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PROCEEDINGS OF
THE CONFERENCE ON
TRANSFER FUNCTION MEASUREMENTS
AND
REACTOR STABILITY ANALYSIS
HELD AT
ARGONNE NATIONAL LABORATORY
ARGONNE, ILLINOIS
MAY 2-3, 1960

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PREFACE

The Conference on Transfer Function Measurements and Reactor Stability Analysis was held at Argonne, Illinois, on May 2-3, 1960. The meeting was sponsored by the Reactor Engineering Division of Argonne National Laboratory in response to a request by the Chicago Operations Office of the USAEC, acting on a recommendation made in a meeting of the Advisory Committee on Reactor Safety by Dr. Richard Doan, ACRS member.

The conference provided an exchange of current information between knowledgeable people working in the subject field. Analytic methods were discussed, and experimental measurements and analyses of operating thermal and fast reactors were reviewed to stimulate thinking in connection with the testing and analysis of fast reactor stability. The safety of such reactors is, of course, of great interest at the present time. At the suggestion of the USAEC, the meeting was kept small to achieve an informal atmosphere and promote free discussion.

There was some international participation as shown by the list of attendees included in the appendix. However, most of the papers were presented by representatives of American installations.

This document contains the conference proceedings, and includes the papers presented together with records of the discussion periods held following each paper. The material is presented chronologically in the same session format as used at the conference since papers in a given session are related. The session titles are self-explanatory with respect to the subject matter covered in each session.

It should be noted that most of the papers are verbatim copies prepared in advance by the authors for inclusion in the proceedings. In some cases, authors composed their papers by editing the transcript of their oral presentation. The conference editors assume all responsibility for any errors, and apologize for any mismatching of names to recorded statements or any misquoting which may have crept in due to the difficulty of obtaining a perfectly accurate stenographic record. It is felt that the informal nature of the meeting induced a free exchange of ideas which offsets many times over any minor errors in the proceedings. In fact, the indications are that such meetings could profitably be held at least once a year.

The session secretaries, J. C. Carter, C. A. Pesce, W. B. Loewenstein, and H. A. Sandmeier, also served as conference editors, and their efforts are gratefully acknowledged. Special recognition is also due Mr. A. B. Krisciunas for making and carrying out the physical arrangements of the conference.

James A. DeShong, Jr.
General Chairman

SESSION I

TRANSFER FUNCTION USE IN REACTOR DYNAMIC STUDIES

Monday Morning, May 2, 1960

Chairman: J. A. DeShong, Jr.

Secretary: J. C. Carter

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Secretary: C. Pesce

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Chairman: F. Martens

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SESSION I

TRANSFER FUNCTION USE IN
REACTOR DYNAMIC STUDIES

Chairman: J. A. DeShong, Jr.
Secretary: J. C. Carter

ARGONNE NATIONAL LABORATORY
CONFERENCE ON TRANSFER FUNCTION MEASUREMENTS
AND REACTOR STABILITY ANALYSIS

INTRODUCTORY REMARKS

Bernard I. Spinrad, Director of Reactor Engineering
Argonne National Laboratory
Argonne, Illinois

Since I am to make introductory remarks, I thought I would go for about three minutes into the history of the subject of the conference to explain why we have here a heterogeneous group of physicists, control engineers, and mathematicians for a conference on reactor stability.

I guess it all started because, when reactors were first brought up as objects to be investigated, the members of the various disciplines which had been engaged in their original design each thought they had invented something. Thus the physicists who had worked on criticality properties, and the mathematicians who had worked out the ways of computing them, and instrument engineers who had helped monitor them, all discovered that reactors (not very surprisingly) had dynamic properties which were subject to change; and all discovered that this was a very interesting subject for further investigation.

By now, the field has bifurcated into two major portions. The one which has more emphasis these days is the analysis of reactor response under various dynamic conditions at various power conditions to determine the stability of a large power system. However, there is another classical use of the dynamic behavior of reactors which is also being covered in this meeting - by classical I mean anything more than 10 years old - and this is the fact that a reactor will respond to an absorber located within it in a way which is standard (if still not completely understood in terms of the equations that are used to convert reactivity to absorption). This standard response gave rise to the secondary art of the use of nuclear reactors at low power as measuring instruments for neutronic absorption and other neutronic properties.

All these problems are embedded in the framework of control system theory, as are so many things in this modern technological world, and probably this conference has been long overdue. Thank you.

WELCOME

Louis A. Turner, Deputy Laboratory Director
Argonne National Laboratory
Argonne, Illinois

It is my very pleasant task to bid you welcome to Argonne. I suspect it is superfluous. I hope the atmosphere will be one that will speak for itself. But just for the record, you are indeed welcome.

I think that this conference is a timely one. I think these developments are among the most interesting and the most exciting in the reactor work that I have heard about for a long time. The late Enrico Fermi is reported to have said once that what the United States needs is more reactors that blow up, and more atomic bombs that don't. Well, of course, he was asking for boldness in design and experiment. Maybe you fellows are taking the fun out of it, but on the whole, it is a lot cheaper, and I think the experimentation and design can be much bolder now that you know how to sneak up on it gradually without having drastic consequences.

I hope the deliberations in this room will ensure the future. You are very welcome to Argonne.

REACTOR TRANSFER FUNCTIONS

J. P. Franz
Westinghouse Electric Corporation
Pittsburgh, Pennsylvania

Transfer functions, a standard technique in control engineering analysis, have a number of advantages. They simplify the study of stability by the use of highly developed analytical and graphical methods. In addition, analytically derived transfer functions or measured experimental transfer functions are in a practical sense interchangeable in the analysis procedure. Transfer functions thus provide a language well suited for the comparison of analytical predictions and experimental results when the system behavior is linear.

To extend the transfer function method to the analysis of nonlinear systems, describing function methods have been introduced. Considerable information on these techniques is available.¹

It is only recently that Smets² presented the describing function of a nuclear reactor. As the describing function method has been quite useful in other applications, it warrants some consideration in the study of reactor kinetics. Some review of transfer function techniques is useful before defining and illustrating the describing function.

The analysis of reactor control using transfer functions was given early justification as a result of the experimental work of Harrer, Boyer, and Krucoff in measuring the transfer function of the Argonne CP-2 reactor.³ In automatic reactor control design, an analysis using small signals is a generally accepted method. By assuming small sinusoidal reactivity and neutron level variations, the reactor kinetic equations can be linearized to obtain the reactor transfer function. Similar transfer functions for the power plant components and control system can be determined and synthesized to determine the degree of stability of the reactor and its control system.

Concurrent with this stability analysis, it is common practice to study the reactor and plant transient performance on an analog computer or plant simulator. The reactor kinetics solved on the analog are the familiar equations

¹ Truxal, Control Engineers' Handbook. McGraw-Hill Book Co., New York (1958).

² H. B. Smets, The Describing Function of Nuclear Reactor, IRE Transaction on Nuclear Science, NS-6, No. 4, (December 1959).

³ Harrer, J. M., R. E. Boyer, and D. Krucoff, Transfer Function of Argonne CP-2 Reactor, Nucleonics, 10, No. 8, p. 32, (1952).

$$\frac{dn}{dt} = \frac{\rho n}{\ell} - \sum_{i=1}^6 \frac{dC_i}{dt} \quad (1)$$

and

$$\frac{dC_i}{dt} = \frac{\beta_i n}{\ell} - \lambda_i C_i \quad (2)$$

The equations are linear except for the product reactivity times neutron level (ρn). This nonlinearity is readily solved by the simulator. The simulator thus serves as a means of refining stability predictions and offsets the small signal limitations of a linearized transfer function analysis. The simulator can also be used to show the nonlinear effects of large sinusoidal reactivity signals. With such techniques available, the principal incentive for making use of the describing function is to have a simple analytical check.

A describing function is the ratio of the amplitude of the fundamental component of the output of a nonlinearity to the amplitude of the sinusoidal input to the nonlinearity. One of the restrictions of the describing function is that only the fundamental component of the output of the nonlinearity may contribute to the input signal through the feedback loop. H. B. Smets² has shown that the describing function for a nuclear reactor can be represented as a linear system in series with a nonlinear amplifier. The block diagram for this is

Expressions for these can be obtained from an analytical solution of the reactor kinetic equations. In such a general solution Ackasu⁴ shows that if

$$\rho = \mu \sin wt \quad (3)$$

then

$$n(t) = n_0 e^{\mu |Z|} \sin (wt + \phi) \quad (4a)$$

$$\ln(n/n_0) = \mu |Z| \sin (wt + \phi), \quad (4b)$$

where

⁴ Z. Akcasu, General Solution of the Reactor Kinetic Equations with Feedback, Nuclear Science and Engineering, 3, 456-467 (1958).

$$Z(s) = \frac{1}{s \left(\ell + \sum_i \frac{\beta_i}{s + \lambda_i} \right)} \quad (5)$$

and is the transfer function of the linearized reactor kinetic equations.⁵ If $\mu |Z|$ is small, then

$$n(t) = n_0 [1 + \mu |Z| \sin (wt + \phi)] \quad , \quad (6)$$

which is the form for small-signal, linear analysis. Although the expression in (4a) may only be an approximate solution of the reactor kinetics equations, it can be used to illustrate the describing function method.

The describing function method represents the relation between the input ρ and the output n as linear and nonlinear elements in cascade. In the block diagram the linear element is Z and the nonlinear element is an amplifier with gain $K(x)$ where x is the output of the linear element. In Laplace transform notation,

$$x(s) = \rho(s) Z(s). \quad (7)$$

In the time domain, the magnitude and phase of x are $\mu |Z|$ and ϕ at any given frequency w :

$$x(t) = \mu |Z| \sin (wt + \phi) \quad . \quad (8)$$

From equation (4a),

$$n = n_0 e^x. \quad (9)$$

We note that the relation between x and n involves no phase shift and is simply a nonlinear amplifier, $K(x)$. The determination of $K(x)$ presented in footnote 2 will be briefly described.

A sinusoidal input to the nonlinear element will produce an output response which can be determined from a Fourier series expansion. This describing function technique can only be applied when harmonics generated in the nonlinear elements are attenuated by the linear elements of the system.

⁵ J. P. Franz, Pile Transfer Function, AECD-3260 (1949).

If ρ is a sine wave (equation 3), x is a sine wave with magnitude $\mu |Z|$. The Fourier series for $n(t)$ is

$$n(t) = \frac{A_0}{T} + \frac{2}{T} \sum_{r=1}^{\infty} A_r \sin rwt, \quad (10)$$

$$\text{if we assume } x = \mu |Z| \sin wt. \quad (11)$$

Next we determine the coefficients A_0 and A_1 :

$$\begin{aligned} A_0 &= \int_0^T n(t) dt \\ &= n_0 \int_0^T e^{\mu |Z| \sin wt} dt, \end{aligned}$$

so that

$$\frac{A_0}{T} = 1 + \frac{1}{2!2} \mu^2 |Z|^2 + \frac{3}{4!8} \mu^4 |Z|^4 + \dots \quad (12)$$

Further,

$$A_1 = \int_0^T n(t) \sin wt dt,$$

so that

$$\frac{2A_1}{T} = n_0 \left[1 + \frac{1}{3!4} \mu^2 |Z|^2 + \frac{1}{5!8} \mu^4 |Z|^4 + \dots \right] \quad (13)$$

Thus, if

$$K(x) = \delta n / \delta x, \quad (14)$$

then

$$K(x) = n_0 \left[1 + \frac{\mu^2 |Z|^2}{8} \right]. \quad (15)$$

The accuracy of the describing function depends on attenuation of the harmonics by the linear elements. This is an important limitation when considering that the low-power reactor transfer function in the range of frequencies from 0.1 to 100 radians per second can be relatively constant.⁶ Much of the simplicity of the describing function technique is lost when consideration must be given to the presence of harmonics in the reactivity.

In most stability problems, however, there are thermal and hydraulic time constants which introduce attenuation. Harmonics present in neutron level are thus attenuated and may not introduce harmonics in the reactivity signal. With such attenuation in a feedback loop, the describing function may often be used without introducing excessive error. It is in such applications that the describing function can be a simple useful method.

Discussion of Paper
Presented by Mr. Franz

MR. WASSERMAN:

There is one situation which isn't clear to me in the application of this particular technique. That is a situation where you do have a feedback loop, and the feedback itself has no linearity associated with it. What happens to this technique under such conditions?

MR. FRANZ:

I have to qualify my answer because I don't see immediately why the technique is still not useful. It, of course, would depend on the feedback nonlinearity, but for most types of feedback, I wouldn't see why it would not apply. I think the answer is that you will have to try it and see.

MR. SANDMEIER:

About this feedback being nonlinear: I think in the fast reactor (EBR-I) analysis, I had a nonlinear feedback assuming limited bowing. I computed these coefficients, assumed different inputs of negative magnitudes, and cut it off at the proper point. Maybe you are interested in this work, which appears in ANL-6014.

⁶ Corben, H. C., Analysis of Zero Power Reactor Transfer Functions, Nuclear Sci. and Eng. 6, 461-465 (1959).

MR. BAKER:

Can you give me the name which sounded to me like Smitz and the reference?

MR. FRANZ:

H. B. Smets, and I can give you two references. One is Peaceful Uses of Atomic Energy, 1958, Volume II, Geneva Paper, 1958 conference. The other reference is Transaction of Professional Group on Nuclear Science, IRE, December, 1959, NS6, No. 4.

MR. THOMPSON:

I believe the complete work is in his doctor's thesis at MIT, which is sponsored by Dr. Gyftopoulos, who is the next speaker on the program.

MR. AKCASU:

I worked on this same subject long ago, and I published a paper in 1958. I attacked the same problems, using a sinusoidal input, and I obtained the gain of the reactor defined as the ratio of the first harmonic to the input, and the result I obtained contains a series like this, but not exactly. There are terms which depend on the transfer function evaluated at the double frequency.

MR. FRANZ:

That introduces, for example, the constant term I was eliminating by definition.

MR. AKCASU:

I have other terms which depend on what I call the $Z(2\omega)$, which is the transfer function evaluated at the double of the input frequency. Can you tell me why? I doubt if that expression takes into account the effect of the second harmonic on the output of the first harmonic. It is neglected, I guess. That term is not negligible.

MR. FRANZ:

That is a good question. There is an immediate question concerning the validity of a describing function approach. What you say can be quite true and yet not be inconsistent with the use of the describing function as just presented.

MR. DE SHONG:

I would like to add a comment here that I think might satisfy. I believe that normally one assumes or rather neglects all of the harmonics other than the fundamental in the describing function approach. And this frequently, if not always, is justified by the fact that the system at some point becomes a low-pass filter so to speak, which has the effect of eliminating the second and higher order terms in the complete closed loop.

MR. FRANZ:

That is the assumption inferred in obtaining a describing function. One neglects the higher harmonics in the feedback. If this assumption in a particular application is not a valid assumption, you have to expect an error. That is answering your question by giving you a question.

MR. COHEN:

I think there is still a point that Akcasu was trying to make. It is one thing to neglect the higher harmonic, but there are also generalizations of the subharmonic, and particularly the constant term, which is part of the nonlinear effect and is really generated at the same time that the second harmonic is generated, so it is not always completely clear that you can neglect the higher harmonics by merely saying that the system will have a low-pass filter characteristic.

MR. WASSERMAN:

Dick Cohen raises the question or at least the suggestion that there are subharmonics generated. I remember in Smets' work (correct me if I am wrong) that he comes to a quite positive conclusion that in nonlinearities associated with lower power reactors, there could not be any subharmonics generated. What is the story?

MR. GYFTOPOULOS:

Smets did come to this conclusion that he states about subharmonics. Also, I think the difficulty here arises from the following fact: In order to apply this method, particularly in feedback cases, you have to use in some cases some nonlinear feedback in the feedback group, and then you cannot express explicitly the Laplace transform or the frequency-dependent function of the signal which is fed back in terms of the Laplace transform signal itself, because if you know the term, you can find the Laplace transform of the second. And what Smets suggested is that you evaluate it graphically, and that is how the higher harmonics might come in. But it doesn't show up in analysis.

MR. AKCASU:

I would like to make my point clear on the blackboard, and what I meant. I didn't mean the subharmonics. What I wanted to remark was that when one finds an approximate solution like this, it is true that the solutions you have written are exponential. But there are terms here which start with μ^2 (at blackboard) and continue to the second harmonic.

Now, when you expand this, then you obtain terms containing μ, μ^2, μ^3 , and so forth. If you want to calculate the amplitude of the first harmonic, with this order of magnitude, then since this is also a second-order term, you get another term here which is also of the same order of magnitude, and which depends on this 2ω . Therefore, to be consistent, if you want to go to the second approximation here (this is really the second approximation, because you don't ignore the terms which depend on the square of this quantity here), you must take into account this factor, which gives another term here of the same order of magnitude.

The effect of the upper harmonic on the amplitude of the first harmonic was this. If you stop here, I call this the first approximation, and there is nothing wrong. If you go one step further to obtain the second approximation for the gain, then this term has to be taken into account because it gives a term which is also of the same order of magnitude. That is what I found. This was checked with other approaches.

MR. FRANZ:

When you wrote down (2ω) , did you mean $(\sin 2\omega)$ or the quantity (2ω) ?

MR. AKCASU:

This (writing on board) is a factor which depends on (2ω) . I don't remember it by heart, of course. There is a term here which is proportional to μ^2 , and depends on the value of (2ω) . This is the second harmonic. When you expand this term, it gives you a term which depends on the $Z_{(2\omega)}$. If this is neglected here, your result here is not consistent because you take into account the second order terms in one place, and you ignore it in the other place, which is the same order of magnitude.

That is what I wanted to say. I didn't mean subharmonics. When you solve this, you don't get subharmonics.

MR. COHEN:

In addition to the $Z_{(2\omega)}$, there is a Z of zero.

MR. FRANZ:

We have it dropped out by definition.

MR. AKCASU:

I don't mean the constant term. I mean the gain of the first harmonic. Thank you.

MR. DE SHONG:

I think we have made our point here. When you use describing functions, you want to be sure and look at all the possibilities.

MR. LANGMUIR:

I am confused by a point. Maybe this would help. I get the impression that $K(x)$ contains nonlinearities, so if it is linear, $K(x)$ becomes constant. I don't see how that reflects in the formula.

MR. FRANZ:

If the μ terms were small, they would be negligible compared to 1 in equation 15, leaving only the term n_0 .

MR. LANGMUIR:

Why would it not be small for a linear system?

MR. FRANZ:

For a system to be linear, we have to assume small amplitude (small μ). What is left gives the linear reactor transfer function. In other words, as you suggest, it would be small in a linear system.

The other point we were making was that, in other elements in the system, the ability to feed back harmonics would be very small in many systems, so we would not expect to have second-order harmonics. Another point is that if the feedback elements are treated as nonlinear, the approach becomes more complex. The describing function is an approximation which provides a simple tool when harmonics can be neglected. With a particular system or problem the describing function may not be adequate nor provide much simplification. More exact analytical treatment, such as in Mr. Akcasu's paper,⁴ where he integrates the equations, is strongly suggested. From the discussion, it would seem that there are many facets of the describing function method as applied to reactors which could be the subject of further investigation

TRANSFER FUNCTION REPRESENTATION OF NUCLEAR POWER PLANTS

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Cambridge, Massachusetts

Introduction

The concept of transfer function representation of nuclear power plants has proved a very fruitful tool both from the points of view of dynamic analysis and the interpretation of experimental data.

This fact can be best ascertained by considering the dynamics of a nuclear power plant, shown schematically in Fig. 1.

Fig. 1.
Schematic of Nuclear Power Plant.

Suppose all parameters of the plant have consistent equilibrium values prior to time $t = t_0$, when one or more of the parameters are perturbed intentionally or unintentionally. Two questions arise:

1. Will the plant parameters seek equilibrium again, namely, is the system stable?
2. If the system is stable, what are the intermediate states its parameters assume between the two equilibria?

In order to find answers to those questions during the design phase of the nuclear plant, one starts by describing analytically the processes partaking in the dynamic operation of the plant. The principles of conservation of particles, mass, energy, momentum, etc., at all times are the basis of the formalism. The resulting equations relate the plant parameters, like neutron population, temperatures, pressures, flows, and load, to the independent variables: time and phase space. The equations are, generally, nonlinear, and/or linear, with variable coefficients, partial differential

equations. The solution of such a system of equations, which would answer the questions of stability and transient response, is extremely difficult if not impossible.¹ Consequently, one either resorts to machine-aided computations or one tries to simplify the analytical model of the plant without introducing excessive errors. The latter approach seems, to us anyway, more attractive.

The simplification of the analytical model is accomplished in two steps:

First, the phase-space dependence of the dynamic equations is integrated out, formally at least, by reformulating the problem in terms of appropriately defined derived variables,² like reactivity, neutron density amplitude, and average temperatures. The derived variables are customarily bilinear averages of the physical parameters of the plant and are related by ordinary differential equations with respect to time only. Those equations are still nonlinear and/or linear with variable coefficients.

Next, the ordinary differential equations are linearized by considering only small amplitude variations of the variables involved. Thus, the differential system can be reduced to a set of algebraic equations by the use of the Laplace transform technique. The algebraic equations can be solved explicitly to yield the desired relationship between any two variables of the plant, which are arbitrarily postulated as the input and output, respectively. This relationship is the transfer function.

Now, the questions of stability and transient response can be answered very elegantly by examining the characteristic modes of the transfer function and without ever finding the inverse Laplace transform. Bode, Nyquist or root-locus plots are very helpful in this respect. Furthermore, the characteristic modes of the transfer function are not related to the a priori postulated input and output of the system. Therefore, the arbitrary definitions of input and output do not limit the generality of the transfer function.

Apart from the fact that the use of the Laplace transformation leads to explicit solutions of systems of linear differential equations, it also affords a physical interpretation in terms of real frequencies. Such an interpretation makes possible the experimental verification of the implications derived from the analysis of transfer functions, as well as the experimental measurement of important parameters of the nuclear power plant. Oscillation tests, autocorrelation or cross-correlation techniques can be used very profitably for this purpose.

¹ Cohen, E. R., Some Topics in Reactor Kinetics, Proc. Second Geneva Conference, 11, 302 (1958).

² Henry, A. F., Application of Reactor Kinetics to the Analysis of Experiments, Nuclear Sci. and Eng., 3, 52 (1958).

Of course, one might object that the concept of transfer function, no matter how elegant, is derived from such a simplified model of the plant dynamics that its implications are unreliable. This objection is quite valid, particularly when one is interested in the details of solutions of the dynamic equations for large amplitude variations of the variables involved. At the same time, though, the applicability of the concept of transfer function in nuclear power plants is not as restricted as the approximations introduced during its derivation tend to indicate. This assertion is based on the following facts:

1. The boundedness and stability of the solutions of a system of ordinary differential equations begins always with the examination of the type of stability at the equilibrium points of the differential system.^{3,4} The equilibrium points are derived from the linearized equations and the type of stability is deduced from the characteristic modes which are identical with the characteristic modes of the transfer function.⁵ Consequently, the concept of the transfer function is very important, even when the exact solution of nonlinear differential equations is investigated.

2. The nonlinear reactor equations are usually formulated in terms of the time-dependent amplitude of the first spatial mode of the neutron density. It can be shown⁶ that the neglect of the modes of higher order introduces, particularly in cases of localized perturbations,¹ a larger error than the linearization of the dynamic equations. Therefore, the applicability of both the linear and the nonlinear approaches is limited.

3. The experimental data, which constitute the ultimate criterion of any theory, are in very good agreement with the theoretical predictions derived from transfer function analysis.

4. The implications of the transfer function concept can be used very profitably even in cases where the physics of certain dynamic aspects of new nuclear plants is not well understood. Transfer function measurements allow one to explore the unknown effects safely during the development of the plant. The Boiling Water Reactor⁷ is an example of this approach.

³ Malkin, I. G., Certain Questions on the Theory of the Stability of Motion, Am. Math. Soc. Transl. No. 20, (1950).

⁴ Liapunov, M. A., Problème Général de la Stabilité du Mouvement, Ann. Fac. Sci. Univ. Toulouse, 9 (2), 203-474 (1907).

⁵ Smets, H. B. and E. P. Gyftopoulos, Topological Methods in Reactor Kinetics, Nuclear Sci. and Eng., 6, 341 (1959).

⁶ Devooght, J., Nonlinear Reactor Dynamics, M. Sc. Thesis in Nuclear Eng. MIT, (June 1959).

⁷ Kramer, A. W., Boiling Water Reactors, Addison-Wesley Publishing Co., Reading, Massachusetts (1958).

Components of Nuclear Power Plants

The major components of a nuclear power plant, in general, are: the nuclear reactor, the heat exchanger, the turbogenerator, and the condenser. Associated with those components are auxiliary devices, such as valves, pumps, and electronic controls (see Fig. 1).

The transfer function of each component is not universal. Its form depends on the structure, the mode of operation of the component, and the mental picture that one has about the sequence of events during dynamic operation.

The following sections present typical transfer functions of various components or processes of nuclear power plants and some conclusions that may be derived by means of simple manipulations.

Neutron Dynamics

The time-dependent equations for the neutron and delayed neutron precursors are

$$\frac{d}{dt} N(t) = \frac{\rho - \beta_e}{\Lambda} N(t) + \sum_i^m \lambda_i C_i(t) + S(t) \quad (1)$$

$$\frac{d}{dt} C_i(t) = \frac{\beta_{ie}}{\Lambda} N(t) - \lambda_i C_i(t) \quad (i = 1, 2 \dots m). \quad (2)$$

Equations (1) and (2) can be derived either from a one-group model⁸ or from general transport theory balance equations.² The neutron generation time, Λ ,⁹ rather than the neutron lifetime, ℓ , is used because, in most problems of practical interest, the reactor is perturbed through changes of the destruction rate. Therefore Λ , and not ℓ , is constant. Furthermore, when the concept of the generation time is used, the equations for the delayed neutron precursors do not contain the multiplication factor or reactivity.

If reactivity is assumed as the input to the reactor and neutron level amplitude as the output, two transfer functions can be derived from equations (1) and (2).

⁸ Glasstone, S., Principles of Nuclear Reactor Engineering, D. Van Nostrand Co., New York (1955).

⁹ Lewins, J., The Use of the Generation Time in Reactor Kinetics, Nuclear Sci. and Eng., 7, 122 (1960).

1. Transfer Function of Critical Reactor

The source $S(t)$ is zero. The linearization of equations (1) and (2) and transformation of the time variable into frequency yields:

$$\frac{\bar{N}}{\bar{\rho}} = \frac{N_0}{s \left[\Lambda + \sum_i^m \frac{\beta_{ie}}{s + \lambda_i} \right]}. \quad (3)$$

The same transfer function has been derived by various authors after introducing certain approximations in the basic kinetics equations.¹⁰ The approximations are not necessary in the present derivation because of the use of the concept of the generation time.

2. Transfer Function of Subcritical Reactor

The source $S(t) = S_0 = \text{constant}$. The transfer function is

$$\frac{\bar{N}}{\bar{\rho}} = \frac{N_0}{\Lambda s + \sum_i^m \frac{\beta_{ie}s}{s + \lambda_i} + \Lambda S_0}. \quad (4)$$

It is evident from equation (4) that a subcritical reactor does not act as an integrator, as when $S_0 = 0$. This fact is important when interpreting experimental data from reactors with built-in sources, like the heavy water reactors.

Equivalent Delayed Neutron Groups

The transfer function of the reactor provides the means for approximating the m delayed neutron precursors by $k(<m)$ equivalent groups.

The basic idea is to evaluate the constants of the equivalent delayed neutron groups (λ_j^* , β_j^*) so that the transfer functions, of the exact and approximate systems, are almost identical over broad ranges of frequency. This idea may be implemented by means of the method of moments approximation.

More specifically, the approximate transfer function is

¹⁰Soodak, H. and E. C. Campbell, Elementary Pile Theory, John Wiley & Sons, Inc., New York (1950).

$$\frac{\bar{N}}{\bar{\rho}} = \frac{N_0}{s \left[\Lambda + \sum_{j=1}^k \frac{\beta_j^*}{s + \lambda_j^*} \right]} \quad (5)$$

If a high-frequency approximation is desired, the functions (3) and (5) may be expanded into power series around $1/s = 0$ and the coefficients of the first $(2k)$ powers of $1/s$ taken as equal. Thus $(2k)$ equations result:

$$\sum_{j=1}^k \beta_j^* \lambda_j^{*q} = \sum_{i=1}^m \beta_{ie} \lambda_i^q \quad [q = 0, 1, \dots, (2k - 1)], \quad (6)$$

from which λ_j^* and β_j^* can be determined. A similar procedure can be followed for small frequencies but then the power series expansions should be performed around $s = 0$. Thus it is found that

$$\sum_{j=1}^k \beta_j^* \lambda_j^{*-q} = \sum_{i=1}^m \beta_{ie} \lambda_i^{-q} \quad [q = 1, 2, \dots, 2k]. \quad (7)$$

Furthermore, the large and small-frequency approximations can be combined to yield a better overall approximation of the actual transfer function. The $2k$ equations that must be solved are

$$\left. \begin{aligned} \sum_{j=1}^k \beta_j^* \lambda_j^{*q} &= \sum_{i=1}^m \beta_{ie} \lambda_i^q & [q = 0, 1, \dots, (k - 1)] \\ \sum_{j=1}^k \beta_j^* \lambda_j^{*-p} &= \sum_{i=1}^m \beta_{ie} \lambda_i^{-p} & [p = 1, 2, \dots, k] \end{aligned} \right\} \quad (8)$$

It should be pointed out that the small-frequency approximation is good for very slow transients while the large frequency approximation is valid for fast transients.

Reactor Core Thermal Dynamics

The reactor core thermal dynamics refers to the relationships between the power, coolant input temperature, and coolant flow, and the temperature distribution throughout the reactor core. It is also related to the effects of the temperature distribution on criticality or reactivity.

The thermal dynamics transfer functions are not universal. They depend on the particular reactor in question and the conceptual model that one forms about the sequence of events involved.

The presentation of all the models that have been derived for different reactors is beyond the scope of this paper. The interested reader is referred to references 11 through 17. In order to discuss the methodology of the development of thermal dynamics transfer functions, only a distributed and a lumped parameter model are given.

Transfer Functions of a Distributed Parameter Reactor Core

Consider a heterogeneous reactor with fuel, moderator and one-phase coolant regions. The heat exchange is mostly one-dimensional from the fuel region perpendicularly to the coolant, through the moderator. The reactor is assumed as made of groups of three slabs (see Fig. 2), without any loss of generality, because such a geometry can be adapted to many reactor designs.¹⁸

Fig. 2.
Reactor Core Model

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- ¹¹ Schultz, M. A., Control of Nuclear Reactors and Power Plants, McGraw-Hill Book Co., New York (1955).
 - ¹² MacPhee, J., The Stability of Multipass Reactors, Nuclear Sci. and Eng. 4, 200 (1958); The Kinetics of Circulating Fuel Reactors, Nuclear Sci. and Eng. 4, 588 (1958).
 - ¹³ Skinner, R. E. and D. L. Hetrick, The Transfer Function of a Water Boiler Reactor, Nuclear Sci. and Eng. 3, 573 (1958).
 - ¹⁴ Remley, M. E. et al., Kinetic Experiments on Water Boiler Reactors, Proc. Second Geneva Conf. 11, 447 (1958).
 - ¹⁵ Beckjord, E. S., Dynamic Analysis of Boiling Water Reactors, ANL-5799 (1958).
 - ¹⁶ Ackroyd, R. T., Stability Considerations in the Design of Fast Reactors, Proc. Second Geneva Conf. 12, 230 (1958).
 - ¹⁷ O'Neill, T. J., Heat Transfer Transient Equations for Graphite Reactors, Proc. Second Geneva Conf. 11, 269 (1958).
 - ¹⁸ Larson, C. L., Heat Transfer in a Cross Flow Nuclear Reactor, Nuclear Sci. and Eng. 4, 607 (1958).

Assume that the inputs to the reactor core are power, coolant velocity and coolant input temperature. Consider two temperature coefficients of reactivity. One is associated with the mean fuel temperature and another with the mean moderator temperature. Under those conditions the transfer functions of interest are the ones shown schematically in Fig. 3.

Fig. 3. Reactor Core Block Diagram.

The transfer functions are derived from the space and time-dependent partial differential equations. It is assumed that:

1. The heat is released in the fuel slabs and carried away by the nearest coolant slabs.
2. The heat is uniform throughout the reactor core.
3. The heat is transferred by conduction in the fuel and moderator regions and by convection in the coolant region.
4. The temperature across the coolant region is constant, because of turbulence of the fluid.
5. The cross section of the coolant slabs is varied so that the outlet coolant temperature is equal for any slab.
6. The heat transfer coefficient h is proportional to $v^{0.8}$.

The details of the derivation of the transfer functions are given in reference 19. It is found that:

¹⁹ Gyftopoulos, E. P. and H. B. Smets, Nuclear Power Plant Transfer Functions, Nuclear Eng. and Sci. Conf., Preprint 53; Chicago (1958) Distributed Parameter Reactors, Nuclear Sci. and Eng. 5, 405 (1959).

$$\frac{\bar{\theta}_\ell}{\bar{Q}} = \frac{y_{12}}{h \cos h(qy_0)} \left[\frac{s+b}{vf_1} - 1 \right] \left[1 - e^{-f_1 \ell} \right] \quad (9)$$

$$\bar{\theta}_\ell / \bar{\theta}_0 = e^{-f_1 \ell} \quad (10)$$

$$\frac{\bar{\theta}_\ell}{\bar{v}'} = \frac{Q_0 y_{12}}{v^2 h f_1} \left[0.8 (s - vf_1) + b \right] \left[1 - e^{-f_1 \ell} \right] \quad (11)$$

$$\begin{aligned} \frac{\bar{T}_{AVm}}{\bar{Q}} = y_{12} & \left[\frac{\tanh(qy_0)}{h q y_0 \left\{ 1 + \frac{kq}{h} \tanh(qy_0) \right\} \cosh(qy_0)} \right. \\ & \left. \cdot \left\{ \frac{1 - e^{-f_1 \ell}}{f_1 \ell} + \frac{s+b}{vf_1} \left[1 - \frac{1 - e^{-f_1 \ell}}{f_1 \ell} \right] \right\} + \frac{\cosh(qy_0) - 1}{k q^2 y_0 \cosh(qy_0)} \right] \end{aligned} \quad (12)$$

$$\frac{\bar{T}_{AVm}}{\bar{\theta}_0} = \frac{\tanh(qy_0)}{f_1 \ell q y_0 \left[1 + \frac{kq}{h} \tanh(qy_0) \right]} \left[1 - e^{-f_1 \ell} \right] \quad (13)$$

$$\frac{\bar{T}_{AVm}}{\bar{v}'} = \frac{Q_0 y_{12} \tanh(qy_0)}{h q y_0 v} \cdot \frac{\frac{1 - e^{-f_1 \ell}}{f_1 \ell} \left[0.8 - \frac{0.8s+b}{vf_1} \right] + \frac{0.8s+b}{vf_1}}{1 + \frac{kq}{h} \tanh(qy_0)} \quad (14)$$

$$\frac{\bar{T}_{AVf}}{\bar{Q}} = \frac{vb}{\ell q_2^2 y_{12}} \cdot \frac{\ell \frac{\Lambda}{\pi} - 1 + e^{-\frac{\Lambda}{\pi} \ell}}{\Lambda^2 h} \sinh^2(q_2 y_{12}) + \frac{\alpha_2}{k_2 s} \quad (15)$$

$$\frac{\bar{T}_{AVf}}{\bar{\theta}_0} = \frac{v}{\ell y_{12}} \left[\frac{1 - e^{-\frac{\Lambda}{\pi} \ell}}{q_2 \Lambda} \sinh(q_2 y_{12}) \right] \quad (16)$$

$$\frac{\bar{T}_{AVf}}{\bar{v}'} = \frac{Q_0 \sinh(q_2 y_{12})}{h q_2} \left[0.8 \frac{1 - e^{-\frac{\Lambda}{\pi} \ell}}{\Lambda \ell} + \frac{0.8s+b}{v \Lambda} \left(1 - \frac{1 - e^{-\frac{\Lambda}{\pi} \ell}}{\Lambda \ell} \pi \right) \right], \quad (17)$$

where

$$f_1 = \frac{s}{v} + \frac{b}{v} \cdot \frac{\frac{kq}{h} \tanh(qy_0)}{1 + \frac{kq}{h} \tanh(qy_0)}$$

$$\Lambda = s \frac{q_2 k_2}{q_1 k_1} \sinh(q_1 y_1) \cdot \sinh(q_2 y_{12}) + (s+b) \frac{q_1 k_1}{h} \sinh(q_1 y_1) \cdot \cosh(q_2 y_{12})$$

$$+ s \cosh(q_1 y_1) \cdot \cosh(q_2 y_{12}) + (s+b) \frac{q_2 k_2}{h} \cosh(q_1 y_1) \cdot \sinh(q_2 y_{12})$$

$$\pi = v \frac{q_2 k_2}{q_1 k_1} \sinh(q_1 y_1) \cdot \sinh(q_2 y_{12}) + v \frac{q_1 k_1}{h} \sinh(q_1 y_1) \cdot \cosh(q_2 y_{12})$$

$$+ v \cosh(q_1 y_1) \cdot \cosh(q_2 y_{12}) + v \frac{q_2 k_2}{h} \cosh(q_1 y_1) \cdot \sinh(q_2 y_{12})$$

$$q_1^2 = s / \bar{\alpha}_1 \quad .$$

All the other symbols are defined in the nomenclature.

The transfer functions are transcendental and very cumbersome to use, which is typical of distributed parameter systems. However, both graphical¹⁹ and analytical^{20,21} methods have been devised for their approximation in terms of rational functions and pure delays. Two typical approximations are

$$\frac{\bar{\theta}_l}{\bar{\theta}_0} = e^{-\frac{b l}{v}} + \frac{(1 - e^{-\frac{b l}{v}})(1 + \tau_1 s)}{(1 + \tau_2 s)(1 + \tau_3 s)} e^{-\frac{s l}{v}} \quad (18)$$

$$\frac{\bar{\theta}_l}{\bar{Q}} \propto \frac{1}{1 + \tau_4 s} \frac{1 - \tau_5 s}{1 + \tau_5 s} \quad (19)$$

²⁰ Sandmeier, A. H., Time Constant of Cylindrical Fuel Pin, Nuclear Sci. and Eng., 5, 186 (1959).

²¹ Iriarte, M., Heat Release in Cylindrical Fuel Elements, Nuclear Sci. and Eng., 7, 26 (1960).

The time constants τ_i are complicated, implicit functions of the geometry and the steady-state parameters of the reactor. They can be found by trial-and-error matching of the exact functions, over a broad frequency range, along the $s = j\omega$ axis. Particular examples of the exact and approximate transfer functions $\bar{\theta}_\ell / \bar{\theta}_0$ and $\bar{\theta}_\ell / \bar{Q}$, for $s = j\omega$, are given in Figs. 4 and 5, respectively.

Fig. 4.
Exact and Approximate Temperature Transfer Functions for $s = j\omega$ (ω in hr^{-1}).

Fig. 5.
Exact and Approximate Power Transfer Functions for $s = j\omega$ (ω in hr^{-1}).

Transfer Functions of a Lumped Parameter Reactor Core

Consider a reactor with two temperature coefficients of reactivity, r_1 and r_2 , related to two representative core temperatures, T_1 and T_2 , respectively. Assume a model independent of spatial coordinates and whose thermal dynamics is describable by Newton's law of cooling, namely,

$$\frac{dT_1}{dt} = a_1 N - g_1 T_1 \quad (20)$$

and

$$\frac{dT_2}{dt} = a_2 N - g_2 T_2 \quad (21)$$

Both the production coefficients a_i and the heat removal coefficients g_i are constant. The block diagram of this reactor is shown in Fig. 6, where

$$\frac{\bar{T}_1}{\bar{N}} = \frac{a_1}{s + g_1}; \quad \frac{\bar{T}_2}{\bar{N}} = \frac{a_2}{s + g_2}; \quad H(s) = r_1 \frac{\bar{T}_1}{\bar{N}} + r_2 \frac{\bar{T}_2}{\bar{N}}. \quad (22)$$

Fig. 6.
Reactor Core Reduced Block Diagram.

It is interesting to discuss the stability of the reactor for various values of the temperature coefficients r_1 and r_2 . For the purposes of this discussion the effects of the delayed neutron precursors are neglected from the neutron transfer function, which reduces to

$$\frac{\bar{N}}{\bar{\rho}} = \frac{N_0}{\Lambda s}. \quad (23)$$

The application of Nyquist's or root-locus criterion of stability reveals the following:

1. If both temperature coefficients are positive ($r_1 > 0$, $r_2 > 0$), the reactor is obviously always unstable.

2. If both temperature coefficients are negative ($r_1 < 0$, $r_2 < 0$), the reactor is always stable.

3. If one temperature coefficient is positive and one is negative ($r_1 > 0$, $r_2 < 0$), then, when

$$r_1 a_1 g_1 + r_2 a_2 g_2 < 0; \quad r_1 a_1 g_2 + r_2 a_2 g_1 < 0 \quad (24)$$

the reactor is always stable. However, if

$$r_1 a_1 g_1 + r_2 a_2 g_2 > 0; \quad r_1 a_1 g_2 + r_2 a_2 g_1 < 0, \quad (25)$$

which implies that $g_1 > g_2$, the reactor is stable only when the neutron density is

$$N_0 < \frac{\Lambda g_1 g_2 (g_1 + g_2)}{r_1 a_1 g_1 + r_2 a_2 g_2}. \quad (26)$$

Those results are summarized in graphical form in Fig. 7. The graphs clearly indicate the well-known fact that the importance of a temperature coefficient is determined by its promptness or delayedness which, in the present case, is the inverse heat removal coefficient.

Fig. 7. Regions of Stability of Reactor with Two Temperature Coefficients. Lumped Parameter Model.

Stabilizing Effects of Heat of Radioactive Decay

Part of the heat of radioactive decay of fission products is transferred directly to the coolant. This fact is usually omitted from the analytical model of the thermal dynamics, even though it has a favorable influence on the stability of the reactor.

Consider a water reactor with plate-type fuel elements. Assume a lumped parameter model with one temperature coefficient associated with the coolant. Neglect the heat exchanger coupling.

If all the heat of fission is produced in the fuel, the feedback transfer function is¹¹

$$H(s) = \frac{k}{s^2 + \left[\frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{2}{\tau_0} \right] s + \frac{2}{\tau_1 \tau_0}} \quad (27)$$

The root locus of the characteristic modes of the overall transfer function is as shown in Fig. 8. Evidently, the reactor becomes unstable above a certain power level indicated by the crossover points (A, A') of the locus.

Now, if a fraction α of the total heat is delivered directly to the coolant, it can be easily proved that the feedback transfer function is

$$H(s) = \frac{k\alpha\tau_1 \left(s + \frac{1}{\alpha\tau_1}\right)}{s^2 + \left[\frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{2}{\tau_0}\right]s + \frac{2}{\tau_1\tau_0}} \quad (28)$$

The reactor is stable for all power levels (Fig. 9), provided that

$$\alpha > \frac{1}{1 + \frac{\tau_1}{\tau_2} + 2\frac{\tau_1}{\tau_0} + \sum_i^m \frac{\beta_{ie}}{\Lambda} \left[\sum_j^m \lambda_j - \lambda_i \right]} = \alpha_0 \quad (29)$$

For $\Lambda \leq 10^{-4}$, condition (29) yields $\alpha_0 \leq 1.2\%$, and $\alpha > \alpha_0$ is satisfied by many reactors. Therefore, it is ascertained that the heat of radioactive decay has a stabilizing influence.

Fig. 8. Root Locus of Characteristic Modes of Heterogeneous Reactor with One Temperature Coefficient. All Heat is Released in the Fuel.

Fig. 9. Root Locus of Characteristic Modes of Heterogeneous Reactor with One Temperature Coefficient. A Fraction of Heat (α) is Transferred Directly to the Coolant.

Effects of Delayed Neutrons on Stability

It has been shown, quite rigorously,²² that the delayed neutron precursors influence favorably the stability of a reactor. This fact can also be illustrated by means of the transfer function concept as follows:

Consider, as an example, the case of the EBWR. The block diagram of the reactor is shown in Fig. 6,⁷ where

$$H(s) = \frac{k_1(1 + \tau_1 s)}{(1 + \tau_2 s)(1 + \tau_3 s)(1 + \tau_4 s)(1 + \tau_5 s)} \quad (30)$$

Suppose that the delayed neutron precursors are omitted from the neutron transfer function. The root locus of the characteristic modes of the over-all transfer function is then as shown in Fig. 10. The power level at which instability occurs is, to a very good approximation, proportional to the ratio of the distances of the crossover point A from the poles and zeros (stars and noughts) of the locus, respectively.

Fig. 10
Root Locus of Characteristic Modes of
EBWR without Delayed Neutrons.

Now, suppose the delayed neutrons are included in the neutron transfer function and examine their effects on the root locus. It is evident that the asymptotic behavior of the locus will not be affected because m negative poles and m negative zeros have been introduced (see Fig. 11). However, the center of gravity G (Fig. 10) of the pole-zero configuration has been shifted to G' because the difference between the poles and zeros introduced by the delayed neutrons is

²² Gyftopoulos, E. P. and J. Devooght, Effects of Delayed Neutrons on Nonlinear Reactor Stability, to be published.

$$\begin{aligned}
\sum_i^m (\text{poles}) - \sum_i^m (\text{zeros}) &= - \left[\sum_i^m \lambda_i + \sum_i^m \frac{\beta_{ie}}{\Lambda} \left(\sum_j^m \lambda_j - \lambda_i \right) \right] + \sum_i^m \lambda_i \\
&= - \sum_i^m \frac{\beta_{ie}}{\Lambda} \left(\sum_j^m \lambda_j - \lambda_i \right) < 0 \quad . \quad (31)
\end{aligned}$$

This shift changes the crossover point from A to A' and simple inspection and comparison of the loci, with and without delayed neutrons, reveals immediately that the new power level for instability is higher. Hence, the delayed neutrons have a stabilizing influence on the reactor.

Fig. 11
Root Locus of Characteristic
Modes of EBWR with Delayed
Neutrons.

It should be pointed out that the use of the EBWR is incidental and the previous procedure and conclusions are applicable to any reactor model.

Heat Exchanger Dynamics

The dynamics of heat exchangers is extremely involved both from the physical and the analytical standpoints. In spite of the fact that heat exchangers have been in use for many decades, it is only recently that the problem of their dynamic operation has attracted the attention of various authors. A complete discussion of recent developments and an extensive bibliography can be found in reference 23.

²³ Hansen, P. D., The Dynamics of Heat Exchange Process, ScD. Thesis, M. Eng. MIT, (Feb. 1960).

Transfer functions of heat exchangers have appeared in the literature.^{11,24-26} Both lumped and distributed parameter models have been considered, and the transfer functions have been derived under a variety of assumptions. Unfortunately, there is practically no experimental information available with which the analytical models can be evaluated. This is particularly true for variable flow conditions.

In what follows, the transfer functions of a counter flow heat exchanger (Fig. 12) are presented. These functions were derived¹⁹ from a distributed parameter, tube-and-shell model (see Fig. 13), under the following assumptions:

1. The primary coolant does not change phase.
2. The flow is turbulent in both fluids.
3. The heat conduction along the axial direction of the tube and the shell is zero.
4. The heat conductivity along the radius of the tube and the shell is infinite.
5. The heat losses to the surroundings are zero.
6. The heat transfer coefficients are independent of temperature for each region.
7. The boundaries between economizer-boiler and boiler-superheater are fixed.

Fig. 12.
Counter Flow Heat Exchanger.

²⁴Paynter, H. M. and Y. Takahashi, Counterflow and Parallel Flow Heat Exchangers, Trans. ASME 749, (May 1956).

²⁵Takahashi, Y., Transfer Function of Heat Exchange Processes, Automatic and Manual Control, Butterworth Scientific Publications (1952).

²⁶Sutton, R. W., A Study of the Transient Behavior of Heat Exchangers, AERE-LEO/A 63 (1953).

Fig. 13.
Heat Exchanger Model.

The derived transfer functions are shown in Figs. 14 and 15. The details of the derivations can be found in reference 19.

Fig. 14. Superheater Transfer Functions.
Distributed Parameter Model.

Fig. 15.
Counter Flow Heat Exchanger Transfer
Functions. Distributed Parameter
Model.

Again, the transfer functions are transcendental but can be approximated by rational functions and pure delays.

It should be pointed out that the dynamic behavior of heat exchangers is due to thermal capacities and mass transports. Therefore, the open loop response to flow or temperature changes is monotonic and non-oscillatory. However, in natural-circulation circuits, stable and unstable oscillations may occur due to couplings of thermal and mechanical effects.

Turbogenerator Dynamics

The dynamics of turbogenerators is also very difficult to describe and not so well understood. The problem is complicated because it is related to transient thermodynamics, a field which has not been fully explored yet.²⁷

Approximate transfer functions have been derived and some typical examples are subsequently presented.

Consider the turbogenerator shown schematically in Fig. 16. Transfer functions for the unit are shown in Fig. 17. They are derived under the following assumptions:

1. The pressure in front of the throttling valve is constant.
2. The turbine angular speed is regulated by a speed controller.
3. The alternator has no dampers.
4. The alternator is connected to an infinite bus bar.
5. The alternator flux linkages are not affected by speed changes.

Fig. 16. Schematic of Turbogenerator Unit.

²⁷ Brown, D. H., Transient Thermodynamics of Reactors and Process Apparatus, Nuclear Eng. and Sci. Conf., Preprint 81, Philadelphia (1957).

Fig. 17. Turbogenerator Transfer Functions.

The valve opening-turbogenerator power transfer function is

$$\frac{\bar{P}}{\bar{E}} = \frac{k}{1 + \tau_t s} \quad (32)$$

and is derived in reference 28.

The shaft speed-throttle opening transfer function is

$$\frac{\bar{E}}{\bar{\omega}} = \frac{e^{-\mu s}}{s \left[\tau_s + \frac{\tau'_s}{1 + \tau_r s} \right]} \quad (33)$$

and is derived in reference 29.

The power-shaft position transfer function is given in reference 30:

$$\frac{\bar{\delta}}{\bar{P}} = \frac{\frac{k_1}{s^2 + 2\xi\omega_n s + \omega_n^2}}{1 - \frac{k_1 k_2}{s^2 + 2\xi\omega_n s + \omega_n^2} \left[\frac{k_3 F(s)}{1 + \tau_a s - k_3 k_6 F(s)} \right] \left[k_5 - \frac{k_4 k_6}{1 + \tau_a s} \right]} \quad (34)$$

²⁸ Hoyt, P. R. et al, Dynamic Field Tests of a Steam Turbine, ASME paper 57-IRD-15 (1957).

²⁹ Willems, P., Effect du Contrôle à Asservissement de la Turbine d'un Groupe sur la Stabilité de son Alternateur, Revue E 1, 4, 75 (1954).

³⁰ Heffron, W. G. and R. A. Phillips, Effect of a Modern Amplidyne Voltage Regulator on Turbogenerators, AIEE Trans., 71 III, 692 (1952).

Here, $F(s)$ = voltage comparator transfer function.

If the effects of speed changes were included (assumption e) the shaft position-power transfer function would have been much more complicated.³¹

Other authors have also treated the problem of generator dynamics under a variety of assumptions.^{32,33}

Condenser Dynamics

For all practical purposes, the dynamic operation of a condenser is similar to the operation of a heat exchanger. Therefore the previous discussion on Heat Exchanger Dynamics is directly applicable to the problem of condensers.

Conclusions

Representative transfer functions of the major components of nuclear reactor plants have been described and some useful techniques for their manipulation illustrated. Both distributed and lumped parameter models have been considered.

It is shown, by simple methods, that both heat of radioactive decay and delayed neutron precursors have a stabilizing influence on reactor dynamics.

Transfer functions of pipes, mixing volumes and auxiliary equipment are not discussed, but can be found in the literature.^{11, 23, 34-36}

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- ³¹ Mesarovic, M., and I. Obradovic, Influence of a Supplied Power System on the Frequency Control, C.I.G.R.E. (1956).
 - ³² Riaz, M., Analogue Computer Representation of Synchronous Generators, AIEE Trans. 75, III, 1178 (1956).
 - ³³ Breedon, D. B., and R. W. Ferguson, Fundamental Equations for Synchronous Machines, AIEE Trans. 76, III, 297 (1957).
 - ³⁴ Thaler, G. J. and W. A. Stein, Transfer Function and Parameter Evaluation for D. C. Servomotors, AIEE Trans. 74, II, 410 (1956).
 - ³⁵ Ezekiel, F. D., and H. M. Paynter, Computer Representation of Engineering Systems Involving Fluid Transients, ASME paper 56-A-120 (1956).
 - ³⁶ Chestnut, H. and R. W. Mayer, Servomechanisms and Regulating System Design, John Wiley & Sons, Inc., New York (1948).

The usefulness of distributed parameter models may be inferred from the following facts:

1. The transfer functions introduce phase shifts larger than 90° for large frequencies. This is not the case for lumped parameter models. Therefore the transcendental functions or their approximations should be used in questions of stability.
2. The transfer functions require more than one time constant for their representation. This is important when interpreting experimental data because certain characteristic frequencies may be attributed to non-existing physical phenomena.
3. The initial transient response is better represented by distributed parameter models. Disturbances do not appear to propagate instantly through the system, as is the case with a lumped parameter model.

Of course all the advantages of distributed parameter models are gained at the expense of simplicity.

Finally, it should be emphasized that each component participates effectively in the plant dynamics only over a limited frequency range. Therefore, in analyzing the plant operation, one may concentrate on definite frequency ranges and ignore effects which happen to be ineffective in that range.

Nomenclature

Section - Transfer Functions of a Distributed Parameter Reactor Core

$$b = \frac{hA^*}{M'c'}$$

c' specific heat of coolant

h heat transfer coefficient at the coolant-moderator interface

$k = k_1, k_2$ conductivity of moderator and fuel, respectively

ℓ average length of coolant slabs

v, v' coolant velocity and velocity increment, respectively

y_1, y_{12} thickness of moderator and fuel slab, respectively

$$y_0 = \frac{\alpha}{k} \left[\frac{k_1 y_1}{\alpha_1} + \frac{k_2 y_{12}}{\alpha_2} \right]$$

A^* effective surface of heat exchange between moderator and coolant

M_f, M_m, M'	mass of fuel, moderator, and coolant, respectively
Q_0, Q	steady-state and incremental heat source density in the fuel, respectively
T_{AVf}, T_{AVm}	average temperature of fuel and moderator, respectively
α_1, α_2	thermal diffusivity of moderator and fuel, respectively

$$\frac{1}{\alpha} = \frac{M_m \frac{1}{\alpha_1} + M_f \frac{1}{\alpha_2}}{M_m + M_f}$$

θ_0, θ_ℓ	input and output coolant temperature, respectively
ρ^*, ρ	reactivity

Section - Stabilizing Effects of Heat of Radioactive Decay

τ_1	fuel time constant
τ_2	fuel-coolant interface time constant
τ_0	coolant transit time

Section - Effects of Delayed Neutrons on Stability

τ_i	time constants, functions of pressure and power
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Section - Heat Exchanger Dynamics

$$a_i = \frac{h_i A_i}{M_i c_i}$$

$$b_1 = \frac{h_1 A_1}{M_3 c_3}; b_2 = \frac{h_2 A_2}{M_2 c_2}; b_3 = \frac{h_3 A_3}{M_4 c_4}$$

c_1, c_2, c_3, c_4	specific heat of water, hot fluid, tube, and shell, respectively
h_1, h_2, h_3	heat transfer coefficient on the water side, hot fluid side, and shell side, respectively
ℓ, ℓ', ℓ''	superheater, boiler, and economizer length, respectively
v_1, v_2	water and hot fluid velocity, respectively

A_1, A_2, A_3	surface area of tube on water side, of tube on hot fluid side, and of shell, respectively
F	fluid flow through mixing volume
M_1, M_2, M_3, M_4	mass of water, hot fluid, tube, and shell, respectively
V	mixing volume
$\theta_{10}, \theta_1 \ell$	inlet and outlet water temperature, respectively
$\theta_{20}, \theta_2 \ell$	inlet and outlet hot fluid temperature, respectively

$$f_1 = \frac{1}{v_1} \left[s + a_1 \left[(s+b_2)/(s+b_1+b_2) \right] \right]$$

$$f_2 = \frac{1}{v_2} \left[s + a_2 \left[(s+b_1)/(s+b_1+b_2) \right] \right] + a_3 \left[s/(s+b_3) \right]$$

$$f_3 = \frac{1}{v_1} \left[a_1 b_2/(s+b_1+b_2) \right]$$

$$f_4 = \frac{1}{v_2} \left[a_2 b_1/(s+b_1+b_2) \right]$$

$$f_5 = s + a_2 \left[(s+b_1)/(s+b_1+b_2) \right] + a_3 \left[s/(s+b_3) \right]$$

$$\alpha = \frac{f_2 - f_1}{2}, \quad \beta = \frac{1}{2} \left[(f_1 + f_2)^2 - 4f_3f_4 \right]^{\frac{1}{2}}$$

$$\sinh \lambda = \beta / \sqrt{f_3 f_4}$$

Discussion of Paper
Presented by Mr. Gyftopoulos

MR. ERGEN:

In the equations which you had, about in the middle of your talk, I think, you did not consider the transfer of the heat from one region, which was one time constant, into the other time constant. The heat was generated simultaneously in both regions, is that right?

