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CREEP ANALYSIS
OF STATICALLY INDETERMINATE BEAMS

W. H. Sutherland and V. B. Watwood Jr.

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W. H. Sutherland and V. B. Watwood, Jr

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Physics and Engineering Division

June 1970

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by

W. H. Sutherland and V. B. Watwood, Jr.

ABSTRACT

A general finite element analysis procedure has been defined which will permit the creep analysis of statically indeterminate beams of arbitrary cross-section using the same structural idealization as used for conventional beam theory. Furthermore, any incremental functional representation for the creep law can be accommodated. Provision has been made in the formulation to consider the time-dependent loading condition resulting from material swelling and time-varying mechanical and thermal loads. Results are presented for a cantilever beam and a conceptual Fast Test Reactor (FTR) flow duct.

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INTRODUCTION

To design adequately the FFTF core, it is necessary to predict accurately the radial and axial deformation of all core structural components during the operational life of the reactor. Furthermore, refueling tolerances require that the position of the flow ducts be known when the core restraint mechanism is released. Preliminary analysis using elastic beam theory predicts relatively large deformation of the flow ducts for projected burnup when the core restraint mechanism is released. These deformations are caused primarily by the swelling gradients existing across the flow ducts and result in large reaction loads at the points of application of the restraint mechanism. Elastic theory assumes that the resulting stresses in the duct wall do not relax and, thus upon release of the restraint mechanism result in outward radial movement of the top of the flow duct.

However, the severe environmental conditions, high temperature and irradiation levels, existing during the operation of the reactor seriously invalidate the results obtained using elastic beam theory. These extreme environmental conditions make it necessary to consider primary and secondary thermal creep, and transient and steady-state irradiation induced creep. These mechanisms for stress relaxation are expected to reduce the duct wall stresses and, thereby reduce the radial deformation of the flow ducts when the restraint mechanism is released.

This report presents the development of an analysis procedure which will permit the creep analysis of any statically

indeterminate beam of arbitrary cross section. Arbitrary creep laws can be accommodated and all loads may be assumed to be time dependent. The end result of this analysis will be the capability to predict the complete time history creep and static response (i.e., displacements, strains, stresses, moments, shear forces, and reactions) for any multiple-supported beam with arbitrary cross section.

SUMMARY AND CONCLUSIONS

A general finite element procedure has been defined which will permit the creep analysis of beam structures using the same structural idealization as used for the conventional elastic beam theory. Furthermore, any arbitrary creep law of the general functional form

$$\Delta \epsilon_e^C = F(\sigma_e, \epsilon_e^C, \phi, T, \Delta t)$$

may be used. Provision has been made in the formulation to accommodate the time-dependent loading condition resulting from material swelling as well as transient mechanical and thermal loads. Results from two sample problems are presented. Excellent correlation has been achieved between the finite element solution and a stationary creep solution for a cantilever beam. The conceptual FTR flow duct with variable cross sections and multiple supports (considering the effects of time dependent swelling loads and transient and steady-state irradiation induced creep) is typical of the beam problems which can be solved with CRASIB.

DISCUSSION

GENERAL METHOD OF ANALYSIS

The principle of virtual displacements is stated independently of the constitutive equation to be used. The principle for a three-dimensional body assuming infinitesimal

deformation theory becomes,

$$\int_V \epsilon_{ij}^* \sigma_{ij} dV = \int_S T_i u_i^* dS + \int_V B_i u_i^* dV. \quad (1)$$

σ_{ij} represents a set of stresses in equilibrium with given surface tractions T_i and body forces B_i .

$$T_i = \sigma_{ij} n_j \text{ on } S_T \quad (2)$$

$$\sigma_{ij,j} + B_i = 0 \text{ in } V \quad (3)$$

where n_j = components of the outward normal to the surface S_T

S_T = surface on which the traction (T_i) are specified.

u_i^* represents an arbitrary small variation of the displacements from the equilibrium configuration which vanish on S_u and ϵ_{ij}^* is a compatible set of strains $\epsilon_{ij}^* = \frac{1}{2} (u_{i,j}^* + u_{j,i}^*)$. Observe that the virtual displacements u_i^* need not follow from the stresses σ_{ij} and it therefore follows that σ_{ij} and ϵ_{ij}^* are not necessarily connected by the prescribed constitutive equation. Verbally the principle states that the work done by the internal forces equals the work done by the surface tractions plus the work done by the body forces for any small displacement variation from the true one which vanishes on the surfaces where displacements are specified.

Utilization of the above principle in conjunction with the Rayleigh-Ritz method permits the derivation of the finite element relationships necessary to analyze the creep behavior of statically indeterminate beams with arbitrary cross-sections.

Assume the constitutive equation to be used is given by

$$\sigma_{ij} = C_{ijmn} \epsilon_{mn}^e \quad (4)$$

where C_{ijmn} = elastic material constants

ϵ_{mn}^e = elastic strains

In the analysis of the flow duct the total strains (ϵ_{mn}) are given by the following sum

$$\epsilon_{mn} = \epsilon_{mn}^e + \epsilon_{mn}^c + \epsilon_{mn}^T + \epsilon_{mn}^S$$

where ϵ_{mn}^c = creep strains (5)

ϵ_{mn}^T = thermal strains

ϵ_{mn}^S = swelling strains.

Substitution of the elastic strains into the principle of virtual displacements (1) gives upon neglecting the body forces B_i

$$\begin{aligned} & \int_V \epsilon_{ij}^* C_{ijmn} \epsilon_{mn} dV - \int_V \epsilon_{ij}^* C_{ijmn} \epsilon_{mn}^c dV \\ & - \int_V \epsilon_{ij}^* C_{ijmn} \epsilon_{mn}^T dV - \int_V \epsilon_{ij}^* C_{ijmn} \epsilon_{mn}^S dV \\ & = \int_S T_i u_i^* dS. \end{aligned} \quad (6)$$

To derive the necessary finite element relationships from Equation (6), the Rayleigh-Ritz procedure requires that a certain functional form be assumed for the displacements and the principle of virtual displacements is then used to make the best choices of undetermined constants which have been incorporated into the general functional form. The general functional form assumed for the displacements is limited only by the condition that the displacements must satisfy the displacement boundary condition on S_u and continuity requirements.

The resulting matrix equations which emerge can be represented symbolically as follows,

$$\underline{K}_{\sim} r_{\sim} = R_{\sim}^c + R_{\sim}^T + R_{\sim}^S + R_{\sim}^a = R_{\sim}^i + R_{\sim}^a \quad (7)$$

where $\underline{K}$ = stiffness matrix for the assembled structure.

