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ANNUAL PROGRESS REPORT

"Plasma Instabilities and Wave Excited
by an Anisotropic Energy Distribution
Produced by Electron Cyclotron Resonance"

U.S. Atomic Energy Commission
Contract No. AT(11-1) 1695

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ANNUAL PROGRESS REPORT

Plasma Instabilities and Waves Excited by an Anisotropic Energy
Distribution Produced by Electron Cyclotron Resonance

Contract No. AT(11-1) 1965

U.S. Atomic Energy Commission

- Staff during all or a portion of the contract year -

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I. INTRODUCTION

J. L. Shohet

The research performed under this contract involves the study of plasmas produced by electron cyclotron resonance heating in a magnetic mirror device. Specifically, the work has centered on four major problems:

I. The measurement of the intensity and polarization of free-free bremsstrahlung from the plasma and the theoretical problem of relating the results to the electron energy distribution.

II. The behavior of the plasma produced by cyclotron resonant heating when subjected to microwave radiation at a frequency displaced from the cyclotron frequency. Both experimental and theoretical results have been obtained.

III. The investigation and identification of an electromagnetic instability.

IV. Numerical simulation and experimental diagnostic techniques used in conjunction with the above work.

This report summarizes the progress in the above areas during the period of December 15, 1969 to December 14, 1970 and outlines a program for further theoretical and experimental effort.

The free-free bremsstrahlung measurements have been continued. The radiation spectrum has been calculated using the Heitler scattering cross section for various anisotropic-relativistic velocity distributions. The spectrum of the radiation parallel and perpendicular to the d.c. magnetic field has been calculated and observed experimentally. The calculations are now being extended to consider the polarization distribution of the X-ray flux and an experiment is currently in progress to attempt measurement of the polarization.

Off resonance heating has been extensively studied, both theoretically and experimentally. The data obtained are in corroboration with the results on this type of heating first obtained by R.A. Dandl at Oak Ridge.

Basically, below resonance, the plasma is expelled from the device. Above resonance, a finite heating rate is measured. The upper off-resonance heating rate and the lower off-resonance power required to expel the plasma agree with the prediction of a theoretical model.

The electromagnetic instability previously observed in this experiment has been studied further. The instability is accompanied by strong microwave emission, axial particle loss, fluctuations in diamagnetism, and intense X-ray bursts.

The parameters needed to produce the instability were determined from experimental measurements. In addition, current research is designed to measure the wave number of the instability, which should complete the identification and provide a physical mechanism for it.

Plasma numerical simulation has progressed well. Heating of a small number of particles in the resonance plane has shown evidence of heating with low values of density. The code is currently being revised, in cooperation with the Los Alamos Scientific Laboratory, to include the effects of a non-uniform axial magnetic field and a 2 1/2 dimensional representation. Resonant and/or off resonant heating will be studied in this way.

In addition, single particle orbits in magnetic mirror field under the influence of microwave fields will be studied using a hybrid computer. Perpendicular energy measurements have been improved by the construction of a better, shielded, diamagnetic loop. Since density measurements made by microwave techniques in a hot plasma are often subject to question, an independent measurement of the density distribution by probes or optical methods has been made and the results are presented in this report. Experimental improvements have included the extensive use of probes, and other particle sensors. This has permitted spatially resolved measurement of plasma density, temperature, and floating potential has been made. The d.c. magnetic field and magnetron

high voltage supplies have been filtered and the ripple at both has been reduced by an order of magnitude. Low power (~ 100 watt) CW microwave S-band sources have been constructed. These have been used for upper and lower off resonance heating.

Extensive filtering and shielding, together with the elimination of ground loops and improved pulsing circuitry have reduced the electrical pickup of noise to a new minimum.

A new experimental facility has become operation. It is a 4" diameter glass vacuum system with a set of coils that can be connected to produce a mirror field ($R = 3.8$) or a cusp. Plasmas are produced by a dc discharge, 400 kHz, 1 kW, CW, inductive heating, 10 MHz, 1 kW, CW, resistive heating, or by ECRH using either a 50 W, 3.0 GHz, CW source, or a 10 kW, 10 μ sec, 3.25 GHz pulsed source at 120 pulses/sec. The device is being used to study ion cyclotron resonance heating (using the 400 kHz source), and lower off-resonance heating (using the 10 MHz source).

Section II describes the experimental apparatus and diagnostics. Sections III through VI discuss the status of the work in the areas previously mentioned.

II. APPARATUS AND DIAGNOSTICS

The basic experimental device is shown schematically in Fig. 1. The double ended vacuum system has a base pressure of about 1×10^{-7} torr. Helium is continuously admitted through a variable leak so that the pressure during operation is between 2 and 10×10^{-4} torr. The dc magnetic mirror field is formed by a set of 24 pancake coils which allow variation of the mirror ratio from 1.02:1 to 2.38:1. A digital voltmeter across a calibrated series resistor permits accurate measurement of the magnet current, and the shape and strength of the field has been calculated to a high precision from the geometry of the coils by computer.

Microwave pulses of 1.5 μ sec duration for plasma resonance heating are produced by a 100 kW, 8.5 GHz magnetron. The magnetron operates at a repetition rate of typically 360 Hz, and a special two-pulse system allows the magnetron to be pulsed twice in rapid succession during each period. A second microwave system capable of producing 20 kW at pulse lengths up to 1 msec is being developed. The microwaves pass through a hybrid junction and enter the vacuum chamber through two ports located 180° apart in the midplane of the device. The microwave cavity is formed by the cylindrical vacuum chamber (15.4 cm in diameter) and by two perforated copper end plugs (axial separation of 42 cm). Forward and reflected power in the waveguide feeding the cavity is measured with directional couplers.

A new class of experiments has been made possible by the construction of a 50 watt, CW microwave source using a QK61 magnetron that is tunable from 2975 to 3170 MHz. This source and a similar, 100 watt, 2600 MHz CW source borrowed from another group have been used to produce plasmas by electron cyclotron heating as well as for off resonance heating studies. The power is fed into the cavity coaxially, and the cavity is excited by a coupling loop whose position and orientation can be adjusted for optimum coupling. At these low frequencies, it is difficult to get a good

Figure 1

Figure 1

DIAGRAM OF EXPERIMENTAL APPARATUS

impedance match to the cavity, and the reflected power is generally large.

Many of the various diagnostic equipments used to measure plasma parameters have been described previously. However, some new measuring techniques have been developed during the course of the past year these are described as follows.

A. PARTICLE COLLECTOR

For the purpose of measuring the axial and radial particle fluxes, a new type of electrostatic particle collector¹ was used. Figure 2 shows its construction. It consists of fine copper conductors etched 40 to an inch onto a printed circuit board in the pattern shown. The two copper "combs" are completely separate and thus may be biased with different voltages. If they are biased to equal magnitude voltages with opposite polarities both electrons and ions may be collected without producing electric fields more than 1/40" away from the collector.

Figure 2
Particle Collector
40 lines/inch

B. LANGMUIR PROBE MEASUREMENTS IN THE MAGNETIC MIRROR MACHINE

K. A. Connor

The improved circuitry in the thyatron trigger circuitry for the resonant pulsed heating source is shown below. The astable multivibrator has been replaced by the SCR circuit shown.

Figure 3

A modification of this circuit to include the functions of the delayed pulse generator and the pulse amplifiers is being constructed. The machine may be operated in either a single or double power pulse code. The experiments reported below were carried out with a 100 kW single power pulse 300 times per second.

Electron temperature, density, and floating potential were measured with Langmuir probes. The first question to be answered when one wishes to make such measurements is how the probe affects the plasma to be measured. If the plasma is perturbed too much, the information from the probe is useless. One should expect worthwhile results from Langmuir probes in our apparatus for the following reasons. First, there is no noticeable change in the visible light produced when the probe is in place for measurements. Second, the output of a shielded diamagnetic loop shows a change of less than 10% when the axial and radial probes are extended fully into the machine. Third, as is noted below, the radial density profile is approximately symmetric while the probes are

obviously asymmetrically introduced into the plasma. Finally, for the probe to operate such that its data is reasonable, the amount of charge drawn from the plasma must not be an appreciable percentage of the total charge. The total charge is given by

$$Q = neV = 8 \times 10^{-6} \text{ coulombs.}$$

V = plasma volume.

The charge drawn out by the probe is given by

$$\Delta Q = \int_0^{t_p} I_{oi}(0) e^{-t/\tau} dt = 6 \times 10^{-8} \text{ coulombs}$$

τ = decay constant
 t_p = time between power pulses.

Thus, the charge drop due to the presence of the probe is less than 1% of the total.

The probes used consist of a 1/8 inch diameter brass welding rod, approximately a centimeter in length, soldered to the center conductor of an RG 58A/U cable. The cable is placed inside a section of 1/4 inch glass tubing and epoxied at both ends of the glass for vacuum seal.

The circuit shown below was used for the measurements made. R_L was 10 K Ω for the density and temperature measurements and 1 M Ω for the floating potential measurement. C is due to the capacitance of the cable and the input capacitance of the Tektronix 1A6 preamplifier, and is approximately equal to 300 pF.

Figure 4

Figure 5 shows the theoretical magnetic field structure in which the measurements were made. The frequency of the microwave power is 8.54 GHz corresponding to cyclotron resonance at $B = 0.3050 \text{ w/m}^2$. The figure shows where the resonance zones are for two different values of magnet current. Except when indicated, measurements were made with $I_{MAG} = 287 \text{ amps}$.

Figure 6 shows the V-I characteristic of the radial probe at $R = 3.5 \text{ cm}$, 100 μsec after the 2 μsec power pulse. Notice that the high electron current region does not show saturation in the expected manner. This, however, is a common effect in strong magnetic fields. The probe draws more electrons off a field line than are there so electrons have to diffuse across the field lines. The rate at which they diffuse limits the current drawn.

Figure 7 shows the V-I characteristic of the axial probe at $R = 0$, $Z = 15.1 \text{ cm}$, 50 μsec after the power pulse. (Z is measured from the midplane.)

The hot component of the electron temperature has been measured by other techniques--X-rays, diamagnetic loop and $\vec{E} \times \vec{B}$ analyzer. The electron temperature of the cold plasma component can be obtained from the V-I plots using the admittance probe method.² The method works as follows. R_s , the source resistance of the plasma, is the inverse slope of the V-I characteristic at $V = V_f$, the floating potential. The Taylor expansion for R_s gives the result that

$$R_s = \frac{kT_e}{e} \frac{1}{I_{oi}}$$

where: I_{oi} is the ion saturation current

T_e is the electron temperature.

By measuring the slope at $V = V_f$, kT_e can be calculated. Figure 8 shows kT_e vs. time at $R = 3.5 \text{ cm}$, $Z = 15.1 \text{ cm}$. This method was used because it avoids drawing large currents from the plasma.

FIGURE 5 CONTOURS OF CONSTANT MAGNETIC FIELD AMPLITUDE ($R=1.021$)

Location of resonance zones for $I_{MAG} = 287$ amps and $I_{MAG} = 289$ amps

$$|\vec{B}| = \# \times I_{MAG}(\text{amps}) \times 10.42 \text{ w/m}^2$$

Electron temperature of cold component vs. time
 $I_{MAC} = 287$ amps

Figure 8

The electron temperature can be calculated in another way.² If thermal equilibrium is assumed, the current for $V_p > V > V_f$ is given by

$$j(V) = \frac{I(V)}{A} = \frac{I(V_p)}{A} \exp \left[-\frac{e(V-V_p)}{kT_e} \right].$$

When solved for the electron temperature we have

$$T_e = \left[\frac{k}{e} \frac{d(\ln I(V))}{dv} \right]^{-1}.$$

This method, being a bit more clumsy than the first, was used only as a check. For the case of $R = 3.5$ cm and $Z = 15.1$ cm it gave 6.5 ev vs. 6.9 ev for 50 μ sec and 5.9 ev vs. 4.8 ev for 100 μ sec, showing reasonable agreement.

Once we know kT_e , the density may be derived. The ion saturation current may be expressed as follows

$$I_{oi} = \frac{1}{4} n e V^* A \quad (T_i \leq T_e)$$

$$\text{where: } V^* = \left[\frac{8kT_e}{\pi m_i} \right]^{1/2} \quad \text{for a Maxwellian}$$

A = effective probe area.

Figures 9 and 10 show ion saturation current as functions of r , z , and t . The corresponding densities range from 10^9 to 10^{11} cm^{-3} . Notice the approximately symmetric form of the radial profile. The minimum also occurred on the axis of the machine for the case where the resonance zone was determined by $I_{MAG} = 289$ (see Figure 5). The radial profile at different times for this case is shown in Figure 11. This indicates an annular structure to the plasma which has also shown up in the visible plasma light. Similar results have been obtained by Dandl, et al.³ The annular structure has shown up for heating zones both on and off the axis.

The floating potential can be measured directly making R_L large, limited by the input impedance of the Tektronix 1A6 plug-in unit, and C small, limited by cable length and the input

Ion saturation current - radial profile at constant time
 $R = 1.02:1$, pressure = 2.5μ , rep. rate = 300 sec^{-1} ,
 $I_{\text{MAG}} = 287 \text{ amps}$

○ 50 usec
 □ 100 usec
 △ 150 usec

Figure 9

Ion saturation current - radial profile at constant time
 $R = 1.02:1$, pressure = 2.5, rep. rate = 300 sec⁻¹
 $I_{MAG} = 289$ amps

- 15 μ sec
- 50 μ sec
- △ 100 μ sec
- ▽ 150 μ sec
- 200 μ sec

Figure 11

capacitance of the 1A6. A check was made to be sure that the decay of the floating potential was not due to the R-C time constant of the measuring circuit. The circuit time constant was changed using both longer cables to increase C and a 10:1 attenuator to increase R. Both doubling the cable length and increasing the input impedance by 10 resulted in less than a 10% change in the value of the floating potential at all times. The result of the measurements is shown in Figure 12. For the worst case--occurring seldomly--the upper frequency limit of the probe is given by

$$\frac{1}{2\pi fCR_s} = 2 \times 10^3 \text{ Hz.}$$

This is large enough to allow the measurements taken to be useful.

These measurements have shown that it is possible to extend our measuring tools to include Langmuir probes. This allows us to check the results obtained against previous measurements, make spatially localized measurements of plasma parameters, and extend our knowledge of the plasma parameters to longer times. The density measured with probe techniques shows agreement to within an order of magnitude with the fundamental mode microwave cavity technique. The temperature is more difficult to compare since diamagnetic loop and $\vec{E} \times \vec{B}$ analyzer measurements work for times to about 25 μsec after the power pulse while the probe data is for 50 μsec after the power pulse and later. Figure 13 shows kT_e vs. time with an extension to short times for the same data contained in Figure 8, using data from a diamagnetic loop and microwave density measurements under similar conditions. More measurements will be made to check the various techniques more thoroughly.

