto [63]. It was predicted and experimentally verified that as the outlet Froude number approaches zero (Richardson number approaches unity), the spreading pattern (timelines) becomes circular. The experiments show a more jet-like pattern at the highest densimetric Froude numbers. A similarity hypothesis for the temperature profile as a function of depth was postulated in the paper, but not conclusively verified experimentally; actually, it appears difficult to find sufficient physical justification for this postulation. The plot of temperatures along the jet axis shows that the temperatures do not decrease linearly for any of the Froude numbers, but rather decrease at an accelerating rate. The authors attribute this behavior to the variation in Richardson number, which is small near the outlet--promoting mixing and inhibiting surface cooling--but about unity or larger farther along, inhibiting mixing and enhancing surface cooling.

Tamai's experiments [64] were two-dimensional in that the surface discharge channel was the same width as the tank. They concerned vertical distributions of velocity and density as well as axial variations. The vertical velocity profiles were found to be approximately similar and Gaussian for a given densimetric Froude number, but to have a greater width in the buoyant case than in the non-buoyant case. The density profiles were roughly Gaussian and similar only in the developing region (five or fewer channel depths from the outlet), being nearly uniform thereafter; this would indicate a stable stratification between plume and ambient water beyond the developing region. One of Tamai's goals was to measure an entrainment coefficient, but he was unable to do so because of the large variation in local Richardson number.

Two-dimensional tests were also conducted by Wada [65] (not listed in Table II-D-1). He measured values of the entrainment coefficient, but found these varied not only with Richardson number, but also with the slope of the interface, which in turn was affected by entrainment.

Three-dimensional experiments in a somewhat similar range of small densimetric Froude numbers and Reynolds numbers were conducted by Stefan and Schiebe [66]. Both velocity and temperature profiles were recorded throughout the plume. Data have not been reduced sufficiently yet to determine entrainment coefficients, but since the profiles are mostly non-similar, it is expected that the entrainment coefficient will vary from point to point, perhaps as a function of local Richardson number. These experiments demonstrated how the boundary conditions in a steady flow experiment frequently control the stratification conditions in the experimental tank, a situation which is probably paralleled in prototypes, so that the heated water is discharged into an already stratified reservoir. Reference [67] is a more detailed version of [66].

In the high-momentum jets associated with high densimetric Froude numbers at the outlet, the surface discharge canal is filled with more or less homogeneous warm water, and all the mixing occurs in the receiving body of water. On the other hand, at the lower end of the Froude number scale buoyancy becomes so dominant that lake or reservoir water may actually penetrate into the outlet channel and form an arrested cold water wedge beneath the warm water with a stable or slightly unstable interface between. There have been several experimental studies on the related phenomenon of saline wedges beneath fresh water in connection with the problem of salt-water intrusion into river mouths and locks. References to the wedge can be found in Streeter's handbook [19] and in Ippen's book [68], for example. D. F. Harleman is the author of the pertinent chapters. A more recent study dealt with wedges in curved channels [69]. Whether a wedge forms depends on the densimetric Froude number at the outlet, and the mixing depends on this and on the local Richardson numbers.

In the experiments described to this point, the surface discharge canal has entered the reservoir normally to a vertical wall. In field situations, especially in lakes, it often happens that the canal joins a sloping bottom, and this can have significant influence on mixing at the outlet.

Very few experiments have been performed to study the effects of this geometry. Some of the hydraulic models to be described in Chapter III were built with canals on sloping bottoms, but no data were obtained on the effect of the slope. The difference in action between the outlet in a vertical wall and that on a sloping bottom is that in the former case abrupt separation of the outflow is usually forced at the end of the canal, while in the latter case there is a variable point of separation. Only if a cold water wedge forms in the canal does it make no difference how the canal terminates.

A corresponding experiment in the non-buoyant case would be represented by the lower half of a flow of homogeneous fluid through a pipe diffuser. If the densimetric Froude number is large enough, the buoyant jet can be expected to behave similarly. In that case, if the diffuser angle is small, the fluid follows the bottom for great distances, whereas if it is large, separation occurs early and there is much more mixing. This point is illustrated by some experimental results obtained by Wiegel, Mobarek and Jen [72] which are reproduced in Fig. II-12. Here x is the axial distance along the jet and y is the later distance from the centerline in the water surface. The reported temperature concentrations $\frac{T - T_w}{T_o - T_w}$ for slopes ranging from 1:50 to 1:200 for a given x show that the beach reduces the jet mixing considerably, and consequently higher temperatures are found at identical locations at the water surface when the beach is flatter. Another effect of the beach appears to be the wider spread of the plume on the flatter beach. The reported range of densimetric Froude numbers at the outlet was from 5 to 50 with most experiments in the range from 20 to 30. Outlet Reynolds numbers were mostly from 3000 to 6000. Some experiments with outlet channels cut into a beach with a 1:100 slope were also made.

Larger diffuser angles in the non-buoyant case involve flow separation swinging from one wall to another, and this could have no direct counterpart in surface buoyant jets because of lack of symmetry. The separation point from the bottom might be expected to make large excursions with time in the analogous situation. More laboratory work is needed on the effect of sloping bottoms on canals.

It is also possible for a surface canal to enter a water body making a horizontal angle with the shore. The proximity of a boundary to one side of the emerging plume could be expected to distort the plume in this case, but its general direction, which is determined by the momentum of the jet, is not likely to be affected much. This can be illustrated by reference to some of the model studies described in Chapter III.

A buoyant surface jet frequently discharges into an already stratified natural reservoir. Experiments do not usually represent this stratification, which exists because of the annual temperature cycle. However, experimental tanks are limited in size and are usually equipped with some overflow mechanism remote from the outlet which maintains constant volume. As a consequence of this and of the uncontrolled surface cooling in many models, steady state conditions in an experiment are usually associated with stratification in the experimental reservoir, which may be different than that in nature [66]. To avoid boundary effects, experiments are sometimes carried out under unsteady flow conditions, and the results are treated as quasi-steady as was done in the work reported in Refs. [8] and [61].

The mixing processes near a heated water outlet will be influenced by the kind of stratification found in the reservoir. Qualitative aspects of discharges into a two-dimensional, two-layered, non-viscous system have been discussed by Stefan and Schiebe [70, 71]. Various types of flow and flow phenomena influencing outlet behavior were considered for two-dimensional conditions, including cold water wedge penetration into the outlet channel, interfacial wave formation, the internal hydraulic jump, and turbulent jets. Figure II-13 shows some of the possibilities. The basic mechanisms which cause various types of flow near a warm water outlet are primarily controlled by buoyancy and inertial forces, as shown by Stefan [70], and the important dimensionless numbers are the densimetric Froude number and the Richardson number. However, the effects of viscosity and three-dimensionality have not been fully investigated and require further study.

Interfacial stability between a warm water layer and ambient water in two dimensions has been the subject of much research and experimentation. The results of this work on interfacial stability are relevant to mixing in the near field and joining region of surface buoyant jets and to the latter part of the joining region of submerged jets after they reach the surface. The work has some, though not as much, applicability to the far field.

Helmholtz was probably the first to study the interfacial stability of non-viscous two-layered flow assuming uniform but different velocities and densities in the two layers. Esch [73] has studied the same problem assuming that the upper layer is bounded above by a free surface and is slightly less dense and also a similar problem with a sheared velocity profile. Numerous works have been published on the non-viscous-flow stability problem with continuous density profiles since the problem was first dealt with in the classical works of Taylor [74] and Goldstein [75].

The stability of viscous stratified flow first received attention in connection with study of gas-liquid interface stability. Jeffreys [76] was the first to study the viscous effect on the stability of uniform flow. The stability of the gas-liquid interface has since been studied by many people, notably Miles [77], using a more realistic velocity profile. Tchen [78] derived an approximate solution for the viscous stability of the interface between two fluid layers of different fluid properties moving with uniform but different velocities, and many others have worked on similar problems with different boundary conditions.

In general, the densimetric Froude number is of primary importance in stability criteria for two-layer flow. Keulegan [79] and others have included the effects of viscosity from empirical data by introducing a new parameter based on the ratio of Reynolds number to the square of the densimetric Froude number. The cube root of this ratio, called θ, was found by Keulegan to have critical values of 0.127 for $Re < 450$ and 0.178 for $Re > 450$.

If there is no interface, but instead a continuous density and velocity gradient, stability depends on the local Richardson number, previously introduced in Section I-B. Thorpe [80] reported that instabilities of a diffuse interface between two miscible fluids were not observed at local Richardson number values greater than 0.25. The minimum number at the onset of instability was estimated at 0.09. In a fluid of constant density gradient no instabilities were observed. Buoyancy effects appear at Richardson numbers closer to unity according to other theories and measurements, those of Ellison and Turner [9] for example.

Turbulent local diffusivities for mass and momentum are functions of the Richardson number. Correlations have been sought theoretically and experimentally. Ellison and Turner [81] proposed an analytical relationship between Ri and the ratio of mass diffusivity to momentum diffusivity. According to them, if buoyancy has a negligible effect $(Ri \equiv 0)$, the ratio is equal to 1.4. Some effects of buoyancy and turbulence begin to appear at Richardson numbers as small as 0.01 in stable density gradients.

Instead of using the ratio of diffusivities to define mixing in connection with laboratory experiments, an overall entrainment coefficient E for particular flow situations is frequently used. The coefficient was defined in Section I-B as the dimensionless rate of increase of discharge with distance, but may also be given as the ratio of two velocities, as was done by Kato and Phillips [82],

$$E = \frac{u_e}{u_*}$$

where u_e is the entrainment velocity and u_* is the shear velocity of the current which produces the mixing. The overall entrainment coefficient is a function of an overall Richardson number which is obtained by replacing the velocity and density gradients in the original definition with finite values of velocity, density differences, and lengths; for example, Kato and Phillips used

$$\overline{Ri} = \frac{g(\frac{\Delta\rho}{D})}{\rho(\frac{u_*}{D})^2} = \frac{g\Delta\rho}{\rho}\frac{D}{u_*^2}$$

where $\Delta\rho$ is the density jump across the entrainment interface and D is the depth of the mixed layer. The overall Richardson number may be identified with the inverse square of a densimetric Froude number.

Kato and Phillips [82] experimentally applied a constant stress at the surface of an initially quiescent tank of fluid with a uniform density gradient and studied the development of the turbulent layer by entrainment of the underlying fluid. The relationship

$$E = 2.5 \frac{1}{\overline{Ri}}$$

was found to describe the experimental data quite well.

The experiment was stimulated partly by a similar investigation by Ellison and Turner [9] concerned with a surface jet. In that earlier case the overall Richardson number was defined using the overall depth h of the surface current as the characteristic length and the average velocity V of the current instead of the shear velocity. The entrainment coefficient used by Ellison and Turner is the one given in Section I-B, adapted to two-dimensional flow,

$$E = \frac{1}{V}\frac{d(Vh)}{dx}$$

and is found to vary from $E = 0.075$ for $Ri = 0$ to $E = 0$ for $Ri = 0.83$. Wada [65] verified these values approximately for two-layered flow. The value 0.83 is considered a mean for the critical Richardson number. The shape of the plotted curve suggests a form

$$E \propto Ri^{-n}$$

but n has not been specified. In a more recent study by Turner [83] a stirring mechanism was used in the experiments instead of flow and n was found to be equal to one for the transfer of heat and 1.5 for the transfer of salt. The difference may be due to molecular effects. The length scale to be used in the Richardson number was felt to be most meaningfully the size of the large eddies rather than any overall scale.

A similar experiment had already been carried out by Rouse and Dodu [84]. Experimental investigations of the mixing of two parallel streams of dissimilar gases were carried out by Baker and Weinstein [85] investigating the internal turbulence mechanism which produced the apparent exchange. Both jets had an initial velocity distribution profile and zero velocity at the point where the jets made contact. Velocities, turbulence intensities, and densities were measured across the flow at various distances from the outlet. Velocity ratios at the outlet ranged from one to nine. It was found that measurements did not satisfy the similarity hypothesis and that the disagreement was particularly large when the free stream velocities of the parallel jets were about equal.

Evaluation of eddy diffusivities from experimental data often proves difficult [8]. Methods that can be used are given in Chapter VII of [1].

A summary of eddy diffusivities obtained from field measurements is contained in Ref. [86].

Horizontal density gradients are generally much smaller than vertical density gradients. Therefore dispersion in the horizontal direction can be expected to resemble that found in homogeneous waters. This may be larger or smaller than vertical dispersion, depending on Richardson number. Generally, the vertical dispersion or entrainment becomes very important at some place near the outlet, and horizontal dispersion becomes more important in the far field [65]. More research is needed, however, on the relative importance of horizontal and vertical mixing as related to position in the plume.

Wind forces, too, have their major influence in the far field, and they will be discussed under that subject. However, wind can also influence the trajectory of the emitted jet nearer an outlet, especially for small densimetric Froude numbers. Wind shear produces a momentum component in the wind direction, and this can create currents even in the absence of a plume. Since there is little time for shear stress to act close to the outlet, little direct effect can be expected there, but farther along in the joining region wind can produce a curved trajectory or retard or advance the axial motion of the plume, depending on wind direction. No laboratory experiments have been reported on the effect of wind on the near field or joining region.

E. Lake Plumes in the Far Field

As observed in Section II-A, the far field problem in lakes is different from that in rivers and estuaries because in the lake situation, buoyant forces produced by temperature differences, along with wind and other currents when present, are responsible for the motion and spread of heat, while in a river or estuary, temperature in many cases is only a marker of fluid particles carried by the main flow. For that reason, only the lake situation will be discussed here. Many of the model studies to be reviewed in Chapter III deal mainly with the far field, but unfortunately, only a few of them deal with lakes.

An important characteristic of the far field of a thermal plume is the existence of a thermocline along which the local Richardson numbers are usually of sufficient magnitude to inhibit mixing as discussed in Section II-D. At the present state of knowledge, entrainment and mixing can be taken as negligibly small along the thermocline (Wada [65]). The thickness of the plume, therefore, is determined largely by its thickness at the end of the joining region and by its lateral spread. There is usually little difficulty in modeling this no-mixing aspect of the far field, since all that is necessary is an adequate thermocline in the model.

Horizontal spreading in the far field, in the absence of wind and other pre-existing currents, is determined by several factors. The residual momentum in the jet is one of them, but it has been attenuated by viscous shear, spread, and mixing in passage through the near field and the joining region. The density gradient in the horizontal direction now becomes of some significance, its importance relative to jet momentum being measured by the densimetric Froude number, which is the important modeling parameter for this phenomenon. As the densimetric Froude number decreases, the spread becomes more and more circular because of the importance of buoyancy forces as described by Hayashi and Shuto [63]. In modeling it is still necessary to have an adequate Reynolds number, and this requires model distortion, as already observed. Barr [3] has determined empirical curves from two-dimensional experiments which can be used to estimate when the model Reynolds number is large enough to avoid viscous effects in spreading.

Turbulence and mixing do, of course, occur within the plume. These transfer heat in the vertical direction to compensate for surface cooling. There is also turbulent mixing in the horizontal direction which transfers heat much as in a gradient process to the limits of the plume and into the ambient fluid surrounding it. Heat transfer within the plume is measured by the eddy diffusivity, and this is often less in the vertical direction than in the horizontal direction because of the vertical density gradient. Many of the experiments described in Section II-D had as an objective the obtaining of eddy diffusivities. Examples can be found in the work of Jen, et al. [61], Hayashi and Shuto [63], Tamai, et al. [8], Tamai [64], and Stefan and Schiebe [67]. However, the present state of knowledge does not provide consistent numerical values for the diffusivities, and the best that can be done is to assume the values for non-buoyant turbulent jets given in Section II-B for horizontal diffusivities and to use smaller vertical diffusivities. The latter will involve guesswork, with some guidance provided by the values of local Richardson numbers.

Far field models should usually operate in the steady state. This is both a practical matter and a necessity as a consequence of distortion. In a prototype there is a certain rate of cooling to the atmosphere from the plume surface, and this changes slowly as the hours of the day pass. Similarly, wind drift currents are frequently subjected to changes which occur within hours. The detectable limits of the plume will expand or contract and will also be displaced due both to changes in the horizontal density gradient and to horizontal mixing as the surface cooling rate and the wind shear on the water surface change. These changes in the plume are so slow as to be practically unimportant, and it is not really necessary to know how they occur. It is the average steady position of the plume under given meteorological and plant operating conditions that is needed. In a model, time periods are shortened by a factor of at least ten and usually by considerably more. It would be both difficult and costly to operate a model so as to reproduce plume fluctuations.

There is, nevertheless, a certain fascination in examining the unsteady spread of a buoyant plume over a water surface, and rarely, probably, has an occasion to do it been missed. There is one good reason for doing it: During the unsteady spread period the effects from those unrepresentative edges of the model where it has had to be "cut off" because of size limitations are fairly small.

On the other hand, many experiments that are run in the unsteady, expanding plume model to take advantage of restricted boundaries usually have to be completed in minutes, a much more rapid time than even the modeling scale requires. If the far field model is distorted, as it frequently is, the horizontal density gradient becomes much larger in the model than in the prototype. (The time scale is determined by vertical distance, whereas the gradient is determined by vertical over horizontal distance, and the latter is relatively foreshortened.) In model data so obtained the too-large horizontal density gradient will produce both too large a momentum flow and too large an apparent horizontal eddy diffusivity. At the moving front of the plume the mixing may be reduced.

The argument just presented can also be used to show that it is desirable to model surface cooling correctly. If this is not done, the horizontal density gradient will be incorrect, with the consequences just stated. More important, the size of the area affected by a temperature increase will be altered. The difficulty in obtaining consistent values of diffusivities is probably in part attributable to the fact that so many of the experiments described in Section II-D were conducted for unsteady conditions or for incorrect surface cooling. The principal shortcomings of models of the far field have been the failure to model surface cooling and the failure to produce correct steady state boundary conditions.

Another characteristic of the far field is the ease with which significant momentum can be added to the plume by horizontal wind shear at the surface (or by pre-existing lake currents). This is true because of the large surface areas open to wind and the small forces produced by horizontal density gradients. The wind stress produces a vertical velocity profile near the surface, alters the trajectory and the shape of the plume [6, 7], and may produce waves. It also piles up water against the windward shore, creating excess head at that point which forces a return flow at lower depths. If the water reaching the windward shore is still warm, the warm water may be carried to depths well below the depth of the thermocline in the main portion of the plume by the returning flow. (See Sections B and E of Chapter III for examples.)

The vertical velocity profile produced by the wind may reduce the local Richardson number, thus enhancing vertical mixing. Waves may also contribute to enhanced mixing. It is possible for two or more warm layers of different temperatures to form, depending on the depth to which the wind-generated velocity profile penetrates. The depth to which wind-generated profiles penetrate, as well as the stress on the surface, depends on the length of time the wind has been blowing on the water mass, and this is dependent on the fetch, the distance over which the wind has contact with the water surface.

Unfortunately, there has been no laboratory experimentation on the effect of wind on thermal plumes. Experiments are needed to obtain some idea of how deep the vertical velocity profile penetrates and how much momentum is transferred for a given fetch. Also, it is necessary to learn how wind stress effects can be represented in a model. Experiments have, of course, been performed on wind blowing over unstratified water surfaces, and reports from that research can be resorted to for some information. The oceanographic literature contains general information regarding wind effects on circulation patterns. Books by Robinson [87] and Wiegel [88] can be consulted. A paper by Baines and Knapp [89] reports on experiments in a two-dimensional channel for obtaining velocity profiles and turbulence properties. Other laboratory studies were carried out by Plate and Hidy [90, 91] and by Shemdin and Hsu [92], the latter containing a brief review of earlier experimental work.

