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ANOMALOUS TRANSPORT AND STABILIZATION
OF COLLISIONLESS
DRIFT-WAVE INSTABILITIES

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Anomalous Transport and Stabilization
of Collisionless Drift Wave Instabilities *

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ABSTRACT

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Extensive numerical simulations have been carried out to study the enhanced transport processes associated with the collisionless drift wave instabilities. The results indicate that the shear is effective in reducing the anomalous particle diffusion as predicted by theory; however, it is less so for the accompanied electron heat transfer. The quasilinear decay of the density profile is found to be the dominant mechanism for the nonlinear saturation.

Low frequency drift waves produced by the density and temperature gradients, which are inherent for a magnetically confined plasma and can be driven unstable under a variety of conditions, have attracted widespread interest in recent years in view of the fact that the resulting enhanced plasma transport is detrimental to confinement.^{1,2} In this paper, we will report the investigations of the drift wave instability driven by the finite Larmor radius effects in a low- β , collisionless plasma (universal instability) using the electrostatic particle simulation code.³ Predictions of the linear theory on the universal mode^{1,2,4} have recently been qualitatively verified in the Q-machine experiments.⁵ However, a detailed comparison is still lacking. Linear stability criteria for collisionless drift waves in a sheared magnetic field have also been studied,⁶ and there is some preliminary experimental evidence for the stabilization.⁷ Nevertheless, the most important question concerning the mechanism(s) for the nonlinear saturation as well as the magnitude and the scaling of the enhanced plasma transport associated with these instabilities remains unsettled.^{1,8,9} It is, therefore, our purpose to use computer simulation to gain some insight into these areas. To our knowledge, this work represents the first such attempt.

A two-and-a-half dimensional (x, y, v_x, v_y, v_z) bounded plasma model capable of handling a nonuniform system has been developed for our purpose.¹⁰ The system is uniform and periodic in y , and is nonuniform in x where the plasma is bounded between

two conducting walls. The main magnetic field B_0 is perpendicular to the inhomogeneous x direction with $B_{0z} \gg B_{0y}$. Particles reaching the walls are reflected so as not to produce the sheath currents and other undesirable effects.¹⁰

Let us first present a linear theory pertaining to our model. For the case of the universal mode, the governing equation for the perturbed potential $\phi = \tilde{\phi}(x) \exp(ik_y y - i\omega t)$ can be written as⁴

$$\sum_{\alpha} \frac{1}{T_{\alpha}} \left[1 + \frac{\omega + \omega_{\alpha}^*}{\sqrt{2} k_{\parallel} v_{t\alpha}} I_0(\hat{b}_{\alpha}) e^{-\hat{b}_{\alpha}} Z\left(\frac{\omega}{\sqrt{2} k_{\parallel} v_{t\alpha}}\right) \right] \tilde{\phi} = 0 ,$$

$$\hat{b}_{\alpha} = \rho_{\alpha}^2 (k_y^2 - \partial^2/\partial x^2) = b_{\alpha} - \rho_{\alpha}^2 \partial^2/\partial x^2 \quad (1)$$

where α denotes the species, $v_{t\alpha} = (T_{\alpha}/m_{\alpha})^{1/2}$, $\rho_{\alpha} = v_{t\alpha}/\omega_{c\alpha}$, $\omega^* = k_y T_e \kappa / m_i \omega_{ci}$, $\omega_e^* = -\omega^*$, $\omega_i^* = \omega^* T_i / T_e$, $\kappa = - (dn/dx)/n$ and $L_n \equiv 1/|\kappa|$ is the density scale length, $k_{\parallel} = k_y \cos\theta$ where θ is the angle between $\mathbf{k}$ and $\mathbf{B}_0$, I_{ℓ} is the Bessel function and Z is the plasma dispersion function.¹¹ Expanding Eq. (1) in the limit of $k_x \rho_{\alpha} < 1$, we obtain the usual mode equation of the form

$$d^2 \tilde{\phi} / dx^2 + Q(x, \omega) \tilde{\phi} = 0 \quad (2)$$

We will now consider the solutions of Eq. (2) for several different cases. For a shear-free and constant ω^* (or κ) case, Eq. (2) can be solved with the boundary conditions $\tilde{\phi}(0) = \tilde{\phi}(L_x) = 0$, and the resulting dispersion relation is similar to Eq. (1) with $-\partial^2/\partial x^2$ now replaced by $k_x^2 \equiv (m\pi/L_x)^2$ for $m = 1, 2, 3, \dots$. The normal modes are given by

$$\tilde{\phi} = \sin(m\pi x/L_x) \quad (3)$$

For the case of a spatially dependent ω^* without shear, we expand Q in Eq. (2) around the maximum ω^* at x_1^{12} ; i.e., $\omega_\alpha^*(x) = \omega_{1\alpha}^* + \omega_{1\alpha}^{*''} (x - x_1)^2/2$, and obtain