Fluctuations at ~ 40 kHz in both ion saturation current and floating potential were observed near the edges of the plasma with resulting amplitude variations

Floating potential - axial profile at constant time
 $R = 1.02:1$, pressure = 2.5μ , rep. rate = 300 sec^{-1}
 $I_{\text{MAG}} = 287 \text{ amps}$

Figure 12

Figure 13

$$\langle \Delta n \rangle / n = \frac{e \langle \Delta V_f \rangle}{k T_e} = 30-50\%.$$

This diagnostic method has been used effectively in the measurement of off resonance heating parameters.

C. THE DESENSITIVITY OF A MICROWAVE CAVITY TO MEASUREMENTS OF HOT ELECTRON DENSITY

J. L. Shohet*

The PLEIADE plasma device produces a hydrogen plasma by means of electron cyclotron resonance heating. The plasma is then stored in a magnetic bottle. Figure 14 shows a diagram of the device.

The plasma is highly anisotropic. The electrons have higher energies perpendicular to the d.c. magnetic field axis, while, due to the magnetic field gradient and space charge effects, the ions have higher energies parallel to the magnetic field. The average perpendicular electron energy and the average parallel ion energy are both about 1 Kev, producing parallel speeds of about 3×10^7 cm/sec.

The following describes measurements made of the temperature and density of the electrons. First, it should be noted that whenever the ambient gas pressure was kept below about 10^{-5} torr, no optical radiation was observed emanating from the plasma. However, a tungsten pyrometric probe, inserted in the plasma, became hot enough to glow, showing the presence of hot plasma. The computed electron density from this measurement was at least 10^{10} particles/cm³.⁴

Next, the X-rays produced by electrons which penetrated a beryllium window could be used to determine the hot plasma electron temperature. The X-ray measurements showed that the average electron energy was about 1.5 Kev and that the highest electron energy was about 5 Kev, regardless of pressure.⁵ A diamagnetic loop further verified the presence of this hot plasma, in making measurements of stored energy density.⁵ Typical values of this quantity were 10^{13} ev/cm³. Since the X-rays measured the electron energies, we are then able to determine from the diamagnetism that the energetic electron density was again at least 10^{10} particles/cm³, which is in accordance with the pyrometric probe measurement.

*This work was done at the CEN/Saclay (France) while J.L. Shohet was visiting there in 1969-70.

----- QUARTZ TUBE

- A - DIAMAGNETIC COIL
- B - MOVABLE PYROMETRIC PROBE
- C - TM_{010} CAVITY
- D - BERYLLIUM WINDOW FOR X RAYS
- E - $H\beta$ DETECTOR

Figure 14

Assuming an error of 100% on the diamagnetic measurement, and 200% for the energy measurement, the minimum value of hot plasma density is still 10^9 particles/cm³.

An additional measurement of the electron density was made by observing the shift of the resonance of a cylindrical microwave cavity operated in the TM₀₁₀ mode. The mode frequency when the cavity was empty was about 3.7 GHz, while the local cyclotron frequency was about 1.5 GHz. The cavity was oriented so that the plasma column passed along the axis of the cavity and through holes bored through each end. The frequency measuring equipment was able to detect frequency shifts of 1 part in 10^4 , which corresponds to electron densities of 10^8 particles/cm³. The electron density was computed from the frequency shift by use of a perturbation theory for this mode which included the effects of the d.c. magnetic field.⁶

No frequency shift was observed as long as the ambient pressure was kept below 10^{-5} torr. Yet, from the pyrometric, diamagnetic, and X-ray measurements, we know that a hot plasma of density at least 10^9 particles/cm³ was present.

As the pressure was raised, a threshold appeared in which both light and resonant frequency shift were observed. We are thus lead to conclude that cold plasma is produced about this pressure threshold. The integrated diamagnetic signal shows a slight uniform increase as the pressure passed through and beyond the threshold.

Figure 15 shows a summary of the experimental results. The pressure was varied between 5×10^{-6} and 5×10^{-5} torr. The diamagnetic energy density always remained in the range between 10^{13} and 1.8×10^{13} ev/cm³. Frequency shift, which apparently determined cold plasma density, varied between less than 10^8 /cm³ to over 10^{10} /cm³ above the pressure threshold.

These experimental results are consistent if it is assumed that the frequency shift of the cavity is only produced by cold

Fig. 15

plasma, and is insensitive to the presence of the hot plasma component. The following material is designed to give a theoretical basis for the conclusions of the experimental measurements, that is, the hot component of the plasma does not produce frequency shift.

When the plasma is hot, the thermal velocity of the particles begins to become comparable to the phase velocity of the cavity mode. We can then use the warm plasma treatment developed by Bers.⁷

The condition for resonance of a microwave cavity with perfectly conducting walls containing a plasma which is cold and in which only the electrons move is expressed as the following integral equation.

$$\int \left(\frac{1}{4} \mu_0 |\vec{H}|^2 - \frac{1}{4} \epsilon_0 |\vec{E}|^2 + \frac{1}{4} n_0 m |\vec{V}|^2 - \frac{1}{2} n_0 q \frac{(\vec{V}_R \times \vec{V}_I) \cdot \vec{B}_0}{\omega} \right) dv = 0 \quad (1)$$

over the cavity

In equation (1), $\vec{H}$, $\vec{E}$ and $\vec{V}$ are, respectively, the magnetic field and the electric field of the mode and the perturbed velocity due to their action. The term $\vec{V}_R \times \vec{V}_I$ is the cross product of the complex velocity vector in rotating coordinates, viz; $V = V_R + j V_I$. n_0 is the plasma density, $\vec{B}_0$ is the d.c. magnetic field, and q is the electronic charge. When a warm plasma is considered, the resonance condition is changed to

$$\int \left(\frac{1}{4} \mu_0 |\vec{H}|^2 - \frac{1}{4} \epsilon_0 |\vec{E}|^2 + \frac{1}{4} n_0 m |\vec{V}|^2 - \frac{1}{2} n_0 q \frac{(\vec{V}_R \times \vec{V}_I) \cdot \vec{B}_0}{\omega} - \frac{1}{4} \frac{m V_T^2}{n_0} |n_e|^2 \right) dv = 0 \quad (2)$$

The additional term in equation (2) is due to a correction for finite temperature. V_T is the average thermal velocity and is proportional to temperature.^{1/2} n_e is the electron density fluctuation produced by the mode.

If zero plasma density is assumed initially, it can be seen that the last term in equation (2), together with the 4th term, if it appears, tend to cancel the kinetic energy term as density

is increased, thus bringing the condition for resonance back towards that of the empty cavity. The higher the temperature, the greater the cancellation effect.

It is believed that this mechanism is responsible for the lack of observed frequency shift of the TM_{010} mode when only hot plasma is present. Note that the electrons in the plasma are hot in the direction perpendicular to the d.c. magnetic field. The TM_{010} mode, which does not vary along the axis of the cavity does not therefore see the cold parallel component of motion of the hot electrons. Conversely, it would be expected that if the electrons were "hot" along the field and "cold" across the field, the TM_{010} mode might respond as though a cold plasma were present.

The experimental measurements are then quite consistent. It should be further noted that similar effects on microwaves from a multicomponent plasma have also been demonstrated experimentally by others.⁸

D. A NEW EXPERIMENTAL FACILITY

R. J. Pedley

Since the last progress report, a new piece of apparatus has become available. It is a pyrex system, similar in structure to the other available apparatus which is described in a previous progress report.⁹ The pyrex system, shown basically in Fig. 16, although lacking the sophistication of the other apparatus, has certain unique advantages due to its nonconducting wall properties and its transparency. It has a roughing pump, a diffusion pump, and necessary pressure monitoring equipment with a standard helium leak. The brass plates have ports for probes and also serve as electrodes. When a D.C. discharge is used, electrodes can be inserted within the system to increase stability. The magnets are water cooled and are rated at 18 volts, 3.5 Gauss per amp with a maximum of 550 amps. The current source for the magnets is a 3 phase 220 volt arc welder. After modification, the welder can produce 0 to 550 amps at 22 volts D.C. with less than 5% ripple. It was determined that, due to the heating and subsequent change in magnet resistance, a period of about 15 minutes is required for the magnet current to reach equilibrium. At a steady state current of 370 amps, the magnetic field along the center of the system was determined using a small Gaussmeter and a transverse probe. The results are shown in Fig. 17. From this, the mirror ratio was found to be 3.8:1.

Several methods are available for producing and heating plasmas in this system. Some methods produced problems which are unique to the pyrex system. Since the system is new, many methods were investigated so as to determine which were acceptable. The different arrangements are listed.

D.C. discharge - a high voltage supply produces 5 kilovolts at 23 milliamps. When this is applied between the end plates, instabilities in the plasma occur due to varying discharge characteristics caused by pressure gradients within the system. Applying the D.C. voltage between two closer spaced electrodes solves the problem.

PYREX SYSTEM

FIGURE 16

FIGURE 17

A.C. discharge (60 Hz) - supplied by a 10 kilovolt current limited transformer. When this is applied between the end plates, the plasma is again somewhat unstable.

10 MHz discharge - supplied to the end plates by a 1 kilowatt R.F. generator. This produces a stable plasma.

400 kHz inductive heating - several turns of wire around the cylinder near the center are excited by a 1 kilowatt generator.

C.W. microwave heating - radiated into the cylinder at one of the end plates. The supply is a 2.975 - 3.170 GHz, 50 watt magnetron unit. This unit can be pulsed when coupled to a pulse generator. At times, a rectangular screen waveguide was inserted in the cylinder in order to better locate the resonance heating zone where desired.

Pulsed microwave heating - supplied by a 10 kilowatt magnetron unit. The pulses are 10 microseconds in duration at a rate of 120 per second with a frequency of 3.294 - 3.263 GHz.

Using the various heating and discharge schemes, the various plasmas were thoroughly investigated. Considerable data was taken using basic Langmuir probe techniques.¹⁰ Having overcome some of the inherent problems associated with the pyrex system, such as insufficient ground paths and R.F. interference appearing in the probe circuits from the heating source, acceptable data were obtained. Single and double probes of various tip area and shape were used both axially and radially. A typical V.I. characteristic is shown in Fig. 18. Assuming thermal equilibrium, the current in the transition region from V_f to V_p is given by

$$i(V) = i(V_p) \exp \left[-\frac{e(V-V_p)}{kT_e} \right]$$

Solving for the electron temperature gives

$$T_e = \frac{k}{e} \left[\frac{d(\ln i(V))}{dV} \right]^{-1}$$

From Fig. 25, the electron temperature is calculated to be 5.70 eV.

FIGURE 18

Once the electron temperature has been determined, the density (n) can be derived from the electron saturation current:

$$i_{oe} = \frac{1}{4} n e \bar{V}_e$$

where $\bar{V}_e$ is the average electron speed. For a Maxwellian distribution

$$\bar{V}_e = \sqrt{\frac{8kT_e}{\pi m_e}}$$

therefore

$$n = \frac{4 i_{oe}}{e \sqrt{\frac{8kT_e}{\pi m_e}}}$$

From Fig. 18, the density can be calculated to be $5 \times 10^9 / \text{cm}^3$.

Off-resonance heating has been started on this system. At present the results are yet qualitative. A resonance heated plasma is produced by the pulsed microwave unit. The 10 MHz is applied between the end plates, parallel to the magnetic field. When the 10 MHz is turned on the plasma is extinguished. When the power of the 10 MHz is increased the plasma is again present, indicating only that the 10 MHz discharge is producing the plasma. When the 10 MHz is applied to plates perpendicular to the magnetic field, the plasma could not be extinguished. This is evidence in support of the off-resonance heating theory by J.C. Sprott to be found elsewhere in this progress report.

Modifications to the system have recently been incorporated as well as the continuation of the resonance and off-resonance studies. The resonance studies were pursued to determine for what values of the magnetic field and pressure parameters, the condition of resonance could be sustained in the machine. The microwave heating was supplied by the pulsed 3.294-3.263 GHz source radiated from one of the end plates of the system. The determination of the resonance condition or not was arbitrarily

made based on visual observation. The results are shown in Fig. 19. It is noticed that in some cases the resonance is located outside of the mirror. At each point on the figure, an attempt was made to extinguish the plasma with lower off-resonance heating supplied by a 10 MHz R.F. generator.

The location of some of the resonance zones outside of the mirror field prompted some modifications to the system, which are being implemented at the present time. First the entire system is being wrapped in metallic foil in an attempt to create a microwave cavity within the machine. When this is completed the resonance study will be repeated and the results compared. A new microwave radiator is being located at the center of the machine, see Fig. 20. A screen mesh will be placed opposite to the microwave radiator to reflect the radiation back into the mirror field. It is hoped that this location and arrangement will produce stronger electric fields in the center of the machine.

Work on this system is still in the beginning stages, with much remaining to be determined about the system itself. Work will continue on the off-resonance heating with Langmuir probe techniques and with the possible incorporation of directional probes. Also being considered is ion cyclotron heating using the 400 KHz source. Ion temperature would be measured by a monochromator which can determine the broadening of spectral lines caused by the Doppler Effect.

$$F_1 \in \mathcal{L}^1(\mathbb{R}^2) \quad (1.2)$$

FIGURE 20

III. A. ELECTRON CYCLOTRON RESONANCE HEATING EXPERIMENTS

J.C. Sprott and K.A. Connor

1. Measurement of heating efficiency

The theoretical heating model predicted that the electron cyclotron heating efficiency (power absorbed by the plasma/input power to the cavity) should be proportional to electron density at low densities and should approach 100% when the average density exceeds a value given by

$$\bar{n} = \frac{\epsilon_0 m \omega^2}{Q_0 G e^2}$$

where ω is the rf frequency, Q_0 is the unperturbed Q of the cavity, and G is a factor of order unity that depends on the geometry of the field, the location of the resonance zones, and the density distribution.

The cavity Q was measured by means of a 10 mW swept 3cm klystron which injected microwaves through the same ports used for the high power heating. The cavity electric field was measured by a coupling loop. The resulting signal for a typical cavity mode near the heating frequency in the absence of plasma is shown in figure 21. The Q calculated from $F/\delta F$ is about 1000. This value is about a factor of five lower than the Q estimated from $V/A\delta$, where V , A , and δ are the cavity volume, surface area, and skin depth respectively, but the discrepancy is reasonable when one considers microwave leaks in the cavity and contamination of the walls. From the measured Q , we conclude that total absorption of the rf power should occur for $\bar{n} > 10^9 \text{ cm}^{-3}$. This condition is marginally satisfied in the present experiment.

