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STATIC STRESS DETERMINATIONS AND
CRUSHED ZONE MEASUREMENTS
SITE HOBO

by

Robert H. Merrill and Verne E. Hooker

STATIC STRESS DETERMINATIONS AND CRUSHED ZONE

MEASUREMENTS--SITE HOBO ^{1/}

by

Robert H. Merrill ^{2/} and Verne E. Hooker ^{3/}

INTRODUCTION

This report describes the post shot measurements to determine 1.) the static stress field near the shot sites (reported by Merrill) and, 2.) the volume of the crushed zone of rock surrounding the location of the charges fired during Project HOBO (reported by Hooker).

These measurements were made in connection with the Seismic Detection Program of the Atomic Energy Commission under the Memorandum of Understanding No. AT(29-2)-914, Appendix 4, Modification 3, Rock Mechanics Studies Work. The investigation was made in cooperation with the Lawrence Radiation Laboratory, Livermore, California, and other contractors. All field work described in this report was at Site HOBO, Nevada Test Site, Mercury, Nevada (about 80 miles North of Las Vegas, Nevada).

GENERAL GEOLOGY AT SITE HOBO

The geology at Site HOBO has been described in detail and is not within the scope of this report. However, some geologic features influenced the measurements discussed herein, and these features are described briefly.

^{1/} Work on manuscript completed July 1960.

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The general terrain near Site HOB0 is comparatively barren and rugged. The approximate profile of the ground above the tunnel in which the HOB0 tests were performed is shown in figure 1. The depths of overburden above the two tunnel test sites was about 150 and 1200 feet.

The entire HOB0 test area is in volcanic tuff. The tuff is relatively soft and contains small (usually about 1/2 inch diameter) nodules of hard chert or a softer material presumed to be bentonitic. The tuff is massively bedded and the general line of the bedding is shown in figure 1. The tuff is jointed (N45°W and N35°E) and faulted. The strike of the faults (10 were noted in the HOB0 tunnels) is about N50°W; the dip, about 65° to both the Southwest and Northeast. Eight of the faults are displaced parallel to the plane fault; two are bedding plane thrust faults. The major structural feature is a 5-foot-displacement along the general line of bedding that occurred following events BLANCA and LOGAN, see figure 1. It is presumed that all ground above this line was also displaced by these detonations.

STATIC STRESS DETERMINATIONS

The stress in the tuff both near and at a distance from the surface of underground openings was determined by measuring the change in the deformation of borehole before and after stress relieving the hole by over drilling, and by calculating the stress from the relationship between borehole deformation and applied stress. In the derivation of this relationship it is assumed that the medium is perfectly elastic, homogeneous, and isotropic. The derivations for the cases of plane stress and plane strain are presented in Appendix I, part A. The condition for plane stress and a uniaxial and biaxial stress field is used in this investigation.

For a uniaxial stress field and plane stress, the change in the diameters, that is, the deformation, of the borehole, U , is related to the stress in the medium by (see fig. 2, in which $T = 0$)

$$U = \frac{dS}{E}(1 + 2 \cos 2\theta) \quad (1)$$

where

U = deformation (change in diameter) of hole,

d = diameter of hole,

FIGURE 1. - Section of HOB0 Test Site Showing General Line of Geologic Bedding and Profile of Overburden.

FIGURE 2. - Cross-Section of a Hole in a Biaxial Stress Field.

S = applied uniaxial stress,

θ = angle between and direction of deformation and the x axis,

E = modulus of elasticity,

For a biaxial stress field and plane stress, the deformation of the borehole is related to the mutually perpendicular applied stresses, S and T , by (see figure 2 and Appendix I, part A)

$$U = \frac{d}{E} [(S + T) + 2(S - T) \cos 2\theta] \quad (2)$$

The two dimensional stress fields can be resolved into two perpendicular components, P and Q , called the principal stresses. P and Q are characterized by having the directions of the algebraically greatest and algebraically smallest stress. The shear stress is zero in the direction of P and Q (1) 4.

If the borehole deformation across three diameters is measured and the modulus of elasticity of the medium is known, the magnitude and direction of the principal stresses can be computed. If the measured deformations are 60-degrees apart (that is, a 60-degree-deformation-rosette), the relationships between principal stress and deformation are (see fig. 3)

$$P + Q = \frac{E}{3d} (U_1 + U_2 + U_3), \quad (3)$$

$$P - Q = \frac{2E}{6d} [(U_1 - U_2)^2 + (U_2 - U_3)^2 + (U_1 - U_3)^2]^{1/2}, \quad (4)$$

where

U_1, U_2, U_3 = deformations across diameters 60 degrees apart,

and the angle, θ_1 , between the direction at which U_1 is measured and the larger principal stress, P , is

$$\tan 2\theta_1 = - \frac{3 (U_2 - U_3)}{2U_1 - U_2 - U_3} \quad (5)$$

4/ Underlined numbers in parenthesis refer to items in the bibliography at the end of this report.

The deviations for equations 3 through 6 are presented in Appendix I, part B, together with the deviation for the 45-degree-deformation-rosette.

INSTRUMENTATION AND PROCEDURE

The instrumentation used in the field measurement of stress consists of a borehole gage, a strain gage indicator and a diamond-drill. The drill serves two purposes, namely, to drill a hole in which the gage is inserted and to concentrically core around the borehole to produce stress relief.^{5/}

The transducing element in the gage is a strip of beryllium copper on which four resistance-type strain gages are bonded forming a 4-arm bridge, see fig. 4 (3). The strip is actuated by a piston which senses the diameter of the hole. The strain in the strip changes proportionally, to the diametral change and this strain is measured by a conventional strain gage indicator. A calibration curve relating the piston displacement to strain is shown in figure 5. This curve was obtained by displacing the piston in known increments (measured by a micrometer screw) and recording the corresponding strain.