$\underline{r}$ = generalized nodal displacement vector

$\underline{R}^C$ = force vector resulting from creep strains

$\underline{R}^T$ = force vector resulting from thermal strains

$\underline{R}^S$ = force vector resulting from swelling strains

$\underline{R}^i = \underline{R}^C + \underline{R}^T + \underline{R}^S$ = force vector resulting from initial strains

$\underline{R}^a$ = force vector resulting from applied tractions.

The stiffness matrix is derived from the first integral and is the same stiffness matrix derived for elastic beam analysis. The force vectors derived from the thermal and swelling strains are computed in the same manner as for the elastic analysis. The formulation permits nonlinear transverse temperature and swelling distributions. The force vector derived from creep strains is obtained from the following integral:

$$\int_V \epsilon_{ij}^* C_{ijmn} \epsilon_{mn}^C dV. \quad (8)$$

To evaluate the integral it becomes necessary to know the creep strain variation across the flow duct. As shown later, an incremental analysis will be employed and the integral to be evaluated becomes

$$\int_V \epsilon_{ij}^* C_{ijmn} \Delta \epsilon_{mn}^C dV \quad (9)$$

Using the von Mises flow rule the triaxial creep strains can be related to uniaxial creep data by

$$\Delta \epsilon_{mn}^C = \frac{3}{2} \frac{\Delta \epsilon_e^C}{\sigma_e} S_{mn} \quad (10)$$

$$\sigma_e = \sqrt{3/2 S_{ij} S_{ij}} \quad (11)$$

$$\text{where } S_{ij} = \sigma_{ij} - \frac{1}{3} \delta_{ij} \sigma_{kk}. \quad (12)$$

Since the initial elastic stress distribution is known across the flow duct it is possible to calculate $\Delta \epsilon_{mn}^C$ for the first time interval at a fixed number of points uniformly distributed across the flow duct. Subsequent stress distributions at later time intervals can be determined by computing the change in stress at each of these points.

$$\Delta \sigma_{ij} = C_{ijmn} (\Delta \epsilon_{mn} - \Delta \epsilon_{mn}^C - \Delta \epsilon_{mn}^T - \Delta \epsilon_{mn}^S) \quad (13)$$

Knowledge of the stress distribution across the flow duct at each time interval, together with the specified creep law, permits the evaluation of the force vector derived from creep strains.

The general incremental procedure for the solution of creep problems proceeds in the following manner. At time equal to zero (Solution Point 1) the initial loads are applied and the elastic response ($\underline{K} \underline{r}_1 = \underline{R}_1$) calculated. This state of stress (initial elastic stress) is assumed to be constant during the following time interval (Time Interval 1). The creep strains occurring during the time interval can be evaluated at the end of the time interval and related to a force vector as previously discussed. Then for the second solution point new load vectors are computed including the contribution from the creep strains

$$\underline{K} \underline{r}_2 = \underline{R}_2 = \underline{R}_1^i + \Delta \underline{R}_{12}^i + \underline{R}_1^a + \Delta \underline{R}_{12}^a \quad (14)$$

where $\underline{R}_2$ is the total loads existing at solution point 2, $\underline{R}_{12}^i$ is the change in the initial strain "loads" including the

creep during Increment 1, and R_{12}^a is the change in the applied load vector. Observe that the applied loads may be modified at each time interval as well as the loads resulting from initial strains. This feature is essential to permit the modeling of the loading induced by stainless steel swelling in the flow ducts.

The solution procedure applies for incremental values of the displacements and loads and can be shown in the following manner.

$$\text{Let } \underline{r}_2 = \underline{r}_1 + \Delta \underline{r}_{12} \quad (15)$$

where $\Delta \underline{r}_{12}$ is the change in the displacements.

$$\text{This gives } \underline{K}(\underline{r}_1 + \Delta \underline{r}_{12}) = \underline{R}_1^i + \underline{R}_1^a + \Delta \underline{R}_{12}^i + \Delta \underline{R}_{12}^a \quad (16)$$

$$\text{or } \underline{K} \Delta \underline{r}_{12} = \Delta \underline{R}_{12}^i + \Delta \underline{R}_{12}^a \text{ since } \underline{K} \underline{r}_1 = \underline{R}_1^i + \underline{R}_1^a. \quad (17)$$

Knowledge of the change in displacements ($\Delta \underline{r}_{12}$) permits the calculation of the change in total strains ($\Delta \epsilon_{mn}$). Since the change in creep strains ($\Delta \epsilon_{mn}^c$), thermal strains ($\Delta \epsilon_{mn}^T$), and swelling strains are already known for the time interval, the change in stresses ($\Delta \sigma_{ij}$) can be computed as previously indicated. Adding the stresses ($\Delta \sigma_{ij}$) to the previous stress distribution gives the new stress distribution across the flow duct. This state of stress is assumed to be constant during the next time interval (Time Interval 2) and the incremental creep strains are calculated based upon the new stress distribution. This same procedure is repeated until the final time of interest is reached.

SPECIALIZATION TO THE BEAM PROBLEM

Initial consideration has been given to a beam whose orientation is restricted to the x-y plane as shown in Figure 1. Positive direction and number designation for the six degrees of freedom are given in Figure 1.

FIGURE 1. Basic Stiffness Element for Beam Analysis

To derive the necessary finite element relationships (Equation 7) a general functional form must be assumed for the displacements u throughout the beam and v along the beam reference line (cross section centroid). To ensure a linear variation of moment and constant shear within the beam element, the displacement $v(x)$ is assumed to be of the general form

$$v(x) = a_0 + a_1x + a_2x^2 + a_3x^3. \quad (18)$$

Evaluating the displacement $v(x)$ and slope $\frac{dv(x)}{dx}$ at the nodal points i and j permits the solution for a_i in terms of the nodal point displacements and slopes. Denote the generalized nodal displacements (α_i) by $v_i(0) = \alpha_2$; $\frac{dv_i(0)}{dx} = \alpha_3$; $v_j(L) = \alpha_5$; $\frac{dv_j(L)}{dx} = \alpha_6$. The displacement $v(x)$ of the beam reference line may then be written as

$$v(x) = \sum_{i=1}^6 \phi_i(x) \alpha_i \quad (19)$$

$$\begin{aligned} \phi_1 &= 0 & \phi_4 &= 0 \\ \phi_2 &= \left(1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3}\right) & \phi_5 &= \left(\frac{3x^2}{L^2} - \frac{2x^3}{L^3}\right) \\ \phi_3 &= \left(x - \frac{2x^2}{L} + \frac{x^3}{L^2}\right) & \phi_6 &= \left(\frac{-x^2}{L} + \frac{x^3}{L^2}\right) \end{aligned} \quad (20)$$

In addition the assumption is made that v is of a form throughout the beam that the resulting shear strain ϵ_{xy} is everywhere zero. This assumption drops the shear contribution from the virtual work expression and ensures complete continuity at element junctions.