$$Q = A + B^2 (x - x_1)^2$$

$$= \frac{\sum_\alpha [1 + (\omega + \omega_{1\alpha}^*) X_{0\alpha} + \omega_{1\alpha}^{*''} X_{0\alpha} (x - x_1)^2/2] / T_\alpha}{\sum_\alpha \rho_\alpha^2 (\omega + \omega_{1\alpha}^*) (X_{0\alpha} - X_{1\alpha}) / T_\alpha} \quad (4)$$

where $X_{\ell\alpha} = I_\ell(b_\alpha) \exp(-b_\alpha) Z(\omega/\sqrt{2} k_{||} v_{t\alpha}) / \sqrt{2} k_{||} v_{t\alpha}$. The dispersion relation and the localized normal modes are, from Eqs. (2) and (4),

$$A = i\sigma(2\ell + 1), \quad \ell = 0, 1, 2, \dots, \text{ and} \quad (5)$$

$$\tilde{\phi} = H_{\ell}[(i\sigma)^{1/2} (x - x_1)] \exp[-i\sigma(x - x_1)^2/2] , \quad (6)$$

respectively, where H_{ℓ} is the Hermite polynomial and $\sigma = \pm B$. The choice of sign for σ is such that $\text{Im } \sigma < 0$ as required from the proper boundary conditions for outgoing waves at large x .^{6,13} In the presence of shear, Eq. (2) has been solved with a constant ω^* using a perturbative scheme for the instability mechanism, and a criterion for total stabilization has been obtained.⁶ The effect of a variable ω^* on the shear stabilization has also been studied recently.^{12,13} These theoretical predictions will be compared later with the simulation results.

To simulate a collisionless plasma,¹⁴ the standard dipole expansion technique with finite-size particles is used. The guiding centers of the particles are loaded initially according to a prescribed density profile and with spatially uniform temperatures on a 64×32 ($L_x \times L_y$) spatial grid. Maxwellian velocity distributions are also used. The parameters of the simulations are: $m_i/m_e = 25$, $T_e/T_i = 4$, $\lambda_{De}/\Delta = 2$, $\omega_{ce}/\omega_{pe} = 2$, the average number density $\langle n \rangle = 8/\Delta^2$, and $a(\text{rms of a Gaussian particle}) = 1.5 \Delta$. The mesh size Δ is taken as the unit length. All the frequencies are measured in terms of ω_{pe} . These parameters give $k_x \rho_i = 0.12 m$ and $k_y \rho_i = 0.49 n$ where $m, n = 1, 2, 3, \dots$. Exact dynamics for the particle pushing have been used with $\Delta t = 0.5$.

Two different density profiles have been used in the calculations; the exponential profile given by $\bar{n}_0(x, t=0) = \langle n \rangle \kappa L_x \exp(-\kappa x) / [1 - \exp(-\kappa L_x)]$ with $\kappa = 0.07$ and the hyperbolic tangent profile given by $\bar{n}_0(x, t=0) = \langle n \rangle [1 - \kappa_0 \ell_0 \tanh(x - x_0)/\ell_0]$ with $\kappa_0 = 0.1$, $\ell_0 = 24/\pi$, and $x_0 = 32$, are shown in Fig. 1(a). For the exponential profile in which ω^* is constant, the most unstable mode, from Eq. (1), is found to be $(m, n) = (1, 1)$ with $\omega = 0.0064$ and $\gamma = 0.0048$ for $\theta = 88.5^\circ$. For the case of the hyperbolic tangent profile, ω^* has a peak at $x_1 = 36$ where $\kappa_{\max} = 0.12$ and $\kappa_{\max}'' = -0.0033$. Equation (5) gives $\omega = 0.0055$ and $\gamma = 0.0048$ for the most unstable mode $(\ell, m) = (0, 1)$ for $\theta = 88.5^\circ$. Since the two growth rates are the same, we define $\langle L_n \rangle = 1/0.07$ as the average density scale length to describe both systems.

The simulation results of the growth rates for the two density profiles measured in terms of the average density modulation versus time, are shown in Fig. 1(b) where $\langle \bar{n}_1(x, t) \rangle = \int \bar{n}_1(x, t) dx / L_x$ is the spatial average of the most unstable $n = 1$ mode. The agreement with theory is excellent. The instability grows above the thermal noise at $t = 400$ and exponentiates at the expected growth rate. The saturation density modulation is about 10%. The corresponding frequencies are found to be $\omega = 0.004$ for both cases at $t = 1200$ which are smaller than the theoretical estimates. This is due to the quasilinear diffusion to be discussed later.

The spatial structures of $n = 1$ mode are shown in Fig. 1(c). For the exponential profile case, several normal modes in x have

initially been excited which are eventually dominated by $m = 1$ mode at $t = 880$. The deviation of the mode structure from Eq. (3) is again due to the quasilinear diffusion. For the hyperbolic tangent profile case, $\ell = 0$ is the only unstable mode and its structure at the linear stage of the instability, $t = 560$, agrees very well with that given by Eq. (6).

