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Davydov and Chaban\(^{(1)}\) have extended the work of Davydov and Filippov\(^{(2)}\) on the collective excited states of asymmetric even nuclei by including the beta-vibration-rotation interaction in an exact manner.

We have computed the energy levels predicted by Davydov and Chaban's model so that one can compare the theoretical predictions with experimental results. The energies in units of \(\hbar \omega_0\) are given by

\[
\frac{E_{K \nu \ell}(J)}{\hbar \omega_0} = (\nu \ell + \frac{1}{2}) \sqrt{1 + \frac{3}{2} \left(\frac{\mu}{p}\right)^4 \epsilon_K(J)} + \frac{1}{4} \left(\frac{\mu}{p}\right)^2 \epsilon_K(J) + \frac{(p-1)^2}{2 \mu^2}
\]

(1)

where

\(\mu\) is the "nonadiabaticity" parameter, which can take values between 0 and 1;

\(\epsilon_K(J)\) is the energy in units of \(\hbar^2/4B\beta^2\) of the \(K^{th}\) spin J level of a hydrodynamical asymmetric rotor\(^{(2)}\). It is a function of \(\gamma\), the "nonaxiality" parameter,\(^{(2)}\) which varies between 0° and 30°. Tables of these functions have been computed by Moore et al.;\(^{(3)}\)

\(p\) is a parameter related to \(\mu\) and \(\epsilon_K(J)\) by the relation

\[(p-1)p^3 = \frac{1}{2} \epsilon_K(J) \mu^4, \text{ with } p \geq 1;\]

(2)

\(\nu \ell\) is a quantum number (in general not an integer) which is the \(\ell^{th}\) root (the first being the smallest one) of the following transcendental equation

\[
H_{\nu \ell} \left(-\frac{p}{\mu_1}\right) = [2\Gamma(-\nu \ell)]^{-1} \sum_{t=0}^{\infty} \frac{(-1)^t}{t!} \Gamma \left(\frac{t-\nu}{2}\right) \left(-2 \frac{p}{\mu_1}\right)^t = 0
\]

(3)

with \(\mu_1 = \left(\frac{\mu^4}{1 + \frac{3\epsilon_K(J) \mu^4}{2p^4}}\right)^{1/4}\);

(4)

and \(\Gamma(x)\) is a gamma function.
We have also computed the following ratios of energies

\[ RK_\ell(J) = \frac{E_{K\nu}(J) - E_{1\nu_1}(0)}{E_{1\nu_1}(2) - E_{1\nu_1}(0)} \]  

because these are the ones that can easily be compared with experimental results.

The computations were carried out with an IBM-650 computer. Given a value of \( \mu \) and \( \varepsilon_K(J) \), the values of \( p \) and \( \mu_1 \) were obtained by means of equations (2) and (4), respectively. Then the transcendental equation (3) was solved by computing \( H_{\nu_\ell} \) for \( (\nu_\ell)_0 = -0.001 \) and \( (\nu_\ell)_n = (\nu_\ell)_0 + 0.1n \) for \( n = 1, 2, ... \) until \( H_{\nu_\ell} \) changed in sign, and then Newton's method was used to find the root. Each \( H_{\nu_\ell} \) function was computed with a number of terms, \( t = T_\ell \), such that the term \( t = T_\ell + 1 \) changed \( H_{\nu_\ell} \) by less than 1 part in \( 10^4 \).

Before starting to ask for this criterion, the function \( H_{\nu_\ell} \) was computed with a number of terms, \( T \), such that the contribution to \( H_{\nu_\ell} \) of further terms \( T + 1, T + 2, ... \) was monotonically decreasing. A value of \( \nu_\ell \) was called a "root" when \( |H_{\nu_\ell}| \) was less than \( 10^{-4} \). Having found \( \nu_1 \), which is always such that \( \nu_1 \geq 0 \), the next root of \( H_{\nu_\ell} \) was looked for by starting the same process at \( \nu_\ell = \nu_1 + 0.98 \) (because \( \nu_{\ell+1} \geq \nu_\ell + 1 \)). This process could be repeated, and so any number of roots can be calculated. During the first calculations we learned that \( \nu_n = 0, 1, 2, ... \) with an accuracy better than 1 part in \( 10^4 \) when \( p/\mu_1 \geq 4.5 \), so that in these cases the machine was told to make the roots \( \nu_\ell \equiv 0, 1, 2, ... \). Later on, the value of \( p/\mu_1 \) was changed to 4.0, because we found out that still at this value the roots were integers with an accuracy of 1 part in \( 10^4 \). Having the values of \( \nu_\ell \) we computed the energy given by equation (1). Knowing the energies in units of \( \hbar \omega_0 \), equation (5) was used to compute the ratios \( RK_\ell(J) \).

The computations were carried out for

\[ \mu = 1.0, 0.90, ... , 0.40, 0.35, ... , 0.1 \]
\[ \gamma = 8^\circ, 9^\circ, 10^\circ, 12^\circ, 15^\circ, 20^\circ, 25^\circ, 30^\circ \]

for the energy levels with \( \ell = 1, 2 \) and 3 and the following \((K,J)\) values:

\((1,0), (1,2), (2,2), (1,3), (1,4), (2,4), (1,5), (1,6), (2,6), (1,8), (1,10), (1,12)\).

The machine computation took 150 hours.

Table 1 gives the values of \( RK_\ell(J) \) as a function of \( MU = \mu \) (column 1) and \( G = \gamma^\circ \) (column 2). The ratios are identified by two numbers \((\ell, s)\), the first being the root number \( \ell \) and the second one indicating the \((K,J)\) values of the level with the following correspondence:
Table 2 gives the values of $EP = \epsilon_K(J)$ (column 3), $P = p$ (column 4), $-\text{ZETA} = p/\mu_1$ (column 5), $NU \equiv \nu_\ell$ (columns 6, 7 and 8), $T\ell \equiv T_\ell$ (columns 9, 10 and 11) and $E \ell f = E_K \nu_\ell(J)/\hbar \omega_0$ with the same nomenclature for $\ell$ and $f$ as in Table 1 as a function of $MU = \mu$ (column 1) and $G \equiv \gamma^o$ (column 2).

The following nomenclature is used in both tables

$$1.234 \times 10^x \equiv 0.1234 \ 51 + x .$$

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References


TABLE 1

Values of $R_{K\ell}(J)$ as a function of $\mu$ and $\gamma$
TABLE 2

Values of $\epsilon_K(J)$, $p$, $p/\mu_1$, $\nu\ell$, $T_\phi$ and $E_{K\nu\ell}(J)/\hbar\omega_0$ as a function of $\mu$ and $\gamma$
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