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NO-GO THEOREMS FOR DUAL MODELS

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ABSTRACT

Two no-go theorems relevant to the construction of positive-intercept dual models are stated and proved. The theorems explain why one cannot develop a physically sensible model by a simple multiplicative modification of the usual (negative-intercept) N particle dual amplitude.

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In this note we state and prove two impossibility theorems concerning the construction of a dual resonance model with positive intercept Regge trajectories.

In attempts to satisfy superconvergence relations it is a familiar difficulty¹⁾ that there appear ghost states with imaginary coupling constants or imaginary masses or both. It is clear that imaginary coupling constant ghosts are present in the operator factorisation of the N particle dual amplitude of Fubini, Gordon and Veneziano²⁾ and of Nambu³⁾ because of the indefinite metric. Virasoro⁴⁾ has pointed out that when the Regge intercept is equal to one all of the wrong-metric components may become redundant so that it is possible to conjecture that no states with imaginary coupling constants remain in the theory. Because of the positive intercept, however, there remains one imaginary mass state with spin zero. In view of the earlier difficulties there is the suspicion that superconvergence relations are satisfied by virtue of a large compensation by the single ghost state. The theorems proved below indicate the great difficulties in removing the imaginary mass pole.

Another reason for studying positive intercept dual amplitudes, in addition to the fact that of course many physical trajectories have positive intercepts, is the success of the amplitude proposed by Veneziano⁵⁾, Shapiro and Yellin⁶⁾, and Lovelace⁷⁾ to describe elastic $\pi^+\pi^-$ scattering,

$$A(s,t) = - (1+\alpha(s)+\alpha(t)) B_4(\alpha(s)\alpha(t)), \quad (1)$$

where we define $\alpha(s) = \alpha(s)-1$ to conform with the notation adopted below. In order to find the level structure of this model we need to generalise Eq. (1) to an amplitude for N pions. Let us suppose that we may generalise Eq. (1) by writing

$$A_N = F_N(p_i) B_N \quad (2)$$

where $F_N(p_i)$ is some Lorentz scalar function formed from the external momenta p_i , and B_N is the N particle generalised Veneziano amplitude. If such a form is attainable we expect (i) that the degeneracy will be the same asymptotically at high energies as in the generalised Veneziano model; (ii) that the wrong-metric components can be made redundant for special cases exactly as in Reference 4.

Our object here is to show how the constraints of cyclic symmetry and factorisation severely restrict the possible forms of $F_N(p_i)$ in Eq. (2).

Consider the amplitude for six pions. We shall take isoscalar pions since isospin (or SU(3)) can be introduced later by the Chan-Paton procedure⁸⁾ (or its straightforward generalisation to SU(3)). In a three-pion channel a scalar resonance is forbidden by parity conservation but pseudoscalar, vector and axial-vector resonances are allowed.

For the kinematics of the six pion amplitude we take $p_i^2 = m^2$ for $i = 1, 2, \dots, 6$; all external momenta inward so that $\sum p_i = 0$; the shorthand $p_{ij} = p_i^\mu p_{j\mu}$ is used; there are eight independent energy variables which we shall take as

α_{123} , α_{234} , the trajectory functions for the two three-pion channels, together with p_{12} , p_{23} , p_{34} , p_{45} , p_{56} and p_{61} ; the third three-pion trajectory function α_{345} can be related to the eight independent variables by using a Gram determinant constraint.

We consider three possibilities for the spin-parity structure of the resonance at the lowest energy pole in a three-pion channel of the six-pion amplitude: (a) pseudoscalar only; (b) pseudoscalar and vector; (c) pseudoscalar, vector and axial-vector.

(a) Pseudoscalar only. In this case we can prove the following theorem.

THEOREM I: Let

$A_6 = F_6(p_i) B_6(\alpha_{12}, \alpha_{23}, \alpha_{34}, \alpha_{45}, \alpha_{56}, \alpha_{61}, \alpha_{123}, \alpha_{234}, \alpha_{345})$ be a six pion amplitude which factorizes at the poles $\alpha_{123} = 0$ and $\alpha_{234} = 0$ into a form corresponding to the existence of internal pseudoscalar particles only. Thus when

$$\alpha_{123} \approx 0,$$

$$A_6 \approx \frac{1}{\alpha_{123}} B_4(\alpha_{12}, \alpha_{23}) \left\{ \sum_{i=1}^P P^i(p_{12}, p_{23}) P^i(p_{45}, p_{56}) \right\} B_4(\alpha_{45}, \alpha_{56})$$

while its behaviour when $\alpha_{234} \approx 0$ is obtained by a permutation of the variables. Then $P^i(x, y)$ is independent of x and y for all i .