The shear exerted by wind on a water surface is often calculated in accordance with boundary layer theory using an equation in the form

$$\tau_w = C_w \rho_w U_w^2$$

where τ_w is the surface wind shear, U_w is the wind velocity at a specified height, and C_w is a shear stress coefficient. Experimental results on appropriate coefficients C_w are listed by Wiegel [88], Chapter 13. For a rough water surface, Wada [86] proposed $C_w = 0.8 \times 10^{-3}$ if $\rho_w = 1.25 \times 10^{-3}$ g/cm^3 and U_w is the wind velocity measured 10 m above the surface in m/sec. Larger values of C_w are recommended by Roll [93], as shown in Table II-E-1. The turbulent shear caused in the water by wind is frequently expressed in terms of an eddy viscosity. A compilation of values of eddy viscosities derived from field measurements is given by Wada [86]. According to him the relationship applies only if $U_w > 8$ m/sec; below this value the surface has to be considered smooth.

The dynamic roughness z_o has been shown to change rather drastically with wind or shear velocity. Watson [94] gives

$$z_o = \frac{u_*^2}{\alpha g}$$

wherein z_o is given in centimeters, u_* is the shear velocity in cm/sec, and the value of α may vary from 81 to 13, as reported. The uncertainty is probably introduced by the inaccurate determination of the shear velocity. Roll's book [93] is an informative summary on wind-water interaction and contains further references.

Wind also influences cooling, and the extent of the additional cooling can be calculated [100]. As a consequence of the additional cooling and

TABLE II-E-1 - Estimates of Drag Coefficients $C_w \times 10^3$ at 10 m Height above the Sea Surface by Four Different Field Methods and Three Laboratory Methods (after Roll [93])

	LIGHT WINDS (<10 m/sec)		STRONG WINDS (>10 m/sec)	
FIELD MEASUREMENTS				
Wind profiles	(15)*	1.1 ± 0.4	(12)	2.0 ± 0.7
Geostrophic departure	(6)	1.1 ± 0.6	(2)	2.5 ± 0.5
Tilt of water surface	(8)	2.7 ± 1.7	(14)	2.4 ± 0.6
Eddy correlation	(2)	1.45	(1)	2.3
LABORATORY MEASUREMENTS				
Wind profiles	(3)	0.9 ± 0.3	(3)	1.7 ± 0.3
Tilt of water surface	(2)	0.9 ± 0.2	(3)	2.5 ± 0.1
Surface film	(1)	0.9		---

*The numbers in parentheses give the number of sets of observations in each category. The standard deviations indicated represent only the degree of disagreement between different sets of observations and neglect the errors involved in each set.

greater mixing, it is not really necessary to be concerned about strong winds where large waves occur--say over 12 mph--for then the thermal plume practically disappears, anyway. The main problem with using simulated winds in a hydraulic model is that surface cooling by evaporation may then be too large. An adjustment using relative humidities of the air is possible, but this is a problem requiring further study.

F. Numerical Coefficients from Laboratory Models

Experimental laboratory studies are potentially useful for verifying or determining numerical values of certain coefficients or flow parameters which are needed in analytical descriptions of thermal plumes in lakes. Such coefficients or parameters usually describe bulk effects of turbulent motion not obtainable theoretically from basic physical principles or by solution of pertinent equations such as the Navier-Stokes equations or the diffusion equation. A typical example of such a coefficient is the spread coefficient C of a submerged fully developed, homogeneous, turbulent jet; this can be defined as $C = \sigma/s$, where σ is the variance of the Gaussian velocity distribution profile and s is the distance from the nozzle [10].

Numerical coefficients are generally used in and defined by simplified versions of equations referring to the flux of mass, momentum, energy, buoyancy, or heat. Ideally, the value of a coefficient is universal; that is, no attention has to be paid to the order of magnitude of the other parameters used in the same set of equations. This appears to be the case for the coefficient C cited above, the value of which in homogeneous fluids has been determined experimentally to be near 0.081 for an axisymmetric jet and near 0.0109 for a wide (two-dimensional) slot, both having uniform velocity across the outlet width. If a marker such as dye or heat is carried in the jet without changing its density, C for the spread of marker substance is larger by a factor of about two.

The spread and turbulent motion of a non-buoyant jet are primarily inertial effects, and therefore all jet characteristics can be shown to depend on only one experimental coefficient. If other forces come into play and if the geometry of a flow becomes more complicated, a larger number of dimensionless parameters are needed to describe the flow and the universal character of each one is lost. The Darcy friction factor for duct flow, with its dependence on Reynolds number, relative roughness, and shape of the duct, is one of the better known examples. Flow in the thermal plume is a rather complicated process, and several dimensionless parameters are needed for its description. Hence, interdependencies between dimensionless parameters must be expected.

The number and kind of numerical parameters can be obtained using the methods outlined in Chapter I. These include the same dimensionless quantities which must be represented properly in physical models: densimetric outlet Froude number, outlet Reynolds number, Nusselt number for surface heat transfer, turbulent Prandtl number, and others that have been cited before. However, in analytical models it is necessary to introduce additional dimensionless parameters which are convenient to use in a set of equations that can be solved analytically or numerically. The set of new parameters will be a function of the same basic dimensionless quantities already used in physical models. Hopefully, the functional relationship will be a simple one.

Available numerical parameters for thermal plumes are limited to those obtainable from experiments on steady-state, buoyant jets--mostly in the near field. Among these numerical parameters the entrainment coefficient α, as defined in Section II-C, is of particular importance. It is used in a mass flux equation and determines, in a sense, the mixing properties of a jet. Fan and Brooks [28] and Abraham [34] discuss the value of the jet entrainment coefficient for submerged, buoyant jets. According to the first authors α ranges from 0.057 to 0.087, the lower value coming from investigations of momentum jets [10], the latter from those of buoyant plumes with zero initial momentum [26]. The dependence on the buoyant forces and therefore on the densimetric Froude number has been given in more explicit form by Abraham [34] and is reproduced in Fig. II-5.

Application of the entrainment principle to horizontal and buoyant surface jets poses further questions because of the differences in horizontal and vertical mixing in the presence of buoyancy. The horizontal entrainment coefficient (through vertical surfaces) can be considered to be unaffected by buoyancy. Transfer and entrainment through horizontal surfaces can be treated in accordance with Ellison's and Turner's experimental results [81], indicating a reduction of entrainment to zero when the bulk Richardson number approaches a value of about 0.8. This bulk Richardson number is equivalent to the inverse of the square of a local densimetric Froude number. The entrainment coefficient for the vertical transfer E is defined in Section I-B.

Mixing in the zone of flow establishment for a submerged jet is generally different from mixing for a surface jet. In fact, the submerged jet in this zone can be treated in much the same manner as the homogeneous jet using the modified coefficients just discussed.

The mixing of heated water and lake water at outlet canals into lakes is substantially affected by the geometry of the lake shore, the cross section of the outlet canal, and the angle of discharge as well as the dimensionless dynamic parameters. In this situation it is quite impractical to work with local entrainment coefficients. Rather, a bulk entrainment rate given as the total flow rate q at the end of the mixing region relative to the initial heated water discharge q_o would be useful. If the shore of the lake is simply an inclined plane surface, the outlet mixing ratio can be expected to depend on at least five dimensionless parameters:

$$\frac{q_1}{q_o} = f(b_o/d_o,\ d_1/d_o,\ \theta,\ Re_o,\ Fr_o')$$

where b_o is the width of the outlet channel, d_o its depth, d_1 a characteristic thickness of the heated water layer at the end of the mixing zone, θ the slope of the beach, and Re and Fr_o' the Reynolds and densimetric Froude numbers respectively. The geometry of the outlet channel will also have some effect. No experiments to evaluate the functional relation are known of, probably because velocity measurements are difficult to obtain.

Some available measurements [67] referring to a discharge from a rectangular channel into a deep tank with vertical walls are still awaiting analysis. Preliminary results indicate that the reduction in total flow rate or in entrainment caused by buoyant effects as compared to that in the non-buoyant case becomes more and more pronounced with distance.

Available experimental data on temperature distributions from various sources previously cited might be useful in obtaining further information on outlet mixing, but no attempt has been made to extract information of this kind from them.

The outlet mixing process not only produces an increase in flow rate, but also affects the apparent plume width and depth. Equations of the form

$$\frac{y_c}{D_o} = 0.57\,\frac{x}{D_o\,Fr_o^{1/4}} \qquad 18 < Fr_o < 180$$

$$\frac{y_c}{D_o} = 1.25\,\frac{x}{D_o\,Fr_o^{1/4}} - 4.0 \qquad 10 < Fr_o < 60$$

for the width of the plume at the surface have been given by Jen, et al. [61] and by Wood and Wilkinson [62], respectively, and were cited in Section II-D. The latter took their data using fresh water and salt water and at distances $x/D_o \approx 50$. The above equations lump the effects of outlet mixing or entrainment and of jet spreading into one, and for that reason it is doubtful whether a universal expression for all x/D_o values can be obtained. It can be readily seen that the above equations do not apply to small values of x.

No quantitative expressions for the depth of the plume could be found, but available temperature data could be used to obtain such information.

The equations just given for the apparent spread of a plume describe the spread of heat, but not the spread of momentum. Depending on the buoyancy effects, the spread of heat may be greater, equal to, or less than that of momentum. In the homogeneous jet it is about twice, as already indicated. With favorable density gradients the factor may be reduced, while for unfavorable gradients it may be increased. Useful measurements are difficult to obtain because of the tediousness of making velocity measurements.

Another form of spread that is of interest is the path of stream surfaces leaving the outlet. A stream surface encloses a definite discharge rate and hence neglects entrainment. Stream surfaces still spread because of momentum transfer from the central part of the jet to the ambient fluid. Experimental observations of surface streamlines as described by Kashiwamura and Yoshida [60], Hayashi and Shuto [63], Stefan and Schiebe [67], and Jen, et al. [61] were discussed in Section II-D. These experiments are not adequate to make it possible to assign numerical spreading coefficients to the streamlines, principally because bottom spreading is undefined.

Finally, the length of the outlet mixing zone itself is of interest, because it is necessary to specify where the thermal plume becomes largely independent of the complex processes occurring at the outlet. If the characteristics of the jet at this point are known, it may be possible to define a virtual origin of a simpler jet (such as that from a circular or half-circular orifice) which would have produced the same results. The results for this simpler outlet could then be used to extend the computations farther along the plume.

The jet beyond the outlet mixing zone can be described according to characteristics similar to those used before; however, the dependence on the outlet densimetric Froude number and the Reynolds number becomes quite weak, and local values should therefore be substituted. The reason for this is that the cumulative effect of heat loss through the water surface makes it impossible to continue to ignore Nusselt number and temperature ratios.

Average rates of surface heat transfer are sometimes used for computations in this region. An order of magnitude figure for heat loss from a thermal plume might be from 50 to 200 BTU/ft^2,$^\circ$F,day [101] where the temperature difference is that existing between the plume surface temperature and the equilibrium temperature.

The jet in the far field is subjected to wind and currents. The shear stress coefficients necessary in the computation of wind effects have been given in Section II-E.

A description of the thermal plume as a dispersion process essentially requires knowledge of eddy diffusivities and eddy viscosities in both horizontal and vertical directions. Experimental studies on thermal plumes thus far have not produced any such values, and one attempt to find these values [8], limited in scope, has failed. A dependence of local values of the above four parameters on local Richardson numbers has been strongly suggested by various authors. It is not certain, however, whether this is the only dimensionless number to be considered. The inavailability of sufficiently accurate velocity measurements must be blamed in part for the lack of information.

Another point to be made in this connection is the uncertainty involved in correlating laboratory and field eddy diffusivities and viscosities. Scale effects on the turbulence spectra have been shown to exist in the case of the simple submerged jet, indicating that similarities between models and prototype may apply only to time-smoothed values. The situation is probably worse in the far field of the thermal plume. Given this situation it seems desirable to draw on field data rather than on tank experiments for eddy parameters. Some specific values have been listed in a table by Wada [86].

G. Laboratory Instrumentation

Local temperatures and velocities are the output data sought from most experimental plume studies. Copper-constantan thermocouples and thermistor probes are generally used for the measurement of time-smoothed values of temperature. The accuracy of these instruments is generally near 0.1°F, which is frequently considered sufficient. The devices may be installed at fixed points, both horizontally and vertically, or else a single device or a vertical chain of devices may be moved from point to point. It is sometimes sufficient to measure only surface temperatures. This can be done for large areas almost instantaneously using infrared techniques, including both direct photography and image intensification [95]. Infrared techniques are being applied in the model study of Item 22 of Chart III-A-1, for example.

Velocity measurements prove to be quite difficult because of the small velocities occurring in the far field. Values as low as 0.01 fps may be encountered. Special means such as the Bagnold flow meter [96], the tethered sphere probe [97], and an isotopic tracer method [102] have been devised. The process of measurement is tedious, because a separate record must be made for each data point. In turbulent flow the personal judgment of an observer in forming a time average is required, and this is a handicap in obtaining accurate measurements. When near surface velocities only have been required, weighted floats and floating particles or dye streaks have been used to obtain velocities by means of successive photographic exposures.

It might be noted in passing that similar instrumentation has been used in obtaining field data for comparison with hydraulic model results, but field use is complicated by the large scales involved and the vagaries of the weather. Infrared measurements of temperature are useful, but reveal only surface values.

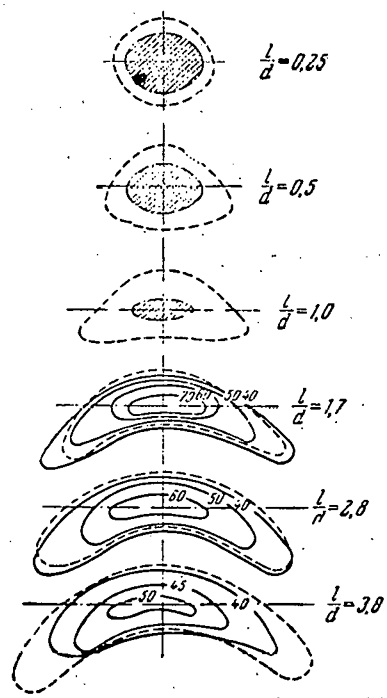


Figure II-1. Cross-sectional configurations of a jet in a cross flow at various distances along the jet centerline (after Abramovich [Ref. 16]).

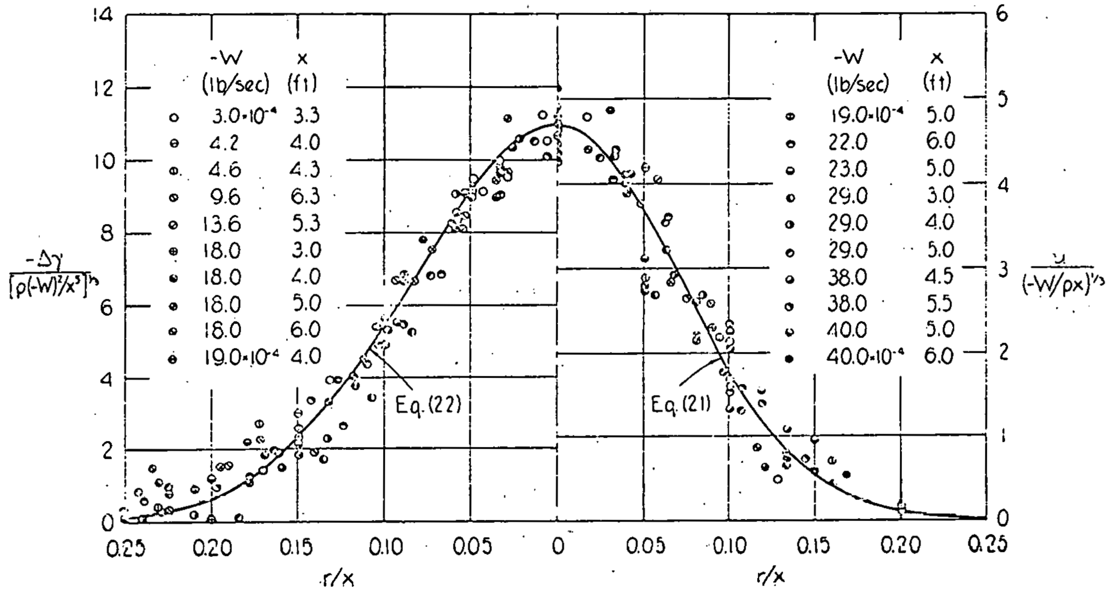


Figure II-2. Distribution functions for axially symmetric convection (from Rouse, Yih and Humphreys [Ref. 26]).

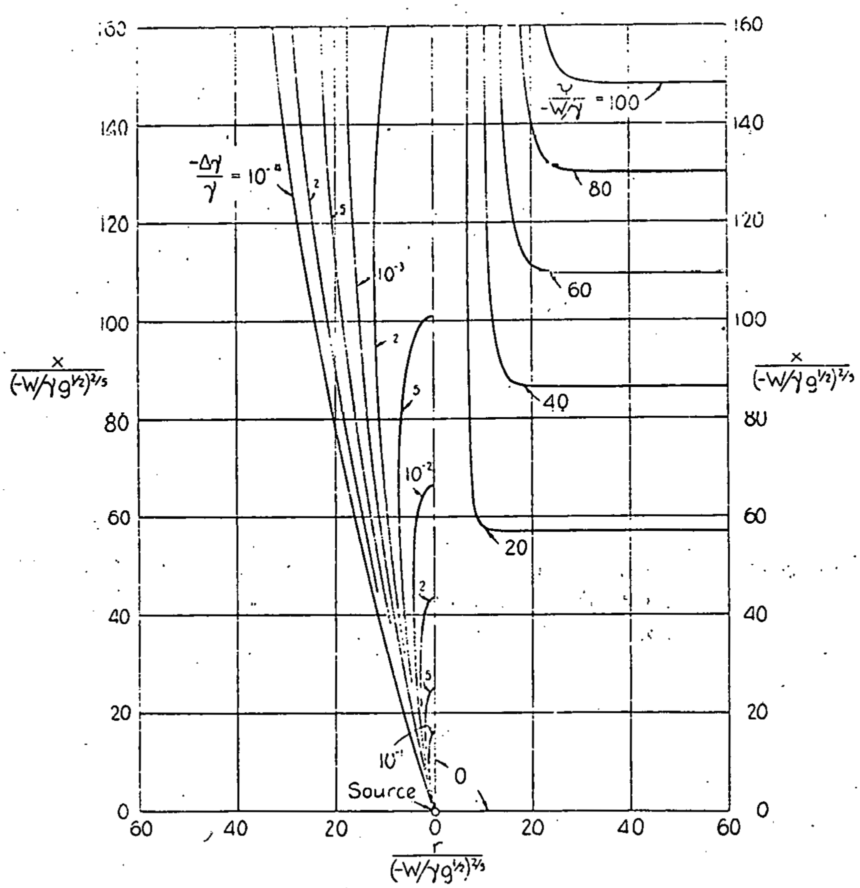


Figure II-3. Convection pattern over a point source (from Rouse, Yih, and Humphreys [Ref. 26]).

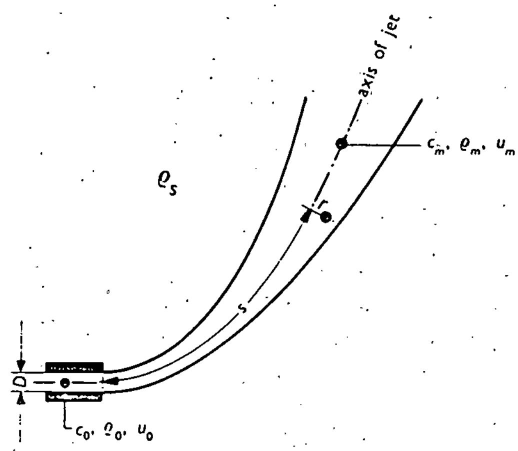


Figure II-4. Definition sketch, horizontal jet (from Abraham [Ref. 25]).

