The Effect of Higher Systematic Multipoles in the High Beta Quadrupoles and Dipoles on the Dynamic Aperture

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1. Introduction

There is some evidence that the higher systematic multipoles in the high beta magnets may cause some loss in dynamic aperture in RHIC. For the assumptions made below, $b_{13}$ and $b_{17}$ in the high beta quadrupoles seem particularly important. It is suggested that in the design of the high beta magnets, some special attention be paid to the effects of the higher systematic multipoles in the high beta magnets; this means, $b_{13}$ and $b_{17}$ in the high beta quadrupoles and $b_{10}, b_{12}, b_{14}, b_{16}, b_{18}$ in the DO dipole in RHIC.

In the design of magnets, the adjustable parameters in the coil design are usually primarily used to control the lower systematic multipoles. As a result, the higher systematic multipoles can be quite large, about 10 times larger than the corresponding random multipoles. These results are based on some previous computer results of P. Thompson and some recent results of R. Gupta. These large systematic multipoles were also observed in the FNAL Tevatron dipoles. The results reported below indicate that $b_{13}$ and $b_{17}$ in the high beta quadrupoles may cause a loss in the dynamic aperture of about 2 mm for the RHIC92 lattice with $6 \beta^* = 6$ insertions.

2. Computed Results

Figure 1 plots the dynamic aperture, $A_{SL}$, as a function of the size of the systematic $b_{13}$ and $b_{17}$ in the quadrupoles. The abscissa, $b_{13}, b_{17}$, is to be interpreted as follows: $b_{13}, b_{17} = 1$ means that $b_{13}, b_{17}$ were set at $q'_{13} = 0.4, q'_{17} = -0.24$, in quadrupole units,
which were the values found by P. Thompson for $b_{13}, b_{17}$ for one particular design of the QF,QD quadrupoles. $b_{13}, b_{17} = 4$ means that $b_{13}, b_{17}$ were increased by a factor 4 over the values they have for $b_{13}, b_{17} = 1$.

\[ b_{13}, b_{17} = 4 \]

Fig. 1: Dynamic aperture versus the systematic $b_{13}, b_{17}$.

The values of $b_{13}, b_{17}$ for $b_{13}, b_{17} = 1$ are about 10 times larger than the corresponding expected random multipoles in the quadrupoles. One sees from Fig. 1 that $b_{13}, b_{17}$ have caused a loss of 2 mm in the dynamic aperture, from $A_{SL} = 18.5$ mm to $A_{SL} = 16.5$ mm. This study used 1000 turn runs for a RHIC92 lattice with $6 \beta^* = 6$ insertions. In this study $b_{13}, b_{17}$ were varied in all the quadrupoles. However, it is assumed that it is the $b_{13}, b_{17}$ in the high beta quadrupoles that is affecting the dynamic aperture.

The effects of the higher systematic multipoles in the dipoles is shown in Fig. 2. The work done in a previous study\(^4\) suggested that with the choice of 5.5 cm for the coil radius of the DO magnet, the higher systematic multipoles in DO would not affect the dynamic aperture with the values used there for the higher systematic multipoles in the dipoles. In
Computed Results

Fig. 2 the size of $b_{12}, b_{14}, b_{16}, b_{18}$ in all the dipoles is varied while $b_{13}, b_{17}$ in the quadrupoles were held constant. From this plot one may conclude that the $b_{12}, b_{14}, b_{16}, b_{18}$ in the dipoles may not be affecting the dynamic aperture unless they get very large.

![Graph showing dynamic aperture versus systematic $b_{12}, b_{14}, b_{16}, b_{18}$](image)

**Fig. 2:** Dynamic aperture versus the systematic $b_{12}, b_{14}, b_{16}, b_{18}$.

A study done with $6 \beta^* = 2$ insertions gave similar results to those found for $6 \beta^* = 6$ insertions, except that the loss in dynamic aperture scales roughly with the size of the dynamic aperture.
3. Comparison of the Systematic and Random Multipoles

For estimating the higher random multipoles, it may be appropriate to use analytical results. Some simple rough analytical results for the rms random multipoles are

\[
b'_n = \left( \frac{2}{N_b} \right)^{\frac{1}{2}} \frac{n + 1}{2} \frac{\epsilon}{R_0} \left( \frac{R_0}{R} \right)^n, \quad \text{dipoles}
\]

\[
c'_n = \left( \frac{2}{N_b} \right)^{\frac{1}{2}} \frac{n + 1}{2} \frac{\epsilon}{R_0} \left( \frac{R_0}{R} \right)^n, \quad \text{quadrupoles}
\]

(3.1)

\(N_b\) is the total number of current blocks in the magnet; \(N_b = 12\) for the RHIC dipoles. \(\epsilon\) is the rms error in the location of the current blocks; \(\epsilon \simeq 0.005\) cm. \(R\) is the average radius of the coil and \(R_0\) is the radius where the multipoles are measured.

For the high beta quads in RHIC, Eq. (3.1) gives for the rms random multipoles

\[
q'_{13} = 0.051, \quad \text{random}
\]

\[
q'_{17} = 0.0088.
\]

(3.2)

This is to be compared with the results of R. Gupta for the systematic \(q'_{13}, q'_{17}\)

\[
q'_{13} = 0.3, \quad \text{systematic}
\]

\[
q'_{17} = 0.08.
\]

(3.3)

The systematic \(q'_n\) are a factor 6 to 10 larger than the corresponding random \(q'_n\).

References

1. P. Thompson, private communication.