The displacement $u_{na}(x)$ at the neutral axis is assumed to be of the general form $u_{na}(x) = b_0 + b_1x$. Evaluating at nodal points i and j and denoting $u_{na_i}(0) = \alpha_1$; $u_{na_j}(L) = \alpha_4$ gives

$$u_{na}(x) = \left(1 - \frac{x}{L}\right) \alpha_1 + \frac{x}{L} \alpha_4. \quad (21)$$

The displacement $u(x,y)$ for any point in the beam is then given by

$$u(x,y) = u_{na}(x) - y \frac{dv(x)}{dx} = \sum_{i=1}^6 \phi_i \alpha_i \quad (22)$$

$$\begin{aligned} \text{where } \phi_1 &= \left(1 - \frac{x}{L}\right) & \phi_4 &= \frac{x}{L} \\ \phi_2 &= -y \left(\frac{6x^2}{L^3} - \frac{6x}{L^2}\right) & \phi_5 &= -y \left(\frac{-6x^2}{L^3} + \frac{6x}{L^2}\right) \\ \phi_3 &= -y \left(\frac{3x^2}{L^2} - \frac{4x}{L} + 1\right) & \phi_6 &= -y \left(\frac{3x^2}{L^2} - \frac{2x}{L}\right). \end{aligned} \quad (23)$$

The assumed displacement field implies that the only nonvanishing strain is ϵ_{xx} . To obtain the stiffness matrices and the force vectors resulting from initial strains, it is necessary to obtain the strain ϵ_{xx} in terms of generalized nodal displacements.

$$\epsilon_{xx} = \frac{\partial u}{\partial x} = \sum_{i=1}^6 \frac{\partial \phi_i}{\partial x} \alpha_i \quad (24)$$

$$\begin{aligned} \text{where } \frac{\partial \phi_1}{\partial x} &= -\frac{1}{L} & \frac{\partial \phi_4}{\partial x} &= \frac{1}{L} \\ \frac{\partial \phi_2}{\partial x} &= -y \left(\frac{12x}{L^3} - \frac{6}{L^2}\right) & \frac{\partial \phi_5}{\partial x} &= -y \left(\frac{-12x}{L^3} + \frac{6}{L^2}\right) \\ \frac{\partial \phi_3}{\partial x} &= -y \left(\frac{6x}{L^2} - \frac{4}{L}\right) & \frac{\partial \phi_6}{\partial x} &= -y \left(\frac{6x}{L^2} - \frac{2}{L}\right). \end{aligned} \quad (25)$$

In matrix form Equation (24) becomes

$$\epsilon_{xx} = \underset{\sim}{g}^{(T)} \underset{\sim}{\alpha} \quad (26)$$

$$\text{Likewise } \epsilon_{xx}^* = \underset{\sim}{g}^{(T)} \underset{\sim}{\alpha}^* \quad (27)$$

where (T) denotes transposition of $\underset{\sim}{g}$.

In addition to the direct displacement field assumptions, some additional assumptions on stresses are made consistent with the usual beam formulation i.e.,

$$\sigma_{yy} = \sigma_{zz} = 0.$$

Substitution of this constraint in the total three-dimensional constitutive relation:

$$\sigma_{ij} = \frac{E}{(1+\nu)} \left[\epsilon_{ij} - \epsilon_{ij}^C - \epsilon_{ij}^T - \epsilon_{ij}^S + \frac{\nu \delta_{ij}}{(1-2\nu)} (\epsilon_{kk} - \epsilon_{kk}^T - \epsilon_{kk}^S) \right]$$

(note that $\epsilon_{kk}^C \equiv 0$ has been used) results in a reduction of this relationship to

$\sigma_{xx} = E [\epsilon_{xx} - \epsilon_{xx}^C - \epsilon_{xx}^T - \epsilon_{xx}^S]$ so that the principle of virtual displacements, Equation (6), is reduced for the beam problem to the following expression,

$$\begin{aligned} \int_V \epsilon_{xx}^* E \epsilon_{xx} dV - \int_V \epsilon_{xx}^* E \epsilon_{xx}^C dV - \int_V \epsilon_{xx}^* E \epsilon_{xx}^T dV \\ - \int_V \epsilon_{xx}^* E \epsilon_{xx}^S dV = \int_S T_i u_i^* dS. \end{aligned} \quad (28)$$

In matrix notation Equation (28) becomes,

$$\begin{aligned} \int_V \epsilon_{\sim}^{*(T)} E \epsilon_{\sim} dV - \int_V \epsilon_{\sim}^{*(T)} E \epsilon_{\sim}^C dV - \int_V \epsilon_{\sim}^{*(T)} E \epsilon_{\sim}^T dV \\ - \int_V \epsilon_{\sim}^{*(T)} E \epsilon_{\sim}^S dV = \int_S u_{\sim}^{*(T)} T_{\sim} dS. \end{aligned} \quad (29)$$

The complete beam structure is assumed to be divided into a finite number of short beam segments. E and α are assumed to be constant within each beam segment. Substituting Equations (26) and (27) into Equation (29) and evaluating the first expression over a single segment gives the following result for the individual element stiffness matrix.

$$\int_V \epsilon^*(T) E \epsilon \, dV = \int_V \alpha^*(T) g E g \alpha \, dV \quad (30)$$

It should be observed that the coordinate system is assumed to be adjusted in such a manner that

$$\iint_A y \, dA = 0 \quad (31)$$

Also $\iint y^2 \, dA = I_z$ where I_z represents the moment of inertia of the beam cross-section about the z-axis. Incorporating these results into Equation (3) gives the following result for a single element:

$$\int_V \epsilon^*(T) E \epsilon \, dV = \alpha^*(T) \underline{k} \alpha \quad (32)$$

where

$$\underline{k} = \begin{Bmatrix} AE/L & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI}{L^3} z & 0 & 0 & 0 & 0 \\ 0 & \frac{6EI}{L^2} z & \frac{4EI}{L} z & 0 & 0 & 0 \\ -\frac{AE}{L} & 0 & 0 & \frac{AE}{L} & 0 & 0 \\ 0 & \frac{-12EI}{L^3} z & \frac{-6EI}{L^2} z & 0 & \frac{12EI}{L^3} z & 0 \\ 0 & \frac{6EI}{L^2} z & \frac{2EI}{L} z & 0 & \frac{-6EI}{L^2} z & \frac{4EI}{L} z \end{Bmatrix} \quad \text{Symmetric} \quad (33)$$