MR. GYFTOPOULOS:

That is right. In this model I did not assume any time delay. However, in the previous example, which was somewhat simpler, this was taken into account. There, three time constants were considered, one for the fuel, one the fuel film coefficient, and one the transit time.

MR. SANDMEIER:

I would like to ask a question. Since all the experts in measurement are here, I wondered are there somewhere reactor data that came off in which one was allowed to insert enough reactivity that harmonics were actually generated in the flux? I am referring, for example, to the experiment which we did at EBWR. When they came into the resonance region the stroke was too large, and the harmonics were generated, producing data which I used. I wondered if such data are available somewhere, in which harmonics were actually generated; in which one was allowed to do that sort of thing?

MR. WASSERMAN:

We did some experiments in which we varied the amplitude of the input at very low power. I will talk a little bit about those this afternoon, so perhaps we will defer it until then.

MR. CORDY:

Again on the KEWB reactor, we modulated and got a lot of data. It hasn't been really looked into, but we have it.

MR. GRACE:

I wanted to ask, what is your interest in generating the transfer functions for the steam system component, the generator, and so forth?

MR. GYFTOPOULOS:

Let me see if I can understand your question. You mean my personal interest?

MR. GRACE:

What is the application of the generation?

MR. GYFTOPOULOS:

Well, I thought, since the topic of my talk was transfer functions of nuclear plant components, I might complete the issue by presenting transfer functions for all the components.

MR. GRACE:

I think that we did find in some of our early studies of power plants, some of the subsequent power plants, that the time constants associated with the components in the secondary side were rather insignificant. That the steam generator could be represented very simply with a two or three-term transfer function, which gave an attenuation of the temperature on the primary side, that is, between heat exchanger inlet temperature to heat exchanger outlet temperature, which is subsequently fed back in the reactor, and that heat exchanger transfer function, which really reflects everything on the secondary side, was quite simple.

MR. GYFTOPOULOS:

Is this experimental or analytical?

MR. GRACE:

All of this was done analytically at first, and then was checked experimentally in the plant at Idaho.

MR. GYFTOPOULOS:

There is one comment I have to make about heat exchangers. The dynamics of heat exchangers are based on heat storage and mass transfer. Therefore, if you analyze only these two phenomena, the chances are you will end up with something which behaves monotonically and introduces some reduction in the gain. However, when you proceed with models which involve also hydraulic or mechanical effects, then it has been shown that you may have coupling between the two and sustained oscillations may develop. This is a case where the model presented in the paper would be useful.

There is another reason why we have an interest in heat exchangers. This is rather from a microscopic point of view as compared to the macroscopic approach of transfer functions, and refers to the type of temperature gradients that one has at the interface between boiling and economizing regions or between boiling and superheating regions. Tube temperature gradients arise because of the change of the characteristics of the heat transfer process. The analysis shows that there is a large tube temperature gradient right at the boundary. Therefore, an oscillating transient constitutes a cyclic thermal load for the tube. We would like to have a better knowledge of this phenomenon. We are doing some work connected with the Enrico Fermi reactor heat exchanger. It is evident that such problems can be handled only with distributed parameter models.

MR. GRACE:

I think it is fortunate in those cases you can separate the stability problems on the secondary side from those on the primary side. Oscillations in heat exchanger systems have been observed due to, perhaps, poor setting of the controller, the water controller. These can be generalized, and once they are designed to be stable, their effect on the primary loop is generally quite small.

MR. GYFTOPOULOS:

You are absolutely right. I didn't read my paper, but there I concluded by saying: "It should be emphasized that each component participates effectively in the plant dynamics only over a limited frequency range. Therefore, in analyzing the plant operation, one may concentrate on definite frequency ranges and ignore effects which happen to be ineffective in that range."

MR. GRACE:

I have one more question. In your treatment of the distributed parameter effects, have you tried sectionalization, say the length of the coolant channel or the length of the boiler tube?

MR. GYFTOPOULOS:

No, because we have been able to integrate the equations for the plane geometry without having to go into this sectionalization. Actually, in a sense, it would amount to the same thing because, if you sectionalize, you might have to take three or four sections, and to the first approximation, each section introduces, say, one time lag. Now, when you have a distributed parameter system, you end up with a transcendental transfer function, and in order to have a good lumped approximation, you need a couple or three time constants. So in effect you get dispersion in characteristic frequencies, in a different way, but you end up with similar expressions.

POWER-VOID TRANSFER FUNCTION MEASUREMENTS
IN A SIMULATED SPERT IA MODERATOR COOLANT CHANNEL*

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Presented by
R. W. Wright

We have been investigating power-void feedback in small oscillation theory, as a possible cause of boiling water reactor instability, by experimental measurements of power-void transfer functions. Purely hydrodynamic oscillations are another possible mechanism for the boiling instabilities. This work is part of the program of the Reactor Safety Branch and is directed toward understanding the kinetic behavior of the SPERT reactors. These measurements have been made with a boiling loop which contains a simulated SPERT IA moderator-coolant channel that is electrically heated and that matches the heat capacity of a channel in the reactor. SPERT IA is an atmospheric pressure, natural circulation, light water-moderated and cooled, tank-type reactor with fully enriched U^{235} -aluminum MTR-type fuel elements. With a 2-ft static head and with the tank water at the free surface saturation temperature, divergent power oscillations of about one-cycle per second frequency were observed from a mean power level of about 450 kw, which occurred when the reactivity compensated by steam void was about $1\frac{1}{2}\%$. Under these conditions the power input to one of the central water channels in the core was about 500 watts, giving a power density of 13 kw per liter. This is the set of conditions for which we have investigated the static and dynamic void behavior in our simulated SPERT IA water channel. Void fraction measurement was by soft X-ray attenuation through a 2-cm² cross section of the channel elevation.

The simple boiling water reactor power-void feedback loop which is considered here is shown in Figure 1. The power response of the reactor to reactivity modulation, or zero-power transfer function, is readily calculable from the reactor kinetic equations. The power-void behavior is the part of the feedback loop, which is unknown and which prevents prediction of the properties of the closed loop system. We have been performing

*This work was done under Contract AT(04-3)-165 with the Reactor Safety Branch of the Division of Reactor Development of the U. S. Atomic Energy Commission. Much of it is reported in more detailed form in a Ramo-Wooldridge report, Kinetic Studies of Heterogeneous Water Reactors, RWD-RL-167, available from the Office of Technical Services, Department of Commerce, Washington 25, D.C.

Fig. 1. Boiling Water Reactor Power-Void Feedback Loop Considered Here.

out-of-pile experiments to measure the power-void transfer function. Internal structure of the power-void system, which may be of importance with the natural circulation flow in SPERT IA, is the void-flow feedback which is shown in dotted lines.

A cross section of the simulated SPERT IA water channel is shown in Fig. 2. The channel has electrically heated aluminum simulated "fuel plates" and constantan alloy edge pieces. These match the heat capacity of the SPERT IA aluminum plate supports and they have too high an electrical resistance to dissipate appreciable electrical power. The channel is assembled by soldering the edges. The water column is 2 ft long and about $\frac{1}{8}$ by 1 in. in cross section. A 17.5-keV Mo K α X-ray beam traverses the thin dimension of the channel for void fraction measurement.

Fig. 2. Cross Section of Simulated SPERT IA Water Channel.

A schematic drawing of the boiling loop which contains the simulated SPERT IA water channel is shown in Fig. 3. The water return path in the loop has 12 times the

Fig. 3. Schematic Drawing of Boiling Loop Containing Simulated SPERT IA Water Channel.

cross-sectional area of the channel. Thus, compared with the channel, the return has low flow resistance, as in a tank reactor. At 500-watt channel power, under boiling conditions, the natural circulation flow rate is about 28 cc/sec. This gives a velocity of 45 cm/sec inside the channel, and is near the peak of the flow vs power curve. The water transit time around the loop is 75 sec, of which about 1 sec is spent inside the channel. The flow rate is measured by a very lightweight, 12-bladed turbine flowmeter with detection by light beam interruption. Measurements at constant channel power showed that the void fraction has very large timewise fluctuations or boiling noise. For example, at 30% mean void fraction, the actual measured void fraction over a 2-cm² elevation of the channel varies from zero to about 70%, with response times as short as 10 milliseconds. The boiling noise decreases at the higher void fractions occurring farther up the channel. The natural circulation flow rate is much less noisy than the void fraction.

With a 2-ft static head and with the channel inlet water at the free surface saturation temperature, purely hydrodynamic oscillations in flow and hence void fraction occurred at constant power levels above about 700 watts, for which the mean flow rate was decreasing with increasing channel power. Another channel, which was fabricated from one sheet of aluminum with a single welded seam and which contained fewer nucleating sites, exhibited hydrodynamic instabilities at various constant power levels well below 700 watts.

Temperature and void fraction profiles along the simulated SPERT IA channel are shown in Fig. 4. These simultaneous measurements were made at 500-watt power with a 2-ft static water head, with the inlet water at the free surface saturation temperature, and with a natural circulation flow rate of 29 cc/sec, resulting in a 47-cm/sec flow velocity in the channel. Under these conditions the head produces 3.2°C inlet water subcooling, and boiling starts about half way up the channel. The void fraction at the channel exit is about 75%; approximately $\frac{3}{4}$ of the way up the channel, where most of the power-void transfer function measurements were made, it is about 50%. The channel wall temperature profile was measured with copper-constantan thermocouples, and corrections were made for appreciable differences in the emf of the different thermocouples at 100°C. The heat transfer coefficient profile was calculated from the measured wall temperatures and from the calculated water temperature in the nonboiling length of the channel and from the calculated local saturation temperature in the boiling length. The nonboiling heat transfer coefficient increased with height in the channel from 0.5 to 0.8 watt/(cm²)(°C). In the boiling region the heat transfer coefficient continued to increase with height to a peak of 1.5 watts/(cm²)(°C) at a void fraction of about 30%, and then fell to about 0.8 watt/(cm²)(°C) near the exit. Changes in power level of 20% produced little change in the shape of the void fraction distribution when the inlet water temperature was adjusted to maintain constant the vertical position of initial boiling.

Power-void transfer function measurements were made by modulating the channel power about the mean power level at a given frequency with small amplitude, usually 10%, and measuring the in-phase and the out-of-phase components of the coherent response of the void fraction at that frequency. The void fraction response amplitude and phase lag were calculated from these two components. Because the desired coherent void fraction response was sometimes as much as two orders of magnitude below the boiling noise level, correlation techniques incorporating a phase-sensitive detector-integrator were used. Initially, measurements were made to determine the range of linear response to modulation amplitude, if any, of the power-void system.

Fig. 4. Temperature and Void Fraction Profiles Along the Simulated SPERT IA Channel.

Measurements of void fraction response amplitude and phase lag vs. power modulation amplitude are shown in Fig. 5. These measurements, made at 1 cycle per second power modulation frequency and at a mean void fraction of about 50%, showed that the void fraction response amplitude was proportional to the power modulation amplitude and that the void phase lag was independent of the modulation amplitude to over 10% power modulation amplitude when the void amplitude was 4%. Earlier measurements at a frequency of $\frac{1}{4}$ cycle per second had given linear response up to 20% power modulation amplitude, for which the void fraction amplitude was 17%. Power-void transfer function measurements were therefore made with 10% power modulation amplitude.

Fig. 5

Void Fraction Response Amplitude and Phase Lag vs. Power Modulation Amplitude at 1 Cycle Per Second Power Modulation Frequency.

The measured void fraction response amplitude vs. power modulation frequency is shown in Fig. 6. These measurements are at 500-watt power and at a position $\frac{3}{4}$ of the way up the channel. The curve is drawn through the circles which show one day's measurements. The x's show data taken a week earlier, and the point scatter gives an indication of measurement reproducibility and day-to-day hydrodynamic system drifts. The measurement errors, which were estimated from the boiling noise scatter

Fig. 6. Void Fraction Response Amplitude vs. Power Modulation Frequency.

in strip chart records, are always less than the size of the plotted points unless flags giving estimated standard errors are shown. The void fraction amplitude approached a constant as the frequency approached zero. The zero frequency response was measured at 0.0083 cycle per second (for a 2-min period). The probable existence of the node at about 2 cycles per second is inferred from its presence in most, but not all, of our measurements of power-void transfer functions. It occurs at such a low void fraction amplitude, however, that it can easily be washed out by the boiling noise and hydrodynamic system drifts, and it can be missed by measurements at the wrong frequency.

The void fraction response phase lag vs. power modulation frequency is shown in Fig. 7. The void fraction phase lag has reached 180° at a frequency of 1 cycle per second. The phase lag does not approach an asymptote at high frequency, but increases approximately linearly with frequency above about 2 cycles per second. Such a phase lag results from a time lag in a system, and the power-void phase lag graphs all show in dashed lines the

Fig. 7. Void Fraction Response Phase Lag vs. Power Modulation Frequency.

phase lag resulting from the time lag which best fits the high-frequency data. Each individual phase-lag measurement is uncertain by an integral multiple of 360° . To check that the void fraction phase lag does not always remain less than 360° , more detailed measurements were made in 7% frequency increments in the neighborhood of 360° -degree phase lag, with results shown in Fig. 8. These data show a smooth progression in phase lag through 360° with increasing frequency. An abrupt return to nearly zero phase lag would have required a reversal in sign of the in-phase component of void fraction response, and this was not observed. This conclusion that the void fraction phase lag does increase beyond 360° with increasing frequency has been strengthened by many power-void transfer function measurements.

Figure 9 shows the void fraction response amplitude vs. power modulation frequency at 300-watt mean power level. This is our best single set of data to show the reality of the node in the void fraction response amplitude curve at about 2 cycles per second. The inlet water temperature was increased to maintain the point of initial boiling at the midpoint of the channel for these measurements.

Fig. 8. Void Fraction Response Phase Lag vs. Power Modulation Frequency.

Fig. 9. Void Fraction Response Amplitude vs. Power Modulation Frequency at 300-watt Mean Power Level.

In order to obtain the power-void transfer function for the entire channel, vectorial spatial averaging of the void fraction response along the channel length is necessary at each frequency. Data for performing this spatial averaging are given in Fig. 10, which shows measurements of the void fraction response amplitude and phase lag, at 1 cycle per second power modulation frequency, vs. channel position, as well as showing for reference the vertical mean void fraction distribution. At one cycle per second, the frequency of interest for the SPERT IA boiling instabilities, the phase shift is nearly constant at about 180° throughout the boiling length of the channel, and this facilitates the spatial averaging. Most of the total channel void fraction response comes from $\frac{1}{6}$ of the channel length, centered at the position of about 40% mean void fraction.

Fig. 10. Mean Void Fraction and Void Fraction Response Phase Lag and Amplitude at 1 Cycle Per Second Power Modulation Frequency vs. Channel Position.

In order to make small oscillation theory stability predictions for the SPERT IA reactor, the measured power-void transfer function must be combined in a feedback loop with the reactor zero-power transfer function relating power to reactivity.

Near 1 cycle per second, the zero-power transfer function for U^{235} and the measured SPERT IA neutron lifetime has nearly constant gain and phase lag, the latter being about 6° . The measured power-void transfer function has a phase lag of 174° at a frequency of 0.95 cycle per second, and hence the phase shift of the feedback loop is 180° at this frequency. The feedback loop gain was calculated as the product of the gain of the zero-power transfer function and the spatially averaged product of the SPERT IA void coefficient of reactivity and the void response amplitude at 0.95 cycle per second. In this calculation it was estimated that the void response in the twelve central fuel assemblies would have a vertical distribution essentially as shown in Fig. 10, and the void response in the peripheral 16 fuel assemblies was negligible because of the relatively low power and low void coefficient in these outer regions. A slight adjustment was made to convert void fraction amplitude measurements at 1 cycle per second to 0.95 cycle per second. The details of this computation are given in Ramo-Wooldridge report RWD-RL-167. These calculations result in a feedback loop gain of 1.7 at 0.95 cycle per second, for which the feedback loop phase-shift is 180° . Thus, according to the Nyquist criterion, the system will be unstable, and power oscillations at a frequency of 0.95 cycle per second are predicted in the reactor at a power level of 450 kw. This is in good agreement with published SPERT IA results that divergent power oscillations of about 1-cycle per second frequency were observed at about 400-kw power under boiling conditions when the static water head was 2 ft. As the gain of the zero-power transfer function is directly proportional to reactor power, a threshold power level for reactor instability of 260 kw would be predicted if the power-void transfer function were independent of power level. However, the reactor power-void transfer function amplitude undoubtedly increases with increasing power, so that the actual instability threshold in SPERT IA should be between 260 and 450 kw.

This transfer function analysis applies to small oscillations only, and does not predict the amplitudes to which large oscillations will grow as the response becomes nonlinear. From a reactor safety point of view, however, our present purpose is the prediction of small oscillations which have a divergent nature, rather than the prediction of the amplitudes to which these oscillations will grow.

An independent check of the quality of the simulation of the void behavior in SPERT IA was made by calculating the reactivity compensated by steam void in the reactor at 450 kw. The spatial distribution of void importance in the reactor was applied to the measured time-average steady-state void fraction distribution in the simulated SPERT IA water channel. The

predicted compensated reactivity of 1.9% Δk at 450-kw power agrees well with reported compensated reactivities of 1.5 and 1.8% Δk at 350 and 500-kw power, and provides additional confidence in the quality of simulation of the void behavior in the reactor.

The success of these out-of-pile measurements and small oscillation feedback theory in predicting the characteristics of the observed SPERT IA boiling oscillations, however, must be qualified by the observation that spontaneous hydrodynamic oscillations sometimes occurred at a steady power level of 500 watts in a water channel which was fabricated by welding. This channel has a relative paucity of nucleating sites in comparison with the soldered channel used for these measurements. These purely hydrodynamic oscillations, however, occurred at a frequency of 0.4 cycle per second rather than at the 1-cycle per second frequency of the observed SPERT IA oscillations. Hydrodynamic oscillations of about 1-cycle per second frequency were observed in the channels assembled both by welding and by soldering, but these occurred at power levels above 700 watts, for which the natural circulation flow rate was decreasing with increasing power. This corresponds to a threshold power level in the SPERT IA reactor of about 650 kw and is not in agreement with the observed thresholds for boiling oscillations.

On the other hand, the static and dynamic void fraction measurements in the simulated SPERT IA moderator-coolant channel and the results of small oscillation theory applied to a simple feedback loop including the neutron kinetics and the power-void behavior are in agreement with observed SPERT IA instabilities, under boiling conditions, in oscillation frequency, threshold power level, and void-compensated reactivity. This would indicate that in the observed SPERT IA power oscillations the moderator-coolant channels in the reactor are not undergoing purely hydrodynamic oscillations, but that these oscillations occur when the feedback loop incorporating power-void feedback with the reactor kinetic equations fulfills the Nyquist criterion for instability in small oscillation theory.

In addition to investigating power-void feedback in small oscillation theory as a possible mechanism for the SPERT IA boiling instabilities, further transfer function measurements have been made to investigate the general properties of dynamic void behavior.

The void fraction response amplitude vs. channel position at several power modulation frequencies is shown in Fig. 11. The slope of the constant-power time-average void fraction distribution, $d\bar{\alpha}(y)/dy$, is also shown for reference. The shape of the vertical distribution curves of void fraction response amplitude is quite similar from 0.1 to 4 cycles per second. The experimental data are well fitted, at 0.1 and 1 cycle per second, by the shape of the $d\bar{\alpha}(y)/dy$ reference curve. Slightly different curves are drawn through the 2 and 4-cycle per second data. These differences may

be caused by measurement errors or hydrodynamic system drifts. The data suggest that the void fraction response may result simply from shifting the constant-power time-averaged void fraction curve, $\bar{\alpha}(y)$, up and down the channel as the point of initial boiling oscillates in response to the modulated power input.

Fig. 11.

Void Fraction Response Amplitude vs. Channel Position at Different Power Modulation Frequencies.

Fig. 12. Void Fraction Response Phase Lag vs. Channel Position at Different Power Modulation Frequencies.

The void fraction response phase lag vs. channel position at several power modulation frequencies is shown in Fig. 12. These curves show that the void fraction phase lag is nearly independent of position along the channel at 1 cycle per second and below. However, the upper part of the boiling section starts the rapid increase in phase lag with increasing frequency at lower frequency than does the lower part. At 4 cycles per second, a void fraction response phase difference exceeding 180° exists across a 6-in. length of the boiling channel. The time lag which would account for the high-frequency phase behavior is about the magnitude of the water transit time from the point of initial boiling to the point of observation.

During the initial power-void transfer function measurements, we observed that the natural circulation water flow, as well as the void fraction, was responding coherently to the power modulation. This coherent response was in addition to the usual flow fluctuations or noise. Measurements of power-flow transfer functions were therefore undertaken simultaneously with the power-void measurements. In a natural circulation system this power-flow response is of interest because of its direct effect upon the power-void transfer function. Measurements gave a range of linearity for the flow response to power modulation which was identical to the linear range of the void response to power modulation.

The flow response amplitude vs. power modulation frequency is shown in Fig. 13. The flow response amplitude rises approximately linearly with increasing frequency to a peak at a little below 1 cycle per second, at which the response is about 8% of the mean flow for 10% power modulation. The flow response, which has been corrected for the flowmeter dynamic response, falls off rapidly at higher frequencies.

Fig. 13. Flow Response Amplitude vs. Power Modulation Frequency.

The flow response phase lag vs. power modulation frequency is shown in Fig. 14. The void fraction phase lag is also shown for reference. In the region of large flow response the flow phase lags the void by about 90° , although at very low frequencies the lag is somewhat less.

Fig. 14. Flow Response Phase Lag vs. Power Modulation Frequency.

The observed flow response to power modulation is consistent with a simple model. The void response to power modulation displaces water. Under the present operating conditions, by momentum conservation, 80% of the water should be displaced out the bottom of the channel and 20% out the top at high velocity. The resulting inlet flow response to void variation is proportional to minus $d\bar{v}/dt$ integrated over the entire channel. At low frequencies, where the void fraction amplitude is independent of frequency, the flow amplitude should increase linearly with frequency from this effect. In addition, the flow phase should lag the void phase by 90° as is observed over most of the frequency range.

In Fig. 15 the predicted flow response from water displacement by void is compared with the measured flow response amplitude. The dashed line shows the predicted flow response for 10% power modulation calculated from the measured void fraction amplitude distribution and the void fraction

Fig. 15. Predicted Flow Response from Water Displacement by Void Compared with Measured Flow Response.

amplitude frequency response measured $\frac{3}{4}$ of the way up the channel. This calculated flow response is proportional to the void fraction response amplitude multiplied by the frequency. The vertical distribution of void fraction response amplitude at one cycle per second was used for spatial averaging, but more data would be needed on the spatial distribution of void phase lag at the higher frequencies in order to make a proper vector spatial average. There are no adjustable parameters in this calculation. The fit with the measured flow response amplitude is excellent below 1 cycle per second for which the void phase lag is nearly constant throughout the boiling length of the channel. This indicates that the flow response is indeed due simply to water displacement by void. Void fraction response phase lag measurements at 4 cycles per second showed a phase difference exceeding 180° in the boiling section of the channel. Thus in this frequency range the void responses at the top and at the bottom of the boiling length of the channel can accommodate each other giving reduced net water displacement and reduced flow response.

A pressure-void and its accompanying pressure-flow transfer function were measured by modulating sinusoidally a slight pressure imposed upon the free water surface in the loop. These transfer functions are of interest in investigating system response to step or periodic changes in pressure and to pressure fluctuations which occur in boiling. They are also of interest because of possible relationship to hydrodynamic instability and for information on basic mechanisms in two-phase flow. Because of the very large effect of pressure modulation upon the void fraction, it was necessary to limit the pressure modulation amplitude to 0.05 psi in order to stay in the linear region of void fraction and flow response. This corresponds to an oscillation amplitude of 0.1°C in the free surface saturation temperature of the water. This modulation produced a void fraction response amplitude of 10% at 1 cycle per second near the frequency of peak void fraction and flow response to pressure modulation. Void fraction was measured $\frac{3}{4}$ of the way up the channel and channel power was 500 watts.

The void fraction and flow response amplitudes vs. pressure modulation frequency are shown in Fig. 16. The amplitudes of both the void

Fig. 16. Void Fraction and Flow Response Amplitudes vs. Pressure Modulation Frequency.

fraction and the flow transfer functions show a large peak at about 0.8 cycle per second with peak amplitudes of about 200 per cent per psi modulation amplitude. (It is coincidental that both responses are numerically equal.) This peak occurs at the same modulation frequency and is similar to the peak in the power-flow transfer function at constant pressure. Below the peak, within the accuracy of the measurements, the flow response appears to be increasing with frequency at a higher rate than the void response by one power of the frequency, as predicted by the water displacement model of flow response. Void and flow response amplitudes decrease identically above the peak.

The void fraction and flow response phases vs. pressure modulation frequency are shown in Fig. 17. In the pressure modulation results, the phase of the void and flow is defined such that when the maximum of the response sinusoid occurs simultaneously with a minimum in the pressure modulation, zero phase shift exists. Below the frequency of peak response, the void fraction phase leads the pressure minimum, but beyond the peak the void fraction phase lag increases steadily. Over a considerable frequency range about the response peak, the flow phase lags the void phase by 90° , but the lag is less at the lowest and highest frequencies. The crossing of the void fraction phase curve by the flow curve at high frequency may only reflect a large difference in void phase across the boiling length of the channel at high frequency.

Fig. 17. Void Fraction and Flow Response Phases vs. Pressure Modulation Frequency.

As in the power modulation case, the amplitudes of the void and flow response to pressure modulation appear strongly correlated. They are in experimental agreement with the water displacement model of flow response up to the frequency of peak response. In contrast with the power modulation case, where the maximum void response amplitude occurs in the zero frequency limit, the void as well as the flow response to pressure modulations appears to be approaching zero in the zero frequency limit. This would be expected for small changes in the steady-state pressure. In view of the similarities in the void fraction and flow response amplitude curves, it will be interesting to see to what extent flow-void feedback affects the response to pressure modulation, and also whether this feedback is related to any of the modes of hydrodynamic instability in two-phase systems.

We have been interested in finding what the physical effects are that determine the characteristics of the measured power-void transfer functions. A preliminary investigation has yielded a simple physical model which is in fair agreement with the general features of the measured transfer functions.

This model consists primarily of shifting the measured steady-state time-average void fraction curve, $\bar{\alpha}(y)$, up and down the channel as the point of initial boiling oscillates in response to the imposed power modulation. It was observed that 20% changes in the steady-state power level from 500 watts produced little change in the shape of the void fraction curve, $\bar{\alpha}(y)$, so that in the zero frequency limit the void fraction response amplitude is certainly given by this vertical oscillation of the void fraction curve. The experimental data of Fig. 11 showed that up to 4 cycles per second, which was the highest frequency measured, the shape of the vertical distribution of void fraction response amplitude is almost independent of frequency and is given by $d\bar{\alpha}(y)/dy$, the slope of the void fraction curve. This result suggested the present model. No further details of the boiling process itself are included in the model.

The amplitude and phase lag of the vertical oscillation, in response to power modulation, of the point of initial boiling in the channel are taken from the power water-temperature transfer function at the point of initial boiling which is calculated by the method of Storrer. This calculation sums vectorially the ac component of the energy input into an element of water as it traverses the nonboiling length of the channel. Consideration is taken of the heat capacity of the channel, including both the simulated fuel plates and supports, the heat capacity of the water, the measured heat transfer coefficient, and the effect of the flow response upon the energy input into the element of water which was small.

Additional void fraction response phase lag is added to the phase lag of the power water-temperature transfer function to account for the water transport time from the point of initial boiling to the point of observation. The measured high-frequency void fraction phase lags are consistent with the water transit times rather than the steam transit times.

The calculated void fraction response amplitude, $\frac{3}{4}$ of the way up the channel, vs. power modulation frequency is shown in Fig. 18. The experimental data previously shown in Fig. 6 are plotted on the graph for comparison. The dotted curve was calculated using a constant heat transfer coefficient in the nonboiling length of the channel. In the solid curve rough account is taken of the measured longitudinal variation in the heat transfer coefficient by using values differing by 25% in the two halves of the nonboiling length. There are no adjustable parameters in the calculation. The agreement with experiment is quite good at the lowest frequencies, but the calculated void response falls off too rapidly with increasing frequency. The high-frequency node observed in some of the experimental data appears in the calculated void response, but at slightly too low a frequency. A secondary node in the calculated response was washed out when account was taken of the longitudinal variation of the nonboiling heat transfer coefficient.

Fig. 18. Calculated Void Fraction Response Amplitude vs. Power Modulation Frequency.

The discrepancy in the calculated high-frequency response is not considered to be a serious reflection upon the model because the high-frequency response is strongly dependent upon the detailed nature of the effective heat capacities and heat transfer coefficients, which were not incorporated precisely into this initial calculation. For example, this calculation was made including the heat capacity of the constantan edge pieces of the channel which simulate thermally the aluminum fuel plate holders in the reactor. These edge pieces contain about $\frac{1}{3}$ of the heat

capacity of the channel. Because the thermal diffusion time from these edge pieces into the center of the simulated fuel plates is about 1 second, which corresponds to a break frequency of 0.16 cycle per second, the edge piece heat capacity should have been excluded from the calculations. Such exclusion increases the calculated high-frequency void response by about 50%.

The calculated void fraction response phase lag vs. power modulation frequency is shown in Fig. 19. The calculated curve includes the break to rapidly increasing phase lag at about 2 cycles per second, which appears in the experimental data. However, in the neighborhood of one cycle per second, the model predicts about 90 degrees too much phase lag. This error appears in the region of high flow response to power modulation. The present preliminary model includes the effect of flow response in the sub-cooled length of the channel, but not as yet in the boiling length. In addition to the direct effect upon the void fraction response of flow variation in the boiling region, the inertial pressures necessary to produce this flow response have been neglected, although the pressure-void transfer function measurements show that these pressure effects are appreciable in this frequency range.

Fig. 19. Calculated Void Fraction Response Phase Lag vs. Power Modulation Frequency.

While the present model predicts successfully the general features of the measured power-void transfer function and indicates the physical effects that very likely determine it, the model has not yet been developed sufficiently for prediction of reactor instabilities. In addition, it includes the empirical information on the nature of the boiling process which is incorporated in the measured time-average steady-state void fraction distribution, $\bar{\alpha}(y)$.

In summary, power-void transfer functions have been measured in a simulated SPERT IA moderator-coolant channel. According to small oscillation theory, these measurements, when combined in a feedback loop with the reactor zero-power transfer function, predict that oscillations will occur in the reactor under boiling conditions at the frequency and at about the threshold power level actually observed. The natural circulation flow response to power modulation results simply from the water displaced by the void response to power modulation, and this response falls off rapidly at the higher frequencies where the void response phase varies across the boiling length of the channel by roughly 180° or more. Pressure-void and pressure-flow transfer functions have been measured. These are highly correlated and have peaks at about the frequency of maximum flow response to power modulation. A simple model predicts the general characteristics of the measured power-void transfer function.

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Discussion of Paper
Presented by Mr. Wright

MR. COHN:

In your diagram of your experimental setup, I noted a sort of ladder network of gate valves in the upper part of your apparatus. I wonder if you could say something about the function of that.

MR. WRIGHT:

They were put in when we first built the loop. Actually, by manipulating them, we can change the effective riser over the channel, and change some of the loop parameters. We have some initial data made at higher flow rates, natural-circulation flow rates, produced this way. We are not yet ready to go into it.

We studied some of the effects of the return path on hydrodynamic instabilities, too.

MR. WASSERMAN:

You mentioned that the void fraction frequency response to power variations was linear up to 20 per cent amplitude in power. Was this half peak-to-peak, or peak-to-peak?

MR. WRIGHT:

This is amplitude.

MR. WASSERMAN:

This is amplitude. So it was really 50 per cent peak-to-peak?

MR. WRIGHT:

Right. Huge linearity which amazed us, and made measurements much easier.

MR. GRACE:

In your studies of hydrodynamic instabilities, and void reactivity instabilities, did you find any situations where the instability of the void reactivity feedback loop could be affected by hydrodynamic variations, that is, where the void reactivity feedback loop is dominating the situation, but nevertheless because of proximity to the natural frequency, the hydrodynamic system, its response, had also to be taken into account?

MR. WRIGHT:

Maybe I am not understanding properly. We didn't have any feedback in our measurements. We measured the power void transfer function.

MR. GRACE:

Right.

MR. WRIGHT:

We just measured it, and we talked about what the reactor would do with this void response in it. Now, a possible corollary of your question may be that we tried to trigger hydrodynamic instability by backing the power off to where the system was just barely stable, and then modulating the power at that frequency. We couldn't trigger the instability at all.

MR. GRACE:

I see.

MR. WRIGHT:

It had no success.

MR. WASSERMAN:

Could you say something very briefly about the instrumentation system you used to discriminate against boiling noise in determining your void fraction? Just give a brief outline of what you did.

MR. WRIGHT:

I will do it quickly in words which will be quicker than a drawing. We took the electrical analog signal of the void fraction from our X-ray system, and we multiplied it, well, we will say we multiplied it by cosine ωt , although actually we used a square wave. We picked up the in-phase component at the modulation frequency in the signal, and then we integrated it. This is more or less a standard sort of technique. We had two channels which did this simultaneously. One which did the multiplication in phase with the modulation, and one which did it out of phase.

MR. KASTEN:

Did you do these measurements at different positions in the channel?

MR. WRIGHT:

Yes.

MR. KASTEN:

Did you get the same response?

MR. WRIGHT:

I showed some graphs showing variations of response with channel position. The steady-state void fraction looks like this, where this is one. Boiling starts here. This is α versus Y , where α is the mean void fraction and Y is channel position. We found that the void fraction response amplitude could be well represented by the slope of this steady-state void fraction curve.

MR. KASTEN:

Did you do measurements at different positions?

MR. WRIGHT:

Yes. Dynamic measurements were made at three positions along the channel. We didn't do continuous measurements.

MR. KASTEN:

You found the same type of response?

MR. WRIGHT:

We found the same type of response in the frequency range of one-tenth to four cycles per second, which surprised us. This is where this model was developed, shifting the void fraction curve back and forth.

MR. KASTEN:

It does seem somewhat surprising inasmuch as we expect the boiling phenomena to be nonlinear, since the bubbles don't collapse, they keep building up.

MR. WRIGHT:

We were very much surprised. The details of boiling are not in the model except as we incorporate them in the steady-state void fraction curve. Another thing that is of interest is that this curve gives a very

erroneous impression of what is actually happening in the channel. The actual void fraction distribution instantaneously in time has very large fluctuations, and this time-average curve is not what is happening. Still, of course, in your transfer function, you are looking for the frequency component at three low frequencies. This is an integration sort of thing. I guess you are doing an averaging process. What is going on doesn't look like the average results at all.

MR. KASTEN:

Thank you.

MR. COHN:

Let me ask, as a preliminary, did you vary your input power sinusoidally or as a square wave?

MR. WRIGHT:

We varied the input voltage sinusoidally, so we had a very slight amount of second harmonic in the power, but this was unimportant. The second harmonic in the void response was rejected by the phase-sensitive detector integrator.

MR. COHN:

I was just wondering whether the use of the square wave as your demodulation frequency would tend to cause an impure measurement by bringing in higher harmonics.

MR. WRIGHT:

The answer is yes, from the odd harmonics. We got into trouble on this in the pressure measurements where we had a synchronous motor driving locked-in gearing, and at certain odd frequencies, all of a sudden the thing would take off. This was because the 60-cycle pickup in the system was always bigger than the coherent response we were looking for. With the square wave demodulation, you have sensitivity to the odd harmonics of the modulating frequency, and we were doing things like picking up the 15th harmonic. When 60 cycles was a 15th harmonic of the modulating frequency, we were picking it up. So currently we are throwing away the switches and putting in sine-cosine pots.

MR. OWEN:

During your work with this natural-circulation loop, were you able to reach a predicted hydraulic instability limit of the loop itself at high enough powers?

MR. WRIGHT:

We can't predict these things.

MR. OWEN:

In other words, consistent rather than predictable.

MR. WRIGHT:

With the soldered channel, the dirty channel, in which we made all of the transfer function measurements, when you got up on the flow versus power curve, up beyond the peak, and started down, the loop started to get unstable. This was predictable and I would say relatively reproducible. But the clean channel was unpredictable and variable in time. The instability regions would kind of wander around. This is something we have to look into a lot more.

MR. OWEN:

You didn't attempt to relate these to any exit void percentagewise?

MR. WRIGHT:

During the hydrodynamic instabilities, the void fraction at a point not far above bulk saturation where the steady-state void fraction is exceedingly noisy, has an interesting appearance. It has periodic sharp spikes of nearly 100% void fraction, oscillating from full to empty. As you get further up the channel, it doesn't get down to empty and it has more random noise. Maybe I am not answering your question.

MR. OWEN:

My intent was to find out if you did have some sort of a criterion for a stability limit for this natural-circulation loop, or whether it was simply an erratic phenomena that you would run into at some unpredictable moment.

MR. WRIGHT:

The high-power instability occurred rather predictably. In the clean channel, the powers for instability below the peak flow rate were rather variable. In the clean channel, I am sure you get various modes of hydrodynamic instability. The bubbles would look different coming out, and the oscillations had different frequencies. As you go up the power scale you go from stable to unstable a couple of times.

MR. OWEN:

Thank you.

MR. BAKER:

My question may seem simple. It seems to me that you are making two points. First of all, you are doing experiments in an out-of-pile loop, and then showing that the reactor actually behaves the way you would think from these experiments. Then you are also trying to explain the results of the experiments by means of a calculation. Well, I understand that you are satisfied on both accounts, but how near do you get to the behavior of the reactor if you use the calculations alone to predict the effect of the voids on the transfer function?

MR. WRIGHT:

They are not good enough. For example, we have 90° too much phase lag at the moment in our model at the frequency of interest for stability, one cycle per second. This model is about two weeks old. We want to improve it.

What amazed us is that the gross features of the power void transfer function could be predicted with such a simple model which did not go into the details of boiling, which, of course, we don't understand.

MR. BAKER:

Just to put myself into perspective, could you write on the board for me the power at which you observe instability, and the power at which you predict it by experiment, and the corresponding frequencies, and can you also say what pure theory gives you for these values.

MR. WRIGHT:

This table gives the oscillation frequency and threshold power level as observed in SPERT IA, under boiling conditions with a 2-ft head, and as predicted by our out-of-pile measurements and by our theoretical model.

	Frequency (cps)	Power (kw)
SPERT IA	1.0-1.2	~400
Experiment	0.95	260-450
Model	0.53	300-450

The SPERT results showed some variation between runs. The frequency shown for SPERT is for very large amplitude oscillations. The frequency of small amplitude oscillations appears from the log power traces to have been about 10% lower.

The experimental power-void transfer function measurements were made at a power corresponding to 450 kw in the reactor, or somewhat above the instability threshold of the reactor. These measurements (giving a feedback loop gain of 1.7) indicate that the reactor would indeed be unstable at this power from power-void feedback, and that it would undergo divergent oscillations at a frequency of 0.95 cycle per second. We have not yet measured the power dependence of the power-void transfer function, so we cannot make an accurate prediction of the threshold power. However, the gain of the zero-power transfer function itself is directly proportional to reactor power, so this power dependence alone gives an instability threshold for the reactor between 260 and 450 kw. The gain of the reactor power-void transfer function undoubtedly increases with power also, so the threshold power for instability from out-of-pile measurement is probably about 400 kw.

We do not consider that our theoretical model of the power-void transfer function is yet good enough for quantitative prediction of reactor instability. What the model does do is reproduce the general features of the measured power-void transfer functions. The model, in its present incomplete form, produces 90° too much phase shift near one cycle per second. It predicts that SPERT IA would be unstable at 450-kw power, but that the oscillation frequency would be only 0.53 cycle per second. The feedback loop gain is 1.5 at this frequency, so an instability threshold between 300 and 450 kw is predicted. As mentioned in the paper, we know that inclusion of the effect of the flow oscillations in the boiling region of the channel will decrease the calculated void fraction phase lag considerably and hence increase the frequency of the predicted reactor oscillations.

COMBINED REACTOR AND POWER SYSTEM ANALYSIS FOR A BOILING WATER REACTOR

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Introduction

To begin with, it is probably worth reviewing briefly a few of the obvious considerations and objectives involved in a combined reactor and power system analysis.

First of all, the objective in making a combined reactor and power plant study is to study the complete, or nearly complete, plant performance rather than to analyze detailed behavior of one component of the plant. The reactor-power system analysis methods referred to here apply to a Boiling Water Reactor (BWR) power system. All significant components of the reactor power plant would be simulated.

An important consideration, however, is the degree of accuracy to be used in the simulation of the various system components. Assuming a limited computer facility is available (of the order of 100 operational amplifiers), it is necessary to exercise some judgment as to how accurately and extensively each component should be simulated. There is, roughly speaking, no point in simulating one component to the n th degree where another component is represented by a very rough approximation as far as their respective effects on the system performance is concerned. What is sought, however, is a well-balanced system representation making use of individual component models that are consistent with the desired overall system accuracy and fall within computer limitations. The use to which combined reactor-power system analysis work may be put is, of course, the study of overall reactor-power plant transients which are used to predict system performance for either operating transient predictions and operating guidance, or to supply data for plant safeguard studies, or to supply data for selecting component equipment during the design phase.

The balance of this discussion will attempt to review some of the methods used to study or analyze the more basic areas peculiar to a BWR plant. The analysis method or models of other non-nuclear components, such as heat exchangers, etc., will not be considered here, even though they are important in the overall system analysis. However, those methods of analysis of the reactor-steam void loop used in system studies of a BWR system will be reviewed.

It is not intended to give specific data for the operation of any one plant. The analytical technique used will, as you will see, be directed towards the utilization of an analog computer, although digital computer programs are, of course, already in use for handling such reactor system transient studies.

General Description of Boiling Water Reactor System

As an example of a boiling water reactor system, I have selected a dual-cycle system (see Fig. 1), which is a representation of the Dresden dual-cycle system. This consists of principal parts as follows; the reactor vessel containing the reactor or core itself, an amount of saturated steam-water mixture, a quantity of subcooled water and a volume of steam. The risers contain a steam-water mixture whose quality varies from one end to the other because of the density head and whose flow is consistent with steam drum and reactor pressures. The steam drum consists of a saturated or subcooled liquid and a large steam volume with little or no steam-water mixing. Downcomers carry saturated or subcooled water. This water is pumped through the steam generators where the addition of subcooling (or decrease in liquid enthalpy) takes place, depending on secondary steam demands, before the water returns to the reactor.

Fig. 1. Dual-Cycle System

In analyzing such a system in detail, in addition to the reactor representation or performance, some special points need to be considered. For example, the delay phenomenon associated with the long steam line must be considered if the pressure regulator loop includes this line length (i.e., pressure controlled by throttling one end of the line with the regulator receiving its signal from a pressure tap at the other end of the long steam line). The effect of transport delays both in the risers and in the recirculation loop (steam drum to the vessel) is important. In a natural-circulation reactor the delay from core outlet to core inlet is of significance. The reactor itself can usually be considered on the one-node basis for adequate gross system behavior studies. A block diagram (Fig. 2) has been prepared to illustrate the information flow that needs to be represented to analyze this type of a BWR system. Each block represents a component model of varying degrees of complexity.

Fig. 2. BWR System Block Diagram

I would like to treat in more detail the fuel model, the thermodynamic pressure model and the void accumulation-void reactivity models that we have used in analyzing such boiling water reactor power systems. In particular in the latter case, I would like to review two separate approaches that have been used.

Reactor Kinetics and Fuel Model

Figure 3 shows the reactor kinetics equations that are generally simulated for a thermal boiling water reactor. Although these equations are derived for a bare thermal homogeneous reactor, they seem to provide

(based on experimental work)¹ a good representation of thermal boiling water reactors in general when considered on a one-node basis. One method of analogue computer simulation of these nonlinear reactor kinetic equations is also shown in Fig. 3.

Fig. 3. Representation of Reactor Kinetics

The fuel model that we are using is that which would apply to a cylindrical type of sintered UO_2 pellet fuel, having a relatively low thermal conductivity. Transfer functions for the heat flux-nuclear flux relationship of such a cylindrical type fuel element have been developed by L. K. Holland and F. E. Tippetts² of General Electric Company and by others, and appears as shown in Fig. 4.

Heat Transfer:

$$\frac{\Delta Q}{\Delta \phi} = \frac{.771}{1 + 13.5 s} + \frac{.171}{1 + 2.17 s} + \frac{.028}{1 + .467 s} + \dots + .03$$

Assumptions: Pellet diam = .50"

$K_{\text{fuel}} = 1.10$

$h_{\text{bond}} = 1000$

$k_{\text{clad}} = 8.0$ (clad thickness = 30 mil)

$h_{\text{film}} = > 10,000$ boiling

Fig. 4

Fuel Models

Fuel Temperature

Temperature at a specific point

$$\frac{\Delta T_i}{\Delta \phi} = + \frac{a_i}{1 + 13.5 s} - \frac{b_i}{1 + 2.17 s} + \frac{c_i}{1 + .467 s} - \dots$$

Average Temperature

¹ J. A. DeShong, Jr., Power Transfer Functions of the EBWR Obtained Using a Sinusoidal Reactivity Driving Function, ANL-5798 (Jan. 1958).

² F. C. Tippetts, Analysis of the Transient Conduction of Heat in Long Solid Cylindrical Fuel Elements for Nuclear Reactors, HW-41896 (May 1, 1956).

The last term represents the prompt heat generation term, assumed to be 3% due to neutron thermalization and gamma energy absorbed by the reactor water moderator. This transfer function assumes a modified Bessel function-type power distribution across the fuel rod, a fuel-to-clad bond coefficient of the order of 1000 Btu/(hr)(ft²)(°F) and a boiling heat transfer film coefficient of about 10,000. The fuel pellet is about $\frac{1}{2}$ in. in diameter. Appropriate constant values are used for fuel and clad conductivities.

A similar transfer function has been developed by Mr. Holland to determine the temperature at any specific point within the fuel, as a function of time. This transfer function is shown in Fig. 4 for the above fuel assumptions. This transfer function consists of a series of terms similar to the transfer function previously discussed for heat flux except that these terms alternate in sign. The coefficients a_i , b_i , etc., are determined for a specific radial location. This series, however, is not rapidly converging and where fast transients are to be studied, it requires many terms for accurate simulation. The validity of these temperature and heat flux transfer functions has been checked by comparison to multinode digital computer heat transfer calculations. Of course, the accuracy of the model representation hinges on knowledge of fuel parameters, such as thermal conductivity. Where one is not dealing with large, rapid, power excursions, the entire fuel model can probably be simplified into a single time constant. In dealing with very rapid power changes, however, the shorter time-constant terms become more significant.

One other aspect of fuel representation is that concerning the Doppler effect. The Doppler effect is the phenomenon of increased neutron absorption with temperature in U^{238} resulting from a broadening of its resonance peak with increase of fuel temperature. The Doppler effect can be quite important. We estimate that of the order of 1% of Doppler reactivity exists at rated power conditions for a typical UO_2 low enrichment, boiling water reactor. Doppler effect has been represented as a linear function of average fuel temperature (a constant Doppler coefficient). To be more accurate, the Doppler coefficient is actually a function of fuel temperature, decreasing somewhat with higher fuel temperatures. In addition, the worth of a given fuel temperature depends upon the flux level in which that fuel exists, and therefore, the Doppler effect, to be more accurate, should be properly weighted by flux. However, in most cases, for most transient work it would be satisfactory and conservative to represent the Doppler effect as simply being proportional to average fuel temperature. In order to determine average fuel temperature, representation has been used as shown in Fig. 4. This involves taking the difference between the heat energy supplied by the reactor flux and the heat energy lost to the water and integrating this energy difference to calculate the average fuel temperature. Subsequently, this average temperature is multiplied by the Doppler coefficient. Accuracy of this model of the Doppler effect is limited by knowledge of the Doppler coefficient.

Thermodynamic Model

It is necessary to develop a thermodynamic model of the steam-water BWR system behavior. The analysis that is developed here follows pretty much along the lines of a method outlined in a paper by Mr. Dale Brown of the General Engineering Laboratory in the General Electric Company.³ That portion of the system illustrated in Fig. 1 consisting of the riser, the steam drum and the reactor vessel may be considered for purposes of illustration to be reduced to a single lumped model. A more accurate method of analysis frequently used is to represent each of these three components as a single node. The combined approach, on the other hand, is satisfactory in many cases and will better serve to illustrate the thermodynamic model development.

For this case, then, the reactor vessel, riser, and drum is lumped together as shown in Fig. 5, and feed flow W_F and recirculation flow W_R are considered to be fed into the bottom of the vessel (the lower plenum) with instantaneous mixing occurring with the rest of the plenum water.

Fig. 5. Simplified System Model

As shown in Fig. 6, the mixed inlet enthalpy of the water feeding into the core is computed from a simple heat-balance equation applied to the lower plenum. The analysis of the balance of the system consisting of the boiling steam-water mixture and the steam volume with exit recirculation flow, w_r , and exit steam flow, w_s , is performed on the basis of writing a liquid and gas mass balance, a volume balance and a system energy balance. These three equations are indicated in Fig. 6. This system of 3 equations contains three unknowns, dM_g/dt , dM_f/dt and dp/dt , and is solved for the latter, the rate of change of pressure with respect to time. The solution is shown at the bottom of Fig. 6.

Certain simplifying assumptions may be made in this solution, depending on volume of steam, M_g , water, M_f , etc., and certain terms including V/J , the flow work term (resulting from writing the energy equation in terms of internal energy), may be neglected. When only part of the steam is in intimate contact with the water, only that part of the total M_g , namely, M'_g , is used in the condensing term. If a more accurate

³ D. H. Brown, Transient Thermodynamics of Reactors and Process Apparatus, ASME Paper #57-NESC-81.

MASS BALANCE

$$\text{Liquid: } w_r + w_F - w_r - w_f = \frac{dM_f}{dt}$$

$$\text{Gas: } w_f - w_s = \frac{dM_g}{dt}$$

VOLUME BALANCE

$$\frac{d(M_g v_g)}{dt} + \frac{d(M_f v_f)}{dt} = \frac{dV_T}{dt} = 0$$

ENERGY BALANCE

Plenum Mixing

$$h_r = \frac{h_s + h_F \frac{w_p}{w_r}}{\left(1 + \frac{M_F}{w_r} s\right)}$$

$$\text{where } s = \frac{d}{dt}$$

$$Q + (w_r + w_f) h_r - w_r h_f - w_s h_g = \frac{dM_g h_g}{dt} + \frac{dM_f h_f}{dt} - \frac{\dot{p}V}{J}$$

$$\text{Now, writing } \frac{dM_f v_f}{dt} \text{ as } M_f \frac{dv_f}{dt} \frac{dp}{dt} + v_f \frac{dM_f}{dt}, \text{ etc,}$$

$$\text{with } h_{fg} = (h_g - h_f) \text{ and } v_{fg} = v_g - v_f,$$

$$\frac{dp}{dt} = \frac{Q - w_r (h_f - h_r) - w_s \frac{h_{fg} v_g}{v_{fg}} + w_F \left(h_r - \frac{h_f v_g - h_g v_f}{v_{fg}} \right)}{\left(M_f \frac{dh_f}{dp} + M_g \frac{dh_g}{dp} - \frac{V}{J} \right) - \frac{h_{fg}}{v_{fg}} \left(M_g \frac{dv_g}{dp} + M_f \frac{dv_f}{dp} \right)}$$

Fig. 6

Thermodynamic Model

model consisting of a vessel node, a steam drum node, and one or more riser nodes is used, it will provide some representation of the delay time that it takes pressure surges in the drum to be felt by the vessel, as well as transport time delay through the risers. Where flashing and condensing action is nonlinear (e.g., in the steam drum, where flashing can occur by condensing action) the thermodynamic system equations may be appropriately modified in a nonlinear manner.⁴

Void Reactivity Effects

Having reviewed the transient thermodynamic model, the last and most important topic to be covered in this consideration of BWR system analysis is that concerning the analysis of void reactivity effects. The original block diagram in Fig. 2 showed a void accumulation and a void reactivity block that converted sensible heat to steam voids, to resultant reactivity effects. Two approaches for analyzing void reactivity phenomena will be discussed.

One of these involves the generation of a void map, a figure such as shown in Fig. 7, which relates void reactivity to core exit quality for

⁴ E. F. Borner and B. F. Cassidy, Analogue Study of Boiler-Reactor Interaction, Nucleonics (May, 1957) p. 84.

Fig. 7

various values of sub-cooling.⁵ The basis for this map lies in the fact that a given reactor power (neutron flux) distribution shape need only have inlet subcooling and core exit quality defined in order to define in turn power magnitude and void accumulation where core inlet flow is fixed. In fact, if any two of the three parameters exit quality, power level, and subcooling are defined, in turn, the third, quality, as well as void accumulation, is determined.

Once, for a given set of conditions, a void accumulation is established and void reactivity may be computed. One method of computing this void reactivity is to work on an axial incremental basis and compute R_g , the void volume, or void area for each unit axial length. Using a set of physics data for the core giving k_∞ and M^2 vs smeared water/fuel ratio, and applying these at each axial unit, then multiplying by local flux, a weighted total change in k_∞ is calculated. Similarly, a total change in M^2 , weighted by flux, is calculated. The void reactivity change from non-boiling to this particular void condition - one point on the void map - is then calculated as Δk void reactivity = $\Delta k_\infty + \Delta M^2 B_g^2$, where B_g^2 is the buckling coefficient. Through the calculation of many such points, a void map as in Fig. 7 is developed.