The gage, together with the bridge, has a sensitivity of 30 micro-inches, which, for a rock with a modulus of elasticity of 3×10^6 p.s.i., corresponds to a change in stress of 20 p. s. i.

The principle of stress-relief (sometimes called strain-relief) and equations 3 and 4 are used to determine the stress in the rock.

The procedure is as follows: An EX-size diamond drill hole (1.5 inch diameter) is drilled to a depth of about 20 feet. The borehole gage is inserted at a depth of about 6-inches or deeper in the hole and oriented at 30 degrees with respect to the horizontal. The rock surrounding the gage is stress relieved by over-drilling concentrically around the EX hole with a 6-inch bit, see fig. 6; the over-drilling is stopped when no further deformation of the EX-hole is indicated (that is, when the rock is stress relieved). Stress relief is accomplished when the gage is over-drilled 1.5 to 2-inches past the gage contacts. Next, the gage is moved 4 to 6 inches past the core depth resulting from the first stress-relief, oriented at 90 degrees and

5/ This method of concentric coring around a borehole was developed by Nils Hast of Sweden (2).

FIGURE 3. - Cross-Section of a 60-Degree-Deformation-Rosette.

FIGURE 4. - Sections Through the Borehole Gage.

600 016

FIGURE 5. - Calibration Curve for the Borehole Gage.

FIGURE 6. - Section of the Gage and Core Barrel in a Borehole.

the stress relief repeated. This procedure is again repeated for a gage orientation of 150 degrees. This cycle of measurement at 30-, 90-, and 150-degree orientation is continued at greater depths in the hole until the deformation shows no further change. At this depth the stress in the rock is not influenced by boundary of the opening from which gage hole was drilled, and hence, only the stress field, that is, the stress before mining, is being measured. Photographs of the drill, gage, and associated equipment are shown in figures 7 and 8.

Solid 6-inch diameter cores were taken at each stress-relief site to provide specimens for a laboratory determination of the modulus of elasticity. To prepare specimens for the laboratory test the cores were cut into cylinders 6-inches long and a 1-1/2 inch diameter (EX) hole was drilled through the core. A borehole gage was inserted in the hole and the specimen loaded in a compression machine. A photograph of a prepared specimen under test is presented in figure 9.

The borehole deformation and the applied stress were recorded at increasing and decreasing stress increments of 100 p. s. i. From these measurements the modulus of elasticity, E , was calculated from equation 1. Sample data from these laboratory tests are given graphically in fig. 10. The ratio of the deformations to the applied stress was always greater over the first increasing half cycle than when the loads were increased or decreased over following cycles. Because the stress determination in the field are made by stress-relief, that is, for a decreasing stress, the modulus of elasticity was obtained from the average of the decreasing slopes of the stress-deformation curves.

The 6-inch specimens from one site disintegrated when drilling the borehole for the gage. Hence, the modulus of elasticity was determined from stress-strain curves obtained from 2 x 2 inch cubes of tuff. The two methods of determining the modulus of elasticity have been compared for other rock types (limestone, marble, salt) and found to be in good agreement.

The magnitude and direction of the in situ principal stresses in the plane normal to the axis of the hole were calculated from the borehole deformation measurements and modulus of elasticity using equations 3, 4, and 5.

To obtain the borehole measurements at a given depth, the following procedure was employed. An average line was drawn through the borehole deformations for each of the three angles of measurement. The borehole deformation at a given depth was read off from these curves. Sample data are presented in fig. 11. The deformations U_1 , U_2 , U_3 , at any depth are obtained from the 30-, 90-, and 150-degree curves. For example, at 30 inches these values are, $U_1 = 850 \mu$ in., $U_2 = 1400 \mu$ in., and $U_3 = 1150 \mu$ in.

IN SITU MEASUREMENTS

Five stress reliefs were made in the walls of tunnels U12e (main drift) and U12e.03 (entry drift). The location of these five reliefs are shown in plan in figure 12. The locations are called sites 1 through 5. Site 1 was near the shot holes used in one phase of the HOBOT tests (holes 11, 14, and 17) and Site 4 was located near another shot hole used in another phase of the HOBOT tests (hole 4). Sites 1 through 5 were all under different depths of overburden; these depths were estimated from topographic maps and are as follows: for site 1, 1200 feet; site 2, 950 feet; site 3, 1400 feet; site 4, 160 feet; and site 5, 300 feet. Sites 2 and 5 were within an area of ground visibly displaced, or broken, by events BLANCA and LOGAN. Sites 1, 3, and 4, were in areas presumed not to have been damaged by these, or other events. Except at site 2, 6-inch-diameter solid cores were taken to provide specimens for laboratory evaluation of the modulus of elasticity. At sites 1 and 5 solid cores were taken from the tuff at the end of the stress relief hole. Because the texture and color of the tuff changed from point to point in strain relief holes at sites 3 and 4, core samples were taken from both the collar (0- to 24-inches from the rib) and at the back (about 180- to 204-inches) of the holes.

Sometimes it was difficult to pull the core barrel from the stress relieved hole. This difficulty was presumed to have been caused by the fact that the 6-inch hole would contract after being drilled to the extent the the core barrel was pinched in the hole. Also, fine dust created by drilling would become wet and adhere to the barrel. It was necessary to abandon site 2 because of the broken nature of the tuff. Also, it was difficult to obtain stress relief measurements at site 5 for the same reason.