Next it is necessary to evaluate the force vector for a single element resulting from creep strains which corresponds to the second expression in Equation (29).

$$\int_V \epsilon^*(T) E \epsilon^C \, dV = \int_V \alpha^*(T) g E \epsilon^C \, dV. \quad (34)$$

If a linear distribution of creep strain is enforced between the end points i and j, then the creep strain ϵ_{xx}^C may be expressed in terms of its nodal point values

$$\epsilon_{xx}^c = \epsilon_{xx,i}^c \left(1 - \frac{x}{L}\right) + \epsilon_{xx,j}^c \frac{(x)}{L}. \quad (35)$$

Substituting Equation (35) into Equation (34) and integrating over the length of the beam gives the following result.

$$\int_V \epsilon_{xx}^c E \epsilon_{xx}^c dV = \alpha \int_V \gamma_{xx}^c \left\{ \begin{array}{l} -\frac{1}{2} \left(\epsilon_{xx,i}^c + \epsilon_{xx,j}^c \right) \\ \frac{Y}{L} \left(\epsilon_{xx,i}^c - \epsilon_{xx,j}^c \right) \\ Y \epsilon_{xx,i}^c \\ \frac{1}{2} \left(\epsilon_{xx,i}^c + \epsilon_{xx,j}^c \right) \\ -\frac{Y}{L} \left(\epsilon_{xx,i}^c - \epsilon_{xx,j}^c \right) \\ -Y \epsilon_{xx,j}^c \end{array} \right\} dA \quad (36)$$

$$\gamma_{xx}^c = E \int \int_A \quad (37)$$

The force vector for a single element resulting from thermal strains and swelling strains may be evaluated in the same manner as the force vectors resulting from creep strains. Denote the thermal strain by

$$\epsilon_{xx}^T = \alpha \Delta T \quad (38)$$

where α = coefficient of thermal expansion
and ΔT = temperature ($^{\circ}\text{F}$) above reference temperature, and
the swelling strain by

$$\epsilon_{xx}^S = \beta \quad (39)$$

Enforcing the condition of a linear distribution of thermal and swelling strains between end points i and j implies the following expressions for ΔT and β

$$\Delta T = \Delta T_i \left(1 - \frac{x}{L}\right) + \Delta T_j \left(\frac{x}{L}\right) \quad (40)$$

$$\beta = \beta_i \left(1 - \frac{x}{L}\right) + \beta_j \left(\frac{x}{L}\right)$$

Substituting Equation (40) into the third and fourth expression in Equation (29) and integrating over the length of the beam gives the following results.

$$\int_V \epsilon_{\sim}^*(T) E \epsilon_{\sim}^T dV = \alpha_{\sim}^*(T) \gamma_{\sim}^T \quad (41)$$

where

$$\gamma_{\sim}^T = \iint_A E \alpha \left\{ \begin{array}{l} -\frac{1}{2} (\Delta T_i + \Delta T_j) \\ \frac{y}{L} (\Delta T_i - \Delta T_j) \\ y (\Delta T_i) \\ \frac{1}{2} (\Delta T_i + \Delta T_j) \\ -\frac{y}{L} (\Delta T_i - \Delta T_j) \\ -y (\Delta T_j) \end{array} \right\} dA, \quad (42)$$

and

$$\int_V \epsilon_{\sim}^*(T) E \epsilon_{\sim}^S dV = \alpha_{\sim}^*(T) \gamma_{\sim}^S \quad (43)$$

where

$$\gamma_{\sim}^S = E \iint_A \begin{Bmatrix} -\frac{1}{2} (\beta_i + \beta_j) \\ \frac{y}{L} (\beta_i - \beta_j) \\ y (\beta_i) \\ \frac{1}{2} (\beta_i + \beta_j) \\ -\frac{y}{L} (\beta_i - \beta_j) \\ -y (\beta_j) \end{Bmatrix} dA. \quad (44)$$

To evaluate the last expression in Equation (29) the displacement vector $u_{\sim}^*$ is expressed in terms of the generalized nodal displacements $\alpha_{\sim}^*$ by using Equations (19) and (22). Expressing the surface traction vector $T_{\sim}$ in terms of its components T_x and T_y gives the following result.

$$\int_S u_{\sim}^*(T) T_{\sim} dS = \alpha_{\sim}^*(T) \gamma_a \quad (45)$$

where

$$\gamma_a = \int_S \begin{Bmatrix} \left(1 - \frac{x}{L}\right) & 0 \\ -y \left(\frac{6x^2}{L^3} - \frac{6x}{L^2} \right) & \left(1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3} \right) \\ -y \left(\frac{3x^2}{L^2} - \frac{4x}{L} + 1 \right) & \left(x - \frac{2x^2}{L} + \frac{x^3}{L^2} \right) \\ \frac{x}{L} & 0 \\ -y \left(\frac{-6x^2}{L^3} + \frac{6x}{L^2} \right) & \left(\frac{3x^2}{L^2} - \frac{2x^3}{L^3} \right) \\ -y \left(\frac{3x^2}{L^2} - \frac{2x}{L} \right) & \left(\frac{-x^2}{L} + \frac{x^3}{L^2} \right) \end{Bmatrix} \begin{Bmatrix} T_x \\ T_y \end{Bmatrix} dS \quad (45a)$$

If the applied surface tractions $T_{\sim}$ are assumed to be only concentrated loads at the nodal points i and j , then Equation (45a) becomes

$$\gamma_{\sim}^a = \left\{ \begin{array}{c} T_{x_i} \\ T_{y_i} \\ -\int y T_{x_i} da \\ A_i \\ T_{x_j} \\ T_{y_j} \\ -\int y T_{x_j} da \\ A_j \end{array} \right\} = \left\{ \begin{array}{c} T_{x_i} \\ T_{y_i} \\ M_{z_i} \\ T_{x_j} \\ T_{y_j} \\ M_{z_j} \end{array} \right\} \quad (46)$$

Assembly of the individual beam elements into the complete structure is accomplished by enforcing the necessary continuity requirements at the nodes. The operation is standard and needs no elaboration here. It is expressed mathematically by the coordinate transformation

$$\begin{matrix} (i) \\ \alpha_{\sim} \end{matrix} = \begin{matrix} (i) \\ \underline{a} \end{matrix} \begin{matrix} r_{\sim} \end{matrix} \quad (47)$$

where $\begin{matrix} (i) \\ \alpha_{\sim} \end{matrix}$ is the i^{th} individual element generalized coordinate vector (same as α before) and $r_{\sim}$ is the vector of independent nodal coordinates of the entire structure. The $\begin{matrix} (i) \\ \underline{a} \end{matrix}$ matrices are rarely formed explicitly but lead to so called "merge" operations.

