Combining multiple BPM measurements for precession AC dipole bump closure

P. Oddo, M. Bai, C. Dawson, J. Kewisch, Y. Makdisi, C. Pai, P. Pile, T. Roser

Presented at the North American Particle Accelerator Conference (NA-PAC 13)
Pasadena, CA
September 29 – October 4, 2013

Collider-Accelerator Department
Brookhaven National Laboratory

U.S. Department of Energy
DOE Office of Science

Notice: This manuscript has been authored by employees of Brookhaven Science Associates, LLC under Contract No. DE-AC02-98CH10886 with the U.S. Department of Energy. The publisher by accepting the manuscript for publication acknowledges that the United States Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce the published form of this manuscript, or allow others to do so, for United States Government purposes.

This preprint is intended for publication in a journal or proceedings. Since changes may be made before publication, it may not be cited or reproduced without the author’s permission.
DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or any third party’s use or the results of such use of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof or its contractors or subcontractors. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.
COMBINING MULTIPLE BPM MEASUREMENTS FOR PRECESSION AC DIPOLE BUMP CLOSURE*

P. Oddo, M. Bai, C. Dawson, J. Kewisch, Y. Makdisi, C. Pai, P. Pile, T. Roser, BNL, Upton, NY 11973, USA

Abstract
For the RHIC spin flipper to achieve a rotating field, it requires operating five AC dipoles as a pair of closed orbit bumps. One key requirement is to minimize the remnant AC dipole driven betatron oscillation outside of the spin flipper by 50 dB [1]. In the past, due to its inherent sensitivity, a single pickup with a direct-diode detector (3D) [4] and dynamic signal analyzer (DSA) were used to measure bump closure by measuring the remnant oscillations. This however proved to be inadequate, as the betatron phase advance between the AC dipoles is non-zero. A method of combining multiple BPMs into a sensitive measure of bump closure has been developed and was tested during RHIC polarized proton operation in 2013. This technique as well as the experimental results will be presented.

INTRODUCTION
A spin flipper for RHIC (Relativistic Heavy Ion Collider) has been developed for RHIC spin-physics experiments. It is needed to cancel systematic errors by reversing the spin direction of the two colliding beams multiple times during a store [2].

The spin flipper system consists of four DC dipole magnets (spin rotators) and five AC dipole magnets (see fig. 1). The aim of this configuration is to produce a rotating field. Multiple AC dipoles are needed to localize the driven coherent betatron oscillation inside the spin flipper [1, 3]. While results from the 2012 run did suggest the presence of a rotating field, the polarization lifetime was degraded with the AC dipoles on. This suggested incomplete bump closure and/or incorrect phase between bumps [1] which lead to reinvestigating the method used to close the AC dipole bumps.

Operationally the AC dipoles form two bumps that minimize the effect of the AC dipoles outside of the spin flipper. The central AC dipole, #3 in figure 1, is common to both bumps. Both AC-dipole bumps operate at the same frequency, but are phase shifted from each other. The convention used when expressing closure in dB is to make the 0 dB reference the strongest AC dipole. In case of the dual bump, AC dipole #4 is used.

BBQ 3D AFE & DSA
Up until the 2013 run the closure was only trimmed using the pickup and direct-diode detector (3D) analog front end (AFE) of the RHIC baseband tune meter (BBQ) processed via a DSA.

The DSA spectrum in figure 2 (green plot) shows that the closure for the AC dipole bumps was 67 dB. However, the spectrum of the most sensitive BPM (fig. 3, dotted plot) shows a closure of only 28 dB. This nearly two order of magnitude difference clearly shows that it’s not possible to determine closure using a single pickup (at a single frequency). The possibility of using a single pickup at multiple frequencies has not yet been fully explored.

COMBINING BPM MEASUREMENTS
Even though discrete Fourier transform (DFT) spectrums, which typically are calculated using the fast Fourier transform (FFT) algorithm, are used here, these

*Work supported by Brookhaven Science Associates, LLC under contract DE-AC02-98CH10886 with the U.S. Department of Energy and RIKEN, Japan.
methods are not strictly DFT/FFT methods. As a matter of fact the BPM magnitude and phase used by software calculations used a sine/cosine fit, which can be thought of as the evaluation of the discrete time Fourier transform (DTFT) or z-transform at a single frequency.

The average response (green trace) is just a simple magnitude average:

$$k_s \sum_{n=1}^{N} |Y_n|$$

(1)

$Y_n$ is the frequency response for BPM $n$ and $k_s$ is a scale factor which normalizes the average to 1 (0 dB) when excited by a single AC. Since this is a magnitude response of all BPMs, all need to be zero for this to be zero and this would only happen for a closed bump. The noise floor for 1024 turn data at store was $\sim$45 dB.

The weighted average (black trace) is a magnitude average weighted by each BPM’s normalized signal to noise ratio squared:

$$\sum_{n=1}^{N} a_n |Y_n|^2$$

(2)

Software calculations for $a_n$ use the mean magnitude of the DFT/FFT bins from 0.025 to 0.48 as the estimated noise value. The RMS error from a fit could also be used. The noise floor for 1024 turn data at store was $\sim$50 dB.

The vector average (purple trace) scales frequency response of each BPM by the normalized signal to noise ratio squared, just as the weighted average, but also counter rotates the phase by measured phase:

$$\sum_{n=1}^{N} a_n e^{-i\phi_n} Y_n$$

(3)

Phase $\phi_n$ is the measured phase of BPM $n$ when it was excited by a single AC dipole. The noise floor for 1024 turn data at store was $\sim$70 dB. While this might look like a good candidate for determining bump closure, because this is a linear combination of BPMs, it is possible for it to be zero for combinations of only two AC dipoles and therefore not useful for definitely trimming bump closure (see eq. 6 of next section).