FIGURE 7. - Photograph of the Concentric Drill.

FIGURE 8. - Photograph of the Core Barrel and Gage.

FIGURE 9. - Photograph of Laboratory Set Up for Measuring Borehole Deformation.

FIGURE 10. - Stress Versus Deformation Curves for Specimen From Site 3.

FIGURE 11. - Representative Curves for Deformation Versus Distance From Collar of Borehole.

FIGURE 12. - Plan of Entry Drift and Main Drift Showing Site Locations for Stress-Relief and Shot Holes Used in Project HOB0.

910 019

DATA

The stress relief data at sites 1, 3, 4, and 5, are presented graphically in figures 13, 14, 15, and 16. In these graphs the deformation at 30-, 90-, and 150-degrees are plotted against the depth of hole at which the relief was made. Because the data for the stress relief at 64-inches and 90-degrees at site 5 was far greater than any other value at any site, it is presumed that this point is in error. Therefore, the data for 90-degree orientation is assumed to more closely fit the dotted line (between depths of 18 and 110 inches) see fig. 16.

The computed modulus of elasticity from each sample is presented by site number in table 1.

TABLE 1. - Modulus of Elasticity.

Site No.	Modulus of Elasticity ($\times 10^6$ p.s.i.)	
	<u>Sample at collar of hole</u>	<u>Sample at back of hole</u>
1	--	0.50
2	--	--
3	0.56	0.47
4	0.16	0.26
5	--	0.46 ^{6/}

^{6/} Determined from stress-strain curves rather than from stress-deformation curves.

RESULTS AND CONCLUSIONS

The general texture and color of the tuff changes from points between the collar and back of stress relief holes at sites 3 and 4. This change in texture and color apparently is accompanied by changes in the modulus of elasticity, see table 1. To compute the stresses from deformations measured at sites 3 and 4, it is assumed that the modulus of elasticity at the collar and back of the holes is the same as the modulus measured in the laboratory, and, at the points between the collar and back, it is assumed that the modulus increases (as in site 4) or decreases (as in site 3) at a constant rate with respect to depth. For example, at a point half way between the collar and back of the hole the modulus is assumed equal to the average of the modulus measured at the collar and back. The stresses at sites 1 and 5 were computed using the modulus from samples at the back of the boreholes. The computed principal stresses at various depths from the collar of the boreholes are presented in table 2.

TABLE 2. - Computed principal stresses at various depths from the collars of boreholes in U12e and U12e.03 drifts.

<u>Site No.</u>	<u>Depth</u>	<u>Stress</u>	<u>P. S. I.</u>	<u>Angle</u>
1	45	1550	850	58 ^{x/}
1	50	1520	740	59
1	60	1330	510	60
1	70	1190	430	67
1	80	1040	550	72
1	90	1120	550	76
1	100	1110	580	76
1	110	1110	570	77
1	120	1150	570	76
3	50	1310	940	66 ^{y/}
3	60	1290	900	54
3	70	1270	850	51
3	80	1190	790	50
3	90	1190	780	57
3	100	1200	780	64
3	110	1200	790	67
3	120	1130	770	67
3	130	1070	750	65
3	140	1080	720	57
3	150	1070	680	50
4	60	390	190	5 ^{y/}
4	70	180	100	5
4	80	170	100	6
4	90	180	110	14
4	100	220	120	22
4	110	250	120	23
4	120	210	140	24
4	130	190	80	21
5	45	1050	610	70 ^{y/}
5	50	1110	390	68
5	60	1240	550	66
5	70	1170	500	67
5	80	990	450	68
5	90	870	350	71
5	100	650	350	73

x/ Degrees clockwise from horizontal line pointing approximately South.

y/ Degrees clockwise from horizontal line pointing approximately North.

FIGURE 13. - Deformation Versus Distance From Collar of Borehole, Site 1.

FIGURE 14. - Deformation Versus Distance From Collar of Borehole, Site 3.

FIGURE 15. - Deformation Versus Distance From Collar of Borehole, Site 4.

FIGURE 16. - Deformation Versus Distance From Collar of Borehole, Site 5.

If it is assumed that the tuff was not subject to tectonic forces the vertical stress field can be estimated from the weight density of the tuff and the thickness of overburden above the various sites, and the horizontal side stress can also be estimated if the degree of confinement, and Poisson's ratio are known. These vertical stresses were computed using a weight density of 0.075 lbs./in.³. The horizontal stress was computed using an estimated Poisson's ratio of 0.33 and assuming complete confinement. The results are presented in table 3. The average measured stress at depths greater than 100 inches are also presented in table 3.

TABLE 3. - Computed and measured residual stresses at various sites in drifts U12e and U12e.03.

<u>Site</u>	<u>Measured Stress</u>		<u>Computed Stress</u>	
	θ (degrees)		Vertical	Horizontal
	P(p.s.i.)	Q(p.s.i.)		
1	1120	570	1050	530
3	1120	730	1250	630
4	220	110	140	70
5	650	350	430	220

The stress field at sites 1 and 4 appear to be in reasonable agreement with stresses computed from theory, hence, it is concluded that the principal residual stresses at site 1 (near holes 11, 14, and 17) are $P = 1120$ p. s. i.; $Q = 730$ p. s. i.; at site 4 (near hole 4) the stresses are $P = 220$ p. s. i.; $Q = 110$ p. s. i., see tables 2 and 3.