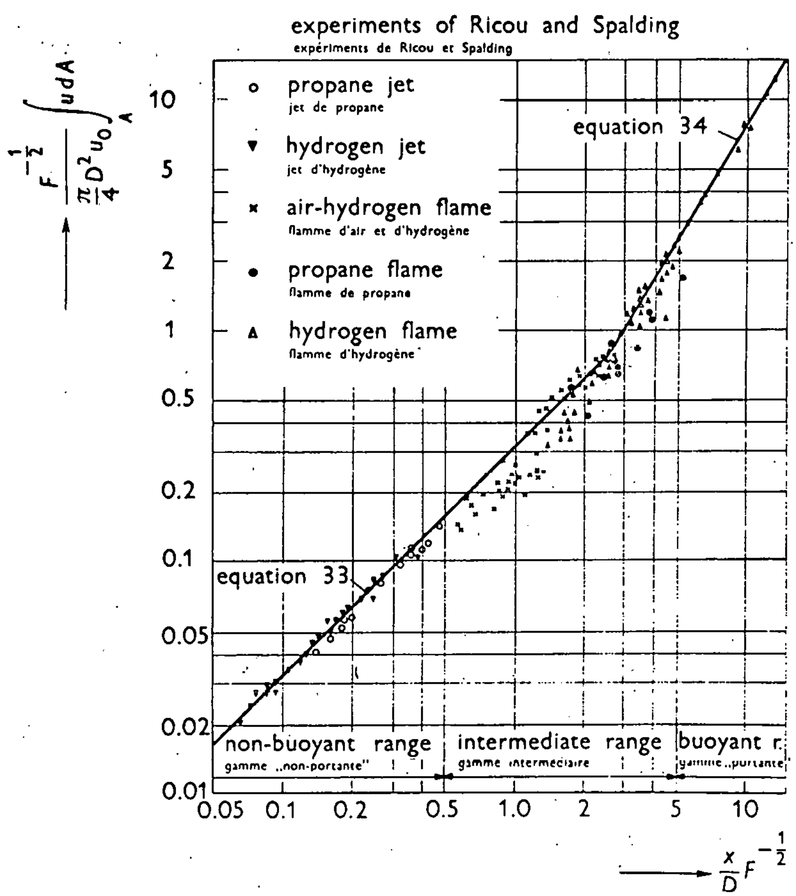


Figure II-5. Entrainment measurements of Ricou, et al. [Ref. 33] (from Abraham [Ref. 25]).

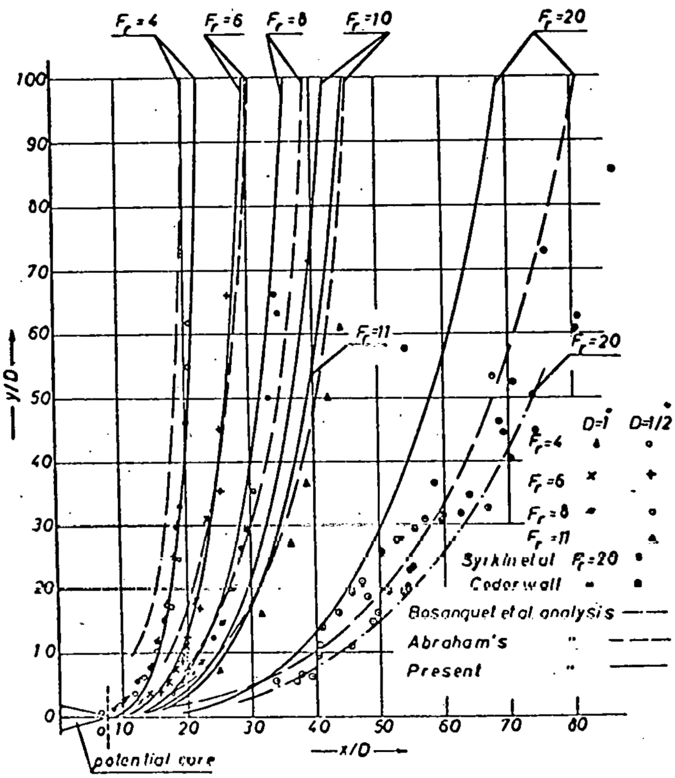


Figure II-6. Coordinate of trajectory of rising plume for various Fr (from Anwar [Ref. 29]).

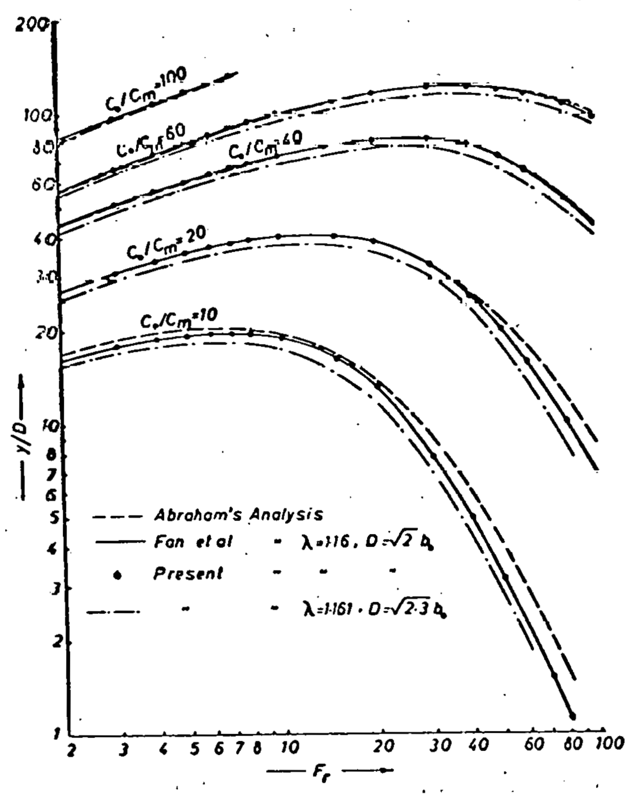


Figure II-7. Variation of y/D with nozzle densimetric Froude number Fr for various centerline dilutions C_o/C_m (from Anwar [Ref. 29]).

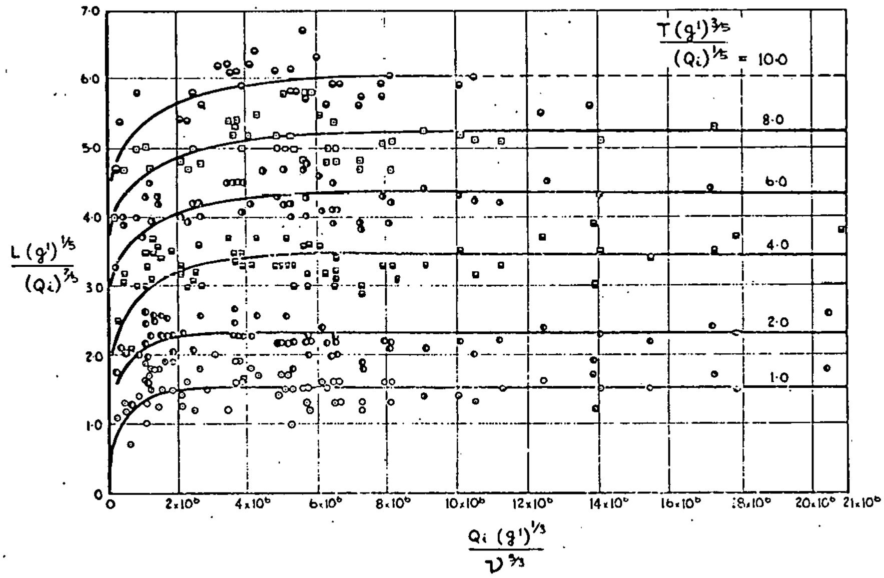


Figure II-8. Spread diagram for the unsteady axisymmetric surface plume (after Sharp [Ref. 51]).

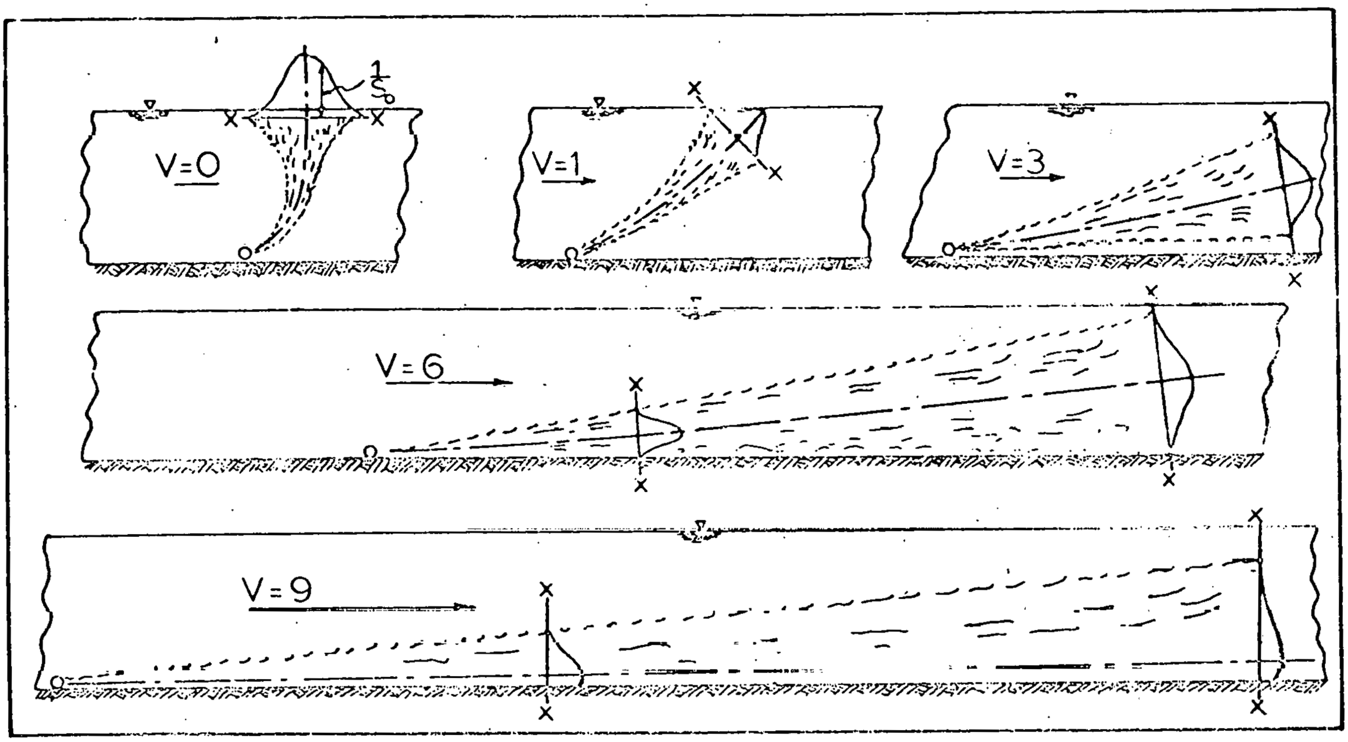


Figure II-9. Schematic representation of the possible effects of a current upon the diffusion pattern of a buoyant horizontal yet (after Pearson [Ref. 58]).

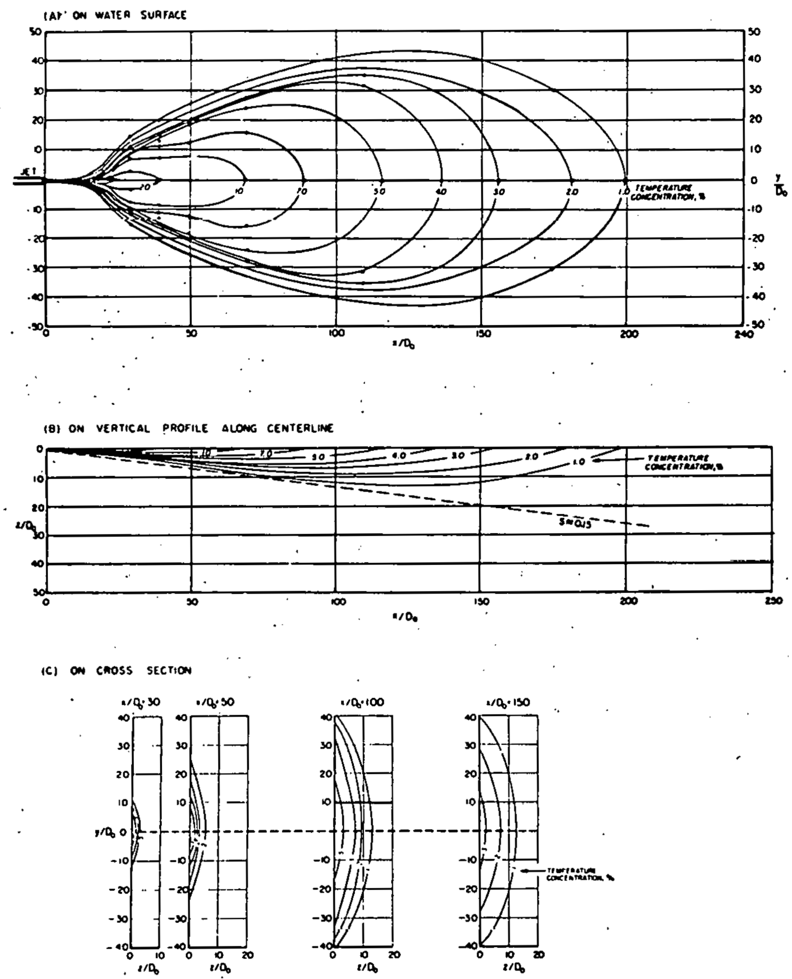


Figure II-10. Example of isotherm patterns from Yen, Wiegel and Mobarek [Ref. 61]. Temperature concentration lines $(T - T_w)/(T_o - T_w)$ for $D_o = 11$ mm, $R = 8300$ to $10,300$, $Fr_o' = 18$ to 24, $u_o = 1.74$ fps.

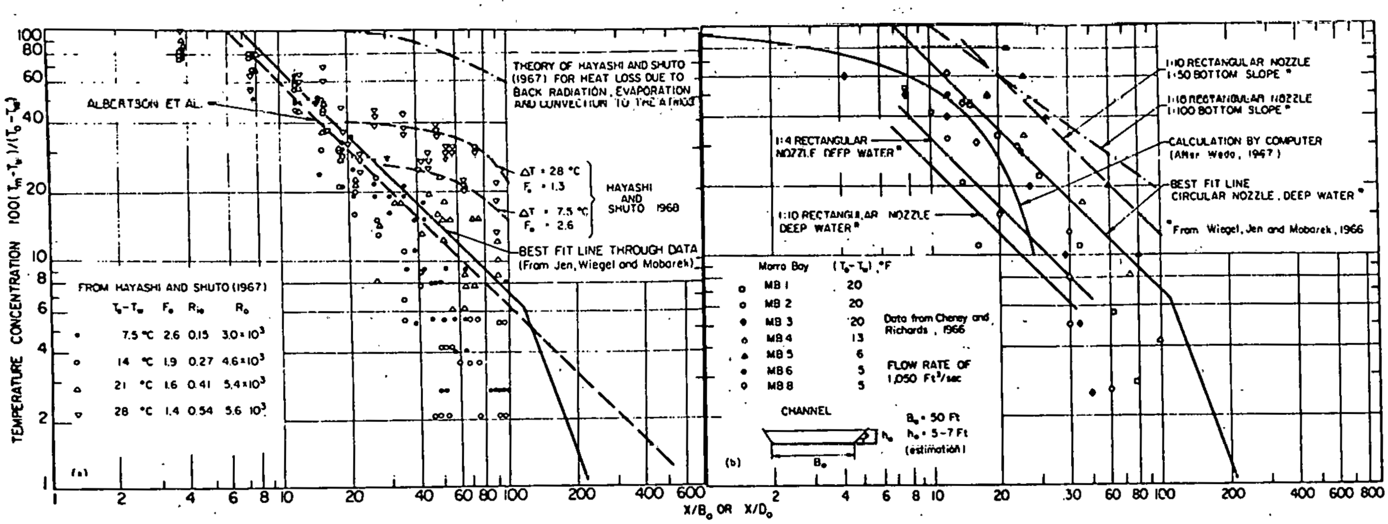


Figure II-11. Variation of temperature concentration along jet axis at water surface (from Tamai, Wiegel and Tornberg [Ref. 8]).

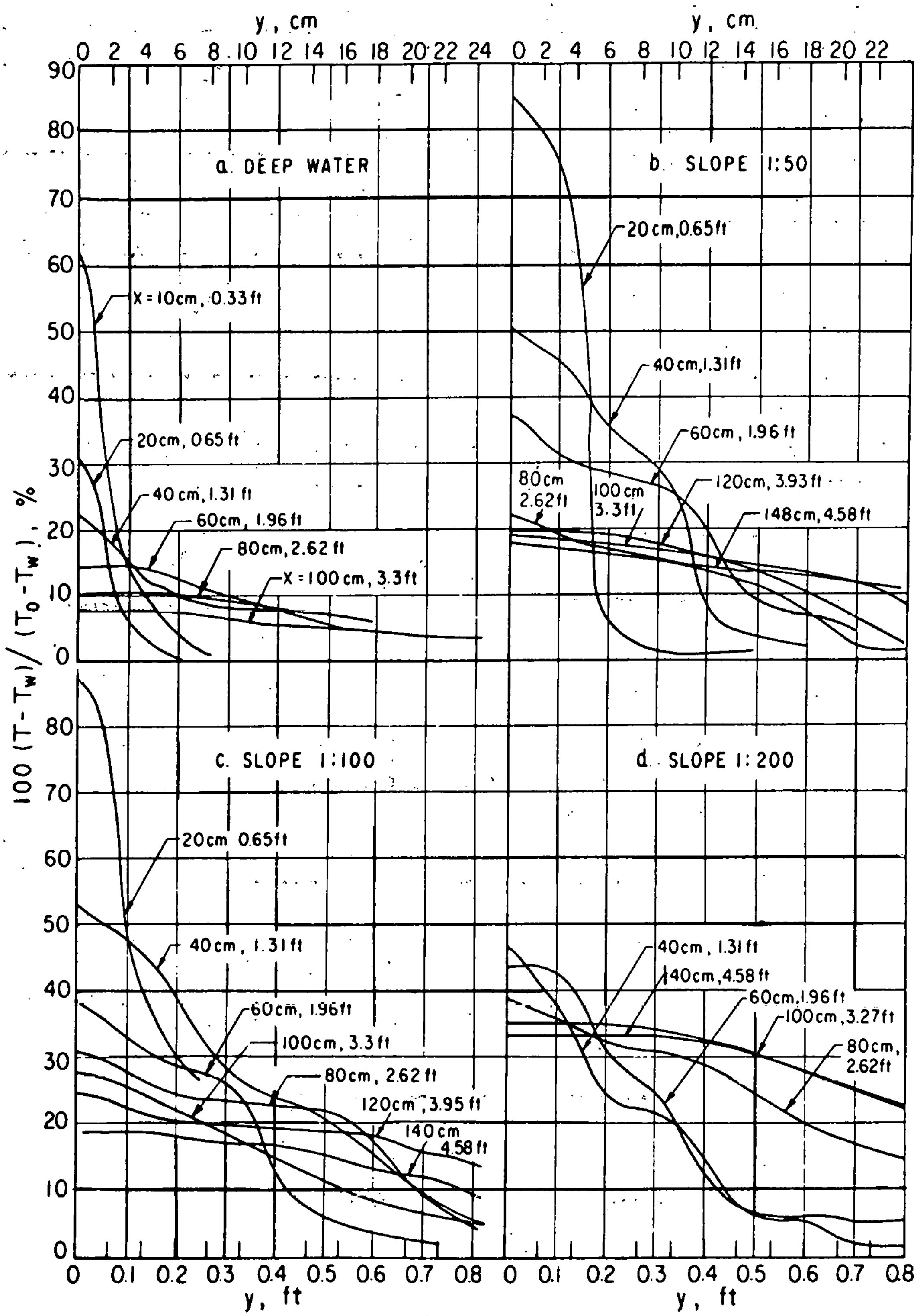


Figure II-12. Lateral distributions of surface percent temperature concentrations for various values of x; deep water, 1:50, 1:100, and 1:200 bottom slopes (after Wiegel, Mobarek, and Yen [Ref. 72]).