The RMS electric field in the cavity in the absence of plasma is given by

$$E_{\text{RMS}} = \frac{P_0 Q_0}{E_{\text{av}} V} = 1.8 \text{ kV/cm for } P_0 = 100 \text{ kW.}$$

We expect negligible reflection of the microwave power for

$$\lambda^3 \ll \frac{8\pi V}{Q}$$

or $f \gg 5.2 \text{ GHz}$. For the normal heating frequency (8.35 GHz), this condition

Typical
Cavity Mode

(a)

Incident Power

Diamagnetic Signal

(b)

1μsec/Div

FIGURE 21

is not well satisfied, and the reflected power is $\sim 33\%$ of the incident power in the absence of plasma. The reflected power drops to $\sim 22\%$ when a plasma with $\bar{n} \sim 10^{10} \text{ cm}^{-3}$ is present.

The measured peak input power to the cavity as determined by measurements of the incident and reflected power is $\sim 40 \text{ kW}$ and has the waveform shown in fig. 19(b). The heating rate as measured by a diamagnetic loop near the midplane is given by

$$\frac{d}{dt} (nV_k T_e) = \frac{BV\epsilon}{\mu_0 NA}$$

where B is the magnetic field strength, ϵ is the loop voltage, N is the number of turns in the loop, and A is the loop area. The diamagnetic signal shown in fig. 21(b) gives a heating rate of $\sim 12 \text{ kW}$ implying an efficiency of $\sim 30\%$, although this is probably an underestimate because of frequency response limitations in the loop and because of power losses by ionization and scattering into the loss cone during the heating pulse. The frequency response of the diamagnetic loop was measured by inductively coupling a sinusoidal voltage to the loop. The upper frequency limit of the loop was $\sim 400 \text{ kHz}$ in agreement with the value calculated from $f_c = R/2\pi L$. A correction was made to the signal in fig. 21(b) to compensate for the bandwidth of the loop. The diamagnetic signal and the ion saturation current to a probe on axis are proportional to the input rf power over the range tested (about a factor of two).

2. Observation of flute instabilities

An anisotropic, low β plasma is expected to be flute unstable if

$$\int \frac{p_{\perp} + p_{\parallel}}{B^2 R r} dl < 0$$

where $p_{\perp}$ and $p_{\parallel}$ are the plasma pressure perpendicular and parallel to $\vec{B}$, R is the radius of curvature of the line of force, r is the distance to the axis of symmetry, and dl is an element of length parallel to $\vec{B}$. The above equation is satisfied in the present experiment, and so we expect to see flute instabilities.

Figure 22 shows a radial profile of density $\langle n \rangle$ and density fluctuation $\langle \delta n \rangle$ 3 msec after the heating pulse as determined from the ion saturation current to a Langmuir probe at an axial position where $B(Z)/B(0) = 1.013$ for a mirror ratio of 1.02. The annular structure appears to be a general feature of the

FIGURE 22

plasma and is independent of the position of the heating zones. The fluctuations occur predominantly in the region where $\frac{dn}{dr} < 0$, and satisfy the condition

$$\frac{\langle \delta n \rangle}{\langle n \rangle} \frac{e \langle \delta V_f \rangle}{kT_e} \approx 100\%,$$

where V_f is the floating potential. We note that the fluctuations occur only on the boundary of the plasma, and that the density gradient near the mirror axis is in the favorable direction for containment. The fluctuations are reasonably sinusoidal with a frequency of ~ 100 kHz just after the heating pulse and a frequency of ~ 10 kHz after 3 msec.

In order to show that these fluctuations are indeed flute modes, the perpendicular and parallel wavelengths were compared using a floating double Langmuir probe with a tip separation of 0.5 cm. The measured electric field in the θ -direction ($\perp$ to $\vec{B}$) and the Z-direction ($\parallel$ to $\vec{B}$) are shown in fig. 23. The parallel electric field is much smaller than the perpendicular field, as would be expected for a flute. An exact measurement of the wavelength was not attempted, although we can conclude from the observations that $\lambda_{\parallel} \gg \lambda_{\perp} > 0.5$ cm.

Flute instabilities can be suppressed by the addition of a linear multiple field (I_{offe} bars) to produce a minimum -B geometry. Calculations were performed to determine the required strength for a quadrupole field. With $R = 1.02$, an absolute minimum B can be obtained in the present apparatus with $\sim 18,000$ amperes per rod. A quadrupole field with 290 amperes per rod was tried with no observable effect on the fluctuations.

The plasma lifetime calculated for this instability, assuming a 100% fluctuation in density, and perfect correlation between δn and δE is given by

$$T = \frac{\lambda}{\langle v \rangle} = \frac{n \lambda B}{\langle \delta n \delta E \rangle} = \frac{\lambda B}{\delta E} \sim 100 \text{ } \mu\text{sec.}$$

The observed lifetime is ~ 1 m sec, and the density distribution remains relatively constant throughout the life of the plasm, suggesting that the radial diffusion caused by these fluctuations is small.

3. CW microwave heating

Since the 100 kW 3 cm source was pulsed, the S-band source had to be used for CW resonant heating. The magnetic field was appropriately decreased so that

FIGURE 23

resonance appeared inside the mirror region.

For this heating, Langmuir probe measurements show that the density increases monotonically with helium gas pressure and microwave power, and that densities of $\sim 10^{11} \text{ cm}^{-3}$ (where $\omega_p = \omega$) can be produced with 100 watts. The electron temperature is 10-20eV and depends only slightly on pressure and on microwave power. The density is peaked near the axis, and fluctuations at $\sim 10\text{kHz}$ with an amplitude of 10-100% occur throughout the plasma. The percentage fluctuation does not depend strongly on gas pressure or on microwave power, in contrast with recent experiments at Los Alamos.¹¹

The microwave source can be pulsed off abruptly for ~ 30 msec and then turned on again. When the heating is turned off, the ion saturation current decays in ~ 10 μsec in contrast to the ~ 1 msec observed for the 100 kW pulsed heating. The decay time is independent of gas pressure and suggests a strong loss mechanism. When the source is turned back on, the ion saturation current builds up much more slowly. At 1×10^{-6} torr, the buildup time is ~ 1 msec, and varies inversely with gas pressure in agreement with the calculated ionization time. The power absorbed by the plasma can be estimated from

$$P \sim \frac{nkT_e V}{\tau}$$

where τ is the decay time after the rf is turned off. Using $n = 10^{11} \text{ cm}^{-3}$, $kT_e = 15 \text{ eV}$, and $\tau = 10 \mu\text{sec}$, the absorbed power is ~ 100 watts suggesting a high heating efficiency as expected theoretically.

The ion saturation current to a Langmuir probe on axis was measured during CW heating as a function of the background helium gas pressure for three different values of incident microwave power. The result is shown in fig. 24. Since T_e does not depend strongly on pressure or power, the density is seen to follow approximately the relation

$$n \propto P/\sqrt{p}.$$

On this curve an ion saturation current of 1 mA corresponds to a density of $\sim 1 \times 10^9 \text{ cm}^{-3}$. When a stub tuner was added to the magnetron output, the ion saturation current could be increased to almost 100 ma for an input power of 100 watts and a pressure of 1×10^{-3} torr. These conditions caused considerable heating of the Langmuir probes.

FIGURE 24

B. CALCULATION OF ELECTRON CYCLOTRON HEATING RATES

J. C. Sprott

1. Theoretical model.

Consider a plasma confined in a cavity of arbitrary size and shape by a static non-uniform magnetic field $\vec{B}(\vec{r})$ in the presence of an rf electric field $\vec{E}_0(\vec{r}, t)e^{i\omega t}$. Assume a plasma density sufficiently low that the waves freely penetrate the plasma and a plasma pressure sufficiently low that the magnetic field is unperturbed (i.e., $\beta = 2\mu_0 p/B^2 \ll 1$).

We define a function ψ which is constant on a flux surface and has a value equal to the magnetic flux enclosed by that surface. The magnetic field lines lie in the surfaces and may or may not be confined in the cavity. An orthogonal coordinate system will be used in which $\hat{\psi}$ is a unit vector normal to a flux surface and $\hat{l}$ is a unit vector in the direction of $\vec{B}$. This system is convenient because plasma energy flows readily in the $\hat{l}$ direction and because in many axisymmetric and linear geometries the coordinate orthogonal to $\hat{l}$ and $\hat{\psi}$ is ignorable.

The power absorbed per unit volume in a resistive dielectric medium can be expressed in terms of a real tensor conductivity $\vec{\sigma}$ as

$$\frac{dP}{dV} = (\vec{\sigma} \cdot \vec{E}) \cdot \vec{E}, \quad (1)$$

or, in our coordinates,

$$\frac{dP}{d\psi} = \int (\sigma_{\perp} E_{\perp}^2 + \sigma_{\parallel} E_{\parallel}^2) dl / B. \quad (2)$$

The perpendicular and parallel conductivities of a cold, tenuous plasma can be expressed in terms of a collision frequency ν for $\nu \ll \omega$ as

$$\sigma_{\perp} = \epsilon_0 \omega_p^2 \nu [\omega^2 + \omega_c^2] / [(\omega^2 - \omega_c^2)^2 + 4\omega^2 \nu^2]$$

and

$$\sigma_{\parallel} = \epsilon_0 \omega_p^2 \nu / \omega^2,$$

where ω_p is the plasma frequency and ω_c is the cyclotron frequency. For $v \ll \omega$, most of the contribution to the integral in equation (2) comes from the resonance at $\omega = \omega_c$, and we can expand $B(l)$ in a Taylor series about its resonance value, retaining only the first order term:

$$B(l) = B_0 + l \left| \nabla_{H_0} B \right|,$$

where $l=0$ is taken where $B = m\omega/e$ and the subscript "0" refers to the value of a quantity at $l=0$. If the electric field and density vary slowly over the width of the resonance, the integral in equation (2) can be evaluated for $-\infty < l < \infty$ to get the result

$$\frac{dP}{d\psi} = \frac{\pi n_0 e E_{\perp 0}^2}{2 B_0 \left| \nabla_{H_0} B \right|}. \quad (3)$$

The corresponding heating rate is

$$\frac{d\bar{W}}{dt}(\psi) = \frac{dP}{d\psi} \int \frac{ndl}{B} = \frac{\pi n_0 e E_{\perp 0}^2}{2 B_0 \left| \nabla_{H_0} B \right| \int ndl/B}. \quad (4)$$

2. Comparison with other works.

The same result can be obtained by a simple phenomenological argument. The average energy gained by an electron during one transit through resonance can be written as

$$\bar{\Delta W} = \frac{1}{2} \pi \Delta v_{\perp}^2 = \frac{e^2 E_{\perp 0}^2}{2m} T^2,$$

where T is the time during which the electron stays in phase with the electric field, or the time required for the electron to cross the resonance region.

Kawamura and Terashima¹² used the above expression to calculate the heating rate in the TP-M machine at Nagoya University (Japan). Lichtenberg, et al.¹³ have also proposed a similar expression, differing only by a numerical factor of order unity.

The transit time of an electron through resonance can be approximated by a method suggested by Guest¹⁴ and by Ard¹⁵:

$$\int_0^T (\omega_c - \omega) dt = \frac{\omega |v_{\perp 0} B|}{B_0} \int_0^T v_{\parallel} t dt = \frac{\pi}{2},$$

to get the result,

$$T = \sqrt{\frac{\pi B_0}{v_{\parallel 0} \omega |v_{\perp 0} B|}}. \quad (5)$$

The resulting energy gain is

$$\overline{\Delta W} = \frac{\pi e E_{\perp 0}^2}{2 v_{\parallel 0} |v_{\perp 0} B|}.$$

Kuckes¹⁶ arrived at an identical result by solving explicitly the equation of motion of an electron that moves through the resonance region with constant parallel velocity in a field with a constant gradient parallel to $\vec{B}$.

The power absorbed by a group of particles is found by multiplying the average energy gained during one transit by the flux of particles through resonance:

$$dP = \overline{\Delta W} n_0 v_{\parallel 0} dA = \frac{\pi n_0 e E_{\perp 0}^2}{2 B_0 |v_{\perp 0} B|} d\psi.$$

The result is identical to equation (3).

Several authors have considered the special case of a straight magnetic field of the form

$$B(z) = B(0)(1 + \beta z^2), \quad (6)$$

in which electrons execute sinusoidal oscillations:

$$z(t) = z_M \sin \omega_{\beta} t,$$

where

$$\omega_{\beta} = \sqrt{\beta} v_{\perp}(0).$$

For a group of electrons that mirror at z_M , the density distribution is given by

$$n(z) = \begin{cases} \frac{n_0 B(z) v_{\perp 0}}{B_0 v_{\perp 0}(z)} & \text{for } z \leq z_M \\ 0 & \text{for } z > z_M \end{cases}$$

where $v_{\perp}(z) = v_{\perp}(0) \sqrt{\beta} \sqrt{z_M^2 - z^2}$.

Substituting into equation (4) gives

$$\frac{d\bar{W}}{dt} = \frac{eE_{\perp 0}^2}{2B_0 \sqrt{\delta}} \frac{r}{r-1}, \quad (7)$$

where $\delta = \frac{z_M^2}{z_0^2} - 1$ and $r = B_0/B(0)$.

Grawe⁶ has treated the same problem by solving explicitly the equation of motion of a relativistic electron in a parabolic magnetic field. In the non-relativistic limit, his result is

$$\frac{d\bar{W}}{dt} = \frac{eE_{\perp 0}^2}{2B_0} \left(\frac{\omega}{\omega_B}\right) J_{\alpha(1-\delta)}^2 [\alpha(1+\delta)],$$

where

$$\alpha = \frac{\omega}{\omega_B} \frac{r-1}{4r} \gg 1.$$

In order to compare the results, consider the case $\delta=1$, for which the Bessel function can be approximated by

$$J_0^2\left(\frac{\omega}{\omega_B} \frac{r-1}{2r}\right) = \frac{2}{\pi} \frac{\omega_B}{\omega} \frac{r}{r-1} \cos^2\left[\frac{\omega}{\omega_B} \frac{r-1}{2r} - \frac{\pi}{4}\right].$$

The solution is a rapidly oscillating function with a mean value of

$$\frac{d\bar{W}}{dt} = \frac{eE_{\perp 0}^2}{2B_0} \frac{r}{r-1},$$

which is identical to equation (7) with $\delta=1$.

If a significant number of electrons mirror exactly at the resonance, it is necessary to generalize the method slightly by leaving the density inside the integral in equation (2), and by introducing a phenomenological resonance width of $v=1/T$ where T is given by equation (5). The calculated heating rate for these particles is then

$$\frac{d\bar{W}}{dt} = 0.21 \frac{eE_{\perp 0}^2}{B_0} \left(\frac{\omega}{\omega_B}\right)^{1/3} \left(\frac{r}{r-1}\right)^{2/3}, \quad (8)$$

in good agreement with the result of Grawe¹⁷ ($\delta=0$) and of Guest¹⁴.