The differences between the magnitude and direction of the measured principal stresses and the computed vertical and horizontal stresses permit some speculation regarding their cause. At site 4, the maximum stress was oriented at an angle less than 45 degrees, that is, the lateral stress was greater than the vertical stress. Whereas, at sites 1, 3, and 5, the angle was greater than 45 degrees. At sites 1 and 5, the direction of the larger principal stress was almost in line with the edge of the mesa. At site 3, the direction is almost perpendicular to the face of the escarpment (see fig. 1). Thus, it appears that the direction of the maximum principal stress at all sites must have been influenced by forces other than gravitational, such as geologic or tectonic forces, or by the forces created by events BLANCA and LOGAN, or by jointing. It is obvious that pre-HARDTACK stress measurements would have provided data from which this conclusion could have been more accurately evaluated.

CRUSHED ZONE MEASUREMENTS

Introduction

The purpose of this study was to determine the size and shape of the crushed zone produced by the detonation of an explosive in volcanic tuff. Measurements were scheduled for holes 4, 11, 14, and 17. The location of these holes is shown in fig. 12.

The crushed zone measuring procedure developed for dry holes 25 to 30 feet deep was used.^{6/} This method consists of removing the crushed material from the shothole with a blowpipe. The volume of the crushed zone is then determined by loading sand or other suitable material in increments of known volume into the hole and measuring the corresponding decrease in the hole depth.

Data from crushed zone measurements in other rock types has shown that crushed zone volume was dependent upon charge size and detonation pressure of the explosive. Also, a correlation existed between peak strain and crushed zone volume.^{6/}

Procedure

The firing of the HOBOT shots blocked the loading holes and it was necessary to redrill these holes to permit the crushed zone measurements. Only two holes were measured (No. 4 and 17) because hole 14 was not redrilled at the time of this test and the top part of hole 11 caved after redrilling, again blocking the hole.

After redrilling to the crushed zone it was found that they were extremely wet and partially filled with a fine silty mud and crushed material. To dry this material and remove it from the hole, air at 110 p. s. i. was blown into the hole through a hose or metal pipe. Hoses of various diameters were used in an effort to create a maximum lift for removal of the particles from the hole. Compressed air was blown into the holes continuously for a period of several days until it became apparent that the influx of ground water would not

6/ ATCHISON, T. C. and TOURNAY, W. E., "Comparative Studies of Explosives in Granite". Bureau of Mines Report of Investigations 5509, 1959, 28 pp.

permit sufficient drying for removal of the crushed particles. Two measurements were taken at this time, the depth of top of the cavity and the deepest penetration of the measuring rods. These measurements, together with other shot data, are given in table 1.

TABLE 1. - Crushed zone data.

	<u>Hole No. 4</u>	<u>Hole No. 17</u>
Date of shot	4/ 3/ 60	3/ 20/ 60
Date of measurement	4/ 16/ 60	4/ 15/ 60
Base of charge prior to shot	104 ft.	101 ft.
Depth to center of gravity of charge	102.2 ft.	100.1 ft.
Charge diameter	30 in.	18 in.
Charge size	967-3/ 4 lb.	103-1/ 4 lb.
Depth to top of cavity	98.0 ft.	97.0 ft.
Deepest penetration by tamp rods after shot	110.7 ft.	99.2 ft.

Results

The crushed zone data for the two shots are given in table 2. The inability to remove crushed material because of wet holes presented serious problems, particularly in hole number 17. In this hole the water level rose at a rate of over 0.5 foot per minute. After cleaning the hole the true depth of the cavity could not be determined with the measuring rods as indicated by the fact that the deepest penetration was less than the center of gravity of the charge. The bottom portion of the cavity undoubtedly contained broken rock and possibly an added amount of silt which had been deposited from solutions higher up in the cavity. Since the bottom portion of the cavity remained filled, volume measurements indicate a smaller crushed zone to charge ratio than actually existed. Because of this difficulty no attempt is made to specify the crater boundary.

The water level in hole number 4 was lowered first to a depth of 102 ft. by bailing and then to 105.2 ft. by compressed air forced through pipes. The material then remaining in the cavity was a wet mud that could not be removed. However, this mud was penetrated by exerting pressure on the measuring rod, forcing it down to a depth of 110.7 ft. This is 2.7 ft. below the depth of the original

11-3/4 inch hole which was drilled to a depth of 108 feet.

The crushed zone measurements following the addition of each volume of gravel were not consistent. This was probably due to the dispersion from settling of the pea gravel into the mud. The best estimate of the crushed zone volume is 150 cubic feet, based on the assumption that the 150 cubic feet of gravel which was added to the zone was completely dispersed within the mud and that the volume of the gravel in the mud was the same as the dry volume of the gravel. Any error resulting from this assumption would make the actual crushed zone larger than 150 cubic feet.

The shape of the crushed zone is less definite. The mid-volume of gravel would place the center of the cavity at a depth of 99 ft. or approximately 3 ft. above the center of the charge. If the diameter of the crushed zone is computed on the basis of incremental rise, 1 ft. above and 1 ft. below the 99 ft. mid volume line the diameter would be 5.8 ft. The data are too inconsistent to permit any further calculation of the crushed zone shape.

TABLE 2. - Volume increment measurements.