PROOF OF THEOREM I: Consider the six pion amplitude

$$A_6 = F_6(p_i) B_6(\alpha_{12}, \alpha_{23}, \alpha_{34}, \alpha_{45}, \alpha_{56}, \alpha_{61}, \alpha_{123}, \alpha_{234}, \alpha_{345})$$

and impose factorization at the poles at $\alpha_{123} = 0$ and $\alpha_{234} = 0$.

B_6 already factorizes into a product of two B_4 's so that we must require that $F_6(p_i)$ factorizes separately. Thus we must have

$$\begin{aligned} F_6(p_i) &= \sum_{i=1}^{\nu_P} P^i(p_{12}, p_{23}) P^i(p_{45}, p_{56}) + \alpha_{123} F_6^{(123)}(p_i) \\ &= \sum_{i=1}^{\nu_P} P^i(p_{23}, p_{34}) P^i(p_{56}, p_{61}) + \alpha_{234} F_6^{(234)}(p_i), \end{aligned} \quad (3)$$

where $F_6^{(ijk)}(p_i)$ remains finite when $\alpha_{ijk} \rightarrow 0$. Now put $\alpha_{123} = \alpha_{234} = 0$ in (3) to find

$$\sum_{i=1}^{\nu_P} P^i(p_{12}, p_{23}) P^i(p_{45}, p_{56}) = \sum_{i=1}^{\nu_P} P^i(p_{23}, p_{34}) P^i(p_{56}, p_{61}). \quad (4)$$

In Eq. (4), all the six variables $p_{12}, p_{23}, p_{34}, p_{45}, p_{56}$ and p_{61} are independent. Since p_{12} does not occur on the right hand side of this equation, it follows that the left-hand side is independent of p_{12} . Thus

$$\mu(x_1, x_2, x_3, x_4) = \sum_{i=1}^{\nu_P} P^i(x_1, x_2) P^i(x_3, x_4) \quad (5)$$

does not depend on x_1 . As a consequence the right hand side of Eq. (4) is independent of p_{23} and therefore so is the left hand side. That is, $\mu(x_1, x_2, x_3, x_4)$ is independent of x_2 as well. An identical argument shows that $\mu(x_1, x_2, x_3, x_4)$ is independent

of x_3 and x_4 also so that it is a constant which we may denote simply by μ . Now from (5) we find on setting $x_1 = x_3$, $x_2 = x_4$,

$$\mu = \sum_{i=1}^{\nu} \left\{ P^i(x_1, x_2) \right\}^2 \quad (6)$$

We show next that (5) and (6) imply that $P^i(x, y)$ does not depend on x or y . Schwartz's inequality applied to (5) gives

$$\begin{aligned} \mu = \sum_{i=1}^{\nu} P^i(x_1, x_2) P^i(x_3, x_4) &\leq \left\{ \sum_{i=1}^{\nu} [P^i(x_1, x_2)]^2 \right\}^{1/2} \times \\ &\times \left\{ \sum_{i=1}^{\nu} [P^i(x_3, x_4)]^2 \right\}^{1/2} = \mu, \end{aligned} \quad (7)$$

so that the inequality is actually an equality.

Therefore,

$$P^i(x_1, x_2) = \lambda(x_1, x_2, x_3, x_4) P^i(x_3, x_4), \quad (8)$$

where λ does not depend on i . Substituting (8) in (5) and using (6), we find $\lambda = 1$. The two sides of Eq. (8) now involve different sets of variables so that $P^i(x, y)$ cannot depend on x or y . Q.E.D.

Concerning the proof of Theorem I, we remark that (i) it includes the possibility of generalizing the familiar four pion amplitude Eq. (1) into an N -pion amplitude of the form Eq. (2) in a consistent manner; (ii) the form of $F_6(p_i)$ is not explicitly needed in the proof so that the theorem applies regardless of whether $F_6(p_i)$ is taken to be a polynomial or not.

(b) Pseudoscalar and Vector. We can extend the result of (a) to the following:

THEOREM II: Let $A_6 = F_6(p_i)B_6$ be a six pion amplitude which factorizes at the poles $\alpha_{123} = 0$ and $\alpha_{234} = 0$ into a form corresponding to the contributions due to internal pseudoscalar and vector particles. Then the vector particles must actually be absent while the contributions due to the pseudoscalar particles are subject to the constraints stated in Theorem I.