3. Application to a magnetic mirror.

Consider a magnetic mirror field of the form

$$B(Z) = \frac{1}{2} B(0) \{ (R+1) - (R-1) \cos KZ \} \quad (-\pi < KZ < \pi)$$

and a loss cone electron energy distribution given by

$$f(W_{\perp}, W_{\parallel}, Z) = \begin{cases} A e^{-(W_{\perp} + W_{\parallel})/kT} & \text{for } \frac{W_{\parallel}}{W_{\perp}} < \frac{RB(0)}{B(Z)} - 1 \\ 0 & \text{for } \frac{W_{\parallel}}{W_{\perp}} > \frac{RB(0)}{B(Z)} - 1 \end{cases}$$

The density distribution is obtained by integration:

$$n(Z) = \int_0^{\infty} \int_0^{\infty} \frac{W_{\parallel}}{\frac{RB(0)}{B(Z)} - 1} A e^{-W_{\perp}/kT} e^{-W_{\parallel}/kT} dW_{\perp} dW_{\parallel} = n(0) \frac{RB(0) - B(Z)}{(R-1)B(0)} \quad (9)$$

The total number of electrons in the flux shell is

$$\int_0^{\pi/K} \frac{n(Z) dZ}{B(Z)} = \frac{\pi n(0)}{KB(0)} \frac{\sqrt{R}-1}{R-1}.$$

Substituting into equation (4) gives a heating rate of

$$\frac{d\bar{W}}{dt} = \frac{eE_{\perp 0}^2}{2B_0(\sqrt{R}-1)} \sqrt{\frac{R-r}{r-1}}, \quad (10)$$

where $r = B_0/B(0)$.

We can define a perpendicular and parallel "temperature" at $Z=0$ by

$$kT_{\perp,0} = \frac{A}{n(0)} \int_0^{\infty} \int_{\frac{R-1}{R}}^{\infty} \frac{W_{\parallel}}{R-1} W_{\perp} e^{-W_{\perp}/kT} e^{-W_{\parallel}/kT} dW_{\perp} dW_{\parallel}$$

to get the result:

$$\begin{aligned} kT_{\perp} &= \left(\frac{R+1}{R}\right) kT \\ kT_{\parallel} &= \left(\frac{R-1}{R}\right) kT. \end{aligned}$$

We expect, therefore, a midplane anisotropy of

$$\theta = T_{\perp}/T_{\parallel} = \frac{R+1}{R-1} \quad (11)$$

The heating rate for an arbitrary anisotropy can be calculated in a straightforward way by generalizing the assumed distribution function, but the resulting equation is quite complicated and unenlightening. Qualitatively, a large anisotropy enhances the heating if resonance occurs near the midplane and reduces the heating for resonance near the mirror throats.

Electrostatic potential gradients can also be included in the model by simply adding a Boltzmann factor, $\exp(-e\Phi(Z)/kT)$ to the density distribution in equation (9).

4. Limitations and discussion.

The use of a steady state, cold plasma conductivity to calculate the heating rate is justified if

$$\omega \gg v = \frac{1}{T} = \sqrt{\frac{v_{Bo} \omega |\nabla_{Bo} B|}{\pi B_o}},$$

or

$$v_{Bo} \ll \omega B_o / |\nabla_{Bo} B|. \quad (12)$$

This condition is generally well satisfied for non-relativistic plasmas in typical magnetic field geometries.

Calculation of the heating rate requires a knowledge of the electric field $E_{\perp o}$. The vacuum electric field is perturbed locally at $\omega = \omega_c$ because of the resonance in the dielectric constant, which in the absence of collisions can be written as

$$\epsilon = \epsilon_o \left[1 - \frac{\omega_p^2}{\omega(\omega - \omega_c)} \right]$$

This local perturbation is negligible provided

$$\int_{-\infty}^{\infty} \text{Im} \left[\frac{\omega}{c} \sqrt{\frac{\epsilon(l)}{\epsilon_o}} \right] dl = \frac{\omega}{c} \int_0^{\omega_p^2 B_o / \omega^2 |\nabla_{Bo} B|} \sqrt{\frac{\omega_p^2 B_o}{\omega^2 |\nabla_{Bo} B| l}} - 1 \quad dl = \frac{\pi \omega_p^2 B_o}{2 \omega c |\nabla_{Bo} B|} \ll 1,$$

$$\text{or } \omega_p^2 \ll \omega c |\nabla_{\parallel 0} B| / B_0 . \quad (13)$$

The low density plasmas produced by electron cyclotron heating generally satisfy this requirement.

In addition to this local perturbation, the mean square electric field in the cavity is reduced because the cavity Q is lowered in the presence of plasma. The input power P_0 is given by

$$P_0 = \frac{\omega \epsilon_0 \overline{E^2} V}{Q_0} + \int \frac{dP}{d\psi} d\psi ,$$

where the first term on the right is the power absorbed by the cavity and the second term is the power absorbed by the plasma. The second term can be written, according to equation (3), as

$$\int \frac{dP}{d\psi} d\psi = \frac{\overline{n} e \overline{E^2} V G}{B_0} ,$$

where

$$G = \frac{\pi}{2V} \int \frac{n_0}{n} \frac{E_{\perp 0}^2}{E^2} \frac{d\psi}{|\nabla_{\parallel 0} B|}$$

is a factor of order unity that depends on the geometry of the field and density distribution. For $n = \text{const}$ and $E_{\perp 0}^2 = \frac{2}{3} \overline{E^2}$, we have

$$G = \frac{\pi}{3} \frac{B_0}{V} \frac{dV}{dB} \Big|_{B_0} .$$

The mean square electric field in the presence of plasma is given by

$$\overline{E^2} = \frac{P_0}{\frac{\omega \epsilon_0 V}{Q_0} + \frac{\overline{n} e V G}{B_0}} ,$$

and the resulting heating efficiency is

$$\eta = \frac{1}{P_0} \int \frac{dP}{d\psi} d\psi = \frac{1}{1 + \overline{n} e / n Q_0 G} , \quad (14)$$

where $n_c = \epsilon_0 m \omega^2 / e^2$. Note that at low densities, the efficiency is proportional to density, but for $\bar{n} > n_c / Q_0 G$, the plasma loading dominates the cavity Q, and total absorption of the rf is expected. At much higher densities ($\bar{n} > n_c$), the rf fields are excluded from the plasma, and the efficiency presumably decreases.

C. OFF-RESONANCE HEATING STUDIES

K.A. Connor and J.C. Sprott

1. Introduction

Recent experiments at Oak Ridge¹⁸ using microwave heating both above and below the electron cyclotron frequency have produced some spectacular effects. Upper off-resonance heating is found to be very efficient, while lower off-resonance heating results in expelling the plasma from the mirror field. Similar experiments in toroidal octupoles at Wisconsin have also indicated the existence of an upper off-resonance heating mechanism, but lower off-resonance heating does not expel the plasma in an octupole with B_θ^2 . As reported, similar effects have been observed in a magnetic mirror here at Wisconsin. During the period covered by this report, we have studied the experimental properties of these effects with the goal of unfolding the mechanisms responsible for them. A theoretical model has been developed that explains the results in terms of an effective collision rate. The model predicts nearly isotropic heating above the cyclotron frequency, and strong parallel heating below the cyclotron frequency. Experimental results are presented which confirm the theoretical predictions.

2. Experimental Results

a. Lower off-resonance heating ($\omega < \omega_c$).

One of the effects noted at Oak Ridge during lower off-resonance heating (hereafter LORH) was an increase in radial diffusion by as much as a factor of 2. To see if this effect accompanies LORH in our magnetic mirror device, a Langmuir probe was used to measure the radial density profile. Figure 25 shows a typical case of $\langle n(r) \rangle$ as determined by the ion saturation current to a Langmuir probe for various values of LORH power, 50 μ sec. after the resonant power pulse. The LORH source is a 50 W, C.W. magnetron tuneable from 2.975 - 3.170 GHz. Note that the characteristic annular structure of the plasma is unchanged. The

Figure 25

profiles show no distinctive differences other than a decrease with increasing power. The highest LORH power indicated (4 watts) is approximately the maximum that could be applied in this case without extinguishing the plasma. The radial profiles at other times after the resonant power pulse evolved in the same way with increasing LORH power.

When Langmuir probes were used to measure $\langle n(r) \rangle$, fluctuations appear on the probe signal as reported in QPR 12, QPR 13, and QPR 14. Figure 26 shows how these fluctuations are affected by LORH. Notice that they are not normally found in the center of the machine with no LORH, but that they appear there when the LORH is applied. Figures 27 and 28 show the time evolution of the probe signal at various radial positions without and with LORH respectively.

A floating double Langmuir probe was used to measure the fluctuations in electric field found on the edge of the plasma with no LORH and on the axis of the plasma with LORH applied. Figure 29 shows the cases of the double probe electrodes perpendicular and parallel to $\vec{B}$. The absence of the fluctuations in the parallel direction indicates the existence of flutes. This condition is to be expected in a mirror confined plasma. The frequency of the observed oscillations is approximately that of the theoretical value for an $m = 1$ flute ($\omega \sqrt{v_{TB}}/r$, where r is distance from mirror axis). Figure 29 shows the results of the same test made when the probe is at the center of the plasma when LORH is applied. The tests in this case show that LORH causes flutes to appear in the center. These fluctuations always appear when LORH is applied but not always with the same amplitude.

A measurement of the axial density profile was made to determine the effect of LORH on it. Figure 30 shows that the axial density peak moves toward the throats of the mirror when LORH is applied. Note that the density gradient to the right of the peak remains approximately the same.

2

Figure 26

Amplitude of Density Fluctuations as Percent
of Density Amplitude vs. Radial Position

50 μ sec after Resonance Pulse

$R = 1.19$

$P = 7 \times 10^{-4}$ Torr

$Z = 15.0$ cm

Figure 27

Time Evolution of Langmuir Probe Ion Saturation Current at Various Radial Positions

For 8.54 GHz Resonant Heating (100 kw-pulsed)

$R = 1.19$

$P = 7 \times 10^{-4}$ Torr

$Z = 15.0$ cm

(a) $r = 0$ cm, .2 v/cm
(b) $r = 1$ cm, .2 v/cm
(c) $r = 2$ cm, .5 v/cm

(d) $r = 4$ cm, .5 v/cm
(e) $r = 6$ cm, .2 v/cm
(f) $r = 7$ cm, .2 v/cm

Figure 28

Time Evolution of Langmuir Probe Ion Saturation Current
For 8.54 GHz Resonant Heating (100 kw-pulsed)
Plus 3.0 GHz LORH (2.5 watts - CW)
 $R = 1.19$; $P = 7 \times 10^{-4}$ Torr; $Z = 15.0$ cm

E

E

(a) No LORH $r = 7$ cm

E

E

← 2.8 msec →

(b) LORH = 2.5 watts $r = 0$ cm

Figure 29

Floating Double Langmuir Probe Measurements
of Fluctuations

Figure 30
 Axial Density Profile
 $R = 1.19$, $P = 5 \times 10^{-4}$ Torr,
 50 μ sec after Resonance Pulse

The electrostatic particle collector described in Section II of this report was first placed on one of the end plugs to monitor axial flux. V-I plots were made with the collector mounted using plasmas produced with LORH. Figure 31 shows a typical V-I characteristic. When the LORH power is increased, the axial flux monotonically decreases. The collector was then placed in the machine as a radial flux collector. Similar V-I plots were obtained. In this case the particle flux increases by about a factor of two with increasing LORH. This increase is observed when fluctuations appear on the Langmuir probe ion saturation current signal early in time ($<50 \mu\text{sec}$). If fluctuations are present but not early in time the collector signals will decrease monotonically just as in the axial case. The decrease in axial flux is just a result of the decrease in density and does not indicate a decrease in the axial diffusion coefficient. In fact, most of the plasma is lost axially from the device, and so the integrated axial flux is a measure of the total number of trapped particles, whether off-resonance heating is applied or not.

The X-rays produced by both wall and free-free bremsstrahlung were measured together using a geiger counter with and without the end plugs in place. As reported previously, the time integrated X-ray flux increases monotonically with LORH power both in the radial and axial directions. The radial increase is usually about a factor of 2 while the axial increase has been as high as a factor of 60 with the end plugs in place. When the end plugs are removed, thus decreasing the X-ray target area for particles moving parallel to $\vec{B}$, the radial increase in X-ray flux is about the same as with end plugs while the axial increase has been no larger than 6. Collimation in the radial direction does not allow X-rays from the end plugs to reach a detector, so one would expect approximately the same flux increase with and without the end plugs.

The increase in X-ray flux can be attributed to the fact that the confinement time is reduced by the LORH. Thus particles are lost, on the average, with higher energies. The larger increase

Figure 31

V-I Characteristics of Particle Collector

1 msec after Resonance Pulse

No LORH

$R = 1.02$

$P = 7 \times 10^{-7}$ Torr

$Z = 15.0$ cm

with the end plugs in indicates that the particles being lost due to LORH strike a more efficiently placed target. That is, particles are less effectively confined and are lost out the mirror throats and strike the end plugs.

The CW magnetron used for LORH was tuneable from 2.925 - 3.170 GHz. When it was tuned through its range, the LORH effect was found to depend on frequency. Depending on the microwave frequency and the position and orientation of the coupling loop, cases were observed where <4 watts of lower off-resonance power would expel the plasma, while other cases showed no effect on the plasma with powers of >50 watts.

The important features of LORH are thus a decrease in density with increasing LORH, the appearance of fluctuations on axis that look like flutes, enhanced radial diffusion, no change in the shape of the radial density profile, enhanced X-ray flux, and change in axial profile.

In an experiment performed on a different mirror machine (see Section II) using plasmas produced by a 10 kW, 10 μ sec, 3250 MHz resonance source, the plasma could be extinguished by applying a 10 MHz, CW, electric field parallel to $\vec{B}$, but not if the same electric field was perpendicular to $\vec{B}$.

b. Upper off-resonance heating ($\omega > \omega_c$).

As reported in QPR #14 an attempt was made to observe upper off-resonance heating (hereafter UORH) using plasmas produced by ECRH with the 50 W, 3.0 GHz C.W. source mentioned above. Any effect present was obscured by electrical noise picked up by the diamagnetic loop. The experimental setup was subsequently modified to minimize electrical noise, and a 10 kW pulsed 3.25 GHz source was built to produce hotter plasmas and thus to improve the signal-to-noise ratio. This source produces 10 μ sec pulses at 120 sec^{-1} while the 8.54 GHz source used for UORH gives 100 kW, 2 μ sec pulses at 360 sec^{-1} .