Hole No.	Increment sand volume cu. ft.	Total added sand volume cu. ft.	Hole depth ft.	Increment at build up ft.	Total build up ft.
4	0.0	0.0	110.7	0.0	0.0
	3.0	3.0	110.7	0.0	0.0
	3.0	6.0	107.6	3.1	3.1
	3.0	9.0	106.0	1.6	4.7
	3.0	12.0	105.3	.7	5.4
	5.0	17.0	105.2	.1	5.5
	5.0	22.0	105.2	--	5.5
	5.0	27.0	105.2	--	5.5
	5.0	32.0	103.2	2.0	7.5
	5.0	37.0	101.3	1.9	9.4
	5.0	42.0	100.3	1.0	10.4
	3.0	45.0	99.8	.5	10.9
	3.0	48.0	99.6	.2	11.1
	3.0	51.0	99.3	.3	11.4
	3.0	54.0	99.1	.2	11.6
	5.0	59.0	99.1	--	11.6
	5.0	64.0	99.1	--	11.6
	5.0	69.0	99.1	--	11.6
	5.0	74.0	99.0	.1	11.7
	5.0	79.0	99.0	--	11.7
	5.0	84.0	98.8	.2	11.9
	5.0	89.0	98.3	.5	12.4
	5.0	94.0	98.1	.2	12.6
	5.0	99.0	98.1	--	12.6
	5.0	104.0	98.0	.1	12.7
	5.0	109.0	98.0	--	12.7
	5.0	114.0	98.0	--	12.7
	5.0	119.0	98.0	--	12.7
	5.0	124.0	98.0	--	12.7
	5.0	129.0	98.0	--	12.7
	5.0	134.0	98.0	--	12.7
	5.0	139.0	97.1	.9	13.6
	5.0	144.0	97.0	.1	13.7
	5.0	149.0	96.9	.1	13.8
	5.0	154.0	90.3	6.6	20.4
	.25	154.25	87.2	3.1	23.5
17	0.0	0.0	99.2	0.0	0.0
	1.0	1.0	98.4	.8	.8
	1.0	2.0	97.9	.5	1.3
	1.0	3.0	97.6	.3	1.6
	1.0	4.0	97.3	.3	1.9
	1.0	5.0	97.0	.3	2.2
	1.0	6.0	95.9	1.1	3.3
	0.25	6.25	95.5	.4	3.7

TABLE 2 Cont'd. - Volume increment measurements.

Hole No.	Increment sand volume cu. ft.	Total added sand volume cu. ft.	Hole depth ft.	Increment at build up ft.	Total build up ft.
	0.25	6.50	95.1	.4	4.1
	0.25	6.75	93.1	2.0	6.1
	0.25	7.0	90.6	2.5	8.6

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ILLUSTRATIONS

Fig.

1. Section of HOBO test site showing general line of geologic bedding and profile of overburden.
2. Cross-section of a hole in biaxial stress field.
3. Cross-section of a 60-degree-deformation-rosette.
4. Sections through borehole gage.
5. Calibration curve for the borehole gage.
6. Section of gage and core barrel in borehole.
7. Concentric drill at site HOBO.
8. Core barrel and borehole gage .
9. Laboratory set-up for measuring borehole deformation.
10. Stress versus deformation curves for specimen from site 3.
11. Representative curves for deformation versus distance from collar of borehole.
12. Plan of entry drift and main drift showing site locations of stress relief and shot holes used in Project HOBO.
13. Deformation versus distance from collar of borehole, site 1.
14. Deformation versus distance from collar of borehole, site 3.
15. Deformation versus distance from collar of borehole, site 4.
16. Deformation versus distance from collar of borehole, site 5.

APPENDIX I

The equations for the deformation of holes in an infinite plate that is perfectly elastic, homogeneous, and isotropic, are presented in Part A of this appendix. The cases for uniaxial and biaxial stress fields in plane stress and plane strain are considered.

The equations relating the principal stresses in a medium to the deformation of a hole as measured along three diameters 45- or 60-degrees apart, are derived in Part B of this appendix. These equations are called the 45- and 60-degree-deformation-rosette-equations.

The symbols used in Part A are as follows:

e_r = radial strain,

e_θ = tangential strain,

$\lambda_{r\theta}$ = shearing strain,

σ_r = radial stress,

σ_θ = tangential stress,

$\tau_{r\theta}$ = shearing stress,

θ = angle measured from direction of applied stress (S) to the x-axis,

u = radial displacement,

v = tangential displacement,

E = Young's modulus,

ν = Poisson's ratio,

a = radius of hole,

d = diameter of hole,

r = radial distance,

S = applied stress,

T = applied stress,

U = deformation (change in length of the diameter) of the hole.

Part A, Case 1 - Deformation of a hole in a plate subjected to a unidirection stress field under the condition of plane stress.

To obtain the deformation of a hole in a plate under the condition for plane stress the Kirsch equation for the stress distribution around a hole are substituted in Hookes' law equations for plane stress and integrated.

Relationship for the strain components and displacements in polar coordinates are

$$e_r = \frac{\partial u}{\partial r}, \quad (1)$$

$$e_\theta = \frac{u}{r} + \frac{\partial v}{r \partial \theta}, \quad (2)$$

$$\lambda_{r\theta} = \frac{\partial u}{r \partial \theta} + \frac{\partial v}{\partial r} - \frac{v}{r}. \quad (3)$$