To make use of this void map concept, core exit quality needs to be computed from a separate thermodynamic core analysis with the core considered on a one-node basis. The manner in which this is done is shown by the equations and block diagram of Fig. 8. Pressure is fed into this model, having already been determined from the previous transient thermodynamic pressure model of the system. In this way, a transient core exit quality is computed, which can then be fed into the void map along with computed subcooling to determine the resultant void reactivity. To account for transport time through the core, or the time effect of a redistribution of voids, a sweep or void redistribution time constant is used as indicated in Fig. 8. As would be expected and as will be seen later, this is only an approximation of the effect, but has been generally

⁵ D. W. Leiby, Analogue Computer Study of the Transient Performance of a Dual Cycle Boiling Water Reactor Nuclear Power Plant, AIEE Conference Paper-57-684, Summer General Meeting, Montreal, Quebec, Canada (1957).

weighting was used. Figure 9 shows the frequency response (solid core) for flux-squared void weighting of the transfer function relating a change in heat flux to a change in void reactivity. This curve is normalized in gain, and frequency is plotted terms of ωT , where T is transit time from boiling boundary to the top of the core. (This T is fairly constant with large changes in power level.) The dashed line represents the fit of a simple simulation network.

Fig. 9

Frequency Response of Weighted Voids vs Power (ϕ^2 Weighting)

In a similar way, Mr. Hogle has developed weighted void transient response and subsequent frequency response for relating the change in pressure to a change in void reactivity. (This would result from the flashing or condensing action following a pressure change.) The frequency response of this relationship is shown in Fig. 10 for the case of flux-squared weighting.

Fig. 10

Frequency Response of Weighted Voids (Flashing or Condensation) vs Pressure

Finally, another frequency response characteristic was derived by him, as shown in Fig. 11, which relates the change in subcooling to a change in flux-squared weighted steam void on void reactivity.

Fig. 11
Frequency Response
of $G(S)$ Weighted
Voids vs Subcooling

A given pressure step increase will effectively produce an increase of subcooling as well as temporary flashing or condensing. The subcooling increase will be temporary, being washed out after a complete loop recirculation time. This temporary subcooling effect of a pressure change is added to the feedwater subcooling quantity and fed into the transfer function for subcooling whose frequency response is shown in Fig. 11. The pressure change is also fed into the transfer function whose frequency response is shown in Fig. 10, which represents the immediate flashing and condensing effects associated with a change in pressure on an intimately mixed steam-water mixture.

Thus, all three transfer functions relating a change in heat flux, pressure and subcooling, respectively, to changes in void reactivity can be summed together, as in Fig. 12. Thus, this void reactivity model of

Fig. 12. J. J. Hogle's Model

Mr. Hogle can be used instead of the void map method previously described and depicted in Fig. 8. Of course, specific values of k_1 , k_2 , and k_3 of Fig. 12 depend on and are determined by the reactor physics and physical parameters of a particular design.

An illustration of the similarity of a BWR system response to a step change in reactivity using these two models, the void map

model and the model of Mr. Hogle, is evident in Fig. 13. The response labeled Mod #2 is that model using the void map concept, and the response of Mod #3 is that using Mr. Hogle's void reactivity model. Each model is used in turn to simulate the void reactivity feedback in the BWR system subjected to the step disturbance in reactivity. (The response labeled Mod #1 is that from a third model for which equations were developed by Mr. L. K. Holland to describe the dynamics of the weight of steam in the core.)

Fig. 13. Analog Computer Traces of Reactor Flux and Void Model Reactivity (ΔK_V) Response to a 10% Step of Rod Reactivity

Conclusions:

A. Using mass, volume and energy-balance equations, thermodynamic models have been developed for BWR systems. Two of the current approaches to relating one-node thermodynamic equations to spatial void reactivity models have been described. These thermodynamic and void models have been suitable to represent the overall system behavior of a BWR power system when used in conjunction with conventional reactor kinetic and fuel models.

B. The void map model is probably more useful for large system changes when the void map itself is simulated using analogue computer function generators. The transfer function model of Mr. Hogle is developed

on a more nearly complete physical basis and is believed to be more desirable for use with reasonably limited BWR system perturbations.

C. Throughout the analyses discussed in this paper, recirculation flow has been assumed as an input variable. Although not described here, system flow equations, whether internal (natural circulation) or external, may be developed to describe transient flow behavior when such is necessary to be incorporated in BWR system studies (e.g., loss of pump power and shipboard applications of BWR's).

D. With regard to system stability it is worth noting that the method of analysis described herein represents the BWR system only insofar as gross nuclear system stability is concerned. It is obvious that detailed hydraulic phenomena, such as core flow pattern transitions, riser instabilities, and core nuclear and hydraulic coupling behavior, are not represented in this type of one-node lumped model. An attempt to represent these phenomena requires considerably more elaborate systems of equations and array of computer equipment.

E. In order to have confidence that the preceding BWR system representation will satisfactorily predict system transient behavior, it is necessary in subsystem studies through the use of detailed subsystem models (e.g., multinode riser system, or multinode nuclearly coupled core) to investigate the adequacy of the simplified models previously discussed or to develop more adequate, yet still relatively simple models for use in the overall system simulation. An alternative, of course, is to establish by experimentation that simplified subsystem models are adequate (e.g., establishment of inherent hydraulic or flow stability in out of pile heat transfer tests of fuel assembly).

F. When reasonably applied the BWR-power system models herein described will satisfactorily predict transient behavior of such systems, based on limited experimental system performance data. Such experimental data have provided verification on an overall system response basis rather than on an individual transfer function basis.

Discussion of Paper
Presented by Mr. Owen

MR. SANDMEIER:

I wonder if you could say a little bit more about how you define the 7.5-sec time constant.

MR. OWEN:

If one sets up the representation, the differential equations for heat transfer, or for temperature distribution through the cylindrical fuel element, and solves these for a step change of heat flux, say, or step change of heat generation made within the fuel, as Mr. Holland has done, one can generate from that solution a series of transfer functions in this manner. I think I can refer you to a paper of his on this if you are interested in the details. This has been done by others, too. Frank Tibbetts from Hanford has done essentially the same thing. The fuel time constant is not particularly critical. There is not a great deal of difference of system behavior if we simply lump this series of time-constant terms altogether into a single equivalent time constant for the fuel itself. The only time it really makes a difference is on the fast excursions, when some of these shorter time constants and prompt terms are quite significant.

MR. GRACE:

By what means were the point-by-point calculations made in the transient response which were then used to get the frequency response? Was this done on a digital computer?

MR. OWEN:

It was done by hand the first time, and on a digital computer after that. The conversion from the transient to frequency response was set up and done on the digital computer.

MR. GRACE:

There is a digital computer code that was developed a year or so ago that we have used quite often for this same purpose. Its designation is ART-03. This enables you to do a transient response as you have done here, with either a step-in power, step-in pressure, or whatever, and then there is a straight one which calculates the frequency response.

MR. OWEN:

That is interesting. Just for this type of a system and including changes in power, pressure, etc.?

MR. GRACE:

Just as you are recommending.

MR. OWEN:

Is that program generally available?

MR. GRACE:

Yes, ART-03. Developed by Johnson and Curley at Bettis.

MR. OWEN:

I see. At the time this was done, I think we wrote a program on 650 to handle this.

MR. GRACE:

This is a 704.

MR. OWEN:

I appreciate hearing about it. Thanks.

MR. OKRENT:

The question is addressed to either of the last two speakers. Does this transient solution assume equilibrium conditions at each point, or does it handle transient hydrodynamic phenomena and so forth?

MR. OWEN:

The one I am referring to does not include transient hydrodynamic phenomena. It assumes a heat balance at each point, essentially a step-by-step steady-state solution. We assume a step change of heat flux coming from the fuel, and the fuel delay phenomena, or the fuel time constant, is represented in the previous block.

MR. STONE:

I wonder if you did any experimental verification of this pressure transfer function?

MR. OWEN:

Not directly. I would say not. We have simply used this to predict a performance, say, for example, with VBWR (Vallecitos BWR) on automatic pressure control. We have used this to predict the performance of the reactor loop itself, and it seems to predict performance consistent with the loop behavior. We have not gotten really a separate check on that specific transfer function.

I would only say, as far as the system representation is concerned, it is consistent.

MR. WRIGHT:

I would like to ask the corollary question to that, namely, the extent to which you have been able to get confirmation of the void response to your heat step inputs and verify the results. You mentioned in your talk that you had had some verification.

MR. OWEN:

Yes, but again it is not specifically from that singly, but only from the integrated system. In other words, we have the model that relates to just pressure, subcooling, and heat flux, and we have not gotten experimental verification on just the transfer function of the heat flux alone, but only for the overall system model.

MR. LIPINSKI:

You mentioned something about automatic pressure control on Vallecitos. Would you like to elaborate a little?

MR. OWEN:

About a year and one-half ago or so, we set up a system to operate the reactor under automatic pressure control. In other words, the central control rod was automatically positioned to hold the reactor pressure constant. We did it really to check out our predictions of the system performance, and I guess, principally, just the system behavior of a BWR under automatic pressure control. It worked pretty much along the lines that we expected.

MR. LIPINSKI:

Did you use sinusoidal frequencies to verify stability or was transient testing the only method used?

MR. OWEN:

This was principally just a transient check, load change performance and so on. Just transient behavior.

METHODS OF INVESTIGATING NOISY POWER REACTORS

Part I

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The incentives for performing tests on power reactors under operating conditions are several:

1. these give information about the safety of the system,
2. it may be possible to improve the performance as a result; and
3. the overall understanding of complexities in power reactor dynamics is increased.

Therefore, with testing playing an important role so far in reactor development, it can be expected to continue to do so

At Argonne, the programs for experimentation with its power reactors, the Boraxes,¹⁻⁵ EBR,⁶ and EBWR,^{7,8} have been extensive and fruitful in results. The boiling reactor experiments will be reviewed here

¹J. R. Dietrich, Experimental Investigation of the Self-limitation of Power during Reactivity Transients in a Subcooled, Water-moderated Reactor, ANL-5323 (1954).

²J. R. Dietrich and D. C. Layman, Transients and Steady-state Characteristics of a Boiling Reactor, ANL-5211 (Feb 1954)

³B. Maxon, O. A. Schulze and J. A. Thie, An Experimental Investigation of Reactivity Transients and Steady-state Operation of the Borax IV Boiling Reactor, ANL-5733 (Feb. 1959).

⁴J. A. Thie, Dynamic Behavior of Boiling Reactors, ANL-5849 (May 1959).

⁵J. A. Thie, An Experimental Investigation of Boiling Reactor Safety, ANL-6135 (1960).

⁶F. W. Thalgott et al., Stability Studies on EBR-I, Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva (1958), Vol 12, p. 242.

⁷J. A. DeShong, Jr., Power Transfer Functions of the EBWR Obtained Using a Sinusoidal Reactivity Driving Function, ANL-5798 (Dec. 1957).

⁸J. A. Thie, Statistical Analysis of Power-Reactor Noise, Nucleonics, 17 (10), 102 (1959).

This is because they offer good examples of making measurements on potentially unstable systems under adverse conditions, i.e., in the presence of "noise." By noise is meant random fluctuations in time for the variables of interest. The presence of noise is not unique in boiling reactors, and the methods discussed here would presumably be applicable to many reactor systems.

Table I shows a variety of experiments which have been performed on boiling reactors, and their results have been reported.^{3-5,7,8} This table describes the experiments in terms of the methods of excitation of transient or oscillatory phenomena. Since it represents only what has been done on these three reactors, the list is by no means exhaustive.

TABLE I

DYNAMIC EXPERIMENTS IN BOILING REACTORS AT POWER

	BORAX II	BORAX IV	EBWR
<u>Transfer functions by oscillatory excitation</u>			
Control rod		x	x
Feedwater valve			x
<u>Transfer functions by step excitation</u>			
Control rod		x	x
Feedwater valve	x		
Steam valve	x		x
<u>Noise excitation</u>		x	x
<u>Ramp excitation</u>			
Control rod	x		
Top reflector	x		

An example of results from one of these experiments, a control rod oscillator excitation of Borax IV,³ is shown in Fig. 1. This shows that noisy fluctuations in power can exceed the desired sinusoidal response to the driving force. Mechanical feasibility considerations generally limit the magnitude of the excitation function, especially at high frequencies. Therefore, a means of obtaining an accurate transfer function under poor signal-to-noise ratio conditions must be

found. The cross-correlation method described below is one powerful technique which has already found applicability in many analogous problems.

Figure 2 shows a simplified block diagram of processes in a noisy reactor. It is seen that the reactor is assumed to be excited by a randomly varying reactivity, which in turn is due to random fluctuations in some physical quantity. For the latter, both incoming water temperature⁸ and the reactor water level⁹ have been suggested as mechanisms for random excitation of voids in boiling reactors. In any event, the physical origin of the noise is "filtered" (i.e., changed in its frequency system) before becoming a reactivity noise.

⁹J. A. DeShong, Jr., Flux, Reactivity, Steam Void, and Steam-Water Interlayer Noise Spectrums in the EBWR, J. Nuc. En. A: Reactor Science, 10, 147-156 (1959)

RUN NO. 7 3/22/57
 POWER RUN WITH OSCILLATOR
 REACTIVITY IN STEAM = 1.645 % $\Delta K/K$
 NO. OF FUEL ASSEMBLIES = 42
 WATER HEIGHT ABOVE FUEL = 2.75 FT

Fig. 1

Fig. 2

In the application of known signals, such as oscillations or steps, to a system such as this, these other driving forces mix with the random ones to give a result which, in linear analysis, is their sum. However, in addition to having a mixture of a random and a regular driving force manifesting itself at the output, it has recently been proposed⁵ that the box labeled "system" in Fig. 2 may possibly have characteristics which are also randomly varying. Thus both external as well as internal noise sources can influence a measurement. An electrical analogue would be a sharply tuned high-gain amplifier, having both a signal and thermal noise at its input, and also having a noisy resistor affecting the gain of the tuned circuit.

Among the methods of excitation in Table I is that of using the noise alone. While not as accurate and informative as other methods, it does have the advantage of experimental simplicity, and sometimes represents the only feasible way of obtaining information. The three steps in obtaining the transfer functions of a system excited by noise are:

1. obtain the frequency spectrum, $P(f)$, of the reactor power;
2. measure, calculate, or estimate the spectrum of the reactor noise, $k_{in.}(f)$; and
3. the transfer function is then

$$G(f) = P(f)/k_{in.}(f) \quad (1)$$

Figure 3 shows typical results of the first step for EBWR at two power levels, as well as for an a priori theoretical simulation of the EBWR at 50 Mw on an analogue computer. While these results were obtained by the Fourier transforms of autocorrelation functions,⁸ it is also possible to obtain them by direct Fourier analyses of the reactor powers.⁹

In a practical engineering sense the results of Fig. 3 are a final answer of considerable significance. It is evident that the reactor under normal operating conditions (undisturbed by experiments) has a resonance,

Fig. 3

Figure 5 shows the results of applying Eq. 1 to Fig. 3 and 4. (The EBWR example used here is presented from pedagogical considerations. The transfer functions shown are in fact those obtained by the rod oscillator method.⁷)

Fig. 4

whose half-width is decreasing as the reactor power increases. At some power a decision, based on safety considerations, is presumably made, that the resonance is as sharp as can be permitted.

However, in a scientific sense, it is of interest to use Fig. 3 to obtain additional information by proceeding with steps 2 and 3 above. Figure 4 shows the reactivity noise spectra measured for the EBWR at two power levels, and also that used on the reactor simulator. (This measurement was possible only because G and P had been independently measured in the EBWR and used in Eq. 1 to obtain k_{in} .) In general, k_{in} is not known experimentally and must be inferred from other experiments, calculated, or estimated.

Fig. 5

RUN NO. 10 3-22-57
 POWER RUN WITH OSCILLATOR
 WATER HEIGHT ABOVE FUEL = 4.25 FT
 NO. OF FUEL ASSEMBLIES = 42
 REACTIVITY IN STEAM = 1.535 % Δ K/K

Fig. 6

The precision to which transfer functions can be obtained depends on the statistical accuracy of the power spectrum and the precision to which k_{in} is known. If $k_{in}(f)$ is assumed slowly varying in the vicinity of a resonance, then the characteristics of the resonance in G are essentially those found in $P(f)$. Should, for some reason, the resonances of G and P differ, the latter would still be of practical value.

In what has been presented so far, noise has been regarded principally as an external perturbation to the systems being measured. However, it is quite possible that this distinction cannot always be maintained. Evidence of this is shown in Fig. 6. This is from a Borax IV test with high water level.³ Although an attempt is being made to obtain the response to a rod oscillator, the reactor is exciting itself into wave trains of oscillations. It is evident here that conventional formulations of transfer functions assuming linear, noise-free systems, break down, and that other definitions of instability must be sought.^{4,5}

This problem along with that of obtaining more information about

TABLE II

PROBLEMS OF BOILING REACTOR DYNAMICS

Deviations from noise-free, linear feedback theory

Noise sources

Four oscillatory modes:

1. Small oscillations rich in harmonics;
2. Resonant reactor transfer functions' details;
3. Oscillations of electrically heated loops;
4. Spatial effects.

the noise source is listed in Table II among other unsolved problems in boiling reactor dynamics. In particular, a quantitative understanding of the absolute amplitude of the noise, and the amplitude probabilities for oscillatory wave trains is needed. Also listed in Table II are various types of instability possibly encountered in boiling reactors. These have been discussed at great length.^{4,10,11}

¹⁰J. W. Weil, et al., Boiling Water Reactor Stability, GER-1468 (1958).

¹¹R. P. Anderson and P. A. Lottes, Boiling Stability, Kerntechnik (to be published).

METHODS OF INVESTIGATING NOISY POWER REACTORS

Part II

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The topic in this part of the paper is the improvement of the signal-to-noise ratio in reactor transfer function measurements by digital cross-correlation analysis. The main problem in reactor transfer function measurements with a periodic reactivity input is to determine the amplitude and the phase of an output signal in the presence of the reactor noise with a given accuracy. Cross-correlation technique proves to be an effective method to improve the signal-to-noise ratio, and thus the precision, in such measurements. In this technique, the noise is filtered out by a certain integration process which can be conveniently performed by a digital computer if the reactor output is digitized, or sampled, at constant time intervals. The main question which arises in the case of the digital computation is the proper choice of the sampling interval and the observation time to attain a given precision in the determination of the observed signal. In answering this question, reactor noise will be assumed to be the only source of error.

The question of detecting a periodic signal in noise using the cross-correlation technique has been subject to several investigations,¹⁻³ mostly in connection with detecting radar signals in the background noise. The aim of this paper, however, is to investigate the cross-correlation technique from the point of reactor applications where the noise spectrum is not flat and contains only very low frequencies ($f \leq 5$ cps). The presented treatment is believed to have some novel features with regard to cross-correlation theory in general.

A typical reactor parameter can be given as

$$y(t) = N(t) + \sqrt{2} A \sin(\omega t + \theta) \quad , \quad (1)$$

where $N(t)$ denotes the reactor noise which is assumed to be an ergodic stationary random process. The second term, $\sqrt{2} A \sin(\omega t + \theta)$, is an

¹Y. W. Lee, T. P. Cheatham, Jr., and J. B. Wiesner, Application of Correlation Analysis to Detection of Periodic Signals in Noise, Proc. I.R.E., 38, 1165-71 (1950).

²J. P. Costas, Periodic Sampling of Stationary Time Series, M.I.T. Research Laboratory of Electronics, Tech. Report No. 156 (1950).

³W. B. Davenport, Jr., R. A. Johnson, and D. Middleton, Statistical Errors in Measurements on Random Time Functions, J. Appl. Phys., 23, 377-388 (1952).

output signal when the input is $\sin(wt)$. Then, A and θ are the magnitude and the phase of the transfer function between the input and the output.

It is assumed that the output $y(t)$ is sampled at M points spaced at intervals of (Δt) (Fig. 1), i.e.,

$$y_n = N_n + \sqrt{2} A \sin(n\alpha + \theta) ; \quad \alpha = w \Delta t$$

Fig. 1
Sampling of the Output and the
Finite Cross Correlation

The following computations are performed on y_n 's in the digital cross-correlation analysis:

$$C_r = \frac{2}{M} \sum_{n=1}^M y_n \sin(n + r)\alpha, \quad (2)$$

which is the average of the lagged products formed by shifting the input by an amount $\tau = r\Delta t$ and then multiplying by the output. The variation of C_r with r is sinusoidal (see Fig. 1).

The ensemble average of C_r is found as

$$\overline{C_r} = \sqrt{2} A \cos(\theta - \alpha r), \quad (3)$$

since the noise is uncorrelated with the input signal, viz., $\overline{N_n \sin(n + r)\alpha} = 0$. The observation time $T = M\Delta t$ is assumed to be an integral multiple of the

signal half-period, viz., $M\alpha = k\pi$, in obtaining Eq. (3).^{*} The variation of $\overline{C_r}$ with r is sinusoidal (see Fig. 1).

The difference between C_r and $\overline{C_r}$, or the deviation from the mean, is entirely due to the presence of the noise:

$$\Delta C_r = C_r - \overline{C_r} = \frac{2}{M} \sum_{n=1}^M N_n \sin(n+r)\alpha \quad (4)$$

or

$$\Delta C_r = \sqrt{2} P \cos(\alpha r + \phi) \quad , \quad (5)$$

where

$$P^2 = \frac{2}{M^2} \sum_{n=1}^M \sum_{m=1}^M N_n N_m \cos(n-m)\alpha \quad . \quad (6)$$

The variation of ΔC_r with r is also sinusoidal. The amplitude (P) and the phase (ϕ) are random quantities in the sense that they would have different values if the above procedure were repeated on the outputs of identical reactors with the same input.

Equation (3) suggests that an estimate of the amplitude and the phase of the output signal can be obtained from Eq. (2) as

$$A^2 = \frac{1}{2} (C_s^2 + C_c^2) ; \quad \tan(\theta - \alpha r_1) = C_c/C_s \quad , \quad (7)$$

^{*} In general,

$$\overline{C_r} = \sqrt{2} A \left[(1-d) \cos(\theta - \alpha r) + e \sin(\theta - \alpha r) \right]$$

where

$$d = \frac{1}{M} \sum_{n=1}^M \cos 2(n+r) \alpha = \frac{\cos(2r+M+1)\alpha \sin M\alpha}{M \sin \alpha}$$

$$e = \frac{1}{M} \sum_{n=1}^M \sin 2(n+r) \alpha = \frac{\sin(2r+M+1)\alpha \sin M\alpha}{M \sin \alpha}$$

where C_s and C_c are two values of C_r at $r = r_1$ and $r = r_2$ which are a quarter of a cycle apart, viz., $(r_1 - r_2)\alpha = (\pi/2)$.** Explicitly,

$$C_s = \frac{2}{M} \sum_{n=1}^M y_n \sin(n + r_1)\alpha$$

and

$$C_c = \frac{2}{M} \sum_{n=1}^M y_n \cos(n + r_1)\alpha$$

If there were no noise, C_r would be identical with $\bar{C}_r$ and Eqs. (7) would yield the exact values of A and θ . An upper bound of the errors for A and θ , when the noise is present, can be found by differentiating both sides of Eqs. (7) and then evaluating ΔC_s and ΔC_c from Eq. (5) as the values of ΔC_r at $r = r_1$ and $r = r_2$. The final result is***

$$\left[\left| \frac{\Delta A}{A} \right|, |\Delta \theta| \text{ (in rad)} \right] \leq \frac{P}{A} \quad (8)$$

** If C_s and C_c are not in quadrature, i.e., $(r_1 - r_2)\alpha = \frac{\pi}{2} + \beta$, where β measures the departure from the ideal spacing ($\pi/2$), the A and θ are given by

$$\tan(\theta - \alpha r_1) = \frac{C_c(1-d) - C_s(e - \sin \beta)}{C_s(\cos \beta + d) - e C_c}$$

$$A = \frac{C_s}{\sqrt{2}} \frac{\sqrt{1 + \tan^2(\theta - \alpha r_1)}}{(1-d) + e \tan(\theta - \alpha r_1)}$$

*** In the case of $M\alpha \neq \pi k$, $(r_1 - r_2)\alpha = \frac{\pi}{2}$, the upper bound of the error becomes

$$\frac{P}{A} \frac{\sqrt{1 + \nu^2}}{1 - \nu^2} \quad \text{where } \nu^2 = d^2 + e^2 = \left(\frac{\sin M\alpha}{M \sin \alpha} \right)^2$$

If $M\alpha = k\pi$, but $(r_1 - r_2)\alpha = \frac{\pi}{2} + \beta$, the upper bound is found as

$$\frac{P}{A} \frac{\sqrt{1 + |\sin \beta|}}{|\cos \beta|}$$

Since P is a random quantity, one defines an expected error ($\bar{\epsilon}$) as

$$\bar{\epsilon} = (1/A) \sqrt{\overline{P^2}} \quad , \quad (9)$$

which will serve as an estimate of the error in an actual measurement. If C_r is regarded as a fictitious noise with a mean-square value $\overline{P^2}$ and $\overline{C_r}$ as the signal, then the expected error may be interpreted as the noise-to-signal ratio after cross correlation.

Expected value of P^2 can be obtained from Eq. (6) as

$$\overline{P^2} = \frac{2}{M} \left[\phi_0 + 2 \sum_{n=1}^{M-1} \left(1 - \frac{n}{M} \right) \phi_n \cos n\alpha \right] \quad , \quad (10)$$

where ϕ_n is the autocorrelation function of the noise, which is defined as

$$\phi_n = \overline{N_q N_{q+n}} \text{ or } \phi(\tau) = \overline{N(t) N(t+\tau)} \quad .$$

The noise-to-signal ratio before the cross correlation is applied is

$$\rho_i = \left(\sqrt{\phi_0}/A \right) \quad , \quad (11)$$

since ϕ_0 is the mean-square value of the noise $N(t)$.

Combining Eqs. (9), (10) and (11) gives

$$\bar{\epsilon} = \frac{\sqrt{2}}{\sqrt{M}} \rho_i \left[1 + 2 \sum_{n=1}^{M-1} \left(1 - \frac{n}{M} \right) R_n \cos n\alpha \right]^{(1/2)} \quad , \quad (12)$$

where R_n is the normalized autocorrelation function, defined as

$$R_n = \phi_n / \phi_0 \quad .$$

One observes in Eq. (12) that the normalized autocorrelation of the reactor noise has to be known to estimate the accuracy of the transfer function measurement with a sinusoidal reactivity input. Figure 2 shows a typical autocorrelation function of EBWR noise.

Fig. 2
A Typical Autocorrelation
Function of EBWR Noise

If the sampling interval is chosen larger than the correlation time**** of the reactor noise, so that $R_n = 0$ for $n \geq 1$, Eq. (12) reduces to

$$\bar{\epsilon} = \frac{\sqrt{2}}{\sqrt{M}} \rho_i \quad (13)$$

In this case, the accuracy increases with the square root of the number of sample points. Increasing Δt above τ_c does not yield any further improvement in the accuracy if M remains the same. Hence, the observation time has no relation to the accuracy as long as $\Delta t \geq \tau_c$.

The following simplified form of Eq. (12) may be used to obtain an approximate value of the error when the sampling interval is shorter than the correlation time:

$$\bar{\epsilon} = \sqrt{\frac{2}{M}} \rho_i \left(1 + 2 \sum_{n=1}^{M-1} |R_n| \right)^{\frac{1}{2}}, \quad (14)$$

which is obtained from Eq. (12) by neglecting n/M compared to unity and replacing $|\cos n\alpha|$ by 1. The approximation made in (14) is realistic for low frequencies for which $\cos n\omega\Delta t$ does not vary appreciably within the correlation time, e.g., for $f \leq (\frac{1}{4}\tau_c)$.

**** Correlation time is defined as the time lag τ_c beyond which the autocorrelation function is essentially zero, e.g.

$$\phi(\tau) = 0 \quad \text{for } \tau \geq \tau_c$$

In the absence of detailed knowledge of the autocorrelation function, the following inequality may be used to evaluate the sum

$$\sum_{1}^{M-1} |R_n| \quad \text{in Eq. (14):}$$

$$|R_n| \leq \cos \frac{\pi}{1 + [\tau_c/n\Delta t]} \quad , \quad (15)$$

where the bracket term $[\tau_c/n\Delta t]$ indicates the least integer not smaller than the $(\tau_c/n\Delta t)$.⁴ Figure 3 shows the variation of the right side of

Eq. (15) and the sum $\sum_{1}^{M-1} |R_n|$ as a function of $(\tau_c/\Delta t)$.

Fig. 3
An Approximate Curve for the
Autocorrelation Function

Frequency Domain Analysis

Frequency domain analysis throws more light onto the mechanisms by which the noise is filtered out. It enables one to introduce the conception of spectral window⁵ into the cross-correlation analysis.

⁴R. P. Boas, Jr. and M. Kac, Inequalities for Fourier Transforms of Positive Functions, Duke Math. J., 12, 189-206 (1945). (See also errata, Ibid. 15, 105-109 (1948).

⁵R. B. Blackman and J. W. Tukey, The Measurement of Power Spectra, Dover Publ., New York (1959).

In this treatment, the double summation in Eq. (6) is replaced by a double integral

$$P^2 = 2 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} B(t) B(t') N(t)N(t') e^{-j\omega(t-t')} dt dt' , \quad (16)$$

where $B(t)$ is defined as the sum of M Dirac functions which are displaced by an interval Δt on the time axis. The magnitude of each Dirac function is taken as $(1/M)$. Explicitly,

$$B(t) = \frac{1}{M} \sum_{n=1}^{M-1} \delta(t - n\Delta t) . \quad (17)$$

By substituting $B(t)$ from (17) into (16), one may verify that the double integration reduces to the double summation in Eq. (6).

Evaluating the ensemble average of P^2 in Eq. (16) and using some of the convolution properties of Fourier integrals yield

$$\overline{P^2} = 2 \int_{-\infty}^{+\infty} W(f') Q(f-f') df' , \quad (18)$$

where $W(f)$ is the Fourier transform of the autocorrelation function $\phi(\tau)$, i.e.,

$$W(f) = \int_{-\infty}^{+\infty} \phi(\tau) e^{-j\omega\tau} d\tau \quad (19)$$

Here $W(f)$ is the power spectral density of the noise.⁶ Since a two-sided spectrum is used, $2W(f)\Delta f$, rather than $W(f)\Delta f$, gives the mean-square value of the noise components in the frequency band Δf .

The function $Q(f)$ in Eq. (18) is the spectral window which is defined as the square of the magnitude of the Fourier transform of $B(t)$:

$$Q(f) = \left| \int_{-\infty}^{+\infty} B(t) e^{-j\omega t} dt \right|^2 = \left[\frac{\sin(M\pi f\Delta t)}{M \sin(\pi f\Delta t)} \right]^2 . \quad (20)$$

⁶J. H. Laning and R. H. Battin, Random Processes in Automatic Control, McGraw-Hill, New York (1956).

Figure 4 shows the shape of the spectral window. One observes that $Q(f)$ is a periodic function of f with a period $(1/\Delta t)$ cps. It is symmetrical about the integral multiples of $(1/2\Delta t)$. The amplitude of the major lobe is unity, whereas those of the minor ones are much less. The amplitude of the first minor lobe, for example, is about $(4/9\pi^2) = 0.045$ when $M \geq 15$. An equivalent band-width (B_e) can be assigned to the major lobes by calculating the width of a rectangle of unit height such that its surface area is equal to the area under $Q(f)$ over a period. This gives

$$B_e = \int_{-(1/2\Delta t)}^{+(1/2\Delta t)} Q(f) df = \frac{1}{T}$$

for large values of M .

Fig. 4
Spectral Window of the
Cross-Correlation Analysis

Hence, $Q(f)$ can be regarded as a train of rectangular windows displaced by $(1/\Delta t)$ cps on the frequency axis, each having a band-width $(1/T)$ cps.

In the light of the spectral window conception, one interprets $\overline{P^2}$ in Eq. (18) as the mean-square value of the noise after it has been passed through a filter whose power transfer function is $Q(f-f')$ (see Fig. 5). Then, one concludes that the cross-correlation technique is equivalent to using a multipass band filter, with pass bands of band-width $(1/T)$ centered at frequencies $f \pm (k/\Delta t)$, where f is the signal frequency. Noise components outside the pass bands are, of course, eliminated.

Fig. 5
Rectangular Window Sequence

Replacing $Q(f-f')$ in Eq. (18) by the rectangular window train, and combining Eqs. (9) and (18) give the second form of the error formula:

$$\bar{\epsilon} = \sqrt{\frac{2}{T}} \rho_i \left[\sum S\left(f \pm \frac{k}{\Delta t}\right) \right]^{\frac{1}{2}}, \quad (21)$$

where $S(f)$ is the normalized noise power spectral density defined as

$$S(f) = W(f)/\phi_0.$$

Figure 6 shows a typical power spectral density of the EBWR noise. By using this curve, one can calculate the expected error when ρ_i , T and Δt are known.

Fig. 6

A Typical Power Spectral Density of the EBWR Noise

The summation in Eq. (21) implies the contributions of the windows falling in the noise spectrum. If Δt is chosen smaller than $(1/2f_{\max})$, where $f_{\max}$ is the highest frequency in the noise spectrum, only one window, which is centered at the signal frequency f , will be found in the spectrum. Then, Eq. (21) reduces to

$$\bar{\epsilon} = \rho_i \sqrt{2 S(f)/T} \quad (22)$$

The last formula indicates that the accuracy increases with the square root of the observation time for the sufficiently small values of the sampling interval. If the latter is decreased below $(1/2f_{\max})$, keeping T constant, no further improvement is gained in the accuracy despite the

fact that the number of sampling points increases. In the limit, M tends to infinity, and the digital cross-correlation analysis approaches the continuous one.

The Confidence of the Expected Error

The confidence is defined as the probability that the actual error in a given trial will be smaller than the expected error. This probability can be determined by studying the distribution of P^2 about its expected value, $\bar{P}^2$. It is assumed that this distribution is a "Chi-square" distribution.⁷ Then, the probability that the ratio $(P^2/\bar{P}^2)$ will exceed a given number depends only on the number of degrees of freedom, K . The latter is defined as

$$K = 2(\bar{P}^2)^2 / \text{var } P^2$$

In Ref. 8, K is shown to be 2. It is found from the tables of Chi-square distribution that the ratio $(P^2/\bar{P}^2)$ exceeds unity with a probability of approximately 30% when $K = 2$. Hence, the confidence of the expected error is about 70%. The confidence becomes roughly 90% when the expected error is increased by 50%.

Concluding Remarks

The mean value of the reactor noise was assumed to be zero in the above treatment. This requires the average reactor power to be held constant during the observation, and to be bucked out before the output is digitized and recorded. In practical applications, these two requirements can be met only approximately due to the shifting of the average power. The effect of the residual constant mean value can be diminished by choosing the observation time as an integral multiple of the signal period. This provision is especially needed at very low frequencies, for which the error due to integration of the average power shift over an incomplete cycle may be large. At high frequencies, the effect of the average power shift is greatly reduced, since the observation time includes many cycles and thereby reduces the effect of the incomplete cycle.

Care should be exercised to insure that two values of the finite cross-correlation function C_r , viz., C_s and C_c , are as close as possible to a quarter of a cycle apart. The expected error tends to infinity, as observed in footnote 3 on page 100, if these two values are half a cycle apart.

⁷R. S. Burrington and D. C. May, Handbook of Probability and Statistics with Tables, Handbook Publishers, Inc., Sandusky, Ohio (1953).

⁸Z. Akcasu, Measurement of Noise Power Spectrum with Direct Fourier Analysis, to be submitted to J. Appl. Phys.

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SESSION II

THERMAL REACTOR MEASUREMENTS
AND ANALYSIS OF STABILITY

Chairman: W. C. Lipinski
Secretary: C. Pesce

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MEASUREMENT AND ANALYSIS OF THE KEWB TRANSFER FUNCTION BY REACTOR MODULATION TECHNIQUES*

Part I

E. R. Cohen
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In many analyses of reactor stability, it is possible to lump the reactor into a single point and describe the system in terms of the total power without giving consideration to the spatial variation of the flux. In a small homogeneous reactor, such as a water boiler, in which the leakage to the reflector is large, a neglect of spatial effects may be unwarranted. Thus, the slowing down time in graphite is approximately 40 μ sec and the thermal lifetime (including the leakage loss) may be of the order of a millisecond. In comparison, the slowing-down time in the core may be only a few microseconds and the thermal lifetime may be only 30 μ sec. Therefore, at frequencies of the order of a few hundred cycles per second, the transfer function of the reactor may be expected to show effects ascribable to reflector delayed neutrons.

If we consider a somewhat idealized model which represents the first step beyond the simplest description, we can describe two neutron densities. Let N be the neutron population of the core and let R be the neutron population of the reflector. We can then write

$$\frac{dN}{dt} = \frac{\beta}{\ell_0} (r-1) N + \sum \lambda_i C_i + \alpha R \quad ,$$

in which ℓ_0 is the prompt neutron lifetime in the core and α is the rate at which neutrons return from the reflector to the core. The equation for the reflector population may be written as

$$\frac{dR}{dt} = qN - \frac{R}{\ell_r} \quad .$$

Here, q is the rate constant for neutron flow from core to reflector and ℓ_r is the reflector neutron lifetime. We can compare this equation with the usual delayed neutron equation

$$\frac{dc}{dt} = \frac{\beta_i}{\ell_0} N - \lambda_i C_i \quad ,$$

*Based on studies conducted for the U.S. Atomic Energy Commission under Contract AT-(11-1)-GEN-8.

and we see that, within the limitations of the model, the reflected neutrons can be considered as the equivalent of an additional delayed neutron group with decay constant $\lambda_7 = 1/\ell_r$ and with an abundance $\beta_7 = \alpha_q \ell_0 \ell_r = (\alpha \ell_r)(q \ell_0)$. The factor $q \ell_0$ is the probability that a neutron, disappearing from the core, will go into the reflector; and the factor $\alpha \ell_r$ is the probability that a neutron which disappears from the reflector will return to the core. The product of these two factors is thus the probability that a neutron produced in the core will go into the reflector and then return to the core after a mean delay time, ℓ_r .

The foregoing description is admittedly crude, although it represents a heuristic basis from which further analysis may proceed. Thus, the effective delayed neutron abundance ought perhaps to represent the probability of a fast neutron from the core passing into the reflector where it is thermalized and then returned to the core; fast neutrons which return from the reflector have so small a delay that they will affect the transfer function only at much higher frequencies than are presently available.

A complete analysis of the transfer function of a reflected reactor should properly include the details of the spatial and energy modes of the system. The function N must then be identified as the amplitude of the fundamental mode, whereas R is the amplitude of the higher mode or modes. The same structure of the equations is still obtained; the interpretation of the coefficients is, however, now to be made in terms of the transition rates and coupling constants between neutron modes rather than physical regions. Although the "reflector neutrons" are not confined entirely to the reflector, they have a distribution which has a node in the core and comprises both "positive" and "negative" neutrons and is orthogonal to the fundamental. Thus, in general, we should be able to interpret experimental transfer functions for reflected reactors in terms of additional delayed neutron groups; the measurement of the KEWB transfer function in the range from 2 to 260 cycles/sec clearly shows this effect.

Discussion of Paper
Presented by Mr. Cohen

MR. KIEHN:

I would like to make a couple of comments. First of all, at one time β_7 was actually measured. It is quite true that the idea of sending neutrons into the core and reflector and then back again has been physically measured. As a matter of fact, when they removed Godiva from the concrete shell to the out-of-doors, β_7 was thought to be a millisecond neutron group and disappeared.

MR. COHEN:

I did not mean to imply that a seventh group of delayed neutrons is a new idea. The experience with the Godiva reactor which had to be placed out-of-doors in order to avoid the effects of reflected neutrons from the laboratory is certainly well known. The neutron reflection from a reflector is, however, much more closely coupled to the reactor, and the purpose of Mr. Cordy's experiments is to evaluate this effect qualitatively.

MR. KIEHN:

There is a burst study system in which you can study this spatial disposition of neutrons with respect to energy groups, and also where and when they were formed. Spatial effects have played a very dominant part in the development of some of these. This is experimental confirmation for some of the ideas that you have talked about.

MR. WASSERMAN:

One of the Argonne people, I think out at Idaho, also has made me aware of some Rossi alpha work being done on a mixed fast thermal system. This was done by using two different time scales in the detection system. They were able to determine separately the lifetime of the core region alone, which was a fast core, and an aggregate lifetime for the entire system, including core and reflector. The mixed fast thermal system was, I guess, a beryllium reflector around U^{235} core at the time.

MR. GRACE:

I would like to mention that in some of our analytical studies of space times kinetics, we have found that when the value of the next higher mode is sufficiently close to the fundamental, say two per cent away, the effect of the delayed neutrons in establishing the flux distribution is quite significant. It retards any flux effect. I will have a slide on that which I will show. But I think this pertains to what you have said about being able to neglect the delayed neutrons.

MR. KIEHN:

I was going to suggest that possibly a stronger convergence in the eigenfunction expansion could be made if the eigenfunctions were those derived from the alpha calculations rather than from the pure space type of calculations.

MR. COHEN:

I think it quite correct to say that the convergence of a calculation for the kinetic behavior of a reactor is improved by using temporal eigenfunctions. My discussion, however, was intended primarily to provide a basis for the interpretation of the experiment, rather than to indicate a technique of analysis. For this purpose, the important point is the qualitative recognition of the effect of higher modes.

MEASUREMENT AND ANALYSIS OF THE KEWB TRANSFER FUNCTION BY REACTOR MODULATION TECHNIQUES

Part II

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Objective

Analysis of transient experiments has indicated that the performance of the KEWB reactor is not adequately described by the conventional reactor kinetic model using six groups of delayed neutrons. This reactor modulation experiment was performed in order to resolve this anomaly and to obtain an independent determination of the ratio of prompt neutron lifetime to the effective fraction of delayed neutrons.

The Theoretical Reactor Transfer Function

The transfer function for a water boiler reactor described by the following system of linear and nonlinear space-independent kinetics equation* has been determined by Skinner and Hetrick.¹

$$\frac{dP(t)}{dt} + \frac{\beta - \rho(t)}{\ell} P(t) - \sum_{j=1}^m \lambda_j C_j(t) = 0 \quad (1)$$

$$\frac{dC_j(t)}{dt} + \lambda_j C_j(t) = \frac{\beta a_j}{\ell} P(t) \quad , \quad (j = 1, 2, \dots, m) \quad (2)$$

$$\frac{dT(t)}{dt} + \gamma T(t) = \kappa [P(t) - P_0] \quad (3)$$

$$\frac{dV(t)}{dt} + \sigma V(t) = G[P(t) - P_0] \quad (4)$$

$$\rho(t) = \rho_1(t) + \alpha T(t) + \phi V(t) \quad (5)$$

The general expression of the transfer function of the physical system described by the solution to the linearized form of these equations can be written as:

*The nomenclature is given in the Appendix.

¹R. E. Skinner and D. L. Hetrick, The Transfer Function of a Water Boiler Reactor, Nuclear Science and Engineering 3 (5), (1958).

$$G_p(i\omega, P_0) = \left[i\omega \ell^* + i\omega \sum_{j=1}^m \frac{a_j}{i\omega + \lambda_j} - \frac{\alpha^* \kappa P_0}{i\omega + \gamma} - \frac{\phi^* G P_0}{i\omega + \sigma} \right]^{-1}, \quad (6)$$

provided the reactivity is expressed in dollars.

This transfer function indicates a unique behavior at angular frequencies greater than any of the system time constants. In this frequency region, the amplitude and phase equations can be written as

$$\left| G_p(i\omega, P_0) \right| = \frac{\delta P_1/P_0}{\rho_0} \sim \frac{\beta}{\omega \ell} \quad (7)$$

$$\angle G_p(i\omega, P_0) = \theta \sim \arctan \left(-\frac{\omega \ell}{\beta} \right) \quad (8)$$

Thus, the experimental transfer function provides a ready means of determining ℓ^* should one be able to modulate at frequencies greater than the largest of the delayed neutron decay controls.

Transient experimentation has, however, indicated that a six-delay group model does not adequately describe the operation of the KEWB reactor. A seven-group model has been proposed,² which describes the neutron lifetime observed in the slower transients as

$$\ell = \ell_0 + \frac{\beta_7}{\lambda_7},$$

where ℓ_0 is the effective lifetime of the neutrons that remain in the core.

At the time of the experiment these seventh-group parameters, ℓ_0 , β_7 , λ_7 , were not well known. All evidence, however, indicated that the lifetime observed during the slower transients was very closely equal to 62.5 microseconds. Thus, one might expect some significant high-frequency behavior in the transfer function that may give an insight to the values of these seventh-group parameters.

Experimental Equipment

The performance of this type of experiment to determine the transfer function of a reactor requires a method by which the stable reactor power may be sinusoidally modulated over a frequency and amplitude region dictated by theory, a means of determining the instantaneous reactor power,

²D. L. Hetrick and D. P. Gamble, Transient Reactivity during Power Excursions in a Water Boiler Reactor, paper presented at the 1958 Winter Meeting of the American Nuclear Society in Detroit, Michigan, December 9, 1958

and a technique by which the time delay between any reactivity perturbation and its corresponding power perturbation may be determined.

The following equipment and methods were developed to perform this reactor modulation experiment on the KEWB reactor.

Modulator

The KEWB reactor oscillator is of the type described by Barrett as the Mechanical Rotary Modulator.³ It basically consists of two nested coaxial cylinders inserted in the central exposure tube of the reactor. The nested assembly is provided with bearings so that the brass outer cylinder, the rotor, is free to rotate about the fixed steel inner cylinder, the stator.

An approximately sinusoidal power modulation occurs as a result of the self-shielding effect created as small rectangular cadmium "shades" mounted on the inner surface of the rotor sweep across sinusoidally shaped cadmium "spots" mounted on the outer surface of the stator. Figures 1 and 2 show the positioning of the spots and shades on their respective cylinders. It should be noted that with this assembly, four perturbation cycles occur for each revolution of the rotor. A typical waveform of the reactivity controlled by this device is shown in Fig. 3.

Fig. 1 Rotor Shade Assembly

³L. G. Barrett, Application of Pile Modulation Measurement Techniques, KAPL-M-LGB-13 (May 22, 1957).

Fig. 2 Complete Stator and Rotor Assemblies

Fig. 3

Input Reactivity Waveform
(9 Rows of Spots and Shades
plus Water-filled Central Bore)

The amount of reactivity is readily changed by the addition or removal of spots. The spots are secured by merely glueing in place, since the cylinder on which they are mounted does not rotate. Further control of the reactivity can be effected by filling the central bore of the stator with water, thus increasing the worth of the spots.

Drive Mechanisms

The first of two separate drive mechanisms consists of a synchronous motor coupled to the rotor through a variable speed transmission and timing belt. With this drive assembly, the speed can be controlled at the reactor console throughout its entire speed range. For the timing pulley configuration used, the frequency range, with this device, was from 1 to 260 cycles per second.

At perturbation frequencies less than 1 cps, the variable speed transmission became slightly unstable. To overcome this situation, a second drive mechanism was used. In this case, a small synchronous motor was coupled to the rotor by a step gear reducer and timing belt. The pulley configuration here was set to obtain frequencies from 0.022 to 20 cps. The two drive assemblies are shown in Fig. 4.

Fig. 4 Installed Reactor Modulator Assembly

The limits on the complete frequency spectrum are established by practical considerations. The lower limit is the lowest frequency at which the drift of the stable reactor power remains negligible during any complete cycle. At the upper limit, the rotor turns at 3900 rpm. Since the rotating assembly was not dynamically balanced, it was believed that vibration problems might have damaged the central exposure tube at higher angular velocities. The four decades in frequency thus available appeared to be entirely adequate for this study.

Timing Signal

The position of the rotor, and thus the relative position of the spots and shades, was determined by observing an electrical pulse triggered by the rotor. Rigidly attached to the rotor was a circular disk with 4 slits cut along radii at 90° intervals. A collimated light beam directed at a photoelectric cell was interrupted by the disk at all angular positions except where slits occurred. Thus four "pips" occurred for each revolution of the rotor, or in other words, one pip for each perturbation cycle. The timing assembly is shown in Fig. 4.

If the position of the pip, with respect to the reactivity waveform, is known, one can compare the pip and the reactor power signal simultaneously and determine the system phase lag. The location of the pip was determined by first bringing the reactor to critical at some arbitrary stable

power with the modulator completely installed, but not rotating. The position of the control rods and thus the relative reactivity was noted. The modulator was then rotated for 4 seconds at its lowest frequency, 0.022 cps, and the new critical rod position determined and recorded. During the 4 seconds of rotation, the pip signal was fed to an oscilloscope and the occurrence of the pip noted as so many seconds of angular rotation after the preceding critical rod position. This procedure was repeated until several complete reactivity cycles had been obtained.

In this manner, a plot of the angular rotation time versus reactivity clearly indicated the position of the pip and its relation to the reactivity waveform. The waveforms thus obtained and a true, best-fit sine wave are shown in Fig. 3.

Reactor Power Detection

The high-frequency region over which the experimental transfer function is to be obtained is such that monitoring of reactor power through thermal neutron detection would result in erroneous data. Two important problems occur in this respect.

The first of these problems is the frequency-dependent attenuation caused by diffusion.⁴ In this case, use of thermal neutrons would lead to an incorrect amplitude of the transfer function. The second difficulty is the long transit period from neutron birth to detection. It turns out, as shown in Section VI, that at even moderate perturbation frequencies this effect is significant.

To overcome this problem, a boron-10 lined, lead-shielded ionization chamber⁵ was wrapped with $\frac{1}{16}$ -in. cadmium sheeting. As a result, a factor of 5 was lost in sensitivity, but at the same time this technique greatly reduced diffusion and transit time errors.

Both a cadmium-wrapped and an unwrapped chamber were used in the experiment. These were placed in the instrument stringer in the graphite reflector approximately 9 in. from the core surface.

⁴A. M. Weinberg and E. P. Wigner, The Physical Theory of Neutron Chain Reactors, University of Chicago Press, Chicago (1958).

⁵C. A. Smith and M. Huntsinger, A Low-cost Uncompensated Ionization Chamber for Monitoring Reactor Power, NAA-AER-Memo-1437 (September 28, 1955).

A detector that was considered, but not used, was a fast fission chamber. The only element that appeared satisfactory for this use was thorium-232. To check its suitability, an ion chamber that would normally be lined with boron-10 was coated to 10 mg/cm² with thorium-232. This chamber was then tested and as a result discarded, due to low sensitivity and a nonlinear power-current characteristic.

Amplification and Recording

A basic philosophy of the design of the instrumentation layout of this experiment was to keep all instrument errors in a region small enough so that correction factors would not have to be applied to the data. Specifically, to obtain ℓ/β to $\pm 15\%$ from phase data (Equation 8), it is necessary to keep the phase error within ± 4 degrees at the break frequency of the reactor.

The maximum perturbation frequency at which it was planned to operate was something less than 300 cps; thus an instrument channel with a bandwidth of 1 kc would introduce a negligible error in amplitude measurements.

The situation is not, however, quite so simple for phase measurements. The phase error situation is discussed in detail in Section IV; it is important to note, however, that for a single time-constant system, the required bandwidth for less than $\frac{1}{2}$ degree phase lag at 300 cps is 34.4 kc.

The electronic equipment in everyday use at the KEWB, have the bandwidth characteristics indicated in Table I.

TABLE I
Instrumentation Bandwidths

Component	Bandwidth (kc)
Electrometer preamp	~ 35
Kintel amplifier	~ 10
CEC 316 Galvanometer	~ 2
Textronix dual beam oscilloscope Model 502	~ 100

With the requirement that this equipment be used and that the phase error be kept to a minimum, the system block diagram shown in Fig. 5 was devised.

Fig. 5 Amplification and Recording System - Block Diagram

All of the components indicated in Fig. 5 are conventional reactor instrumentation components, except perhaps the suppressor. The suppressor provides a means of biasing to zero the signal from the chamber due to the stable reactor power. It is essentially a variable voltage supply connected to the end of the preamp input resistor that would normally be grounded.

Provided as an integral part of the suppressor is a switch which, when closed, will raise the voltage at the grid of the electrometer tube by one per cent of the dc component previously biased to zero. As a result, a signal proportional to the stable reactor power can be recorded.

The calibrated audio oscillator and the single-beam oscilloscope are used to determine the perturbation frequency. The signal from the audio oscillator is beat with the timing pip and a modified Lissajous pattern obtained on the oscilloscope. When the pattern is stable, the modulation frequency is that (or some harmonic) of the audio oscillator.

With the instrument configuration shown in Fig. 5, the phase lag of the reactor can be determined from the oscilloscope photographs without correction of the error introduced by the low bandwidths of the amplifier and galvanometer. At the same time, however, the galvanometer can record many perturbation cycles from which the per cent modulation can be accurately obtained.

Measurement Errors

As indicated in the previous section, the main problem in instrumenting an experiment of this type is the introduction of extraneous phase lags. The most significant of these phase lags are listed, briefly discussed, and their total effect summarized.

Nonlinear Reactor Modulation

To utilize the reactor oscillator technique in an experimental determination of ℓ^* , the conditions of the transfer function derivation must be met. Specifically, the reactor must be operated in the linear region, i.e., the fractional modulation of the reactor power must be small; thus the large signal-to-noise ratio desired for good measurements is limited by the nonlinear effects.

A knowledge of the region at which these nonlinear effects become significant can be obtained from Akcasu's general solution to the reactor kinetics equations without feedback.⁶ These methods can be applied to the water boiler as a result of the negligible temperature and void feedback effects at the perturbation frequencies of interest.

The frequency range that appears to produce the most useful information concerning ℓ^* and the correct group model, is in the region where the effects of six-group delayed neutrons are negligible, i.e., where Equations 7 and 8 apply. If, in fact one considers such a region in a sinusoidally perturbed reactor, the neutron flux can be written as a series of harmonics.

Specifically, if $\rho_1(t) = \rho_0 \sin(\omega t)$, then

$$P(t) = P_0 \left[1 + \rho_0^* |Z_1| \sin(\omega t + \theta_1) - \frac{1}{2} (\rho_0^*)^2 |Z_1 Z_2| \cos(2\omega t + \theta_1, \theta_2) + \dots \right] \quad (9)$$

It was previously mentioned that in a determination of ℓ^* that Equation 8 would be most useful. With this in mind, the main interest is in the error introduced by the higher order harmonics in the phase measurements. For excess reactivities much less than a dollar, only the error introduced by the second harmonic need be considered.

A pure sine wave passes through its mean value at multiples of 180 degrees; thus, if one determines the value of ωt at which the distorted waveform passes through its mean value, he will have a measure of the phase error due to nonlinearities. Specifically, if $P(t)$ in Equation 9 is set equal to P_0 , then

$$\sin(\omega t) = \frac{\rho_0^*}{2} |Z_2| \cos(2\omega t + \theta_2) \quad (10)$$

⁶Z. Akcasu, General Solution of the Reactor Kinetic Equations without Feedback, Nuclear Science and Engineering 3 (No. 4) (1958).

The solution of Equation 10 for the data of this experiment indicates a phase error not exceeding 1 degree.

Neutron Diffusion

The use of epi-cadmium neutrons in determining reactor power greatly reduces amplitude and phase errors due to diffusion effects. An indication of the remaining phase error can be determined from a knowledge of the slowing-down time of an epi-cadmium neutron.

The slowing-down time of a neutron from energy E_1 to E_2 is given by Glasstone⁷ as

$$t = \frac{\sqrt{2M_0}}{\xi \bar{\Sigma}_s} \left(\frac{1}{\sqrt{E_2}} - \frac{1}{\sqrt{E_1}} \right) \quad , \quad (11)$$

where for graphite

$$\xi = 0.158$$

$$\bar{\Sigma}_s = 0.30 \text{ cm}^{-1}$$

Thus, the time required in slowing down from fission energy (~ 2 Mev) to the cadmium cut-off (~ 0.3 ev), is 4.65×10^{-5} sec. The phase shift accounted for by this effect amounts to a lag of 4.3 degrees at the maximum perturbation frequency of 260 cps. It is important to note, however, that this calculation is correct provided all neutrons are at 0.3 ev. The actual experimental error must therefore be less than that calculated.

