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DYNAMICS OF A FAST REACTOR WITH
IN-CORE THERMIONIC CONVERTERS

Fourth Annual Report

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ENGINEERING EXPERIMENT STATION
COLLEGE OF ENGINEERING
THE UNIVERSITY OF ARIZONA
TUCSON, ARIZONA

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THERMIONIC CONVERTERS

Fourth Annual Report

R. L. Brehm
D. A. Ferg
J. G. Guppy
K. D. Kearns
J. M. Kendall

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Department of Nuclear Engineering
The University of Arizona
Tucson, Arizona 85721

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Research Report for the Period 1 September
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PREFACE

This annual report summarizes thermionic reactor system dynamics and control research activities for the period 1 September 1969 to 31 August 1970 performed under USAEC Contract AT-(04-3)-670. Unlike previous annual reports, this report will simply summarize very briefly the research results of the various areas of investigation without detail since the final results of all of these investigations have been or shortly will be reported separately as topical reports.

I. NONLINEAR REACTOR DYNAMICS

Summary

The nonlinear point reactor kinetics model with linear reactivity feedback is studied to determine stable regions of the state space. Geometric and Liapunov techniques are used to determine system stability and effects of system parameters on the stable region.

In studying the one temperature region model with effective lifetime kinetics and a power coefficient, a transformation is obtained which simplifies analysis of the system. Using this transformation, an exact solution is found for parameter values such that the system is a nonlinear center. In addition, a Liapunov function is developed which proves asymptotic stability for an infinite sector of the phase space. The stable region shown by this method is compared to those shown by previous techniques for a given set of parameters.

In applying the method to the two temperature region model with effective lifetime kinetics, a set of trajectories asymptotically approaching lines of constant reactivity is shown to exist. An infinite region bounded by three intersecting planes with the property that all trajectories crossing the planes move toward the interior of the region is determined. The parameter condition required for existence of this region is shown to be identical to the condition for the existence of stable limit cycles stated in a previous paper. Numerical examples studied indicate the region described above is one of bounded trajectories, i.e., all trajectories with finite initial conditions within the region remain bounded.

Although interpretation of results becomes more difficult in higher order models, the set of asymptotes can be found by a straightforward application of the method used on the simpler models. The dimension of the subspace containing these asymptotes is always one less than that of the state space. Further study of the models indicate this subspace is one of constant reactivity.

In each of the models studied, a subspace containing trajectories which approach infinity with constant reactivity is shown to exist. Investigation of numerical examples indicates this subspace contains a very special class of trajectories in that unbounded trajectories not in the subspace approach infinity with increasing reactivity. If this is true, further speculation leads to the possibility that this subspace forms a boundary between stable and unstable regions of the state space.

Pertinent Topical Report

J. M. Kendall, "Stability Domains in Nonlinear Point Reactor Dynamics," Engineering Experiment Station, University of Arizona, Tucson, Arizona, February 1971.

II. STABILITY ANALYSIS OF SYSTEMS WITH TIME DELAY

Summary

A technique based on the Mikhailov stability criterion has been developed to study the effects of time delays on the stability of linear systems. The technique utilizes Descartes' rule of signs and Sturm's theorem to determine regions within the system parameter space in which stability or instability is conditional on the value of the time delay. In these regions, the critical time delay indicating the stability boundary may also be determined.

The method is applied to a seventh-order, in-core thermionic reactor model containing two coolant time delays between the radiator and the core. Stability is found to depend on the sum of the two delays. The results of the analysis reveal the existence of conditional instabilities within physically possible operating regions of the parameter space. Within the conditionally stable regions, several critical time delays are calculated to indicate how stability depends on the value of the time delay.

A number of methods commonly used to study the stability of delayed systems are compared for their accuracy and ease of application. In addition, the results of the exact linear treatment are compared with three commonly applied approximations, all of which attempt to simplify the stability analysis. It is found that none of these approximations necessarily give conservative predictions of the exact stability zones.

Pertinent Topical Report

K. D. Kearns, "Stability of Nuclear Reactor Systems Having Time Delays,"
Engineering Experiment Station, University of Arizona, Tucson, Arizona,
June 1971 (Available December 1970).

III. START-UP ANALYSIS

Summary

An analytical representation of a nuclear reactor containing in-core thermionic devices suitable for transient studies is formulated. The resulting model is applicable to situations involving substantial changes in system operating conditions, for example during start-up transients.

Neutron kinetics and heat transfer are represented by "nodal" descriptions. Contributions from all system regions of importance are retained to produce realistic transient response. The resulting set of equations is coupled to a digital computer integration routine to solve for the dynamic response.

The thermionic converter physics is described by a complex iterative numerical scheme based on a diffusion approximation to the plasma processes. Other thermionic processes included are surface and Schottky effects, plus an accounting of the electrostatic sheaths present. The analysis is extended to include general application to thermionic diodes undergoing transients.