Hookes' law equations for plane stress are

$$e_r = \frac{1}{E} (\sigma_r - \nu \sigma_\theta), \quad (4)$$

$$e_\theta = \frac{1}{E} (\sigma_\theta - \nu \sigma_r), \quad (5)$$

$$\lambda_{r\theta} = \frac{2(1 + \nu)}{E} \tau_{r\theta}. \quad (6)$$

The Kirsch equations for the stress distributions around a circular hole (unidirectional stress field, $T = 0$) are

$$\sigma_r = \frac{S}{2} \left(1 - \frac{a^2}{r^2}\right) + \frac{S}{2} \left(1 - \frac{4a^2}{r^2} + \frac{3a^4}{r^4}\right) \cos 2\theta, \quad (7)$$

$$\sigma_\theta = \frac{S}{2} \left(1 + \frac{a^2}{r^2}\right) - \frac{S}{2} \left(1 + \frac{3a^4}{r^4}\right) \cos 2\theta, \quad (8)$$

$$\tau_{r\theta} = -\frac{S}{2} \left(1 + \frac{2a^2}{r^2} - \frac{3a^4}{r^4}\right) \sin 2\theta, \quad (9)$$

Combining equations 1 and 4, 2 and 5, 3 and 6 yields

$$\frac{\partial u}{\partial r} = \frac{1}{E} (\sigma_r - \nu \sigma_\theta), \quad (10)$$

$$\frac{u}{r} + \frac{\partial v}{r \partial \theta} = \frac{1}{E} (\sigma_\theta - \nu \sigma_r), \quad (11)$$

$$\frac{\partial u}{r \partial \theta} + \frac{\partial v}{\partial r} - \frac{v}{r} = \frac{2(1+\nu)}{E} \tau_{r\theta}. \quad (12)$$

Substituting equations 7 and 8 into 10 gives

$$\begin{aligned} \frac{\partial u}{\partial r} = \frac{1}{E} & \left[\frac{S}{2} \left(1 - \frac{a^2}{r^2}\right) + \frac{S}{2} \left(1 - \frac{4a^2}{r^2} + \frac{3a^4}{r^4}\right) \cos 2\theta \right] \\ & - \frac{\nu}{E} \left[\frac{S}{2} \left(1 + \frac{a^2}{r^2}\right) - \frac{S}{2} \left(1 + \frac{3a^4}{r^4}\right) \cos 2\theta \right] \end{aligned} \quad (13)$$

Integrating equation 13 gives

$$\begin{aligned} u = \frac{1}{E} & \left[\frac{S}{2} \left(r + \frac{a^2}{r}\right) + \frac{S}{2} \left(r + \frac{4a^2}{r} - \frac{a^4}{r^3}\right) \cos 2\theta \right] \\ & - \frac{\nu}{E} \left[\frac{S}{2} \left(r - \frac{a^2}{r}\right) - \frac{S}{2} \left(r - \frac{a^4}{r^3}\right) \cos 2\theta \right] + g_1(\theta), \end{aligned} \quad (14)$$

where $g_1(\theta)$ is an arbitrary function of θ only. Substituting equations 7 and 8 into equation 11 and combining terms gives

$$\begin{aligned} \frac{\partial v}{\partial \theta} = \frac{1}{E} \left[-\frac{S}{2} \left(2r + \frac{4a^2}{r} + \frac{2a^4}{3r} \right) \cos 2\theta \right] \\ - \frac{\nu}{E} \left[\frac{S}{2} \left(2r - \frac{4a^2}{r} + \frac{2a^4}{3r} \right) \cos 2\theta \right] - g_1(\theta). \end{aligned} \quad (15)$$

Integrating equation 15 gives

$$\begin{aligned} v = \frac{1}{E} \left[-\frac{S}{2} \left(r + \frac{2a^2}{r} + \frac{a^4}{3r} \right) \sin 2\theta \right] \\ - \frac{\nu}{E} \left[\frac{S}{2} \left(r - \frac{2a^2}{r} + \frac{a^4}{3r} \right) \sin 2\theta \right] - \int g_1(\theta) d\theta + g_2(r), \end{aligned} \quad (16)$$

where $g_2(r)$ is a function of r only.

Differentiating equation 14 with respect to θ , gives

$$\frac{\partial u}{\partial \theta} = \frac{1}{E} \left[-S \left(r + \frac{4a^2}{r} - \frac{a^4}{3r} \right) \sin 2\theta \right] - \frac{\nu S}{E} \left(r - \frac{a^4}{3r} \right) \sin 2\theta + \frac{\partial g_1(\theta)}{\partial \theta}. \quad (17)$$

Differentiating equation 16 with respect to r , gives

$$\begin{aligned} \frac{\partial v}{\partial r} = \frac{1}{E} \left[-\frac{S}{2} \left(1 - \frac{2a^2}{r^2} - \frac{3a^4}{r^4} \right) \sin 2\theta \right] \\ - \frac{\nu}{E} \left[\frac{S}{2} \left(1 + \frac{2a^2}{r^2} - \frac{3a^4}{r^4} \right) \sin 2\theta \right] + \frac{\partial g_2(r)}{\partial r}. \end{aligned} \quad (18)$$

Substituting equations 9, 16, 17, and 18 into equation 12 and collecting terms gives

$$\frac{\partial g_1(\theta)}{\partial \theta} + \frac{r \partial g_2(r)}{\partial r} + \int g_1(\theta) d\theta - g_2(r) = 0 \quad (19)$$