The high power, short pulse source is ideally suited for UORH because the heating time is short compared with both the ionization time and the scattering time into the loss cone, and so the diamagnetic loop signal should give a reliable measure of the power absorbed by the plasma. The loop circuit was modified to extend its bandwidth to 1 MHz, and this has proved adequate for the measurements made. The loop was also calibrated using resonant heating at 8.54 GHz, and the observed signals were in good agreement with the predictions of the resonant heating theory in QPR #12.

If the 3.25 GHz source is operated so as to be at cyclotron resonance with the $\vec{B}$ -field and the 8.54 GHz source is then triggered by it to provide the UORH, heating will take place under three distinct conditions of temperature and density. These latter quantities decay in time in the afterglow, which should allow us to see at a glance how the UORH qualitatively depends on $\langle n \rangle$ and T_e . We can also vary the pulse spacing continuously. Figure 30 shows the density and temperature as measured by a Langmuir probe in the afterflow of the 3.25 GHz heating. It also shows the average heating rate as derived from the average density ($\langle n \rangle$) and diamagnetic loop voltage (V) as follows:

$$\frac{d\bar{W}}{dt} = \frac{3BV}{2\mu_0 NA \langle n \rangle},$$

where N is the number of turns on the loop (16) and A is the loop area. The density used in this equation was derived from Langmuir probe data so it is a local density rather than the average density that the formula calls for. However, comparisons with cavity perturbation density measuring techniques shows this to be a reasonable choice for $\langle n \rangle$. Figure 33 indicates that $d\bar{W}/dt$ is linearly dependent on $\sqrt{kT_e}$ or velocity. The experiments were carried out at high pressures ($\sim 50\mu$) so it was suspected that the heating indicated by the loop signals was a result of electron-neutral collisions. When the 3.25 GHz source was turned off leaving just

FIGURE 32

HEATING RATE vs. $\sqrt{\frac{kT_e}{e}}$

(SAME CASE AS FIGURE 9)

FIGURE 33

the 8.54 GHz source on a faint glow could be seen at times indicating that resistive heating was taking place.

The 3.0 GHz C.W. source was used to determine the heating rate of the UORH. Figures 34 and 35 show two sets of experimental data for $d\bar{W}/dt$ vs. pressure as determined from the above equation. Note that there is a linear relationship between $d\bar{W}/dt$ and pressure over much of the range of the log-log plot. At lower pressures the heating rate is linearly related to $\sqrt{kT_e}$ or velocity. This suggests that at high pressures, electron-neutral collisions dominate the heating, while at low pressures, some other velocity dependent mechanism becomes dominant.

3. Theoretical Model

a. Upper off-resonance heating ($\omega > \omega_c$)

The heating rate for a cold, tenuous, uniform plasma in a uniform magnetic field can be expressed in terms of a real conductivity tensor as

$$\frac{d\bar{W}}{dt} = \frac{(\vec{\sigma} \cdot \vec{E}) \cdot \vec{E}}{n} = \frac{e^2 E_{\perp}^2 v (\omega^2 + \omega_c^2 + v^2)^2}{m[(\omega^2 - \omega_c^2 - v^2)^2 + 4\omega^2 v^2]} + \frac{e^2 E_{\parallel}^2 v}{m(\omega^2 + v^2)} \quad (1)$$

where $E_{\perp}$ ($E_{\parallel}$) is the perpendicular (parallel) component of the rf electric field, ω is the applied rf frequency, ω_c is the electron cyclotron frequency, and v is the collision frequency ($v \ll \omega$). Equation (1) was used to calculate the stochastic heating of a collection of electrons that traverse a resonance in a non-uniform magnetic field. This was done by integrating equation (1) over the volume of the plasma assuming $v \ll \omega$ so that the dominant contribution to the integral comes from the immediate vicinity of the resonance. The magnetic field was then expanded in a Taylor series about the resonance value, and the resulting heating rate is independent of the collision frequency v . The result was shown to be consistent with the single particle models of Lichtenberg,¹³ Kuckes,¹⁶ and Guest,¹⁴ and with the non-relativistic limit of the results of Grawe.¹⁷

Figure 34
Perpendicular Heating Rate vs. Pressure

Figure 35
 Heating Rate vs. Pressure

For $\omega^2 \gg \omega_c^2$ (extreme upper off-resonance heating), equation (1) becomes

$$\frac{d\bar{W}}{dt} = \frac{e^2 E^2 v}{m(\omega^2 + v^2)} \quad (2)$$

which is the usual result for collisional heating in the absence of a magnetic field. The more general case of ω only slightly greater than ω_c is more complicated because the heating rate depends on the geometry of the field and on the particle trajectories (or density distribution). The heating rate (averaged over a bounce period) is calculated from the expression

$$\langle \frac{d\bar{W}}{dt} \rangle = \int n(l) \frac{d\bar{W}(l)}{dt} \frac{dl}{B(l)} / \int n(l) \frac{dl}{B(l)}. \quad (3)$$

Calculations of the heating rate of a hot, relativistic plasma should take into account the following effects:

- 1) Variation of ω_c with energy: $\omega_c = eB/\delta m$
- 2) Doppler shift of ω due to v : $\omega_D = \omega \pm k v$
- 3) Variation of E over the gyrodiameter of the particle
- 4) Effect of E on the bounce motion
- 5) Non-conservation of the particle's magnetic moment
- 6) Shift in gyrofrequency by perpendicular field gradients (∇B)
- 7) The $\vec{v} \times \vec{B}$ force of the rf magnetic field.

Existing heating models which claim to be relativistic generally consider only a couple of the above effects. By restricting the discussion to the case $\omega^2 \gg \omega_c^2$, the present model can be made relativistic by choosing an appropriate, phenomenological, energy dependent, collision frequency v , which, when substituted into equation (2), gives a heating rate in agreement with experiment.

Physically, v can result from real particle collisions or from some other mechanism that interrupts the simple periodic variation of the particle's energy in a sinusoidal rf electric field. In the weakly ionized plasmas studied here, electron-ion

collisions are negligible compared with electron-neutral collisions. For electron energies greater than a few eV, the electron-neutral collision frequency in helium is

$$\nu_{e-n} = 2.3 \times 10^9 p \text{ (torr)},$$

and the resulting heating rate should vary linearly with neutral pressure in agreement with observations as shown in Figs. 11 and 12. The rf electric field calculated from the input microwave power ($p_0 = 100 \text{ kW}$) and from the unperturbed cavity Q (≈ 1000) is given by

$$E_{\text{RMS}} = \frac{p_0 Q}{\epsilon_0 \omega V} = 1.8 \text{ kV/cm}.$$

The unperturbed Q is used because the off-resonance absorption of power by these cold, tenuous plasmas is quite small. Substituting these values into equation (2) gives a heating rate of

$$\frac{dW}{dt} \text{ (eV/sec)} = 4.5 \times 10^9 p \text{ (torr)}$$

which agrees within experimental error with the heating rates measured at high pressure.

Figure 35 shows, however, that at low pressure the linear dependence on pressure no longer holds, and the heating rate varies with $\sqrt{kT_e}$. This behavior suggests that the bounce frequency or the inverse transit time through a cavity mode ($\nu / \lambda = \omega v / c$), or some other mechanism with a constant mean free path plays the role of an effective collision rate at low pressures. Smullen¹⁹ and Grawe¹⁷ assumed (somewhat arbitrarily) that phase randomization occurs after a bounce time, although Grawe strangely calculates a heating rate less than would be obtained by substituting the bounce frequency (ω_B) for ν in equation (2). Equation (2) with $\nu = \omega_B$ gives a heating rate in agreement with Grawe's prediction at relativistic energies, but Grawe's result decreases sharply with decreasing energy in contrast to the slow energy variation of the

present theory. The heating rate in the present experiment has not been measured with sufficient accuracy to establish that $v=\omega_g$, but the heating rate is of the right order, apparently in contradiction to the much smaller result predicted by Grawe.

b. Lower off-resonance heating ($\omega < \omega_c$)

For $\omega^2 \ll \omega_c^2$, equation (1) predicts a heating rate of

$$\frac{d\bar{W}}{dt} = \frac{e^2 E_{\perp}^2 v}{m \omega_c^2} + \frac{e^2 E_{\parallel}^2 v}{m(\omega^2 + v^2)} \quad (4)$$

Lower off-resonance heating is anisotropic, in contrast to the upper off-resonance case of equation (2). The first (second) term on the right of equation (4) represents the rate of change of perpendicular (parallel) energy. This result is reasonable since the magnetic field reduces the mobility of the electrons in the perpendicular direction and limits the time during which the electron stays in phase with the rf electric field. In a multi-mode cavity, on the average, $E_{\perp}^2 = 2E_{\parallel}^2$, so that for $\omega^2 \ll \omega_c^2$, the parallel heating dominates the perpendicular heating by a factor of $\sim \omega_c^2/\omega^2$. The anisotropic heating should cause enhanced axial diffusion in agreement with observations. In the present experiments, the cavity mode number is rather small (~ 65), and so we expect small changes in frequency to change the ratio of E to $E_{\perp}$ (and thus the diffusion rate). This effect has also been observed. The most convincing experiment, however, is the one in which a 10 MHz electric field was applied perpendicular or parallel to $\vec{B}$ in a mirror system with a plasma produced by ECRH at 3250 MHz. The plasma was expelled with $\vec{E}$ parallel to $\vec{B}$ and unaffected with $\vec{E}$ perpendicular to $\vec{B}$.

The theoretical model can be generalized to arbitrary by defining a heating anisotropy,

$$A = \frac{E_{\parallel}^2}{E_{\perp}^2} \frac{d\bar{W}_{\perp}}{dt} / \frac{d\bar{W}_{\parallel}}{dt} = \frac{(\omega^2 + \omega_c^2 + v^2)(\omega^2 + v^2)}{[(\omega^2 - \omega_c^2 - v^2)^2 + 4\omega^2 v^2]}, \quad (5)$$

such that $A = 1$ represents isotropic heating. Equation (5) is plotted in Fig. 36 as a function of ω/ω_c for $\nu = 0$ and for $\nu = 0.1$. For $\omega \sim \omega_c$ the heating is highly anisotropic with most of the energy in the perpendicular component as expected for resonance heating. For $\omega > \omega_c$, the anisotropy is always greater than one, and approaches unity at large ω . For $\omega < \omega_c/\sqrt{3}$, the anisotropy is less than one, enhancing axial diffusion. For ω only slightly below the cyclotron frequency ($\omega_c/\sqrt{3} < \omega < \omega_c$), we expect heating to occur without enhanced diffusion.

These results are changed if the rf electric field is strongly polarized, and we must also allow for the possibility of different effective collision frequencies in the perpendicular and parallel directions. The expulsion of the plasma in the Oak Ridge Experiments probably results from the strong off-resonance absorption expected and observed for relativistic plasmas. In the present experiment, the plasma is probably easily expelled because of the small mirror ratio (1.02 or 1.19), and because the resonance heating occurs in short pulses after which the large anisotropy can disappear. For relativistic plasmas, the hot electron cyclotron frequency should be used in Fig. 36 so that particles of different energy are heating differently. In a non-uniform magnetic field, such as a mirror, an exact calculation of the heating anisotropy would require an average along the particle trajectories, as indicated in equation (3).

The lower off-resonance power required to expel the plasma in the Oak Ridge experiment can be estimated by assuming a multi-mode cavity with total absorption of both the resonance power (P_R) and off-resonance power (P_{OR}). The plasma is expelled if the midplane anisotropy satisfies

$$\frac{W_{\perp}(0)}{W_{\parallel}(0)} < \frac{R+1}{R-1},$$

where R is the mirror ratio. Neglecting scattering or other processes that cause exchange of perpendicular and parallel energy,

FIGURE 36

the respective energies are determined by the ratio

$$\frac{W_{\perp}(0)}{W_{\parallel}(0)} = \frac{P_R + P_{OR}(1)}{P_{OR}(H)} = \frac{P_R + A P_{OR}/(A+1)}{P_{OR}/(A+1)}$$

where $P_{OR}(1) + P_{OR}(H) = P_{OR}$. Combining these equations with $A = P_{OR}(1)/P_{OR}(H)$ from equation (5), gives

$$P_{OR} = \frac{(R-1)(A+1)}{(R+1)-A(R-1)} P_R \quad (6)$$

For the Oak Ridge case, $A \ll 1$, and the required power is simply then

$$P_{OR} = \frac{(R-1)(A+1)}{(R+1)-A(R-1)} P_R \approx \frac{R-1}{R+1} P_R \quad (7)$$

In the Oak Ridge experiments, $R \approx 2$, and so we expect the required off-resonance power to be about 1/3 of the resonance power in agreement with observations.

The off-resonance power required to expel the plasma in the present experiment can be estimated in a slightly different way:

$$\frac{d\bar{W}_{\parallel}}{dt} = \frac{W_{\parallel}(0)}{t} = \frac{W_{\perp}(0)(R-1)}{t} = \frac{e^2 E_{\parallel}^2}{m\omega^2},$$

where t is the time between resonance pulses (2.8 msec), and E^2 is given by

$$E_{\parallel}^2 = \frac{P_{OR} Q_0}{3\epsilon_0 \omega V}.$$

Solving for P_{OR} gives

$$P_{OR} = \frac{3\epsilon_0 m\omega^3 V W_{\perp}(0)(R-1)}{e^2 \omega t Q_0}$$

Taking $Q_0 = 600$, $R = 1.02$ and $W(0) = 10$ eV, and assuming that the collision frequency measured for upper off-resonance heating holds also for the lower off-resonance case, the calculated power is 2.8 watts, in reasonable agreement with experimentally observed values (Fig. 25).

It may not be necessary to add enough parallel energy to all the particles to put them into the loss cone in order to expel the plasma. A slight axial diffusion, as indicated for example in Fig. 30 could cause flute instabilities resulting in enhanced radial loss as observed experimentally. Furthermore, in a mirror field, there would be a resonance in the heating rate when the parallel rf electric field frequency is equal to the bounce frequency, and the coupling would then become stronger than indicated by equation (4). This effect is probably not very important in the Oak Ridge experiment because the coupling is already very strong, or in the present experiment because the bounce frequency is too small with the relatively cold plasmas produced.

IV. BREMSSTRAHLUNG CALCULATIONS (THEORY)

D. G. S. Greene

The following highlights from the relativistic quantum mechanical derivation of the Heitler²⁰ differential cross section for bremsstrahlung will indicate the techniques of improving the approximations and also indicate the derivation of the percent polarization calculation.