Internal Ion Chamber Effects

There are essentially two effects that become important when an ionization chamber is used in a phase measurement. In the first case, the interelectrode potential must be sufficient so that space charge distortion and resulting ionic recombination is small. Secondly, this potential must be such that the ion transit times are small.

Huntsinger has determined the interelectrode potential as a function of ion current and maximum permissible phase error for the chambers used

⁷S. Glasstone, Principles of Nuclear Reactor Engineering, D. Van Nostrand Company, Inc., Princeton, New Jersey (1955).

in these measurements.⁸ His results, as applied to this experiment, give a maximum phase lag of 1.5 degrees at 260 cps.

Chamber Time Constant

The chamber and lead capacitances combined with the variable input resistor at the grid of the electrometer tube produce a single time-constant system. As such, the phase lag introduced here can be written as

$$\theta_{RC} = \arctan(\omega RC) \quad (12)$$

The experimental error, as computed from Equation 12 for the maximum reactivity data, is less than 1.5 degrees at 260 cps.

Preamplifier and Oscilloscope Bandwidths

The break frequencies of both the preamp and dual beam oscilloscope are such that their combined error will be less than $\frac{1}{2}$ degree for all perturbation frequencies.

Total Measurement Error

The calculated total phase measurement error of this experiment is presented in Fig. 6. It is believed that this plot shows a maximum experimental error, since more than half of its amplitude is made up of the error due to neutron slowing-down time as determined from Equation 11.

Fig. 6

Calculated Maximum Total Phase Measurement Error

Data Treatment

Representative Data

Typical oscillograph traces showing the timing pips and the step proportional to stable power are shown in Figs. 7 and 8. Figures 9 and 10

⁸M. Huntsinger, Ionization Chamber Response for KEWB Pile Oscillator Experiments, NAA-SR-Memo-2295 (November 7, 1957).

show representative photographs of the epi-cadmium signal and the timing pip.

Fig. 7

Representative Oscillograph Data (Indicating Timing Signal)

Fig. 8

Representative Oscillograph Data (Indicating Power Step)

Fig. 9

Representative Oscilloscope Data (Low Frequency)

Fig. 10

Representative Oscilloscope Data (High Frequency)

Data Reduction Techniques

The per cent peak-to-peak modulation is obtained directly from the ratio of the peak-to-peak value of the modulated power to the height of the stable power step. This is valid since the step represents exactly one per cent of the stable power. These measurements were made to a precision of 0.010 in.

The determination of the phase lags were made by digitalizing the photographs and treating the results by a sine-wave fitting program on the IBM-709.

The digitalizing was accomplished by reading the amplitude of the waveform at intervals of 1 mm, starting at the midpoint of the leading edge of the timing pip. These measurements were made to an accuracy of 0.005 in. No smoothing was done in the digitalizing process.

The 709 program input required, in addition to the power data, an accurately known perturbation frequency (distance between pips) and in turn computed the amplitude and phase of the fundamental harmonic of the modulated power. The phase was computed with respect to the timing pip, thus necessitating the addition of a phase correction factor to obtain the phase with respect to the input reactivity. This correction factor was obtained from the measured relationship of the pip with respect to the reactivity waveform.

The only remaining factors necessary to determine the experimental transfer function are the magnitudes of the input reactivities. Two of the input waveforms, nine rows of spots plus water, and three rows and no water, were subjected to the above-mentioned curve-fitting program and their amplitudes obtained as 0.184 and 0.056 dollar, respectively.

Statistical Analysis

The data for the thermal neutron phase lag were not treated statistically, since time did not permit more than one or two measurements at each frequency.

All of the remaining data were treated by Student's "t" test for 95% confidence limits.

Results

Presentation

The experimentally determined transfer function is presented in Figs. 11 and 12. This transfer function is compared with the conventional six-group model¹ and two seven-group models. Seven-group model 1 is that which is best supported by the data and model 2 is that determined from the calculated seventh-group parameters.

Discussion

The frequency region over which the transfer function was obtained necessitated an increase in the magnitude of the input reactivity with the frequency. Four values were used: one row of spots, three rows, five rows

and nine rows, plus the water-filled central bore. In the interest of conserving reactor time, the waveforms of these reactivities were only measured for the three and nine-row cases. It was therefore necessary to normalize the amplitude and phase data taken with the one and five-row inputs to the three and nine-row data.

Fig. 11

Comparisons with Theory -
Amplitude

Fig. 12 Comparisons with Theory - Phase

Diffusion Effects

An interesting sidelight to the objectives of this experiment is presented in Figs. 13 and 14. The transfer functions presented here were obtained under identical amplification and recording conditions; the only difference existed in the cadmium wrapping of the chamber from which the epi-cadmium power signal was taken.

Conclusions and Recommendations

Inspection of Figs. 11 and 12 clearly indicates that the KEWB reactor does, in fact, operate in a manner that is more accurately described by a seven-group, rather than the conventional six-group, model. The

parameters of this seven-group model cannot be accurately determined as a result of the lack of sensitivity of the transfer function to small parameter changes. In addition, there appears to be an appreciable neutron-diffusion effect. Specifically, detection of fast neutrons would probably bring the data close to the model 2 curves in Figs. 11 and 12 and thus provide a good experimental verification of the values predicted by seven-group theory.

Fig. 13

Diffusion Effects on the Transfer Function - Amplitude

Fig. 14

Diffusion Effects on the Transfer Function - Phase

The amplitude data do, however, permit a determination of the neutron lifetime, valid for slow transients, as 61.8 ± 2.4 microseconds for an effective fraction of delayed neutrons of 0.008. This lifetime is computed at the break frequency of the reactor and the limits set by the Students "t" test for 95% confidence. This value of lifetime compares with the value of 62.5 microseconds determined from the experimental inhour equation² and the value of 78.0 ± 3.0 microseconds determined by noise techniques.⁹

⁹C. W. Griffin and J. G. Lundholm, Jr., Measurement of the SRE and KEWB Prompt Neutron Lifetime Using Random Noise and Reactor Oscillation Techniques NAA-SR-3765 (October 15, 1959).

This experiment indicates no serious problems associated with amplitude and phase measurements at frequencies less than 50 cps. The measurement techniques do, however, need improvement at greater perturbation frequencies.

The following is a list of recommendations for such an improvement.

- 1) A brief check of the rod calibration should be made with the modulator installed to determine more accurately the magnitude of the input reactivity.
- 2) The magnitude of the input reactivity, at the high perturbation frequencies, should be increased to improve the signal-to-noise ratio.
- 3) The reactivity waveform should be determined at each reactivity used.
- 4) The waveform of the input reactivity should be more accurately known. In addition to requiring more data points, this would entail development of equipment that would indicate the rotor position in a direct manner. Errors due to timing and rotor acceleration (and deceleration) would then be eliminated.
- 5) The development of a high-sensitivity fast neutron detector would be a great advantage at these frequencies. In lieu of this, however, a refinement in the technique for determining the error contributed by epi-cadmium neutrons is required.

Should these improvements be achieved, one may find it possible to modulate to even greater frequencies and as a result obtain a greatly improved insight into the quantitative aspects of describing the KEWB reactor by seven groups of delayed neutrons.

Nomenclature

$P(t)$	Total fission rate (or power) at time t .
P_0	Stable reactor power (equilibrium level).
$C_j(t)$	Concentration of the j^{th} group of delayed neutrons.
$T(t)$	Instantaneous core temperature difference, at time t , from that at equilibrium power.
$V(t)$	Instantaneous volume difference at STP, of radiolytic gas bubbles, at time t , from that at equilibrium power.
$\rho(t)$	$(k_{\text{eff}} - 1)/k_{\text{eff}}$, the reactivity in "absolute" units at time t , where k_{eff} is the effective multiplication factor.
$\rho_1(t)$	The external input function of reactivity in absolute units.
ρ_0	The magnitude of $\rho_1(t)$.
δP_1	The magnitude of the power perturbation created by $\rho_1(t)$.
ℓ	The effective neutron generation time in the reactor (assumed to have negligible time dependence).
ℓ_0	The effective generation time of neutrons which remain in the core.
β	The effective total fraction of all neutrons which are delayed.
β_j	The effective fraction of all neutrons which belong to the j^{th} delayed group.
a_j	The effective fraction of all delayed neutrons which belong to the j^{th} delayed group, $\sum a_j = 1$.
λ_j	The representative decay constant of neutrons which belong to the j^{th} delayed group.
G	The rate of radiolytic gas formation per unit power for the reactor.
κ	The reciprocal heat capacity for the reactor.
σ	The reciprocal of the characteristic time for loss of radiolytic gas from the core.
γ	The reciprocal of the characteristic time for loss of heat from the core.
α	The temperature coefficient of reactivity in "absolute" units.
ϕ	The void coefficient of reactivity in "absolute" units.
ρ^*	ρ/β , the reactivity in dollars.

ℓ^*	ℓ/β
α^*	α/β , the temperature coefficient of reactivity in dollars.
ϕ^*	ϕ/β , the void coefficient of reactivity in dollars.
ω	The perturbation frequency in radians/second.
R	The preamp input resistance.
C	The total preamp input capacitance.
θ_{RC}	The phase error due to the RC time constant.
Z_1	The amplitude of the transfer function evaluated at the fundamental frequency, ω .
Z_2	The amplitude of the transfer function evaluated at the second harmonic frequency, 2ω .
θ_1	The phase of the transfer function evaluated at the fundamental frequency, ω .
θ_2	The phase of the transfer function evaluated at the second harmonic frequency, 2ω .
$\theta_{1,2}$	$\theta_1 + \theta_2$
ξ	The average logarithmic energy decrement per collision of a neutron.
$\bar{\Sigma}_s$	The average macroscopic scattering cross section.
m_0	Neutron mass.

Discussion of Paper
Presented by Mr. Cordy

MR. WASSERMAN:

There was some work, which Art Baker knows more about than I do, on another fast thermal system in which graphite was used as a reflector. This was done at Harwell. I guess they oscillated the system up to 50 cycles per second and found no evidence of a 7th group, which is more or less in line with the sort of thing you found for the graphite system.

MR. CORDY:

Art, you mentioned earlier that work done in France went up to about 110 cycles and that an effect was observed in this work.

MR. WASSERMAN:

There was some work done by Raievski on a heavy water system in France, where he went up to about 110 cycles per second. He got very large deviations because his chamber was not only a large distance away but was also shielded by heavy water. As I remember, he noted all transport effects. He got in the order of hundreds of degrees of phase shift deviation from what the basic theory or the simplified theory would give.

MR. COHEN:

I don't want to quote on somebody's experiments when he isn't here, but isn't it possible that the D_2O experiments could be confused by the fact that the neutron source added real delayed neutron effects beyond the fission delayed neutrons?

MR. WASSERMAN:

If I remember correctly, I think this was taken into account first in establishing a base point.

EBWR STABILITY ANALYSES

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Introduction

One of the first major efforts of the EBWR experimental program was a stability investigation whose objective was to evaluate maximum stable power output capabilities of the reactor by the use of small-signal linear feedback control theory.

Analytical Transfer Function

The kinetic behavior of EBWR can be described by a set of nonlinear differential equations. The description of the EBWR multiloop feedback system is simplified by using a linearized approximation of small-signal variations about an operating point. A transfer function can then be derived for the linearized system. The transfer function as used here describes the amplitude and phase relation of neutron flux or power output with respect to the sinusoidal reactivity input driving function.

Figure 1 shows the generalized plant block diagram for EBWR. Figure 2 shows the linearized incremental-signal block diagram with appropriate transfer functions for the various elements. The temperature coefficient of reactivity is small compared to the reactivity effects due to power, flashing, and boundary voids. Figure 3, in which the temperature feedback is neglected, shows the simplified reactor model with the resultant transfer function of the steam bypass minor loop. The transfer function, $H(s)$, from reactivity input to reactor power output is given by the following equation:

$$H(s) = \frac{K \delta K \left[n_0 v - \frac{\sigma}{\eta} (b + d) \right] \left[1 + s \left(\frac{(n_0 v \gamma - \sigma d) T_r}{n_0 v \eta - \sigma (b + d)} \right) \right]}{(1 + s \gamma / \eta T_r) (1 + s T_h) (1 + s T_{pv}) (1 + s \tau)} \quad (1)$$

Where:

b	Boundary coefficient, (ft ³)(sec)/psi	γ	Steam constant, lb/psi
d	Boundary coefficient, (ft ³)(sec)/psi	η	Steam constant, lb/psi
T_r	Recirculation time constant, sec	σ	Steaming rate,
s	Laplace operator		lb/(sec)(Mw)
T_h	Heat transfer time constant, sec	τ	Steam transit time constant, sec
T_{pv}	Heat transfer time constant, sec		
n_0	Reactor power, Mw		
v	Power void coefficient (ft ³ /Mw)		
$K \delta K$	Void reactivity coefficient, dollars/ft ³

FIG. 1
PLANT BLOCK DIAGRAM OF E B W R

FIG. 2
LINEARIZED INCREMENTAL - SIGNAL REACTOR MODEL

FIG. 3
SIMPLIFIED REACTOR MODEL

The Bode diagram for equation (1) is shown in Fig. 4 with a straight-line approximation and an exact curve.

The simplified reactor model shown in Fig. 3 is obtained as the result of several approximations. If experimental measurements verified the analytical derivation, then extrapolations could be made to predict reactor behavior at higher powers.

Experimental Technique

The reactor power transfer function, P , is defined as the zero-power transfer function modified by power-to-reactivity feedback.

$$P = \frac{G}{1 + GH} = \frac{\text{Flux Output}}{\text{Reactivity Input}}, \quad (2)$$

where: P = Reactor power transfer function
 G = Zero-power reactor transfer function
 H = Power-to-reactivity feedback

Figure 5 illustrates this schematically.

FIG. 4
ANALYTIC BODE FEEDBACK DIAGRAM

A very high degree of accuracy is required for measurements which are used to calculate H from G and P . Equation (2) can be rearranged to give:

$$H = P^{-1} - G^{-1} \quad (3)$$

FIG. 5
POWER-TO REACTIVITY FEEDBACK SCHEMATIC

At high frequencies, P is nearly equal to the G function. Phase errors of a few degrees and amplitude errors of several per cent in P can change the calculated value of H by large amounts, since H is the difference between two nearly equal quantities, both much larger than H .

A reactivity generator was used to generate an input sinusoidal forcing function, and a wave analyzer was used to measure the amplitude and phase shift of the neutron flux with respect to the reactivity input.

Reactivity Generator

The reactivity forcing function was applied by a special rod drive installed in place of the regular central control rod drive, as shown in Fig. 6. The drive consisted of a variable-displacement bell crank driven by a combination of mechanical and hydraulic transmissions to provide a frequency range from 0.01 to 25 rad/sec. The connecting rod, which was very long with respect to the crank displacement, generated an accurate

FIG 6

sinusoidal control rod stroke and produced the desired sinusoidal reactivity variation. A spring was used to bias out the pressure forces and resonate with the inertia to reduce the horsepower requirements.

Wave Analyzer

The wave analyzer used a correlation technique in which the reactor flux output variation measured by an ion chamber is multiplied by a vector which is correlated to the reactivity driving function in both amplitude and phase. The block diagram of the wave analyzer is shown in Fig. 7. The physical orientation of some of the correlator components is

FIG 7
BLOCK DIAGRAM OF WAVE ANALYZER

shown in Fig. 8. The schematic of the correlator is shown in Fig. 9. It is possible to select a correlation which reduces the fundamental component of the multiplied signal-to-zero, leaving only a small second harmonic plus any noise present in the original flux signal. In Fig. 9 R_2 is used to adjust the amplitude correlation by changing the feed-

back. Phase-shift correlation is attained by a mechanical differential, shown in Fig. 10. Narrow band width is achieved by passing the multiplied signal through a bandpass filter adjusted to pass only the fundamental frequency, as shown in Fig. 11. Under ideal conditions, measurements have been made to better than $\pm 0.5\%$ in amplitude and ± 0.5 degree in phase, and under unity signal-to-noise ratio, the measurements were accurate to $\pm 5\%$ in amplitude and ± 2 degrees in phase.

Fig. 8

Fig 9
CORRELATOR SCHEMATIC DIAGRAM

FIG. 10

FIG. 11
DETECTOR SCHEMATIC DIAGRAM

Experimental Measurements

Transfer functions were measured at different powers and pressures with and without automatic steam bypass control. The data are presented in Table I and in Figs. 12, 13, 14, and 15.

Feedback Transfer Functions

The experimental P functions were determined with an accuracy of about ± 5 per cent in amplitude and ± 2 degrees in phase (which was insufficient), so a means was developed for smoothing the data. If equation (2) is inverted, we have:

CURVE	5.4	9.7	19.9
POWER, mw	5.4	9.7	19.9
PRESSURE, psi	550	550	550
BORON	NO	NO	NO
CENTER ROD, in.	13.6	13.8	23.7
8-ROD BANK, in.	24.1	29.0	33.0
STEAM BYPASS	MANUAL	MANUAL	MANUAL
PEAK REACTIVITY, % FOR 1-in. PEAK STROKE	0.071	0.089	0.043

FIG. 12
EFFECTS OF POWER ON FREQUENCY RESPONSE

CURVE	5.4	5.15	5.1
POWER, mw	5.4	5.15	5.1
PRESSURE, psi	550	300	131.5
BORON	NO	NO	NO
CENTER ROD, in.	13.6	13.8	13.8
8-ROD BANK, in.	24.1	22.9	22.3
STEAM BYPASS	MANUAL	MANUAL	MANUAL
PEAK REACTIVITY, % FOR 1-in. PEAK STROKE	0.071	0.053	0.055

FIG. 13
EFFECTS OF PRESSURE ON FREQUENCY RESPONSE

CURVE	5.4	5.7
POWER, mw	5.4	5.7
PRESSURE, psi	550	550
BORON	NO	YES
CENTER ROD, in.	13.6	23.7
8-ROD BANK, in.	24.1	41.9
STEAM BYPASS	MANUAL	MANUAL
PEAK REACTIVITY, % FOR 1-in. PEAK STROKE	0.071	0.048

FIG. 14
EFFECTS OF ROD BANK POSITION ON FREQUENCY RESPONSE

PARAMETERS	TEST NO. 5 ^a	TEST NO. 7 ^a	TEST NO. 11 ^b	TEST NO. 10	TEST NO. 13 ^b
POWER, mw	19.4	5.15	20.0	19.9	19.94
PRESSURE, psi	550	310	550	305	310
CENTER ROD, in	23.7	23.9	24.0	23.9	23.9
B-ROD BANK, in.	41.3	41.2	33.1	34.7	35.1
PEAK REACTIVITY, %	0.048	0.043	0.043	0.046	0.040
FOR 1-in PEAK STROKE					

a N₃BO₄ ADDED

b AUTOMATIC STEAM BYPASS

FIG. 15
EFFECTS OF MULTIPLE PARAMETER VARIATION ON FREQUENCY RESPONSE

$$\frac{1}{P} = \frac{1 + GH}{G} \quad (4)$$

Multiplying both sides of equation (4) by G gives

$$\frac{G}{P} = 1 + GH \quad (5)$$

Figure 16 shows the result obtained by calculating $(1 + GH)$ from the data for test No. 10 in Table I and plotting it on the log polar plane. On such a plot, it is possible to draw a smooth curve through the experimental points to provide data smoothing. Variation in the normalizing factor is accomplished simply by sliding the entire curve up or down along the $(1 + GH)$ axis. H is then calculated from the following equation:

$$H = \frac{GH}{G} \quad (6)$$

Table II shows the tabulated values of the various experimental feedback transfer functions.

Determination of Power Coefficients and Time Constants

The power coefficients and time constants which describe the reactor can be determined by comparing the amplitude and phase of the experimental feedback transfer function, calculated from closed loop measurements, to the amplitude and phase of the analytic feedback transfer function described by equation (1).

Figure 17 shows the analytic feedback response curves obtained for 20 Mw and 300 psi by fitting the experimental data, represented by small circles. The power coefficients and time constants which describe reactor behavior at 20 Mw and 300 psig are summarized in Table III.

Extrapolation to Higher Power

Table IV shows extrapolated power coefficients and time constants for 40-Mw operation with 33-Mw test data obtained under maximum power output with equilibrium xenon. The calculated closed loop 40-Mw curves are compared to the 33-Mw measurements in Fig. 18.

Investigation of Oscillation Threshold

The reactor would be stable for all values of open loop gain if no time lags were present in the reactivity feedback. In the actual case, there are several time constants associated with the feedback, which result in increased phase shift at higher frequencies. This is shown in Fig. 19, which is a log polar plane plot of the frequency locus of the open loop transfer function, GH , for the 20-Mw, 550-psig experiment designated as Test #4 in Table I.

FIG 16
DETERMINATION OF EXPERIMENTAL GH

TABLE I
POWER FUNCTION MEASUREMENTS

TEST PARAMETERS (EQUILIBRIUM XENON EXCEPT TEST NO. 15)	TEST NUMBER													
	1	2	3	4	5	6	7	8	9	10	11	13	14	15
POWER, mw	ZERO (6 kw)	5.4	9.7	20	20	5.7	5.2	5.2	5.1	20	20	20	33	50
PRESSURE, psig	150	550	550	550	550	550	310	300	130	300	550	310	550	600
BORON	NO	NO	NO	NO	YES	YES	YES	NO	NO	NO	NO	NO	NO	NO
CENTER ROD, in.	13.6	13.6	13.8	23.7	23.7	23.7	23.9	13.8	13.8	23.9	24.0	23.9	41.0	41.0
8-ROD BANK, in.	14.5	28.1	29.0	33.0	41.3	41.3	41.2	22.9	22.3	34.7	33.1	35.1	47.0	47.0
STEAM BYPASS	MANUAL	MANUAL	MANUAL	MANUAL	MANUAL	MANUAL	MANUAL	MANUAL	MANUAL	MANUAL	AUTOMATIC	AUTOMATIC	MANUAL	AUTOMATIC
PEAK REACTIVITY (%) FOR 1-inch PEAK STROKE	0.067	0.071	0.10	0.043	0.048	0.048	0.043	0.067	0.065	0.048	0.043	0.040	—	—

RATIO OF FLUX TO REACTIVITY AMPLITUDES AND RELATIVE PHASE OF FLUX TO REACTIVITY
Ratio is Given in Decibels (Zero Db. = 132) and Phase in Degrees

[illegible]

TABLE II
EXPERIMENTAL FEED-BACK FUNCTION DATA

TEST PARAMETERS (EQUILIBRIUM XENON EXCEPT TEST NO. 15)	TEST NUMBER										
	2	3	4	5	6	7	8	9	10	14	15
POWER, mw	5.4	9.7	20	20	5.7	5.2	5.2	5.1	20	33	50
PRESSURE, psig	550	550	550	550	550	310	300	130	300	550	600
BORON	NO	NO	NO	YES	YES	YES	NO	NO	NO	NO	NO

RATIO OF REACTIVITY TO FLUX AMPLITUDES AND RELATIVE PHASE OF REACTIVITY TO FLUX
Ratio is Given in Decibels (Zero Db. = 0.00755) And Phase in Degrees

FREQUENCY, radians/sec.	Db	Phase	Db	Phase	Db	Phase	Db	Phase	Db	Phase	Db	Phase	Db	Phase	Db	Phase	Db	Phase	Db	Phase	Db	Phase
0.016	—	—	—	—	+2.4	-6	+0.5	-10	-7.6	-21	-7.6	-8	-3.4	-7	-0.8	-9	+1.0	-2	+0.1	-19	—	—
0.032	—	—	—	—	+2.0	+3	+0.4	+3	-7.8	-8	-7.5	-5	-3.3	-2	-1.1	-4	+1.0	+4	-0.4	-5	—	—
0.063	—	—	—	—	+1.9	+7	+0.7	+3	-7.6	0	-9.3	0	-3.2	+1	-0.8	-1	+1.6	+9	-0.4	+8.6	—	—
0.126	-3.4	+6	-1.1	+6	+2.7	+4	+0.9	+5	-7.0	+7	-7.0	+5	-2.5	+5	-0.6	+8	+3.7	+16	+0.7	+14	—	—
0.314	-2.8	+3	+1.9	+5	+3.1	+4	+2.6	+4	-5.5	+3	-6.6	-1	-0.5	+3	+1.6	+6	+6.5	+8	+4.4	+17	—	—
0.63	-1.0	-11	+2.3	-1	+4.6	0	+4.6	-2	-4.5	-21	-5.5	-17	+0.7	-12	+3.2	-15	+7.6	-1	+7.9	+15	—	—
1.26	-1.4	-42	+2.2	-36	+4.8	-23	+4.1	-28	-4.4	-44	-5.9	-55	+0.4	-38	+1.6	-41	+7.5	-21	+8.1	+0.4	—	—
1.89	-1.2	-67	+1.7	-61	+4.6	-42	+4.8	-48	-4.4	-72	-6.2	-75	+0.5	-60	+2.0	-68	+7.6	-36	+7.5	-23	—	—
3.14	-4.0	-105	-0.5	-94	+4.2	-75	+3.4	-84	-7.7	-114	-7.8	-116	-2.4	-99	+1.3	-87	+7.2	-65	+7.2	-67	—	—
4.4	-7.5	-126	-3.8	-113	—	—	+0.9	-114	-11.0	-134	-10.2	-138	-4.8	-116	-1.8	-110	+6.2	-94	+6.3	-97	—	—
6.28	-11.3	-151	-6.6	-139	-1.4	-130	-3.7	-129	-15.4	-153	-14.6	-160	-9.1	-135	-6.9	-131	+2.1	-127	+3.0	-136	—	—
8.6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	-0.3	-147
9.4	—	—	—	—	-7.7	-160	-9.0	-155	—	—	—	—	-16.9	-160	-16.9	-149	-4.1	-158	-5.7	-160	0	-155
10.7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	-2.2	-167
11.6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	-2.4	-170
12.5	-23.4	-185	—	—	-16.3	-183	-12.9	-156	—	—	—	—	—	—	—	—	-8.1	-169	-9.5	-165	-7.4	-175
15.7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	-12.2	-177	—	—	—	—
18.8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	-16.2	-185	—	—	-7.9	-185
25.1	—	—	—	—	-25.1	-224	—	—	-31.8	-205	—	—	—	—	—	—	-23.1	-203	—	—	-25.3	-267

(a) FEEDBACK TRANSFER FUNCTION

(b) POWER TRANSFER FUNCTION

$$H = \frac{1.135(1 + s/0.19)}{(1 + s/0.31)(1 + s/4.4)(1 + s/7)(1 + s/16)}$$

FIG 17
TRANSFER FUNCTIONS FOR 20 mw, 300 psig

TABLE III
SUMMARY OF ANALYTIC FEEDBACK PARAMETERS

PARAMETER	TEST NUMBER					
	2	3	4	8	9	10
MW	5.4	9.7	20	5.15	5.1	20
p psi	550	550	550	300	130	300
γ lb/psi	1.0	1.0	1.0	1.1	1.1	1.1
η lb/psi	6.5	6.5	6.5	9.3	15	9.3
σ lb/sec	6.9	12	26	6.0	5.5	23
V (ft ³ /mw)mw	0.77	1.2	1.5	1.0	1.4	2.2
b ft ³ sec/psi	0.38	0.34	0.19	0.80	2.0	0.57
d ft ³ sec/psi	0.016	0.017	0.010	0.018	0.002	0.010
T _h sec	0.44	0.38	0.29	0.38	0.31	0.23
T _{pv} sec	0.22	0.16	0.14	0.17	0.17	0.14
τ sec	0.084	0.084	0.087	0.083	0.09	0.062
T _r sec	7.1	12	18	19	24	28
K _{OK} \$/ft ³	1.9	1.6	1.8	1.3	1.3	1.5

TABLE IV
COMPARISON OF PREDICTED AND EXPERIMENTAL
FEEDBACK PARAMETERS

	40-MW Extrapolation	33-MW Test
Power-MW	40	33
Pressure - psig	550	550
γ - lb/psi	1.05	1.05
η - lb/psi	6.53	6.53
σ - lb/sec	51.2	42
V - ft ³	2.6	2.1
b - ft ³ sec/psi	0.20	0.23
d - ft ³ sec/psi	0.004	0.007
T _h - sec	0.23	0.23
T _{pv} - sec	0.14	0.17
τ - sec	0.062	0.075
T _r - sec	28	18
K _{OK} - \$/ft ³	1.4	1.6

The point of interest is the magnitude of the open loop gain when the open loop phase shift is -180 degrees. At -180 degrees phase shift, the equation describing the closed loop power transfer function, P , can be written as

$$|P| = \frac{|G|}{1 - |GH|} \quad (7)$$

As $|GH|$ approaches unity, $|P|$ will approach infinity and the reactor flux will oscillate.

The gain and phase margin existing at various powers is plotted in Fig. 20. Extrapolation of both the gain and phase margin show the onset of instability at 66 Mw.

A high-power experiment was performed to determine whether the reactor was stable up to the predicted

66-Mw threshold. At 61.7 Mw, feedwater pump capacity was reached. Power was increased in short bursts to 65 Mw, with reactor water level falling slightly. The reactor remained stable and controllable. The results of transient tests suggest that the phase margin had been reduced to about 5 degrees or less.

Conclusion

The experimental transfer function measurements verified the analytical derivations and provided the necessary information to make higher power extrapolations in a nondestructive test.

A thorough understanding of the kinetic behavior of EBWR has indicated possible reactor operation at 100 Mw with a slightly larger core.

The knowledge gained from the analysis of EBWR stability can be applied to the design of similar plants to produce maximum power at minimum cost.

The experimental and analytic techniques applied to EBWR are not limited to this single type of nuclear power plant. The techniques can be applied to any type of nuclear power plant to achieve economical power production.

FIG. 19
OPEN LOOP FREQUENCY LOCUS FROM TEST NO. 4

FIG. 20
REACTOR GAIN AND PHASE STABILITY

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1. J. A. DeShong, Jr., Power Transfer Functions of the EBWR Obtained Using a Sinusoidal Reactivity Driving Function, ANL-5798 (January 1958).
2. Eric S. Beckjord, Dynamic Analysis of Natural Circulation Boiling Water Power Reactors, ANL-5799 (March 1958).
3. J. A. DeShong, Jr. and W. C. Lipinski, Analyses of Experimental Power-Reactivity Feedback Transfer Functions for a Natural Circulation Boiling Water Reactor, ANL-5850 (July 1958).

Discussion of Paper
Presented by Mr. Lipinski

MR. BAKER:

I was very impressed by your experimental results of amplitude and phase. I would like to ask the following question: how many adjusted parameters were there in developing the curves shown about six slides back?

MR. LIPINSKI:

There is a table that lists these. We can go through it here if you want an accurate number. We have V , b , d , T_h , T_{pv} , τ , T_r , and $K_{\delta k}$. I think that about covers it.

MR. BAKER.

Thank you.

MR. COHEN:

All of these were then fitted?

MR. LIPINSKI:

Well, the fitting is a combination of knowing what some of the values are beforehand in terms of, e.g., time constants, because you have an infinite number of combinations in order to fit this equation to the experimental results. Consequently, you have to have some knowledge beforehand of what the values are for a few of the parameters.

MR. GRACE:

In your closing remarks, about the application of this method to new reactors: did you mean to say that there are methods whereby you can calculate analytically the parameters that go into the equation without having to perform the experiments?

MR. LIPINSKI:

No. My remark specifically was with regard to the sinusoidal method as being a nondestructive type of test as compared to raising reactor power and observing what happens.

MR. GRACE.

It would involve fitting at low power and then extrapolating?

MR. LIPINSKI:

Yes. Perhaps some day we will have enough confidence in our method of making these calculations, and we would use it for designing a completely new plant. The ultimate goal in designing these new plants would be to have complete confidence in the analytical derivations so that the plant would produce a specific power at minimum cost. In the case of EBWR, it was designed for 20 megawatts. An increase by a factor of five to 100 Mw is significant with regard to economy.

MR. THOMPSON:

How far do you think you are along the road actually to making the predictions all of the way for a new reactor? Do you think you have any chance at all? Basically, there are two things that are important in all of this analysis that we are talking about today. The first is that you can predict an instability before you reach it. The second, and even better, of course, is that you can predict an instability before you design it into a reactor. If you can do those two things, you have the entire problem solved, and you certainly can justify the techniques which we are discussing today.

I wondered if you have any idea how far along you are toward the ultimate statement that any instability in a reactor design could be determined before the reactor is built and operated.

MR. LIPINSKI:

We are one-half step closer. Based on experimental test data and heat transfer data of a particular fuel element, one has a closer insight as to what an overall plant would do. To do it on a purely analytical basis is still questionable. If one had a reactor type which was a small deviation from past experiments, one could have confidence in a purely analytical design. But for a completely new concept, there are too many approximations involved.

MR. THALGOTT:

In line with Dr. Thompson's remarks, I believe that it is fair to say that in his first remark one has been able to predict the behavior based on low-power experiments quite well, and that this is one aim. But as far as the second aim is concerned, certainly we can't predict the behavior purely on the basis of theory, but at least we do know now some of the places to look for possible unstable characteristics. I think this is a major part of the battle.

MR. THOMPSON:

It disturbs me a little sometimes to see how far one gets away from reality in terms of analogue computers, digital computers, and block diagrams. One should try to look back occasionally at the experiments. I am pleased to see you are really making an effort at Argonne to try to bring this thing into focus. This is really the way to go about it.

The analytical derivation involves a lot of approximations. One doesn't really know what reliability you can place in the analytical derivations.

HIGH AND LOW POWER SPERT-I TRANSFER FUNCTION MEASUREMENTS *

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Introduction

In this paper, a report is presented of progress in transfer function measurements which are part of the Spert program in reactor kinetics. An initial set of measurements made under various conditions in the Spert I reactor is outlined, the apparatus used is described in some detail, some problems involved in the tests are discussed, and the status of the data analysis is indicated.

Brief Description of the Spert I P-18/19 Reactor¹⁻⁴

Figure 1 presents a cutaway view of the Spert I reactor, which is a facility consisting of an open, unpressurized, unshielded tank, 4 ft in diameter and about 15 ft high, contained within a 10-ft diameter work tank. The core is an early APPR core consisting of plate-type, stainless steel-enriched UO_2 fuel assemblies mounted in a rectangular grid structure. Light water serves as coolant, moderator, and reflector and normally fills the reactor tank to a level of about two feet above the top of the fuel assemblies. No provision is made for continuous forced circulation of the coolant or heat removal.

*This work is also being done in partial fulfillment of the requirements for a D.Sc. degree in Chemical Engineering at MIT under the supervision of Professors T. J. Thompson and E. P. Gyftopoulos.

¹Quarterly Progress Report, Reactor Projects Branch, October, November, December, 1958, F. L. Bentzen, ed., IDO-16537, pp. 11-43 (September 1, 1959).

²Quarterly Progress Report, Reactor Projects Branch, January, February, March, 1959, J. A. Norber, ed., IDO-16539, pp. 25-28, 55-58 (November 20, 1959).

³Quarterly Progress Report, Reactor Projects Branch, April, May, June, 1959, J. C. Haire, ed., IDO-16584, p. 7 (April 12, 1960).

⁴Quarterly Progress Report, Reactor Projects Branch, July, August September, 1959, A. H. Spano, ed., IDO-16606, pp. 3-7 (to be published).

Fig. 1. Spert I APPR Core

The fuel assemblies have a nominal 3 in. by 3 in. cross section and an active length of about 22 in. There are 18 plates per assembly, spaced 0.133 in. apart. Each fuel plate consists of a sandwich of 5-mil stainless steel cladding around a 20-mil meat section of highly enriched UO_2 dispersed in stainless steel, with a small amount of boron carbide added as burnable poison. There are about 515 g of U^{235} per fuel assembly. The fuel sections of the four fuel-poison control rods are similar to the regular fuel assemblies.

Figure 2 presents the core configuration. The positions of the central transient rod (TR), the four symmetrically disposed shim control rods (CR), the reactivity oscillator, and the neutron source are indicated.

For "low" power oscillator tests (50-1000 watts), a boron-lines ion chamber used as a neutron detector was located in either position 76 (as shown) or position 66, while for "high" power oscillator tests (50-600 kw) the chamber was located either just inside or just outside the reactor tank, on a radius passing roughly through position 76.

Objectives of the Initial Spert I Pile Oscillator Tests

Before turning to a description of the pile oscillator apparatus used, it is of interest to summarize the objectives of the initial test series. These were:

1. To determine ℓ/β (the ratio of prompt neutron lifetime to delayed neutron fraction) for the reactor, from small-amplitude, low-power, relatively high-frequency transfer function measurements. The result is to be compared with results derived from short-period step-transient experiments,⁵ poison perturbation tests,⁶ pile-noise measurements,⁷ and calculation.⁶ The purpose of these studies is to evaluate the suitability of various methods for obtaining the value of ℓ/β for use in the prediction of kinetic behavior, as part of the development of standardized acceptance test procedures for new reactors.

2. To determine experimentally the effect of reactivity amplitude on the measured transfer function at low power for comparison with theoretical results.⁸⁻¹² The purpose here is to check the adequacy of approximate theoretical methods for calculation of large amplitude behavior at low power, for use in analyzing large amplitude situations at high power (e.g., near resonances).

Fig. 2. P-18/19 Core Configuration

⁵G. O. Bright, private communication (September, 1959).

⁶A. H. Spano, private communication (April, 1960).

⁷F. L. Bentzen, private communication (April, 1960).

⁸V. H. Rumsey, The Time Variation of Pile Power due to Periodic Modulation of the Effective Multiplication Constant, CRP-233 (PD-217) (June 19, 1947).

⁹Z. Akcasu, General Solution of the Reactor Kinetic Equations without Feedback, Nuclear Science and Engineering, 3 (4), 456-467 (April, 1958).

¹⁰H. B. Smets, Low and High Power Nuclear Reactor Kinetics, D. Sc. Thesis in Nuclear Engineering, Massachusetts Institute of Technology, pp. 173-192 (June, 1958).

¹¹V. Raievski, Study of Neutron Transport by the Modulation Method, D. Sc. Thesis in Physics, University of Paris, pp. 31-34 (April, 1959).

¹²H. A. Sandmeier, The Kinetics and Stability of Fast Reactors with Special Considerations of Non-linearities, D. Sc. Thesis in Physics, Swiss Federal Institute of Technology in Zurich; ANL-6014, pp. 49-66 (June, 1959).

3. To predict the upper limit of stable reactor power for comparison with experimental results and to determine to what extent it is possible to use transfer function measurements to predict measured reactor transient behavior. It is recognized that success in the former objective does not imply success in the latter. A reactor may yield to linearized (or pseudo-linearized) treatment for estimation of the limit of absolutely stable power from results of reactor transfer function measurements at lower powers,¹³⁻¹⁶ but this is no guarantee that the course of dynamic behavior at powers even slightly above this limit can be predicted with accuracy, due to the impact of nonlinearities which manifest themselves only when power amplitudes become quite large. For this reason, it is not self-evident that the course of reactor transients can be adequately predicted from transfer function measurements, since the power levels reached in typical short-period transients are orders of magnitude above the limit of absolutely stable steady powers.¹⁷

4. To gain sufficient experience so that a more definitive effort can be carried out subsequently under more varied and more controlled conditions of power level, pressure, temperature, and coolant flow in the Spert II, -III, and -IV reactors.

Description of Pile Oscillator Equipment^{1,4}

Reactor Oscillator Assembly

Figure 3 presents an exploded view of the reactivity oscillator assembly used. It consists of three concentric parts: (1) the rotor, (2) a supporting housing, and (3) an axially positioned cadmium curtain.

¹³J. W. Weil, et al., Boiling Water Reactor Stability, GER-1468 (General Electric Report) (January, 1958).

¹⁴J. A. DeShong, Jr. and W. C. Lipinski, Analysis of Experimental Power - Reactivity Feedback Transfer Functions for a Natural Circulation Boiling Water Reactor, ANL-5850 (July 1958).

¹⁵B. S. Maxon, O. A. Schulze and J. A. Thie, Reactivity Transients and Steady-state Operation of a Thoria-Urania-fueled Direct-cycle Light-water Boiling Reactor (Borax-IV), ANL-5733 (February 1959).

¹⁶J. A. Thie, Dynamic Behavior of Boiling Reactors, ANL-5849 (May 1959).

¹⁷F. Schroeder, et al., Experimental Study of Transient Behavior in a Subcooled Water-moderated Reactor, Nuclear Science and Engineering, 2 (1), 96-115 (February, 1957).

Fig. 3
Exploded View of Oscillator
Rod Assembly

The oscillator rotor consists of a thin-walled, hollow aluminum cylinder, about $2\frac{1}{4}$ in. OD and a few inches longer than the active length of the core. The rotor cylinder is normally filled with water when in the reactor. A 20-mil layer of cadmium is embedded in a 165° angular segment of the cylinder wall. Thus, the effective multiplication constant of the core is modified as a function of the angular position of the rotor. Due to the particular neutron flux distribution in the "corner" position 22 (Fig. 2), the shape of the reactivity-vs.-rotor-orientation curve is found to be nearly sinusoidal, as shown later.

The housing consists of a hollow aluminum cylinder slightly larger than the rotor. Upper and lower end boxes attached to the housing permit it to be clamped in the core grid structure in the same manner as the fuel assemblies.

The curtain consists of another hollow aluminum cylinder, slightly larger in diameter than the housing cylinder. A 20-mil layer of cadmium running the full 360-degree circumference is embedded in the wall of the cylinder. The curtain is shorter than the oscillator rotor and can be moved by means of a lead screw axially between the top core clamp and lower grid plate of the core. The curtain serves to provide, within a certain range, a continuous control of the amplitude of the reactivity variation.

Figure 4 is a photograph of the assembled oscillator taken at the end of the test series. (The corrosion of the metal surface is due to the use of untreated cooling water in the final high-power test, as discussed later.)

Fig. 4

The reactivity wave shape of the oscillator was determined from the measured periods resulting from incremental angular displacements of the rotor. Figure 5 shows the reactivity wave shape obtained with an 18-in. curtain located at its topmost position. The peak-to-peak amplitude is 7ϕ . In this figure is presented, also, a curve representing a least-squares fit of the data to a sine wave plus its first two harmonics. The equation for this curve indicates a second harmonic content of less than 2% and a third harmonic content of less than 3%.

Fig. 5

REACTIVITY WAVE SHAPE
Oscillator with 18-inch Curtain
at Top in Position 2-2 in
Normal P-18/19 Core
at Ambient Temperature

Thus the wave shape of the oscillator is a good approximation to a simple sine wave and therefore places only a modest demand on the instrumentation and data processing for filtering out the effects of the harmonics. Furthermore, additional tests showed that the wave shape was not significantly affected by removal of the curtain or by changes in core temperature or void content.¹

Figures 6 and 7 show the results of measurement of the reactivity amplitude for various positions of a 12- and an 18-in. curtain. Movement of the 12-in. curtain is seen to provide a continuous variation of reactivity amplitude from about 4.5ϕ to 22ϕ peak-to-peak, while the 18-in. curtain provides a range from about 2ϕ to 7ϕ .

Fig. 6. PEAK-TO-PEAK REACTIVITY AMPLITUDE
vs
DOWNWARD DISPLACEMENT OF CURTAIN

Oscillator in Position 6-6 in Normal P-18/19
Core at Ambient Temperature.

For a meaningful comparison of transient behavior predicted from oscillator results with transient behavior observed experimentally, it is important that the presence of the oscillator does not modify the major shutdown mechanisms of the reactor significantly. Flux traverses made with and without the oscillator in the core indicated¹ that, except in the outermost fuel assemblies, there was no significant change in the flux distribution, suggesting that the distribution of energy-dependent shutdown mechanisms in the most important region of the core, at any rate, was affected little by installation of the oscillator.

Fig. 7. PEAK-TO-PEAK REACTIVITY AMPLITUDE
vs
DOWNWARD DISPLACEMENT OF 18-inch
CURTAIN
Oscillator in Position 2-2 in Normal P-18/19
Core at Ambient Temperature.

Instrumentation and Drive System

Two instrumentation systems were employed in the reactor transfer function measurements. The first, so-called "raw" data, system involved the recording on a common oscillograph record of (a) the amplified signal from the neutron detector, (b) the same signal with the dc component bucked out and the ac component amplified for more accurate determination of amplitude and phase, and (c) an amplified phase-reference signal derived from a sine potentiometer geared to the main oscillator drive shaft. This raw data system was used in all tests.

The second, or "nulling" data, system was similar to a system first used in reactor work at KAPL¹⁸ and later at ANL.¹⁹ This system, which is inherently capable of accuracies in phase and magnitude measurements of at least 0.5° and 1%, respectively, is illustrated schematically in Fig. 8. The current signal from the ion chamber is passed through the series combination of a sine potentiometer and an adjustable resistance. The sine pot is driven in synchronism with the oscillator, but its phase may be adjusted by means of a differential gear. At the wiper of the sine pot, a voltage appears which, when passed through a bandpass filter tuned to the oscillator frequency, consists of two ac components of generally unequal phases and magnitudes. By proper adjustment of the phase of the

¹⁸R. T. Frost and R. J. Schemel, PTR Zero-Power Transfer Function, KAPL-MEMO-RTF-3 (August 1955).

¹⁹J. F. Boland, R. R. Smith, and F. W. Thalgott, Experimental Techniques Used in Stability Studies on EBR-I, Mark III, Trans. Am. Nucl. Soc., 1 (2) Paper 6-1 (December, 1958).

Fig. 8. SCHEMATIC DIAGRAM OF NULLING CIRCUIT
Reactivity Oscillator Test Spert-I P-18/19 Core

sine pot and the magnitude of the adjustable resistance), these two components may be made equal and opposite to each other, so that a null signal is achieved at the output of the bandpass filter. When this condition is attained, the phase of the neutron signal is simply related to the phase of the sine potentiometer, and the ratio of ac to dc currents from the chamber (equal to the fractional neutron amplitude) is simply related to the resistances in the circuit, as indicated in Fig. 8. This nulling system was used for all the low-power tests, but could not be used in the high-power tests for reasons to be discussed later.

Figure 9 presents a complete block diagram of the signal circuits used in the transfer function measurements.

Figure 10 presents a schematic diagram of the mechanical system. A drive motor and variable-speed transmission provide speeds from less than 0.01 cps up to 18.5 cps. In an instrument gearbox the oscillator shaft motion is transmitted to the sine potentiometers of the raw data and nulling systems. The drive motor, transmission, and gearbox are mounted at the top of the reactor tank, as shown in Fig. 11.

Using various relay and synchro links, all operation of the oscillator equipment was carried out remotely from the Spert I control room, located one-half mile from the reactor building. Figure 12 shows a general view of the control room. In the right half of the photograph are the main reactor console and instrument racks; in the left half of the photograph are the oscillator console and instrument rack.

Fig. 9. SIGNAL INSTRUMENTATION FLOW SHEET

Reactivity Oscillator Tests Spert-I P-18/19 Core

Fig. 10. MECHANICAL SYSTEM FLOW SHEET

Reactivity Oscillator Tests Spert-I
P-18/19 Core.

For the high-power tests, fuel-plate-surface and coolant temperatures, coolant flow rates, and pressure were recorded at various points in the reactor along with the power variations.

Summary of Tests and Discussion of Some Practical Problems

Figure 13 summarizes the conditions of the transfer function tests as divided into three categories: (a) low-power, small-amplitude, (b) low-power, large-amplitude, and (c) high-power, small-amplitude tests. In the first and second categories, the tests were carried out at powers between 50 and 1000 watts, with most of the runs at about 750 watts. Reactivity amplitudes used were approximately 7ϕ , 13ϕ , and 22ϕ peak-to-peak. (An additional test at 30ϕ is tentatively scheduled.) The bulk water temperatures for most of these tests ranged between 10 and 20°C, with one small-amplitude test carried out at 50°C.

Fig. 11

Fig. 12

MEAN POWER	PEAK-TO-PEAK REACTIVITY AMPLITUDE	
	LOW	HIGH
LOW	50-1000 WATTS 7-13 CENTS 10-50°C	50-1000 WATTS 22-30 CENTS 10-20°C
HIGH	50-600 kw 7-13 CENTS 50-100°C	

Fig. 13

TABLE OF OSCILLATOR TESTS
Spert I P-18/19 Core

In the absence of any normal provision for cooling, heat removal in the high-power tests was achieved by a combination of two ad hoc methods: (a) by running softened and deionized cold water directly into the bottom of the reactor tank, stirring, and permitting the heated water to pass into the work tank by flowing over a weir located about $2\frac{1}{2}$ ft above the top of the core, and (b) by running additional unprocessed water through coils of copper tubing immersed in the reactor tank. Using all the water available in this way permitted the steady removal of about 500 kw of heat while maintaining the bulk water temperature in the tank at about 75°C. To increase the heat removal capacity to 600 kw while lowering the bulk water temperature to 60°C, both the processed and the unprocessed water were run directly into the reactor tank in the final run of the first test series. (Because of the unprocessed water, corrosion of all aluminum surfaces, including parts of the oscillator, resulted during this test.)

Attempts to run at simultaneously higher powers and higher temperatures than those mentioned increased the boiling noise to such an extent that interpretation of raw data records became impossible. The boiling noise occurred principally in the form of dips in the power level, having an amplitude of the same order as that of the imposed oscillations and half-widths typically a few tenths to one second. The dips occurred at irregular intervals, with a frequency which increased as either the power level or the bulk water temperature was increased. Figure 14 shows a raw data record taken at 500 kw mean power, 75°C bulk water temperature, a frequency of about 0.03 cps, and with an input reactivity amplitude of about 13¢ peak-to-peak and a fractional neutron amplitude of about 16% peak-to-peak. The power dips are evident.

These power dips caused ringing of the bandpass filter in the nulling circuit. This rendered the nulling circuit useless when the frequency of the dips approached that of the oscillator frequency. As a result, only raw data could be taken at conditions more severe than 100 kw power and 50°C bulk water temperature.

The power dips observed in these tests appeared to be essentially thermal-hydrodynamic in origin rather than the result of a closed feedback loop between reactivity and power. Raising the power under conditions of

- 1 Phase Reference Signal
- 2 Same as 1, but Amplified and Clipped
- 3 Total Neutron Signal
- 4 Amplified ac Component of Neutron Signal
- 5 Same as 4, but Amplified and Clipped

Fig. 14. PILE OSCILLATOR RAW DATA FOR SPERT-I P-18/19 REACTOR

500 kw, 75°C Bulk Water Temperature

0.03 cps, 13° Peak-to-peak Reactivity Amplitude

bulk boiling from 50 kw (which was the highest power at boiling at which transfer function data could be taken) to 200 kw increased the frequency of the power dips markedly (and the amplitude somewhat), but there was no evidence of damped oscillations heralding an approach to a reactivity-feedback type of instability.

The data for the oscillator tests are currently being analyzed. To obtain a reliable value of ℓ/β , various corrections to the low-power transfer function data are being applied. Due to the large fluid friction developed between the oscillator rotor and its housing at the higher frequencies, a correction to the measured phase must be made to account for mechanical torsion of the system. This correction was measured in a mockup of the oscillator system in the laboratory and found to increase monotonically with frequency, increasing sharply above 7 cps to a value at 18.5 cps of 1.9 degrees at 20°C, and about one-third of a degree less at 80°C (due presumably to the negative temperature coefficient of water viscosity).

Another correction must be made to account for the finite frequency response of the detector circuit, determined by the RC time constant of the circuit. The capacitance was measured directly and also by measuring the transient response of the circuit with a known resistance in it

In use of the nulling data system, the resistance of the detector circuit is varied from run to run, and account must be taken of this in computing the electrical correction to apply to the measured transfer function. At a given frequency, the correction increases as the fractional neutron amplitude decreases. The correction is therefore smallest for the large-reactivity-amplitude tests and vice versa. For the 7¢ peak-to-peak reactivity-amplitude runs, the electrical correction at 18.5 cps is about 2.3 degrees..

Because the oscillator is located on one side of the reactor and the detector on the other side, a correction must be made for the time required for the effect of the oscillator movement to be propagated across the core. A very crude preliminary calculation indicates that this correction is probably less than 1 degree at 20 cps. Another correction is required to account for the finite propagation time associated with neutron diffusion through the layer of water between the core and detector. A crude preliminary calculation indicates that this effect will be of the order of 1 degree at 20 cps for a two-inch water gap between core and detector.

Preliminary examination of the low-power data indicates that the ℓ/β values from oscillator and from transient results agree within 15% or better.

Preliminary examination of the results of the high-power tests showed a significant change from the low-power transfer function for powers above about 300 kw. At 600 kw and 60°C the suggestion of a

resonance begins to appear at about 0.1 cps. The computed feedback function has a general shape similar to those found for the EBWR,¹⁴ falling off sharply above about 0.2 cps.

Raw data records have also been taken of pile noise (with no oscillation) up to powers of 600 kw. It is planned to process these records by the auto-correlation technique.

More complete details of the measured results and their analysis must await a later meeting.

Acknowledgments

The cooperation and assistance of many members of the Spert staff in this effort is gratefully acknowledged. Specifically, appreciation is due to W. E. Nyer, S. G. Forbes, T. R. Wilson, F. L. Bentzen, A. H. Spano, J. C. Haire, and G. F. Brockett for making available facilities and assistance in carrying out the program, as well as for being available for consultation on various aspects of it. Especial thanks go to G. F. Brockett, K. E. Krauter, and R. Heffner for assistance in the design of the instrumentation; to L. A. Stephan, T. M. Quigley, H. L. Whitener, J. D. Siegwarth, R. L. Johnson, and R. W. Garner for assistance in reactor operation and data taking; to A. R. Kroeger, C. T. Howard, J. R. Brown, K. D. Jackman, and others for assistance in the assembly, maintenance, and operation of the instrumentation; and to Marie McKenzie for assistance in data reduction.

Discussion of Paper
Presented by Mr. Wasserman

MR. ZIVI:

It appeared that the little blips you have always occurred at the same half-cycle of the modulation. Is that correct ?

MR. WASSERMAN:

No, that is not correct. You can't see the dips which ran off the paper. If one could look at the complete neutron signal, one would see they occur just as much in one direction as in the other.

MR. ZIVI:

Was it the appearance of these blips that led you to the conclusion that your oscillations were hydrodynamic rather than neutron feedback in origin?

MR. WASSERMAN:

Well, there were two things. The nature of the blips was one of them. There was no evidence of any kind of overshoot, which I would have expected from that kind of a feedback process. The other thing was, as I said, the fact that raising the power level at boiling from 50 to 200 kilowatts did not produce a very drastic change in the nature of the boiling noise, except for very marked increase in the frequency of the blips. This suggested that we might have some coupling with the reactivity feedback; but this was still not the dominant effect as far as I could tell.

MR. ZIVI:

As you increased your power from 50 to 200 kilowatts, what was your inlet water temperature doing ?

MR. WASSERMAN:

It was constant because we were at boiling for the test.

MR. ZIVI:

Could you refresh my memory as to the relative dimensions between the P-core of Spert-I and the A-core of Spert-I ?

MR. WASSERMAN:

The A-core had 28 fuel assemblies of 17 fuel plates each. The P-core had only 19 fuel assemblies of 18 fuel plates each. Thus, the P-core was a smaller, more compact core.

MR. WRIGHT:

Do I understand that these tests are always made with substantial subcooling: something like 60 degrees?

MR. WASSERMAN:

No. Bulk water temperatures in the tank ranged from about 50°C to boiling.

MR. WRIGHT:

Did you do some tests at boiling?

MR. WASSERMAN:

Yes. The tests I was just discussing were boiling tests.

MR. WRIGHT:

Do the fuel elements have a stainless cladding?