As the instability develops, the density profile undergoes a quasilinear change in time due to the $\tilde{E} \times \tilde{B}$ drift as shown in Fig. 1(a). Initially, the diffusion takes place at around the maximum wave amplitude, then gradually evolves into a plateau, and finally settles into a new stable configuration. The density profiles then oscillate around the new equilibrium with approximately twice the drift frequency because of the existence of the large amplitude waves in the system by this time. The profiles shown for $t = 1600$ roughly correspond to the equilibrium states. The solutions from Eq. (5) have confirmed this observation; i.e., these final configurations are indeed linearly stable. In general, the diffusion of the electrons is found to be slower than that of the ions. The amplitude of the modes has reached the maximum at the time of the plateau formation with $e\bar{\phi}_1/T_e = 25\%$ and 15% , respectively, for the two cases. Note the initial noise level here is about 1.5% .

Let us now examine the energy transport processes associated with the instability which were first pointed out by Coppi.¹⁵ The simulation results have shown that in the linear stage there is energy transfer from the electron parallel temperature to the waves through inverse Landau damping in the unstable regions.

However, as the waves grow in amplitude causing the quasilinear diffusion, a significant amount of electron heating has been observed elsewhere. After the plasma reaches the stable configuration, the heating also stops. The electron heat transfer patterns normalized by the initial temperature $\langle T \rangle_{e||}$ are shown in Fig. 1(d). The energy loss from the unstable regions is about 10% of the total parallel electron thermal energy for the exponential case and 7% for the other. More than half of that amount, however, is transported by the waves back to the electrons in other regions causing large heat transfer. This is due to the quasilinear change in density which alters the local dispersions; however, the linear wave absorption is also responsible for the heating of the hyperbolic tangent case.¹⁶ Most of the remaining amount goes to the ambipolar drift, and relatively little is kept by the waves. No ion heating has been detected, except for the ion drift parallel to B_0 . In Fig. 1(e), the average electron velocity distribution functions corresponding to those in Fig. 1(d) are shown along with the measured phase velocity. The resonance regions are evidently broadened by the instability. This is also the case for the local distributions.

The nonlinear behavior of the instability reported here does not seem to agree with the various existing theories.^{1,8,9} Instead, the quasilinear diffusion in space is the dominant saturation mechanism. The theoretical analysis has been initiated with encouraging results and will be reported later.¹⁶

Let us now look at the shear stabilization of drift waves using the hyperbolic tangent density profile. The sheared magnetic field is introduced by adding $B_s(x) = \hat{y} B_0 (x - x_1) / L_s$

to the main magnetic field with the shear scale length $L_s = 800$, 320 , and 128 . We choose $x_1 = 36$ so that $B_s = 0$ at the maximum κ . As the shear length decreases, we have observed that the unstable mode becomes more and more localized with decreasing amplitude. The partially stabilized drift waves again cause the density profile change around the region where the wave amplitude is maximum. The instability eventually saturates when the density settles into a new stable configuration. The resulting density profiles at the time of the saturation are shown in Fig. 2(a) for various shear lengths. The effect of the shear stabilization is evident. The detailed comparison with the linear stability theory will be given later.¹³ The pattern of the associated energy transport is similar to the shear-free case, although the amount of the electron energy loss, as well as the cross field transport, decreases with shear. The net energy loss of the electrons again goes mostly to the ambipolar drift with no observable ion heating, except for the parallel drift motion. From these results, we therefore conclude that the saturation for the partially stabilized drift waves is also due to the quasilinear particle diffusion.

To estimate the particle diffusion and heat conductivity coefficients, diffusion equation, $\partial(\bar{n}_o, \bar{T}_{ell})/\partial t = (D_\perp, k_{e\perp}) \partial^2(\bar{n}_o, \bar{T}_{ell})/\partial x^2$ is used. The average diffusion and heat conductivity coefficients for the hyperbolic tangent profile with and without shear are shown in Fig. 2(b).

For the shear-free case, the diffusion coefficient is about 25% of the values given by the Bohm diffusion,

$D_i \sim cT_e/16 eB_0$, and the turbulent diffusion, $D_i \sim \gamma/k_i^2$, by Kadomtsev.¹ This is also true for the shear-free exponential case. The Pearlstein and Berk criterion for the total stabilization by shear⁶ is $\langle L_n \rangle / L_s \geq 0.24$ for our case. The results shown in Fig. 2(b) approach this limit asymptotically if extrapolation is used. However, our results differ from the estimate given by Ref. 12, which sets a rather stringent limit for the stabilization due to the peaking in ω^* .

As shown in Fig. 2(b), the heat conductivity of the electrons is much greater than the particle diffusion and it is also less sensitive to shear.

For the results reported here, we have used a small mass ratio with essentially only one unstable mode. The simulations using large mass ratio with many unstable modes are being investigated using a newly developed model¹⁰ and the results will be reported later.

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Fig. 1(a). Time evolution of the density profiles.

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Fig. 1(b). Growth of the density modulation for $n = 1$ mode.

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Fig. 1(c) Mode structures for $n = 1$ mode.

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Fig. 1(d). Parallel electron heat transfer patterns.

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Fig. 1(e). Average velocity distributions.

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Fig. 2(a). Saturation density profiles for various shear lengths.

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Fig. 2(b). Diffusion and conductivity coefficients vs. shear.