A. The transition probability per unit time is:

$$w = \frac{1}{h} |K_{FO}|^2 \rho_F \quad (1)$$

$$\text{where: } K_{FO} = I \left(\frac{V_{FI} H_{IO}}{E_0 - E_I} + \frac{H_{FII} V_{II0}}{E_0 - E_{II}} \right) \quad (2)$$

K_{FO} is the matrix element describing the transition from initial state O, to final state F. It consists of two interactions, V and H. V is the interaction of the electron and the atomic field; H is the interaction of the electron and the radiation field which produces the photon K. I is the intermediate state when V follows H. A photon is present in state I. II is the intermediate state when V precedes H. There is no photon in state II. The summation is over all possible intermediate states of both positive and negative energies and over all electron spin directions,

E_0 = total energy of the incident electron (zero subscripts refer to the incident electron and no subscript refers to the scattered electron),

ρ_F is the density of final states,

$$\rho_F dE = \frac{pE d\Omega dE K^2 d\Omega_K dK}{(hc)^3 (hc)^3} \quad (3)$$

p = electron momentum times c (the speed of light).

The cross section is:

$$ds = w L^3 / v_o, \quad (4)$$

where: L^3 is the volume occupied by the colliding particles. This is the

volume over which the wave functions are normalized. L^3 is set equal to 1. The calculation of V_{ij} and H_{ij} is the heart of quantum electrodynamics. Without the details of the derivation:

$$H_{ij} = \text{coef. } (u_i^* \alpha u_j) \quad (5)$$

$$V_{ij} = \text{coef. } (u_i^* u_j) / |\vec{p}_j - \vec{p}_i|^2 \quad (6)$$

where: u_k is the wave function of the k^{th} state, $\vec{p}_k$ is the momentum (times c) of the electron in the k^{th} state (Here $\vec{p}_j - \vec{p}_i = \vec{q}$ is the momentum transferred to the nucleus), and α is the component of $\vec{\alpha}$ in the direction $\vec{q}$, which is the direction of the polarization of K . $\vec{\alpha}$ is a 4×4 matrix which is related to the Pauli matrix. It appears in the relativistic wave equation:

$$H\psi = (\vec{\alpha} \cdot (\vec{p} - e\vec{A}) + \beta U_0 + e\phi)\psi. \quad (7)$$

The summation to yield $|K_{PO}|^2$ follows the techniques leading to the Klein-Nishina formula. In particular, terms such as $(\vec{\alpha} \cdot \vec{p}_K)(\alpha)$ appear which yield $\vec{p}_K \cdot \vec{q}$ during the summation procedure. These dot products are summed over the two final polarization states to yield the Heitler differential cross-section for bremsstrahlung.

B. Approximations in this derivation and the corrections are as follows:

u_k are taken as plane waves (the Born approximation).

This is a good approximation at high energies where:

$$i \gg \xi \text{ and } i \gg \xi_0,$$

$$\text{where: } \xi_i = \frac{ze^2}{h\nu_i}. \quad (8)$$

V is the first order term of an expansion in powers of ξ_i . A difficult way to correct for this is to calculate the higher order terms in the expansion. Consider two possible intermediate states and calculate terms like HHV, HVH, VHV, etc. This is a monumental task of many weeks. Furthermore, there is no guarantee that additional corrections such as HHVV, HVVV, etc., because of the multiplicity of such terms, are not equally important. An easier way to correct is to compare to the calculations of Sommerfeld at non-relativistic

energies (Sommerfeld uses the exact non-relativistic wave functions). A comparison shows that the angular dependence remains the same, but the total intensity has to be multiplied by the Elwert factor:

$$f(\xi, \xi_0) = \frac{\xi}{\xi_0} \frac{(1 - \exp(-\xi_0))}{(1 - \exp(-\xi))} \quad (9)$$

This correction is labeled "fix" on figures 37 and 38. $f(\xi, \xi_0)$ is always greater than 1 and increases toward lower incident electron energies and toward higher photon energies. At the characteristic temperatures considered, the correction due to $f(\xi, \xi_0)$ at the electron energies which produce the most photon intensity is as much as 10% (see figure 37). Considering the total photon intensity for all electron energies, the correction is about 6% (see figure 38). However the absolute value of photon intensity is not easily determined, whereas the measured quantity, the slope of the log of photon intensity versus photon energy, is not greatly affected by $f(\xi, \xi_0)$. The slope of the corrected curves are slightly less by about 0.8% (see figure 38).

A second approximation is that the nucleus has infinite mass. This correction was discussed previously.²¹

A third approximation is that the nucleus is a point charge scattering center. Screening corrections involve a recalculation of V_{1j} given in equation (6). In general

$$V_{1j} = \int \psi_1^* V \psi_j d\tau \quad (10)$$

For equation (6) V was taken as the coulomb potential for a point charge:

$$V = Z e^2 / r_{12}. \quad (11)$$

A screened potential which considers the atomic electrons has the form:

$$V = \frac{Z e^2}{r_{12}} \exp(-r_{12}/a), \quad (12)$$

where for helium: $Z = 2$

$$a = .30 a_0 = .30 \pi^2 / m c^2.$$

Figure 37 (a)

Photon Intensity versus Electron Energy
for various photon energies

$$\text{Distribution } X(3.1) = \text{EXP}(-E(3.1 \cos^2 A + 31.0 \sin^2 A))$$

Figure 37 (b)

Figure 38 (a)

Normalized X-Ray Spectra

for Various Electron Energy Distributions

Perpendicular Spectra

18

Electron Energy Distributions:

$$12 \sim \exp [-X(1.55)]$$

$$1 \sim \exp [-X(3.1)]$$

$$32 \sim \exp [-X(15.5)]$$

$$10 \sim \exp [-X(21.7)]$$

$$18 \sim \exp [-X(31.0)] = I$$

$$X(kT_H) = E_0 / (kT_H \cos^2 A + 31.0 \sin^2 A)$$

Figure 38 (b)

Parallel Spectra

Log (Intensity from Distribution / Intensity from Isotropic Dist.)

Photon Energy (keV)

The V_{1j} resulting from this screened potential differs from equation (6) in that the denominator q^2 is replaced by $q^2 + \alpha$, where α is $(\hbar/a)^2$. $a = 0.3 a_0 = 3 \times 10^{-11}$ meters.

The equation for the x-ray intensity containing the screening correction has been integrated over the angles of the scattered electron (θ and ϕ) and programmed for numerical integration over the incident electron angles (polar angle, A , and azimuthal angle, B) and energy (E_0) to yield the x-ray spectrum.

Bremsstrahlung from electron-electron encounters (where the electrons are bound to the atom), and the nonzero size of the nucleus may be considered as corrections to the screening correction. These corrections would require a more accurate expression for the atomic wave functions, which in turn would modify V , leading to additional smaller correction terms in the denominator of equation (6).

C. To calculate the polarization, consider the terms containing $\vec{p} \cdot \vec{e}$ which were described following equation (7). $\vec{e}$ is first taken in the $\vec{z}$ direction to yield $dI(P_z)$, then in the $\vec{\theta}$ direction to yield $dI(P_\theta)$.

Percent polarization is defined as:

$$P(K) = \frac{\int dI(P_z) - \int dI(P_\theta)}{\int dI} \quad (13)$$

where the integrations are over $d\theta$, $d\phi$, dA , dB , and dE_0 . $P(K)$ is calculated for the perpendicular X-rays. The parallel X-rays have no plane polarization (consider the cylindrical symmetry). Figure 39 shows some graphs of $P(K)$.

If the integrations of equation (13) are not carried out over E_0 , the result is $P(E_0, K)$ which is shown in figure 40. $P(E_0, K)$ is not observable, but serves to show that polarization due to low energy electrons is opposite to the polarization due to high energy electrons. However, the polarization due to the entire distribution of electron energies is determined by the lower energy electrons for the "low" energy distributions studied. This accounts for the higher characteristic temperature electron energy distributions producing less polarization than the lower temperature distributions in figure 5. The dashed asymptotes on figure 40 are at electron energies equal to the electron rest energy plus the photon energy. No real photons may be produced below this energy. The asymptotic values of $P(E_0, K) = 1$, the classical value for

Polarization vs. Photon Energy

Figure 39(a)

Electron Energy
Anisotropy Increasing
↑

$$X(KT_H) = -E_0 / (KT_H \cos^2 \lambda + 31.0 \sin^2 \lambda)$$

$$\text{Distribution Function} = \exp(-X(KT_H))$$

Polarization vs. Photon Energy

Figure 39(b)(continued)

Figure 39 (c)(continued)

Figure 40 (a)

Polarization versus E_0
for various photon energies

$$\text{distribution} = \exp(-E_0 / (3.1 \cos^2 A + 31.0 \sin^2 A))$$

Figure 40 (b)

Polarization versus E_0
for various photon energies

Beam Distribution

Figure 40 (C)

the polarization of the perpendicular X-rays from non-relativistic electrons.

Note that polarization measurements will help in distinguishing between the distributions $X(kT)$ which gave nearly identical X-ray intensity anisotropies in figure 38(b). Other coincidences in X-ray intensity anisotropies may be resolved by polarization measurements.

D. Additional Theoretical Considerations

Two problems related to free-free bremsstrahlung from a plasma containing relativistic electrons have been considered: 1) polarization of the bremsstrahlung, and 2) corrections to the bremsstrahlung spectra due to the finite mass of the nucleus. Both problems were programmed for simultaneous computer calculation.

To express the polarization, the cross-section for bremsstrahlung is derived following the technique of Heitler without averaging over the final states of polarization. These two final states of polarization yield two cross-sections: σ_1 and σ_2 . Bremsstrahlung emission perpendicular to the magnetic field results from $\sigma_1 = \sigma_z$ and $\sigma_2 = \sigma_\theta$ and has the intensities:

$$I_z = \int I_0 dN V_0 d\sigma_z \quad \text{and} \quad I_\theta = \int I_0 dN V_0 d\sigma_\theta \quad (1)$$

where: I_z = intensity of radiation polarized in the z direction,
 I_θ = intensity of radiation polarized in the θ direction,
 I_0 = constant of proportionality, and
 dN = density of incident electrons having velocity V_0 .

The polarization is:

$$P(h\nu) = (I_z - I_\theta) / (I_z + I_\theta) \quad (2)$$

Preliminary approximate calculations indicate that P may be a measurable fraction.

The finite mass correction results from four considerations: 1) the density of final energy states for photon and electron is altered by the final energy states for the nucleus; 2) the probability of populating these final energy states of the nucleus is a product of the nuclear wave functions; 3) on deriving the Heitler cross-section the incident electron energy no longer equals the recoil electron energy plus the photon energy, so certain cancellations are no longer possible, resulting in additional terms; and 4) the energy of the recoil is less by the amount of energy taken by the recoil nucleus. For the range of energies examined (electrons up to 1 MeV and photons up to 80 Kev) these effects for a helium nucleus produce corrections to the cross-section of the order of .01%. Moreover, the corrections are largest where the cross section is smallest.

E. BREMSSTRAHLUNG MEASUREMENTS AND COMPARISON TO CALCULATIONS

D.G.S. Greene

Figure 41 shows a top view of the experimental apparatus. The plasma at H is confined in the long evacuated cylinder I by the mirror magnetic field from coils J. The mirror ratio is 1 to 1.19 which corresponds to a loss cone angle of 66.5° . Axial X-rays are measured by the counter F through port E. Radial X-rays are measured through port B. The counters are beryllium faced NaI scintillator crystals backed by a 6810A photomultiplier tube. The pulses from the phototube are shaped and then counted by a pulse height analyzer to yield an intensity versus energy spectrum for the X-rays.

A computer program was used to calculate the magnetic field due to the field coils J for a current of 400 amperes, as reported in the 1968 annual progress report.²² Since the magnetic field is proportional to the current in the field coils, a contour of constant magnetic field could be relabeled with the value of the magnet current which would produce resonance heating on that contour. This has been done for a resonant field of 3050 gauss in Fig. 42. The magnet current was measured by the voltage drop across a series resistance. The voltage in millivolts multiplied by $20/3$ gives the magnet current in amperes. It was found that on increasing the magnetic field the plasma would go out at 56.30 mv; on decreasing the field the plasma would ignite at 55.20 mv and go out at 46.00 mv or sometimes as high as 47.00 mv.

1. The scintillator-photomultiplier tube-pulse height analyzer combination was calibrated using various radioactive sources. At different analyzed gain settings, the graphs of the X-ray energy versus analyzer channel number were found to be curves. On log-log paper these graphs were straight lines. These straight lines were interrelated in that the gain versus the inverse slope of the log-log graph was a straight line. See Fig. 43.

When the X-ray source was too intense, pulse pile up resulted. Pile up means that two or more smaller pulses arrive simultaneously

Bremsstrahlung Experimental Apparatus

(Top View Cut-Away)

- H plasma
- I confining evacuated stainless steel cylinder
- M end plates of confining cylinder
- G tungsten limiter
- J magnetic field coils
- K movable copper end plug
- A movable brass plug
- B,L,E,N,D ports
- C,F photomultiplier tubes; shaded parts are NaI scintillators

Figure 41

Typical Calibration Curves for X-ray Spectrum Measurements

Fig 43

and are recorded as a larger pulse. This tends to increase the channel numbers of the X-rays and so move the characteristic peaks from the calibrating radioactive sources to larger channel numbers. The percent of dead time for the analyzer is a measure of the frequency of arrival of X-rays. For dead times less than 10% no displacements of calibration peaks nor changes of slope were observed. For later measurements dead times were kept to less than 2% and usually much less by limiting X-ray intensities with pinholes.

The phototubes were always shielded with both lead foil and netic, conetic, and soft iron shields so that calibration curves in the axial and radial positions were indistinguishable, even with the magnetic field on.

2. Tests to determine whether the bremsstrahlung occurs from wall encounters or free-free interplasma encounters were conducted. Referring to Fig. 41 counter C was aligned with port B and the diametrically opposite port D using a laser beam. Pinholes limited the field of observation to the brass plug A. As A was inserted into the cylinder the X-ray intensity sharply increased. See Fig. 44. The slope of the X-ray spectra stays nearly constant as the plug is inserted, perhaps becoming only slightly flatter at the discontinuity of Fig. 44. With the brass plug removed and as counter C was moved in the z direction or in the up and down direction, the X-ray intensity again sharply increased as the field of observation began to include the confining cylinder wall at the edge of port D. See Fig. 45. The X-ray intensity dropped to background intensity at sufficiently large displacements such that the phototube could no longer see through the pinhole at port B into the plasma. The asymmetry appeared because the pinhole at B was not carefully set perpendicular to the confining cylinder. The asymmetry was later shown to be reversible by purposefully misaligning the pinhole.