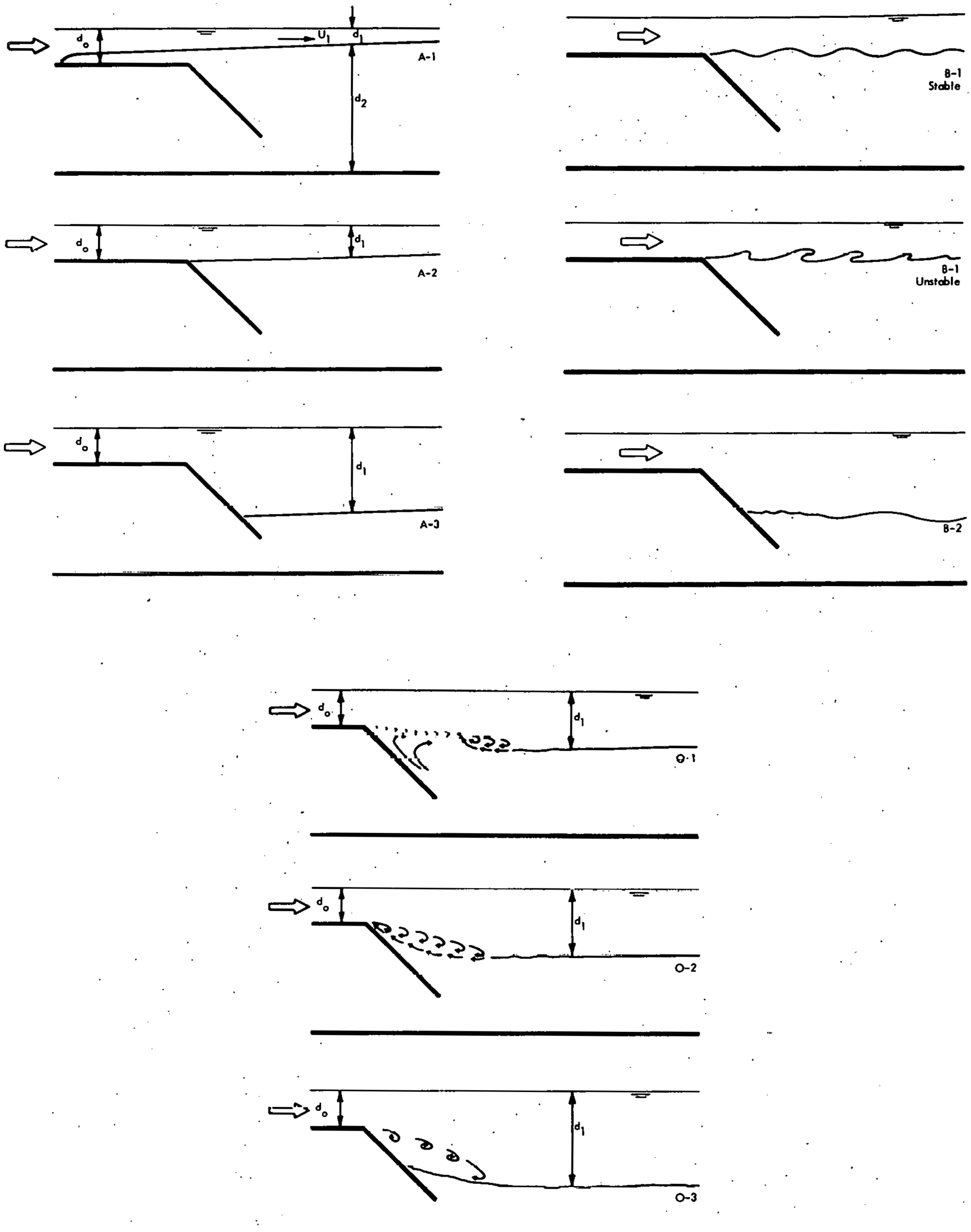


Figure II-13. Possible flow patterns at a two-dimensional, two-layered buoyant surface jet outlet (after Stefan [Ref. 70]).

III. CASE STUDIES OF HYDRAULIC MODELS

A. Introduction

Chart III-A-1 contains a list of steam electric generating plants in the United States, both fossil and nuclear fueled, for which hydraulic model studies of the dispersion of heated condenser cooling water were made. Also included are notations as to whether field data have been or will be collected at the site subsequent to plant construction. There are only a few instances where model and field data are comparable. Apparently there will be many more in the future.

The plants in the list have been grouped according to the laboratory which reported the model study. From the viewpoint of this survey they could have also been grouped by type of water body (lake or reservoir, river, or estuary) or by type of model. Examination of Chart III-A-1 shows that most modeling has been conducted for estuary conditions and very little for lakes or reservoirs. The reasons for this are not clear, but they may be associated with a belief that computational procedures introduced for cooling ponds can be applied to lakes without paying much attention to the dynamics of the flow, which is of great importance in rivers and estuaries. Also, hydraulic models have been used for studying estuary and river hydraulics without temperature differences for many years and, since temperature was considered mostly as a fluid particle marker in these problems, established modeling practices could be applied. For lakes, temperature differences provide part of the driving force and new procedures for dynamic modeling were required.

Only two items in Chart III-A-1 refer directly to thermal dispersion in lakes. These are Items 24 and 25. The comparisons achieved between model and prototype are discussed in some detail in Sections B and C which follow.

Some data were available to make a model-prototype correlation for Indian Point Station, Unit. No. 1, Item 1, and for the Chalk Point Steam Plant, Item 8 of Chart III-A-1. A discussion of these may be found in Sections D and E. These are both tidal situations, although Indian Point has only small tidal effects and no density variation resulting from salinity, while salinity is quito important at Chalk Point.

Item 17 of Chart III-A-1, the Valley Power Plant, is located in Milwaukee, Wisconsin, on the Menominee River and an adjacent navigation canal very near to Lake Michigan. Field studies have just been completed and will be reported on in comparison to the model data as noted in Chart III-A-1. The purpose of the model study was to investigate recirculation of warm water and the possibility of drawing cold Lake Michigan water into the thermal dispersion area in the river through the cold water wedge. It has been reported in a private communication that the correlation between model and prototype is generally good. The induced cold water flow from Lake Michigan into the lower stratified layer turned out to be somewhat greater than predicted in the model study and this has been beneficial.

These are the only hydraulic model studies of record in the United States that can be compared with field data at this time. There has, of course, been foreign experience in this area also. However, most power plants in Europe and elsewhere are located on rivers or estuaries, and no extensive effort has been made to search out model data. Experience with the modeling of heated water discharges at the Hydraulics Research Station, Wallingford, Berkshire, England, was reported on by Ackers [2, Ch. 6]. His article presents the various modeling principles used and gives comparisons between field and prototype data for a power plant on the Severn estuary in England, for the Hong Kong harbor power stations, and for other problems. More information on the Hong Kong harbor power plants can be found in Ref. [98]. Practice at the University of Strathclyde in Scotland is illustrated by Ref. [99].

Japanese experience at the Central Research Institute of Electric Power Industry has been extensive. Condenser cooling water is frequently discharged to bays on the coast. Wind and tidal effects have been included in the Japanese studies, an example of which is Wada's paper [86].

A summary of Chinese practice was published by Chen in 1965 [100]. It contains the practical modeling criteria used and a comparison of experimental and prototype flow patterns for a cooling pond. The agreement appears to be quite good by present standards.

In general, there is considerable difficulty in finding comparable model and field data. Even when the field data were available in advance, not enough attention was paid to fitting the model to the meteorological conditions, especially wind. In some cases, plant load was variable during field trials and the time scale between model and prototype could not be adjusted to take this into account. When the model study is completed before the plant is in service, other difficulties arise. For example, the plant design may be changed from that contemplated when the model study was made.

The comparisons of field data with hydraulic model experiments made in the following sections show that if the model is properly designed and operated in accordance with the proper physical relationships, the model and prototype will behave sufficiently similarly to permit obtaining at least qualitative data. Data regarding the direction of heat dispersion can certainly be obtained from a physical model. Data regarding the distance required to reach a given isotherm and the thickness of the warm layer depend more critically on matching meteorological conditions and on correctly modeling mixing conditions near the outlet. Even here, a range of predicted values may be obtained by varying the input conditions to the model.

CHART III-A-1

PHYSICAL (HYDRAULIC) MODEL STUDIES OF DISPERSION OF CONDENSER COOLING WATER
(Numbers in brackets in Chart refer to References on page III-8)

	Plant	Location	Laboratory	Field Data for Model Comparison	Remarks
1.	Indian Point Station (3 units) Consolidated Edison Company of New York, Inc.	Hudson River Indian Point, N. Y.	Alden	Yes- Unit No. 1 only	Other units not yet constructed. Ref. [1] reports some field data obtained by others. Ref. [2] contains pertinent laboratory data.
2.	Millstone Nuclear Power Station (2 units)	Long Island Sound Waterford, Conn.	Alden	?	
3.	Peach Bottom Atomic Power Station (3 units) Philadelphia Electric Co.	Rock Run Creek Peach Bottom, Pa.	Alden	Planned Later	Ref. [3] summarizes model study results.
4.	Calvert Cliffs Nuclear Power Plant (2 units) Baltimore Gas & Electric Co.	Chesapeake Bay Lusby, Md.	Alden	Planned Later	Premodelling drogue studies, field studies by Baltimore Gas & Elec. Co., Sheppard T. Powell, and Academy of Natural Sciences.
5.	Beaver Valley Power Station & Shippingport Atomic Power Station (2 units) Duquesne Power Company	Ohio River Shippingport, Ohio	Alden	No	Ref [4] outlines model studies.
6.	James A. Fitzpatrick Nuclear Power Plant Stone and Webster	Lake Ontario Scriba, N. Y.	Alden	Planned Later	Premodelling data and follow-up is planned by Stone & Webster
7.	Seabrook Nuclear Station	Hampton Harbor Seabrook, N. H.	Alden	?	

CHART III-A-1 (Cont'd)

	Plant	Location	Laboratory	Field Data	Remarks
8.	Chalk Point Steam Power Plant Potomac Electric Power Co.	Patuxent River (estuary) Chalk Point, Md.	Alden	Yes	Ref [5] contains some field measurements. Ref. [6] describes model study.
9.	Morgantown Potomac Electric Power Co.	Potomac River (estuary)	Alden	Planned Later	Both the Chesapeake Bay Institute and Potomac Elec. Power Co. plan field measurements.
10.	Dickerson Potomac Electric Power Co.	Potomac River Fredrick, N. J.	Alden	Yes	Field data by PEP Co. was not obtained so as to be comparable with model.
11.	Cronby Philadelphia Electric Co.		Alden	?	
12.	Cardinal Steam Plant Ohio Power Co.	Ohio River	Alden	No	
13.	Petersburg Steam Power Plant Indianapolis Power & Light Co.	White River	Alden	No	
14.	Oyster Creek Nuclear Plant Jersey Central Power & Light Co.	Barnegat Bay New Jersey	Alden	?	See also items 33 and 34.
15.	Union Beach Nuclear Plant Jersey Central Power & Light Co.		Alden	?	See also items 33 and 34.
16.	Consumers Power Co.	Cooling Pond Midland, Michigan	Alden	?	
17.	Valley Power Plant Wisconsin Electric Power Co.	Menomonee River Milwaukee, Wisc.	MIT	Yes	Ref. [7], Model Study. Report on field studies planned for publication in ASCE Jour, probably late 1970 by Stone and Webster Engr. Corp.

CHART III-A-1 (Cont'd)

	Plant	Location	Laboratory	Field Data	Remarks
18.	Pilgrim Nuclear Power Station Boston Edison Co.	Cape Cod Bay Plymouth, Mass.	MIT	?	Ref. [8], model study.
19.	Cumberland Steam Plant Tennessee Valley Authority	Cumberland River	TVA	Planned Later	
20.	Browns Ferry Nuclear Plant Tennessee Valley Authority	Tennessee River Wheeler Reservoir	TVA	Planned Later	
21.	Arkansas Nuclear One The Arkansas Power & Light Co.	Arkansas River (Dardanelle Reservoir) Russellville, Ark.	H-R-S	?	Ref. [9]
22.	Pittsburg Power Plant Pacific Gas & Electric Co.	San Joaquin & Sacramento River Delta area San Francisco Bay (estuary)	C of E San Francisco Bay model	Yes	Report planned for publication later in 1970.
23.	Montezuma Nuclear Plant Pacific Gas & Electric Co.			?	
24.	Allen S. King Plant Northern States Power Co.	St. Croix River (Lake St. Croix) Bayport, Minn.	SAFHL	Yes	Ref. [10], model study.
25.	Madison Gas & Electric Co.	Lake Monona Madison, Wisc.	Univ. Wisconsin	Yes	Ref. [11] and [12]
26.	Weston Powerplant (135 MW) Wisconsin Public Service Corp.	Wisconsin River Rothschild, Wisc.	Univ. Wisconsin	Yes	Field survey, summer 1969. Model study under way.
27.	Point Beach Nuclear Plant (994 MW) Wisconsin Electric Power Co.	Lake Michigan Point Beach, Wisc.	Univ. Wisconsin		

CHART III-A-1 (Cont'd)

	Plant	Location	Laboratory	Field Data	Remarks
28.	Surry Nuclear Power Station Virginia Electric Power Co.	James River (Estuary) Hog Island	C of E WES	Planned Later	Model study conducted in existing James River Model. Field study will be by Pritchard.
29.	Salem Nuclear Generating Station Public Service Electric & Gas Co. of Newark	Delaware River (Estuary) Artificial Island	C of E WES	?	Model study conducted in existing Delaware River model. Ref. [13]
30.	(may be alternate to 29) Public Service Elect & Gas Co. of Newark	Delaware River (Estuary) Cohansey River	C of E WES	?	"
31.	6 proposed and 11 existing plants Philadelphia Electric Co.	Delaware River Trenton To Delaware Bay	C of E WES	?	"
32.	Tidewater Oil Co.	Delaware River Delaware City, Del.	C of E WES	?	"
33.	3 existing plants New Jersey Power & Light Co.	Hackensack River (lower part)	C of E WES	?	Model study conducted in existing New York Harbor model.
34.	Proposed plant New Jersey Power & Light Co.	Raritan Bay Conaskank, N. J.	C of E WES	?	"
35.	Existing plant Corps of Engineers	Narragansett Bay Providence, R.I.	C of E WES	?	Model study conducted in existing Narragansett Bay model.

References for Chart III-A-1

[1] Effect of Indian Point Cooling Water Discharge on Hudson River Temperature Distribution. Report submitted by Quirk, Lawler, and Matusky Engineers to Consolidated Edison Company of New York, Inc., February 1969. (The report deals with a mathematical model, but field data assembled in Figs. 1, 2, 3, 11, 12, 13, 14, and 17 with the existing plant in operation are of use in comparisons with hydraulic model studies.)

[2] Hydraulic Model Study, Indian Point Generating Station, Part II. Report submitted by Alden Research Laboratories to Consolidated Edison Company of New York, Inc., covering studies conducted in 1965 and 1966. (The report deals with the first proposed enlargement of the original plant. Figures 10, 11, 12, and 13 contain temperature measurements with only the original plant in operation and can be compared with field data.)

[3] Moyer, S., and Reney, E. C., "Thermal Discharge from Large Nuclear Plant," Proceedings, ASCE, Sanitary Engineering Division, November 1969, pp. 1131-1163.

[4] McAllister, R. J., Neale, L. C., and Brodfeld, B., "Control of Thermal Effects at Beaver Valley Station," Proceedings, ASCE, Power Division, June 1970, pp. 287-298.

[5] Carter, H. H., The Distribution of Excess Temperature from a Heated Discharge in an Estuary. Chesapeake Bay Institute, The Johns Hopkins University, Technical Report 44, October 1968. 39 pp.

[6] Chalk Point Model Study for Potomac Electric Power Company. Alden Hydraulic Laboratory. (Report describes model studies conducted in the period from May 1962 to October 1963 for the proposed fossil fuel plant at Chalk Point.)

[7] Harleman, D. R. F., and Stolzenbach, K. D., A Model Study of Thermal Stratification produced by Condenser Water Discharge. Hydrodynamics Laboratory, Massachusetts Institute of Technology, Report No. 107. 1967.

[8] Harleman, D. R. F., and Stolzenbach, K. D., A Model Study of Proposed Condenser Water Discharge Configurations for the Pilgrim Nuclear Power Station at Plymouth, Massachusetts. Hydrodynamics Laboratory, Massachusetts Institute of Technology, Report No. 113. 1969.

[9] Riesbol, H. S., and Wend, F. H., Therma-Hydraulic Study--Arkansas Cooling Reservoir, ASCE Meeting Preprint 1136 (Memphis, Tennessee, January 1970).

[10] Silberman, E., and Stefan, H., Effects of Condenser Cooling Water Discharge from Projected Allen S. King Generating Plant on Water Temperatures in Lake St. Croix. St. Anthony Falls Hydraulic Laboratory, University of Minnesota, Project Report No. 76. 1964.

[11] Niemeyer, J. A., Modelling of Power Plant Cooling Water Discharges into Lake Monona. M.S. Thesis, Department of Civil Engineering, University of Wisconsin. 1969. 93 pp.

[12] Zeller, R. W., Cooling Water Discharges into Lake Monona. Ph.D. Thesis, University of Wisconsin, 1967. (This thesis reports on field data from an existing plant and proposes a math model to fit it.)

[13] Simmons, H. B., "Use of Models in Resolving Tidal Problems," Proceedings, ASCE, Hydraulics Division, January 1969, pp. 125-146.

B. Allen S. King Plant

The Allen S. King Plant is given as Item 24 in Chart III-A-1. Condenser cooling water from this plant is returned via a surface discharge canal to Lake St. Croix. The lake in the vicinity of the discharge is sketched in Fig. III-B-1. Lake St. Croix is actually part of the St. Croix River, which forms the boundary between Minnesota and Wisconsin and drains into the Mississippi River, where a dam controls the flow. This part of the river is called a lake because at times other than during floods the average discharge is such that mean velocities are of the order of 0.015 fps and the water surface is nearly level.

The King plant has been in operation for two summers at its initial and present capacity of about 550 megawatts. This requires somewhat less than 700 cfs of condenser cooling water raised 17° F. The plant was designed so that the warm water returned to the lake would be stratified for maximum surface cooling. The plant operating permit requires that water at the discharge point not exceed 86° F. Hence, cooling towers are used during adverse weather conditions. With the use of cooling towers there is little stratification during hot summer weather. However, permission was granted to operate the plant without cooling towers for occasional one-day periods to collect field data under adverse conditions. Data were collected during three such one-day periods in the summer of 1969, when the plant load was essentially constant near the design capacity. Some isotherms from these field tests are plotted in Figs. III-B-2 through III-B-5. Water in the discharge canal at the outlet was about 10 ft deep under the test conditions, and the lake was 35 to 40 ft deep.

A physical model study of the dispersion of warm water in Lake St. Croix from the King plant under stratified conditions was completed before the plant permit was issued (Ref. [10], Chart III-A-1). In the model, salt and fresh water were used to represent cool and warm water, respectively. Only surface dispersion patterns were measured (by photographing dyed fresh water fronts), and temperatures were calculated from the dispersion studies. Although a wind of 6 mph was used in calculating temperatures, there was no wind in the model studies, and the dispersion patterns apply to calm conditions. Figure III-B-1 shows the surface dispersion patterns as solid lines for model conditions as close as possible to those of the field studies indicated in Figs. III-B-2 through III-B-4. (Figure III-B-1 is not contained in Ref. [10], Chart III-A-1, but is taken from the original data from which that report was prepared.)