Substitution of Equations (32), (36), (41), (43), (45), and (47) into Equation (29) gives the discretized form of the principle of virtual displacements for the assembled structure.

$$r_{\sim}^{*(T)} (\underline{K}_{\sim} r_{\sim} - R_{\sim}^C - R_{\sim}^T - R_{\sim}^S - R_{\sim}^a) = 0 \quad (48)$$

where
$$\underline{K} = \sum_{i=1}^n \underline{a}^{(i)(T)} \underline{k}^{(i)} \underline{a}$$

$$\underline{R}_c = \sum_{i=1}^n \underline{a}^{(i)(T)} \underline{\gamma}_c^{(i)}$$

$$\underline{R}_T = \sum_{i=1}^n \underline{a}^{(i)(T)} \underline{\gamma}_T^{(i)}$$

$$\underline{R}_S = \sum_{i=1}^n \underline{a}^{(i)(T)} \underline{\gamma}_S^{(i)}$$

$$\underline{R}_a = \sum_{i=1}^n \underline{a}^{(i)(T)} \underline{\gamma}_a^{(i)}$$

and $\underline{k}^{(i)}$, $\underline{\gamma}_c^{(i)}$, $\underline{\gamma}_T^{(i)}$, $\underline{\gamma}_S^{(i)}$, and $\underline{\gamma}_a^{(i)}$ are the individual element stiffness matrix and force vectors, respectively.

Since the virtual displacements $\underline{r}_\sim^*$ are arbitrary and independent, for the Expression (48) to vanish the quantity in parenthesis must vanish.

Therefore,

$$\underline{K}_\sim \underline{r}_\sim = \underline{R}_c + \underline{R}_T + \underline{R}_S + \underline{R}_a \quad (49)$$

which represents the finite element relationships necessary to analyze the creep behavior of statically indeterminate beams with arbitrary cross-section.

To compute the reactions multiply the total stiffness matrix $\underline{K}$ times the total displacement vector $\underline{r}_\sim$ and then subtract the total force vector $\underline{R}_c + \underline{R}_T + \underline{R}_S$. The above calculation yields directly the reactions at constrained nodal points and returns the applied loads $\underline{R}_a$ at nodes where specified.

Incremental Procedure

Equation (49) applies for incremental values as previously shown and, therefore, it is possible to define an

incremental creep solution routine. For the beam formulation as previously developed the only nonvanishing stress and elastic strain components are σ_{xx} and ϵ_{xx}^e . For this simplified stress field, the effective stress and deviatoric stresses as defined in Equations (11) and (12) are given by

$$\sigma_e = \left| \sigma_{xx} \right| \quad (50)$$

$$S_{xx} = \frac{2}{3} \sigma_{xx}; S_{yy} = -\frac{1}{3} \sigma_{xx}; S_{zz} = -\frac{1}{3} \sigma_{xx}. \quad (51)$$

Substituting results from Equation (51) into Equation (10) gives the following result for the incremental creep strains ($\Delta \epsilon_{xx}^c$).

$$\Delta \epsilon_{xx}^c = \Delta \epsilon_e^c \operatorname{sgn}(\sigma_{xx}) \quad (52)$$

Note that shear strains remain zero. As indicated previously the contribution to ϵ_{yy}^c and ϵ_{zz}^c do not produce stresses.

If the incremental effective creep strain is assumed to have the following functional dependence

$$\Delta \epsilon_e^c = F(\sigma_e, \epsilon_e^c, \phi, T, \Delta t), \quad (53)$$

then Equation (52) specialized to the beam problem becomes

$$\Delta \epsilon_{xx}^c = F(\sigma_{xx}, \epsilon_{xx}^c, \phi, T, \Delta t) \operatorname{sgn}(\sigma_{xx}). \quad (54)$$

To evaluate the incremental creep strain ($\Delta \epsilon_{xx}^c$) through the cross section of the beam it becomes necessary to know σ_{xx} , ϵ_{xx}^c , ϕ , T as a function of position in the cross section. All of which are either specified or have been explicitly exhibited and are therefore available.

The general incremental procedure for the solution of creep problems proceeds in the following manner:

STEP 1: Apply the initial set of loads designated by $R_1 = R^T + R^S + R^a$ at time ($t = 0$) and solve for the initial elastic

stress distribution at each end of the beam ($\sigma_{xx_i}, \sigma_{xx_j}$).

At time ($t = 0$) the force vector R^C is identically zero. However, the force vectors $R_{\sim}^T$, $R_{\sim}^{S_{\sim}}$ and $R_{\sim}^a$ are all operative for the initial time step and are given by Equations (42), (44), and (47), respectively. Substituting the load R_1 into Equation (49) and solving for the initial elastic nodal displacements $r_{\sim 1}$ gives the following result.

$$r_{\sim 1} = \underline{K}^{-1} R_1 \quad (55)$$

To avoid recomputing the inverse of the stiffness matrix $\underline{K}^{-1}$ for each time step, the following procedure for solving the equation $\underline{K} r_{\sim 1} = R_1$ was used. First, decompose the stiffness matrix into the following form

$$\underline{K} = \underline{U}^T \underline{W} \underline{U}$$

where $\underline{U}$ is an upper triangular matrix with ones on the diagonal and $\underline{W}$ is a diagonal matrix. Then solve for $r_{\sim 1}$ by the following steps.

$$\underline{U}^T \underline{W} \underline{U} r_{\sim 1} = R_1$$

$$\underline{W} \underline{U} r_{\sim 1} = \underline{U}^{T^{-1}} R_1 = y_{\sim}$$

Evaluating $y_{\sim}$ is equivalent to solving $\underline{U}^T y_{\sim} = R_1$ which can be accomplished by forward substitution since $\underline{U}^T$ is a lower triangular matrix. Then

$$\underline{U} r_{\sim 1} = \underline{W}^{-1} y_{\sim} = z_{\sim}$$

$z_{\sim}$ is easily evaluated since $\underline{W}$ is a diagonal matrix.