Similar tests were made on the axial X-rays. Up and down displacement and radial displacement of the counter F produced the same characteristic graphs as in Fig. 45. In this case the X-ray

Insertion of Brass Plug

Fig 44

Variation in Position of Photomultiplier Tube for Viewing Plasma

Fig 45

intensity sharply increased as the limiter G entered the field of observation. Corresponding to the radial insertion of the brass plug A, a perforated copper end plug K was inserted. However, as the plug entered the mirror throat, the plasma was extinguished. On withdrawing the plug some distance the plasma reignited. Fig. 46 is a graph of this effect as a function of plasma resonance heating location. Axial X-rays increased sharply on insertion of a target into port L. Large deflecting magnets placed outside the confining magnetic fields were meant to deflect the electrons escaping axially from striking either of the end plates M. No changes in X-ray intensity or slope of spectra were observed on employing these magnets.

In conclusion it is assumed that only free-free bremsstrahlung will be observed when the fields for counters C and F are blank spaces and not metallic targets.

3. Radial spectra through both ports B and N and axial spectra were observed for various conditions of the plasma. A coincidence gating circuit allowed X-ray counts only during the power pulse and for about 20 microseconds thereafter. Background radiation during that time was negligible. Photographs of the pulse height analyzer display were taken. See Fig. 47. The photographs were measured with a movable stage microscope. This measured data was supplied to a computer programmed to make calculations and corrections to yield final data for a graph of X-ray intensity (from the plasma source) versus X-ray energy. The following corrections were made:

1. The decade markings on the photograph did not form an exactly rectangular grid and so corrections for distortion were employed such that the measured positions of the decade markings would be adjusted to lie on evenly spaced horizontal lines. This also would correct for any tilt of the photograph on the microscope stage.
2. Channel number was translated into X-ray energy by considering the calibration curves referred to in part A.

Condition of Plasma as a Function of End Plug Position for Various Resonant Heating Locations

Fig 46

Drawing of Photograph of Pulse Height Analyzer Display

Fig 47

3. Absorption corrections due to the mylar window covering the port, the air path, and the beryllium window of the NaI crystal were made. The correction due to the air path was the largest of all the corrections considered in either the theoretical calculations or the experimental measurements.
4. The efficiency of the NaI crystal was calculated.
5. The pinhole geometry and volume of plasma observed were considered.

Graphs of the final data for the resonance heating zone located at the center of the plasma and the lowest pressure for a sustained plasma are given in Fig. 48 for axial X-rays and radial X-rays viewed at the plasma's center. Figure 49 shows a graph of the central radial X-ray spectra for various locations of the resonance zone. The axial X-rays remain essentially unchanged for these different locations. The off-center radial X-ray spectra viewed from port N (which is located 15.2 cm from the center of the plasma and 7.6 cm from the mirror throat) have not yet been analyzed.

4. Calculations of X-ray spectra have been reported in previous progress reports.¹⁰ The experimental axial and radial spectra of Fig. 48 have a greater vertical separation and also diverge much more rapidly than any of the calculated spectra reported so far. Electron energy distributions with characteristic temperatures of above 160 keV are required to produce theoretical spectra with either sufficient vertical separation or sufficient divergence to match the experimental results, but at such high electron temperatures the X-ray spectra are much too flat.

Although the perpendicular experimental spectrum is matched quite well by the perpendicular theoretical spectrum for a loss cone electron energy distribution with characteristic temperature of 31 keV and loss cone angle of 67.5° , the parallel spectrum is not at all matched. See Fig. 49. Thus with the given theoretical calculations, the experimental results cannot be matched by a single electron temperature for the extended plasma volume.

X-ray Spectra

Fig 48

Parallel X-ray Spectra for Various Magnet Currents

Fig 49

However a variety of models with regions of different electron temperatures can be used to match the experimental results of Fig. 48. For example: a two component model consisting of 31 keV plasma in the heating zone and 15.5 keV plasma filling the remaining volume between the magnetic mirrors produces the spectra marked by (x) on Fig. 48.

E. Proposed research in an effort to find a unique model of the plasma to match Fig. 48 will include:

1. Analysis of the off-center spectra taken through port N.
2. Analysis of the central radial spectra for heating not in the center of the confining cylinder.
3. Axial insertion of a scintillator probe to determine electron energies as a function of position.

F. THEORY OF POLARIZATION MEASUREMENT

P.A. Raimbault

The most convenient method with which to study polarization of X-rays is scattering. It is easy to explain what is happening in scattering by using classical electromagnetic theory (Thomson Scattering).

The X-rays are considered as electromagnetic waves. They act on the electrons basically through their electric field vectors. As a result the electrons become oscillators which emit X-rays themselves.

Consider the diagram shown in Figure 50. The following equations are those of the electric field and dipole intensities of the emitted (scattered) radiation of the electron.

$$E_{\theta} = \frac{E e^2 \sin^2 \theta}{r m c^2}$$

r is the classical electron radius. The dipole intensity is:

$$I_{\theta} = \frac{I e^4 \sin^2 \theta}{r^2 m^2 c^4}$$

with components

$$I_{\theta y} = I_y \frac{e^4 \cos^2 \phi}{r^2 m^2 c^4}$$

$$I_{\theta z} = I_z \frac{e^4}{r^2 m^2 c^4}$$

Note that $I_{\theta z}$ is constant and that $I_{\theta y}$ depends on ϕ . If we look only in the y direction then we shall see only one component of the field.

If we want to study the polarization with a non-polarized source we must use two scatterers. See figure 51. In position (1) we see a maximum of intensity; in position (2) a minimum which ought to be zero. In fact, however, this theory does not apply when the energy of X-rays is comparable to the rest energy of the electrons (0.51 MeV). We must then use the theory developed by Heitler derived from the Dirac relativistic quantum mechanics.

Figure 50

Figure 51

Figure 52

Preliminary Experiments.

A study of the scattering of X-rays by a block of graphite was undertaken. Figure 52 shows a diagram of the apparatus. This preliminary experiment permitted us to determine if scattering with graphite was appropriate. The detection circuitry was such that a range of energies could be observed, because it is expected that the polarization can be a function of energy.

The range of energy of X-rays we can observe will be determined by using two single channel counters.

Example	1st count	> 100 KeV
	2nd count	> 200 KeV

By subtraction we can obtain the number of counts for energies between $100 \text{ KeV} < E < 200 \text{ KeV}$. However, it was noted that the background level was comparable to or greater than the scattering intensity from the source. The conclusion is that in spite of the shielding, the background level was very important (2 inches of lead could absorb 99% of the X-rays coming from the source).

A second polarization experiment was tried with the source by using double scattering but the results were still not significant.

V. INSTABILITY STUDIES

S.J. Gitomer*

An instability which has previously been discussed in these progress reports has been studied further. The instability is accompanied by strong microwave emission, axial particle loss, fluctuations in diamagnetism and intense x-ray bursts.

In order to enable the unambiguous identification of this instability, a number of questions need to be answered: (1) How does the heating zone location effect its existence? (2) How does the emitted rf frequency scale with the applied heating frequency? (3) What are the plasma parameters at the time the instability occurs? After it occurs? (4) What is the plasma spatial density distribution? The measurements reported here attempt to answer these questions and will enable definitive comparison to be made with relevant instability theories.

In general features, the strong microwave emission occurs in bursts (0.1-1.0μsec long) within 5μsec after the applied heating ends. The frequency of the radiation falls in a band below the microwave heating frequency. The instability is found to occur for only midplane and off axis heating. The dependence on background pressure appears to favor the lower pressures ($6-10 \times 10^{-4}$ Torr) for which discharge is possible.

The instability was observed to occur for both 1.02 and 1.19 mirror ratios. The heating zone for instability appeared to be near the midplane for the 1.02 case; however, a $\pm 1\%$ magnet current ripple made this conclusion uncertain. At $R = 1.19$, however, the $\pm 1\%$ ripple doesn't affect the heating zone location appreciably. Figure 53 shows the range of heating zones over which instability was produced for a heating frequency of about 8.525GHz. (Mod $\vec{B}$ contours of about 0.962-0.998)

The microwave source was mechanically tuned through its range (8.5-9.2GHz) and the bands of frequencies associated with the instability were measured using a microwave spectrum analyzer. A typical such spectrum is shown in figure 54. The applied heating frequency is clearly indicated. The plot in figure 55 shows the variation of instability frequency divided by applied heating frequency as a function of applied heating frequency. For both mirror ratios, the frequency associated with the instability scales roughly linearly with the applied heating frequency. The possibility of the excitation of a particular cavity mode (as noted

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FIGURE 53. CONTOURS OF CONSTANT MAGNETIC FIELD AMPLITUDE ($R = 1.190$)
SHOWING REGION OF INSTABILITY PRODUCTION (CROSS-HATCHED)

FIGURE 54. TYPICAL SPECTRUM SHOWING PLASMA MICROWAVE EMISSION FOR A BAND OF FREQUENCIES BELOW THE HEATING FREQUENCY. (HEATING PULSE FREQUENCY = 8.52GHz IS ALSO SHOWN)

FIGURE 55. NORMALIZED INSTABILITY FREQUENCY VERSUS APPLIED HEATING FREQUENCY (R = 1.02 and 1.19)

in reference 23 is ruled out by this behavior.

A number of diagnostic measurements were made to characterize the plasma for parameters yielding instability. Figure 56 shows electron density variation with time as determined using probe and microwave methods. The microwave method yields an average of the total (cold and hot) electron density while the Langmuir probe method yields a local value of the cold electron density. We will make the reasonable assumptions²⁴ that the microwave power pulse is of too short a duration to appreciably change the electron density and that the electron density is only hot at the end of the heating pulse. This implies a ratio $n_C/n_H < 1$. At the end of the heating pulse, n_C/n_H rises rapidly to a maximum value of ~ 10 at $t \approx 50$ microseconds, and then falls slowly to a final value ~ 0.3 just before the next heating pulse.

The radial and axial dependence of the plasma cold electron density (determined using Langmuir probes) is shown in figures 57 and 58 for two times in the afterglow. Since the cold electron component is produced by ionization of neutral helium atoms by the hot electrons, these plots display the hot electron spatial distribution also.

Previously²⁴ temperature and anisotropy measurements were made using plasma diamagnetism and $\vec{E} \times \vec{B}$ energy analyzer information. It was found, however that for the plasma studied here (1) the diamagnetism could not always be interpreted as stored energy in the plasma due to the complication of plasma motion and (2) no appreciable low energy electron signals were detectable using the $\vec{E} \times \vec{B}$ analyzer. Temperature measurements were made instead using Langmuir probes. Figure 59 shows the decay of cold electron temperature with time. This decay is not inconsistent with those shown in figure 56 for the total and cold electron densities. An estimate of hot electron temperature of 10 keV is made based on previous x-ray measurements. X-ray measurements of the hot electron energy distribution were attempted with ambiguous results. These measurements are being pursued further.

Estimates of anisotropy may be made from (1) the axial probe density data and (2) the formula $\langle W_H \rangle / \langle W_L \rangle = (R-1)/(R+1)$ where R = axial mirror ratio. These yield anisotropy values of the order of 100 ($R = 1.02$) and 10 ($R = 1.19$). These values probably represent lower limits on the actual anisotropy.

The only remaining piece of information needed is instability wave number. This will be determined using a fixed loop probe and a movable axial loop probe

FIGURE 56. ELECTRON DENSITY VERSUS TIME - USING FUNDAMENTAL MODE SHIFT AND LANGMUIR PROBE METHODS.

FIGURE 57. ELECTRON DENSITY VERSUS
RADIUS ($z = 15.0 \text{ cm}$)
FOR TWO TIMES IN THE AFTERGLOW.
($R = 1.19$, $f = 8.52 \text{ GHz}$)

FIGURE 58. ELECTRON DENSITY VERSUS AXIAL COORDINATE
 ($\tau = 0.0$ cm) FOR TWO TIMES IN THE AFTERGLOW.
 ($R = 1.19$, $f = 8.52\text{GHz}$)

FIGURE 59. ELECTRON TEMPERATURE
VERSUS TIME LANGMUIR PROBE DATA
($r = 4.0\text{cm}$, $z = 15.0\text{cm}$)

following the method used by Ikegami, et al.²⁵

Our previous theoretical identification of the instability (an electromagnetic mode) failed to consider the spatial variation of plasma density and the presence of a background of cold plasma. Following the work of Ikegami, et al.,²⁵ Jacquinet, et al.,²⁶ Lee and Crawford,²⁷ and others, an assessment of this instability will be made using this new information to obtain a more clearcut identification.

VI. COMPUTER SIMULATION

A. MULTIPLE PARTICLE SIMULATION

J. L. Shohet

Experimental research has shown that effective heating of a plasma is possible by means of electron cyclotron resonance.²⁸ When the r.f. heating field is oriented perpendicularly to a uniform d.c. magnetic field, highly anisotropic plasmas are produced. Theory to explain the heating mechanism is largely based on three assumptions. They are: (1) At any given instant of time, it is equally likely for a particle to be decelerated as it is to be accelerated. (2) Since a net heating occurs; some sort of collisional process must "throw" the electrons into the proper phase to produce a net heating effect. (3) After heating, a non-Maxwellian distribution is often the result, usually having a collection of "hot" and "cold" particles.

It is the purpose of this work to show that a computer simulation of a plasma in the resonance (heating) plane will satisfy the above three assumptions. The basic method of solution follows the individual motions of a collection of "super-particles" in a two-dimensional region. Super-particles are essentially clouds or collections of real particles, and have the same charge-to-mass ratio as real particles. Both super-ions and super-electrons will be used.

The interaction region is immersed in a d.c. magnetic field. The external a.c. electric field may be made either uniform or into a plane section of the TE₁₁₁ mode of a microwave cavity. Initially, all particles are placed where desired in the interaction plane. From their initial positions, we may obtain the self-consistent electric field.²⁹ The initial velocities of the super-particles are given a Maxwellian distribution.

Each super-particle is assumed to obey the following finite difference equation, which includes a relativistic mass correction:

$$v_{j+1,x} = \frac{\beta\delta + \gamma}{1 + \beta^2} \quad \text{and} \quad v_{j+1,y} = -\frac{\beta\gamma + \delta}{1 + \beta^2}$$

where

$$\beta = \frac{q}{m_0} \frac{\Delta t B_0}{2} \quad \alpha = \left(1 - \frac{v_{j,x}^2 + v_{j,y}^2}{c^2} \right)^{1/2}$$

$$\gamma = v_{j,x} + \frac{q}{m_0} \Delta t \propto E_{j,x} + \beta v_{j,y}$$

$$\delta = v_{j,y} + \frac{q}{m_0} \Delta t \propto E_{j,y} - \beta v_{j,x}$$

m_0 is the rest mass; B_0 is the d.c. magnetic field, Δt is the time step and q is the charge. $v_{j,x}$ and $v_{j,y}$ are the known x and y components of velocities. The subscript $j+1$ refers to the values of quantities Δt units of time later. The electric field in the above expressions is the sum of the self-consistent and the external r.f. electric fields. Once the new velocities are determined, the changes in position may be obtained to produce the motion. After each calculation, the new averages of particle kinetic and potential energy are computed, and the velocity distribution is examined to observe its changes.