Unfortunately, when the model study was undertaken, the details of the discharge canal were not known. In building the model, the discharge point was selected about 500 ft downstream from where it was finally constructed, and the discharge canal was modeled to follow a straight line from the plant to this point. To make the model and field data comparable, the canal outlet and dispersion pattern in Fig. III-B-1 were shifted bodily by 500 ft to the north and rotated by 28 degrees so that the model outlet location and direction would agree with those of the actual outlet. The broken lines shown the transposed dispersion pattern. Only the two lines of equal time of dispersion closest to the outlet are shown because the transposition of the patterns makes shore interference effects too great on the others; even the second line from the outlet is probably incorrect because of these shore effects.

The contours in Fig. III-B-1 represent equal times of dispersion from the end of the canal. If it is assumed that the stratification is strong (large Richardson number), then there is little interfacial mixing beyond the outlet and all heat loss is through the water surface. Under these conditions, the contours are also isotherms.

Figures III-B-2 through III-B-5 have been plotted by superimposing isotherms from field data over the transposed model isotherms so that direct comparisons can be made. The comparison are fairly good, especially close to the outlet, but they show immediately that winds, which were neglected in the model, have a very important influence on surface spreading. Wind effects are especially prominent in Figure III-B-4, where the wind is blowing along the length of the lake. Figures III-B-2 and III-B-3, in which the wind is not so directly along the length, show less influence of the wind. Figure III-B-5, obtained from data taken on the same day as those in Fig. III-B-4, shows that at greater depths, 5 ft in this case, the dispersion is little affected by direct wind action. (Model dispersion was constant with depth through the stratified layer because of the absence of wind and the comparative unimportance of interfacial mixing and shear.) No field data were obtained at depths between 3 in. and 5 ft, and so it is not known to what depth the wind influences the dispersion patterns.

An interesting anomaly is shown in Fig. III-B-5. Warm water has accumulated at a 5 ft depth near the north shore of the lake. This is apparently the result of down-welling due to the surface transport of warm water seen in Fig. III-B-4. The distorted isotherm pattern at the outlet may be evidence of the return flow from this down-welling. A warm water pocket is also found near the north shore in the measurements taken at 10 ft depth, but it has disappeared in the measurements taken at 15 ft depth. No other warm water is found at 10 ft, and none at all occurs at 15 ft. A similar anomaly may be found in the data at 5 and 10 ft depths corresponding to Fig. III-B-2. This may be a remnant of an earlier wind in another direction. (Many other interesting phenomena, such as the cold water wedge at the discharge canal outlet, may be found in the field data, but these are not directly pertinent to the dispersion study.)

The depth of the warm water layer was not measured in the model for the field conditions corresponding to those in Figs. III-B-2 through III-B-5. However, it was measured for condenser cooling water flow rates of 400 cfs (winter) and 1500 cfs (two units, ultimate plant). From these results, shown in Table II of Ref [10], Chart III-A-1, it is estimated that the model would have predicted depths of 3 to 4 ft close to the outlet and 1 ft near the far shore with no mixing at the outlet. Field data show the depth to have been 5 to 8 ft at the outlet and less than 5 ft further out, but there appears to have been considerable mixing at the outlet even though a cold water wedge occurred in the canal outlet for all field tests.

Temperature calculations in Ref. [10], Chart III-A-1, based on Fig. III-B-1, were made assuming a dew point temperature of 80° F, an ambient lake temperature of 80° F, a canal outlet temperature of 97° F, a wind of 6 mph, and no mixing at the canal outlet. Chart 19, Appendix H, of Ref. [10], Chart III-A-1, shows the calculated isotherms (interpolated from the solid lines of Fig. III-B-1 and others like them) for these conditions. It should be noted in Fig. III-B-4, for example, that even though the condenser cooling water enters the canal at about 95° F as measured in the field tests, it has already cooled to about 92° F at the canal outlet because of the wind and the low humidity. In the model study calculations represented by Chart 19, no allowance was made for cooling in the canal, and adverse conditions were generally selected for the model study report so that the worst possible temperature distributions would be known. The model isotherm temperatures have been recalculated using the method of Ref. [10], Chart III-A-1, for the field data shown in Figs. III-B-2 through III-B-4. Results are shown in the following table.

TABLE III-B-1

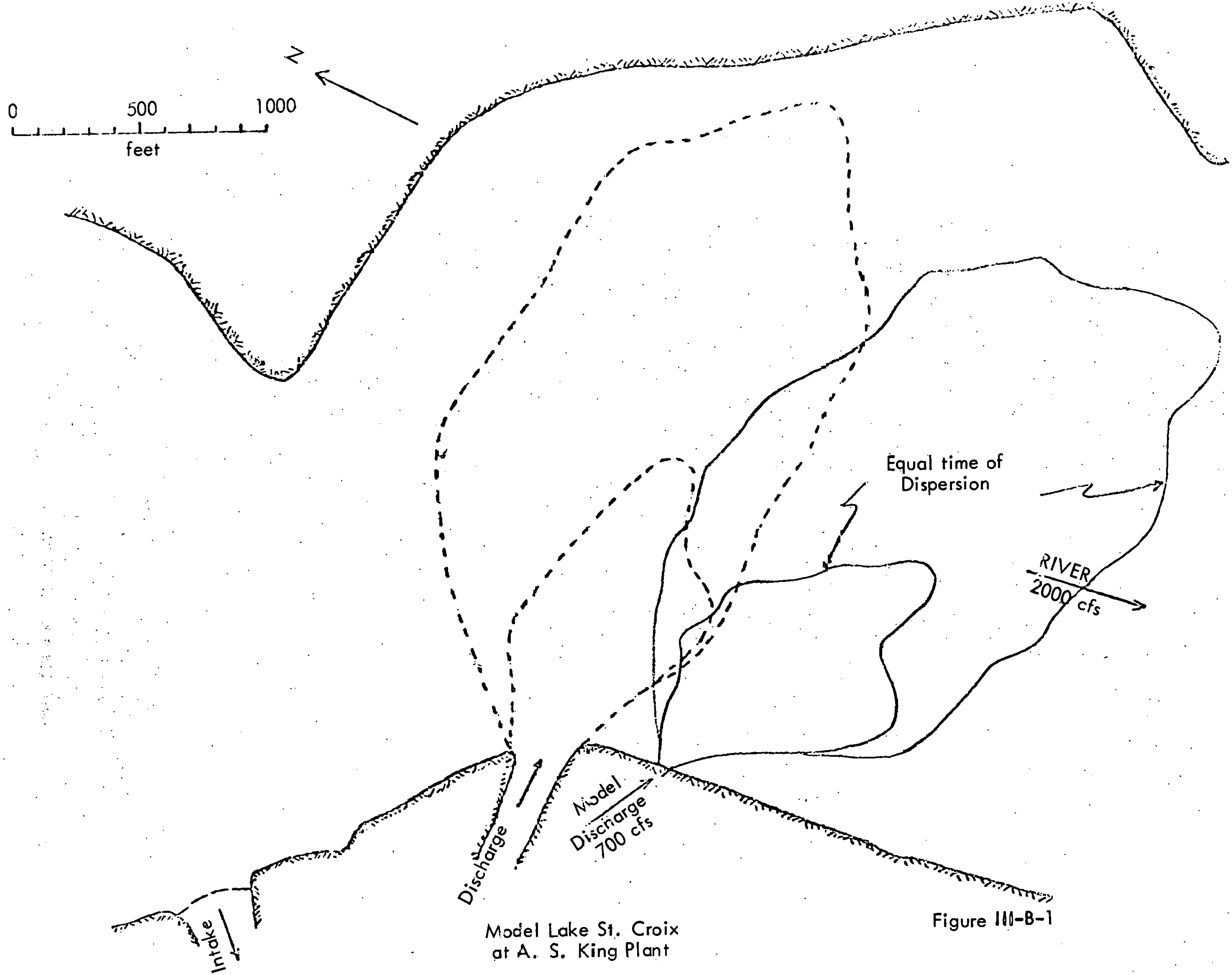
CALCULATED TEMPERATURES ON ISOTHERMS IN MODEL

Fig. No. III-	Dilution Water at Outlet due to Mixing cfs	Dew Point Temp. °F	Ambient Lake Temp. °F	Canal Outlet Temp. after Mixing °F	Wind Speed mph	Isotherm Closest to Outlet °F	Isotherm Farthest from Outlet °F
B-2	350	73.5	78.0	83.3	9	83.0	82.0
B-3	350	65.0	79.0	87.7	6	87.0	85.5
B-4	700	75.5	78.0	85.0	10	85.0	84.0

The rate of surface heat loss appears to be greater than would have been calculated from the model dispersion patterns even for these less severe conditions. There are several reasons for this. Perhaps the most important one is that the model computations assume that the warm water is homogeneous for the full depth of the stratified layer and that the layer moves at a uniform, average velocity. In actuality, the warmest water is probably right at the surface, with decreasing temperatures to the bottom of the stratified layer, and the warmest water near the surface is probably spreading more rapidly than the underlying warm water because of its greater temperature and the surface wind shear. If enough information were available to enable calculation of the speed of the surface layer, a better calculation of the isotherm temperatures could be made.

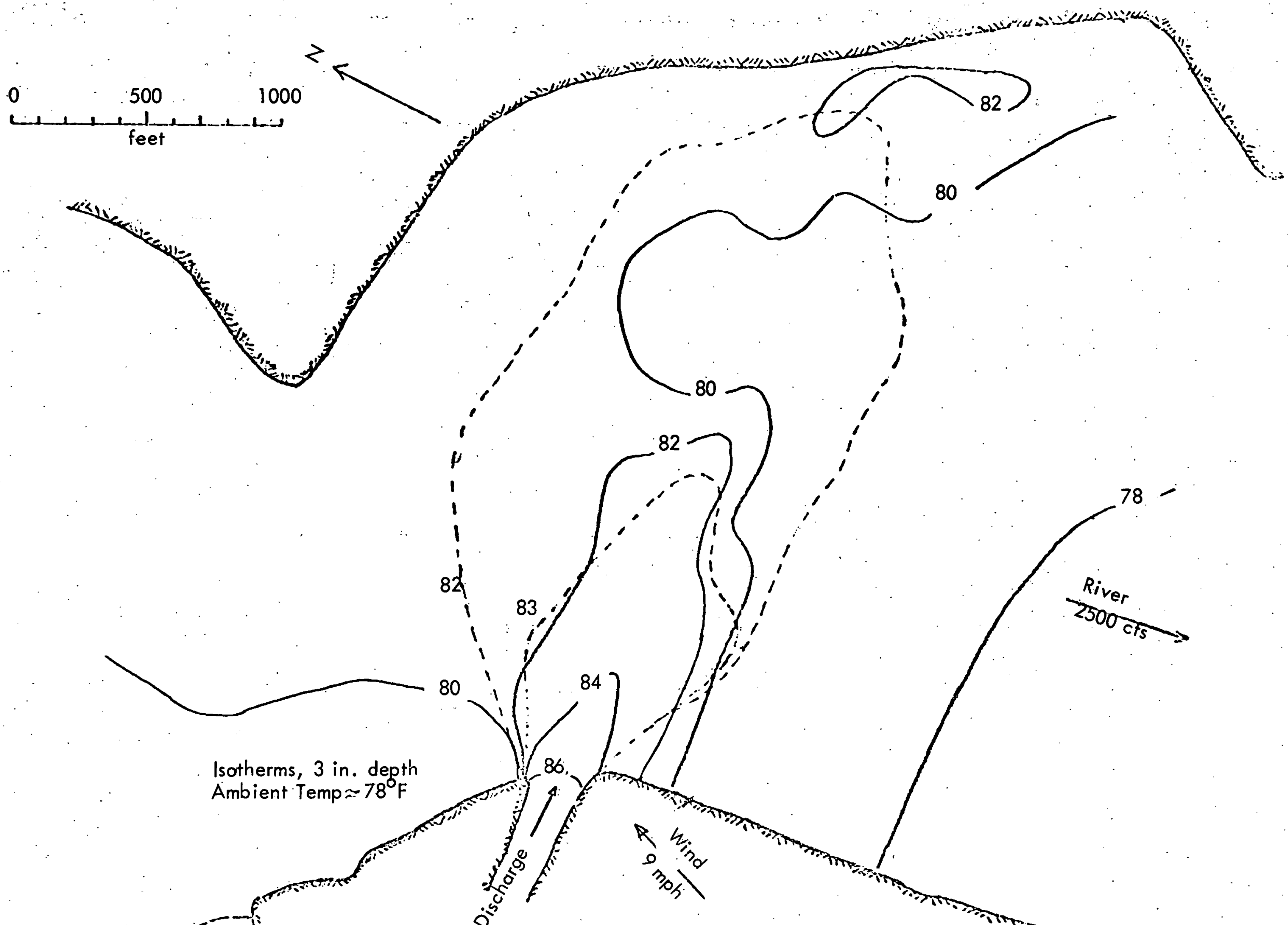
Perhaps the most important conclusions to be drawn from the comparisons herein are as follows.

1. It is very important to include provision for winds in model studies of dispersion.
2. The physical model study would probably have agreed with field data well enough under calm conditions if measurements were available for such conditions.
3. Near the surface, field data should be taken at depths spaced more closely than in the 1969 tests.



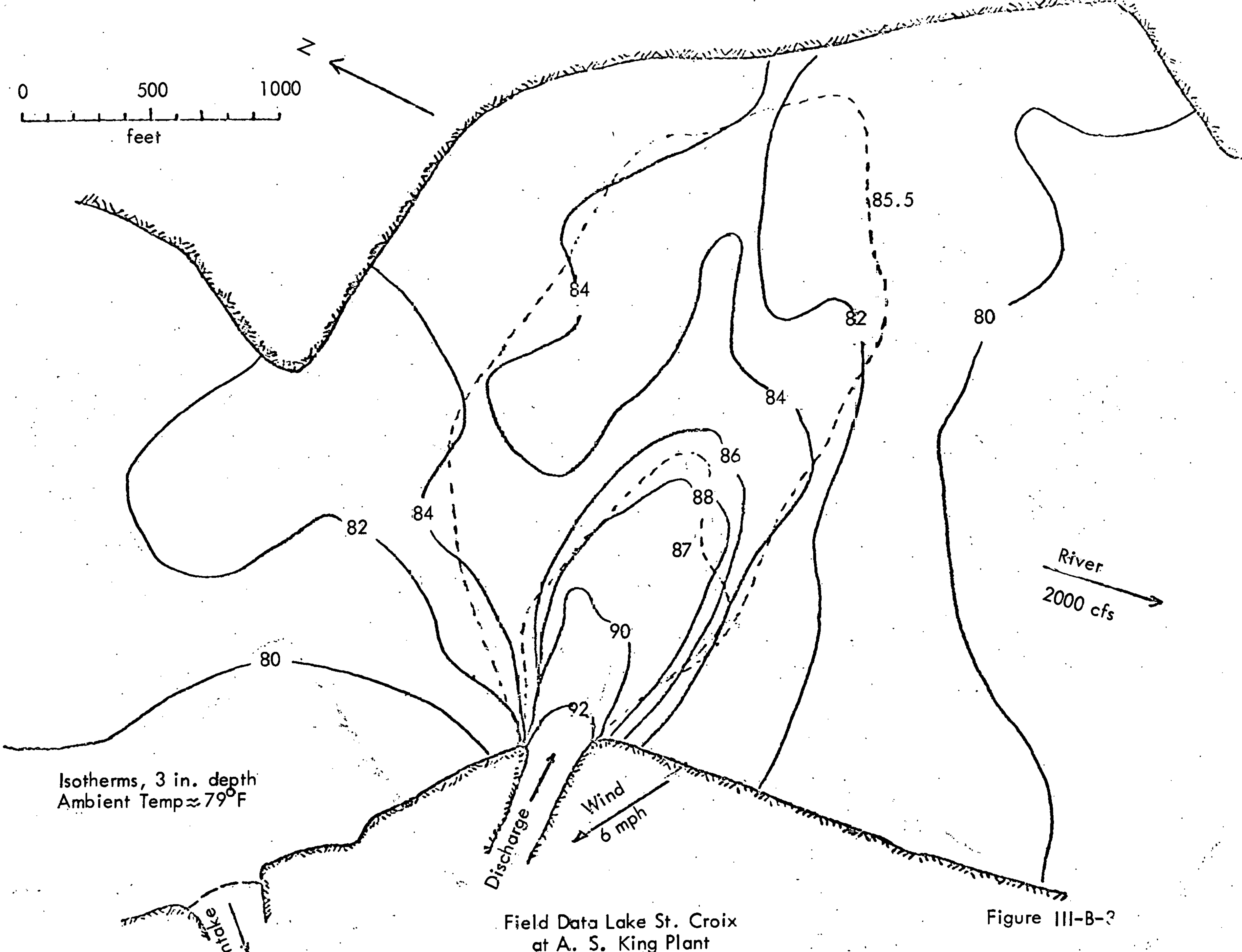
Model Lake St. Croix
at A. S. King Plant

Figure III-B-1



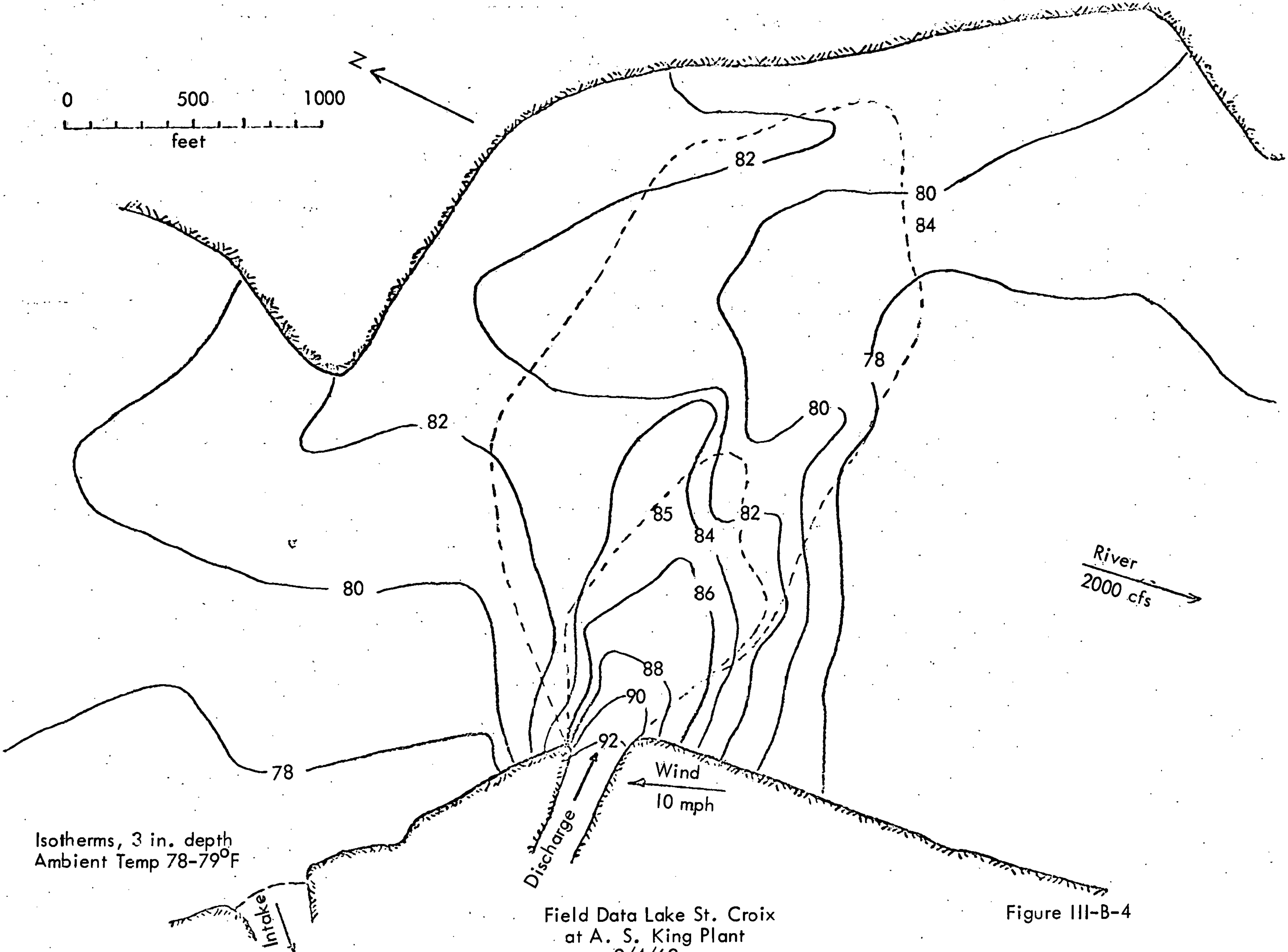
Field Data Lake St. Croix
at A. S. King Plant
7/30/69