MR. WASSERMAN:

Uranium dioxide.

ANALYSIS OF A REACTIVITY INSTABILITY EXPERIMENT WITH
BOILING AND NONLINEAR ANALYSIS OF SPATIAL STABILITY
AND FLUX TILT TRANSIENTS

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Analysis of a Reactivity Instability Experiment with Boiling

Two methods of analysis of boiling reactor stability have been developed and tested experimentally. One, in its simplest form, can be carried out by hand calculation and is suitable for survey studies. The other employs a large-scale computer and provides accurate results expediently. Both methods employ parametric values obtained from theory and from out-of-pile experiments. Thus, experimental data from core operation are not required for the prediction of instability.

In this brief talk, it is intended to explain the essence of these methods of analysis and to describe the results of an experiment designed to test the theory.

The first method, developed by S. G. Margolis and S. Kaplan,¹ involves the division of the core into a nonboiling section and a boiling section, along the path of coolant flow (the extension to more sections is straightforward). The boundary between the sections is fixed, and the actual, moving boiling boundary is assumed to lie inside the nonboiling section at an infinitesimal distance from the boundary of the fixed section. In the resulting analytical model it is found that the effect of a moving boiling boundary is of second order, which can therefore be neglected in any analysis concerned only with the onset of instability.

A block diagram of the reactivity feedback loop is illustrated in Fig. 1. Reactor power output is shown divided between the nonboiling and boiling sections. Reactivity feedback is derived from the average enthalpy of the boiling section; the less important average enthalpy of the nonboiling section also may be included.

Application to typical cases in which the nonboiling regions get the larger share of power reveals that the principal feedback path involves heating of the coolant in the nonboiling section and transport of the heated

¹S. G. Margolis and S. Kaplan, Transfer Functions for Boiling Reactor Stability Calculation, WAPD-T-1158 (To be published in Transactions of American Nuclear Society, Vol. 3, No. 1.)

coolant into the boiling section, where it can have a strong reactivity effect. Thus, the coolant transport delay between the two sections is a principal contributor to the phase lag of the loop.

Fig. 1

The second method of analysis makes use of a modification by S. O. Johnson, et al., to the ART digital computer transient code (by J. E. Meyer²), which allows the determination of frequency response from transient response through a Fourier transformation. The method allows as many as thirty sections axially (the direction of coolant flow) in each of five radial regions. Previously calculated power and void coefficient distributions are entered as input information. The code is used to obtain the total reactivity feedback transient resulting from a step change in core power, for constant inlet temperature. The input amplitude is adjusted to minimize nonlinear effects. The necessary transformation from transient response to frequency response is then carried out by the machine to obtain the reactivity feedback transfer function. The ordinary reactor kinetics transfer function is used to complete the loop transfer function, which is then examined for stability.

A power reactor experiment was designed and carried out to test the methods of analysis. This experiment involved the measurement of the closed loop reactor transfer function by means of a rod oscillator at a sequence of fixed operating pressures, all done at the same power level and flow rate. The pressure was reduced, increasing the boiling rate, and thus gradually approaching the predicted conditions for stability.

²J. E. Meyer, R. B. Smith, H. B. Gelbard, D. E. George, and W. D. Peterson, ART - A Program for the Treatment of Reactor Thermal Transients on the IBM-704, WAPD-TM-156 (November 1959).

The sequence of measured transfer functions revealed a resonance which grew in amplitude and approached an asymptotic frequency as the pressure was reduced. The final change in pressure resulted in a self-sustained oscillation of power, i.e., with no motion of the rod oscillator. The frequency of oscillation was 5.1 cps. The amplitude was limited to between 10% and 30% of the operating power level, but was randomly modulated at frequencies in the neighborhood of $1/2$ cps. With the amplitude so limited by the nonlinearities of the system, the oscillation was allowed to persist for approximately four hours.

The theoretical phase and amplitude curves, calculated by the two methods described, are shown in Figs. 2 and 3. Note that the phase crossover point occurs at 8 cps for the 2-section model and at 5.5 cps for the 30-section model in Fig. 2, whereas the instability actually occurred at 5.1 cps. The amplitude curves (Fig. 3) show loop gains of 1.5 and 2.5 at the critical frequencies of 8 and 5.5 cps.

Fig. 2

Phase Shift Boiling Reactor Reactivity Feedback Loop

Another calculation was done by I. Kohlberg, after the experiment, which took into account the partially depleted condition of the core in the determination of power and void coefficient distributions; the result was a phase crossover frequency of 5.9 cps and a loop gain slightly less than unity. This latest result suggests that the observed oscillation may actually have been a noise-induced excitation of a very sharp resonance.

Fig. 3
Magnitude
Boiling Reactor Reactivity Feedback Loop

More comparisons of theory with clean interpretable experiments must be done. In addition, the effect of hydrodynamics and pressure variations must be added to the theory to determine their effect on reactivity instability, and to find conditions under which hydrodynamic instability may predominate. Nevertheless, results presented here do suggest that reactivity instability in boiling reactors is predictable.

Nonlinear Analysis of Spatial Stability and Flux Tilt Transients

In reactor cores which are large in size and heavily loaded with fuel and burnable poisons, the neutron coupling between core regions is poor. This results in high flux tilt sensitivity to nonuniform perturbations and a tendency for spatial oscillations of the flux distribution. Not only are long-term xenon oscillations of concern, but short-term spatial instabilities, in which the temperature or void coefficient of reactivity and delayed neutrons may have an effect, must be considered. Furthermore, flux tilt transients in cores which are stable but poorly coupled require analysis if the core is to be subjected to nonuniform perturbations, such as unbalances in coolant flow or control rod positions.

The problems of space-time reactor kinetics may be attacked by the development of model analysis techniques, coupled region (space mesh)

techniques, or a combination of the two. Both modal and coupled-region analysis methods have been applied to the problem of xenon oscillations.³⁻⁷ More recently, Harris, Kaplan and Margolis, at Bettis Atomic Power Laboratory, have further developed and extended modal analysis methods to analyze delayed neutron effects, xenon transients, and nonlinear effects. Some results of this work have recently been reported,⁸⁻¹⁰ and additional results will be presented at the June 1960 meeting of the American Nuclear Society.^{11,12} It is intended to summarize the results of this recent work.

In cores of PWR size and larger, it has been found that the delayed neutrons cause a significant delay in the response of flux distribution to nonuniform perturbations.⁸⁻¹⁰ This effect was revealed in an analysis of a uniform bare core of highly enriched fuel and water moderator, using a simple one-energy group model and one group of delayed neutrons. Extension to two energy groups and six delayed groups has also been carried out. The analysis involves first-order perturbation theory and a modal expansion.

³J. N. Grace, M. A. Schultz, and T. E. Fairey, Inherent Reactor Stability (Fundamental Mode Xenon Oscillations), Proceedings 1955 Conference on Nuclear Engineering, U.C.L.A., May 1955, California Book Co., Ltd., Berkeley 4, California.

⁴A. G. Ward, The Problem of Flux Instability in Large Power Reactors, CRRP-657, AECL-345 (1956).

⁵A. F. Henry and J. D. Germann, Oscillations in the Power Distribution within a Reactor, Nuclear Science and Engineering, Vol. 2, No. 4 (July 1957).

⁶D. Randall and D. S. St. John, Xenon Spatial Oscillations, Nucleonics 16, No. 3 (March 1958).

⁷J. N. Grace, M. A. Schultz, Diffusion-coupled Oscillations with Xenon Reactivity Feedback, Transactions of American Nuclear Society, Vol. 1, No. 1 (June 1958).

⁸D. R. Harris, S. Kaplan, S. G. Margolis, Modal Analysis of Flux Tilt: Transients in a Uniform Reactor, Transactions of the American Nuclear Society, Vol. 2, No. 2 (November 1959).

⁹D. R. Harris, S. Kaplan, S. G. Margolis, Modal Analysis of Flux Tilt: Transients in a Nonuniform Reactor, Transactions of the American Nuclear Society, Vol. 2, No. 2 (November 1959).

¹⁰S. Kaplan, S. G. Margolis, Delayed Neutron Effects during Flux Tilt Transients, Nuclear Science and Engineering, Vol. 7, No. 3 (March 1960).

¹¹S. G. Margolis and S. Kaplan, Nonlinear Effects on Spatial Power Distribution Transients and Oscillations with Xenon Reactivity Feedback, to be published in Transactions of American Nuclear Society, Vol. 3, No. 1 (June 1960).

¹²S. Kaplan, The Property of Finality and the Analysis of Problems in Reactor Space-Time Kinetics by Various Modal Expansions, to be published in Transactions of American Nuclear Society, Vol. 3, No. 1, (June 1960).

The analysis indicates that the asymptotic amplitude of any mode of flux tilt, for a given perturbation, is inversely proportional to the difference between the eigenvalues of the fundamental mode and the mode in question; thus, the increase of the eigenvalue difference ($1/\Delta\lambda$) may be called the flux tilt sensitivity. (The eigenvalue λ may be interpreted as the k_{eff} of the mode in question.) The transient response of the time coefficient of the first overtone mode is shown in Fig. 4 for cores having different flux tilt sensitivities. Note that there is a prompt jump in the time response, followed by a delayed transient caused by delayed neutrons. The larger the flux tilt sensitivity, the more important is the delayed neutron effect. Six delayed groups were used in all cases, but the result of the model having one delayed group was included in one case for comparison. These results indicate that delayed neutron effects should be included in any short-term spatial transient or stability analysis.

Modal analysis techniques have been applied to a uniform ring reactor for investigation of spatial stability and transient response when xenon feedback is the dominant reactivity effect.¹¹ The characteristic time delays in this analysis were such that the delayed neutrons could be neglected. The fundamental and first overtone modes of the two-group diffusion operator were employed. Typical results of this analysis are shown in Figs. 5 and 6. These results revealed the increase in eigenvalue difference (increased coupling) required for stability as flux level is increased, (Fig. 5), and show the stabilizing effect of a negative temperature coefficient of reactivity on the flux tilt transient, for different flux levels and eigenvalue differences (Fig. 6).

The analysis of spatial xenon transients in a uniform ring reactor was extended to include the effect of nonlinearities.¹¹ Again, the modal expansion was in terms of the eigenfunctions of the two-group diffusion operator, including the fundamental and the two (degenerate) first overtone modes. The products of perturbations of the fundamental and first overtone variables were retained. The resulting coupled nonlinear equations were solved with a general-purpose analog computer. Solutions were obtained for different combinations of average flux level and eigenvalue difference. However, the important conclusions are revealed by the typical results shown in polar diagrams of Fig. 7. Here, the amplitude of the flux (the distance from the center of the diagram) is plotted as a function of azimuthal position in the ring reactor. Four plots are shown at successive times during a complete cycle of continuous spatial oscillation. The significant conclusions brought out here as to the effect of nonlinearities are that the amplitude of flux tilt is limited and, furthermore, that there is a definite precession of the flux around the core. This precession is attributed to coupling between the variables of the two first overtone modes and the fundamental xenon mode. These effects have been observed in the testing of the Shippingport core.

Fig. 4 Transient Flux Tilt in Annular Core Caused by Step Temperature Unbalance (490°F to 510°F)

Fig. 5. Stability Boundary for Ring Reactor

Note: Ordinate is eigenvalue difference for core including its equilibrium poisoning of xenon

Fig. 6. Xenon Transients in Uniform Ring Reactor Following Rod Unbalance for Varying Amounts of Temperature Feedback

Fig. 7. Polar Flux Plots at Successive Times during 23.3 Hour Cycle

Modal analysis involving the eigenfunctions of the diffusion operator (λ modes) is certainly not new. However, the results of the analyses described here have revealed at least two effects not previously disclosed. These are:

1. the effect of delayed neutrons on short-term flux tilt transients; and
2. the precession of the flux caused by nonlinearities during xenon oscillations.

A more complete presentation of the delayed neutron analysis appears in the March 1960 issue of Nuclear Science and Engineering; and the nonlinear analysis, including motion pictures of transients portrayed on an oscilloscope in polar form, will be presented at the Chicago ANS meeting in June 1960.

An inconvenience encountered in the use of λ modes as a basis for expansion is that the expansion coefficient (time function) of each mode usually cannot be solved independently of the others and depends on the number of terms retained in the series. For this reason, other modes of expansion have been examined. A study by S. Kaplan¹² has revealed that the desired property (called finality), where each expansion coefficient is determined independently of the others and independently of the number of terms retained, is achieved only when the modes chosen are eigenfunctions of an operator derived from the complete set of equations describing the system. For example, in the analysis of short-term space-time transients the precursor equations must be used together with the time-dependent diffusion equations to derive the operator defining the modes of expansion. If a temperature coefficient of reactivity is included, the appropriate equations must also be represented in the basic operator. Similarly, in the analysis of long-term transients the xenon and iodine equations are included in defining the modes of expansion.

When the set of modes is chosen such that the property of finality is obtained, another desirable consequence is realized. The eigenvalues of the operator (which are, in general, complex) reveal directly the transient characteristics and the stability of the system. The sign and the magnitude of the real part of each eigenvalue indicate stability or instability and the rate of convergence or divergence, while the imaginary part is the frequency of oscillation.

Modes which are eigenfunctions of an operator derived from all of the equations of the system, thus having the property of finality, we identify as the "natural" modes of the system. An analogy exists between the use of the natural modes in space-time kinetics and the familiar approach to the analysis of ordinary differential equations. A characteristic equation of a lumped parameter kinetics system is derived from the Laplace transform

of the complete set of differential equations of the system. When this is done, the roots of the characteristic equation describe the stability and transient performance of the system.

The development of means of calculating these complex modes in heterogeneous reactor configurations is a difficult task. A one-dimensional digital computer code is being prepared for the calculation of natural modes of the spatial xenon problem. The experience thus obtained will be applied to the development of two-dimensional codes.

Thus far, modal analysis techniques applied to simple geometries have provided an expedient means of understanding factors contributing to spatial stability and transient behavior, revealing such effects as that of delayed neutrons and nonlinear precession. It is not clear at this stage, however, that the ultimate design tool for the analysis of space-time kinetics in complicated cores will be based on modal techniques, since this depends on complexities yet to be encountered. The ultimate design method may involve a combination of mode calculations on a digital computer with feedback computed on the analog computer, using a continuous-time, finite-difference spatial representation. Alternatively, it may prove fruitful to use both time and space differencing for direct digital solution.

I have attempted here to give a summary of the space-time reactor kinetics work being done at Bettis Atomic Power Laboratory. I hope you will forgive my trying to cover so much material in so little time, which has limited the presentation to word descriptions. However, more complete explanations of the various parts of the work have been and will be given in the literature by the individual investigators.

Discussion of Paper
Presented by Mr. Grace

MR. LIPINSKI:

Very interesting. I have a comment to make. You implied that you tend to have faith in these predictions because of earlier experience with frequency response methods in reactor plant kinetics. I am assuming you were referring to pressurized reactors. Am I correct?

MR. GRACE:

Yes. I said we have had luck in predicting control system performance and xenon oscillations.

MR. LIPINSKI:

Primarily, reactors were pressurized to avoid the inclusion of the void phenomena in the reactor at that time?

MR. GRACE:

That is true.

MR. LIPINSKI:

So now we are trying to familiarize ourselves with the problem.

MR. GRACE:

Actually, the inclusion of bubbles is an order of magnitude more complicated than the systems we have had to analyze to date. I should point out that the hydrodynamics have yet to be included in our reactivity feedback analysis. We are confident that in this experiment we did not have any hydrodynamic oscillations. However, a truly complete analytical model would include both effects.

MR. AKCASU:

What I am going to tell you here is something that is underway at Argonne to explain the types of oscillations which are observed in boiling water reactors. We don't claim that we have found the right model, but these are speculations which are partly supported by concrete studies. First of all, it was shown by Rice that, if one has a narrow band pass filter which is excited by a white noise, the output displays sinusoidal oscillations which have a frequency that is equal to the frequency of the band pass filter.

The sinusoidal oscillations are amplitude modulated, and the modulation frequency is equal to that of the band width of the band pass filter. We know that, when the reactor power level approaches to the instability power level, the resonance peak becomes very narrow. The reactor then behaves like a narrow band pass filter which is excited by the reactor and the noise. This type of a model, the band pass filter model, does not explain the type of diverging oscillations.

We developed a model which has not been supported by experiments. We didn't have time to perform the experiments. In this model we assume the value of the resonance peak and the band width of the band pass filter. We further assume that the width of the peak and the magnitude of the band width are both random functions of time. This is a variable damping resonance circuit. If this model is adopted, one can mathematically show that, after a certain power level, instability in the mean square can exist; although the mean value of the reactor is probably stable, the mean square is not stable.

Suppose the fluctuations increased exponentially with time; this does not mean that the reactor reaches the instability power level in the classical sense. The reactor is still stable in the classical sense and only the fluctuations increase exponentially. This model will explain these types of oscillations. There are some quantitative formulas that do explain and check this model. One of them gives the expected number of maximums in a certain envelope per unit time related to a certain band width. This has been checked in an actual reactor. We used the transfer function of the EBWR and excited it by white noise. Since we knew where the resonance peak was from past oscillator tests, we counted the expected number of maximums and calculated, using the formula given by Rice, the number of maximums related to this band width.

Using the formula given by Rice, we checked this band width and obtained a good fit with the known band width of the resonance peak. Therefore, we feel that this type of oscillation will introduce into the reactor the concept of stability in the mean square. So far that is where we stand.

MR. HARRER:

Mr. Grace, I was wondering if you could give us the magnitude of the Δk that was represented by your high frequency oscillations. I think that would be interesting because that could be tied up possibly with the low frequency variation. Is that possible?

MR. GRACE:

The amplitude of the reactivity effect during the sustained oscillations, I believe, was in the neighborhood of $2 \times 10^{-4} \delta k$. Were you thinking that this might affect the modulation?

MR. HARRER:

Well, it is possible it is a very simple effect. If it were large enough, you would expect some sort of side effect.

MR. GRACE:

A beating effect caused by nonlinearity.

MR. HARRER:

You could do it with a simulator or anything that you wanted if you had large enough oscillations. That is why I wondered what size the oscillation was.

MR. GRACE:

It was quite small. I don't think it could have caused any beating of that kind.

MR. HARRER:

It has to be some fraction, like fifty cents.

MR. GRACE:

Oh, no. It was nothing like that. It was around $2 \times 10^{-4} \delta k$. We did note another thing, though, and that is that the flow distribution varied across the core, under these boiling conditions. We know we didn't have hydrodynamic chugging, but, the flow distribution as measured by the outlet flow devices indicated that it was shifting around the core at this same frequency. This may not have been related, but I would like to pursue this further.

MR. STORRER:

I would like to ask a question about your selection of modes. You said that you had λ modes, and so on. Don't you really have to take the eigenfunctions or separable solutions? I would think, for instance, your sine mode and cosine mode are not eigenfunctions.

MR. GRACE:

Right. The sine and cosine modes are eigenfunctions only in simple geometries where the Helmholtz equation applies, and those which we term λ modes are for complicated geometries. Furthermore, the eigenvalues of the overall dynamic system, which would involve the diffusion operator combined with the xenon equations, are still different. These are what we call the complex p modes, or the natural modes of the system.

MR. STORRER:

Omega modes

MR. GRACE:

The omega modes as we define them do not include the xenon equations. We call the natural modes of the system the p modes, which include all of the equations. The notation gets a little awkward. We are trying to be consistent with our terminology. I think we will keep it straight.

THE DYNAMICS OF LARGE BOILING REACTORS

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(Presented by Mr. Beckjord)

Introduction

Reactors having core dimensions which are large compared to the neutron migration length have a freedom in spatial flux distribution not present in small reactors. Spatial flux oscillations driven by xenon have been studied in detail by Ward,¹ who predicted the effect in large reactors, by Henry and Germann,² and by Randall and St. John.³ Quantitative descriptions were developed in these papers, which agree closely with oscillation thresholds and periods found in operational experience. Unfortunately, the elegant eigenvalue expansion technique used in the papers of Ward and Randall obscures the role of the operator in causing the oscillations. The purpose of this paper is to demonstrate the operator's decisive part.

First an analysis of large reactor dynamics previously presented by us⁴ will be reviewed. This analysis is based on the following assumptions:

1. Changes in important reactor variables are small during a neutron lifetime.
2. Perturbations from the steady state are small enough that second-order terms may be neglected.
3. The coolant flow is confined to channels along one dimension.
4. The rate of change of power is small compared to the total power divided by the transit time of coolant across the core.
5. The reactivity in a channel is a single-valued function of the variables affecting it, and has a nonzero slope in the vicinity of the operating point.

¹A. G. Ward, The Problem of Flux Instability in Large Power Reactors, CRRP-657 (1956).

²A. F. Henry and J. D. Germann, Oscillations in the Power Distribution within a Reactor, Nuclear Science and Engineering 2 (1957).

³D. Randall and D. S. St. John, Xenon Spatial Oscillations, Nucleonics, (March 1958).

⁴E. S. Beckjord and W. H. Harker, The Internal Dynamics of Large Reactors, ANS Conference Oral Paper, June 1959.

For simplicity, a one-energy group and one-delay group description of reactor kinetics is used, but this is readily extended.

Dynamics of the Channel

We consider first the dynamics of a single channel. The reactor usually has a negative power coefficient of reactivity, which is related to the stored heat energy in the channel. The reactivity variation, ρ , can be expressed as

$$\rho(t) = - \int_{\infty}^t K(\tau) [P(\tau) - P_0] d\tau \quad , \quad (1)$$

where K is the power coefficient of reactivity kernel, P_0 the steady-state channel power, and P the disturbed power. The reactor kinetic equations are of the form

$$\frac{dP}{dt} = \frac{1}{L} (\rho - \beta) P + \lambda C \quad (2)$$

and

$$\frac{dC}{dt} = -\lambda C + \frac{\beta}{L} P \quad , \quad (3)$$

where L is the neutron lifetime, λ the time decay constant of delayed neutrons, β the yield ratio of precursors, and C the channel power resulting from delayed neutron precursors. For linear analysis, equation (1), (2), and (3) can be combined to give

$$\left[Ls + \frac{\beta s}{s + \lambda} + P_0 K_1(s) \right] P(s) = D(s) P(s) = E(s) \quad (4)$$

where $P(s)$ is the Laplace transform of power deviation, $E(s)$ the transform of applied channel disturbances, and $D(s)$ the channel Laplace operator function.

If the roots of $D(s)$ are located in the left half of the frequency plane, the system is stable, and unstable if the roots are in the right half-plane.

Dynamics of the Large Reactor

In a large reactor, many channel sections are coupled by neutron diffusion. The equation of channel section "i" becomes

$$\left[D_i(s) + \sum_j \Gamma_{ij} \right] p_i(s) - \sum_{j \neq i} \Gamma_{ij} p_j = E_i(s) \quad , \quad (5)$$

where Γ_{ij} is the fraction of power-producing neutrons escaping from channel "i" which die in channel "j." The net leakage from channel "i" out of the core is $\Gamma_{ii} p_i(s)$.

The effect of control systems, thermal loops and reactor operators can be included in the channel equations. If the effect depends only on local power and acts locally, it appears as an additional term in the bracket which is multiplied by p_i in Equation (5). If the effect depends on the power in other channels, then additional terms in the summation of p_j appear.

The stability of this system can be studied by locating the roots of the matrix operator for all channel sections.

Xenon Oscillations

We turn now to consideration of the effects of xenon on reactor dynamics and to application of the preceding analysis. Changes in xenon concentration are described by the xenon and iodine growth equations:

$$\frac{dX}{dt} = \lambda_I I + \gamma_X \Sigma_f \phi - \lambda_X X - \sigma_X X \phi \quad (6)$$

$$\frac{dI}{dt} = \gamma_X \Sigma_f \phi - \lambda_I I \quad , \quad (7)$$

where ϕ is neutron flux, gammas are yields, sigmas are cross sections, and lambdas are decay constants. In a typical case for a flux of 3×10^{13} , the preceding equations may be integrated to give the ratio of reactivity, poisoned by xenon to small flux changes:

$$\frac{p(s)}{\phi(s)} = K_X \frac{1 - 6 T_s}{\left(1 + T_s\right) \left(1 + \frac{T_s}{4.35}\right)} \quad (8)$$

$$T_s = 3.45 \times 10^4 \text{ sec} = 9.6 \text{ hours}$$

where K_X depends upon the fission and absorption cross sections.

We will now form the D operator function for a small, one-channel section reactor. The time scale of xenon transients is so long that the reactor is essentially in a steady state insofar as delayed neutron and negative power coefficient effects are concerned. Therefore, we may write

$$\left[P_0 K_1 + P_0 K_X \frac{1 - 6 T_s}{\left(1 + T_s\right) \left(1 + \frac{T_s}{4.35}\right)} \right] p(s) = E(s) \quad . \quad (9)$$

Clearly the system is stable if $(1 + 1/4.35)K_1 - 6 K_x > 0$. This statement is equivalent to the stability condition derived by Ward. If the power coefficient K_1 is too small, the reactor operator can stabilize the system by adjustment of control rods. This action effectively makes K_1 large enough to fulfill the condition of stability.

Let us consider a large reactor, having two symmetrical channel sections coupled by neutron diffusion. First, we will examine the case of local control by reactor operator, and secondly, the case of control by total power. In local control, the reactor operator applied a correction to reactivity $P_0 K_{0p}$, which applies only in the node, p_1 , where the power deviation occurs. Hence the system is described by the equation

$$\begin{bmatrix} P_0(K_1 + K_{0p} + K_x(s)) + \Gamma_{11} + \Gamma_{12} & -\Gamma_{12} \\ -\Gamma_{12} & P_0(K_1 + K_{0p} + K_x(s)) + \Gamma_{11} + \Gamma_{12} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}. \quad (10)$$

To examine system stability, we evaluate the determinant of the matrix, which is the characteristic. The determinant is

$$\left[P_0(K_1 + K_{0p} + K_x(s)) + \Gamma_{11} + 2\Gamma_{12} \right] \left[P_0(K_1 + K_{0p} + K_x(s)) + \Gamma_{11} \right].$$

The factors are similar except that the first has both a net leakage term and a coupling term, while the second factor has only the net leakage term. Apart from leakage which tends, as a prompt negative power coefficient, to stabilize, the stability of the two-section reactor differs little from the small reactor as may be seen by inspecting the characteristic factors.

When the reactor is controlled by moving rods together in a block on the basis of a total power signal, the system stability is radically altered. The control signal in each section is proportional to p_1 and p_2 . The system is then described by the equations

$$\begin{bmatrix} P_0\left(K_1 + \frac{K_{0p}}{2} + K_x(s)\right) + \Gamma_{11} + \Gamma_{12} & -\Gamma_{12} + \frac{P_0 K_{0p}}{2} \\ -\Gamma_{12} + \frac{P_0 K_{0p}}{2} & P_0\left(K_1 + \frac{K_{0p}}{2} + K_x(s)\right) + \Gamma_{11} + \Gamma_{12} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}. \quad (11)$$

It is noted that the term K_{0p} appears both on and off the main diagonal. The characteristic factors in this case are

$$\left[P_0(K_1 + K_{0p} + K_x(s)) + \Gamma_{11} \right] \left[P_0(K_1 + K_x(s)) + \Gamma_{11} + 2\Gamma_{12} \right].$$

By inspection of these factors, it is apparent that the first is similar to the factors in the case of local regulation. The second factor, however, does not contain the K_{0p} term which has dropped out because of its placement

in the matrix. The reactor will therefore be stable if the power coefficient is large enough. If this stability condition is not met, the reactor will oscillate, because of the total power method of control. An experiment which demonstrates this behavior has been performed on a large pressurized water reactor.⁵ It is concluded that the large reactor having a threshold of xenon instability can be controlled by the reactor operator at powers above the threshold if he applies corrections locally. Spatial instability, however, is caused by the method of total power control.

Discussion of Paper
Presented by Mr. Beckjord

MR. MOREWITZ:

Local control is all very well in theory, but how do you accomplish this in practice? How do you do this in a uniformly enriched core where you have many rods and there are many, many local situations? The external instrumentation to the core isn't a really adequate guide to an operator to permit him to program the rods locally in any intelligible way.

MR. BECKJORD:

Well, the Dresden reactor, I believe, is the first large power reactor which has effective local monitoring. There are 64 ion chambers in the core spread around.

MR. MOREWITZ:

Actually in the core?

MR. BECKJORD:

In the core. There are four layers of sixteen each. Effectively you do look at local power disturbances. They were put in because of the possibility, that was envisioned several years ago, of a spatial flux instability

⁵WAPD-BT-12, April 1959, Shippingport International Power Operation Test, p. 181.

in the Dresden reactor due to hydrodynamic instability or something of this sort; therefore, local monitoring was put in and at great expense.

Also, I would like to say that recent tests have been completed showing that there is no evidence in the reactor of a spatial flux instability. It is a very solidly behaving core. Getting back to the question, this is one way you could do it.

MR. MORRISSEY:

I would like to comment on Mr. Morewitz' comment. It is kind of a circular situation. The fellows who design instruments say why should we design instruments any better? I just came from a two-day instrumentation meeting, and found that three power reactors were being built without a single piece of in-core instrumentation in them. The reason for this is that the instrument designers don't believe you need them any more. I think more work like Mr. Beckjord's would result in a push in this development.

MR. BECKJORD:

I would say that core ion chambers are not easy to build. This is the reason.

MR. LIPINSKI:

It is really not a question of the instrument people deciding they don't need the in-core ion chambers. It is the people who want to make the measurements. It is a question of supply and demand. If there is a demand, there will be a supply.

MR. THOMPSON:

It seems to me that when you gradually, say, lose reactivity in your reactor, you are going to have to pull some sort of control rods. You said that this type of control would lead to a tilt. What you are really doing is pulling a local rod which starts the tilt, and then you pull another rod somewhere else. Therefore, what you are really doing is pushing on the edge of the top which is tilting a little and trying to keep it from tilting too much. You always have a tilt. You create a tilt by removing one rod. You are almost forced to do it by your rules. After you once create the tilt, you have to knock it down. The operator is continually playing the game with the reactor. Isn't that what it amounts to?

MR. BECKJORD:

When this experiment was performed, they had to achieve, in order to run the experiment, an initial tilt. The point I want to make is that to begin with, the reactor must have had enough xenon in it for it to be unstable. Then when the tilt was put in, it must have been handled locally. From the reactor flux trace, you see it steadied out rather nicely using local control.

It was only after the block control was used that you achieved these beautiful out-of-phase oscillations.

MR. THOMPSON:

You are forced to create a tilt and then forced to take it out again. Isn't it a series of operations like that in time?

MR. BECKJORD:

Well, if you were operating with enough xenon to overcome the negative coefficient, it is very likely.

MR. THOMPSON:

Thank you.

MR. LIPINSKI:

I would like to make a comment. EBWR is having a very extensive in-core program which will enable us to measure fuel temperatures. Some of the phenomena that we note on the first go-around will be covered in a second experimental investigation.

We will have better data with regard to what is happening internally in that reactor. Most of our measurements previously were external.

MR. GRACE:

I think that in-core instrumentation that you propose to do in experimental and research reactors is a good thing for investigating this phenomena. I want to make sure Mr. Beckjord would not propose allowing a design to be built which is naturally unstable spatially, and depend on local control to keep the thing stable. You would rather design the system such that it is absolutely stable spatially rather than depend on local control.

MR. BECKJORD:

Yes. We started out on this coupled reactor business in order to investigate the possibility of spatial instabilities in the Dresden reactor. In working with the Dresden reactor, we thought that we could show that the operator was both part and parcel in causing reactor instability. We thought that this would be a very nice experiment to explain, perhaps not in all the variations of it, but at least the basic instability.

MR. GRACE:

How do you get the coefficients? This is the sort of coupled region method I was referring to in my paper. There always exists this question of how do you calculate the coefficient. Do you use a simple two-region model?

MR. BECKJORD:

In this particular case, actually, the coupling coefficients for the simple two-region job turns out to be not critical because they do not enter into the instability calculations unless they were very large. If the coupling coefficients were very large, they would tend to stabilize because the diffusion process acts as a prompt negative power coefficient. But in a practical case, where you are considering more nodes, you would calculate the coupling coefficients just by taking the numerical approximation of the diffusion operator.

The coefficients are calculated by using the Laplace operator. This is the way we have used to calculate them.

MR. GRACE:

Do you integrate the current over the boundary between the two regions and get your coefficient by taking the average flux difference between the two?

MR. BECKJORD:

No, to date we have not done it. We have just taken a rough five-point difference. A nine-point difference has also been used.

DIGITAL CALCULATIONS OF SPATIAL STABILITY RELATED TO PWR

P. F. Lacy, WAPD (Not available for publication)

v

v

v

SESSION III

FAST REACTOR MEASUREMENTS
AND ANALYSIS OF STABILITY

Chairman: D. Okrent

Secretary: W. B. Loewenstein

OSCILLATOR TESTS IN BRITISH FAST REACTORS

A. R. Baker
A.E.R.E.
Harwell, England

Introduction

Oscillator experiments are performed in power reactors in order to investigate their stability. Such experiments give information about the various contributions to the power coefficient of reactivity and the delay times associated with them. The oscillator experiments performed in the zero-power fast reactors, Zeus and Zephyr, are described in this paper. Brief mention is made of the experiments proposed for the 60-Mw Dounreay Fast Reactor. A description of an experimental to measure the Doppler effect is also included, because it required the same type of equipment as oscillator tests of stability and because the Doppler effect, if large, would have been an important contributor to the power coefficient.

Zeus Oscillator Experiments

If a reactor, initially operating at a steady power level, is subjected to a sinusoidal change in reactivity, the power will oscillate sinusoidally about the steady level. The amplitude and phase of the power oscillation produced by a given amplitude of reactivity oscillation depend on the frequency. The complex function of frequency relating power oscillation to reactivity oscillation is called the transfer function.

A transfer function analyser was built and tested in Zeus Core 3A. Zeus was a zero-power mockup of the Dounreay Fast Reactor. This work had two purposes:

1. To test the principles of the design of an analyser for Dounreay and to test the technique of using it.
2. To measure the zero-power transfer function in Zeus for comparison with the zero-power transfer function for Dounreay. The comparison will test the accuracy of both analysers.

A sinusoidal oscillation in reactivity was produced in Zeus by rotating a half-cylinder of U^{235} in the core. The power level of the reactor was measured by means of a large fission chamber used as a current chamber. The dc component of the current, proportional to the steady power level, was first backed off. The amplitude and phase of the remaining ac component was measured with the transfer function analyser.

Rotary Oscillator

The sinusoidal variations of reactivity were produced by rotating an asymmetrical section of a fuel element inside one of the standard fuel element channels. The channel chosen was 6 in. from center, which was close to the point of maximum flux gradient. The rotor was just over 20 in. long and consisted of a brass tube on a concentric shaft of $\frac{3}{8}$ -in. mild steel. Half the annular space between tube and shaft was loaded with fuel pellets (45% U^{235}), cut in half along a plane through the axis of cylindrical symmetry. The other half of the space was filled with similar pieces of natural uranium to act as counterbalance. The enriched and natural uranium were kept in alignment by two longitudinal aluminum vanes let into slots in the shaft. The completed rotor was supported inside a guide tube by bearings level with the two core matrix plates just above and just below the central section of the core. The rotor was turned by means of a motor and a gearbox mounted on the movable concrete shields above the reactor.

The motor was a Velodyne (type 74) operated over a speed range from 20 to 2000 rpm. The speed was reduced by a permanently engaged worm reduction gear and further by an epicyclic reduction gear. A manually operated spur gear change took off the drive either before or after the second reduction gear. In high gear, the rotor drive covered the range from 0.03 to 3 cycles per second; in low gear, slower than 0.04 cycle per second.

Electronics

The electronic equipment used in Zeus was similar to that built subsequently for the Dounreay reactor. An account of the Dounreay equipment has been published by Harrison.¹ The Zeus electronic apparatus was used later in Zephyr to measure the prompt neutron lifetime. Only minor modifications were made to the apparatus for the experiments on Zephyr, but the high-frequency limit was extended to 50 cycles per second.

A block diagram of the electronics used in Zeus is given in Fig. 1. A large fission chamber² containing about 10 g of U^{235} was used to provide a current proportional to reactor power. This chamber was mounted in the movable graphite shield above the reactor. When the reactor was operating at about 50 w, the mean ionisation current in the chamber was $2 \mu a$ and the

¹Harrison, D., A Transfer Function Analyser for the Dounreay Fast Reactor, AERE-RIC2/6 (1957).

²Aves, R., Baker, A. R., Barnes, D., and Smith, R. D., A Large Fission Chamber Designed for Use as a Current Chamber, AERE-R/M 119 (1957).

amplitude of the oscillating components was 1 to 10% of this. The current signal was converted to a voltage signal, about 50 v, by means of an analogue-computer amplifier (type 1469A). The steady dc component was backed off by means of an adjustable stable voltage. Then, after further amplification of the alternating component, an amplifier, of gain minus one, was used to produce a second output 180° out of phase with the first.

Fig. 1. Block Diagram for Transfer Function Analyser

These two outputs formed the two inputs to the transfer function analyser. The amplitude and phase of a sinusoidal voltage of low frequency can be measured with the aid of sine and cosine resolver potentiometers, but at 3 cycles per second such resolvers have a limited life and at 50 cycles per second they are impracticable. An analyser that did not use sine potentiometers was therefore constructed from units of a mechanical reference generator made by Solartron Ltd. This was comprised mainly of a high-frequency generator, modified to operate at 2 kilocycles per second, modulated by the signal from the reactor. There were two separate synchronous detector circuits.

The balanced modulator gave a substantially carrier-free side-band output. This output was fed to a synchro-resolver (Smith's type S. 11). The resolver, driven by a Magslip receiver mechanically connected to the

reactivity rotor, in effect multiplied the modulated signal by $\sin \theta$ and $\cos \theta$, where θ was the attitude of the rotor. The two outputs from the resolver were separately detected by the Solartron unit.

The final detected outputs contained direct components proportional to the in-phase and quadrature components of the reactor power oscillations. At high frequencies, these signals could be registered directly on meters. At low frequencies, the signals were integrated over a whole number of cycles by further computer circuits. The start and stop of integration was controlled by interlock relays operated by a cam on the resolver shaft. This same cam was used as a revolution counter and also operated a solenoid-controlled stopwatch for accurate speed measurements.

The meters of the two integrators were calibrated in terms of the percentage modulation of power by means of a sinusoidal test signal. This signal was produced by a reference generator (Smith's type HM21/1) directly geared to the resolver. It will be shown later that day-to-day variations in the measured amplitudes for a given reactivity oscillation were due to the inadequacy of this calibration system.

Experimental method

Static calibration of the effectiveness of the rotor was carried out in Core 3A. The rotor was turned, manually, to 16 different attitudes, as indicated by a protractor temporarily mounted on the drive shaft, and the reactor kept at constant power by adjusting one control rod. The sequence of control rod settings was analysed and could be represented by $0.453 \cos \theta - 0.0085 \cos 2 \theta \pm 0.0045$ standard cm of control rod. This showed that the oscillations to be imposed would be adequately sinusoidal. Subsequent absolute calibration of the control rods permitted conversion of these figures, the total swing of reactivity during one revolution being 1.5×10^{-4} , peak-to-peak.

During this test, the effective maximum and minimum perturbation attitudes of the rotor were determined. Whilst working near the midpoint, with the maximum rate of change of reactivity with angle, the reactor was used as a sensitive detector of back-lash in the rotor drive. Such back-lash amounted to rather less than 1° . Phase errors due to incorrect alignment of parts were investigated by reversing the sense of rotation.

Any given series of transfer function measurements was always started with the rotor spinning at a moderate speed, about $\frac{1}{2}$ to 1 cycle per second, and the reactor brought up to a steady power of about 50 w, with the Ra-Be source removed. This enabled the operator to steady the mean power level without interference from the oscillations. Thereafter, the operator's task was to maintain the same level, any adjustment of control rod position being the minimum necessary and made, preferably, between

runs on the analyser. The criterion of steadiness was that the mean level did not drift more than about 1% during any run. A run consisted either of many cycles at high speed lasting 1 or 2 minutes or even of only one cycle lasting several minutes. To assist the operator, a chart recorder displayed a backed-off and amplified trace of reactor power such that 10% variation covered full scale. Readings were taken first at a selection of high speeds and were then taken at progressively slower speeds. It was during the latter stage that the operational difficulties set a practical limit to the length of cycles that could successfully be observed. Very few measurements were made at periods longer than 500 seconds, despite the excellent stability of Zeus.

At each frequency, several readings were taken of both components of the response, similar readings being taken with the direction of rotation reversed. To avoid transient disturbance of the power level, such reversals were always made when the rotor was, as near as could be judged, in a position of extreme perturbation effect. Changes of speed were also made at the position of maximum or minimum perturbation effect.

Whenever the rotor had to be stopped it was found advisable, especially at slow speeds (where the oscillating power has a phase lag of 90° with respect to the reactivity oscillation), to interrupt the steady rotation at one extreme of rotor position and then motor it rapidly through 90° before bringing it to rest in a mean perturbation position. If this precaution were not taken, the power level either had an offset of up to 10%, which had to be corrected, or had a drift with a time constant of about 20 minutes, which also had to be compensated by adjusting the control rod.

The inverse procedure of motoring out rapidly from midposition to an extreme, in order to start a slow run, was never successfully achieved.

Results

The response of the reactor to a sinusoidal reactivity change was measured on several different days over a range of frequencies from 0.002 to 3 cycles per second. Long-term drifts were encountered in the experimental equipment, which gave rise to discrepancies of up to 10% in apparent amplitudes from day to day. Over shorter periods (half-day sessions), results at all frequencies were self-consistent and reproducible to within about 3% in amplitude and 2° in phase angle.

The long-term drift was thought to be due to variation in the gain of the head and main amplifiers. The calibration system only tested the gain after the main amplifier. (This fault has been rectified in the apparatus built for Dounreay reactor, which should have an accuracy much better than the specification: accuracy of 5% in amplitude, and 5° in phase). To avoid the uncertainty in the calibration, all the results in Fig. 2 have been

normalised to unit amplitude for fast oscillation. The theoretical curve, also normalised to unity for fast oscillations, was calculated from the delayed neutron data recommended for Dounreay. This theoretical curve is shown in polar form in Fig. 2.

Fig. 2. Reciprocal Zero-Power Transfer Function

The principles of the design of the transfer function analyser built for Dounreay and the experimental method were both found satisfactory in Zeus. The zero-power transfer function for the Dounreay Fast Reactor should be similar to that for Zeus Core 3A, given in Fig. 3.

Fig. 3. Transfer Function. Experimental Points, Core 3A;
Calculated Curves, Mean of all Cores or D.F.R.

Zephyr Oscillator Experiments

The zero-power fast reactor Zephyr had a cylindrical core, 15 cm high and 15 cm in diameter, containing about 15 kg of plutonium. The core was located at the center of a hexagonal pillar of natural uranium extending through the full 91 cm height of the reactor and having a mean radius of 11.6 cm. The main blanket (outer radius of 45 cm) of the system used during the oscillator tests consisted of graphite, instead of the uranium of the original version of the reactor. Experimental results for the graphite version are given in a paper by Smith and Sanders.³

³Smith, R. D., and Sanders, J. E., Experimental Work with Zero-energy Fast Reactors. Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva (1958), Vol. 12, p. 89.

The mean prompt neutron lifetime for this mixed fast-thermal system was about 10^{-5} sec. It can be seen from Fig. 2 of a paper by Corben⁴ that an increase of lifetime from zero to 10^{-5} sec changes the phase lag of the transfer function at a frequency of 50 cps from 0° to 30° . The corresponding change in amplitude, from Corben's Fig. 1, is a few per cent. It is evident that a measurement of the phase of the transfer function near 50 cps will yield an accurate estimate of the prompt lifetime of the graphite-blanketed Zephyr.

The transfer function analyser used for the lifetime measurement was the same equipment as used on Zeus. In this case the sinusoidal reactivity change was produced by rotating an eccentric cadmium vane past a stationary semicircular vane, also of cadmium. (The arrangement was similar to that of the tuning condenser in a radio set.) The rotor assembly was situated in the graphite blanket.

The phase lag was measured as a function of oscillating frequency in the range from 5 to 50 cps. The value obtained for the prompt neutron lifetime was given by

$$T_0 = 6.7 (1 \pm 0.1) \times 10^{-6} \text{ sec,}$$

in agreement with an independent method. A full description of these experiments has been published by Ingram, Mc Culloch and Sanders.⁵

Doppler Measurement in Zeus

The measurement of the Doppler effect in Zeus is included in this paper because this effect might give rise to a positive prompt power coefficient in future power-producing fast reactors and also because the electronic equipment used was similar to that required for oscillator tests. A full report has been published by Baker and Jaques⁶ and the experiments are described only briefly here.

The Doppler effects in U^{235} , U^{238} and iron were studied by observing the difference between the reactivities controlled by hot and cold (similar) samples at the center of Core 3B. It was necessary to use an oscillator method for measuring the small change in the presence of reactor drift

⁴Corben, H. C., Analysis of the Zero-power Reactor Transfer Function, Nuclear Science and Engineering, 6, 461 (1959).

⁵Ingram, G., Mc Culloch, D. B., and Sanders, J. E., Neutron Lifetime Measurements in the Fast Reactors Zeus and Zephyr, J. Nuclear Engineering and Reactor Science, 10, 22 (1959).

⁶Baker, A. R., and Jaques, T. A. J., A Measurement of the Contribution of the Doppler Effect to the Temperature Coefficient of Reactivity in a Fast Reactor, AERE R/M 168 (1958).

and noise. Small changes, attributed to the Doppler effect were observed in U^{235} and U^{238} of the same order of magnitude as the noise, but these were not great enough to lead to a serious prompt power coefficient in a fast reactor.

Experimental Method

Two samples of similar size were mounted eighteen inches apart inside a hollow drive rod, which was oscillated vertically in the central channel. The drive rod was attached to a pneumatically operated piston of 18-in. stroke. When the piston was up, the lower sample was at the center of the reactor; when down, the upper sample was at the center. When this drive rod was made to execute square-wave oscillations of 60-sec period, fluctuations in the reactor power of the same period were observed. This periodic fluctuation with both samples cold was due to the drive rod itself and the slight difference in mass between the two samples; the change in amplitude on heating one sample was a measure of the Doppler effect.

The reactor power was measured by means of the large fission chamber² containing about 10 g of U^{235} coated on 100 plates situated in the upper graphite shield of Zeus.

An output signal proportional to the amplitude of the power oscillation was obtained by means of the electronic equipment described below. Two series of readings were obtained, one series with both samples cold and the other with the upper one hot. The shift in the mean was a combination of the Doppler effect and the effect of displacing the sample by thermal expansion of the drive rod. A separate experiment was performed to calibrate the output signal of the integrator in terms of reactivity. In order to obtain the effect of heating, the integrator reading with the sample hot was compared with the mean of readings for the cold condition before and after.

The shift in the mean for the empty drive rod was subtracted from the other mean shifts to give the effect of the samples alone. The principal cause of the shift on heating was the expansion of the apparatus (which led to displacement of the samples). The correction for thermal expansion was estimated from measured thermal expansions and perturbation data.

Electronics

A block diagram for the electronics is given in Fig. 4. The cyclic operation of the whole system depended on a crystal oscillator producing at its output 50 pulses per second. In the control box, the pulse rate from the oscillator was first divided by 3, using a Miller paralysis circuit. Pulses, at $16\frac{2}{3}$ per sec, were then fed to three scales-of-ten in series. Three thousand pulses from the crystal oscillator (i.e., 60 sec) produced

a complete cycle of the system. Trigger pulses, derived from the final two scales-of-ten, were used to control the pneumatics and the integrator. (Events could be initiated by the control box at any multiple of 1% of the period.)

Fig. 4. Block Diagram for Electronics of Doppler Experiment

The current from the fission chamber measuring reactor power was first backed off approximately. The remaining current was amplified using a dc amplifier. A recorder with center-zero connected to the output of this amplifier was used to keep the reactor power steady during the experiment. Full-scale deflection of this recorder was about 2% of the mean power.

A final output, proportional to the amplitude of oscillation of reactor power, was obtained by means of two additional dc amplifiers and a change-over switching circuit. The output of the first amplifier was connected to the input of a second similar one of gain minus one. The outputs of these two amplifiers were therefore equal in magnitude but opposite in sign. A changeover switch, actuated by the control box, connected the input of the third amplifier, arranged as an integrator, to these two outputs alternately for half a period. This operation is equivalent to multiplication of the chamber current by a square wave, thereby eliminating the nonperiodic components of reactor power.

The amplitude of the integrator output depended on the phase difference between the trigger pulses to the solenoid-operated pneumatic valves and to the electronic switching circuit. The advance in phase of the change-over pulse with respect to the pneumatics pulse was variable in units of 1% of the oscillator period. The phase advance was adjusted to give the maximum signal.

Conclusions

The experimental results for U^{235} are consistent with zero Doppler effect. For natural uranium, a small negative temperature coefficient was observed, corresponding to increased capture in U^{238} with increasing temperature, given by

$$d\sigma_c/dT = 0.0006 \pm 0.0002 \text{ mb/}^\circ\text{C}.$$

The estimated Doppler temperature coefficient for the whole reactor Zeus was zero within experimental error. The accuracy of this estimate is limited by the accuracy of the estimate of the effect of thermal expansion of the apparatus. An upper limit can, however, be placed on the magnitude of the Doppler coefficient (which might be positive or negative) of 10×10^{-7} per $^\circ\text{C}$. This limit is somewhat less than that quoted by Kato and Butler (16×10^{-7})⁷ after performing a similar experiment in ZPR-III at the Argonne National Laboratory.

Oscillator Tests in the Dounreay Fast Reactor

The oscillator equipment installed at Dounreay is a development of the equipment tested on Zeus and Zephyr. The design was simplified in several ways in order to reduce the number of setting-up and operating adjustments and to reduce drifts. One such simplification was made possible by the reduced frequency range specified for the Dounreay equipment, 0.002 to 3 cps instead of 0.001 to 50 cps. The carrier frequency was reduced from 2000 cps to 50 cps, and it became possible to use synchronous choppers as modulator and demodulator. A boron-10 rotor is installed in place of the U^{235} rotor used in Zeus, to reduce the cooling problem in a power reactor. A full description of the Dounreay equipment has been given by Harrison.¹

The Dounreay oscillator equipment will be used to check the stability of the reactor by the methods described by Bethe.⁸ The transfer function will be measured at essentially zero power and then at successively higher powers. At every stage it will be possible to predict a range of higher powers throughout which instability will not occur.

⁷Frost, R. T., Kato, W. Y., and Butler, D. K., Measurement of Doppler Temperature Coefficient in Intermediate and Fast Assemblies. Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva (1958), Vol. 12, p. 79.

⁸Bethe, H. A., Reactor Safety and Oscillation Tests APDA-117 (1957).

In such a test the oscillating component of reactor power is related (by means of the zero-power transfer function) to an oscillating reactivity change which is the sum of two effects, the impressed reactivity and the feed-back component. The oscillating power itself contributes this feed-back component because of the existence of a power coefficient of reactivity. The power coefficient of the Dounreay Fast Reactor can be ascribed to five effects:

1. axial expansion of fuel elements,
2. bowing of fuel elements in a temperature gradient,
3. expulsion of coolant from the core due to coolant expansion,
4. radial movement of fuel elements due to expansion of the lower support plate, and
5. blanket effects (which may be further subdivided as 1-4).

The first two effects are approximately simultaneous and follow the power variation with only a small delay time. The other effects are listed in order of increasing delay time.

The various components of the power coefficient have been estimated theoretically, and the behavior of the reactor in various types of accident has been studied using an analogue computer. The results of the oscillator tests will be used to check the model of the reactor used in these safety tests.

The bowing of fuel elements is of great importance because it is a contributor to the power coefficient which is both positive and prompt. In order to reduce bowing, a central core structure, called the tube-nest, was introduced into the design. Each fuel element channel in the tube-nest is somewhat farther from the core axis than the corresponding holes in the upper and lower support plates. The fuel elements are therefore bowed outwards initially by this arrangement, and no inward bowing should be possible. Because of engineering tolerances on components, slight bowing of some elements may be possible. The measurement of the transfer function at power should establish whether the tube-nest is fulfilling its function.

So far the Dounreay Fast Reactor has been operated at powers up to 1 kw and it has been possible only to measure the zero-power transfer function.

Acknowledgements

The Zeus oscillator tests were performed by D. D. McVicar; the Zephyr oscillator tests by G. Ingram, D. B. McCulloch and J. E. Sanders. The oscillator was designed by R. B. Stephens; the modified version for Dounreay by D. Harrison. The Doppler experiment was performed by T. A. J. Jaques and the author.

Discussion of Paper Presented by Mr. Baker

MR. WASSERMAN:

What, in the synchro system that was used, do you know limited it to only 50 cycles per second; or was it in the mechanical system associated with the rotor that prevented it from going higher in Zephyr? Do you happen to know why they couldn't get over 50?

MR. BAKER:

As I said earlier, it was Sanders, Ingram, and McCulloch's experiment, but my guess is that the synchro would be able to go faster, but the home-made rotor system was the limiting factor.

PROGRESS IN FREQUENCY RESPONSE MEASUREMENTS AT LOS ALAMOS

Part I

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The topics that will be covered here will include only the results of zero-power transfer function measurements made at Los Alamos for a number of different reactor assemblies. The results, some of the measurement techniques, and some of the problems involved in making the measurements will be discussed.

The zero-power transfer function may be determined analytically from the differential equations describing the prompt and delayed neutrons. These differential equations contain fundamental delayed neutron data and a parameter described as the neutron lifetime.

Figures 1 and 2 give the theoretical results for the one-point transfer function as determined from the six-group delayed neutron data measured by Keepin. These figures are the same ones that appeared in *Nucleonics*.¹ The amplitude and phase quantities are presented as a function of reactor lifetime, and represent analytic solutions to the reactor kinetic equations for point geometry. The graphical points shown in the figures are not experimental points, but merely represent the computational plotting points.

Fig. 1

¹Keepin, G. R., and Wimett, T. F., Reactor Kinetic Functions: A New Evaluation *Nucleonics* 16 (10), 86 (October 1958).

Fig. 2

To confirm these analytic results, a number of experimental measurements over a limited range of frequencies have been made. Figures 3 and 4 are essentially reproductions of the first two figures, but in this case a number of experimental points for zero-power systems have been superimposed upon the theoretical curves. The ANL data have been published previously; the data for the Little Eva assembly (a U^{235} core surrounded by a U^{238} reflector) and the Kiwi-A assembly (a system containing enriched uranium and having a moderately long lifetime of 2×10^{-4} sec) are presented here for the first time. The data from the Water Boiler are buried in old LASL reports.² The data seem to be in agreement with the

Fig. 3

²deHoffman, F, Criticality of the Water Boiler, Number of Delayed Neutrons and Dispersion of the Neutron Emission per Fission, LA-183 (1944)

Fig. 4

one-point model described by the inhour equation, at least within the experimental scatter. Similar zero-power measurements made for a few plutonium systems will be presented below.

For the Little Eva and Kiwi-A assemblies, the experimental determination of the zero-power transfer function was accomplished by means of a similar technique. A calibrated control rod was oscillated in a linear fashion by means of a remote hydraulic servo supplied by a command signal derived from a commercial "servo-scope." The output neutron signal was detected with a boron-lined chamber, amplified, and compared to the input signal through a visual oscilloscope display system. The "servo-scope" gave an arbitrary phase shift to the input signal, which allowed the phase to be measured by a conventional modulated null technique. The frequency range of these measurements was limited to the range from 0.1 to 4.0 cps.