The specific problem consisted of a collection of 900 each, super-ions and super-electrons. The d.c. magnetic field was set to 3000 Gauss and the r.f. field amplitude was set to 100,000 volts/meter. By varying the number of real particles per super-particle; the plasma frequency could be changed. Results for the case when $\omega_{pe}/\omega_{ce} \approx .01$ are shown in Figure 60.

The total kinetic energy of the electrons increases with time. The ions do not increase in energy, and, when the heating is removed, the electron kinetic energy remains constant. Figure 61 shows the electron velocity distribution at 3 different times. The solid lines are Maxwellians, whose temperatures are fixed by the relation: $1/2 = \langle v^2 \rangle$, where $\langle v^2 \rangle$ is the average of v^2 . Close fits to the Maxwellians are seen. The selfconsistent field (collisions) is necessary for this result; without it, a drift, but no heating, was observed. When the plasma frequency was increased, the velocity distribution rapidly becomes non-Maxwellian, and the ions are accelerated as well. Under these conditions, collisional effects have begun to dominate³⁰ and are over-emphasized from reality by about the ratio of the number of real particles per super-particle. In order to observe heating at higher plasma frequencies, it is necessary to increase the number of super-particles, thereby lowering the number of real particles per super-particle for a given value of plasma frequency.

The program may be easily modified to obtain results based on a theoretical model for r.f. confinement. This is done by removing the d.c. magnetic field and using the external r.f. field as one plane in a TM_{011} rectangular cavity.

TOTAL KINETIC
ENERGY - JOULES

R.F. CYCLOTRON
HEATING
REMOVED
(CASE A)

R.F. CYCLOTRON
HEATING
REMOVED
(CASE B)

Figure 60

A dramatic confinement is observed, closely following the theoretical shape of the confinement region predicted by the theory. The effects of coulomb collisions do not appear to destroy the confinement.

Currently a scheme to increase the number of super-particles is being developed. This will result in a lowering of the ratio of the number of real particles per simulation particle. The increase will be done by batch processing groups of simulation particles. That is groups of say, 1000 simulation particles will be stored on a random access file. Each group may be taken by the main program and its charge divided up among the appropriate mesh points. After each group has been so processed, the Poisson algorithm will be used to obtain the electric field. Then a similar sequential operation will be used to find the new positions and velocities of the simulation particles.

In addition a 2 1/2 dimensional code¹ will be used to study cyclotron heating in a variable magnetic field. The simulation particles will be assumed to be in the x-z plane. The d.c. magnetic field will vary and point in the z-direction, thus producing a resonance region which is a curve in x-z space. The y direction will be considered by assuming no variation with y, thus making the canonical momentum in the y direction a constant.

This scheme has the advantage of permitting the use of the 2-dimensional Poisson code, but allows consideration of 3-dimensional (really 2 1/2 dimensional problems). It is anticipated that both off-resonant and resonant heating will be studied with this method.

B. SINGLE PARTICLE ORBITS

S.K. Nijhawan

The University of Wisconsin has a hybrid computer which is a combination of AD 256 analog computer and SDS 930 digital computer. An attempt is being made to simulate the motion of a single charged particle in externally applied electric and magnetic fields. The simulation will be done in a three dimensional plane, i.e., x, y, z plane. The projection of the path of the charged particle will be studied in the xy, xz, and yz planes on a cathode ray oscilloscope and will be plotted with the help of an x-y recorder.

Since we can generate nearly any function of space and time in the hybrid computer, attempts will be made to study very many kinds of electric and magnetic fields.

The equations of motion of the charged particle to be solved are the following.

$$\frac{dV_x}{dt} = \frac{q}{m} [V_y B_z(x,y,z,t) + E_x(x,y,z,t)] \quad (1)$$

$$\frac{dV_y}{dt} = \frac{q}{m} [-E_y B_z(x,y,z,t) + E_y(x,y,z,t)] \quad (2)$$

$$\frac{dV_z}{dt} = \frac{q}{m} [0 + E_z(x,y,z,t)] \quad (3)$$

We shall be taking the z axis as the direction of magnetic lines of force.

Attempts will be made to study the motion of the charged particle in a real magnetic mirror field like the one used in the experiment, but the values of electric and magnetic fields will also be changed as desired to examine the validity of the program and to observe differences in the motion of the particle from one orbit to another.

The hybrid computer has also a simulation language in its digital portion where the differential equations are solved by numerical integration methods and the results are converted into proportional voltages for plotting on an x-y recorder and/or a cathode ray oscilloscope. The digital program has been written

and the digital solution will also be obtained.

It is very natural that the problem must be scaled since it is impossible to simulate the large velocities, high frequency electric fields, and small radii of gyration of the particle orbits exhibit in reality. The approximate values used for scaling would be the following:

$$v_x = 10^8 v_{cx}$$

$$v_y = 10^8 v_{cy}$$

$$v_z = 10^7 v_{cz}$$

$$x = 2 \times 10^{-4} x_c$$

$$y = 2 \times 10^{-4} y_c$$

$$z = z_c$$

$$c \text{ time } \tau = 10^{10} t$$

where the symbol c represents the variable in the computer and v_x, v_y, v_z, x, y, z represent the velocities and positions in x, y, z directions. τ and t are computer and real time, respectively.

The differential equations of the motion of a charged particle after scaling and assuming the value of $\frac{q}{a}$ as 1.75×10^{11} become the following

$$\frac{dx_c}{d\tau} = 50 v_{cx}$$

$$\frac{dy_c}{d\tau} = 50 v_{cy}$$

$$\frac{dz_c}{d\tau} = \frac{1}{1000} v_{cz}$$

$$\frac{dv_{cx}}{d\tau} = 17.5 [v_{cy} B_z(x_c, y_c, z_c, \tau) + \frac{E_x}{10^8} (x_c, y_c, z_c, \tau)]$$

$$\frac{dv_{cy}}{d\tau} = 17.5 [v_{cx} B_z(x_c, y_c, z_c, \tau) + \frac{E_y}{10^8} (x_c, y_c, z_c, \tau)]$$

$$\frac{dv_{cz}}{d\tau} = 17.5 \left[\frac{E_z}{10^8} (x_c, y_c, z_c, \tau) \right]$$

The following various cases of electric and magnetic fields will be examined first.

Case 1

$$B_z = \text{const} = 3 \times 10^{-1} \text{ webers}$$

$$\omega = \frac{q}{m} B_z$$

$$E_x = E_0 \cos \omega t \text{ where } E_0 \sim 10^5 \text{ volts/meter}$$

$$E_y = 0 = E_z.$$

Case 2

$$B_z = B_0 (1 + \alpha |z|)$$

where $\alpha = 2 \frac{(M-1)}{L}$ where M is the mirror ratio and 2L is the total length of the machine

$$E_x = E_0 \cos \omega t \text{ and } \omega = \frac{qB_0}{m}$$

Such a magnetic field cannot exist in reality since it does not satisfy the Maxwell equations. However so long as α is small B_z is a good approximation of the actual field. Subsequently, the other components of B will be added to be consistent with Maxwell's equations.

Case 3

We then shall try to simulate when the field is an actual magnetic mirror field as shown in Fig. 62.

The magnetic field will be generated in the digital computer and the continuous equations will be solved in the analog computer.

Three different kinds of external electric fields will be tried.

i) $E_x = E_0 \cos \omega_c t$ where ω_c is the gyration frequency in the midplane. By changing the value of B_0 the position of heating zone could also be changed in the device.

ii) LORH. We shall utilize two electric fields; one whose frequency is the same as the cyclotron frequency and the other whose frequency is so low that with that frequency no resonance exists.

Figure 62

iii) Upper off-resonance heating: The second electric field would be of 30° high ω_A frequency such that cyclotron resonance occurs nowhere in the mirror.

A scaled hybrid computer diagram is given on the next page.

fig 62. Main diagram.

VISITS OF LABORATORY STAFF, PUBLICATIONS AND CONFERENCE PAPERS PRESENTED DURING THE PERIOD OF THIS REPORT

J. L. Shohet and A. J. Hatch, Eigenvalues of a Microwave Cavity Filled with a Plasma of Variable Radar Density, J. Appl. Phys. 41 2610 (1970).

S. J. Gitomer and J. L. Shohet, Electromagnetic Instability in an Electron Cyclotron Resonance Plasma, Phys. Fluids 13 413 (1970).

J. C. Sprott, K. A. Connor and J. L. Shohet, Off Resonance Heating of Mirror Confined Plasmas. Plasma Physics, submitted for publication.

J. L. Shohet, Numerical Simulation of Electron Cyclotron Heating of a Plasma, Physics Letters 32A, 511 (1970).

D.G.S. Greene, J. C. Sprott and J. L. Shohet, Free-Free Bremsstrahlung for Loss Cone Distributions of Relativistic Electrons, presented at the 12th Annual meeting, Division of Plasma Physics, American Physical Society (1970).

J. C. Sprott, Theory of Off-Resonance Heating, presented at the 12th Annual meeting, Division of Plasma Physics, American Physical Society (1970).

K. A. Connor, J. C. Sprott and J. L. Shohet, Experiments on Off-Resonance Heating, presented at the 12th Annual meeting, Division of Plasma Physics, American Physical Society (1970).

J. L. Shohet, C. Brisset, R. Geller and B. Jacquoit, Desensitivity of a Microwave Cavity to Hot Plasma, presented at the 12th Annual meeting, Division of Plasma Physics, American Physical Society (1970).

J. L. Shohet, Numerical Simulation of Electron Cyclotron Heating of a Plasma, University of Wisconsin PLP Report No. 362.

A. J. Hatch and J. L. Shohet, Computer Simulator of Radiofrequency Confinement of a Plasma, University of Wisconsin, PLP Report No. 365.

J. C. Sprott, Electron Cyclotron Heating of an Anisotropic Plasma, University of Wisconsin PLP Report No. 322.

J. L. Shohet, C. Grisset, R. Geller and B. Jacquoit, The Desensitivity of a Microwave Cavity to Hot Plasma, Physics Letters, submitted for publication.

J. C. Sprott, Theory of Off Resonance Heating, University of Wisconsin, PLP Report No. 373.

J. L. Shohet, Collisionless Damping in a Cold Spatially, Non-Uniform Plasma, University of Wisconsin PLP Report No. 391.

J. L. Shohet, Numerical Simulation of Electron Cyclotron Heating and Confinement of a Plasma, Proc. 4th European Conference on plasma and Controlled Fusion, Rome (1970).

REFERENCES

1. This collector was developed by D.E. Lencioni and others in the plasma physics group at the University of Wisconsin and is reminiscent of the "Plasma Eater" described by Alexeff and Neidigh (I. Alexeff and R. V. Neidigh, Nuclear Fusion 3, 23 (1963)).
2. F.F. Chen in Plasma Diagnostic Techniques, R.H. Huddleston and S.L. Leonard, Eds. (Academic Press Inc., New York, 1965) Chap. 4.
3. Oak Ridge National Laboratory Thermonuclear Division Semiannual Progress Report for the period ending December 31, 1968.
4. R. Bardet and R. Geller, Nuclear Fusion.
5. R. Geller, B. Jacquot, and C. Jacquot, Proc. of the 4th European Conference on Controlled Fusion and Plasma Physics, Rome (1970).
6. A. F. Kuckes, Phys. Fluids.
7. W.P. Allis, S.J. Buchsbaum and A. Bers, Waves in Anisotropic Plasmas, The MIT Press, Cambridge, Mass., 1963, Chapter 7.
8. R.A. Dandl, et al., Thermonuclear Division, Semiannual Progress Report, Oak Ridge National Laboratory, 196 .
9. AEC Report No. COO-1695-26, QPR No. 10.
10. J.C. Sprott, Electrostatic Probe Techniques, PLP88, Univ. of Wisconsin, Plasma Laboratory.
11. D. Michael, J. McLeod, and H. Dreicer in Los Alamos Scientific Laboratory Annual Status Report #LA 4351-MS.
12. T. Kawamura and Y. Terashima, in Institute of Plasma Physics, Nagoya University Annual Review, page 103 (1968).
13. A. J. Lichtenberg, M. J. Schwartz, and D. T. Tuma, Plasma Physics 11, 101 (1969).
14. G. E. Guest, in Oak Ridge National Laboratory report ORNL-4150 (1967), page 29.
15. W. B. Ard, in Oak Ridge National Laboratory report ORNL-4401 (1968), page 41.
16. A. F. Kuckes, Plasma Physics 10, 367 (1967).
17. H. Grawe, Plasma Physics 11, 151 (1969).
18. Oak Ridge National Laboratory, Thermonuclear Division Annual Report for 1969, Chapter 4.
19. L. D. Smullin, Phys. Fluids 8, 1412 (1965).

20. W. Heitler, Quantum Theory of Radiation, 3rd edition (University Press, Oxford, 1954).
21. University of Wisconsin Report No. COO-1695-26, Quarterly Progress Report No. 12, March 15, 1970.
22. AEC Report No. COO-1695-14.
23. A. Bers & Speck, M.I.T., Research Laboratory of Electronics, Quarterly Progress Report 91, Oct. 15, 1968.
24. S. J. Gitomer, Ph.D. Thesis, University of Wisconsin, June 1969.
25. H. Ikegami, H. Ikezi, T. Kawamura, H. Momota, K. Takayama, and Y. Terashima, Plasma Physics and Controlled Nuclear Fusion Research, Volume II, 1969, p. 423 (Conference Proceedings, Novosibirsk, Aug. 1968).
26. J. Jacquinet, S. Kawasaki, C. Leloup, J.P. Poffe, M. de Pratis, J. Ripault and F. Waelbroeck, "On Controlled Fusion and Plasma Physics" Utrecht, 1969.
27. J. C. Lee and F. W. Crawford, Jour. Geophysical Res. 75, 85 (1970).
28. M.C. Becker, R.A. Dandl, H.O. Eason, A.C. England, R.J. Kerr, and W.B. Ard, Nucl. Fusion, 345 (1962).
29. R.W. Hockney, Phys. Fluids 9, 1826 (1968).
30. R.W. Hockney, Proc. Conf. of Computational Physics, Culham (1969).