Figure III-B-2



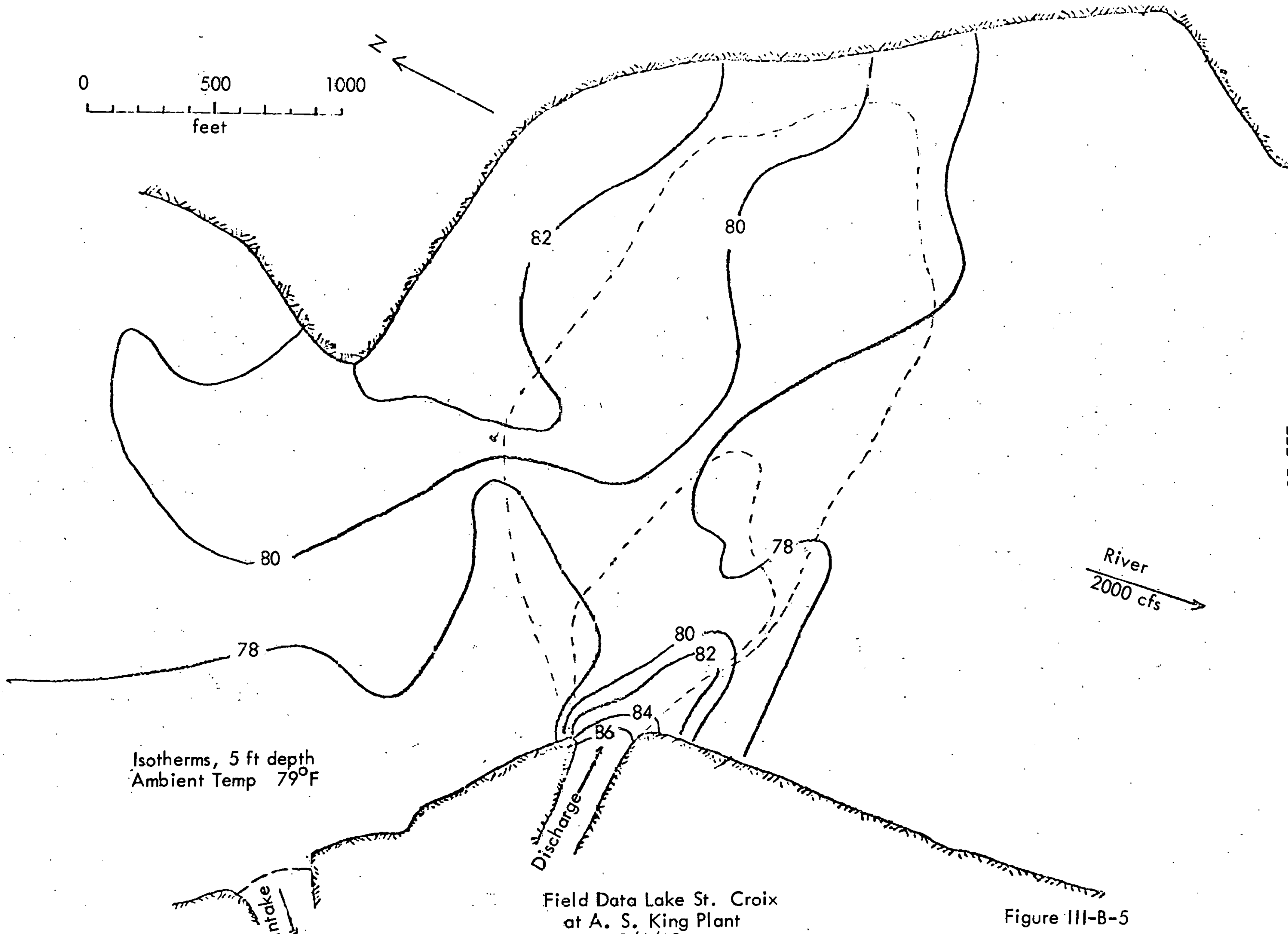
Field Data Lake St. Croix
at A. S. King Plant
8/20/69

Figure III-B-3



Field Data Lake St. Croix
at A. S. King Plant
9/4/69

Figure III-B-4



Field Data Lake St. Croix
at A. S. King Plant
9/4/69

Figure III-B-5

C. Madison Gas and Electric Company Plant

This plant is identified as Item 25 on Chart III-A-1. Condenser cooling water from this 190 megawatt fossil fuel plant is returned via two surface discharge canals to Lake Monona. The lake in the vicinity of the discharge is sketched in Fig. III-C-1. (It should be noted that the baffle is in place in winter only to protect the ice surface and is removed in summer.) The depth for the first 300 ft from the outlet is about 1 meter, but it then drops rather rapidly to the order of 10 meters. A physical model was constructed to study proposed changes at the plant, particularly the effect of doubled heat rejection rates on dispersion and temperature patterns.

Two sets of summer field data were chosen for verifying the model, and it is these data which are pertinent to this report. Table III-C-1, from Ref. [11], Chart III-A-1, tabulates field as well as model parameters used in the comparison. There were two repetitions of the model tests for each set of field data. Reference [11], Chart III-A-1, states that these two sets of field data were chosen for modeling because the winds on those dates (W and SW) were such that wind currents could be reproduced in the model by appropriate water discharges through the model. It was not considered feasible to model other field data that were available because of the inability to model wind currents.

A distorted model was used with a horizontal scale of 1:100 and vertical scale of 1:15. Heated water was used for modeling the discharge and the temperature was controlled by maintaining the densimetric Froude number at the outlet the same in the model as in the prototype, and by requiring that $\underset{\text{model}}{\underline{\Delta\rho/\rho}} = \underset{\text{prototype}}{\underline{\Delta\rho/\rho}}$. The discharge from the canals occurred at densimetric Froude numbers considerably larger than unity so that the discharge was jet-like. The other modeling ratios are:

(Field Discharge) = 5,810 x (Model Discharge)
(Field Velocity) = 3.88 x (Model Velocity)
(Field Time) = 25.8 x (Model Time)

These ratios may be applied in Tables III-C-1 and III-C-2.

Path lines were obtained by use of drogues for comparison between field and model. The warm water layers were measured at about 1 meter thick in the field (about 0.2 ft model) and the drogues were set to operate at half these depths. Figures III-C-2 to III-C-5 show the field test results and corresponding model test results for the two separate model tests for each field condition. The drogues were followed sequentially rather than simultaneously so that there appears to be some crossing of paths.

In addition to path lines, comparisons consisted of field and model measurements of directions of the jet centerline β , velocity along the centerline, and temperature along the centerline, all at about mid-depth of the warm layer. These quantities were measured against distance S along the centerline and are given for field and model in Table III-C-2 taken from Ref. [11]. (The angle β is measured clockwise from a base line normal to the outlet centerline at discharge.)

Isotherm maps at 0.1 ft depth in the model were also prepared, but apparently there was a dearth of field data for comparison. The author of Ref. [11] indicates that such correlations as he made were not good.

The model was also designed to reproduce winter conditions within the baffle shown in Fig. III-C-1. There was some difficulty in obtaining the density change in the model corresponding to the change in water temperature which occurs on passing through 4° C. (The model was kept in a room at normal temperatures, even though cold lake water was drawn into the model to represent ambient conditions.) A comparison between field and model was made for vertical temperature distribution inside and outside the baffle along its length. The comparison given in Ref. [11] is not good. Somewhat better results were obtained in retests during February of 1970 in which the lake surface was covered with a layer of insulating styrofoam. These results were made available in a private communication from Professor John A. Hoopes of the University of Wisconsin.

The method of operating the model was determined by cut and try to make the model reproduce field conditions as closely as possible. The model was moderately successful in doing this, as may be seen in Table III-C-2 and Figs. III-C-2 to III-C-5. The conclusion to be drawn is that a jet-like discharge from a surface cooling water canal into a lake can be modelled with some confidence, both as to the mean properties of the surface jets and the depth of the warm water layer near the outlet.

TABLE III-C-1

GENERAL DATA SUMMARY
(Summertime Data)
(All Velocities and discharges scaled to model)

Characteristic	Field	Model	Model
Modelling the data of	8-30-66	---	---
Lake temperature	24.6°C	23°C	17°C
Livngst. outfall temp.	33.1°C	32°C	27.7°C
Blount outfall temp.	30.7°C	28.6°C	24.9°C
Livngst. discharge	7.36 gpm	7.4 gpm	7.4 gpm
Blount discharge	2.86 gpm	2.9 gpm	2.9 gpm
Lake current velocity	1.6 cm/sec	1.7 cm/sec	1.6 cm/sec
Date of model test	---	8/30-68	10-5-68
Modelling the data of	7-8-66	---	---
Lake temperature	27.5°C	23°C	19.5°C
Livngst. outfall temp.	36.9°C	35°C	32.9°C
Blount outfall temp.	33.5°C	30.3°C	27.7°C
Livngst. discharge	7.7 gpm	7.7 gpm	7.7 gpm
Blount discharge	7.2 gpm	7.2 gpm	7.2 gpm
Lake current velocity	1.8 cm/sec	1.8 cm/sec	1.8 cm/sec
Date of model test	----	8-30-68	9-10-68

TABLE III-C-2

JET CHARACTERISTICS
DATA SUMMARY

Simulation of 7-8-66 Field data
1. 8-30-68 Test data
2. 9-10-68 Test data

Note: Velocities scaled to model

	$S_{℄}$, ft			$Y_{℄}$, ft/sec			β, degrees			$T_{℄}$, °C		
	Field	1	2	Field	1	2	Field	1	2	Field	1	2
Livingston	202	200	200	.141	.16	.18	90	119	125	none	28.5	27.5
	305	300	300	.107	.13	.16	90	115	119		28.5	26.6
	409	400	400	.089	.11	.14	90	104	111		26.7	26.0
	510	500	500	.083	.10	.14	90	90	95		26.7	25.9
		600	600		.08	.09		70	56		26.0	25.6
Blount	188	200	200	.257	.24	.24	90	70	74	---	27.5	26.5
	281	300	300	.135	.18	.22	88	69	68	---	27.0	25.7
	371	400	400	.097	.08	.19	81	63	66	---	27.5	25.5
	581	600	600	.083	.08	.12	57	72	71	28	26.6	25.1
	787	800	800	.076		.08	33	65	62	---	26.8	25.1

(Table III C-2 cont.)

TABLE III-C-2 (Cont.)

JET CHARACTERISTICS
DATA SUMMARY

Simulation of 8-30-66 Field Data
1. 8-30-68 Test Data
2. 10-5-68 Test Data

Note: Velocities scaled to model

	$S_{℄}$, ft			$Y_{℄}$, ft/sec			β, degrees			$T_{℄}$, °C		
	Field	1	2	Field	1	2	Field	1	2	Field*	1	2
Livingston	200	200	200	.137	.12	.23	77	124	115	27	26.7	23.6
	297	300	300	.143	.10	.21	71	120	110	26.5	26.7	23.6
	393	400	400	.105	.09	.18	69	112	103	26	26.7	22.6
	481	500	500	.091	.09	.16	65	91	93	25.5	26.0	21.0
	566	600	600	.038	.08	.12	62	86	84	25	25.5	
Blount	200	206	200	.110	.17	.21	none	79	70	26	27.6	23.0
	300	300	300	.051	.13	.20	"	71	72	25	27.2	22.9
	400	400	400	.048	.10	.19	"	67	74	26	26.5	23.0
	500	500	500	.046	.08	.14	"	68	71	26	26.4	22.1
	600	600	600	.030			"	74	84	25	26.2	22.0