The digital description of the reactor model is tested against a comparable analog computer simulation and shown to yield better accuracy. The complex thermionic analysis is compared to a simpler converter physics description and found to be far superior in predicting the electrical characteristics of the converter for large changes in operating conditions. The thermionic analysis is also compared with transient experimental diode data over wide ranges of converter operation and

shown to produce excellent agreement. Application of the model to system start-up is described for two postulated start-up approaches encompassing either constant diode voltage or constant emitter temperature.

This thermionic reactor model is shown to be useful in obtaining an understanding of the overall system dynamic behavior during large changes in system operating conditions. Furthermore, since the thermionic analysis can be decoupled easily from the system model, separate application to studies involving only transient diode operations may be accomplished.

An important finding of these analytical studies is that under certain conditions, the results obtained assuming a spatially averaged temperature distribution, especially for the emitter surface, may not be sufficiently accurate to represent all of the important aspects of diode transient behavior. Analytical studies involving the complete reactor model demonstrate that simple control methods may be adequate to produce reasonable response during system transients.

Pertinent Topical Report

J. G. Guppy, "Analytical and Experimental Studies Relating to the Simulated Start-up of In-Core Thermionic Reactor Systems, Engineering Experiment Station, University of Arizona, Tucson, Arizona, June 1970.

IV. MULTIPLE INPUT-MULTIPLE OUTPUT CONTROL STUDIES

Summary

A computational procedure is formulated for the feedback synthesis of a linear multivariable control system with input, output cross-coupling. The feedback synthesis technique which is developed results in a closed-loop interacting transfer matrix containing the desired transfer functions between all permutations of the system outputs and the external control inputs. The technique depends on previously established decoupling techniques for the state-variable feedback design of noninteracting multivariable control systems.

Specifically, the study provides a condensed review of the literature pertaining to decoupling techniques and incorporates the necessary information and formulation required to compute a pair of decoupling state feedback and gain matrices for a multivariable system. A variational decoupling technique is developed which represents a logical mathematical procedure for decoupling a given multivariable system into all possible transfer function relationships between the system outputs and the control inputs.

The variational decoupling technique is used to obtain a restricted set of decoupling matrices. Each pair of decoupling matrices will decouple the system so that each system output is affected by a different control input without repetition over the set of decoupling matrices. The entire set of decoupling matrices are combined into one pair of state-feedback and gain matrices so that the resultant closed-loop

transfer matrix contains the desired control input, system output cross-coupled interaction.

General results for the interacting state-feedback matrix indicate that the matrix does not contain constant gain elements. Consequently, a new form for a state-variable feedback control law is introduced where the simple product of the state-feedback matrix and the state vector is interpreted as a convolution integral over the product.

The applicability of the feedback synthesis technique is indicated by designing a control system for a single diode representation of an in-core thermionic reactor system. Results indicate that a very simple pair of state-feedback and gain matrices will result in a closed-loop control system which can satisfy the following constraints. The diode voltage is restricted to a $\pm 4\%$ variation about an equilibrium value, the thermal power and diode electrical-power dependency is decoupled for sudden load changes, and the thermal shocks on the collector and emitter surfaces are limited to $60^\circ\text{C}/\text{minute}$. The set of input perturbations applied separately to the control system consist of 1) a $2\text{c}/\text{second}$ reactivity ramp for a time interval of 20 seconds, 2) a 20% step change in the diode load resistance, and 3) a $1^\circ\text{C}/\text{second}$ coolant temperature increase over a time interval of 20 seconds.

Pertinent Topical Report

D. A. Ferg, "Multiple Input-Output Feedback Synthesis Incorporating Cross Coupling," Engineering Experiment Station, University of Arizona (To be published, 1971).

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PH D STUDENTS AND FACULTY SUPPORTED BY USAEC CONTRACT AT-(04-3)-670

Students	1966 6 9 12	1967 3 6 9 12	1968 3 6 9 12	1969 3 6 9 12	1970 3 6 9 12	1971 3 6 9 12	Dissertation Topic
T. Schmidt	— ↑ —	— — A P	P P — A	— D T J P P ↓	— A J —	— — — —	Nonlinear Oscillations and Stability
J. Guppy	— ↑ —	— — A —	— — — A	— P — — P —	— D P — — — T A↓	J A — — —	Thermionic Reactor Start-up
K. Kearns	— — —	— — — —	— — — A	— ↑ — — —	— A — TP D ↓	A — — J —	Nonlinear Stability and Time Delay Systems
D. Ferg	— — —	— — — —	— — — —	— — — ↑ —	— A P — —	TA — — J DP ↓	Multiple-Input Multiple-Output Control Studies
J. Kendall	— — —	— — — —	— — — —	— — — — —	↑ — — — T D ↓	A — P J —	Nonlinear Stability
Faculty							
M. V. Davis	↑ — — —	— — — A P	— — — — ↓	— — — — —	— — — — —	— — — — —	
D. L. Hetrick	↑ — —	— P — A P	— P P — A	— P J P P ↓	— A J —	— — — —	
R. L. Brehm	↑ — —	— — — A P	— — — A P	— — J P P	P — A P P P	J AP — — —	

Symbols: D Dissertation
 T Topical Report
 J Journal Article
 P Paper Presented
 A Annual Report
 ↑ ↓ Period of Contract Support