A number of essentially zero-power studies have been made on small, well-reflected plutonium cores. In these experiments rotational control rod motions were used instead of linear motions. As the core sizes were relatively small (6 x 6-in. cylinders), the flux distribution generally was not linear over the volume occupied by the oscillator control element. In all cases the control element was located outside the core in the reflector with approximately two inches of reflector between the core surface and the edge of the controlled element; the oscillator rods were located symmetrically with respect to the core midplane, however.

Due to the nonlinear fall-off of flux with position, all rotated oscillator rods contributed appreciable amounts of higher harmonics to the fundamental reactivity input frequency. The first oscillator rod consisted

of an eccentrically rotated rod; the second rod consisted of two eccentrically rotated holes and allowed a double reactivity frequency for a given rotational frequency; the third rod consisted of a cylinder with symmetrically milled flats, which gave both the double frequency advantage and allowed a dynamically balanced rod to be constructed. The results of the reactivity calibrations for the three rods mentioned above are given in Fig. 5. Note that the per cent of second harmonic content seems to be proportional to the total reactivity amplitude.

Fig. 5

The influence of the higher harmonic content of the exciting signal (reactivity) may be determined, in principle, by analytic techniques. Practically, it would be best if the harmonic content of the input signal was low. If the quadrature method of analyzing the frequency response of the reactor system is used, then the even harmonics are eliminated and the odd harmonics are suppressed. The angular reactivity as a function of rotational position as actually measured for the oscillator rod with symmetrically milled flats is given in Fig. 6.

For the wide-band, fast reactor systems, mechanical motions with an upper frequency input limit of, say, 100 cps barely cover the midband frequency range at zero power. A noise analysis may be made to gain some insight into the higher frequency region of the assembly transfer. Such a technique was used to cover the frequency region from 30 cps to 30 kcps for one of the small plutonium systems. The amplitude modulus of the

Fig. 6

zero-power transfer function was flat within this region. However, at zero power, this may be a negative result, as random noise would yield a similar flat-with-frequency amplitude distribution.

Boron-coated ion chambers and large scintillator current devices were used as flux detectors, and for low level work medium bandwidth amplifiers (1 kc on the most sensitive ranges used) were used to boost the current amplitudes to useable levels. For the fast reactor systems with a very wide inherent bandwidth, noise becomes a serious problem at low power levels. Using detection systems and amplifiers of a few kc bandwidth, signal-to-noise ratios of the order of 1 in 10 were observed when low pass filters were not used. When these signals and noise were fed into an integrating quadrature device of a few kc bandwidth, the noise and higher harmonics were satisfactorily rejected, without the use of low pass filters.

The quadrature technique and the results obtained by it for the small plutonium systems deserves further mention. The technique and machines will be discussed by its designer, Mr. E. O. Swickard.

PROGRESS IN FREQUENCY RESPONSE MEASUREMENTS AT LOS ALAMOS

Part II

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The following is a description of a reactor oscillator being used at Los Alamos to determine zero-power transfer functions of plutonium systems. The purpose of the analyzer is to apply a small, sinusoidally varying δk of known magnitude to a reactor and measure the amplitude of N_1/N_0 and the phase angle between δk and δn ; here N_0 is the average neutron level, and $\delta n = N_1 \sin(\omega t - \alpha)$. The frequency range over which the analyzer was designed to operate is from 0.01 to 10 cycles per second, a range including the frequencies where instabilities could be expected. Figure 1 is a schematic diagram of the analyzer.

Fig. 1 Reactor Transfer Function Analyzer

A rotating oscillator rod supplies the sinusoidally varying δk because of its asymmetry about the axis of rotation. Several rod geometries have been tried by experimenters to achieve sinusoidal reactivity disturbances. The rod used in test of this analyzer will be described later.

The motor and variable speed reducer shown in Fig. 1 is a Graham transmission unit. The motor is a $\frac{1}{3}$ -horsepower, 1800-rpm synchronous type. The output speed of the reducer may be varied from 0 to 650 rpm. Prior to incorporation into the equipment, output speed drift over the

operating range was checked and found to be less than 0.2%. A small electric motor driving the speed-changing mechanism permits changes in speed to be made remotely. The reducer is connected to the oscillator rod and timing mechanism through a 6:1 remotely operated gear box and a right-angle drive. Light sources, a slotted disc and photo diodes generate pulses at certain rod positions and the pulses are fed to the computer circuit.

An ionization chamber in the neutron flux of the reactor delivers a current to the computer circuit, which is proportional to the neutron flux and hence to power level. The chamber used was of the boron-coated variety. A gamma-ray scintillator was tried briefly and appeared to give satisfactory results.

The computer circuit operates on its input signals and displays on two output voltmeters values from which the relation of $\delta n/N_0$ to δk may be readily obtained.

The five amplifiers shown in the schematic diagram Fig. 2 are high-gain operational amplifiers. The particular ones used are George A. Philbrick Research model USA-3. They have an open loop gain of about 10^7 .

Fig. 2

In Fig. 2 the chamber current, i_n , is made up of components given by the following equation:

$$i_n = I_0 + I_1 \sin(\omega t - \alpha) + \text{noise} \quad , \quad (1)$$

where

I_0 = current from the average neutron flux

I_1 = amplitude of the sinusoidally varying current

ω = frequency of oscillation

α = lag angle of neutron flux sinusoid relative to input reactivity sinusoid

and "noise" is the random fluctuation of current from any cause but primarily from the statistical variation of power level.

R_0 is adjusted so that the component of e_1 due to I_0 is -5 volts. The explanation for the particular value of -5 volts is made when the operation of A_2 amplifier is described. Proper adjustment of R_0 is made by changing R_0 until oscillations of the null meter are centered about the meter null. As will be shown, very precise adjustment of R_0 is not required.

$$I_0 R_0 = -(-5)$$

$$R_0 = 5/I_0 \quad (2)$$

$$e_1 = -i_n R_0 \quad (3)$$

Substituting Equations (1) and (2) into Equation (3) gives

$$e_1 = -\frac{5}{I_0} [I_0 + I_1 \sin(\omega t - \alpha) + \text{"noise"}] = -5 - \frac{5I_1}{I_0} \sin(\omega t - \alpha) \quad (4)$$

In the last part of Equation (4) the noise term has been dropped, for its contribution to the computer output is very small. The reason that noise can be neglected is because of later integration of the signal. The integral of truly random noise over a long time is zero; the integral over a short time might not be exactly zero, but will be small. Conceivably it might be necessary to rerun a point that lay outside the expected range because of the small probability that there is correlation between noise and ω .

A_2 is a summing amplifier and is used to buck out the I_0 component of the input signal. The cancelling is done by changing the gain of the input amplifier, A_1 , (by changing R_0) until $10 I_0 R_0 = 50$ volts. The figure of 50 volts was arbitrarily chosen as a comfortable point at which to operate an amplifier with a maximum allowable input voltage of 100 volts. Since the 50-volt reference input resistor to A_2 is 10 times the input resistor of e_1 , then e_1 should be -5 volts for cancellation.

$$e_3 = - \left[\frac{\pi}{0.1} e_1 + \frac{\pi}{1}(50) \right] \quad (5)$$

where resistances are in megohms. Substituting the expression for e_1 in Equation (5) gives

$$e_3 = \pi \left[50 + \frac{50 I_1}{I_0} \sin(\omega t - \alpha) - 50 \right] = \frac{50\pi I_1}{I_0} \sin(\omega t - \alpha) \quad (6)$$

The reason for the factor of π in the A_2 feedback resistor is that subsequent integration introduces a $1/\pi$ factor in the results, and by inserting π in the gain along the way, the $1/\pi$ is cancelled. The factor π was eliminated from the final output in order to simplify the equations used to calculate I_1/I_0 and α .

$$e_4 = -e_3 \quad . \quad (7)$$

After commutation, e_3 and e_4 are fed into the integrating amplifiers A_4 and A_5 . By proper selection of the commutation scheme, the outputs of A_4 and A_5 are voltages proportional to (time in seconds) $\times (I_1/I_0 \cos \alpha)$ and (time in seconds) $\times (I_1/I_0 \sin \alpha)$, respectively.

Figure 3 shows timing of the commutation for the A_4 input. The heavy line, e_5 , is obtained by switching alternately between e_3 and $-e_3$ at times when the reactivity sinusoid $\delta k = K_1 \sin \omega t = 0$, i.e., commutation occurs when ωt is an integral multiple of π . Similarly, the A_5 input, shown as the heavy line in Fig. 4, is obtained by commutation of e_3 and $-e_3$ when ωt is an integral multiple of π plus $\pi/2$.

$$e_7 = \frac{-1}{(10^6 \text{ ohms})(10^{-6} \text{ farad})} \int e_5 dt = - \int e_5 dt \quad (8)$$

$$e_8 = \frac{-1}{(10^6 \text{ ohms})(10^{-6} \text{ farad})} \int e_6 dt = - \int e_6 dt \quad (9)$$

The constant of integration is represented by voltage across the feedback capacitor and is made zero by shorting the capacitor immediately prior to making a run.

Fig. 4

Fig. 3

Integration of both e_5 and e_6 is carried out over an integral number of cycles, m , of $K_1 \sin \omega t$. The start-stop circuit connects A_4 and A_5 to e_5 and e_6 , respectively, at the start of a cycle of $K_1 \sin \omega t$, and disconnects the m cycles later.

$$\begin{aligned}
 e_7 &= - \int_0^{2m\pi/\omega} e_5 dt = -2m \int_0^{\pi/\omega} \frac{50\pi I_1}{I_0} \sin(\omega t - \alpha) dt \\
 &= - \frac{100\pi m I_1}{I_0} \int_0^{\pi/\omega} (\cos \alpha \sin \omega t - \sin \alpha \cos \omega t) dt \\
 &= - \frac{100\pi m I_1}{\omega I_0} \left[-\cos \alpha \cos \omega t - \sin \alpha \sin \omega t \right]_0^{\pi/\omega} \\
 &= - \frac{100\pi m I_1}{\omega I_0} \left[2 \cos \alpha \right] = -100t \frac{I_1}{I_0} \cos \alpha, \quad (10)
 \end{aligned}$$

where t = time in seconds = $2\pi m/\omega$.

In similar fashion,

$$\begin{aligned}
 e_6 &= - \int_0^{2m\pi/\omega} e_6 dt = - \frac{100\pi m I_1}{\omega I_0} - \left[\cos \alpha \cos \omega t - \sin \alpha \sin \omega t \right]_{\pi/2}^{3\pi/2} \\
 &= - \frac{100\pi m I_1}{\omega I_0} \left[2 \sin \alpha \right] = -100t \frac{I_1}{I_0} \sin \alpha. \quad (11)
 \end{aligned}$$

From Equations (10) and (11),

$$\frac{I_1}{I_0} \cos \alpha = - \frac{e_7}{100t} \quad (12)$$

and

$$\frac{I_1}{I_0} \sin \alpha = - \frac{e_8}{100t} \quad (13)$$

A timer, operated by the start-stop circuit, measures in seconds the time elapsed for m cycles of $K_1 \sin \omega t$. Division of meter readings e_7 and e_8 by $100t$ gives the two vector components of I_1/I_0 and α .

Assuming, then, that $\delta k = K_1 \sin \omega t$, that the value of K_1 is known, and that I_1 and I_0 are proportional to n_1 and N_0 , the transfer function has been obtained.

Two methods of commutation have been tried to date. One method uses light sources, a slotted disc and photo diodes. The other employs a mechanical commutator connected to the right-angle drive.

The start-stop circuit is composed of logic elements and operates the integrator input relays. The analyzer operator pushes a start switch when he is ready to make a run, but the relay is not actuated until the circuit receives the next pulse from the timing circuit generating pulses at time $\delta k = 0$. In like fashion, at the termination of a run, the relay is de-energized by the first pulse from the timing pulse generator received after the operator has pushed the stop switch. This scheme is employed to ensure that integration takes place only over an integral number of cycles. Figure 5 is a photograph of the computer circuit cabinet.

The chief advantage of the analyzer just described is the relative ease and speed with which points of the transfer function curve can be obtained. Maximum integration time is 100 seconds - the period of a 0.01-cps oscillation.

Precise adjustment of R_0 for cancelling out the I_0 component of i_n is not necessary, since integration of a plus signal for half a cycle followed by integration of a minus signal for the next half cycle cancels any dc component. For the same reason, variation of I_0 during a run is cancelled to a first order approximation. Random fluctuations in reactor power, i.e., "noise," does not affect the output appreciably, because integration will eliminate random components in the signal i_n . There is the slight probability that the noise during a particular run is similar in character to the ω frequency and this correlation would cause error.

The possibility of harmonics in the reactivity input has been mentioned. Such harmonics will show up in the i_n expression and, if large enough, make analysis of the results difficult. Integration eliminates even harmonics from the signal since integration of $\sin 2 \omega t$, $\sin 4 \omega t$, etc., over a half-period of the fundamental frequency yields zero. Odd harmonics could contribute to error, but experimental results indicate that they are not present to a bothersome degree with the type of oscillator rod used.

The use of an operational amplifier on the output of the ionization current chamber results in negligible phase shift due to chamber capacitance. Since the input of an operational amplifier is virtually at ground potential, the R of the RC product is just the resistance of the connecting cable. There is no large input resistor in the amplifier as is usual with many dc amplifiers.

There are several limitations inherent in the analyzer. In any mechanically oscillated system, the range of frequencies is limited at the upper end by bearing and vibration difficulties. With a dynamically

Fig. 5

Computer Circuit Cabinet

balanced rod, frequencies of 100 cps or above should be attainable. The balancing problem can be solved in some applications by using a rod which introduces two cycles of reactivity disturbance for one mechanical revolution, for instance, a round rod with diametrically opposite milled flats.

The operation of the circuitry will accommodate, with one exception, frequencies far in excess of those attainable with the mechanical drive. The exception is the commutating relay. The relay used has about a one-millisecond dead time during switching and can, at 10 cps, introduce about 3% error for some α 's. Use of brush-type commutators reduces error from this source.

The operational amplifiers, provided they are of quality design and manufacture, will give rise to no appreciable error due to moderate changes in vacuum tube characteristics. Feedback and input resistors as well as feedback capacitors can, however, cause error if they are unstable or have large temperature coefficients. Capacitor leakage can be another source of error. High-quality components which are satisfactory are available.

Fig. 6

Operation of the computer circuit was checked by connecting a sine potentiometer to the right-angle drive and substituting the output of the potentiometer for the ionization chamber current at the A_1 input. Several angles set into the potentiometer over the range from 0 to 90° were computed by the circuit with an error of less than $\frac{1}{2}^\circ$. The test runs were made at several frequencies between 0.02 and 3 cps.

The analyzer was tested on a critical assembly at the Pajarito site of the Los Alamos Scientific Laboratory. The assembly, LCX II, was a mockup of a reactor under development by the Laboratory's K-Division. It was a fast assembly with a calculated mean prompt neutron lifetime of 10^{-8} sec. Surrounding the core was a stationary reflector and outside the stationary reflector was a movable iron annular reflector used for reactivity control. Rods moving independently in holes in the annular reflector provided vernier reactivity control. One of these rods was replaced by an oscillator rod and the transfer function of the assembly was measured from 0.025 to 2.8 cps. Mechanical difficulties prevented operation beyond this range. Figure 6 shows geometry of the oscillator rod.

The reactivity versus angular position of the rod was measured and the results are plotted in Fig. 7. It is seen that the reactivity input is appreciably different from a sinusoid. The addition of 18% second harmonic

to a fundamental gives a curve which closely fits that of Fig. 7. Comparison of test data with theory would indicate that the second harmonic probably had little influence on results. Harmonics higher than the second did not seem to be present.

Fig. 7

Figure 8 is a plot of the experimental data and also the curve computed from theory by Keepin and Wimett.

Fig. 8

Figure 9 is a plot of the data obtained during zero-power experiments on LAMPRE I. Amplitude values agree with theory to about 1%. The phase error is $3\frac{1}{2}^\circ \pm \frac{1}{2}^\circ$ and is believed to have been caused by improper setting of the timing pulse generator. During these runs, signal to noise ratio was 1:9.

Fig. 9

Discussion of Paper
Presented by Mr. Kiehn and Mr. Swickard

MR. THALGOTT:

My question is addressed to Bob Kiehn. You showed some examples of different types of oscillator rods. In the one example I saw there, was this rod, which you also said was similar to the one we used on ZPR-III, a hollow cylinder?

MR. KIEHN:

In the actual experiment the rotated element was a solid rod rather than a hollow cylinder. The figure presented could lead to the incorrect and reverse interpretation, as the cross-hatching is located ambiguously.

MR. THALGOTT:

We did some playing around with several rods like this. We got a much lower high harmonic content out of a rod which had an off-center poison.

MR. KIEHN:

The large second harmonic content measured for the oscillator rod used in the LAMPRE experiment is due to the large relative size of the rod in comparison with the core meanfree path. The rod diameter itself was about one-half the core diameter. This rod was separated from the core by half a core diameter.

MR. BAKER:

Do you make statistical checks on your analysis of the results of these rods to ensure that your second harmonic component is indeed significant?

MR. KIEHN:

Second harmonic response is rejected, and therefore no statistical analyses have been made.

MR. BAKER:

If you are really interested in knowing what is the second harmonic, and I agree that you don't worry very much in this case, then all you have to do is to check whether the standard error of your readings from the fitted curve is significantly reduced on including an extra component in your fitted curve. And this is a standard statistical procedure. By this means, you can show whether there is any evidence for second, third, fourth, or fifteenth, because you keep on fitting an extra component until the introduction of an extra fitted parameter doesn't significantly reduce the average deviation of your points from your fitted curve.

MR. KIEHN:

In this work no analytical reductions using more than the fundamental frequency component have been completed.

MR. DESHONG:

We have heard several different versions of measuring devices here in the conference, and I thought at this point it might be well to point out one thing that I don't think has been emphasized sufficiently. I think Mr. Swickard did mention the fact that he was dealing largely with white noise, specifically in the flux. This is not always the case, so that one really, in order to evaluate his system, has to have some sort of a figure of merit. Earlier Mr. Akcasu indicated that each of these methods could be thought of as a spectral window, the width of which determined the amount of the noise which one would see effectively in the output after all of this work is done to the signal. For this reason, then, it is necessary, in order to evaluate the accuracy, to know what your output or flux noise spectrum is, that is, whether or not it is white. In fact, in a boiling reactor such as EBWR, it is far from white. Then, as well, the transfer function itself from reactivity to flux frequently has large variations in

magnitude producing large variations in the signal content of the output. These two effects together define the signal-to-noise ratio which obviously can vary widely with frequency.

For instance, in the boiling reactor you find very large noise amplitude at low frequency, which is the very region where you also have very low signal output. So in setting or determining what your accuracy is, you have to take into account not only the noise at the frequency of measurement, but the signal amplitude possible as well.

MR. WRIGHT:

Swickard's system is the same system which we have been using in our power void transfer measurements where the boiling noise level was very high. We have had a little experience on some of these points which may be relevant. First of all, we have checked the effect of noise, and of dc system drift (which in our measurements is one in which you can fall over the cliff on real easily). The easiest way is to run it with no modulation for a while, and then turn the modulation on and look at the difference. Also, we have been setting a standard operating procedure which we think is more helpful and more informative than looking at the meter output. That is to record continuously the integrator output of your recorder. This gives you a measurement of your boiling noise level. In our measurements, we take the average slope of this record and see how much you can shift it in the boiling noise, so we have a noise measurement on each dynamic measurement or transfer function response measurement. You have to do a zero modulation to check for the other things such as drift in your equipment, which can be very bad. Incidentally, I think you can get a somewhat higher sensitivity this way, too, by measuring the average slope over a long period. We often ran for, say, 15 minutes or so in cases of low signal-to-noise ratio, and think that we had sensitivities such that we could detect signals of about one per cent of the noise level. I have some strip chart records here if anybody is interested in seeing some of these points.

MR. BECKJORD:

I just wanted to mention that some stability experiments were completed on Dresden about two weeks ago. We did run oscillator measurements, and were faced with having a rod which has finite positions at which you can move it. It is a notch system with about eight inches between the notches, and we felt also in planning the experiment some time ago (not knowing what sort of noise we would have to deal with) that we would not make any attempt to use either a filter or an electronic system to make the phase and magnitude measurements. Instead, we would make our experiments, record the results, and use a numerical data process in order to determine the spectral content, so to speak.

There are several advantages to this method. For one thing, it takes very little reactor time to do it. This is an advantage when you are trying to get a reactor up to power. Also, it eliminates not only the sine and cosine, but also the changeover switch which may be an advantage in some cases.

In regard to noise, by using a numerical scheme for data processing, you can, to almost any desired degree (that is, to the degree of the accuracy of your reactor tracing) eliminate noise by autocorrelation technique.

MR. HUMMEL:

This has probably already been answered. I wanted to ask about this fission neutron noise you were talking about. Did you say it is worse in the plutonium reactor than the U^{235} fuel reactor?

MR. KIEHN:

No, I didn't want to imply that the noise was worse in the plutonium system. I wanted to state that in the measurements presented the source strength was of the order of 10 microwatts. Thus, as the measurements were made at a power level of 1 watt, a system multiplication of 10^5 is implied. The system lifetime was of the order of 10^9 ; hence in the region of low-power operation, the system has a large bandwidth. At low power level this bandwidth is not reduced by thermal feedback. The neutron response detected, therefore, contained a large number of unattenuated high-frequency components generated by random neutron fissions. The instantaneous sum of these high-frequency components exceeded the measured system response to the driving frequency, if these components of higher frequency were not removed by filtering. In the detection system used, no such means of filtering was utilized, except that of the quadrature integration.

The one-watt power level was an operating limit because of the lack of core cooling. The system temperature was about 100°C due to alpha heating. This temperature had been prescribed as a "cold-critical" upper limit.

RECENT EBR-I STABILITY STUDIES

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Before I present the results of our latest experiments in EBR-I, I would like to discuss very briefly the nature of the problem which existed in the earlier loadings. While steady-state operation was reasonably smooth and posed no serious problems, it was nevertheless recognized that self-sustaining power oscillations could be initiated quite easily. An excellent example of this is given in Fig. 1. The dotted curve consists of data taken from an actual power trace. Note that the period between peaks is long enough to permit the exercise of manual control. In one particular instance, the oscillation was sufficiently severe to cause the reactor to shut down on a power scram. Usually such oscillations could be initiated through sudden, rather modest, changes in coolant flow or by bringing the reactor to power on too short a period.

Fig. 1.

Since the physical origin of such behavior was not at all understood, some concern for the safe operation of fast reactors was expressed. To provide more analytical information, two series of experiments were conducted: one in May of 1955 and the other in November of 1955. In the November experiments, a dramatic demonstration of the prompt positive power coefficient resulted in a partial meltdown of the reactor core. The results of these experiments have been subjected to a great deal of scrutiny by many investigators. While no one model has received anything like universal acceptance, the consensus of opinion of those working in the field is that the instability arose through the combination of a prompt positive power coefficient of reactivity which in steady-state operation was

obliterated by a much larger but slower acting negative component. The source of the positive component has long been attributed to the inward bowing of fuel rods during an increase in power. The source of the delayed negative component has never been precisely defined, but it seemed clear then, as it does now, that it must have been the result of some peculiarity in the reactor structure. Since both of these undesirable features seemed to be of a mechanical rather than nuclear origin, it was postulated that the instability could be eliminated through basic changes in design. To establish the validity of this postulate and to obtain more definite fundamental information, the damaged core was removed and was replaced by one in which the radial movement of fuel was restricted by a system of stabilizing ribs and tightening rods. An additional feature consisted of a valving arrangement which permits either series or parallel coolant flow.

The principal experimental approach used to evaluate the performance of the new core consisted of a very comprehensive series of transfer function measurements. These have been supplemented whenever necessary by power coefficient measurements, flow reduction tests and both steady-state and transient temperature measurements.

Since everyone here is no doubt familiar with transfer function theory, it will not be necessary to go into any great detail in matters of definition. I would like, however, to describe very briefly the formalism we have used in presenting our results. The basic expression with which we work is given in Fig. 2. Here G_0 is the zero-power transfer function of zero-power gain, G is the measured gain, and $-H$ is the feedback arising from temperature and time dependent expansion effects. From this expression, it is clear that when the feedback $-H$ is equal to $1/G_0$, the reactor gain G goes to infinity. In other words, true resonance results when the feedback and the inverse zero-power gain are equal in magnitude and opposite in phase.

Fig. 2.

$$G(s) = \frac{G_0(s)}{1 - G_0(s) H(s)}$$

$$\frac{1}{G(s)} - \frac{1}{G_0(s)} = -H(s)$$

The effect of feedback on gain is illustrated graphically in Fig. 3, which describes the feedback associated with the simplest model possible, that of a process acting with a single time constant T_1 and a power coefficient X . The average power about which the reactor oscillates is P .

Fig. 3.

The inverse zero-power gain always lies in the first quadrant. The effect caused in the measured gain by the feedback is illustrated by the vector diagram. For all situations, $1/G$ will be larger than $1/G_0$. Thus, the measured gain is always less than the respective zero-power gain. If the power increases, the $1/G$ vector increases, and the gain G must decrease. Thus, the net result of a process having a negative power coefficient acting with a single time constant will be a gain that is something less than the corresponding zero-power gain at that frequency.

The effect of introducing a second time constant into the denominator is illustrated in Fig. 4. The feedback now enters the third quadrant and approaches zero at high frequencies with a phase lag of 180° . At frequencies where the feedback lies in the third quadrant, vector addition can now lead to a resultant value of $1/G$ smaller than $1/G_0$. Assuming a linear feedback model, increasing the power increases the magnitude of the feedback without changing the phase. At some combination of power and frequency, the phases and amplitudes of $-H$ and $1/G_0$ will reach cancellation. Here resonance will occur. It is easy to see (for the situation illustrated, which represents actual EBR-I data) that the resonance power will be extremely high.

Fig. 4.

Using these concepts as a background, I would like now to discuss some of our actual results. Basically, the experimental program may be divided into two phases: one dealing with the fully-ribbed rigid core and one dealing with various configurations in which stabilizing ribs have been removed.

To begin with, transfer function measurements were carried out at various power levels to test the linearity of the feedback. For the most part, the results were consistent with a strongly negative power coefficient whose time dependence is dictated by two time constants. In principle, it should have been possible to predict the feedback, hence the performance, of the reactor at full power from low-power data, since the feedback ideally is a linear function of power. In practice, efforts in this direction were hampered by a nonlinearity in the power coefficient. Furthermore, a decided tendency for the high-frequency phase of the feedback to decrease with increasing power was noted. Such deviations from the idealized linear feedback model caused extrapolations to be conservative, i.e., the reactor was actually found to be more stable at full power than extrapolations from low power indicated.

The use of transfer function data taken at 465 kw and 100-gpm flow, and the application of the Nyquist stability criterion indicate that the reactor would be stable for operation below a hypothetical power level of 19 megawatts and a flow rate of 100 gpm. A similar treatment of data taken at full power and full flow (1200 kw and 290 gpm) indicated that the reactor would never reach instability regardless of how high in power the reactor was operated.

A dramatic illustration of the differences between M-2 and M-3 feedbacks under extreme operating conditions is given in Fig. 5. In the case of M-2, it is easy to see how the addition of feedback and $1/G_0$ vectors can lead to amplitude and phase cancellation. It is equally clear why the resonance power level for M-3 under these conditions is extremely high.

Fig. 5.

In the course of our measurements, we have consistently observed that the measured feedback depends on the reactor inlet temperature. A measure of this effect is shown in Fig. 6, in which the feedback is given as a function of inlet temperature for both series and parallel flow. In both cases, i.e., for series and parallel flow, an increase in the inlet temperature effected an increase in the amplitude of the feedback for all frequencies. The phase of the feedback seemed to be unaffected by changes in temperature. Such behavior suggests that the physical process responsible for this dependence operates through the power coefficient. Independent measurements of the power coefficient as a function of inlet temperature substantiated this premise. Increasing the inlet temperature increased the magnitude of the power coefficient in approximately the same proportion as the increase of the feedback amplitude. The explanation of this effect will most likely be found in terms of a temperature-dependent system of clearances which, depending on the inlet temperature, modifies the expansion-reactivity-feedback process.

Fig. 6.

It is also clear from the data given in Fig. 6 that changes from series to parallel flow and vice versa effected no significant change in the feedback. For both high and low inlet temperatures, the feedbacks measured under series and parallel flow conditions were essentially identical. These results strongly suggest that feedback effects having an origin in the radial blanket and which enter the feedback model in series flow through a transport lag are insignificant.

In an effort to increase heat transport across the flow separator, blanket rods at the outer edges of two fuel subassemblies were removed and were exchanged for fuel rods taken from locations on the opposite side

of the core. This procedure modified the core outline to the extent that one-third of the core-blanket interface consisted of fuel and blanket rods immediately separated by the flow divider. The results of these studies are given in Fig. 7. For both high and low inlet temperatures, and for series and parallel flow, no significant change in the feedback was effected by the increase in heat transfer between core and blanket. These results substantiate the conclusion that feedback effects arising in the radial blanket are unimportant, and very definitely weaken the arguments advanced for strong blanket feedback effects as an explanation for the M-2 instability.

Fig. 7.

The results of studies carried out under parallel flow at low inlet temperature have shown that the feedback is not significantly affected by loosening the tightening rods in the fuel and blanket subassemblies. As shown in Fig. 8, a small and probably not significant difference was noted in the feedback amplitudes at low frequency. At high inlet temperature, two significant differences were found: the amplitude of the feedback decreased as the core was loosened, and for a given frequency the phase of the feedback increased. Both observations are consistent with the introduction of a small positive power coefficient component.

The results of all experiments conducted up to this point, while not at all spectacular, very clearly lead to definite conclusions, the most important of which is that the physical processes responsible for the instability noted in M-2 have been completely eliminated through relatively elementary changes in mechanical design. In particular, no evidence of the prompt positive or delayed negative power coefficients so obviously present in M-2 was observed.

Fig. 8.

The final phase of the instability studies consisted essentially of an attempt to introduce a positive power coefficient component through the inward bowing of fuel or blanket rods. Up until this point, bowing effects were eliminated or were at least greatly reduced through a system of stabilizing ribs and tightening rods. A cross-sectional view of a typical fuel or blanket subassembly is given in Fig. 9. Note that each rod is separated from its neighbors by ribs which consist of thin zirconium wires spot-welded to the Zircaloy cladding. Each rod has three equally spaced ribs, each approximately 20 in. long, running the entire length of the blanket and fuel sections. Expansion of the centrally located tightening rod forces the rods outward to effect a rigidly locked array. Under these conditions, inward bowing is prohibited.

Fig. 9.

To encourage rod bowing, eighteen-inch sections of the ribs were removed. One-inch sections were left intact at upper and lower ends to provide proper action for tightening rods. A cross-sectional view of a typical tightening rod is given in Fig. 10. The outer portion of the tightening rod consists of a slotted tube containing a series of modified Woodruff keys welded along the interior. The keys in turn ride in ways milled in the solid portion of the rod. Movement through thread action of the slotted tube relative to the solid rod results in an expansion which in turn forces the fuel or blanket rods outward into a rigid array.

Fig. 10.

The first experiment consisted of an attempt to introduce changes in the feedback through the shearing of ribs from all blanket rods in a typical blanket subassembly. The results at full power - full series flow and for high and low inlet temperatures are summarized in Fig. 11.

Fig. 11.

In both cases, the feedback appeared to be unaffected by the shearing of ribs. The effects of a positive component arising from blanket rod-bowing would have been noted through decreases in the amplitude and increases

in the phase of the feedback; no such changes were observed. Although only $\frac{1}{12}$ of the blanket was sheared, it is nevertheless clear that the extrapolated effect for a fully sheared blanket would still be small.

Before I present the results of the sheared fuel rod experiments, I would like to discuss the effect of fuel rod tightness on rod-bowing. Inward rod-bowing, as we recognize it in the ideal case, involves the bowing of a fuel rod towards the high-flux center during a power increase or, conversely, away from the high-flux center during a power decrease. The reason for this is simply that, during a power increase, the temperature differential across a fuel rod increases. The inner surface, which is heated a little more than the outer surface, expands relative to the outer surface. Under ideal circumstances, i.e., when the top and the bottom of the rod are rigidly fixed, the result of the differential expansion is a bow in the direction of the higher flux. Fuel is moved towards a region of greater worth and the effect is clearly positive.

If, however, the top of the rod is fixed and the bottom is free to move radially, it does not follow that the reactivity effect will be positive. Rather, it seems likely that the deformation accompanying differential expansion will cause the bottom of the rod to move outward. Such an effect will, no doubt, be negative. In the practical case where a rod tip accidentally bears on an outer edge of a bottom plate-locating hole, the rod tip can no longer move outward and inward rod-bowing will result. Thus, for a collection of rods fixed at the top, but free at the bottom, the net reactivity effect may be either positive, negative or even zero, depending on the exact location of the rod tips with respect to their bottom plate-positioning holes.

For a system in which the rods are fixed at the bottom but are essentially unrestrained over the rest of their length, including the rod handles, it is not clear exactly what form the deformation will take, although it seems more likely that the net effect will be an inward movement of fuel. If the rods are unrestrained at both top and bottom, the most likely movement accompanying deformation will be outward at the bottom, except for those rods bearing on the outer edges of positioning holes.

It is clear then that the successful observation and interpretation of feedback effects arising from fuel rod bowing depends to a very large extent on how well the rods are restrained at top and bottom. As it turned out, promoting a satisfactory degree of tightness proved to be a major difficulty encountered in the bowing experiments.

In all, a series of thirteen different experiments were carried out. Some of these involved the effects of additional increments of sheared rods, some involved different attempts to promote a more rigid arrangement, and some involved small refinements intended to increase the magnitude of the positive effect. The actual sequence of experiments is given in Table I.

TABLE I

1. Reference Core, 1425-1441, 9 BR in outer two rows stripped. Conventional TR in D-37.
2. Runs 1529-1544, 9 BR and 12 FR stripped. First non-expandable LR in D-37.
3. Runs 1553-1573, 9 BR and 27 FR stripped. First non-expandable LR in D-37.
4. Runs 1580-1602, Replaced LR by fuel rod. 8 BR and 28 FR stripped. First non-expandable LR in D-21.
5. Runs 1637-1654, 8 BR and 28 FR stripped. Installed flow-restricting wires. First non-expandable LR in D-21.
6. Runs 1655-1671, 8 BR and 28 FR stripped. Second non-expandable LR in D-21.
7. Runs 1685-1701, 8 BR and 28 FR stripped. Installed loosened conventional TR in D-21.
8. Runs 1710-1725, 36 FR stripped. Conventional TR in D-21, Core tight.
9. Runs 1726, 1742, Returned to a loading of 7 BR and 28 FR stripped. Two expandable TR in D-20 and D-31.
10. Runs 1765-1780, All fuel and blanket rods in D and C sheared. Four expandable TR. New ion chamber.
11. Runs 1781-1798, All fuel and blanket rods in D and C sheared. Four expandable TR. Old ion chamber.
12. Runs 1841-1856, All fuel and blanket rods in D and C sheared. First non-expandable LR in D. Second non-expandable LR in C.
13. Runs 1857-1872, All fuel and blanket rods in D and C sheared. Four wedge-action TR.

Unfortunately, there is insufficient time to describe in detail each experiment. It will suffice to say, however, that with two exceptions, which I will point out, the sequence is such that a successively larger positive effect should have been initiated. Perhaps the most satisfactory method of presenting the results in concise form is to consider the amplitude and the phase of the feedback for individually selected frequencies. The results for 0.02 cps are given in Fig. 12. As successive increments of fuel rods were sheared, and as additional refinements were made, the amplitude of the feedback decreased. Similarly, it may be noted that the phase of the feedback increased. It is particularly worth noting that as the second sub-assembly was sheared (experiment 10), a sharp drop in amplitude and a correspondingly sharp increase in feedback phase was observed.

Finally, for the most successful system of tightening rods (experiment 13), the amplitude reached its lowest value while the corresponding phase reached a near maximum.

Fig. 12.

For experiment 7, in which the rods were essentially loose, a decided tendency for the feedback to return to reference can be seen. Such behavior may be interpreted as experimental support of the importance of end restraint in rod-bowing. The same effect, but to a somewhat smaller degree, is noted for experiment 12. In this case, an examination of all rods in both hexes revealed the accidental loss of three one-inch rib sections. Under such conditions, the degree of tightness of the core may be questioned.

While the remaining data points do not follow a concise pattern, the trend towards decreased amplitude and increased phase is unmistakably clear. Essentially, the same conclusions may be reached from a consideration of data taken at 0.04 cps, shown in Fig. 13. Again, the trend towards decreased amplitude and increased phase is clear. The effect of rod looseness in experiment 7 is again apparent from the tendency of the feedback to return to reference. Even at the higher frequencies, as shown in Fig. 14, where the reliability of the feedback separation is poor, the effect, while less obvious, may still be seen.

A more graphic illustration of the magnitude of the shearing effect at low inlet temperature is given in Fig. 15. In this case, the reference feedback is compared directly with the feedback obtained for the situation in which the maximum positive effect was reached, i.e., experiment number 13. The decrease in amplitude and the increase in phase, while apparent at all frequencies, is particularly striking at the lower frequencies.

Fig. 13.

Fig. 14.

Fig. 15.

That the feedback is measurably affected by the removal of ribs seems beyond question. At high inlet temperatures, as shown in Fig. 16, for which more feedback is to be expected, exactly the same behavior was observed. Again, the decrease in amplitude and the increase in phase are unmistakably clear.

Fig. 16

A measure of the effect responsible for these changes may be obtained by subtracting the reference from the modified feedback. The results of such a subtraction are given in Fig. 17. Here the feedback associated with the rib-shearing lies almost entirely in the second quadrant, proving the effect to be positive. While no claim can be made for accuracy, because of a lack of information at low frequencies, a crude extrapolation to zero frequency results in a value of 0.030 ih/kw for the rod-bowing coefficient. Bode analysis of the feedback shown in this figure indicates clearly a frequency dependence of a single time constant of 1.87 sec, a value in good agreement with that calculated for fuel rod-expansion effects.

Fig. 17.

The successful separation of the positive effect may be used as a basis of extrapolation for the performance of the reactor at higher levels of power. Assuming the positive feedback associated with the fully sheared core to be three times that measured for two subassemblies, the Nyquist stability criterion can be applied. The results of such an analysis are given in Fig. 18. Here the product G_0H is plotted for various hypothetical values of power as a function of frequency. From the rearranged transfer function expression, it is clear that when G_0H is equal to unity, the amplitude of the transfer function goes to infinity or, in other words, the reactor goes into resonance. From the data shown, it is easily seen that the resonance power is close to nine times nominal full power or about 10 megawatts. The frequency at which resonance will occur is in the vicinity of 0.14 cps.

Fig. 18.

The predicted performance of the fully sheared reactor at nominal full power and at resonance is given in Fig. 19. While the extrapolated fully sheared performance at full power is somewhat less stable than the performance for two sheared subassemblies, it is nevertheless clear that the reactor may be safely operated under fully sheared conditions, even at full power. For higher levels of power, the reactor would become increasingly more unstable until open resonance is reached at about 10 megawatts.

Fig. 19.

A measure of the importance of the positive effect on the stability of the reactor may be inferred from the results given for the transfer function at nominal full power and at nine times full power for the fully ribbed and rigid core. The full-power experimentally measured transfer function is given by the circles in Fig. 20. The transfer function extrapolated to nine times full power is given by the dashed curve. At low frequencies and nine times full power, the reactor would be extremely stable.

Fig. 20.

For frequencies greater than about 0.3 cps, the reactor tends a little towards instability. However, operation at the hypothetically high power level of nine times full power would still be possible. With sheared ribs, operation at this level would, of course, be impossible. The effect, then, of the positive component is one of lowering the resonance power level.

From the results I have just cited, it seems clear that under special conditions of core tightness, the stripping of ribs from fuel rods leads to the introduction of a positive power coefficient component which can logically be explained only on the basis of the inward bowing of fuel rods. On the other hand, it must be admitted that we have failed to substantiate these conclusions with independent measurements of the power coefficient. The reasons for this rather serious contradiction are unknown at this time. If both channels of information are accepted as equally reliable, one is tempted to accept the conclusion that rib shearing involves a strongly delayed negative effect which, because of the low frequency at which it appears, cannot be detected through transfer function measurements.

Discussion of Paper
Presented by Mr. Smith

MR. KIEHN:

I would like to ask a practical question. Suppose that you had an oscillator measurement of some reactor at some given power level and that you suspected there might be an instability at some higher power level. How far would you feel safe in advancing the power level to make the next frequency response measurement? A factor of three, a factor of ten?

MR. SMITH:

You are presuming that you have made only one measurement, is that right?

MR. KIEHN:

Well, let's take the case of one measurement, and then go to the case of more measurements.

MR. OKRENT:

I think, as legal counsel (laughter), I can say you need not answer the question if you choose not to.

MR. SMITH:

I think he asked for an opinion.

MR. KIEHN:

You are the one that has to do it. I want to know how safe you feel doing it.

MR. SMITH:

I think I would pay a great deal of attention to what this one measurement indicated. If there was any tendency towards instability, I would be quite concerned with it. If it was extremely stable, indicated a single-pole model, or double pole, with a very strong negative power coefficient, I would be inclined to be a little less concerned.

MR. BAKER:

On this point, surely Bethe shows the answer to this problem in his discussion (in APDA-117). One extrapolates upwards assuming that the power coefficient is independent of power. If, on this basis, the reactor appears to be safe at a power, a factor of 10 higher, it would be reasonable to raise the power by a factor of two or three.

MR. COHEN:

Isn't this just a problem of the approach-to-critical. You are extrapolating essentially your inverse multiplication to zero. You go to something like a half or a third of the addition required to reach the extrapolated critical in any one step. This is a standard limitation in most of the safeguards reports on critical experiments. I think the same criterion could be applied here.

MR. SMITH:

Except for nonlinearities. It is possible that you may have a non-linearity appearing at some slightly higher power level, and this non-linearity may be in a direction causing your prognostications to be conservative. You can't know this in advance.

MR. COHEN:

That is why you go to one-third or one-half of the distance to critical. It is the same thing as the inverse multiplication extrapolation. You don't know that that is linear.

MR. STORRER:

Mr. Bethe also pointed out that if by accident you reached a critical power level where you are unstable, even then nothing happens. If you have an instability, you can have an oscillation with a long period, usually. You

don't expect a very high frequency instability, and secondly, the amplitude of the oscillation goes up exponentially at a very low rate, for a very long period. So that even if you are unstable, there is really no great danger. This was explained by Mr. Bethe in APDA-117.

MR. CHAUMONT:

You have not yet explained the delayed negative coefficient required to obtain the oscillation of the EBR-I.

MR. SMITH:

As I understand the question, you are asking whether we understand the delayed negative coefficient?

MR. CHAUMONT:

Yes.

MR. SMITH:

I think we do know its source, but I would rather not comment at this time.

MR. THALGOTT:

I have one comment on Bob Kiehn's question. I think there are very few experimentalists here who will believe one measurement absolutely. So with one measurement, I think one would tend to be fairly cautious. If one, however, has a series of measurements, which indicates that no instability is developing or at least gives an indication that the behavior is linear, then I think within the usual cautiousness of approach to critical, you can go ahead, but not with one measurement. In other words why try to overextend your extrapolations on stability measurements when we do the same sort of thing, as Dick says, in critical measurements, almost every day on critical experiments.

MR. GRACE:

I think what really saves the day here are the nonlinearities when you finally attain this unstable situation. As I mentioned yesterday, we actually did go to an unstable situation under boiling conditions, and allowed this thing to oscillate continuously for some three or four hours. Actually, the theoretical gain at the operating condition was some 50 per cent higher than what was required to just maintain the threshold of oscillation. The higher the gain would have been, the larger the amplitude that would have been sustained. The amplitude in this case was quite small and tolerable despite having a gain that was about 50 per cent higher than was necessary.

A SHORT DESCRIPTION OF RAPSODIE

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"RAPSODIE" is an experimental fast breeder reactor, fueled with an alloy of enriched uranium, plutonium and molybdenum. Its expected power is 20 Mw (thermal). It will give us experience in fast reactor design. It will eventually be used to do fast irradiations for testing the fuel of later projects.

The presence of plutonium gives a low β_{EFF} of 0.00475. The neutron lifetime is 7.10×10^{-8} sec.

The cooling system includes two loops: sodium in the first, and NaK in the second. No production of electricity is planned and the NaK loop is aircooled.

Two methods are followed to study the dynamic behavior of the reactor: both start with the same formulation and the same model. The first makes use of a code developed for the digital computer Mercury. The second makes use of an analogue computer to simulate the plant.

Results already obtained concern essentially thermal shocks in the sodium loop or in the fuel elements, but inherent stability of the reactor is being studied, though we encounter some difficulties in particular to find a correct formulation of a possible bowing effect.

ANALYSIS OF THE POWER FEEDBACK RELATIONS IN FAST REACTORS

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This paper describes a method which APDA proposes to use in the analysis of the results of the oscillator tests, which will be run on the Enrico Fermi Reactor. This method provides a quick and easy check on the agreement between experimental and predicted feedback characteristics. It will be illustrated by an analysis of some of the experimental data obtained from the EBR-I, Mark III tests. In the second part of the paper, a simple stability criterion is presented, which, besides being applicable to any reactor type, indicates without lengthy calculations that the Fermi reactor will be stable, at least if the feedback is not drastically different from what we calculate. The detailed kinetic behavior of the Fermi reactor and plant is being investigated by both analytical and analog methods. Two simulators are being used; The first, owned by PRDC and built by the Holley Carburetor Company, simulates the whole reactor plant, including part of the water-steam cycle. It uses a simple lumped reactor model, and is suitable for the study of rather slow transients corresponding to normal operation of the plant. The second, which is owned by The Bendix Corporation, simulates only the reactor, uses a sectionalized reactor model, and can handle the faster transients which are pertinent to safety and stability studies.

Analysis of the Power-Feedback Relations

The reactor transfer function at power P is given by

$$G(i\omega) = \frac{G_0(i\omega)}{1 + PX(i\omega) G_0(i\omega)} \quad (1)$$

where $G_0(i\omega)$ is the zero-power transfer function. Both $G(i\omega)$ and $G_0(i\omega)$ are measured in the course of oscillator tests, and from them the feedback transfer function $X(i\omega)$ can then be determined as a function of frequency:

$$X(i\omega) = \frac{1}{P} \left[\frac{1}{G(i\omega)} - \frac{1}{G_0(i\omega)} \right] \quad (2)$$

Since $X(i\omega)$ is a function of the flow, we shall use the notation $X(i\omega, \lambda)$, where λ is the ratio of a reference flow, as for instance full flow F_0 , to the flow F under consideration:

$$\lambda = F_0/F \quad (3)$$

*Assigned to APDA by contract with Belgonucleaire SA, Brussels, Belgium.

If the feedback were perfectly linear, $X(i\omega, \lambda)$ would be independent of power and inlet coolant temperature. In practice, it will be somewhat dependent on these quantities, since properties of the materials of which the reactor is constructed and reactor configuration are to some extent temperature and power dependent. In our analysis we assume linearity, and we shall consider the effect of nonlinearities as we go along.

Let us now compare the experimental values of $X(i\omega, \lambda)$ with calculated values. With the linear assumption, $X(i\omega, \lambda)$ can be expressed by the following volume integral:

$$X(i\omega, \lambda) = \int_V \rho(\bar{r}) T(\bar{r}, i\omega, \lambda) d\bar{r} \quad , \quad (4)$$

where the integral is carried over all the feedback producing regions. The quantity $\bar{r}$ represents any point in the coolant, fuel or any structural component, and $T(\bar{r}, i\omega, \lambda)$ represents the transfer function between power and temperature at point $\bar{r}$. Also $\rho(\bar{r})$ represents the temperature coefficient at point $\bar{r}$. You will notice that it does not contain $i\omega$. This means that we assume that the reactivity effect caused by a local temperature change is instantaneous. This assumption is valid for all transients other than the most severe accidents, where mechanical inertia introduces a lag between the temperature increase and the resultant expansion.*

In principle, the integral (4) can be calculated. In practice it is evaluated as a sum of several integrals, one for the coolant, another for the fuel, and one for each structural component that may affect the reactivity by inducing distortions in the reactor. A similar integral can be written for the reactivity effect of fuel element bowing. In APDA-132² we present a method that allows the calculation of $T(\bar{r}, i\omega, \lambda)$ in analytical form, with only minor assumptions, for the whole range of frequencies. These calculations, which are quite lengthy, presume the exact knowledge of all material properties and heat transfer coefficients. In principle, $\rho(\bar{r})$ can be evaluated by a time-independent perturbation method.**

*J. C. Carter¹ has postulated a model for the feedback of EBR-I, Mark III where such a lag is introduced, not primarily by the inertia, but by a succession of stepwise slips between surfaces in contact. In our analysis we postulate that such slips do not take place. When applying this analysis to EBR-I, Mark III, we realize that any real reactor may choose to ignore our postulates.

¹J. C. Carter, D. W. Sparks, and J. H. Tessier, The Internal Feedback of EBR-I, Mark III, ANL-6124 (February 1960).

²F. Storrer, Temperature Response to Power, Inlet Coolant Temperature and Flow Transients in Solid Fuel Reactors, APDA-132 (June 5, 1959). (Parts of this report were presented at the summer meetings of the ANS in 1958 and 1959.)

**The calculation of $X(i\omega, \lambda)$ can also be performed by a simulator when the frequency is not too high. A sectionalized model of reactor heat transfer evaluates $T(i\omega, \lambda)$ for a few discrete regions with reasonable accuracy. Independently calculated values of ρ for the same regions are made available to the simulator which performs the integral (4) numerically. The same simulator usually also handles the neutron kinetics so that it evaluates equation (1) as well. The simulator works directly in the time domain, rather than in the frequency domain, so that its use is not restricted to oscillatory transients.

For some contributions to the feedback, such as the effect of changes in the coolant density, * where the temperature coefficient is directly proportional to the local danger coefficient, the calculation of $\rho(r)$ is straightforward.

For such contributions as overall core expansion and local core distortions, the calculation is much more complicated and uncertain, since heating at a point $\bar{r}$ of the structure, even outside the active fuel region, can induce distortions over a large region, which is important reactivitywise. In this case, the calculation of ρ at this single point $\bar{r}$ involves the knowledge of the danger coefficients and the determination of the deflections throughout this latter region.

It is not a simple job to calculate thermally induced deflections of such a complicated structure, which is usually statically indeterminate, and it requires an exact knowledge of structural details such as tightness and tolerances. These considerations are particularly pertinent to fast reactors for which overall or local thermally-induced perturbations of the geometry contribute a large fraction of the total power coefficient, because of the high neutron leakage within and from a fast core, and because of the high ratio of temperature to power.

In the case of a small fast reactor, it would be a rather sensational achievement if one succeeded in one set of calculations to obtain values of $X(i\omega, \lambda)$ which would perfectly match the experimental values for the whole range of frequencies and flows, not to speak of the secondary nonlinear effects of power and inlet coolant temperature. These lengthy calculations, which require a digital or analog computer, should probably be repeated several times after rather intuitive adjustments of several parameters before a good fit could be obtained.

With these considerations in mind, we decided to try a simpler approach. If, as a first step, one considers only very low frequencies, or more specifically the asymptotic behavior of $T(\bar{r}, i\omega, \lambda)$ and $X(i\omega, \lambda)$ when ω tends to zero, the complexity of the calculations is so reduced that a computer is no longer needed. Matching of the asymptotic behavior of experimental and calculated values of $X(i\omega, \lambda)$ for the whole range of flows still yields considerable information that can be used to adjust the value of some parameters and to gain understanding of the feedback mechanism. Having thus learned to walk, we will be in a much better position to attempt to run, i.e., to extend the calculation to higher frequencies.

Here is how it works. As is shown in Ref. 2, the frequency dependence of $T(\bar{r}, i\omega, \lambda)$, which is the solution of a set of partial differential equations and thus cannot be described by any combination of time constants

*This is the main contributor to feedback in water moderated reactors.

over the whole range of frequencies, can be described by a single first-order time constant at very low frequencies if terms of the order of ω^2 are neglected, viz.,

$$T(\bar{r}, i\omega, \lambda) = \frac{T(\bar{r}, 0, \lambda)}{1 + i\omega\tau(\bar{r}, \lambda)} \quad (5)$$

The following alternate expressions are also valid up to first order in ω :

$$T(\bar{r}, i\omega, \lambda) = T(\bar{r}, 0, \lambda) e^{-i\omega\tau(\bar{r}, \lambda)} \quad (6)$$

and

$$T(\bar{r}, i\omega, \lambda) = T(\bar{r}, 0, \lambda) [1 - i\omega\tau(\bar{r}, \lambda)]. \quad (7)$$

Substituting (7) in (4), one obtains

$$X(i\omega, \lambda) = X(0, \lambda) [1 - i\omega T_X(\lambda)] \quad (8)$$

where

$$X(0, \lambda) = \int^V \rho(\bar{r}) T(\bar{r}, 0, \lambda) d\bar{r} \quad (9)$$

is the zero frequency or steady-state power coefficient and

$$\tau_X(\lambda) = \frac{\int^V \tau(\bar{r}, \lambda) \rho(\bar{r}) T(\bar{r}, 0, \lambda) d\bar{r}}{\int^V \rho(\bar{r}) T(\bar{r}, 0, \lambda) d\bar{r}} \quad (10)$$

It can be seen that at low frequencies, the frequency dependence of $X(i, \omega, \lambda)$ can also be described by a first-order time constant $\tau_X(\lambda)$ [expression (8) is similar to (7), and could also be written in a form similar to (5) or (6)] and that the time constant of the resultant of components working in parallel is the weighted value of the time constants of these components. The asymptotic behavior of $X(i, \omega, \lambda)$ when ω tends to zero can be described for any value of λ by the two quantities $X(0, \lambda)$ and $\tau_X(\lambda)$, the amplitude and the time constant. We will now briefly outline how these can be calculated.

The amplitude of the transfer function of power to temperature, which gives the steady-state temperatures for a unit power, can be obtained for any λ by a steady-state heat transfer calculation. Assuming that all material properties and heat transfer coefficients are independent of temperature and flow over the range of interest, their dependence upon λ and $\bar{r}$ is as indicated in the first column of Table I. Here, T_c is the coolant temperature, and T_{f_1} and T_{f_2} are those components of the fuel temperature which in steady state correspond respectively to the temperature difference between fuel and neighboring coolant and to the temperature of the neighboring coolant. These two components are considered separately because they have different λ dependences and time constants. The temperature of a structural component is indicated by T_s . The function $f(\bar{r})$ describes the spatial dependence of

TABLE I

	Amplitude	Time Constant
$T_c(\bar{r}, \lambda)$	$f(\bar{r})\lambda$	$\tau_f + h(\bar{r})\lambda$
$T_{f_1}(\bar{r}, \lambda)$	$g(\bar{r})$	τ_f
$T_{f_2}(\bar{r}, \lambda)$	$f(\bar{r})\lambda$	$2\tau_f + h(\bar{r})\lambda$
$T_f(\bar{r}, \lambda) = T_{f_1} + T_{f_2}$	$g(\bar{r}) + f(\bar{r})\lambda$	$\frac{\tau_f g(\bar{r}) + 2\tau_f f(\bar{r})\lambda + h(\bar{r}) f(\bar{r}) \lambda^2}{g(\bar{r}) + f(\bar{r})\lambda}$
$T_s(\bar{r}, \lambda)$	$f(\bar{r})\lambda$	$\tau_f + \tau_s + h(\bar{r})\lambda$
$X(\lambda)$	$a + b\lambda$	$\frac{c + d\lambda + e\lambda^2}{a + b\lambda}$

coolant and structure temperatures in steady state for unit power and λ . Inserting these values of T in expression (9), we obtain the amplitude $X(0, \lambda)$ of the power coefficient:

$$X(0, \lambda) = a + b\lambda \quad (11)$$

Remember that in practice the integral (9) would be evaluated as a sum of integrals, one for each temperature listed in Table I, since the temperature coefficient function $\rho(\bar{r})$ is discontinuous at interfaces between media. For example, the constant term of (11) comes from T_{f_1} , and is given by

$$a = \int_{V_f} \rho_f(\bar{r}) g(\bar{r}) d\bar{r} \quad , \quad (12)$$

where V_f is the volume occupied by the fuel.