*Estimated

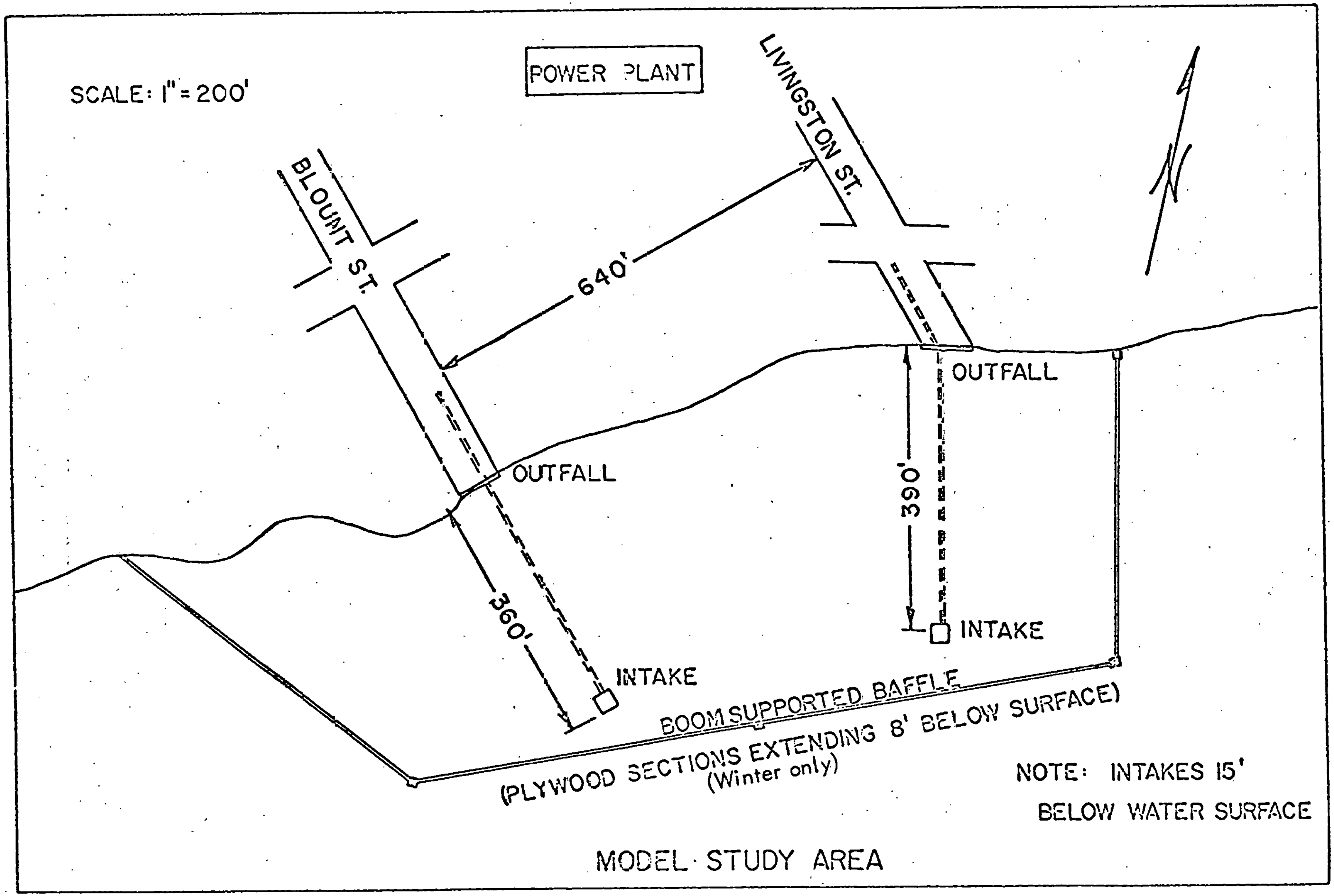


FIGURE III-C-1

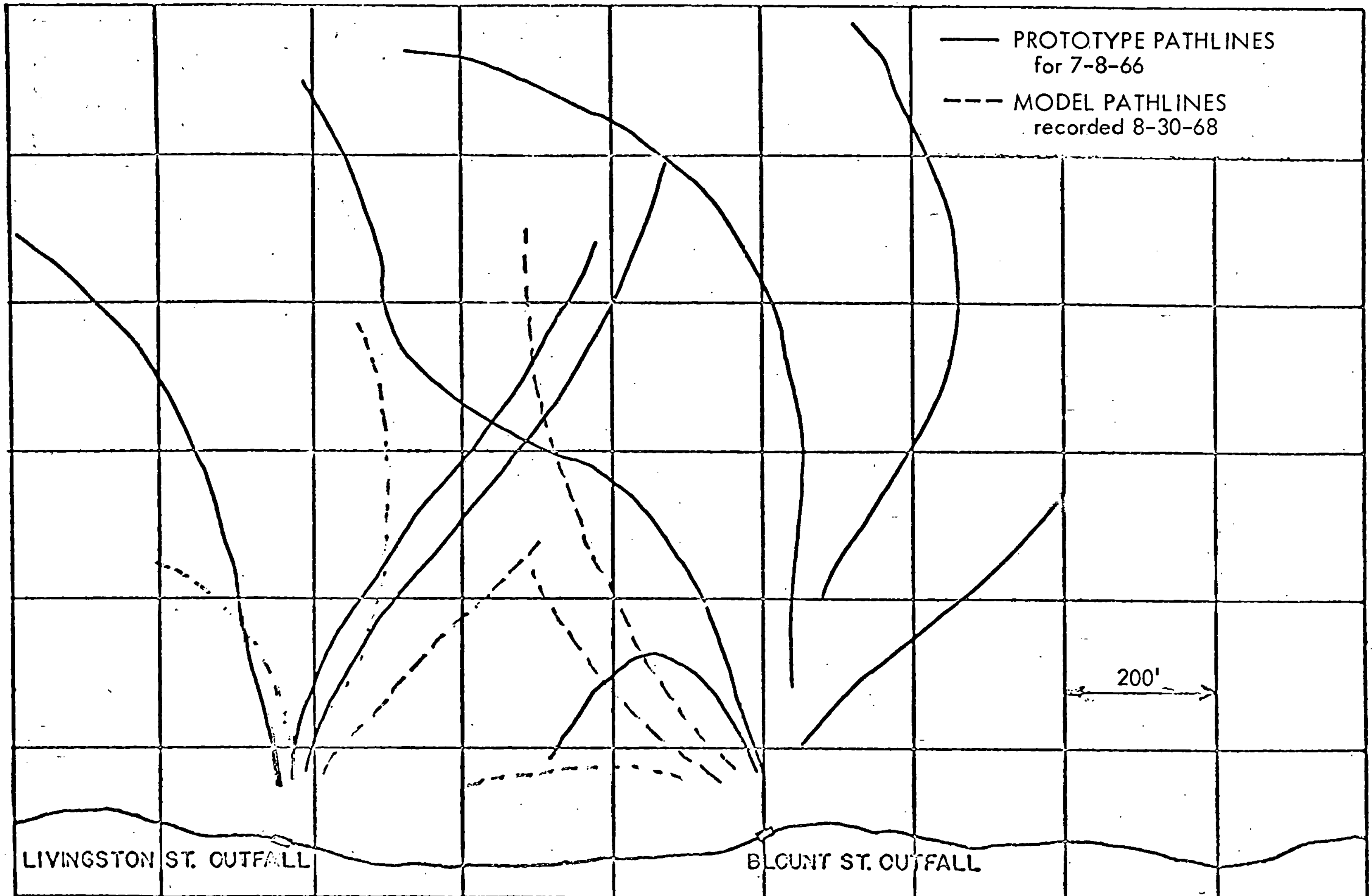


FIGURE III-C-2

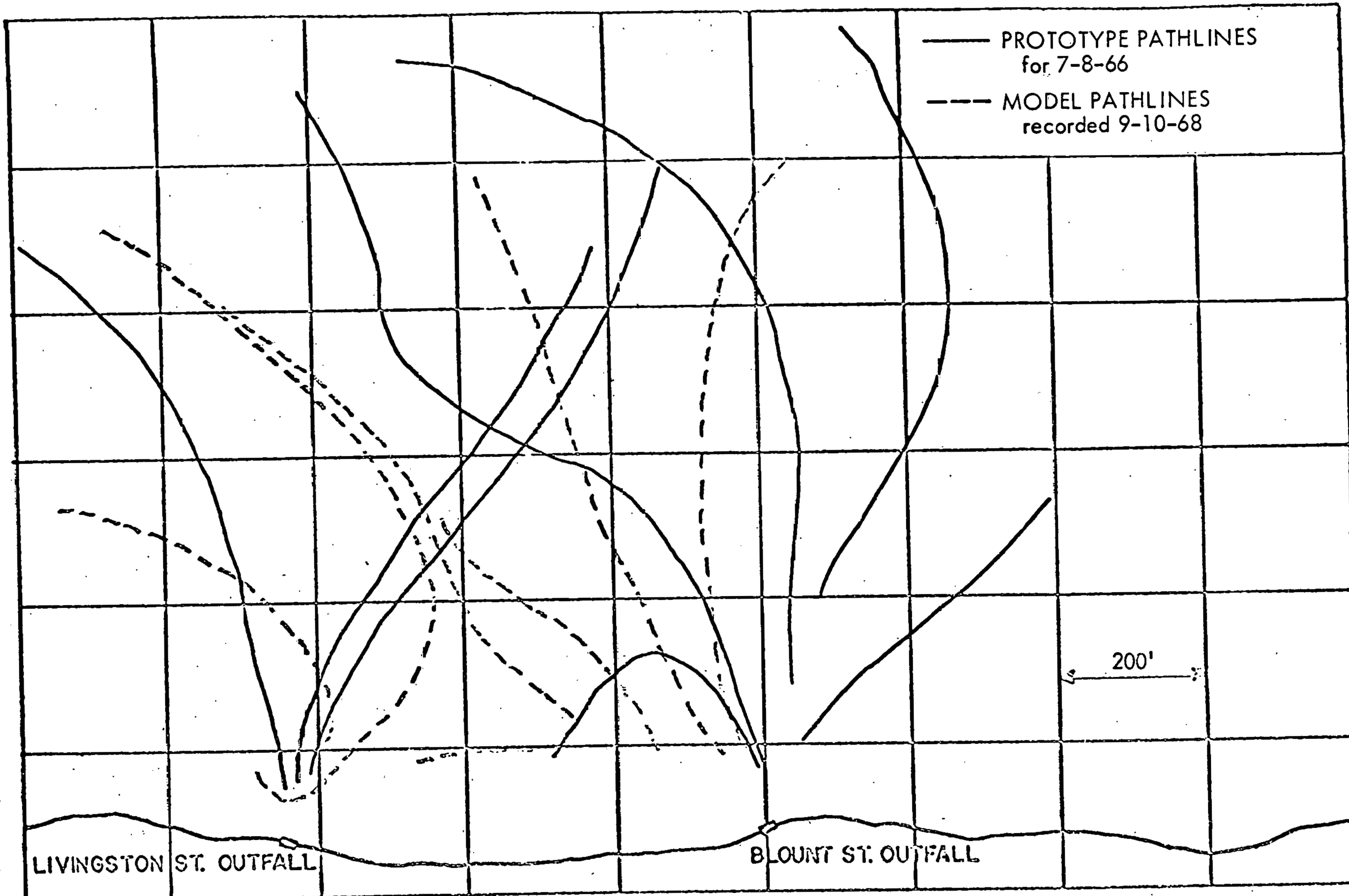


FIGURE III-C-3

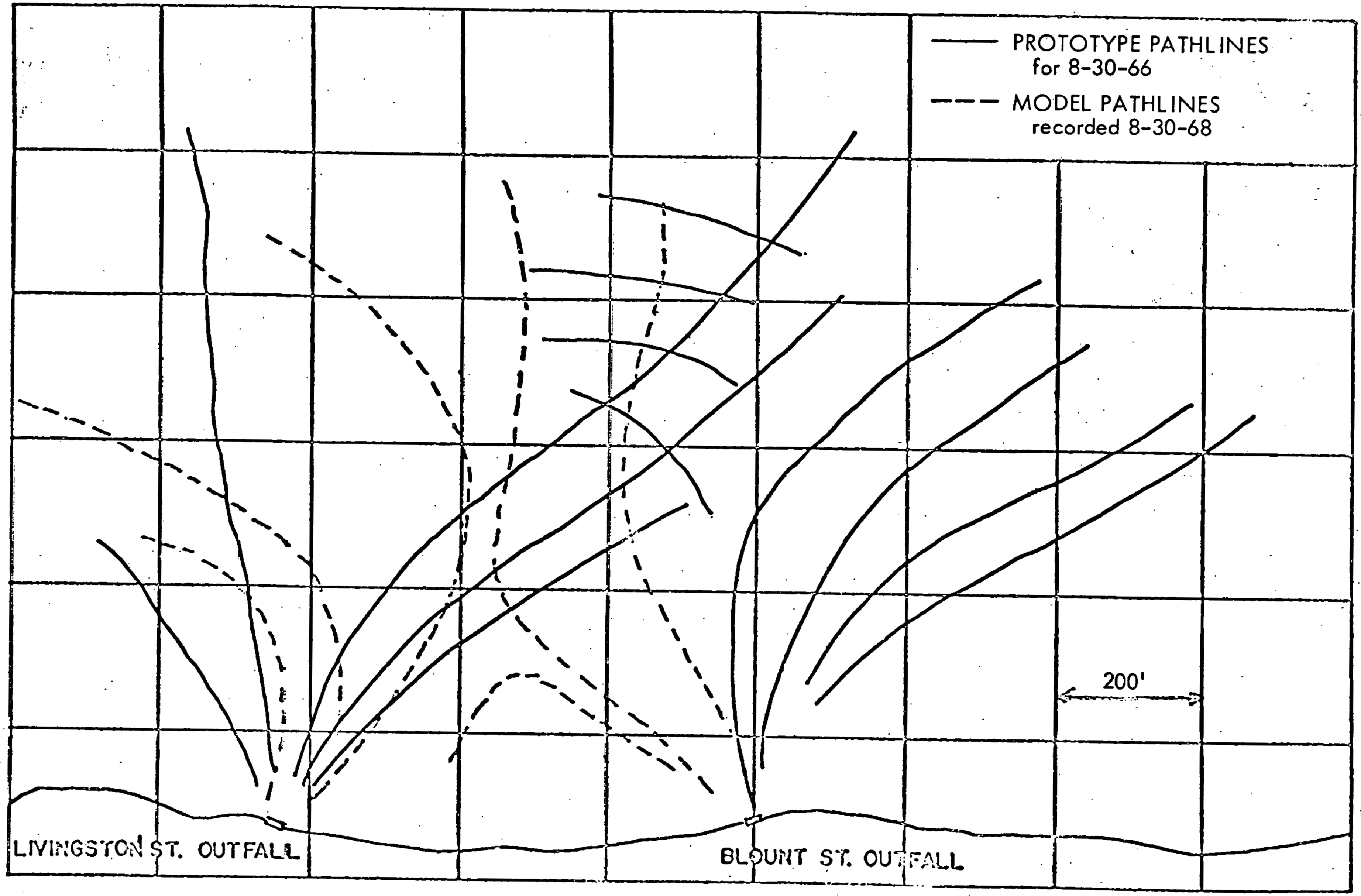


FIGURE III-C-4

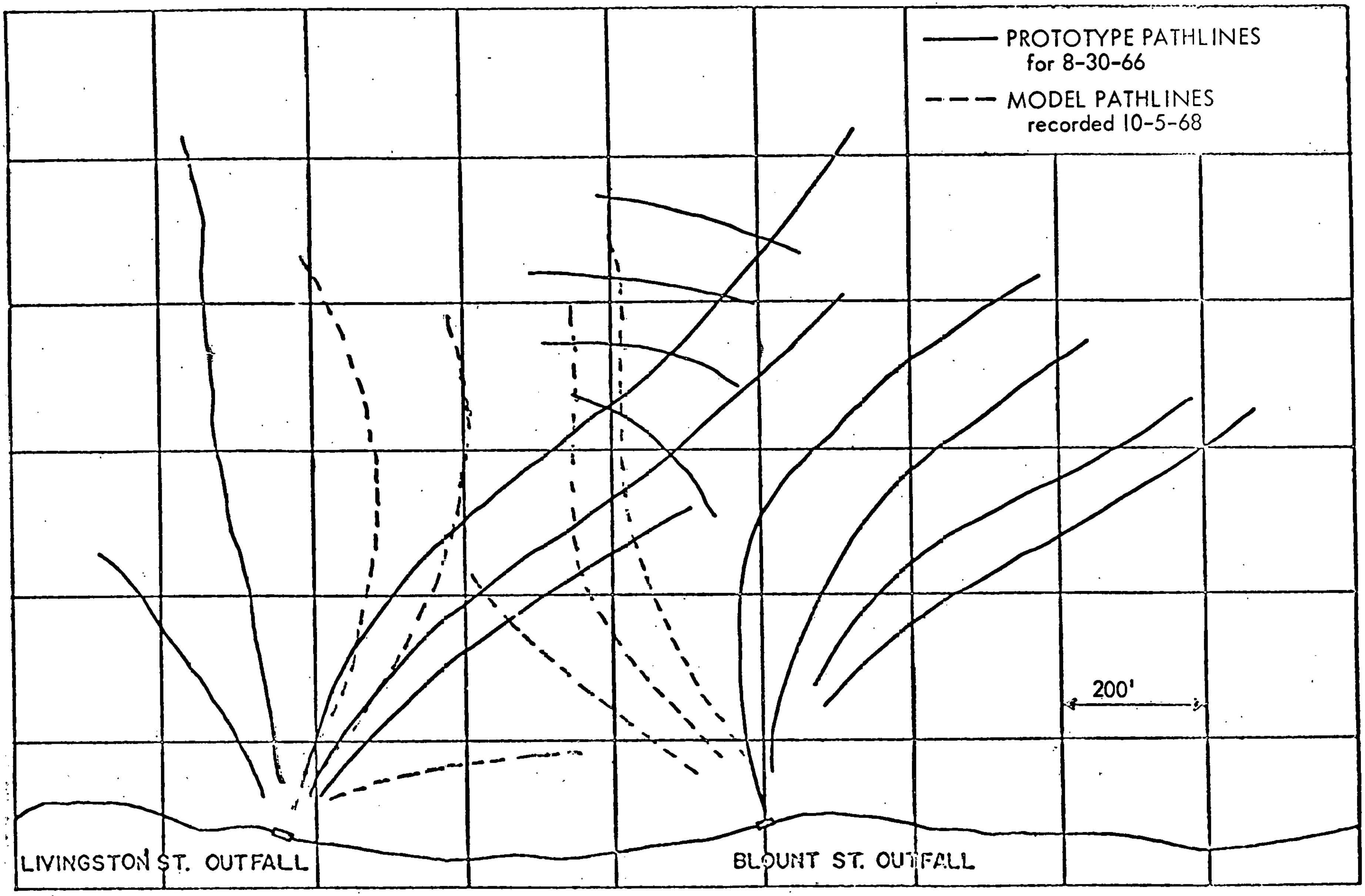


FIGURE III-C-5

D. Indian Point Station

The Indian Point Station is given as Item 1 on Chart III-A-1. Condenser cooling water from an existing fossil fuel plant of about 285 megawatts electrical capacity is returned via a surface canal to the east bank of the Hudson River. The tide exerts an influence at this place in the river, and surface temperature patterns due to warm water emission may be seen to wander upstream and downstream with the tide.

Field data on temperature distributions are presented in Ref. [1] of Chart III-A-1. Surface temperatures were obtained by Texas Instruments, Inc., on April 6, 1968, using radiometric instruments. They are given in Figs. 11-14 of Ref. [1] of Chart III-A-1. Figure 13, representing conditions at mid-flood tide, is particularly useful because nearly corresponding hydraulic model data were obtained during an earlier model program. The plant was at full thermal load in the model study and was at full load in the field study for 5 hours preceding the measuring time, following many hours at about 0.6 load.

The hydraulic model data presented in Ref. [2], Chart III-A-1, were obtained mainly for use in studying means of abating recirculation problems associated with adding a unit to the plant. However, the model was run with only the existing plant in operation in one series of tests, and temperature data were recorded on 100 ft grids (50 ft in some places). Figures 10 through 13 of Ref. [2], Chart III-A-1, show these data at the surface and at 2 ft, 5 ft, and 10 ft depths, respectively, for what is called "incoming" tide. It is assumed this means mid-flood tide. Since the model was being operated to allow study of recirculation, the data cover an area extending about 1300 ft upstream of the discharge canal outlet, but only 200 ft downstream. Data-taking extended 600 ft riverward out of a width of about one mile.

Comparison of Fig. 13 of Ref. [1], with Fig.10 of Ref. [2], Chart III-A-1, shows considerable similarity, although in some locations differences are apparent also. In the model the temperature rise just outside the outlet has dropped to only about 4° F over ambient, even though the plant raised the temperature by 15.5° F. In the field data the closest isotherm to the plant (on a small-scale map) is 8° F over ambient. However, in both cases about 500 feet normal to the shore are required to reduce the temperature rise to 1° F. In the model study the upstream rise is limited to just over 2° F everywhere, since all warmer water apparently is being mixed and carried downstream. In the field data, regularly spaced isotherms extend upstream beginning from the 8° F maximum. Because of the short downstream distance in the model, nothing can be said about comparisons in this direction.

Field data obtained by Northeast Biologists, Inc., using temperature sensors located on a grid plan to provide both surface and sub-surface records are also presented in Ref. [1], Chart III-A-1. (Only a small portion of the data is presented in this reference.) Figures 1 and 2 show average data over one tidal cycle for surface dispersion on two different days. These again are qualitatively similar to the data in Fig. 10 of Ref. [2], Chart III-A-1, showing the 1° F temperature rise curve about 500 to 600 ft riverward from the outlet. The temperature rise extends almost symmetrically upstream and downstream from the outlet, contrary to the pattern in the model, which shows little rise upstream even though the model was operated under flood tide conditions. Figure 3 of Ref. [1], Chart III-A-1 shows isotherms in a vertical cross section opposite the outlet for early flood tide. Model data from Figs. 10 through 13 of Ref. [2], Chart III-A-1, may be superimposed on Fig. 3. When this is done, the comparison is again qualitatively correct, although it does seem that the model surface temperatures near the outlet are too low.

The conclusions to be drawn from this comparison are that the model and the field data are qualitatively comparable, but that there was apparently too much mixing near the outlet in the model. A somewhat stronger stratification appears to have occurred in the field than occurred in the model, and this permitted the warm water to float upstream with the tide in the river but not in the model.

E. Chalk Point Steam Power Plant

This power plant at Chalk Point is Item 8 in Chart III-A-1. A fossil fuel plant of 710 megawatts electrical capacity in two units has been in operation since late 1964 on the west bank of the Patuxent River near Benedict, Maryland. At this point, the Patuxent River is an estuary, since it is influenced by the tide and at the same time has a density profile produced by salt water underlying fresh water. The mean tidal range is about 1.5 ft near the plant. With two-unit operation, about 1100 cfs of condenser cooling water raised about 11.5° F is required for summer operation. The cooling water intake is a short distance downstream from the plant and the discharge canal outlet is upstream, about 9000 ft from the intake (about one tidal excursion). The discharge canal projects upstream into the river for about 1000 ft from shore, with raised banks making a 45° angle with the shore. The river depth is generally 10 ft or less except in the navigation channel, and the average flow is several tens of thousands of cfs.

A hydraulic model study of the plant intake and discharge canals and of a section of the river extending about 20,000 ft upstream and downstream was conducted before the plant was built (Ref. [6], Chart III-A-1). The model scale was 1:300 horizontal and 1:30 vertical. The primary purpose of the model was to study recirculation, and so temperature isotherms in the river obtained in the model are largely for the area between intake and discharge. One set of isotherms is presented upstream of the discharge canal, and a few downstream temperatures are presented for oyster bed locations near both shores. Model data are largely for two-unit operation.

The model was operated with homogeneous fluid except that the condenser discharge water was heated. Model data reduction was accomplished through the use of the unmodified Froude law. No account was taken of either the horizontal salinity gradient between intake and discharge or the vertical salinity gradient, and heat was used largely as a fluid marker, much as dye was used. It was easier to get numerical data from temperature probes than from any type of dye concentration probe. The heated discharge did rise to the surface, of course, and, depending on the tidal phase, became more or less well mixed with the estuary waters.

Field data were obtained by personnel of the Chesapeake Bay Institute during the period September 26 to October 4 1967 for a stretch of the Patuxent River from about 6 miles upstream of the Chalk Point plant to about 3.5 miles downstream while both units were in operation. The data are summarized in Ref. [5], Chart III-A-1. These field data were obtained to check a mathematical model of the plant operation rather than the hydraulic model. Although there are some points of comparison with the hydraulic model, comparisons are difficult because of the limited objective of the model study and because salinity effects were excluded from the model. (It is possible that additional field data may have been obtained by the owner, Potomac Electric Power Company, but no access has been had to those data if they exist. In any event, the last-mentioned difficulty with the model tests would still exist in comparison with any other field data.)

Field temperatures were obtained by continuous sample withdrawal from 0.5 meter depth as a boat traversed across the river. Traverses were taken at ten sections: two between the intake and the discharge, five above the discharge, and three below the intake. Temperatures were recorded as a function of time for a three- to four-hour period on each of two or three days during the test period at each section, the recording period being chosen to include the passage of a temperature front due to tidal action. Temperatures were recorded as excesses over a base temperature for the given phase in the river at a given time. Base temperatures were determined through a complex analysis of the temperature records and through the use of surface cooling data obtained in the hydraulic model study.

It is reported in Ref. [5], Chart III-A-1, that vertical temperature profiles were also obtained to determine the representativeness of the surface traverses. The vertical profile data are not reported, but the following is a summary of pertinent information.

> Within about 500 yards of the discharge, there is no significant vertical temperature variation. This result is expected because of mixing at discharge where the speed is about one fps (densimetric Froude number approximately unity) and because the condenser cooling water is drawn in at a salinity excess of 1.0 to 1.5 per cent over the normal salinity at the discharge point. This result is in disagreement with the hydraulic model tests, which show maximum temperature at the surface, because there was no salinity difference in the model.
>
> Upstream of the discharge point, the cooler temperature is at the surface because of cooling to the atmosphere as the water is convected upstream by the flood tide. The temperature excess near the bottom is estimated at 25 per cent greater than near the surface. This result is also in disagreement with the hydraulic model result, which shows maximum temperature at the surface, for the same reason as above.
>
> Downstream of the discharge point, the surface waters are warmer than the deeper waters by 2 to 3° F within the first tidal excursion and 1° F within the next. These excess surface temperatures account for the net convection of heat down the river. These results are in partial

agreement with the hydraulic model results insofar as it is possible to check them. (Ref. [6], Chart III-A-1, Fig. 3 and Appendix A-2, for example.)