Frequency Response of multiple AC dipoles

From previous treatments of transverse beam motion due to a single AC dipole [5] the basic equation of motion can be thought of as a discrete sampled time system, but the actual sampling is done by a particle or bunch.

Extending the model for an arbitrary number of AC dipoles in the time domain is:

$$\begin{align*}
    y_0[n] &= M \cdot y'[n-1] + e[n] \\
    y_0'[n] &= M \cdot y'[n-1] + e'[n]
\end{align*}$$

(4)

$$\begin{align*}
    e[n] &= g_{ac} \sum_{k=1}^{N} M_{kn} \cdot i_k[n] \\
    e'[n] &= g_{ac} \sum_{k=1}^{N} M_{kn} \cdot i_k'[n]
\end{align*}$$

(5)

Figures 4 and 5 show the weakest and strongest BPM spectrums and the spectrums for different methods of combining BPMs for polarized proton beam at injection and store respectively when excited by a single AC dipole (#4). In all cases the response is normalized to place the AC dipole peak at the driven frequency (0.49) at 0 dB. This is done as convenient way to visualize the signal to noise ratio. BPM data was taken with 1024 turn data records. For the 2013 run, 4096 turn records were available and these longer records would improve the signal to noise ratio squared, just as the weighted average, but also counter rotates the phase by measured phase.
Functions \( y_0[n] \) and \( y'_0[n] \) are the beam position and slope at center of the last AC dipole, \( e[n] \) and \( e'[n] \) are the effective stimulus of all AC dipoles, matrix \( M \) is the transport matrix for a full turn, \( g_{acd} \) is a constant which converts magnet current into deflection (units A\(^{-1}\)) and matrix \( M_{00} \) is the transport matrix from AC dipole \( k \) to the last AC dipole \( N \).

These equations illustrate that superposition can be used and that each AC dipole stimulus includes the path or optics to the reference point (center of last AC dipole).

Although not presented here, eqns. 4 and 5 can be transformed into the frequency domain. However, since this is a linear time invariant (LTI) system, certain assumptions can be made. The first is that the BPM position is a superposition of AC dipole currents:

\[
Y_n(z) = \sum_{m=1}^{M} H_{nm}(z) I_m(z)
\]  

(6)

\( H_{nm}(z) \) is the transfer function from AC dipole \( m \) to BPM \( n \), with \( M \) the number of AC dipoles.

Similarly, based on eqn. 5, BPM position is also a superposition of the error signals:

\[
Y_n(z) = G_n(z) E(z) + G'_n(z) E'(z)
\]  

(7)

\( G_n(z) \) and \( G'_n(z) \) are the transfer functions from AC dipole stimulus (or bump error signal) to BPM position \( n \).

While both eqns. 6 & 7 were shown as continuous functions in z-transform form, actual calibrations are done at a fixed frequency \( z = e^{(2\pi k_0)} \). The calibration process then just becomes a matter of multiplying complex constants. The transfer functions \( H_{nm} \) are also not calculated from a model, but a result of BPM position and AC dipole current measurements. For a single AC dipole, on one at a time (superposition) eqn. 6 becomes:

\[
H_{nm}(z) = \frac{Y_n(z)}{I_m(z)}
\]  

(8)

Not shown, but it’s also possible to derive these transfer functions from stepped AC dipole data.

**Closing AC dipole bumps using calculated correction factors**

Once the transfer functions have been determined, eqn. 6 can be used to solve for the precise currents needed to close a bump by setting \( Y_n = 0 \) and solving for currents (or correction factors). Since the spin flipper contains two bumps, both need to be calculated separately using:

\[
H_{n3} = 2k_2 H_{n2} - k_1 H_{n1}
\]  

(9)

\[
H_{n3} = 2k_4 H_{n4} - k_3 H_{n5}
\]  

(10)

These equations assume a value for the shared AC dipole \( #3 \) and calculate correction factors \( (k_1, k_2, k_3, \text{ and } k_4) \) for the other four magnets assuming ideal settings.

Two methods have been used to calculate these correction factors. One involves a matrix operation using all BPMs at once for eqn. 9 and 10. The other method is a cutting method that involves iteratively calculating average correction factors for all combinations of BPM pairs and removing BPM pairs that produce correction factors that deviates more than four standard deviations from the average.

**Measuring bump closure by estimating AC dipole stimulus components**

A direct consequence of eqn. 7 is that the total AC dipole stimulus or bump error components \( (E, E') \) can be calculated using pairs of BPMs using:

\[
E = p_n Y_n + p_m Y_m
\]  

(11)

\[
E' = p'_n Y'_n + p'_m Y'_m
\]  

(12)

The \( E \) and \( E' \) used are the ideal values calculated from the magnet current (based on eqn. 5). The scaling convention used is to make both \( E \) and \( E' \) equal 1 when excited by AC dipole \#4 alone. Constants \( p_n \), \( p_m \), \( p'_n \), and \( p'_m \) are then calculated using matrix operations while iterating over all BPM pairs (each of BPM \( n \) and \( m \)). These constants are summed and scaled to produce coefficients for each BPM so that producing the \( E \) and \( E' \) response only requires a single pass over the BPM data. The scaling used depends on which BPM pairs were used. When using all BPM pairs, the scaling should use the normalized signal to noise ratio squared to improve the resultant signal to noise ratio. When only using the final pairs used to calculate the bump correction factors, then the coefficients only need to be normalized by the number of pairs.

**CONCLUSION**

It is possible to combine multiple BPM measurements to close the AC dipole bumps. Using the calculated correction factors produced good results during the 2013 run. There are also numerous ways to verify closure, and this still includes the 3D pickup and DSA.

**REFERENCES**