PROOF OF THEOREM II: Considering successively the residues at $\alpha_{123} = 0$ and $\alpha_{234} = 0$ now gives

$$\begin{aligned}
 F_6 &= \sum_{i=1}^{\nu_v} V^i(p_{12}, p_{23}) V^i(p_{45}, p_{56}) D_{456}^{123} + \sum_{i=1}^{\nu_p} P^i(p_{12}, p_{23}) \times \\
 &\times P^i(p_{45}, p_{56}) + \alpha_{123} F_6^{(123)} \\
 &= \sum_{i=1}^{\nu_v} V^i(p_{23}, p_{34}) V^i(p_{56}, p_{61}) D_{561}^{234} + \sum_{i=1}^{\nu_p} P^i(p_{23}, p_{34}) \times \\
 &\times P^i(p_{56}, p_{61}) + \alpha_{234} F_6^{(234)}, \quad (9)
 \end{aligned}$$

in which

$$D_{pqr}^{lmn} = \begin{vmatrix} p_{lm} & p_{lq} & p_{lr} \\ p_{mp} & p_{mq} & p_{mr} \\ p_{np} & p_{nq} & p_{nr} \end{vmatrix} \quad (10)$$

is the spin factor which signifies the presence of the l^{th} pole.

In Eq. (9) we put $\alpha_{123} = \alpha_{234} = 0$ and $p_1^\mu = p_2^\mu$. This leaves three independent variables which we take as p_{45} , p_{56} , p_{61} in

$$\begin{aligned} & \sum_{i=1}^{\nu_P} P^i(m^2, k^2) P^i(p_{45}, p_{56}) = \\ & \sum_{i=1}^{\nu_V} V^i(k^2, M^2 - m^2 - p_{56}) V^i(p_{56}, p_{61}) [a(p_{56}, p_{61}) + b(p_{56}) p_{45}] + \\ & + \sum_{i=1}^{\nu_P} P^i(k^2, M^2 - m^2 - p_{56}) P^i(p_{56}, p_{61}) , \end{aligned} \quad (11)$$

in which $M^2 = -\alpha_{123}(0)/\alpha'$ and $k^2 = (M^2 - 5m^2)/4$. We have displayed the fact that in this limit D_{561} is linear in p_{45} with coefficient $b(p_{56})$

$$b(p_{56}) = -(1/4)[M^4 - 6M^2 m^2 + m^4 - 4M^2 p_{56} + 4m^2 p_{56} + 4p_{56}^2] . \quad (12)$$

We now impose the restriction

$$\sum_{i=1}^{\nu_V} V_i(k^2, M^2 - m^2 - p_{56}) V_i(p_{56}, p_{61}) \neq 0 \quad (13)$$

in Eq. (11). If this restriction is relaxed then the vector particles decouple in the limit $p_1^\mu = p_2^\mu$; this special case can be handled by keeping all momenta unequal and assigning values to the arguments in Eq. (9).

Putting $p_5^\mu = p_6^\mu$ in Eq. (9) gives

$$\sum_{i=1}^{\nu_P} P^i(m^2, k^2) P^i(m^2, k^2) = \sum_{i=1}^{\nu_P} P^i(m^2, k^2) P^i(M^2 - 2m^2, k^2) \quad (14)$$

where we used the symmetry $P^1(xy) = P^1(yx)$ implied by the isoscalar nature of the particles. Putting on the other hand $p_{56} = k^2$ in Eq. (11) shows that the expression

$$\sum_{i=1}^{\nu_P} P^1(m^2, k^2) P^1(p_{45}, k^2) \quad (15)$$

is linear in p_{45} , the coefficient of p_{45} being proportional to

$$b(k^2) = - (1/16)(M^2 - m^2)(M^2 - 9m^2). \quad (16)$$

If $b(k^2) \neq 0$ and $M^2 \neq 3m^2$, Eq. (16) contradicts Eq. (14). The special cases where $b(k^2) = 0$ or $M^2 = 3m^2$ can be handled as follows. For $M^2 = 3m^2$ put $p_{2\mu} = p_{3\mu}$, $\alpha_{123} = \alpha_{234} = 0$ in Eq. (11) to find that

$$\sum_{i=1}^{\nu_P} P^1(m^2, k^2) P^1(p_{45}, p_{56}) \quad (17)$$

is independent of p_{45} , which contradicts Eq. (11). For the threshold situation $M=3m$ put $p_{56} = p_{61} = m^2$ in Eq. (10) to find $a(m^2, m^2) = b(m^2) = 0$ and hence

$$\sum_{i=1}^{\nu_P} P^1(m^2, m^2) P^1(m^2, p_{45}) = \text{constant}. \quad (18)$$

Now, putting $\alpha_{123} = \alpha_{234} = 0$, $p_2^\mu = p_3^\mu$, $p_{61} = M^2$ in Eq. (11) and using Eq. (18) gives that the expression, Eq. (17), is a constant. Returning to Eq. (11), if we choose a value to p_{56} such that

$$b(p_{56}) = - (p_{56} - m^2)(p_{56} - 7m^2) \quad (19)$$

is non-zero then Eqs. (11) and (18) are contradictory.