Since $g_1(\theta)$ is a function of θ only, and $g_2(r)$ is a function of r only, the terms containing θ and the terms containing r must be equal to zero, thus

$$\frac{r \, dg_2(r)}{dr} - g_2(r) = 0, \quad (20)$$

$$\frac{d^2 g_1(\theta)}{d\theta^2} + g_1(\theta) = 0, \quad (21)$$

where total derivatives are used because g_2 is a function of r only, and g_1 is function of θ only. Integrating equation 20 gives

$$g_2(r) = Cr. \quad (22)$$

Integrating equation 21 gives

$$g_1(\theta) = A \sin \theta + B \cos \theta. \quad (23)$$

As there can be no tangential displacements on either the x or y axis the boundary conditions are

$$v = 0 \text{ when } \theta = 0 \text{ for all values of } r;$$

$$v = 0 \text{ when } \theta = \frac{\pi}{2} \text{ for all values of } r.$$

Substituting equations 22 and 23 and the boundary condition into equation 16 gives

$$A = 0, B = 0, C = 0.$$

Because the constants in equations 22 and 23 are zero, the arbitrary functions $g_1(\theta)$ and $g_2(r)$ are zero; thus, the displacements u and v , from equations 14 and 16, are

$$u = \frac{1}{E} \left[\frac{S}{2} \left(r + \frac{a^2}{r} \right) + \frac{S}{2} \left(r + \frac{4a^2}{r} - \frac{a^4}{r^3} \right) \cos 2\theta \right] \\ - \frac{\nu}{E} \left[\frac{S}{2} \left(r - \frac{a^2}{r} \right) - \frac{S}{2} \left(r - \frac{a^4}{r^3} \right) \cos 2\theta \right], \quad (24)$$

$$v = - \frac{1}{E} \left[\frac{S}{2} \left(r + \frac{2a^2}{r} + \frac{a^4}{r^3} \right) \sin 2\theta \right] \\ - \frac{\nu}{E} \left[\frac{S}{2} \left(r - \frac{2a^2}{r} + \frac{a^4}{r^3} \right) \sin 2\theta \right]. \quad (25)$$

Equations 24 and 25 describe radial and tangential displacements anywhere in the medium. To obtain the change in diameter, U , of the hole, (deformation) substitute $r = a = \frac{d}{2}$, into equation 24, thus

$$U = \frac{Sd}{E}(1 + 2 \cos 2\theta) . \quad (26)$$

Part A, Case 2 - Deformation of a hole in a plate subjected to a unidirectional stress under the condition of plane strain.

To obtain the deformation of a hole in a plate in plane strain the Kirsch equations (equations 7, 8, and 9) are substituted in the Hookes' law equations for plane strain which are

$$e_r = \frac{1}{E} [(1 - \nu^2) \sigma_r - \nu(1 + \nu) \sigma_\theta] , \quad (27)$$

$$e_\theta = \frac{1}{E} [(1 - \nu^2) \sigma_\theta - \nu(1 + \nu) \sigma_r] , \quad (28)$$

$$\lambda_{r\theta} = \frac{2(1 + \nu)}{E} \lambda_{r\theta} . \quad (29)$$

Equations 1, 2, 3, 7, 8, and 9, are valid under conditions of both plane stress or plane strain. Hookes' law equations for plane strain differ from the plane stress equations only by the terms $(1 - \nu^2)$ and $(1 + \nu)$. Therefore, the method of solution and the boundary conditions are valid for both cases and the radial and tangential displacements anywhere in the medium are

$$u = \frac{1 - \nu^2}{E} \left[\frac{S}{2} \left(r + \frac{a^2}{r} \right) + \frac{S}{2} \left(r + \frac{4a^2}{r} - \frac{a^4}{r^3} \right) \cos 2\theta \right] \\ - \frac{\nu(1 + \nu)}{E} \left[\frac{S}{2} \left(r - \frac{a^2}{r} \right) - \frac{S}{2} \left(r - \frac{a^4}{r^3} \right) \cos 2\theta \right] , \quad (30)$$

$$v = - \frac{1 - \nu^2}{E} \left[\frac{S}{2} \left(r + \frac{2a^2}{r} + \frac{a^4}{r^3} \right) \sin 2\theta \right] \\ - \frac{\nu(1 + \nu)}{E} \left[\frac{S}{2} \left(r - \frac{2a^2}{r} + \frac{a^4}{r^3} \right) \sin 2\theta \right] . \quad (31)$$

To obtain U , $r = a = d/2$ is substituted into equation 30, thus

$$U = \frac{Sd}{E} (1 - \nu^2) (1 + 2 \cos 2\theta) .$$

Part A, Case 3 - deformation of a hole in a plate subjected to a biaxial stress field and for the condition of plane stress.

The equations for the stress distribution around a hole are obtained by superposition and equations 7, 8, and 9, become

$$\sigma_r = \frac{S+T}{2} \left(1 - \frac{a^2}{r^2}\right) + \frac{S-T}{2} \left(1 - \frac{4a^2}{r^2} + \frac{3a^4}{r^4}\right) \cos 2\theta , \quad (32)$$

$$\sigma_\theta = \frac{S+T}{2} \left(1 + \frac{a^2}{r^2}\right) - \frac{S-T}{2} \left(1 + \frac{3a^4}{r^4}\right) \cos 2\theta , \quad (33)$$

$$\tau_{r\theta} = - \frac{S-T}{2} \left(1 + \frac{2a^2}{r^2} - \frac{3a^4}{r^4}\right) \sin 2\theta . \quad (34)$$

The solution for the displacements is obtained by the same method as used for Case 1; the displacements are

$$u = \frac{1}{E} \left[\frac{S+T}{2} \left(r + \frac{a^2}{r}\right) + \frac{S-T}{2} \left(r + \frac{4a^2}{r} - \frac{a^4}{r^3}\right) \cos 2\theta \right] \\ - \frac{\nu}{E} \left[\frac{S+T}{2} \left(r - \frac{a^2}{r}\right) - \frac{S-T}{2} \left(r - \frac{a^4}{r^3}\right) \cos 2\theta \right] , \quad (35)$$

$$v = - \frac{1}{E} \left[\frac{S-T}{2} \left(r + \frac{2a^2}{r} + \frac{a^4}{r^3}\right) \sin 2\theta \right] \\ - \frac{\nu}{E} \left[\frac{S-T}{2} \left(r - \frac{2a^2}{r} + \frac{a^4}{r^3}\right) \sin 2\theta \right] . \quad (36)$$

To obtain the deformation of the hole, $r = a = d/2$ is substituted in equation 35, thus

$$U = \frac{d}{E} [(S + T) + 2(S - T) \cos 2\theta] . \quad (37)$$

Part A, Case 4 - deformation of a hole in a plate subjected to biaxial stress and for the conditions for plane strain.