Finally

$$r_{\sim 1} = \underline{U}^{-1} z_{\sim} = \underline{K}^{-1} R_1$$

is evaluated by backward substitution since $\underline{U}$ is an upper triangular matrix. Therefore, once the stiffness matrix has

been decomposed and the matrices $\underline{U}$ and $\underline{W}$ saved, subsequent incremental displacements can be solved for by performing the above two matrix operations on the incremental load vector and then performing a backward substitution using the upper triangular matrix $\underline{U}$.

The total strain (ϵ_{xx}) for the first step can be computed for each element from α_1 using Equation (26)

$$\epsilon_{xx} = g_{\alpha}^{(T)} \alpha_1 \quad (56)$$

where α_1 for the element in question is obtained from Equation (47). Since it is only necessary to know the strains at the nodal points, Equation (26) may be evaluated at the nodal points.

$$\begin{aligned} \epsilon_{xx_i} &= \left[-\frac{1}{L} \frac{6y}{L^2} \frac{4y}{L} \frac{1}{L} -\frac{6y}{L^2} \frac{2y}{L} \right] \alpha_1 \\ \epsilon_{xx_j} &= \left[-\frac{1}{L} -\frac{6y}{L^2} -\frac{2y}{L} \frac{1}{L} \frac{6y}{L^2} -\frac{4y}{L} \right] \alpha_1 \end{aligned} \quad (57)$$

The initial elastic stress distribution at each end of the beam element is then given by

$$\begin{aligned} \sigma_{xx_i} &= E \left(\epsilon_{xx_i} - \epsilon_{xx_i}^T - \epsilon_{xx_i}^S \right) \\ \sigma_{xx_j} &= E \left(\epsilon_{xx_j} - \epsilon_{xx_j}^T - \epsilon_{xx_j}^S \right) \end{aligned}$$

where ϵ_{xx}^T and ϵ_{xx}^S are given by Equations (38) and (39), respectively and must be interpreted in terms of end point values.

STEP 2: Assuming that the stress is constant during the time interval, it is possible to calculate the incremental creep strain distribution across each end of the beam element using Equation (54).

$$\Delta \epsilon_{xx_i}^C = F \left(\sigma_{xx_i}, \epsilon_{xx_i}^C, \phi_i, T_i, \Delta t \right) \text{sgn}(\sigma_{xx_i}) \quad (59)$$

$$\Delta \epsilon_{xx_j}^C = F \left(\sigma_{xx_j}, \epsilon_{xx_j}^C, \phi_j, T_j, \Delta t \right) \text{sgn}(\sigma_{xx_j})$$

STEP 3: From the incremental creep strains $\Delta \epsilon_{xx_i}^C$ and $\Delta \epsilon_{xx_j}^C$ it is possible to calculate the incremental force vector γ^C given by Equation (37).

STEP 4: Sum the incremental force vector over all elements resulting from incremental creep strain, additional incremental thermal strain, additional incremental swelling strain, and additional incremental applied loads and denote the sum by

$$\Delta R_{\sim 12} = \Delta R_{\sim 12}^C + \Delta R_{\sim 12}^T + \Delta R_{\sim 12}^S + \Delta R_{\sim 12}^a. \quad (60)$$

STEP 5: Solve for the incremental nodal displacements $\Delta r_{\sim 12}$ using the previously generated total elastic stiffness matrix $\underline{K}$.

$$\Delta r_{\sim 12} = \underline{K}^{-1} \Delta R_{\sim 12} \quad (61)$$

From the incremental nodal displacements $\Delta r_{\sim 12}$ calculate the incremental change in reactions $\Delta R_{\sim \text{eact}12}$ by

$$\Delta R_{\sim \text{eact}12} = \underline{K} \Delta r_{\sim 12} - \Delta R_{\sim 12}^C - \Delta R_{\sim 12}^T - \Delta R_{\sim 12}^S.$$

Note the nonzero values in the above equation correspond to the incremental change in reactions at constrained nodal points and the applied incremental loads $\Delta R_{\sim}^a$ at nodal points which were loaded.

STEP 6: Compute the incremental change in total strain from the incremental nodal displacements using Equation (57).

$$\Delta \epsilon_{xx_i} = \begin{bmatrix} \frac{-1}{L} & \frac{6y}{L^2} & \frac{4y}{L} & \frac{1}{L} & \frac{-6y}{L^2} & \frac{2y}{L} \end{bmatrix} \Delta \alpha_{\sim 12} \quad (62)$$

$$\Delta \epsilon_{xx_i} = \begin{bmatrix} \frac{-1}{L} & \frac{-6y}{L^2} & \frac{-2y}{L} & \frac{1}{L} & \frac{6y}{L^2} & \frac{-4y}{L} \end{bmatrix} \Delta \alpha_{\sim 12}$$

STEP 7: Knowing the incremental change in total strain from Step 6 and the incremental creep strains from Step 2, together with the additional incremental thermal strains and swelling strains from Step 4, it is possible to compute the change in stress ($\Delta\sigma_{xx}$).

$$\begin{aligned}\Delta\sigma_{xx_i} &= E \left(\Delta\epsilon_{xx_i} - \Delta\epsilon_{xx_i}^c - \Delta\epsilon_{xx_i}^T - \Delta\epsilon_{xx_i}^S \right) \\ \Delta\sigma_{xx_j} &= E \left(\Delta\epsilon_{xx_j} - \Delta\epsilon_{xx_j}^c - \Delta\epsilon_{xx_j}^T - \Delta\epsilon_{xx_j}^S \right)\end{aligned}\quad (63)$$

STEP 8: Add the incremental stresses to the initial elastic stress distribution giving the new stress distribution on each end of the beam element. Return to Step 2 with the new stress distribution and repeat the procedure for the next increment in time.

The incremental procedure outlined above is highly sensitive to time increment selection in the sense that too large a time increment produces instability. The maximum permissible time interval at each step depends upon the specific form of the creep law (Equation 54) that is used, the problem geometry and the specific loading conditions. In addition, this optimum choice varies as the solution progresses. Thus, it is desirable to compute the size of time increment internally in the computer.