The time constants for the temperatures, as given in the second column of Table I, can be inferred from Ref. 2 (in particular Paragraph VII-3). For example, the time constant for T_{f_1} is τ_f , the fuel time constant, which is independent of position and flow. The calculation of the other time constants is somewhat more difficult. Let us look at the time constant of T_c , for example. Just as the time constant of the power coefficient can be derived by proper weighting of the time constants of the various feedback-producing points acting in parallel, the time constant of the coolant temperature at any point $\bar{r}$ can be obtained by looking at all the power-producing points $\bar{r}'$ upstream of point $\bar{r}$, which contribute to the coolant temperature at point $\bar{r}$. From Ref. 2 one can compute the time constant pertaining to heat transfer and heat transport from the fuel in $\bar{r}'$ to the coolant in $\bar{r}$. It is

$$\tau(\bar{r}, \bar{r}') = \tau_f + \left(1 + \frac{\tau_f}{\tau_c}\right) \tau \quad , \quad (13)$$

where τ_f and τ_c are the fuel and coolant time constants as defined in Ref. 2, and τ is the transport time of the coolant from $\bar{r}'$ to $\bar{r}$. The first term, τ_f , which is independent of flow and position, is also the first term of the resultant time constant of T_c (see Table I), while $h(\bar{r})$ is the properly weighted second term of equation (13) for $\lambda = 1$. If $\bar{r}$ is replaced by the cylindrical coordinates r and z , we obtain, assuming symmetry with respect to the z axis,

$$h(r, z) = \left(1 + \frac{\tau_f}{\tau_c}\right) \frac{1}{v(r)} \frac{\int_0^z (z - z') q(r, z') dz'}{\int_0^z q(r, z') dz'} \quad , \quad (14)$$

where $v(r)$ is the coolant velocity at radius r for $\lambda = 1$, where $q(r, z')$ is the power distribution, and where $z' = 0$ represents the upstream boundary of the power producing region. For example, if z is at the outlet of a channel within which $q(z')$ is symmetrical about the $z/2$ point, equation (14) gives

$$h(r, z) = \left(1 + \frac{\tau_f}{\tau_c}\right) \frac{z}{2v(r)} = \frac{\tau}{2} \left(1 + \frac{\tau_f}{\tau_c}\right)$$

and the time constant of $T_c(r, z)$ is $\tau_f + \frac{\tau}{2} \left(1 + \frac{\tau_f}{\tau_c}\right)\lambda$, which is obtained in paragraph VII-3 of Ref. 2 for $\lambda = 1$. For T_{f_2} or T_s , the time constant is that of the coolant with which it is in contact, plus the time constant τ_f or τ_s for transfer of heat from the coolant to the fuel or structure. Physically the time constant $\tau_f + \tau_s + h(\bar{r})\lambda$ of T_s can be visualized as the sum of the time constant τ_f for the heat transfer from fuel to coolant in any upstream point, of the flow dependent effective* average transport time $h(\bar{r})\lambda$ of the heat by the coolant from the power producing region to the vicinity of the structure, and of the time constant τ_s for heat transfer from the coolant to the structure. One should remember that it is only at very low frequencies that these time constants and transport times can be used to determine the time response. This, and also the fact that they can be combined by a simple addition, is the reason for the relative simplicity of this asymptote analysis. The time constant of T_f can be obtained by proper weighting of those of T_{f_1} and T_{f_2} .

Substituting the amplitudes and time constants of the temperatures listed in Table I into expression (10), one can obtain $\tau_x(\lambda)$, which will be of the form

$$\tau_x(\lambda) = \frac{c + d\lambda + e\lambda^2}{a + b\lambda} \quad , \quad (15)$$

where a , b , c , d and e are constants. The denominator is, of course, $X(0, \lambda)$. By taking a close look at how (15) was obtained, one can see that:

*"Effective" because it is larger than the average coolant transport time by a factor $(1 + \tau_f/\tau_c)$.

$$c/a = \tau_f$$

$$d/b = \tau_f + \text{a properly weighted combination of } \tau_f \text{ and various structure time constants } \tau_s.$$

$$e/b = \text{a properly weighted effective average transport time of heat between power producing and feedback producing points for } \lambda = 1.$$

It should be noted that formula (15) is valid only if the time constant τ_f of all fuel elements (in core and blanket) is the same, as is the case in EBR-I, Mark III. If this is not the case, as in the Fermi reactor, a slightly different formula would be obtained.

For a homogeneous liquid fuel reactor the same formulas apply, with $\tau_f = 0$. For a reactor where the coolant contributes practically all the feedback, the constants a and c are zero.

Let us examine what information we can extract from the expressions (11) and (15) for $X(0, \lambda)$ and $\tau_x(\lambda)$. If all temperature coefficients have the same sign, both $|X(0, \lambda)|$ and $\tau_x(\lambda)$ are increasing functions of λ , or decreasing functions of the flow, since $d/b > c/a$. For very high flows (λ very small), X and τ_x tend respectively to a and τ_f . Only the T_{f1} component of the fuel temperature contributes then to the feedback. For very low flows (large λ) the main contributions to X and τ_x are respectively $b\lambda$ and $\frac{e}{b}\lambda$. For any λ , it is useful to know the magnitude and individual time constant of each of the components of the total power coefficient $a + b\lambda$, especially if any component is positive. The fastest component is the constant a (time constant τ_f), then come the various components of $b\lambda$: the coolant contribution (time constant $\tau_f + h_1\lambda$),* the T_{f2} contribution (time constant $2\tau_f + h_2\lambda$), and then the various structure contributions (time constant $\tau_f + \tau_s + h_s\lambda$).

Now that we have seen how $X(0, \lambda)$ and $\tau_x(\lambda)$ can be calculated, and have established the nature of their λ dependence, let us see how they can be determined from the $X(i\omega, \lambda)$ experimentally obtained from the oscillator tests. It can easily be seen that**

$$\left. \begin{aligned} X(0, \lambda) &= \lim_{\omega \rightarrow 0} |X(i\omega, \lambda)| \\ \tau_x(\lambda) &= \lim_{\omega \rightarrow 0} \frac{\phi_x(i\omega, \lambda)}{\omega} = \lim_{\omega \rightarrow 0} \tau_b(i\omega, \lambda) \end{aligned} \right\} \quad (16)$$

where $\phi_x(i\omega, \lambda)$ is the phase of $X(i\omega, \lambda)$. The left-hand sides can best be obtained graphically, by plotting the right-hand sides versus ω for all the frequencies at which $X(i\omega, \lambda)$ has been determined experimentally, and

* h_1 , h_2 and h_s are properly weighted values of $h(\bar{r})$. We assume the structure contributions to lag the T_{f2} contribution, since in general $\tau_s > \tau_f$ and for most structural components $h_s > h_2$.

** τ_a and τ_b are hereby defined for later use.

extrapolating to zero frequency. The following relations are, of course, also valid, since $\lim_{\omega \rightarrow 0} \phi_x(i\omega, \lambda) = 0$

$$\left. \begin{aligned} X(0, \lambda) &= \lim_{\omega \rightarrow 0} \frac{|X(i\omega, \lambda)|}{\cos \phi_x(i\omega, \lambda)} \\ \tau_x(\lambda) &= \lim_{\omega \rightarrow 0} \frac{\tan \phi_x(i\omega, \lambda)}{\omega} = \lim_{\omega \rightarrow 0} \tau_a(i\omega, \lambda) \end{aligned} \right\} \quad (17)$$

Plotting the right-hand sides of (17) together with those of (16) makes the extrapolation somewhat easier, because those of (16) usually* approach the limit from below, and those of (17) from above. The reason for this is that the right-hand sides of (16) and (17) would be frequency independent if $X(i\omega, \lambda)$ varied respectively according to the transport lag model

$$X(i\omega, \lambda) = X(0, \lambda) e^{-i\omega \tau_x(\lambda)} \quad (18)$$

and according to the first-order time constant model

$$X(i\omega, \lambda) = \frac{X(0, \lambda)}{1 + i\omega \tau_x(\lambda)} \quad (19)$$

over the whole range of frequencies, and that in fact the actual frequency dependence of $X(i\omega, \lambda)$ is usually intermediate between those two models.

As an illustration, Figs. 1 and 2 show plots of (16) and (17) for some operating conditions of the EBR-I, Mark III core. Besides the flow rate,

Fig. 1. Power Coefficient Amplitude EBR I - Mark III

Fig. 2. Power Coefficient Time Constant EBR I-Mark III

*We say "usually" because this is not the case when some time constants of the individual components of $X(i\omega, \lambda)$ acting in parallel have widely different orders of magnitude. We shall come back to this later.

Fig. 3. Effect of Flow on Power Coefficient of Reactivity EBR I - Mark III

The steady-state power coefficient $X_{SS}(\lambda)$, which should theoretically be the same as the zero-frequency power coefficient $X(0, \lambda)$ was also obtained as a function of flow by steady-state measurements. The steady-state power coefficient was measured at full flow ($\lambda = 1$), and also the reactivity change with varying λ was measured at constant power P_0 .

If we write

$$X_{SS}(\lambda) = a_{SS} + b_{SS} \lambda \quad , \quad (20)$$

the first experiment gives $a_{SS} + b_{SS}$, and the second experiment gives $P_0 b_{SS}$. These two quantities allow the determination of a_{SS} and b_{SS} , and the resulting linear expressions for $X_{SS}(\lambda)$ are plotted on Fig. 3 with the label "steady state" for both series and parallel flow. Quite unexpectedly, these steady-state values appear to be much larger than the zero-frequency values. The only possible explanation for this discrepancy is that a component $X_d(\lambda) = X_{SS}(\lambda) - X(0, \lambda)$ of the power coefficient is affected by such a large time delay of the conductive type, that it is completely attenuated even at the lowest frequency of the oscillator tests. This time constant $\tau_d(\lambda)$ must be much larger than $(1/\omega)_{\min}$, which is about 16 sec. In fact the so-called zero-frequency values $X(0, \lambda)$ and $\tau_x(\lambda)$ are false limits. If the oscillator tests had been run at lower frequencies, and the results plotted on Figs. 1 and 2, all the curves would probably evidence a sharp increase at the low-frequency end. This is what was referred to in the last footnote. The curves on Fig. 1 would tend to $X_{SS}(\lambda)$ and those of

which is an essential variable, other parameters are indicated on the figures. The flow can be either in parallel through core and radial blanket, or in series through the radial blanket and then through the core. The core can be loose or tight. The power and inlet coolant temperature are also noted so that any nonlinearity would be detected in the analysis of the data.

In Fig. 3, the values of $X(0, \lambda)$ obtained as in Fig. 1 for various flow rates are fitted by a straight line $a + b\lambda$ (see Eq. 11) for series and parallel flow, for an inlet temperature of 220°C and a tight core configuration. These are the lines labeled zero frequency. The value $\lambda = 1$ corresponds to 300 gpm.

Fig. 2 would tend to

$$\tau_{SS}(\lambda) = \frac{\tau_f(\lambda) X(0, \lambda) + \tau_d(\lambda) X_d(\lambda)}{X_{SS}(\lambda)}, \quad (21)$$

which would be approximately

$$\tau_d(\lambda) \frac{X_d(\lambda)}{X_{SS}(\lambda)} \text{ since } \tau_d(\lambda) \gg \tau_x(\lambda)$$

The plot of $X(i\omega, \lambda)$ as a function of ω on the complex plane would look roughly like two adjoining half circles. The false limits and the true zero-frequency or steady-state limits are obtained by extrapolating along one or the other half circle. The component of the power coefficient with a very large delay must have its origin in the heating of a heavy structural component which markedly distorts the core structure. It certainly must be of the form $b\lambda$, so that $a_{SS} = a$, $b_{SS} = b_d + b$. Also the constant a must be the same for both series and parallel flow, since it is caused by the T_{f1} component of fuel temperature, which does not depend on flow distribution. Hence the four straight lines of Fig. 3 should have the same value a at $\lambda = 0$.^{*} That one actually has a slightly different value results from the large extrapolation from $\lambda = 1$ (full flow) to $\lambda = 0$ (infinite flow) and from our reluctance to cheat. We confess, though, that if the three other lines

Fig. 4. Effect of Flow on Power Coefficient Time Constant EBR I - Mark III

intersect exactly at $\lambda = 0$, we were fully aware that they should do so, while selecting a best fit to the experimental data. After this concession to honesty, let me point out some other interesting conclusions that can be reached by looking at this figure. The fastest responding component a of the total power coefficient (time constant τ_f) is seen to be negative,** and this fact is very important with respect to stability. Furthermore, at full flow, this component is about 60% of the so-called zero-frequency component, which is still relatively prompt, and about 40% of the total power coefficient.

Figure 4 shows a fit of the experimental points $\tau_x(\lambda)$ for series flow and 220°C inlet coolant temperature.

It can be seen that at each flow the points corresponding to the highest

* This is also obvious from the fact that at infinite flow the coolant and the structure (the latter if gamma heating is negligible) are essentially isothermal.

** In this graph a positive ordinate corresponds to a negative power coefficient.

powers (about 1200 kw for full flow, 500 kw for $\frac{1}{3}$ flow) are bunched up very close to the curve. For lower powers the scatter is much larger, probably because at low power-to-flow ratios, the feedback $PX(i\omega, \lambda)$, as given by equation (2), is obtained as a small difference of large experimental quantities. Besides the scatter, $\tau_x(\lambda)$ also appears to be higher on the average at low power-to-flow ratios. This is probably an evidence of some non-linearity of the feedback with power (or temperature). However, the degree of scatter indicates that the results of the oscillator tests are not accurate enough to allow a detailed investigation of the nonlinearity toward lower powers.

The curve of Fig. 4 can now be fitted by an expression of the form (15). Since a and b are already determined from Fig. 3, we can now calculate c , d , and e . You can see that they all will have the same sign, and that $\frac{d}{b} > \frac{c}{a}$. In particular at $\lambda = 0$:

$$\tau_x(0) = \frac{c}{a} = \tau_f \approx 2.75 \text{ sec.}$$

This is considerably larger than the calculated fuel time constant. This discrepancy may be due to the large extrapolation from $\lambda = 1$ to $\lambda = 0$, but on the other hand it may be an indication that the heat-transfer coefficient is smaller than was calculated. This discrepancy also illustrates the usefulness of this method of analysis. We have indicated how the various coefficients a , b , c , d , and e can be calculated (see for instance Eq. 12) and how they can be determined experimentally. A comparison of these two sets of values gives valuable information as to how well the feedback mechanism is understood, and how well the material properties are known. Adjustments can and should be made before one can hope to be successful in matching calculation with experiment over the whole range of frequencies. It should also be noted that all the data presented in Figs. 3 and 4 were obtained by simple and fast hand calculations. Besides the determination of the five constants, we gathered additional information, as indicated in the discussion of Figs. 3 and 4.

If temperatures are measured during the course of oscillator tests, this method of analysis can be extended still further. In EBR-I, Mark III, many thermocouples were located in the fuel and coolant, so that the transfer function of power to temperature could be determined at many points within the reactor. From these values of $T(\bar{r}, i\omega, \lambda)$, one can determine $T(\bar{r}, 0, \lambda)$ and $\tau(\bar{r}, \lambda)$ by a method similar to that given in equations (16) and (17) and illustrated by Figs. 1 and 2 for the power coefficient.

At these points the values of the functions listed in Table I can then be determined. This makes possible an intermediate check on some of the parameters involved in the feedback. As an example, Fig. 5 shows six experimental values of the time constant of the coolant temperature close to

the center of the core. The power in kw is again indicated for each point. The two runs at 877 kw were made with different oscillator rods. As suggested by Table I, the data have been fitted by a straight line $\tau_f + h(r)\lambda$. This gives for the fuel time constant τ_f a value of about 2 sec, which is also larger than the calculated value, but smaller than the value 2.75 sec obtained from Fig. 4. The discrepancy can be explained by some inaccuracy in the measurements and by the large extrapolation to $\lambda = 0$. A similar analysis of the other temperature data would give more information on the value of τ_f . For this particular point the value of $h(\bar{r})$ is seen to be about 1 sec. It can be used as a check on the parameters appearing in equation (14).

In addition to the comparison between experimental and calculated values, the experimental data on the power coefficient and the various temperatures can be checked for internal consistency, with the help of equations (9) and especially (10). A first qualitative check would consist in ascertaining whether or not $\tau_x(\lambda)$ appears to be a weighted average of the various temperature time constants. If, for example, $\tau_x(\lambda)$ was consistently larger than the latter values, one should look for another mechanism that could induce a larger lag in the feedback. Such a mechanism could be the previously mentioned slip-and-stick model of J. C. Carter or the more delayed heating of some structural component. A comparison of Figs. 4

Fig. 5. Effect of Flow on Temperature Time Constant EBRI - Mark III

and 5 seems to indicate, however, that the time constant of the feedback is consistent with the time constant of temperatures within the core. Note that we are dealing here only with the relatively prompt component of the feedback and not with the delayed component $X_d(\lambda)$ evidenced in Fig. 3.

I hope to have made clear in the first part of this paper that considerable information can be gained by an asymptotic analysis of the feedback, and eventually of the temperature response, for various flow rates. Of course,

once agreement has been obtained between experiment and calculation at low frequencies, it is desirable to extend the analysis to higher frequencies by the method indicated at the beginning of this paper.

Stability Criterion

One of the objectives of the second part of this paper is to offset any concern which may have been caused by a statement made in the first part. It is well known that the stability of nuclear reactors subject to small

perturbations can be ascertained in terms of the Nyquist criterion, which involves the knowledge of $G_0(i\omega)$ and $X(i\omega, \lambda)$ over the whole range of frequencies. It was indicated in the first part of this paper that it may be very difficult to accurately calculate the latter quantity. It is true that the stability or instability of any reactor can be safely determined in the course of oscillator tests, but it still is highly desirable to ensure the stability of a reactor in the course of the design. It will now be shown that for many reactors, including EBR-II and the Enrico Fermi reactor, stability can be predicted without detailed knowledge of $X(i\omega, \lambda)$ over the whole frequency range. A stability criterion is given, based on the knowledge of the zero-frequency feedback characteristics defined above and on a simple assumption as to the frequency dependence of the feedback.

The Nyquist criterion indicates that, for a given flow, the reactor will be unstable at and above a critical power P_c , if for $P = P_c$ there is a frequency ω_c for which the denominator of equation (1) vanishes.

This can be expressed in terms of amplitude and phase, the feedback separate from the neutron characteristics as:

$$P_c |X(i\omega_c, \lambda)| = \frac{1}{|G_0(i\omega_c)|} \quad (22)$$

and

$$\phi_x(i\omega_c, \lambda) = \pi - \phi_{G_0}(i\omega_c) \quad , \quad (23)$$

where ϕ_x and ϕ_{G_0} are the phase lags of X and G_0 , respectively. Instead of (23) the following expression may be used:

$$\tau_b(i\omega_c, \lambda) = \frac{\pi - \phi_{G_0}(i\omega_c)}{\omega_c} \quad (24)$$

where τ_b is ϕ_x/ω as defined in equation (16). For any reactor, the right-hand sides of equations (22) and (24) can be calculated as a function of the argument ω_c , with good accuracy, since they involve only neutron kinetics parameters. Hence at the critical point for instability, i.e., when equations (22) and (24) are satisfied, the left-hand sides are related to ω_c in the same way as the right-hand sides. This relationship between $P_c |X(i\omega_c, \lambda)|$, $\tau_b(i\omega_c, \lambda)$ and ω_c is represented on Fig. 6 for the Enrico Fermi reactor, and also for a thermal reactor with the same delayed neutron characteristics but a neutron lifetime of 10^{-4} sec. The two curves diverge only for frequencies beyond the range where instability is most likely.* Since the dollar was chosen as unit of reactivity, the intermediate part of the curve, where the abscissa is unity, would not be affected by a different value of the delayed neutron fraction. At low frequencies the

*At high frequencies the amplitude $|X|$ of the feedback is so attenuated and $\frac{1}{|G_0|}$ is so large that instability is not likely.

curve would only be slightly affected by the different delayed neutron characteristics of other fissionable isotopes.^{3,4} Hence for all practical purposes the curve of Fig. 6 is almost universal, regardless of the reactor type of interest.

Fig. 6. Relationship Between τ , $P|X|$ and ω at Instability

If a reactor is observed to be unstable at a frequency ω_c under some conditions of flow and at a critical power P_c , the value of $P_c|X(i\omega_c, \lambda)|$ and $\tau_b(i\omega_c, \lambda)$ can be read directly on the graph. Actually, for this particular application it would be preferable to use a graph containing the same information as Fig. 6, but with ω_c as the abscissa. For example, EBR-I, Mark II was observed to be unstable at 95 gpm, $P_c = 550$ kw and $\omega_c = 0.185$ rad/sec. From the graph one can read: $P_c|X(i\omega_c, \lambda)| = 57$ cents and $\tau_b(i\omega_c, \lambda) = 14$ sec.

In addition to this rather trivial application, this graph can be used as a stability criterion.

If the feedback followed exactly a transport lag model as given by expression (18), one would have, for any frequency,

$$|X(i\omega, \lambda)| \equiv X(0, \lambda) \quad (25)$$

and

$$\tau_b(i\omega, \lambda) \equiv \tau_x(\lambda) \quad , \quad (26)$$

³ G. R. Keepin and T. F. Wimett, Reactor Kinetic Functions: A new evaluation. Nucleonics, 16 (10), 86 (1958).

⁴ H. C. Corben, Analysis of the Zero Power Transfer Function. Nuclear Sci. and Eng. 6, 461 (1959).

where the right-hand sides are the asymptotic feedback characteristics introduced in the first part of this paper. It is clear that if its feedback followed such a model, a reactor would be stable or unstable depending on whether the point corresponding to the asymptotic feedback characteristics was on the left or on the right of the curve on Fig. 6

The proposed stability criterion is based on the following assumptions concerning the frequency dependence of the feedback.

$$\left. \begin{aligned} |X(i\omega, \lambda)| &\leq X(0, \lambda) \\ \tau_b(i\omega, \lambda) &\leq \tau_x(\lambda) \end{aligned} \right\} \quad (27)$$

If these assumptions hold, a reactor is certainly stable if the point with coordinates $PX(0, \lambda)$, $\tau_x(\lambda)$ is on the left of the curve on Fig. 6. The converse is not necessarily true, however. If the point was on the right of the curve, a detailed determination of P , $|X(i\omega, \lambda)|$ and $\tau_b(i\omega, \lambda)$ as a function of frequency would be required before any conclusion could be reached regarding the stability. The curve described by the point representing the left-hand sides of the expressions (27) would have to be followed as a function of frequency, from the initial point $PX(0, \lambda)$, $\tau_x(\lambda)$ until it crosses the curve of Fig. 6. If it crosses this latter curve for a value of ω lower than that indicated on the curve, the reactor is stable, and conversely.

The assumptions of equations (27) are certainly valid for most reactors. They possibly would not be valid if the total negative power coefficient consisted of a large positive prompt component, and of a larger negative delayed component, as was the case in EBR-I, Mark II, or if the feedback was not given by an expression such as (4). This would be the case, for example, if at some frequency the feedback were amplified by a mechanical or hydrodynamic resonance. Note that if the feedback is linear with power, $PX(0, \lambda)$ is the so-called power override, or reactivity required to bring the reactor from zero power to power P . at constant inlet coolant temperature.

For the Enrico Fermi reactor the calculated values of $PX(0, \lambda)$ and $\tau_x(\lambda)$ at full power and full flow are respectively 50 cents and 0.7 sec. As shown on Fig. 6, this point is well on the stable side of the curve. The safety margin is seen to be large, from 50 to 97 cents* for $PX(0, \lambda)$ and from 0.7 to 18 sec for $\tau_x(\lambda)$. The assumptions of equations (27) mean that for increasing values of ω the point would move toward the left and the bottom of the graph, well away from the curve. Even if the assumptions of equations (27) were not valid, they should be violated quite drastically if the point were to reach the curve, because of the large safety margin. At any rate, with the feedback mechanism of the Fermi reactor as we understand it, i.e., without any net positive component in the power coefficient,

*For the Fermi Reactor the total available excess reactivity is 92 cents, so that the power override can certainly not exceed this value

the assumptions of equations (27) are certainly valid, and the simple stability criterion that we have obtained is just as good as that which would be obtained by a detailed calculation of $X(i\omega, \lambda)$ based on the same feedback mechanism. The conclusions are only as good as the physical assumptions, but for given assumptions the simple method is more conclusive, because its simplicity leaves no room for errors.

From the analog studies made at APDA, it appears that at full flow, $\tau_b(i\omega, \lambda)$ decreases very rapidly from its zero frequency value of 0.7 sec to 0.4 sec at 0.2 rad/sec, and then decreases much more slowly. This is very similar to what was said earlier about EBR-I, Mark III. The time constant 0.7 sec is really $\tau_{SS}(\lambda)$, and 0.4 sec is $\tau_x(\lambda)$. [See equation (21).] The delayed component of the power coefficient is caused in the Fermi reactor by the expansion of the holddown plate, which has a time constant of several minutes. The amplitude of this component, however, is quite small. When investigating the stability at other than full flow, the λ dependence should be taken into account. As can be seen from expression (15), $\tau_x(\lambda)$ increases somewhat with decreasing flow, while $PX(0, \lambda)$ decreases somewhat, if P is reduced in the same proportion as the flow in order to keep the same coolant temperature rise. For instance, at 40% flow our calculations show $\tau_{SS}(\lambda) = 1.18$ sec and $\tau_x(\lambda) = 0.6$ sec. For many reactors now in operation, both fast and thermal, the point representing the asymptotic feedback characteristics is on the left of the curve of Fig. 6, so that these reactors satisfy the stability criterion presented here. In boiling water reactors the power override, which is a good measure of the magnitude of $PX(0, \lambda)$, and which consists mostly of the reactivity compensated by the voids, can attain many dollars. The zero-frequency point is then well on the right of the curve and a detailed analysis of the frequency dependence of $X(i\omega, \lambda)$ is required to guarantee stability.

If plutonium or U^{233} are substituted for U^{235} in a reactor, and if the power coefficient is about the same when absolute units of reactivity are used, the numerical value of this power coefficient, and the abscissa of the points representing the feedback characteristics on Fig. 6, will be multiplied by about a factor of two, if the dollar is used as unit of reactivity. Since, as was said before, the curve of Fig. 6 will remain approximately unchanged, this substitution would reduce the critical power level for instability by about a factor of two.

In concluding this paper, I would like to thank the personnel of the Idaho Division of Argonne National Laboratory for providing us with the wealth of experimental data obtained with EBR-I.

Discussion of Paper
Presented by Mr. Storrer

MR. SMITH:

I wasn't aware of Mr. Storrer's conclusions. Mr. Thalgott and I came to the conclusion a few weeks ago that there was some unexplained process with a very long time constant. It didn't make any sense at all. Do you have any idea what this is? What part of the structure?

MR. STORRER:

No. I think, though, that there is no question that there is one. There is almost a factor-of-two difference between the zero-frequency and steady-state power coefficients. It could be due to vessel expansion or to something else, I don't really know.

MR. SMITH:

That is the only possible explanation.

MR. STORRER:

The long time constant may correspond to the time it takes for the vessel to expand and let the core expand somewhat.

MR. GRACE:

I wonder if this difference might be explained by the rest of the plant not having attained equilibrium. There is this very strong regenerative feedback effect through your primary circuit. I am not sure what your complete circuit looks like. This, in some cases that we have studied, has involved an amplification of the feedback effect by at least a factor of five in the weakest case, and at full power. In the lower end of the power range, where the attenuation of the temperature in going through the heat exchanger in our system is smaller, this multiplication of the feedback approaches a factor of fifty. What enters into this, of course, are the delay times of the complete plant all of the way down to the turbine in determining just how long it takes to establish this regenerative feedback through the primary circuit.

MR. THALGOTT:

I think that we do have a situation similar to this, but not precisely so. The flow through EBR-I, instead of going through a heat exchanger, and being a relatively fast-acting thing, is through a reservoir tank which holds 3,000 gallons of coolant. Our maximum flow rate is 300 gallons per minute.

The flow is up through the reactor, and there is a considerable amount of structure above the reactor core and blanket which is certainly affected by the outlet coolant temperature without having to go through the whole circuit, and the structure is rather complex up there. I would expect that changes in temperature would have a reactivity effect, but it is a little difficult to say precisely what. This is what we have to look at.

KINETICS STUDIES ON EBR-I AND EBR-II

Part I

D. Okrent
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Having been one of those who tried to calculate the physics of EBR-I Mark III before we ran experiments on it, I think I am a little bit less optimistic than Mr. Storrer in connection with the idea that EBR-II and Fermi will prove so dull after we have some experiments. I think the geometric motions particularly will bedevil the man who tries to get a quantitative fit from first principles, meaning no adjustable parameter-marked expansion or bowing.

I would like to discuss a few calculations, some of which I have had a peripheral connection with, and with some of which no connection. The first of these, performed by J. C. Carter, gives some preliminary results on the kinetics of EBR-II.

To refresh your memory, I would like to refer to Fig. 1.

Fig. 1

The reactor has a large number of subassemblies, hexagonal in shape, which are supported at the base of the reactor. There is a lower blanket, core, upper blanket, and a radial blanket. There is no support at the top of these subassemblies, so that they are free to move as long as they are not touching a neighbor.

The coolant flow is upwards, so that the base which supports the subassembly should stay at the inlet coolant temperature. We expect an extremely long time constant in this particular reactor for any effects due to recirculation.

Table I gives preliminary estimates as to the relative reactivity effects of different parts of the reactor. The expansion of the uranium is important essentially only in the core. Uranium expansion effects are small elsewhere.

TABLE I

REACTIVITY EFFECT PER $^{\circ}\text{C}$ ($\times 10^{-5}$)

	Lower Blanket	Lower Gap	Core	Upper Gap	Upper Blanket	Inner Blanket	Outer Blanket
Uranium Expansion	.024		.509		.024	.066	.014
Radial Structure Growth			1.02				
Density Change of Coolant	.21	.1	.597	.1	.21	.20	.017
Density Change of Structure	.021		.039		.021	.022	.003

The term labeled radial structure growth was arbitrarily taken as twice the axial expansion in this calculation; other times, positive or negative values have been assigned to it. It is a parameter used to describe unknown radial motions. The coolant is seen to have its major effect in the core, but its effect is not trivial in other parts of the system. We will observe later that one gets appreciable feedback from the upper part of the blanket,

since the coolant flowing through the core has been heated and has appreciable temperatures in this region. Below the core, it provides a negligible contribution.

We will find also that the inner and outer radial blankets are trivial in their contributions.

Table II shows some of the estimated feedbacks for the different regions, i.e., lower blanket, lower gap, core, upper gap, upper blanket, and inner and outer blankets. The contribution in $\delta k/k$ due to the uranium or coolant is listed.

One of the most interesting things to be observed is that the phase in the core for uranium and sodium is rather similar. Also in the upper gap and the upper blanket, where one gets appreciable additional contributions of reactivity due to the sodium, the phase again is not very much

TABLE II

		$\omega = .1 \text{ rad/sec}$		$\omega = .5$		$\omega = 5$	
		ϕ	$-\Delta k/k_{\text{eff}}$	ϕ	$-\Delta k/k_{\text{eff}}$	ϕ	$-\Delta k/k_{\text{eff}}$
L. B.	Δk_{U}	-46.8	.000	- 36.6	.000	- 72.4	.000
	Δk_{Na}	-47.2	.000	- 37.9	.000	- 89.6	.000
L. G.	Δk_{Na}	-45.2	.000	- 37.9	.000	- 89.7	.000
Core	Δk_{U}	-42.5	.216	- 26.5	.116	- 41.9	.073
	Δk_{Na}	-42.7	.138	- 26.9	.073	- 43.6	.045
	$\Delta k_{\text{Rad. Exp.}}$	-42.7	.285	- 26.9	.151	- 43.6	.094
U. G.	Δk_{Na}	-42.9	.058	- 27.5	.030	- 47.7	.018
U. B.	Δk_{U}	-45.5	.007	- 42.8	.004	-127.6	.001
	Δk_{Na}	-43.5	.059	- 29.3	.030	- 52.7	.018
I. B.	Δk_{U}	-61.5	.007	- 83.8	.002	- 87.0	.001
	Δk_{Na}	-67.5	.006	- 87.8	.002	-119.5	.001
O. B.	Δk_{U}	-63.0	.002	-106.3	.001	- 88.3	.000
	Δk_{Na}	-64.3	.003	-107.1	.001	- 88.6	.000
Total		-43.6	.79	- 29.3	.41	- 46.0	.238

different from that in the core. It must be noted that these are reactivity-weighted phases, performed in the manner that Mr. Storrer has outlined, but done automatically for us on the analogue computer. So we are considering the average contribution for any particular region. And while there might be a considerably greater phase difference between the coolant leaving the top of the upper blanket, and the same coolant in the core, the former region doesn't contribute much reactivity in this model. Hence, the EBR-II looks like a relatively simple reactor within the confines of this thermal model.

If the EBR-I had the same structure as we have assumed here, and if there were no feedback due to supports above the upper blanket or things of this sort, one would expect it to have generally similarly response characteristics to those outlined here for EBR-II.

Of course, the coolant flow is lower in the EBR-I, and one might get somewhat bigger phase changes between core and upper blanket, but the trends should be similar.

Figure 2 presents the results of the calculation of the feedback at full power and full flow by this model of EBR-II. The zero-power curve is given for reference. The difference between the two calculations depends on whether or not a reactivity loss due to radial structural expansion is assumed. When active, such an effect is assigned the time response of the core coolant. If it had had the opposite sign, in other words, if it added reactivity, this curve would tend to move back towards the zero-power

Fig. 2

curve. Everything happening of importance here has about the same time constant, the same phase behavior.

What we will see soon is that some of the early calculations for the EBR-I, where we took the same simple picture of the reactor in advance of experiments, has a similar "uninteresting behavior."

Now Fig. 3 shows how the phase of the power behaves in this calculation of EBR-II. Here is the zero-power curve, and here is the phase with no radial expansion. If one assigns a reactivity loss to radial expansion, the result is this.

Fig. 3

Figure 4 shows the results of an early calculation on EBR-I. Here is the zero-power curve. It looks a little different because this figure goes down to much lower frequencies. The amplitude for full power and flow

Fig. 4

just stays below the zero-power curve, and not much of interest is observed. The same similarities exist for phase.

Unfortunately, when the EBR-I, Mark III was run, it exhibited a curve more like this, with a slight bump in amplitude, which the very simple lumped axial model was not sufficient to explain.

In the absence of success with this relatively simple picture for the EBR-I, a variety of approaches have been taken. Carter has already reported at an ANS meeting on his success in fitting a host of EBR-I, Mark III experiments with a synthetic model, which originated by adding a factor empirically to the axially lumped thermal model, and was developed into a friction-stiction picture which could be described by differential equations leading to the same factor. Using only a pair of parameters, fixed once and for all by a single experiment, this model gave remarkable agreement with most of the experiments on the rigid core, including some non-linear effects.

Fred Storrer later produced an analytic solution of the axial and radial variation-type thermal model, and John Long of the Idaho Division of ANL applied this in calculating the temperature distributions which were measured in the EBR-I, getting very good agreement, as indicated in the next several figures.

Figure 5 reports on a measurement on the axis of the fuel pin near the core center. We see the amplitude and phase of the temperature are in very good agreement with calculation.

Fig. 5

Temperature on Axis of a Fuel Pin, 1.7 in. above Core Center. Full Power, Full Flow, Low Inlet Temp.

Figure 6 shows the coolant amplitude and phase at the core center. I would like to note that near the core center we have phases of 60 or 70 degrees.

Fig. 6
Temperature in Coolant. Core Center.
 $\frac{1}{3}$ Power, $\frac{1}{3}$ Flow, High Inlet Temp.

In Fig. 7, where measurements are reported for the exit from the upper blanket at full flow, the phase is not very different from that in the core. However, at one-third flow, which is in Fig. 8, where there is considerably greater transport lag at the same position, the phase is closer to 200 than to 80 degrees. So we have experimental evidence of transport lag, in good agreement with calculation. One hoped that on assigning reactivities corresponding to the various temperatures in the manner that Mr. Storrer outlined, good agreement would be obtained with transfer function experiments. However, in the first effort made by Mr. Long, wherein he did not assign any structural effects, but took reactivity contributions only from the thermal model, he did not get adequate agreement.

Fig. 7
Temperature in Coolant. 7 in.
above Core. $\frac{1}{3}$ Power, Full
Flow, High Inlet Temp.

Fig. 8
Temperature in Coolant.
7 in. above Core. $\frac{1}{3}$ Power,
 $\frac{1}{3}$ Flow, High Inlet Temp.

Now, actually, Thalgott and I were just discussing this yesterday, and there is some feeling that if one assumes that there is a structural effect due to the coolant leaving the top of the upper blanket, which does not seem impossible when you consider how the reactor is built and clamped, one could get both the greater amplitude and the shift in the phase required to move Long's calculations into closer agreement with experiment. I have a feeling that eventually we may find ourselves with two rather different models, each of which has some empirical aspect, both agreeing with experiment in large part.

The last thing I would like to discuss very briefly is the relatively good agreement between experiments on bowing effects and the crude calculations that have been made. Pierre Clauzon of Saclay, who was here for a year, made some calculations of the bowing effect, using the simplest method that we could assign for each aspect of the problem. By assuming that the fuel element was pin supported at each end, calculating the amount of bowing that would occur in accordance with the power distribution, and assigning reactivities on the basis of one-group perturbation theory, he estimated the shift in feedback, or the shift in transfer function, which should occur for the EBR-I, Mark III with bowing permitted.

I might note that at full power and full flow, this calculation predicts an equivalent change in core radius, due to bowing of only 0.037 cm. So these are really very small motions that can produce the effects that Mr. Smith was talking about.

If one extrapolates the measurements made with two hex subassemblies sheared to six subassemblies, the $\delta k/k$ shift at zero frequency turns out to be of the order of 0.0022 from experiment. From the simple calculation, this number came out 0.0026. So, as regards order of magnitude, there is reasonably good agreement.

Another area of good agreement arose in considering blanket bowing. Experimentally there were no changes in transfer function observed within error when a certain number of blanket rods were sheared and allowed to bow. The same was predicted by theory.

Figure 9 shows the results of an early "bowing" experiment. The squares represent a tight core; the circles represent 12 core elements sheared. The differences are very small. But there is a decrease in amplitude of the transfer function at low frequencies, and a slight increase at the higher frequencies in accord with a slight positive coefficient. Figure 10 shows how the phase gets slightly more negative at low frequencies.

Fig. 9
Amplitude of Feedback

Fig. 10
Phase Angle of Feedback

If one does a calculation of the reactor behavior with bowing, he finds about the same effect. Figure 11 will show the results of several calculations having different time constants for the bowing feedback.

Fig. 11
Amplitude of Feedback for
Several Time Constants

Very slight dependence on the time constant is observed. All of these relatively simple calculations seem to show a systematic deviation from experiment in that the frequency at which the no bowing and bowing transfer functions cross is smaller experimentally. The variation in time constant was performed to see whether one could get agreement between experiment and theory. Clearly this is not the explanation. I, perhaps, should note that the experimental uncertainties are relatively great and that the crossover point is poorly defined.

Figure 12 will show the same sort of result. Again, there is very little dependence of the phase on the time constant. The full line indicates what the calculation predicts if all the core were permitted to bow. The reactor would still be stable according to the calculation.

Fig. 12
Phase Angle of Feedback for
Several Time Constants

KINETICS STUDIES ON EBR-I AND EBR-II

Part II

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We are in the same situation that Mr. Storrer is as far as how interesting our reactor appears likely to be. Of course, in view of this discussion about delayed negative coefficients, perhaps we are being a little too optimistic, because possibly something will show up in structure that we don't at present anticipate. Certainly from the standpoint of what we have taken into account so far, nothing very exciting seems to be in prospect. Most of the effort so far has been directed towards understanding what is generally called the thermal feedback, that is, simply what the response of the reactor to unrestrained thermal expansion would be. This, of course, involves calculation of the temperature distribution in the reactor, and also the reactivity change associated with expansion of its various parts.

We are using two different calculational techniques. We have an analogue setup, and also a digital computer setup that uses Mr. Storrer's method of integration, and we are comparing the two. We have to depend on the analogue computer to handle nonlinear effects in oscillator experiments and also to make calculations for rod-drop experiments, while the Storrer type of calculation is suitable for a linear system which is oscillated.

So far the agreement between the Storrer calculation and the analogue setup seems to be good for a rather crude division of the pins in the reactor, even up to frequencies of ten radians per second (which somewhat surprises me).

I might explain, for those of you who are not familiar with it, that the EBR-II fuel element is a pin-type element consisting of a uranium pin, a sodium bond region, and a stainless steel clad. Ninety-one of these pins are contained in hexagonal subassemblies which are 2.3 in. across flats. The calculations I am referring to are for individual pins, and then it is necessary to sum up various representative pins over the whole reactor to obtain the complete feedback.

In addition to the thermal feedback, there is expected to be a small amount of bowing. We haven't really put our backs into trying to understand what this is liable to do to the reactor yet. Previous calculations made for the EBR-II Hazards Report indicate that the effect would be very small. The subassemblies are spaced by buttons, with very small clearances, so it is expected that the overall effect can certainly not be very large. It is at present about the only appreciable source of nonlinearity that seems possible.

In addition to the temperature calculation, we have experiments under way at Idaho at present in ZPR-III to obtain a better mapping than previously available of various reactivity coefficients. The fuel, sodium, and steel are being studied experimentally. The calculation of sodium and steel reactivity coefficients is quite a thorny problem because of the effect of the scattering resonances that disturb the spectrum. Although theoretical work is underway on this problem, because of the difficulty of the calculations, we are very interested in obtaining a rather good mapping, particularly for the sodium, which does have quite an important reactivity effect, comparable to the fuel.

A source of uncertainty in getting even the thermal feedback is what the radial expansion of the core is going to be because it depends on what clearances are between the subassemblies, and while we feel that probably there won't be much clearance at all at the operating temperature, this might turn out to be an adjustable parameter, because it is a big effect. The radial expansion effect is potentially the order of magnitude of that of the axial expansion. It is not twice as big because of the fact it varies axially because it depends largely on the sodium temperature.

Experimental work is probably not going to get under way for about another year yet. The reactor is probably not going to go critical before about the end of the year. Power operation won't ensue until some time subsequent to that, so it is expected it will be at least another year before measurements can be made at power.

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SESSION IV

PILE OSCILLATOR REACTIVITY MEASUREMENTS

Chairman: F. Martens

Secretary: H. A. Sandmeier

DETERMINATION OF IRRADIATED FUEL CROSS-SECTIONS BY REACTOR MODULATION

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Introduction

For a number of years AERE and AECL have been carrying out a joint program of measuring the change in reactivity of reactor fuels as a function of irradiation. For these studies the irradiations were carried out at Chalk River and the reactivity measurements at Harwell. To facilitate these measurements, a reactivity-measuring facility has been built at Chalk River. Those who have contributed to the program are W. J. Henderson, G. L. Keech, R. M. Pearce, D. H. Walker, A. Okazaki and myself.

General Description

The reactor is a pool type using 10 plate MTR-type fuel elements, light water moderator, and a canned graphite reflector. Heavy water and beryllium were excluded from the core to cut down photoneutron production by gamma rays from the irradiated samples.

Fig. 1. Cross Section of Core

The chamber is shielded in the direction of the core by approximately 5 cm of lead.

Figure 1 shows a horizontal cross section of the reactor core. At the center is an aluminum tube used to guide the sample carrier. It is surrounded by plug-in fuel elements, then plug-in graphite elements, and then an 18-in. thickness of graphite.

The ionization chamber used to obtain a signal for analysis is situated in the graphite reflector, approximately 50 cm from the center of the core. Its efficiency for detecting a neutron from a fission in the core is approximately 3×10^{-4} . The gamma-ray efficiency of the chamber is low, being 2×10^{-11} amp in a gamma field of 1 r/hour. The

Fuel elements surrounding the sample tube may be removed and replaced with graphite plug-in units or else the volume allowed to fill with

water. In this manner the ratio of the reactivity worth of fissile material to that of a pure absorber may be altered. If the internal reflector is water, this ratio if calculated to increase by a factor of ten at the center as the radius of this reflector is increased from 4.5 to 13.5 cm. However, as the ratio is increased, the sensitivity drops. This sensitivity for U^{235} is calculated to decrease by a factor of five for this change in radius.

The neutron energy spectrum at the sample can also be altered by changing the internal thermal column. However, the presence of the guide tube filled with water controls the spectrum to a large extent. Measurements have been made in collaboration with C. B. Bigham of the spectrum at the center of the guide tube when fuel is immediately adjacent to the tube. We refer to this as the one-hole system. Cadmium ratios taken with indium and manganese foils yield an r value of 0.025 if the Maxwellian portion of the spectrum is assumed to have the same temperature as the water. The r value is essentially equal to the fraction of the total neutron density which lies in the $1/E$ portion of the spectrum. Until now we have only used this one-hole system.

Fig. 2
A Vertical Cross Section
of Core

Figure 2 shows a vertical cross-sectional view of the reactor core. The aluminum tube passing through the center of the core guides the sample carrier. The carrier is 6 ft long and is centered in the guide tube by means of Teflon bearings.

Our measurements are made by comparing the effect on the reactor of two samples which are alternately placed at the center of the core. They are separated by a distance of approximately two feet, the height of the core. Thus, when one is at the center, the other is one foot outside the core. In order to eliminate the effect of scattering, we have tried to have the sample either at the center, where scattering has no effect, or else well away from the core. Thus, the sample is moved with a square-wave motion having a dwell period in the core of 18 sec and a transit time of 2 sec.

To obtain accurate measurements, the transit time must be maintained constant. With the present system it is measured to be reproducible to 0.05 sec. The sample carrier attaches to a drive shaft, 10 ft long, which extends to a hydraulic piston situated above the pool. This shaft may be disconnected easily either at the carrier or at the piston.

I should emphasize that the sample carrier oscillates in the tube filled with water. This is necessary, as we use the water above the guide tube as shielding when handling irradiated samples. The presence of the water complicates the system. First, it is necessary to ensure that all air bubbles are removed from the carrier; otherwise the results will be very erratic. Secondly, samples which might undergo dimensional changes during irradiation must be placed in sealed containers so that these changes will not be measured when the sample is oscillated. For some carrier designs the presence of water in the tube has the advantage that it will reduce the size of an undesired scattering signal from a sample not at the center.

Five control rods are used, four of which have absorbers similar to those used in the Bulk Shielding Reactor. Two are always completely withdrawn during operation. The core is trimmed so that the other two only extend a short distance into the core. There is a fine rod, worth approximately 1 mk, located in one of the graphite plug-in units next to the fuel.

Analysis of the Signal

Because of the square-wave motion of the carrier, Δk_{eff} is also a square-wave function except for the possible addition of spikes during the transit process. I will discuss these later.

To obtain a measurement of the difference in reactivity of the two samples we measure the amplitude of the fundamental component of the modulated flux. Our procedure is similar to that which has previously been used at Harwell and Chalk River. The steady component of the current from the ion chamber is backed off and the remaining current is multiplied by a sine function and integrated over an integral number of cycles to obtain the amplitude of the fundamental component of the flux. At the same time, the current is multiplied by a cosine function and integrated to obtain a measure of any transit signal caused by unequal transit times. Colmer, Wiblin, and Woollen have pointed out that the integration should be started at the maximum of the sine and cosine functions since this eliminates the effect of linear drifts on the integrated answer and minimizes the answer due to any drifts varying as t^2 .

The fundamental component of the flux lags the fundamental of Δk_{eff} by a phase angle θ . The multiplying sine function is adjusted to be the phase angle of the sine-cosine potentiometer until the cosine integration is a minimum when the sample has a small scattering cross section.

Figure 3 shows the circuit used to analyze the signal. The backed-off ion-chamber current is amplified by a vibrating reed electrometer amplifier and then further amplified by an operational amplifier. The output

of this amplifier is fed to one end of a sine-cosine potentiometer. The other end of the potentiometer is fed from an inverter. This allows the multiplying sine and cosine functions to oscillate around zero voltage. The output of the function multipliers are fed into integrators.

Fig. 3. Circuit

All operations are controlled by the motion of one shaft. This shaft drives the sine-cosine potentiometer, starts the integration, and controls the motion of the hydraulic piston which moves the samples.

Sample Carrier

Earlier I stated that the samples are held in a 6-ft long carrier. Two types of carriers currently in use are shown in Fig. 4.

I mentioned earlier that reactivity spikes can occur during sample transfer. As an example, this will happen if we are comparing two aluminum samples and the space in the carrier between the samples is full of water. On sample transfer we effectively exchange temporarily a void at the center for water, which causes a large change in k_{eff} . I should point out that when one sample is leaving the core the other is just entering.

Both of these carriers have been designed to minimize the size of this reactivity spike. This is accomplished by having the composition along the tube as uniform as possible. On the left is shown a Lucite carrier. This carrier is primarily for samples which have been prepared

Fig. 4. Sample Carriers

as a dilute mixture in Lucite. The main body is Lucite. Holes extend in from both ends into which the samples are placed, followed by Lucite spacers and end caps. This construction holds the samples firmly in place, which is a strict requirement for accurate measurements in this system. To this carrier two Lucite bearing rings have been added near the center, which are thought to have improved the accuracy of the measurements. Since Lucite is very similar to water, this carrier may be introduced into the core while the reactor is critical. I have already mentioned that care must be taken to eliminate bubbles of air. Since the end plugs have to be screwed in, this carrier is not good for active samples.

On the right is shown a stainless steel carrier. This carrier is simple to load, as the samples and spacers are dropped in from the top. The carrier is submerged in the pool during this loading operation to eliminate air bubbles. Irradiated samples will be removed from a submerged castle and dropped into the tube. The samples and spacers are kept in place by a push rod which is part of the drive shaft. This carrier has the advantage that the spacers may be picked to match the samples. A Teflon liner has been added to offset the reactivity effect of the stainless steel. To unload the carrier it is set into a vertical holder, tilted horizontally, and the samples and spacers ejected by a jet of water.

Figure 5 shows a typical pile response curve taken using the stainless steel carrier. An increase of power is in the downward direction. Two aluminum samples, one 1 in. in diameter and the other

Fig. 5
Modulation Trace
with 1" D Spacer

0.985 in. in diameter and 20 cm long, were oscillated. One-inch diameter aluminum rods were used as spacers. The difference in volume of the two samples was 1.88 cc. The spikes have been discussed previously and are caused in this case by rings machined on the samples to hold them in position and by nonuniformities in the stainless steel. The height of the spikes is 0.4% of the mean signal.

Fig. 6. Modulation Trace with
0.753" D Spacer

Figure 6 shows the trace obtained when the same two samples oscillated with a 0.753-in. diameter aluminum spacer. Large transit spikes now are present of a height equal to 10% of the mean signal. However, they are found not to affect the answer. With the use of the 1-in. diameter spacer, the integrated answer in $\Delta k/k$ was $(447 \pm 5) \times 10^{-7}$, and with the use of the 0.753-in. diameter spacer the answer was $(450 \pm 5) \times 10^{-7}$.

Accuracy

A series of measurements were made using the Lucite carrier and comparing a 24-gm sample of Al + 336 μg of boron with a pure Al sample. Since the vertical flux distribution is not necessarily symmetric, each measurement consisted of two readings, one made with each sample in the upper position. From this series of measurements the accuracy of an individual measurement was determined to be $\pm 0.04 \text{ mm}^2$ for 160 watt-min of reactor operation. For this carrier the sensitivity is 0.6 mk/cm^2 of absorber and thus the accuracy of measurement may be

quoted in terms of $\Delta k/k$ as 2.4×10^{-7} for this irradiation. This is to be compared with the calculated limit of 1.2×10^{-7} , considering the limit to be reached when the signal-to-noise ratio equals one.

A large number of samples were measured with the stainless steel carrier. The accuracy was determined to be $\pm 0.2 \text{ mm}^2$ for 400 watt-min of reactor operation. The sensitivity in this carrier was 0.22 mk/cm^2 of absorber. The accuracy can then be stated in terms of $\Delta k/k$ as 5×10^{-7} , which is to be compared with the calculated value of 0.8×10^{-7} .

The better accuracy obtainable with the Lucite carrier has been ascribed to the bearing rings added near the center and to the fixed center spacer.

To attain the quoted accuracy with the reactor operating at 10 watts, the time to load the carrier, oscillate it, reverse the sample positions, and re-oscillate, was 35 minutes for the Lucite carrier and one hour for the stainless steel carrier.

The Program

Our main program is the measurement of the change of reactivity of fuels with irradiation. This involves oscillating samples, then irradiating them, and then remeasuring their reactivity effect on the reactor. To date we have only made the pre-irradiation measurements at Chalk River.

Fig. 7. U to 0.6 n/kb

We are interested in carrying these measurements to higher irradiations.

Figure 7 illustrates the type of measurement which interests us. This is a measurement of the reactivity change of natural uranium as a function of irradiation from our joint program with AERE. The abscissa is the irradiation level in n/kb, extending up to 0.6. The ordinate is the change in the effective absorption cross section in barns per initial fissile atom, which is a function of the changing composition of the sample, the relative worth of fast to slow neutrons in the reactor core in which the samples are measured, and the changing flux in the sample. The top curve shows the experimental results, and the bottom that predicted by

Table I lists some of the samples which have been prepared. These include natural uranium, uranium oxide, and thorium metal. These samples are being irradiated to a maximum of 2.5 n/kb. Samples of Th containing an amount of U^{233} equal to 1/85 of the Th^{232} content are also being irradiated up to 2.5 n/kb. This quantity of U^{233} is equal to the equilibrium amount which would exist if Th^{232} were not being destroyed. These samples will allow a better study of the buildup of fission products than in the pure Th samples. In the latter, large reactivity changes occur because of the growth of U^{233} . This is accompanied by large flux depressions in the sample which complicate the interpretation of the measurements.

TABLE I

a. Fuel Samples for Irradiation

Material	Weight (gm)	Size (cm)	Upper Limit of Irradiation (n/kb)
Natural U	150	1.5 D x 6	2.5
UO ₂	210	1.5 D x 12	2.5
Th	51	1.0 D x 6	2.5
Th + U ²³³	55 gm Th 645 gm U ²³³	1.0 D x 6	2.5
Th + Al	100 gm Th	3.5 D x 10.2	1.0

b. Calibration Samples

Material	Weight	Size, cm
Th + Al + U ²³³	100 gm Th 590 gm U ²³³	3.5 D x 10.2
Th + Al + U ²³³	100 gm Th 400 gm U ²³³	3.5 D x 10.2
Th + Al + U ²³³	100 gm Th 200 gm U ²³³	3.5 D x 10.2

Samples of U, UO₂, Th, Th + U²³³, required with varying amounts of poison and fissile matter.

Another series of Th samples has been made with Th dispersed in Al. They will be irradiated to 1 n/kb. Because of the dilution of Th in these samples, large flux depressions will not occur as U^{233} builds up, and thus the interpretation of the results is simplified.

To assist in the analysis samples have been made containing a range of amount of U^{233} . These will not be irradiated. More samples have yet to be made with U, UO_2 , and Th containing varying amounts of poison and fissile material.