The equations for the stress distributions around a hole in a plate subjected to a biaxial stress field, (equations 32, 33, and 34,) are substituted in equations 27, 28, and 29. Then, the procedures outlined under Case 1 are followed to obtain

$$U = \frac{(1 - \nu^2)d}{E} [(S + T) + 2(S - T) \cos 2\theta] . \quad (38)$$

Part B, Case 1 - 60-degree-deformation-rosette-equations.

The equations for the deformation of a hole in a plate in biaxial stress fields are used to relate the stresses in the plate to the deformations measured across three diameters of the hole. It is assumed that the principal stresses P and Q can be substituted for the stresses S and T in equations 37 and 38.

The symbols for Part A are the same in Part B; the following symbols are also used:

U_1 = deformation measured across a diameter in the hole,

U_2 = deformation at angle 60 degrees from U_1 ,

U_3 = deformation at angle 120 degrees from U_1 ,

θ_1 = angular direction of U_1 with respect to the maximum principal stress P, ^{4/}

K = constant,

A = constant,

B = constant.

^{4/} See figure 3 for a 60-degree-deformation-rosette.

The equations for the deformation of a hole in a plate in a biaxial stress field is (see equation 37)

$$U = \frac{d}{E} [(S + T) + 2(S - T)\cos 2\theta] .$$

If we let $K = d/E$, $A = P + Q$, $B = P - Q$, and substitute P and Q for S and T , equation 37 becomes

$$U = KA + 2KB \cos 2\theta . \quad (39)$$

To obtain U_1 , U_2 , and U_3 ,

$$U_1 = KA + 2KB \cos 2\theta_1 , \quad (40)$$

$$U_2 = KA + 2KB \cos 2\theta_2 , \quad (41)$$

$$U_3 = KA + 2KB \cos 2\theta_3 , \quad (42)$$

For a 60-degree-rosette, $\theta_2 = \theta_1 + 60^\circ$ and $\theta_3 = \theta_1 + 120^\circ$; and equations 40, 41, and 42, become

$$U_1 = KA + 2KB \cos 2\theta_1 , \quad (43)$$

$$U_2 = KA + 2KB \cos 2(\theta_1 + 60^\circ) , \quad (44)$$

$$U_3 = KA + 2KB \cos 2(\theta_1 + 120^\circ) , \quad (45)$$

The cosine of the sum of the angles in equations 44 and 45 is

$$\cos 2\theta_1 + 120^\circ = -\frac{1}{2} \cos 2\theta_1 - \frac{\sqrt{3}}{2} \sin 2\theta_1 , \quad (46)$$

$$\cos 2\theta_1 + 240^\circ = -\frac{1}{2} \cos 2\theta_1 + \frac{\sqrt{3}}{2} \sin 2\theta_1 . \quad (47)$$

Substituting equations 46 and 47 in equations 44 and 45 and solving for A and B gives

$$A = \frac{U_1 + U_2 + U_3}{3K} , \quad (48)$$

$$B = \frac{\sqrt{2}}{6K} [U_1 - U_2]^2 + (U_2 - U_3)^2 + (U_1 - U_3)^2]^{1/2} . \quad (49)$$

Substituting equations 46 and 47 in equations 43, 44, and 45, and solving for θ_1 gives

$$\tan 2\theta_1 = \frac{-\sqrt{3}(U_2 - U_3)}{2U_1 - U_2 - U_3} \quad (50)$$

Because $A = P + Q$ and $B = P - Q$, the sum and difference of the principal stresses are

$$P + Q = \frac{E}{3d} (U_1 + U_2 + U_3), \quad (51)$$

$$P - Q = \frac{\sqrt{2}E}{6d} [U_1 - U_2]^2 + (U_2 - U_3)^2 + (U_1 - U_3)^2]^{1/2} \quad (52)$$

Hence, the principal stresses in the medium can be obtained from a measurement of U_1 , U_2 , and U_3 . The angle between the principal stress P and the angle at which U_1 is measured is obtained from equation 50.

The conditions for plane strain can be obtained by dividing equations 51 and 52 by the term $(1 - \nu^2)$.

Part B, Case 2 - 45-degree-deformation-rosette-equations.

The solution for the 45-degree-deformation-rosette is obtained by the same steps as were used for the solution of the 60-degree-deformation-rosette. The solution for the sum and difference of the principal stresses are

$$P + Q = \frac{E}{2d} (U_1 + U_3), \quad (53)$$

$$P - Q = \frac{E}{2\sqrt{2}d} [(U_1 - U_2)^2 + (U_2 - U_3)^2]^{1/2}, \quad (54)$$

and the tangent of twice the angle between the principal stress P and the deformation U_1 is

$$\tan 2\theta_1 = \frac{2U_2 - U_1 - U_3}{U_1 - U_3} \quad (55)$$

Again, for plane strain, equations 53 and 54 can be used by dividing by the term $(1 - \nu^2)$.

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