It appears reasonable that for a given problem the area subjected to the greatest stress is the most critical. Therefore, the time interval selected is chosen in such a manner that the incremental creep strain ($\Delta\epsilon_{xx}^c$) in that element is equal to a certain fraction η_0 of the total elastic strain (ϵ_{xx}^c) already existing in that element. In general the optimum value of η_0 must be established for each different

problem as well as how this value should be changed during the solution. Several schemes have been tried and research is continuing on this subject.

In the case of creep under constant load a steady-state condition (stress state does not vary with time) may be reached, after which creep strains and deformations can be extrapolated to the final time of interest without taking incremental steps. The following test may be used to judge when such an extrapolation is feasible:

$$\frac{(\Delta\sigma_{xx})}{\Delta t} \max \leq \rho \quad (64)$$

A numerical quadrature formula is utilized to perform the area integrals over the cross-section of the beam. These integrals occur in the force vector calculations given by Equations (37), (42) and (44). All of these integrals are of the general form

$$\iint_A f(y) \, dA = \iint_A f(y) dy dz \quad (65)$$

Expressing the thickness $h(y)$ of the beam in the z -direction as a function of y and the extremes of y as $y_{\min}$ and $y_{\max}$, the above integral may be expressed as

$$\iint_A f(y) \, dy dz = \int_{y_{\min}}^{y_{\max}} h(y) f(y) dy \quad (66)$$

Assume that the distance $y_{\max} - y_{\min}$ is divided into five equal intervals. Using a Newton-Cotes quadrature formula employing a fifth order interpolation formula, the above Integral (66) may be written

$$\int_{y_0}^{y_5} g \, dy = \frac{5w}{288} \left[19 g(y_0) + 75 g(y_1) + 50 g(y_2) + 50 g(y_3) + 75 g(y_4) + 19 g(y_5) \right] \quad (67)$$

where $g = h(y) f(y)$ and w denotes the width of the interval. If more numerical accuracy is required, then the depth of the beam may be subdivided into a multiple of five equal intervals. Generalizing Equation (67) gives

$$\int_{y_0}^{y_0 + nw} g \, dy = \frac{5w}{288} \left[19g(y_0) + 75 g(y_1) + 50 g(y_2) + 50 g(y_3) \right. \\ + 75 g(y_4) + 38 g(y_5) + 75 g(y_6) + 50 g(y_7) \\ + 50 g(y_8) + 75 g(y_9) + 38 g(y_{10}) \dots \\ + 38 g(y_{n-5}) + 75 g(y_{n-4}) + 50 g(y_{n-3}) + 50 g(y_{n-2}) \\ \left. + 75 g(y_{n-1}) + 19 g(y_n) \right]$$

where n is a multiple of 5.

APPLICATIONS

To establish the validity of the finite element procedure and to demonstrate the generality of the results to be expected, two illustrative problems are presented.

Cantilever Beam

The first problem considered was a cantilever beam (length - 100 in., depth - 10 in., width - 1 in.) with a concentrated vertical end load ($P = 2000$ lb). The finite element idealization used was a single element along the length with 20 subdivisions through the depth. The particular creep law and the modulus of elasticity (E) for this problem were taken as

$$\Delta \epsilon_{xx}^c = 6.4 \times 10^{-18} \sigma_{xx}^{4.4} \Delta t \operatorname{sgn}(\sigma_{xx}) \\ E = 20.0 \times 10^6$$

Figure 2 shows the distribution of longitudinal stress through the cross-section at the support. The initial stress

FIGURE 2. Redistribution of Longitudinal Stress at the Support

distribution is linear and for this particular creep law the steady-state stress distribution is achieved at an elapsed time of 0.0031 hr. The CRASIB solution converges exactly to the stationary creep solution given by Hult.⁽¹⁾ Stationary creep is used here to denote a creep state for which the space distribution of stress in a body remains constant with respect to time.

Figure 3 shows the variation of longitudinal stress with time at the support on a fiber adjacent to the neutral axis and on the extreme boundary fiber. Notice that it requires about three times as long for the stress on the inner fiber to approach the stationary creep value as on the outer fiber. However, the solution procedure remains stable and converges to the stationary creep solution as long as the incremental creep strain is forced to be less than the elastic strain for each time interval. For this particular problem the incremental creep strain was required to be less than 0.3 times that of the elastic strain.

Figure 4 shows the initial elastic strain and incremental creep strain at the support. Also shown are the total strain, the total creep strain at 0.0031 hr when the steady-state stress distribution is reached, and some intermediate stages of creep strain. Figure 5 gives the maximum beam deflection as a function of time. This simple problem demonstrates the type of accuracy which can be achieved with CRASIB. It required 37 sec on the UNIVAC 1108 to calculate 57 time iterations and to produce the solution.

Conceptual FTR Flow Duct

The second problem considered was the analysis of a conceptual design for an FTR driver flow duct assumed to be

FIGURE 3. Longitudinal Stress Variation with Time at the Support

FIGURE 4. Strain Distributions at Support

FIGURE 5. Cantilever Beam Creep Deflection

located in Row 6 of the FTR flow duct assembly. Figure 6 shows the moment of inertia variation along the length of the flow duct. It should be observed, however, that the definition of the moment of inertia for a cross-section does not adequately define the cross-section for a creep analysis. To define adequately the cross-sections for the beam elements the following information was provided.

Round Tube (Nodes 1 to 4)

Outside diameter 2.9 in.
Wall thickness 0.18 in.

Hexagonal Duct (Nodes 4 to 5, 7 to 8, 28 to 29,
32 to 34)

Outside flat to flat dimension 4.715 in.
Wall thickness 0.190 in.
Outside corner radius 0.372 in.

Hexagonal Duct (Nodes 5 to 7, 8 to 28, 29 to 32)

Outside flat to flat dimension 4.615 in.
Wall thickness 0.140 in.
Outside corner radius 0.303 in.

The flow duct orientation was such that the bending occurred about an axis extending from corner to corner. The program computes all necessary cross-sectional information from the above input values. The sections with larger moments of inertia represent the thickened pad regions where the lower grid plate and restraint mechanisms interact with the flow duct. A further assumption has been made that the fuel pins within the flow duct do not contribute significantly to the overall stiffness of the flow duct.

Figure 7 shows the finite element nodal structural idealization that has been used. Thirty-three elements were used to model the flow duct with the material and section

FIGURE 6. Moment of Inertia Variation

FIGURE 7. Longitudinal Stress Distribution

properties assumed to be constant within each element. The active core region extends from Node 10 to Node 27 and the element breakdown is refined in this region to accommodate the rapid variation in environmental conditions.