For the mass degenerate case $M=m$, put $p_{56} = -M^2$ in Eq. (11) to find

$$\sum_{i=1}^{\nu_P} p^i(M^2, -M^2) p^i(-M^2, p_{45}) = \text{constant}. \quad (20)$$

Putting $\alpha_{123} = \alpha_{234} = 0$, $p_2^\mu = p_3^\mu$ and $p_{61} = -M^2$ in Eq. (9) and using Eq. (20) gives

$$\sum_{i=1}^{\nu_P} p^i(M^2, -M^2) p^i(p_{45}, p_{56}) = \text{constant}. \quad (21)$$

Finally, choosing p_{56} such that

$$b(p_{56}) = p_{56}^2 - M^4 \quad (22)$$

is non-zero, we find a contradiction between Eqs. (11) and (21).

Q.E.D.

(c) Pseudoscalar, vector, and axial-vector. For the axial-vector there are two independent couplings to three pseudoscalars. Inclusion of axial-vector contributions gives an additional term

$$\begin{aligned} & \sum_{i=1}^{\nu_A} [A_1^i(p_{12}, p_{23})(p_1+p_2)_\mu + A_2^i(p_{12}, p_{23})(p_1-p_2)_\mu] \times \\ & \times [g_{\mu\mu'} - \frac{(p_1+p_2+p_3)_\mu (p_1+p_2+p_3)_{\mu'}}{M^2}] [A_1^i(p_{45}, p_{56})(p_4+p_5)_{\mu'} + \\ & + A_2^i(p_{45}, p_{56})(p_4-p_5)_{\mu'}] \end{aligned} \quad (23)$$

in the first expression for F_6 in Eq. (9) above and a similar term in the second expression. The analogue of Theorem I for case (a) and Theorem II for case (b) is incorrect for case (c) since we can give an explicit counterexample. The counter example is provided by

$$F_6(p_i) = \text{Tr}(\underline{p}_1 \underline{p}_2 \underline{p}_3 \underline{p}_4 \underline{p}_5 \underline{p}_6) \quad (24)$$

with $\underline{p}_i = \gamma^\mu p_{i\mu}$. Equation (24) may be obtained from the model of Bardakci and Halpern⁹⁾ and Mandelstam¹⁰⁾ (BHM). The BHM model possesses an $SU(6,6) \times O(3)$ mass spectrum; in particular its quark spin factor possesses $U(2,2)$ symmetry giving rise at $\alpha_{123} = 0$ and $\alpha_{234} = 0$ in the six pion amplitude to two pseudoscalars, two vectors, and one axial-vector. The mass spectrum of the model is very unphysical, corresponding to $M_\pi = M_\rho = M_{A_1}$ and although imaginary mass states are avoided half of the states have imaginary coupling constants.

We conjecture, however, that the systematic inclusion of quark spin, as in the BHM model, is the only possibility for writing a positive intercept amplitude in the form of Eq. (2). Breaking the unphysical high symmetry is clearly very difficult without the addition of more terms (compare footnote 18 of Reference 9). For example, we have proved in (b) above that one cannot, while preserving the form of Eq. (2), remove parity doubling in the BHM model so as to keep only vector and pseudoscalar contributions at the ground state pole.

We conclude by noting that approaches of other authors to the positive intercept problem violate the starting assumption

of our no-go theorems either (i) by breaking exact Veneziano duality for narrow resonances, or (ii) by taking a superposition of terms of the form F_N . As an example of the first alternative, Carlitz-Kislinger fixed J-plane cuts¹¹⁾ have been incorporated into an N particle dual amplitude by Carlitz, Ellis, Freund and Matsuda¹²⁾, who construct an ingenious model similar to the BHM model but with no parity doubling and with duality properties essentially different from those of the Veneziano model; the mass spectrum is still unphysical with $M_\pi = M_\rho$. A good example of a superposition of terms is the proposal by Olive and Zakrzewski¹³⁾ of an N pion amplitude which satisfies the bootstrap condition at internal pion poles and which reduces to the expected four pion amplitude^{5,6,7)}; the model contains a superposition of satellite terms and consequently has a higher asymptotic degeneracy¹⁴⁾ and more ghosts than the Veneziano model. Superpositions of terms have been considered also by Montvay¹⁵⁾, by Green and Heimann,¹⁶⁾ and by Chaichian, Dorren, Rittenberg, Rubinstein and Squires¹⁷⁾ and by Panchapakesan¹⁸⁾.

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