Qualitatively, the surface temperature patterns are similar in model and field in that higher temperature water is shown to be carried upstream on flood tide and downstream on ebb tide. However, there are very apparent discrepancies in details. Figure 3 of Ref. [6], Chart III-A-1, shows surface isotherms (and also isotherms at the 5-ft depth) in an approximately 13,000 ft reach from just below the intake to just above the discharge at ebb tide. Figures 7 and 8 of Ref. [5], Chart III-A-1, give surface temperatures obtained in the two cross-stream traverses in this reach for the channel center and for each bank throughout a quarter tidal cycle. Comparison at ebb tide shows that the temperature rise at midstream is less in the model and that there is little change between midstream and banks in the model, whereas there is considerable such variation in the field data. Unfortunately, the model data used in this comparison were taken from an early phase of the model study when the discharge canal outlet had not yet been placed in the position it now occupies.

A somewhat similar comparison can be obtained upstream of the discharge for a slack water period following flood tide. Figure 6 of Ref. [6], Chart III-A-1, gives surface isotherms (and isotherms at the 5 ft depth) for the model extending about 12,000 ft upstream from the discharge canal outlet, while Figs. 12 and 13 of Ref. [5], Chart III-A-1, give field data for two sections in the reach covered by the model figure. Again, field data are given on the surface as a function of time for midstream and at the two banks. In midstream, model and prototype temperature rises are comparable (but vertical profiles are not, as previously noted), but again the field data show temperature variations across the stream which are more prominent than in the model. Measurable temperature effects in the model extend upstream beyond the limits of the model, as do the field data.

Model data for surface temperatures at two points, one near the east bank and one near the west bank, about 24,000 ft downstream of the discharge point in Tables A-2-a, b, c, and d for Ref. [6], Chart III-A-1 can be compared with the surface temperature data given in Fig. 11 of Ref. [5], Chart III-A-1. Field temperature excess is measured at about 2° F and model excess at about 1° F. Temperatures near the bottom were also measured in the model at these same points and are nearly equal to surface temperatures at all times.

In the field tests an estimate of recirculated water quantity was made using Rhodamine B dye. Recirculation was estimated at about 9.5 per cent. In the model it was determined that intake temperatures would be increased by 1.0 to 1.5 degrees F due to recirculation, and this can be considered to be comparable with the field measurement.

An interesting anomaly mentioned in connection with wind currents in Section B of this chapter can also be detected as a result of river currents in the hydraulic model tests of the Chalk Point Plant. Temperatures were measured at two locations a few hundred feet apart just upstream of a point jutting out from shore near the east bank about 17,000 ft downstream from the discharge point. Measurements were taken near the surface, near the bottom (6.0 to 7.5 ft depth), and near mid-depth. Tables A-2-a through A-2-d of Ref. [6], Chart III-A-1, show that temperatures at mid-depth, particularly, and also near the bottom are considerably greater than the surface temperatures near the end of and following ebb tide. This can be looked upon as the result of downwelling due to river currents flowing against the point.

It can be concluded that although some measure of qualitative agreement between model and prototype was obtained, this was no more than could have been expected intuitively. The model was not operated in a manner that could be expected to produce quantitative comparisons with field temperature measurements, and so the comparisons were not good. Even in this estuary-type flow, where mechanical mixing is quite important, it is still necessary to consider density variation a dynamic variable in planning a model study.

IV. STATUS OF HYDRAULIC MODELING OF THERMAL PLUMES

A. General Requirements for Plume Models

It may be seen from the preceding chapters that hydraulic models of thermal plumes may serve several purposes. It is equally apparent that a single model cannot always serve all purposes. The case studies referred to in Chapter III, for example, were largely applicable to the far field, even though this was not always recognized by those conducting the studies.

To completely model a given plume, it is ideally necessary to model three regions--the near field, the joining region, and the far field. These may be differentiated by the physical phenomena which they do not hold in common. In the near field (close to the outlet), entrainment of ambient fluid is very important. In the joining region, entrainment is still important, but buoyancy, surface cooling, and convection are also important. In the far field, it is surface cooling, dispersion, and convention which are most significant.

In most situations these regions cannot all be combined in one physical model because the vertical scale of the model needs to be large enough to allow room for differentiating stratified layers, to prevent surface tension forces from controlling the dynamics of flow, and to assure turbulent Reynolds numbers. If the horizontal scale were then the same as the vertical scale, the model surface area would be so large as to make the model unwieldy and to defy placing it under a roof where the environment could be controlled. By modeling separately the far field, which occupies by far the largest part of the area, a distorted model can be used, thus reducing the required surface area. Entrainment, however, as seen in Chapter II, can only be modeled at an undistorted scale. Hence, at least one other model is required to cover the near field and the part of the joining region where entrainment is important.

Since entrainment alone is the outstanding characteristic of the near field, it can be modeled very simply. It is only necessary to maintain geometric ratios and to use two miscible fluids of different densities (for example, fresh water into salt water or helium gas into air) to model densimetric Froude number. Such a simple model cannot include the joining region, however, because of the other physical phenomena which become important there. By using a more complicated model, the near field and joining region may be combined so that a minimum of two models could suffice for a given plume. Table IV-A-1 summarizes some of the modeling problems.

When parts of a plume are modeled separately, it is very important to place the proper boundary conditions on the separate models. Unfortunately, this is not always easy from a practical point of view. The importance of boundary conditions at the end of the near field as determined by the joining region is illustrated in Fig. II-13, for example. As another example, in the far field the initial thickness and momentum of the plume are determined by the end conditions in the joining region.

It should not be necessary in every case to make hydraulic models of all regions of a plume. It might be possible, for example, to model only the near field and joining region and to use a mathematical model for the far field. Or it might be possible to construct only a far field hydraulic model, using arbitrary thickness, momentum, and densimetric Froude numbers as input from the unmodeled joining region. Unfortunately, as discussed in Section F, Chapter II, not enough numerical data are at hand to make this procedure really practicable. It is just this sort of partial modeling which is required, however, for obtaining numerical values of pertinent parameters.

B. Practical Considerations in Hydraulic Modeling

Apart from the theoretical and technical considerations which were outlined in the previous section, there are certain practical considerations which control the kinds and numbers of models that can be built and which limit the number of details that can be studied. One of the most important of these is cost.

There are many variable factors which determine the cost of a hydraulic model study. Model construction may involve temporary housing for the model and new heating facilities for producing warm water. In another situation, the model may be placed in an existing space and require only a heat exchanger for obtaining warm water from an existing heating facility. In some models the atmosphere over the model may be controlled; in others, it may be ignored. Many model study costs are hidden by re-using existing facilities or by drawing on engineering talent of the sponsor. Finally, it is almost

TABLE IV-A-1

HYDRAULIC MODELING OF THERMAL PLUMES

Model	Most Important Physical Phenomena	Important Parameters*	Remarks
Near Field (Outlet Region)	Entrainment	Geometry; densimetric Froude no.; local Richardson nos.; boundary conditions at end of model	Can be modeled with any two miscible fluids of different density. Undistorted scale required.
Joining Region	Entrainment Buoyancy Surface Cooling Convection	Boundary conditions at upstream and downstream ends of model; rate of surface cooling; local Richardson nos.; pre-existing stratification; pre-existing currents; wind	May be combined with near field model to eliminate problem of upstream boundary condition. This complicates the near field model.
Far Field	Surface Cooling Convection Dispersion	Boundary conditions at upstream and downstream ends of model; rate of surface cooling; densimetric Froude no.; pre-existing currents; wind	Distorted model usually required. Latter part of joining region may be placed in far field model, but combination with near field is impracticable.

*It is assumed that suitably large Reynolds numbers are obtained in each case.

impossible to obtain financial records of projects from organizations conducting model studies.

Some cost data are available for the model studies associated with the Allen S. King Plant discussed in Chapter III. There were two separate models, first a far field model and then a detailed model of the discharge canal and outlet. The far field model was placed in a tank 17 ft by 50 ft in plan. The horizontal scale was 1:500 and the vertical, 1:30. No provision for control of the atmosphere was made and fresh and salt water were used as the experimental fluids. It was necessary to build a salt water mixing and circulation facility. This far field study cost about $42,000, of which about $15,000 went into constructing the model and appurtenances for controlling and recirculating salt water. About eight months were required from beginning of the model study in July of 1964 to delivery of the final report.

The discharge canal model was constructed at a uniform 1:40 scale and included 2000 ft of canal. It used warm and cool water. The warm water was obtained by heat exchange from the building heating plant; the model was operated mostly during the summer. The cost of this model study was about $28,000. A more modest study of the outlet only would cost considerably less, of course. These studies at 1970 prices would probably cost a quarter to a third more.

The use of fresh and salt water in a model is advantageous in cases where heating requirements would otherwise be very great. Modeling heat loss is very difficult when using salt water and is usually accomplished by calculation. Conductivity probes used for measuring salt concentration are a little more cumbersome to use than are thermistor or thermocouple probes. Also, there are corrosion problems associated with models using salt water. The model must be continually inspected and repairs have to be made often to prevent damage to adjacent equipment. The model has to be destroyed when funds are no longer available for its upkeep.

A model using warm water is capable of meeting the technical requirement for properly modeling surface cooling. However, use of warm water also requires that the bottom and sides of the modeling basin be insulated. Otherwise, heating of the bottom could easily upset the model balance. In controlling surface cooling, shifting the operating temperature range with respect to equilibrium temperature over the model is useful, but care must be taken to maintain the proper density differences in the water.

If photographic records of flow patterns are to be taken, the color and texture of the model is of importance.

C. What Can Be Accomplished with Hydraulic Model Studies

It is desirable to look at the possible uses of hydraulic models for studying thermal plumes by considering each region of the flow separately. For the far field, the discussion will be limited to the lake situation.

The Near Field - Models are particularly useful for complicated outlet geometries at specific sites. They can define the trajectory of the plume and show the possible ranges of heat spread, as well as temperature and velocity distributions in the plume for given input and outlet conditions. They are useful for both submerged and surface buoyant jets.

Models may also be used to study the fundamental physics of the near field as was outlined in Chapter II. They can reproduce mixing and entrainment as a function of geometry and other variables, and provide numerical values to be used in estimating boundary conditions for a subsequent model of the joining region.

Modeling in the near field requires only that the scale be undistorted, that the Reynolds number be large enough to create turbulent flow, that the densimetric Froude number be specified, and, in some cases, that the end boundary condition be known. If there is a current past the outlet, the ordinary Froude number may also have to be specified.

The Joining Region - The rising plume from a submerged outlet is part of the joining region and is subject to study in an undistorted model. Several model studies of this phenomena were listed in Chapter II, Section C. Models in the joining region also define the trajectory and spread of the plume between a surface outlet or rising plume and the far field. Little separate attention has been paid to modeling in this region, the joining region being tacitly included in either near or far field models. It is better included in the former because of the undistorted scale, but then the additional physical phenomena listed in Table IV-A-1 must be considered.

The Far Field - A model of the far field in a lake situation may be expected to define the spreading pattern for heat, given the densimetric Froude number and the input conditions to the plume in the far field. However, there are many secondary problems in far field modeling which must be considered. These have already been discussed and include surface cooling, pre-existing stratification, pre-existing currents, and wind currents. All far field models for a given densimetric Froude number and given input conditions are alike as long as the physical boundaries are remote from the plume.

Far field models may also be used to obtain numerical values for vertical and horizontal eddy diffusivities in plumes and for studying the effects of wind and other currents on plume development. It is a moot question as to whether numerical values for diffusivities in a model are comparable with field values. This subject was mentioned in Chapter II, Section F.

D. Requirements and Possible Goals for Future Model Studies

In the previous section, the general utility of hydraulic models was summarized. It is obvious from that summary and from the case studies reviewed in Chapter III that hydraulic modeling of thermal plumes is not merely an application of a set of scale ratios, but requires judgment as to the phenomena to be modeled. Comparisons between model and prototype are more likely to be qualitatively than quantitatively correct at the present state-of-the-art. It is also evident from the review in Chapter II that, although the basic physical phenomena associated with thermal plumes are now apparently understood, not enough well-controlled model studies have been conducted to make possible the tabulation of numerical coefficients for use in the applicable equations and the extension of the coefficients to full-scale problems. What can be done to improve this situation?

First, more comparisons between field data and model study results need to be made, with complete analysis of the data and with special attention given to some of the detailed points enumerated in the remainder of this section. Such comparisons should be given high priority.

Secondly, a program of fundamental studies extending those already described in Chapter II needs to be carried out to determine the importance and influence of each of the dimensionless numbers in each region of the flow on the flow pattern. Numerical values of coefficients and ranges of values of the various parameters are needed. Specifically, the following topics for further study in hydraulic models can be listed by regions in the flow:

Near Field

1. The parameters which control entrainment near a surface buoyant jet outlet are still not completely known. It is therefore impossible to specify entrainment coefficients in that region. For high enough densimetric Froude numbers, modifications of the results for submerged, buoyant jets can be used, but as the densimetric Froude number decreases, the feedback from further along the jet and perhaps other factors become important. Wada [65] has attempted to relate entrainment to the local Richardson number in this region, but finds that the relation is non-unique. In any event, the local Richardson number is not a practical parameter to use since it has to be determined as part of the flow solution. More information is needed on entrainment for the surface buoyant jet both for constructing future hydraulic models and for use in mathematical models.

2. As was brought out in Chapter II, the bottom slope of the receiving body of water, if it is flat, can have great influence on entrainment and spread of a surface buoyant jet near its outlet. The effect of slope has hardly been considered in previous works and requires detailed model study to determine its effect on entrainment and spread.

3. In the past, it has been tacitly assumed that an outlet canal is little affected by the angle, other than the 90 degrees its centerline makes with the shore. This assumption may be incorrect; secondary currents, for example, may be introduced by such a configuration. Additional secondary flow problems are probably created when both a bottom slope and shore angle other than 90 degrees occur together. Because of lack of symmetry, it may be difficult to specify numerical values for entrainment coefficients and other parameters in these cases, but the problem should be studied further.

4. Although some work has been done for discharge of submerged buoyant jets into receiving water which is already stratified, additional research has to be done on this problem for a broader range of the controlling dimensionless parameters.

Similar research is required for the surface buoyant jet case. Entrainment, plume trajectory, and spread parameters need to be studied.

5. It has been stated that if Reynolds number is sufficiently large, it should have little effect on mixing at an outlet. It is still necessary to define more precisely what "sufficiently large" is in the case of both surface and submerged buoyant plumes, in spite of some work that has already been done. And, if the model Reynolds number cannot be made "sufficiently large," the effect of that parameter on comparison between model and full-scale entrainment must be known.

Joining Region

6. A problem that has not been studied sufficiently concerns the transition from a rising plume issuing from a submerged outlet to the essentially horizontal plume of the far field. It is especially important to be able to predict the thickness and temperature of the initial horizontal plume. To do this, entrainment in the rising plume must be known as a function of several pertinent variables, including the relative depth of the submerged outlet. Secondary influences on the rising plume such as cross currents and bottom or other solid boundary proximity need to be considered.

7. By definition, the joining region continues until entrainment is relatively unimportant. In a prototype, such a definition is of no importance. In a hydraulic model, however, it is necessary to recognize this limit because it marks the true beginning of the region where a distorted model may be applied. It is necessary to study entrainment (as well as interfacial friction and other plume characteristics) in the joining region to find parameters which will define the beginning of the far field. In the same context, it is also important to know what influence feedback from the far field has on fixing the terminus of the joining region.

Far Field

8. In a prototype thermal plume, the plume eventually becomes undetectable because of surface cooling and dilution by

mixing and spreading. In a hydraulic model, even with distortion, physical size limitations frequently prohibit reaching the detectable limits of the plume in a steady state situation. It is then necessary to know what boundary conditions are to be applied in the model to substitute for the correct boundary conditions on surface cooling and mixing in the prototype. Furthermore, by distorting the far field model, the necessary boundary conditions are complicated by altering the horizontal gradients of surface cooling and of density. It may be useful under these conditions to simply by-pass these surface gradients by modeling the no cooling case ($T_E = T_o$ or fresh water into salt water) and then calculate the effects of cooling. The quantitative laws for applying boundary conditions in physical models of the far field require detailed investigation.

9. Another problem requiring study in connection with surface cooling involves the techniques for reproducing in a hydraulic model the surface cooling and other surface thermal effects that occur in nature. Is it sufficient, for example, to simply produce in the model the equilibrium temperature of the prototype atmosphere? Or must radiation effects in depth be modeled also?

10. It has seemed to investigators of far field thermal plume problems that it is reasonable to ask for numerical values of horizontal and vertical eddy diffusivities. Yet, repeatable values have not been obtainable from different hydraulic model tests. The reasons for this were speculated upon earlier, but the fact remains that it is still desirable to attempt to obtain numerical values for these coefficients. One of the problems has been the lack of data on velocity profiles and eddy viscosities. Since these may differ from one model to another, such data should be obtained at the same time as temperature profiles are obtained to define local Richardson numbers. A further problem is that it is not really known whether diffusivities obtained from a model have a definite, scaled relation to those obtained from a prototype, and this problem requires further investigation.

11. It has been customary to think of the far field as a two-layered, density stratified problem. Although prototype plumes are frequently of this type, they need not be. Density gradients will

exist over part of the depth. An exact hydraulic model would reproduce such prototype conditions, of course. But since models are far from exact, the influence of such density anamolies on model results obtained from two-layered systems need to be considered or else means of operating models with anamolies have to be found.

12. Some investigations need to be conducted on viscous shear in the far field. It may not be of importance, but existing experimental results cannot be used to support or refute this supposition.

13. The question of the importance of Reynolds number arises in the far field as it does in the near field. Viscous shear discussed in item 12 is undoubtedly related to Reynolds number. Barr's congruency diagrams [3] have been referred to previously as providing a basis for estimating when the Reynolds number is large enough to neglect in the far field. But these have been determined for a slightly different physical problem--lock exchange flow using fresh and salt water in two dimensions only--and require verification for the application to thermal plumes. These diagrams are also said to be useful for correcting for too small a Reynolds number by using an appropriate distortion. This application, too, requires verification.

14. It has been observed in Chapter III that wind currents, especially, are responsible for difficulties in comparing model and prototype results. No model studies have been conducted on wind or other currents. It is necessary to develop techniques for representing wind effects in producing currents in model studies. This must be done without upsetting the surface cooling modeling discussed earlier. This problem is applicable to joining region models as well as to those of the far field. Methods for modeling other pre-existing currents also need to be developed.

15. Finally, one of the real problems in hydraulic modeling is the tediousness associated with acquiring data, especially velocity data. Techniques have to be developed for obtaining velocity or momentum spread data at least as rapidly as data for heat or mass spread.

Not all of the research topics stated above are of equal importance. In the near field and joining region, the most important problems are those associated with entrainment (items 1 and 7), and study of these could well be undertaken in a single research project. The remaining items in the near field are probably in the approximate order in which they should receive attention, although item 5 on Reynolds number effects could well be studied in a combined project with items 1 and 7.

In the far field, questions on boundary conditions (item 8) are of fundamental importance. Research on modeling wind currents (item 14) is of importance in modeling specific sites. These should receive considerable priority in experimental studies. Problems in diffusivity, eddy viscosity, and interfacial shear (items 10 and 12) have been under study together at the St. Anthony Falls Hydraulic Laboratory (Ref. [67]), and application for further support of the work has been made. Reynolds number effects (item 13) are still under study by Barr and his colleagues at the University of Strathclyde in Glasgow, Scotland.

Several other individuals and laboratories have been engaged in experimental work on some of these topics. Since their work was referred to in Chapter II, no further listing will be attempted here. Most of the installations participating in earlier work have certain facilities, equipment, and instrumentation available and, generally speaking, they would be in the best position to undertake further experimental work. It must be recognized, however, that some of the problems discussed may not be capable of being completely answered, considering the present state of knowledge, regardless of who or what installation performs the research. Partial answers, where complete answers cannot be obtained, will be useful, however.

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