The longitudinal and transverse temperature and flux profiles are given in Figures 8, 9, 10 and 11. The longitudinal values are assumed to apply at the mid-plane of the flow duct and the transverse values represent linear gradients across the flow duct. Figure 9 shows two possible transverse temperature gradients immediately above the active core.

To complete the definition of the environmental conditions it was necessary to define the swelling and irradiation-induced creep laws for 20% cold worked Type 316 SS. Typical expressions for these laws are as follows:

Swelling law

$$\frac{\Delta V}{V} = 1.01 \times 10^{-36} (\phi t)^{1.69} \left[\exp(-7800/RT) - 5480 \exp(-25,300/RT) \right]$$

where $\frac{\Delta V}{V}$ = % change in volume

T = Temperature (°K)

R = 1.9872 cal/mole -°K

ϕt = Fluence (neutrons/cm²) with energy >0.1 MeV.

Irradiation induced creep law

$$\epsilon_{XX}^c = 10^{-8} \sigma_{XX} \left[1 - \exp(-\phi t/10^{20}) \right] + 4.6 \times 10^{-30} \sigma_{XX} (\phi t)$$

Additional information concerning irradiation induced creep is available in the indicated References 2 through 6. Primary and secondary thermal creep were neglected for this problem because of the low temperatures.

FIGURE 8. Longitudinal Temperature Variation

FIGURE 9. Variation of Transverse Temperature Gradient

FIGURE 10. Axial Flux Variation

FIGURE 11. Transverse Flux Gradient

For this problem CRASIB was used to predict the flow duct response for one complete reactor cycle, including thermal cool down and mechanical release of the restraint mechanism at the end of the cycle (9600 hr). Figure 12 shows the multiple constrained transverse displacements for different loading conditions and elapsed times. Observe the relative magnitudes of the in-core displacements for the thermal load case (0 hr) and the combined thermal and swelling load case (9600 hr without creep) (-3 mils versus +57 mils). The initial thermal response in the active core region is outward from the center of the reactor core giving the desired negative reactivity effect, but the bowing induced by the time-dependent swelling gradients is inward and 20 times larger at 9600 hr than the initial thermal bow if irradiation induced creep is disregarded. With the consideration of irradiation induced creep, however, the large bowing induced by swelling does not develop and the deformed shape of the flow duct at 9600 hr considering creep is as shown in Figure 12. Also shown on Figure 12 is the deformed shape at 9600 hr when the reactor is shut down (thermal unload 9600 hr). Figure 13 shows the transverse displacements after thermal and mechanical unload with and without creep. Note that for total unload the restraint mechanism is released at the top two nodes. Without creep the end deflection of the flow duct is outward 1.62 in. whereas with creep the end deflection is only 0.068 in. Figure 14 gives the maximum in-core displacement variation with time with and without creep.

Figure 7 also gives the distribution of the maximum longitudinal stress along the length of the flow duct for different loading conditions. Observe that the stresses are continuous from element to element with the exception of the

Distance from Flow Duct Bottom, in.

FIGURE 12. Transverse Displacement Variation

FIGURE 13. Transverse Displacement After Thermal and Mechanical Unload

FIGURE 14. Maximum In-Core Displacement Variation

elements where the cross-sections change. Considering creep, note that the swelling and thermal stress at 9600 hr is not as high as the initial thermal stress. This result is shown better in Figure 15 where the variation with time of the maximum longitudinal stress (Node 28) is plotted with and without creep. With creep, the maximum longitudinal stress at 9600 hr has not reached its original value (initial thermal stress). Without creep, the increase in longitudinal stress induced by swelling would cause the flow duct to be deformed plastically before 9600 hr. Figure 16 shows the variation of the longitudinal stress through the cross-section of the beam at Node 28 where the maximum stress occurs. Note the initial linear distribution of thermal stress and the subsequent distortion of the linear distribution induced by nonlinear swelling through the cross-section. When creep is considered, this large distortion does not occur and the stress at 9600 hr is less than the initial thermal stress. Figure 17 shows the residual stress distribution at Node 28 after thermal and mechanical unload with and without the consideration of creep. Observe the complete reversal in signs for the residual stress field depending upon whether creep is considered or not.

Figure 18 shows the variation of the maximum restraint load at the support pad immediately above the active core. If creep is considered the maximum restraint load occurs at the initial time and results from the initial thermal load. To show the effects of the stiffness of the flow duct on the restraint loads, all elements above Node 4 were assumed to have the cross-sectional properties of the support pads. This stiffening of the flow duct increased the initial maximum restraint load from 1440 lb to 2160 lb. Figure 19 shows the effect of the variation of the temperature gradients (Figure 8) above the active core on the maximum restraint loads. The

FIGURE 15. Maximum Longitudinal Stress Variation

FIGURE 16. Maximum Stress Distribution (Node 28) Before Unload

FIGURE 17. Residual Stress Distribution After Thermal and Mechanical Unload

FIGURE 18. Variation of Maximum Restraint Load Above Core

FIGURE 19. Maximum Restraint Loads for Different Transverse Temperature Gradients

maximum restraint load is decreased from 1440 lb to 1260 lb if a linear decrease in temperature gradient from the top of the active core is assumed. Figure 20 shows that the variation of temperature gradients above the active core has a small effect on the transverse displacements after thermal and mechanical unload.

Because of the uncertainty associated with the irradiation induced creep law, a parameter study was made on the effects of the creep coefficients on the flow duct structural response. Figure 21 shows the transverse displacements after thermal and mechanical unload where the creep coefficients for the transient and steady-state terms were multiplied by 6 and divided by 10. This type of variation completely reverses the direction of the spring back after mechanical unload. Figure 22 shows the effects of this same creep coefficient variation on the maximum longitudinal stress. Here again, the importance of the accurate determination of the creep coefficients is vividly demonstrated. CRASIB should be very useful to the materials engineer trying to design some meaningful experiments to characterize both thermal and irradiation induced creep.

These results demonstrate the wide range of information that CRASIB can provide to assist in the design of the flow ducts and the restraint system for the FTR. Furthermore, CRASIB is not limited to these particular problems but can predict the creep behavior of any statically indeterminate beam with arbitrary cross-sections.

FIGURE 20. Transverse Displacement After Thermal and Mechanical Unloading for Different Transverse Temperature Gradients

FIGURE 21. Transverse Displacement for Different Creep Equations

FIGURE 22. Maximum Longitudinal Stress for Different Creep Equations

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