Samples of aluminum alloys of U^{235} , U^{233} , and Pu^{239} are also being irradiated to measure the fission product absorption after an irradiation of 2 n/kb. These samples are listed in Table II. They contain approximately 1 gm of fissile material. The accuracy of this measurement of the fission product absorption will be approximately 5%. The estimated barns per initial fissile atom, bifa, after the irradiation are listed as 176 for U^{235} , 31 for fission products, and 75 for the Al. The initial and final U^{235} contents will be determined to 0.1% by the chemists. The reactivity changes in the aluminum are being followed by dummy samples.

TABLE II

Σ_a FISSION PRODUCTS AT 2n/kb		
Samples: Al alloys 1.5 cm D x 12 cm		
<u>U^{235}</u>	<u>U^{233}</u>	<u>Pu^{239}</u>
1 gm	1 gm	1.5gm
Accuracy desired $\pm 5\%$		
For U^{235} at 2n/kb in BIFA		
U^{235}	F.P.	Al
176	31	75
CALIBRATION		
For U^{235}		
Additive to Al	Amount, gm	
U^{235}	0.25 $\rightarrow$ 1.5	
B	.01 $\rightarrow$.06	
$U^{235} + B$	0.4 of U^{235}	
	.0015 $\rightarrow$.006 of B	

Samples containing varying amounts of U^{235} , boron, and boron plus U^{235} have been made to assist in the analysis. I list only those for U^{235} .

The chemists have also extracted the fission products from one of the uranium samples used in the joint AERE-AECL program, and the absorption will soon be measured.

A considerable amount of chemistry is involved in these experiments. Most samples are made in duplicate and one destroyed for analysis. After irradiation the samples are also chemically analyzed.

Figure 8 illustrates the type of sample which is being used. The top and bottom fittings are dictated by the irradiation facility into which it fits. The cobalt monitor is used to measure the irradiation. Because of the odd shape it is necessary to place it inside the can shown on the left in order to oscillate it, and to make the measurement independent of dimensional changes. Assembly of the irradiated samples in these cans will be done remotely in a cave.

Fig. 8
Sample + Can

Discussion of Paper
Presented by Mr. Craig

MR. WASSERMAN:

I would like to make a very brief comment about your system. It again was quite similar to many of the other systems used, in that it is an integrated type of cross-correlation system. I met a man at Oak Ridge by the name of Colomb who had been at the swimming pool reactor in Switzerland. He described one additional wrinkle to a system of this sort.

You made mention that in your system you take some pains as to the phasing of your switching and so on, in order to help eliminate the effects of drifts on your results. Mr. Colomb mentioned that someone at Reactor Limited, Switzerland, had done an analysis (I think Mr. Christensen suggested it might have been the gentleman by the name of Schmid who did this) in which, by multiplying the signal from the chamber by a square-wave signal where the amplitude of successive changes went as the binomial coefficients, he was able to make a much better discrimination against the effect of reactor drift than by any other means that they had tried. I wondered whether this had been brought to your attention.

MR. CRAIG:

No, we were not familiar with this. Thank you for telling us.

MR. WASSERMAN:

I thought I would get it on the record.

THE AETR OSCILLATOR SYSTEM

H. A. Morewitz and S. G. Carpenter
 Atomics International
 Canoga Park, California

(Paper Presented by Mr. H. A. Morewitz)

Introduction

The Advanced Epithermal Thorium Reactor (AETR) concept is being developed by Atomics International for the Southwest Atomic Energy Associates (a group of southwest utility companies). The main purpose of the present phase of the effort is to obtain basic reactor physics data for U^{233} -thorium systems in the intermediate and fast neutron energy regions. To this end, a critical facility is presently under construction and will begin operating sometime this summer. A honeycomb, split-table critical assembly machine, similar in design to the ZPR-III at ANL, will be used to investigate the reactor spectrum average cross sections of a variety of materials over a range of spectra. The spectra will be varied by changing the ratio of fuel to moderator in the core. Since only 25 kg of U^{233} is available from the AEC for our use, it has been necessary to design a multiregion, spherical assembly in which the U^{233} resides in a small central "Test Region." The critical assembly consists of the spherical test region surrounded by spherical shells of buffer, decoupler, thermal driver and reflector, as shown in Fig. 1.

Fig. 1.

The driver region material is arranged so that springs take up the thermal expansion between the fuel plates and the solid polyethylene moderator. To reduce the temperature coefficient of reactivity further, the hydrogen-to-uranium ratio of the driver has been chosen so that the reactivity remains constant for small changes in the ratio. In addition, the assembly room is temperature controlled to within $\pm \frac{1}{2}$ F.

Because of the limited region in which the flux and adjoint are similar to those of a full-scale core, an oscillator type reactivity experiment is required.

A "square-wave" oscillator experiment will be performed on the assembly to measure the reactivity coefficients of a variety of materials in the Test Region. However, most of the materials to be tested have large cross sections and are of importance in the driver region. Thus, rapid transport of the sample through the driver-reflector, buffer-decoupler regions, combined with cadmium shielding of the sample entry tube in these regions will be required. In addition, signal artifacts produced during the sample transit time must be eliminated from the data.

Mechanical Oscillator

The oscillator mechanism consists of two pneumatic cylinders which drive lead-filled shafts which position the samples in and out of the assembly. The lead-filled shafts are split for ease of closure of the split-table assembly machine. They contain two pockets, one each for a standard (or dummy) and a sample, so that reactivity differences can be measured. The lead filling eliminates neutron streaming out of the test region through the sample transfer hole and provides an unmodified spectrum in the test region.

In order to reduce self-shielding and to study the self-shielding and mixing effects of the materials of interest, the samples have been made in the form of circular foils whenever possible. These are one-half inch in diameter and usually 20 mils thick. They stack into 2-in. long cylindrical sample containers. Flux plots within the sample container can be obtained for those foil materials which can be activated. The flux can also be measured by interspacing the sample material with other foils.

The oscillator mechanism inserts and withdraws the sample into the assembly in ~ 100 msec. (This is a small fraction of the 30-sec oscillator period and tends to reduce signal artifacts obtained during each transit.) The pneumatic cylinder system is adjusted to position the sample rapidly, but with little bounce at each end of its travel.

Electronic Instrumentation

An instrumentation system has been developed which controls the period of the square-wave oscillator and collects and displays the periodic signal so that signals produced during the sample transit time can be observed.

This system is similar in principle to the "RC Memory Commutator" developed by Beard and Skomal¹ for use in geophysics research. More recently, an improved analog device has been used by D. M. Hunten² in studying the spectra of the twilight. The present system is built from commercial units and utilizes digital storage.

Principle of Operation

A block diagram of the system is shown in Fig. 2. The BF_3 -filled ion chamber is located in the reflector region of the assembly near the location of maximum flux. It detects the pile power level and presents this information to a wide bandpass filter set to attenuate strongly the steady power level and to pass only the modulated components. This latter component is amplified by a vibrating reed electrometer whose output feeds a voltage to pulse converter. The pulse converter is an integrator so designed that the input voltage frequency modulates a blocking oscillator. The output frequency (0-100 kc) is proportional to the input voltage (0-1 volt) to within 0.01%. The pulses from the converter are fed to a 200-channel time analyzer. The pulses from the converter are fed to a 200-channel time analyzer.

Fig. 2. Reactivity Oscillator Instrument System

The same timer which triggers the instroke and outstroke of the mechanical sample oscillator also controls the channel widths of the time analyzer. The instroke is triggered simultaneously with the start of channel 1 and the outstroke is triggered simultaneously with the start of channel 100. Thus each cycle is divided into 200 equal time increments, the average value of the signal and noise in each increment being stored in a single channel. The equiphase data from successive periods of oscillation are accumulated in the same storage channels so that the coherent signals add constructively, and the noncommensurate frequencies, and noise add

¹C. I. Beard and E. N. Skomal, Rev. Sci. Instr. 24, 276 (1953).

²D. M. Hunten, Can. J. Phys. 38, 346 (1960).

destructively (i.e., fill all channels uniformly). Thus the original signal-to-noise ratio builds up as the square root of the number of repetitions. After 100 complete cycles the signal-to-noise ratio has built up a factor of 10 and the data has been summarized in a form useful for further digital computation.

A PROPOSED FUEL ASSAY REACTOR

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(Paper Presented by Mr. Martens)

There is a need for a device that can test nuclear reactor fuel elements nondestructively to determine the total amount of both fissionable and parasitically absorbing materials in a completed, unirradiated fuel element. The device itself is a reactor, called a Fuel Assay Reactor.

Such a reactor would be extremely valuable for several reasons. In the present state of reactor science, there are still certain discrepancies between experimental and theoretical results. Part of the trouble is due to the lack of accuracy in determining the content and distribution of fuel in the elements. The manufacturing of fuel elements requires many different processes, some of which cause nonuniform fuel distribution within the element. It is also possible that neutron poisons may be introduced into a fuel element inadvertently. For example, there have been cases where the welding materials have introduced poisons of high cross section into elements. Accurate post-assembly assay of the fuel and absorbers within fuel assemblies would help immensely in making accurate reactivity calculations.

The other very important reason for having such a reactor concerns economics. The transfer of responsibility of all fissionable material to partial control by private industries poses financial accountability problems. A system for assaying the fuel content of assembled elements would eliminate one troublesome source of discrepancies. In this sense, such a system would perform in the nuclear industry the same function as an assay office in the precious metals trade.

There are a number of assemblies presently used in the USA for assaying materials used in reactors. All of these, however, are used in extremely specialized cases, such as the testing of components designed for insertion into particular production piles. But a versatile assembly to assay accurately the amount and distribution of fuel and absorbers in a wide variety of shapes, sizes and enrichments presents a much greater problem than the construction of one of these specialized units. This paper discusses the major problems in the design of a versatile reactor.

The choice of the type, geometry, moderator, etc., depends on several factors, such as sensitivity, long-term stability, and the size of elements to be tested. A general statement may be made that for a bare reactor of given height, maximum sensitivity to absorption exists when the

system is critical at minimum effective thermal absorption. Maximum sensitivity to fission neutron creation occurs for minimum critical mass.

Perhaps the most severe limitation is imposed by the size of the fuel elements on samples to be tested. If only very small samples are to be tested, the choice of type would probably be the light water-moderated, highly enriched uranium reactor, because criticality could be achieved with a smaller critical mass than with a graphite system. However, if the samples are very large, the choice might be either a graphite or a heavy water-moderated assembly. Not only would the critical mass necessary to achieve criticality in the large reactor be smaller with either graphite or heavy water as compared to light water, but the parasitic absorption of the moderator would also be much less. Thus the sensitivity to both fission and absorption would be increased. Beryllium has not been considered because it appears that if a solid moderator is desired, graphite would have certain advantages, especially with regard to cost.

The geometry and sensitivity are best studied by looking at equations relating the reactivity change to a perturbation in a pile. If the change that is introduced is small, perturbation theory may be used to give the change in reactivity:

$$\frac{\Delta k}{k} = \frac{\int_{\text{sample}} [\nu \delta \Sigma_F \phi_{th} \phi_f^* - \delta \Sigma_a \phi_{th} \phi_{th}^*] dV_p}{\int_{\text{core}} \nu \Sigma_F \phi_{th} \phi_f^* dV} \quad (1)$$

The equation is valid only for small values of $\Delta k/k$, so there is no appreciable shift in the flux or adjoint flux patterns. Because this must be small, a comparison test must be used. That means a standard for each type of element must be produced.

The equation has two unknowns, $\delta \Sigma_a$ and $\delta \Sigma_F$, so two tests must be made to separate the two unknowns. The ratio of the fast and thermal importance functions must be changed for these two tests, and the greater the change in ratio, the better will be the discrimination.

To illustrate how this ratio may be changed, consider putting a sleeve which absorbs some of the incident thermal neutrons around the test position. This will lower the thermal flux, and also the thermal importance. The fast importance remains essentially unchanged.

The sensitivity to both absorption and fission is increased if the sample is placed in a region of high thermal flux. This may be accomplished by introducing an internal reflector region which is free from fuel. This internal reflector is often referred to as a "flux trap."

Not only will this "flux trap" increase sensitivity, but it also provides a fuel-free region in which the changes in the adjoint flux distributions may be made.

The most promising of reactivity determinations for fuel assay work is the "automatic flux control" (AFC) method first suggested by D. R. de Boisblanc. In this method the flux level of the reactor is kept constant by means of a servo system which regulates a lightly absorbing control rod. Any change in the reactivity of the pile will result in a change in the position of the rod. The magnitude and direction of this position change will be a measure of the magnitude and sign of the reactivity perturbation.

Using the AFC method as with a pile oscillator, one must replace a standard with a stock fuel element while making the measurements. The initial or zero condition of the reactor is determined with the standard fuel element in place, and the perturbation is introduced by the variations between the stock and the standard elements. The oscillator method requires that data be taken continuously. A channel containing the standard and the test elements has to be moved forwards and backwards in a periodic fashion. Not only is this system mechanically complex, but it introduces large perturbations caused by the end fittings, which are usually made of aluminum or zirconium. These large perturbations may well overshadow the small perturbation caused by changes in fuel and poison content in the active section of the test element.

In the AFC method it is not necessary to accumulate data during the sample transfer period; in fact, often the data accumulation must be stopped during this period to allow the reactor to regain equilibrium after a change is made.

For safety, the insertion of reactivity must not exceed a certain rate. Since the rate of increase of the neutron flux is directly dependent on the rate of reactivity insertion, one must insert the test element at a certain speed consistent with the maximum expected deviation of reactivity from the standard element. This does not cause any difficulties in the AFC method. In the oscillator method, however, one might get a very low frequency from which one might not be able to extract the pertinent information. The pile period method is very unlikely to be of advantage, since it is quite dependent on the reactor remaining extremely stable for a long period of time; a convenient schedule for the transfer of large elements would make reactor stability difficult to maintain.

The AFC method is about 10 to 15 per cent more sensitive than the others. If there were good reasons for choosing one of the other methods, this advantage would not be significant. However, with all of the other reasons set forth, we conclude that the AFC method is the best reactivity-measuring technique for this type of experiment.

The precision expected from the reactor measurements is extremely high. There is a theoretical limit to the precision of reactivity measurements because of the statistical variation of the neutron population within the reactor core. This population is generally considered to remain constant, but in reality it varies about a mean value according to well-known statistical processes:

$$\sigma = 0.89/\sqrt{f} \quad . \quad (2)$$

Dr. Cohn has shown that the standard deviation in making a single reactivity measurement is

$$\sigma = 5.1 \times 10^{-6} / \sqrt{wt} \quad , \quad (3)$$

where w is the power in watts, and t is the testing time in seconds. The standard deviation from the exact value of σ will be small if either the power or the testing time is large.

As in many other engineering problems, there are restrictions placed on both values. We cannot operate the reactor at too high a power, because the amount of activity induced in the sample must be kept low. Furthermore, we would like to eliminate all effects of temperature upon reactivity. Thus, an actual "zero power" reactor is needed. Most reactivity-measuring facilities operate at a level of approximately 100 sec. The error of a single measurement under these conditions is obtained from Eq. (3).

$$\sigma = \frac{5.1 \times 10^{-6}}{\sqrt{10 \times 100}} = 1.6 \times 10^{-7} \Delta k/k. \quad (4)$$

Since one must make two measurements - one with the standard and one with the test element - the error or standard deviation from the difference between those two measurements is $\sqrt{2}$ times the single error, or $2.3 \times 10^{-7} \Delta k/k$.

It is not practical to try to decrease the experimental error below the statistical limit expressed in Eq. (2). The largest error in the measurement is in strong control of the limit, because the total error in any measurement is the root-mean-square of all errors. If we assume that the experimental and statistical errors add up to a total error of $10^{-6} \Delta k/k$, we can obtain the precision with which a measurement may be made.

To see what a total error of $10^{-6} \Delta k/k$ means in terms of the error in determining the grams of fissionable material in the test element, let us rewrite Eq. (4) in a way that relates $\Delta k/k$ and the mass of fissionable material m :

$$\frac{\Delta k}{k} = \alpha \frac{\Delta m}{m} \quad , \quad (5)$$

where

$\frac{\Delta k}{k}$ = the change in the reactivity of the reactor when a perturbation changes the critical mass m by an amount Δm ,

α = constant depending on the sensitivity to the perturbation of the test reactor.

For a reactor designed to make reactivity measurements, the value of α will be of the order of unity. Using unity as the value of α and a total expected error of $10^{-6} \Delta k/k$, we obtain for the error in detectable fissionable material Δm , from Eq. (5),

$$\Delta m = m \frac{\Delta k}{k} = 10^{-6} m \quad . \quad (6)$$

Thus the error is one milligram for each kilogram contained in the reactor, five milligrams for a reactor with five kilograms of critical mass. This precision will remain constant regardless of the size of the sample which is inserted. Actually, the precision will be far greater than is necessary for this type of test.

An error in the content of a standard would be a systematic error, and consequently would affect the accuracy. Therefore it is necessary to obtain elements whose fuel content is known to a fraction of one per cent. This problem is difficult, but not impossible. The standard must have the identical dimensions and composition as the test elements, but it need not be fabricated exactly as the test element was. The standard element will not be placed in a hot, liquid bath, and the fuel-cladding bond is therefore unimportant. Because there will be no heat generation, there will be no demand for good heat transfer characteristics. No physical strain will be on the element, so welding may be eliminated if a method of keeping the element mechanically intact can be found. It may even be possible to replace an alloy with foils of uranium and the alloying materials. With these allowances it should be possible to construct excellent standards.

Other factors that tend to increase the total error are positioning variations, spurious reactivity shifts within the reactor, and instrumental errors. To place alternately test and standard elements of the type used in present-day reactors means shifting and positioning a heavy load in the test region. Even with a well-designed mechanical system, there will undoubtedly be some vibration and shifting of elements within the reactor core. These shifts will cause minute reactivity changes that affect the results. If the element is not placed in precisely the same position each time a change is made, there will be some error.

There may also be virtue in distributing the fuel through the length of the reactor in such a way that the flux distribution along the axis of the reactor is uniform. Goertzel¹ has shown that such a flux distribution results in a lower critical mass than the uniform fuel distribution. Sandmeier² has made a number of calculations to increase the sensitivity of this system by changing the fuel distribution. There is probably a great deal of virtue in a nonuniform fuel distribution, especially along the length of the fuel assay reactor. The equations in the theoretical section of this report neglected the effect of scattering by the test element. This assumption is true only at a point of zero flux gradient. With an element that reaches almost the entire length of the reactor, such a condition does not exist, and differences in the scattering properties would appear to be differences in the fuel or poison content. If a uniform flux distribution were achieved, these effects would be eliminated. And further, the requirements on the axial positioning of the fuel element would be greatly decreased.

By changing the importance functions of the thermal and fast neutrons in the central region, a discrimination can be made between the lack of fuel and the inclusion of poison. Theoretically it would be possible to distinguish between several different types of poison by making several different tests with several different importance function ratios. However, in practice it will probably only be possible to distinguish between the lack of fuel and the presence of poison. In this case the inclusion of U^{238} would appear as a neutron poison and the total amount of U^{238} would not be measurable in this system. The method cannot distinguish between the various types of fissionable material, that is, between Pu^{239} , U^{235} or U^{233} . The system is only sensitive to the quantity $\nu \Sigma_F$. Consequently, it would be possible to substitute the proper amount of other types of fissionable material and balance the known fission absorption by the inclusion of other poisons. At the present time this problem appears to be purely academic, since the amounts of Pu^{239} and U^{233} available are still extremely small. At some later time this problem may become serious.

In order to determine the fuel distribution in the test element in the axial direction, a set of fast and slow neutron detectors has to be placed along the length of the reactor near the test position. By reading accurately the output of these detectors, the neutron flux distribution may be determined with sufficient accuracy to tell if there is any fuel variation along the length of the fuel element.

¹Gerald Goertzel, Minimum Critical Mass and Flat Flux. J. Nucl. Energy, 2, 193-201 (1956).

²Henry A. Sandmeier, On the Maximization of a Fuel Assay Reactor, American Nuclear Society Transactions, 1960 Annual Meeting, June 13-15.

Conclusion

The concept of a fuel assay reactor is most certainly feasible. There are no great theoretical problems which remain unsolved that would prevent the design of such a system. The state of the art has advanced sufficiently so that such a system might be built with a reasonable effort and investment of materials. The information that could be gained with such a reactor would certainly help the atomic energy program at the present time and may be mandatory in the not too distant future.

Discussion of Paper Presented by Mr. Martens

MR. CRAIG:

You are dealing with very large samples, and I understand you are moving them very slowly. I wonder if you considered moving two samples at one time, much as Pierce Walker did some years ago where they oscillated around $H/4$, say on the same bar, and separated by half the height of the reactor? This way there is no signal to develop between the two samples. You are now interested in moving them fast, and could always get rid of the large reactivity changes.

MR. MARTENS:

Well, we would probably move the two samples simultaneously, but on the completed fuel element you are going to have end fittings. At some time or other, there is going to be a condition in which the reactivity of the system is considerably different from the quiescent state, with the fuel elements in, even though you do move the two elements simultaneously.

MR. CRAIG:

I don't think these two elements are separated by $H/2$, half the height of the reactor. It would be essentially constant throughout.

MR. MARTENS:

You are thinking in terms of systems with samples that are short compared with the length of the reactor.

MR. CRAIG:

That, or else you could have them oscillating beside each other. I agree that it depends on the length of the fuel element that you are talking about.

MR. MARTENS:

There are ways in which this can be reduced. I don't think it can be made to go to zero, primarily because of the end fittings. There are always things on the end. The engineers put them on there to hold them in place, to mess up the physicist.

MR. MOREWITZ:

There is some information in the AEC programs where people have tried to develop these fuel assay reactors and tried to use them as useful tools. They spent considerable amounts of money in this area, and found that the statistics of the situation were such that all they could say was that they had fuel in the fuel elements. To be quite crude about it, the accuracy these things are capable of may be shown to be very good theoretically, but in at least two cases that I know of it turned out in the long term to be quite impractical. The exact cause of this instability in the reactor, or what have you, remains for someone else to judge, but, in as-saying PWR fuel elements before they were installed in Shippingport, all that could be said was that each fuel element was about like the next fuel element. There is no such thing as five-milligram precision, or even five-gram precision in what this thing was capable of doing. The figures that prove that these things were wonderful things to build looked very close to the sort of figures that you presented here today.

MR. MARTENS:

I just recently saw some figures by some people from Westinghouse. They showed an accuracy of 2 grams, and of fuel, and it was a very small amount of absorber.

MR. MOREWITZ:

If I recall correctly (this is subject to several years' passage of time), about all they could get accurately was that the integral value of the amount of fuel that went into PWR was about what they thought they had and not the amount in the individual fuel element. The scattering in the answers from the individual fuel elements was sufficiently large that the gentlemen who were doing this problem added all the numbers up, and then finally said, well, this is about right. You could have said that without bothering to do the experiment.

MR. COHEN:

Do you have any suggestion as to why they were apparently off by a factor of 100 in their accuracy?

MR. MOREWITZ:

No. However, there was another reactor designed for a similar purpose in the Naval Reactor Program. It was used for many years, and had about the same sort of general behavior. It might very well pay to take a hard look at what this experience is.

MR. MARTENS:

We can get fairly close to the theoretical limit on small samples.

MR. MOREWITZ:

I am talking about six-foot long fuel elements.

MR. MARTENS:

For big samples, I will admit there are going to be more problems. I am not completely familiar with what has been done, but Westinghouse did quote ± 2 grams quite recently. I don't know the reference, but I could find it for you. However, this was in a critical mockup not specifically designed as a fuel assay reactor. If they can get to two grams in such a system, I think you can do a lot better in a system which is primarily built and designed for this.

MR. MOREWITZ:

The systems I was talking of were specifically designed as fuel assay devices. There has been a recent paper from Westinghouse Commercial in which they have talked of building a fuel assay reactor, again with about the same kind of figures that you are talking about here. It certainly would pay to take a good look at the data that comes from the earlier machines in which I think the original builders expressed optimism as to how good these things were going to be. Afterwards they had to make excuses when it didn't work out well.

MR. MORRISSEY:

Was the figure of two grams that you gave precision or accuracy? I got confused somewhat.

MR. MARTENS:

The two grams, I think, is accuracy.

MR. MORRISSEY.

I think this might be the largest problem. I don't think that excellent precision means anything to people running fuel if you can't guarantee they will be paid by the AEC for the fuel they return.

MR. MARTENS

You are right Precision is the limiting factor. The precision, I believe, is more than enough.

MR. BAKER:

I would like to make a comment about the remarks of Mr. Morewitz. We have in England on both Zephyr and Zeus measured perturbation effects, as we call them, just by the very method I have described on the board, and we did in fact achieve $\Delta k/k$ of the order of 10^{-7} . The difference between our system and yours is that we have a human system. The operator merely kept moving control rods to balance the sample in the middle of the reactor.

Now as to the point of whether or not it is possible to achieve this in a big system, I think that if we had lowered a fuel element into the reactor on a piece of string, the measurement would have been quite feasible. Obviously, we would never be allowed to lower fuel in such a crude way, however. I am sure we could have moved the control rods in Zeus, losing reactivity to balance the insertion of this string-held fuel element. I feel that we would have achieved something like 10^{-7} in reactivity because Zeus control rods were replaceable to about two-thousandths of an inch. I think it is quite feasible.

Whether you can do it with a thermal reactor is quite another matter, because you do have these effects like nitrogen to be concerned with. You are saying that your method is as good as the oscillator technique, provided a new unexpected drift does not occur. Well, in Zeus and Zephyr there weren't any unexpected drifts, and we were able to do well without an oscillator. However, I am not sure that in thermal reactors and reactors of other sorts that you are in such a desirable state of affairs.

DETERMINATION OF REACTOR ZERO POWER KINETICS PARAMETERS BY FLUX NOISE ANALYSIS

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This paper will describe some studies of the frequency spectra of the statistical power fluctuations in nuclear reactors. The experimental aspects have been reported in a paper in the May, 1959 issue of Nuclear Science and Engineering, and the theoretical aspects are covered in another paper which is to appear in a forthcoming issue of the same journal. Here I shall discuss the theory first.

The theory of reactor noise has been investigated by a number of workers. However, the treatments which they have presented have been rather complicated. I wish to propose an alternative treatment which I believe to be much simpler, especially when applied to more complicated systems. The results obtained agree with those obtained previously, where applicable.

We may begin by postulating a fictitious "noise equivalent" neutron source of random strength in the reactor. This source represents the fluctuations in the number of neutrons available to the reactor caused by the natural statistical fluctuations in the rates of neutron absorption, leakage, and fission. At any instant this source may be either positive or negative, representing, respectively, an excess or a deficiency from the average. The characteristics of this source may be readily calculated using conventional random noise theory; the resulting fluctuations in neutron population then being obtained through the use of the source transfer function. The development uses one-group, space-independent, bare reactor kinetic theory. To simplify the calculation, the fluctuations in neutron population are assumed to be small compared to the average, and the delayed neutron fraction β is neglected in comparison to unity.

The production, absorption, and leakage of neutrons in a reactor may be considered analogous to the flow of electrons in a thermionic diode, because all of these processes obey Poisson statistics. Therefore, the magnitude of the noise-equivalent source may be obtained from the Schottky formula, as shown on next page. This formula was originally developed to calculate the noise in a temperature-limited thermionic diode, but may be written for the reactor case as shown. The summation is taken over all possible types of nuclear reactions which may occur in the reactor. The "spectral density" is the mean square amplitude of that part of the noise contained in a given frequency band, divided by the band width in cycles per second. It will be noted that the spectral density of the noise equivalent source is independent of frequency. This will hold true up to frequencies of the order of the reciprocal of nuclear transition times.

SCHOTTKY FORMULA

For Electrons: $\langle |I|^2 \rangle = 2 q^2 \bar{m}$

$\langle |I|^2 \rangle$ = Spectral density of current noise, amps² second

q = Charge carried by one electron, coulombs

$\bar{m}$ = Average number of electrons flowing per second

For Neutrons: $\langle |S_0|^2 \rangle = 2 \sum_i q_i^2 \bar{m}_i$

$\langle |S_0|^2 \rangle$ = Spectral density of noise equivalent source, neutrons² second

q_i = Net number of neutrons produced in the occurrence of one nuclear reaction of type i

$\bar{m}_i$ = Average number of reactions of type i occurring in the reactor per second

The specific reactions contributing to the noise-equivalent source are tabulated below:

Nature of Process	Average Rate of Occurrence	Net Number of Neutrons Produced
Nonproductive Absorption including Leakage	$\frac{n}{\ell^*} \frac{A}{A+F}$	-1
Fission giving Rise to N Prompt Neutrons	$\frac{n}{\ell^*} \frac{F}{A+F} P_N$	$N-1$
$k_{eff} = 1 = \frac{\bar{\nu} F}{A+F}$ $\sum_N P_N = 1 \quad \sum_N N P_N = (1-\beta) \bar{\nu} \approx \bar{\nu}$		

Here the only reactions considered are nonproductive absorption, leakage, and fission. Other possible reactions, such as the generation and decay of delayed neutron precursors, are a fraction β weaker than the others and thus are neglected. Here A is the macroscopic cross section for all nonproductive neutron absorption including leakage, F is the macroscopic cross section for fission, n is the total number of neutrons in the reactor, and ℓ^* is the prompt neutron lifetime. The quantities F and A are subject to a criticality constraint as shown, where $\bar{\nu}$ is the average number of neutrons, both prompt and delayed, produced per fission. The quantity P_N is the probability that N prompt neutrons will be produced in any one fission, subject to the constraints shown.

With this information at hand, the spectral density of the noise equivalent source may be calculated as shown:

NOISE - EQUIVALENT SOURCE

$$\langle |s_0|^2 \rangle = \frac{2n}{\ell^*(A+F)} \left[A + F \sum_N (N-1)^2 P_N \right] = \frac{2n}{\ell^*} \left[\frac{\overline{\nu^2} - \overline{\nu}}{\overline{\nu}} \right]$$

EQUIVALENT REACTIVITY FLUCTUATION

$$\langle |\rho|^2 \rangle = \left(\frac{\ell^*}{n} \right)^2 \langle |s_0|^2 \rangle$$

$$\langle |\rho|^2 \rangle = \frac{2\ell^*}{n} \left[\frac{\overline{\nu^2} - \overline{\nu}}{\overline{\nu}} \right]$$

The Schottky formula yields the first expression, while the application of the criticality condition and the constraint on the P_N shown previously give rise to the second expression. Here

$$\overline{\nu^2} = \sum_N N^2 P_N.$$

The spectral density of the equivalent reactivity fluctuations may be obtained as shown because a small reactivity fluctuation ρ can be looked upon as supplying to a critical reactor a source of strength $\rho n / \ell^*$ neutrons per second, so long as the fluctuations in power level remain small. The spectral density of the reactivity fluctuations thus obtained is also shown.

We may digress a moment and use the results obtained thus far to derive a quantity of considerable practical interest, namely, the error due to noise in a criticality measurement. This has been mentioned by the previous speaker. The calculation may be done as shown on next page.

The quantity we require is the standard deviation in the reactivity averaged over τ seconds, where τ is the measuring time. This is given by the average square of the reactivity integrated over a time τ from a large number of samples. This has been evaluated by Edgar Bennett of ANL as shown, reducing the subsequent expression when the spectral density of the reactivity fluctuations is independent of frequency. Substituting the previously determined value for that quantity results in the indicated expression. This may also be put in the convenient forms shown, the last one resulting from the fact that the quantity $\frac{\overline{\nu^2} - \overline{\nu}}{\overline{\nu}}$ has a value of ~ 0.8 for U^{235} . The order of magnitude in these expressions indicates that noise is not a serious hindrance to accurate criticality measurements.

ERROR IN A CRITICALITY MEASUREMENT

$$\sigma^2(\tau) = \left[\frac{1}{\tau} \int_0^\tau \rho(t) dt \right]^2$$

Where $\sigma(\tau)$ = Standard Deviation in Reactivity Averaged Over τ Seconds

$$\sigma^2(\tau) = \frac{1}{2\pi^2 \tau^2} \int_0^\infty \frac{1 - \cos 2\pi f \tau}{f^2} \langle |\rho|^2 \rangle df = \frac{1}{2\tau} \langle |\rho|^2 \rangle = \frac{\ell^*}{n\tau} \left(\frac{\overline{\nu^2} - \bar{\nu}}{\bar{\nu}} \right)$$

$$\sigma(\tau) = \frac{1}{\sqrt{Q}} \left(\frac{\overline{\nu^2} - \bar{\nu}}{\bar{\nu}} \right)$$

Q = Total Number of Fissions Occurring During Time τ

$$\sigma(\tau) = \frac{5.1 \times 10^{-6}}{\sqrt{P\tau}}$$

P = Reactor Power in Watts

Now let us return to the main development with a look at the zero-power source transfer function:

SOURCE TRANSFER FUNCTION

Kinetic Equations:

$$\frac{dn(t)}{dt} = -\frac{\beta}{\ell^*} [n_0 + n(t)] + \sum_i \lambda_i [C_{i0} + C_i(t)] + S(t)$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{\ell^*} [n_0 + n(t)] - \lambda_i [C_{i0} + C_i(t)]$$

$$\frac{\beta n_0}{\ell^*} = \sum_i \lambda_i C_{i0}, \quad \frac{\beta_i n_0}{\ell^*} = \lambda_i C_{i0}$$

$$\text{Let } n(t) = \underline{n}(f)e^{i2\pi ft}, \quad C_i(t) = \underline{C}_i(f)e^{i2\pi ft}, \quad S(t) = \underline{S}(f)e^{i2\pi ft}$$

Transfer Function

$$T(f) = \frac{\underline{n}(f)}{\underline{S}(f)} = \frac{\ell^*}{2\pi j f L + B}$$

Where

$$L = \ell^* + \sum_i \frac{\lambda_i \beta_i}{(2\pi f)^2 + \lambda_i^2}, \quad B = \beta - \sum_i \frac{\lambda_i^2 \beta_i}{(2\pi f)^2 + \lambda_i^2}$$

The source transfer function, as its name implies, relates fluctuations in reactor power level. Its development proceeds in a familiar fashion and starts with the one-group kinetic equations shown, in which the neutron population n and the delayed neutron precursor population C_i are considered to be composed of steady and fluctuating parts. Since the reactor is critical, there is no steady source. Working from these equations and the corresponding ones obeyed by the steady parts alone, and assuming sinusoidal fluctuations, the reactor transfer function may be derived as shown. It will be noted that L and B approach ℓ^* and β , respectively, when the frequency becomes large compared to the delayed neutron decay constants λ_i .

With these steps completed, the spectral density of the fluctuations in reactor neutron population may now be calculated:

OBSERVATION OF PILE NOISE

Pile Noise

$$\langle |n(f)|^2 \rangle = |T(f)|^2 \langle |S_0|^2 \rangle = \frac{2n\epsilon^2}{(2\pi fL)^2 + B^2} \left(\frac{\overline{\nu^2} - \overline{\nu}}{\overline{\nu}} \right)$$

Chamber with Efficiency ϵ Absorbs $\frac{\epsilon n}{\epsilon^2}$ Neutrons Per Second, Passes Current $\frac{\epsilon Q n}{\epsilon^2}$ Amperes

Where Q = Charge Per Neutron, Coulombs

Pile Noise Component

$$\langle |I_p|^2 \rangle = \frac{2n\epsilon^2 Q^2}{\epsilon^2 [(2\pi fL)^2 + B^2]} \left(\frac{\overline{\nu^2} - \overline{\nu}}{\overline{\nu}} \right)$$

Chamber Noise Component

$$\langle |I_c|^2 \rangle = 2Q^2 \frac{\epsilon n}{\epsilon^2}$$

Total Noise

$$\langle |I|^2 \rangle = \langle |I_p|^2 \rangle + \langle |I_c|^2 \rangle = \frac{2n\epsilon^2 Q^2}{\epsilon^2} \left[1 + \frac{\epsilon}{(2\pi fL)^2 + B^2} \left(\frac{\overline{\nu^2} - \overline{\nu}}{\overline{\nu}} \right) \right]$$

However, this is not the complete story, as the pile noise must be observed in order to be meaningful, so we must consider the process of observation. This is commonly done by means of a detector such as a boron-loaded ionization chamber, which absorbs some of the neutrons in a reactor and produces an electrical signal. Assume that this detector collects a fraction ϵ of all the neutrons lost from the reactor in any manner. The average current passing through the chamber will then be as shown, where Q is the charge transferred per neutron absorbed. Superimposed on this steady current there will be fluctuating currents which arise in two ways. The pile noise component results from the fluctuations in the reactor neutron population to which the chamber is exposed. The chamber noise component results from the statistics of the detection of the randomly arriving neutrons. Its spectral density can be obtained directly from the Schottky formula as shown. Since these two components are uncorrelated to the first order, their spectral densities add in quadrature, giving the total current noise in the chamber output. Thus the relative proportion of pile noise to chamber noise in the chamber output depends on the chamber efficiency ϵ . Thus ϵ must be adequate if the pile noise is to predominate, and if the accuracies of criticality measurement previously mentioned are to be attained. The actual requirement depends on the frequency range of the detector and need not be exceeded by a great deal, because addition in quadrature very rapidly suppresses the effect of the smaller of two components.

The last formula constitutes a complete description of pile noise, within the approximations used in the derivation. As previously mentioned, it is exactly equivalent to the expressions derived by the other workers who have used methods that are quite different, and may be manipulated according to conventional noise theory to describe any type of statistical experiment in a reactor. It is evident that the information provided by such experiments is that contained in the square modulus of the transfer function for the given reactor.

The experimental work to be described has been done on various low-power experimental reactors and critical assemblies at Argonne National Laboratory. Since these reactors operate at such a low level that power effects on reactivity do not appear, the shape of the low-frequency portions of their transfer functions would depend only on fairly well-known delayed neutron parameters, and thus would be of little interest. However, the shape of the high-frequency roll-off portion of the transfer function is strongly dependent on the quotient β/ℓ^* , commonly known as the Rossi α . Since prompt neutron lifetime measurements were of interest to the experimental programs of some of the reactors involved, it was decided to concentrate attention in this area.

The measurements were made by exposing an ionization chamber to the reactor flux, and then analyzing the fluctuations in the current signal from this chamber with a tunable bandpass filter to obtain the noise spectrum. As previously mentioned, the chamber efficiency must be adequate in order to make the experiment feasible. To achieve this, the chamber is heavily loaded with boron and is placed in a high-flux region in the reactor. The chamber used in these experiments was a parallel circular plate-type loaded with about four grams of B^{10} , in the form of both boron coating on the plates and BF_3 gas filling. Insertion of this chamber for the measurements removes a few tenths of a per cent of reactivity.

A schematic of the electronics used in these measurements is shown in Fig. 1. The output of the chamber is taken to a cathode follower pre-amplifier using a 12 J7GT tube operated with reduced filament and plate voltages to reduce grid current. A battery and potentiometer provide a bias voltage in the grid circuit to oppose the voltage drop across the grid resistor produced by the chamber current and thus to hold the tube at the correct operating point. The output of this preamplifier feeds the intermediate amplifier, which in turn feeds the analyzer. The 6SJ7 voltage amplifier stage in the analyzer drives the bandpass filter, consisting of the components L, C and R shown inside the dotted line. Different combinations of L and C can be switched in to obtain a wide range of resonant frequencies - from 2.3 to 530 cycles per second in the latest version of this equipment. The low-frequency limit is determined by the characteristics of available components, while the high-frequency limit was selected to be around that point where the pile noise in the reactors tested becomes small compared with the chamber noise, due to the roll off of the transfer function.

Fig. 1. Simplified Schematic of Pile Noise Analyzer

The inductance L is a high Q audio choke, while the condensers C are paper condensers ranging up to 12 microfarads. For each combination, R is chosen to be about 10 times the resonant impedance of the tuned circuit in order to prevent loading by the preceding stage. The Q of the tuned circuit varies from 10 to 50 for different settings.

The amplitude of the frequency components selected by the filter is measured by a feedback-type linear vacuum tube voltmeter. In theory, since the mean square amplitude is of interest, a square-law voltmeter should be used. However, it can be shown that the reading of a square-law voltmeter and the square of the reading of a linear voltmeter, when averaged, will always be proportional on a pure Gaussian signal. The linear voltmeter thus gives the spectrum correctly, and is a bit more convenient than the square-law circuit. The indicating meter has a large condenser which could be switched across it to provide a time constant of about 1 second, and a shunt to provide a signal for a 10-millivolt Brown strip chart recorder. To eliminate hum, all supply voltages for the equipment are obtained from dry batteries, except for the filaments of all tubes but the preamplifier, which are powered by a storage battery. Other precautions to avoid hum include the grounding of the setup at one point only (at control console) to avoid ground loops.

In order to obtain a correct noise spectrum it is necessary to make corrections for the frequency response of the equipment. That is dependent upon many factors, including the resonance curves of the filters,

the frequency response of the amplifier, and the collection properties of the chamber. To allow for all these factors, the system was calibrated with white noise in the form of a 40-curie Co^{60} source. With the chamber exposed to the source, readings were taken at all the filter settings to be used in subsequent runs. Then, by dividing readings obtained from a reactor by readings obtained for the Co^{60} source at corresponding frequencies, the reactor noise spectrum would be directly obtained. Calibration has also been done by placing the chamber in a very low-flux portion of a reactor. In such a case, the pile noise component is small and a white noise spectrum is obtained. Care must be taken to operate the chamber in a saturating condition and to use the same polarizing voltage for all runs. The voltage used is only just enough to give saturation in order to reduce microphonics in the chamber.

Since the amplitude of each frequency component of the noise is itself fluctuating, it is necessary to average it over a period of time to obtain an accurate reading. Thus, during each run, voltmeter output is recorded for about 3 min at each frequency setting. The recorder trace is read at about 1-sec intervals and the points are averaged to give the reading.

Since the frequencies observed are large compared to the delayed neutron decay constants, the noise spectrum has the shape $A + B/(\alpha^2 + \omega^2)$. In order to extract a value of α , the spectrum is fitted to this three-parameter function by means of a special least squares code written for the IBM-704 Computer.

Figure 2 shows a typical noise spectrum obtained in this manner. This was obtained on an earlier version of the equipment, where the frequency range was not quite so wide. Both the roll off of the pile noise component and the tail due to the white noise component can be clearly seen. The fitted curve is also shown. The accuracy shown for α is typical.

Fig. 2.

In conclusion, this method has proven to be a convenient and accurate means for observing the high-frequency portion of a reactor transfer function and for obtaining experimental values for prompt neutron lifetime. The method is at its best at frequencies which are too high for reactor oscillation, autocorrelation measurements, and similar low-frequency techniques. However, it must be noted that the simple relationship between the observed α and the resulting lifetime is based on one-group bare reactor kinetics, and thus might be expected to break down in more complicated reactors. This matter is under investigation at the present time.

With your permission, I should like to show quickly some calculations that I have done on what reactor kinetics would look like in the two-group space-independent model. Assume a reactor with a lifetime in the thermal group of 60 microseconds and a lifetime in the fast group of 20 microseconds. Then the corresponding one-group lifetime, i.e., that

which would be measured in a ℓ/v -uniform poisoning measurement, is the sum of these, or 80 microseconds.

Fig. 3. Amplitudes of Two-group Thermal-source Transfer Functions.

In this system there are four source transfer functions, covering all combinations of thermal and fast sources and thermal and fast groups. The amplitude and phase curves of all these functions closely follow the corresponding one-group curves out to frequencies well above the β/ℓ^* roll off. The amplitudes and phases of these transfer functions are shown in Fig. 3 thru 5, and the corresponding pile noise spectra are shown in Fig. 6. It will be noted that the amplitudes, and especially the phases, show interesting deviations from one-group behavior at the higher frequencies.

Finally, I might mention that the behavior of the experimental phase data shown by Mr. Cordy for thermal and fast fluxes suggests, in the light of Fig. 5, that a fast source was operative, and thus that the major effect of his oscillator was to vary the fission rate rather than the total absorption.

Fig. 4.
Amplitudes of Two-group
Fast-source Transfer
Functions.

Fig. 5. Phase Shifts of Two-group Transfer Functions.

Fig. 6. Two-group Pile Noise Spectra.

Discussion of Paper
Presented by Mr. Cohn

MR. STORRER.

Am I right in my understanding that what you talked about today, and what Thie talked about yesterday, are two entirely different things. Not only do you have a noise-equivalent source, but you assume that the only thing that produces noise are statistical fluctuations in the neutron absorption and so on

You have white noise, you have high statistical fluctuations because you are at low power, and you don't have any reactivity fluctuations because in the zero-power reactor nothing is moving very much. On the other hand, in a high-power reactor, the source noise would be quite low because you have so many neutrons present. Secondly, the source of noise is really the reactivity due to coolant going by, and in the boiling water you have bubbles, and so on. There the noise is not white, and you cannot use it for transfer function measurements. Is this right? There are two clearly distinct cases?

MR. COHN:

I should have mentioned explicitly, perhaps, that this is strictly a zero-power development, and thus the neutronic noise is the only noise that matters here. It is true that the noise that is observed in boiling reactors is to a large extent bubble noise. However, I think, for example, if you had a reactor in which mechanical noise was not too prevalent, you could still observe this sort of behavior, and if you had a resonance in a transfer function you would observe a very large amplification of the neutronic noise.

You can still use this technique for transfer functions, but, if you have mechanical noise, the source spectrum would undoubtedly be more complicated.

MR. STORRER:

Is it not true that when you have very many neutrons, as in a power reactor, that the statistical fluctuations are very small?

MR. COHN:

They are small only relatively. You see, the root-mean-square noise amplitude goes as the square root of the power, so the absolute noise amplitude increases, but it decreases relative to the steady power.

MR. STORRER:

Thank you.

MR. CORDY:

I think perhaps I don't understand what you are saying here, because I can't relate it to our KEWB experiment. I feel that the data I showed yesterday on the thermal neutrons as compared to the epicalm neutrons, indicated that the thermal neutrons have spent a finite amount of time diffusing to that energy. This time is essentially constant for all measurements. As we increase in frequency, this time occupies a greater number of degrees. I can't see any reason for the 180° asymptote.

MR. COHN:

This comes out of the formulas. It is not too easy to explain intuitively. Remember, although this is a two-group model, it is still a space-independent model.

MR. COHEN:

Mr. Cohn, I think your last remark is the one that explains the difference, because I feel quite confident that the delay that Mr. Cordy has measured is due to the spatial effects and represents an effect that is directly related to the fact that his detectors are in the reflector of the reactor. This sort of behavior would not be identifiable at all in your single-region multigroup description.

MR. COHN:

That is, perhaps true, especially since Mr. Cordy's curves did not go out to high enough frequencies to see what they would do. I am just conjecturing this on the basis of my calculations which may be out of line. I might state that this points up one advantage of pile noise measurements. Odd phase effects due to localized spatial effects that are not taken into account in the model are cancelled out due to the incoherence of noise. Remember, if you are measuring noise, and you have a detector in the core you are looking at noise originating in many parts of the core. Since these are all incoherent, the phase effects could be expected to cancel out.

MR. CHAUMONT:

Moore said in his paper it will be possible to obtain the power coefficient with noise spectrum measurement. What do you think about it?

MR. COHN:

You could get the same information from this type of measurement that you could get from an oscillator measurement that so many people here have reported. As I mentioned, the technique that I have used is essentially a high-frequency technique, and the filtering out of wave components could be somewhat awkward at frequencies of much less than one cycle. However, the autocorrelation technique that Joe Thie used might be considered the low-frequency complement of this technique.

MR. KIEHN:

I have a question. Have you extended this noise work out to the region where you do have a finite multiplication, not an infinite multiplication? This region is of particular interest to some of us in the plutonium work. We have quite a large built-in source, and at power levels of a few watts, the multiplications are maybe of the order of 500 to 5000. A typical experimental problem in this region occurs when we use wide band amplifiers to measure current signals, and utilize these devices through standard rate circuitry. We find out the incidence of accidental rate scrams is very large in the vicinity of multiplications of $1/\beta$. The incidence decreases at lower and higher multiplications. I don't understand this. I wondered if somebody did.

MR. COHN:

I have not looked into this. I have attempted to do measurements at subcritical but I have not been successful, mainly because the noise signal from the reactor doesn't compete too well with the background noise in the apparatus under those conditions. For example, I have worked mainly with water reactors in which the microphonics from the pump are a source of interference. If the problem were of great interest, if it were beat on very hard, I am sure fundamentally there would be no bar to obtaining the results.

MR. FRANZ:

This is an academic problem, but it may be of practical interest. If you were to take your source transfer function, and calculate it for, say, a shutdown reactivity of β , it would extend that last break point out, so as to double it. This might be an interesting kind of experiment if you had an accelerator to get lots of neutrons and modulate them into a reactor.

MR. COHN:

This might be a good way to get your effective β , which is rather hard to get other ways.

MR. FRANZ:

No, I think this would be a way of measuring the shutdown.

MR. COHN:

Yes, excuse me. I was mistaken. Your reactivity measurement referred to β so this is true. It would be a way of getting the shutdown. This could actually be considered analogous to pulsed source measurements.

MR. COHEN:

The roll-off point is merely extended by the ratio of $1 - r$, where r is the negative reactivity, so that you still can't separate β from the Rossi α component. That is, you still can only detect β / ℓ .

MR. WASSERMAN:

I want to mention for the record that several fellows out at Spert, including Benson and Brockett, have recently begun a program of using a system somewhat similar to Mr. Cohn's, which they are trying to work up to make measurements of ℓ / β in the same fashion. I think preliminary results have shown as good promise as your results have indicated previously.

CLOSING REMARKS

J. M. Harrer
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Gentlemen, it is nice to see that there is still somebody here to hear the closing remarks. The idea of having someone close the meeting with sum-up remarks is one that has been followed quite frequently. The idea here is to say something that most everyone agrees with, because this is the point of no return, supposedly no back talk.

I thought perhaps it might be interesting to those that are still here, and for the record, to remember that the transfer function work is not very new, but even so, has not developed as rapidly as it might have. I think the first time the use of the transfer function as an analytical tool in reactor work came to my attention, was through work at Chalk River. The transfer function was used to study the delayed neutron groups. The Chalk River people were working in a classified area. About the same time, Langsdorf at ANL was working with what they called the pile oscillator. He was using the equipment to measure small values of reactivity. He obtained extreme accuracy, because he used what we might call a cross-correlation method. His signal was independent of spurious reactor flux variations.

People working in these early areas were not publishing very widely. The first time I heard of this transfer function being applied was by Piggott and Owens at GE, who suggested setting design parameters from the reactor transfer function data, which is really the primary use of any such data.

Piggott and Owens presented the argument that the way to control a nuclear reactor was to make the closed system transfer function chart one in which the frequency function continued straight down in amplitude from the delayed neutron curve. They rejected any response in the prompt neutron region. The theory was that this would be effective for small δk variations and easily control the reactor with a low-power rod system. This met with a lot of objection from various people who wanted to build faster systems at that time. M. A. Schultz and I went to see Piggott and Owens and from their work and from some of our own, Monty decided to run through a calculation of transfer functions and to do some analytical work in his group at WAPD. Joe Franz of WAPD picked up the ball and did some of the initial work for water reactors.

I think the first transfer function measurement made in the unclassified area was at Argonne in the CP-2 reactor, in which case we tried to go beyond the frequency point Langsdorf was using and beyond that which had been covered at Chalk River. If possible, we wanted to measure the prompt neutron lifetime. It was a poor experiment. It was a probing experiment, and the real purpose was to see whether we could run a better one. The follow-up experiment to that was to be a run on the ZPR-I, a water critical, in which we were going to try to measure the lifetime of the STR core which was mocked up at the time. That experiment was also terminated very rapidly by taking a reactor apart. Dr. Zinn decided he didn't want to fool around with the reactor any more. It never lasted long enough for us to get the measurement.

I wonder whether the business didn't really suffer a setback at that time, because we would have tried the rotating rod to vary reactivity, and we would have probably noticed some of these effects that you have been describing in this session.

I was very happy to see so many good experimental approaches described at this session. The work that was initially done at Chalk River was very similar: the technique used was very similar to the one that most people used in their measurements now - a sine and cosine multiplier computer with the integrator. The method that we have been using for measurement at ANL, and which has also been adopted by several people, is the analyzer method, in which you get your data point at each frequency somewhat more quickly.

I was a little disappointed in the fact that we didn't have more of the technique type of papers. I would have liked to have seen more discussion of the difficulties in measurements, because I think this is the important item. I think we ran the gamut all the way from extremely careful analysis to crude results in experiments, and tried to force correlations of them. I think that a lot of work remains to be done in this field to improve data analyses.

If I have any other criticism of the field of transfer function analysis as it stands today, it would be that we still haven't come to an agreement on terminology. I honestly think that we ought to. I have no objection to people using feedback theory and some terminology method that they wish to select, but I do think it is unnecessary in a knowledgeable group to explain your method, if you are using one that is relatively standard.

The use by certain individuals of computing aids, such as the log polar plot which was suggested originally at MIT (I don't know whether or not they suggested it; at least they put it out), and the degenerates from that which have been used here at Argonne, are all computing aids, such as the Nyquist diagram, or the linear polar plot. All of these are computing

aids, but I think the information that you finally obtain for the benefit of all should be put in the language that is of the art, which is the log frequency plot versus log amplitude, accompanied by the phase versus frequency plot. Then one can see how the results did come out and can readily compare the result with some standard in his own mind.

I would like to run over one or two high points. The first session was outstanding because of the fact that there were several results of experiments described. The breakdown of the SPERT-I kinetics into a fuel channel response measurement is important. I did not attend that session, but I understand that some of the results of experiments were extremely good.

In the afternoon session, I was particularly impressed by the fact that Grace did have some experimental results, and I should really wish that he would publish the data so that we could all look at it. As you know, I commented on his paper at that time, and I would like to see the results of that boiling experiment where the pumped system became unstable, because I think that many of us need that sort of stimulation to get some new ideas to investigate.

I refer to the work described by Akcasu, for example. If he had not had available the data taken on ANL reactors, I doubt whether many of the ideas that he is working on now would have been brought forth.

Going on to the fast reactor work, I am always a little startled when we come to fast reactors because when you compare them to the slow reactor or the boiling and zero-power reactor work, you find there is a great difference in terminology completely. Now, here we are working in a field which is supposedly the negative feedback field. Still in the fast reactor, we are dealing continually with the elimination of the negative feedback. When you deal in a system in which a positive feedback occurs you must be very careful to remember that the negative feedback analysis which we consider standard does not apply.

One must realize that if you have a zero feedback, for example, as a result of a positive and a negative feedback, the reactor goes back to its theoretical character, and beyond that point, where you have excessive positive feedback added for some reason, the reactor is basically unstable. Now, a system that is basically unstable can't be controlled. This is why we need the transfer function - to design an overall control system which is stable.

On the other hand, we should be very careful in drawing conclusions about how the system will work or how the system might act as a stable system from just the fact that positive feedback is beginning to creep into the picture.

If I were to conclude anything from the session, I think that we are now up to date on where we stand in transfer function analysis. I think that we can certainly conclude one happy thing from the session, and that is that the cross correlation represented by the transfer function, which is a measure of the output as a result of some known input, is definitely a very powerful tool, and I look forward to at least a few years in which this technique will actually produce some very significant results.

And now, gentlemen, on behalf of the chairman, may I say that we are happy that you came. Please get your papers in, and could we have a round of applause for the hard-working group in organizing this, all the chairmen and the general chairman, Mr. DeShong.

APPENDIX

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CONFERENCE ON TRANSFER FUNCTION MEASUREMENTS
AND REACTOR STABILITY ANALYSIS

ARGONNE NATIONAL LABORATORY
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May 2 and 3, 1960

APPENDIX

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