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ELECTRON IMPACT EXCITATION OF HEAVILY IONIZED ATOMS

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TO MY WIFE, BATOOL, AND MY PARENTS

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ELECTRON IMPACT EXCITATION OF HEAVILY IONIZED ATOMS

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ABSTRACT

We use the Coulomb-Born approximation to calculate the total cross section for the electronic excitation of ionized atoms. When the incident energy is high and the atom is highly ionized we can use the eikonal and classical approximations simultaneously. In this case the scattering amplitude reduces to a one dimensional path integral. The path integral, in turn, can be reduced, to any desired accuracy, to a sum over integrals which can be done analytically. Using this procedure, we have calculated the total cross section for the heavily ionized atoms Fe^{15+} , Co^{16+} , Ni^{17+} , and Cu^{18+} .

K. M. Watson

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INTRODUCTION

Electron impact excitation of various types of atomic ions has been examined in different approximations¹ with or without the inclusion of exchange. Most of the calculations are done in the Coulomb-Born approximation, however. The Coulomb-Born approximation differs from the ordinary Born approximation in that plane waves are replaced by Coulomb waves. The Coulomb-Born approximation for the excitation of positive ions should yield more accurate results, especially for high values of ionic charge Z , than the ordinary Born approximation for the excitation of neutral atoms. This follows from the fact that for high Z the perturbation potential, which is taken to be the interelectronic interaction, is much smaller than the Coulomb potential of the ion.

Consider an ionized atom with one electron in its external shell. As we shall show later, the Coulomb-Born amplitude, without exchange, has the form:

$$T_{fi}^{C.B.} = \int d^3r X_{\vec{k}}^{(-)*}(\vec{r}) V_{fi}(\vec{r}) X_{\vec{k}}^{(+)}(\vec{r})$$

where $X_{\vec{k}}^{(-)}$ and $X_{\vec{k}}^{(+)}$ are the Coulomb wave functions representing the incident and scattered electrons respectively. Here the "effective" interelectronic interaction V_{fi} is defined by:

$$V_{fi} \equiv \int U_f^*(\vec{r}') \frac{1}{|\vec{r} - \vec{r}'|} U_i(\vec{r}') d^3r'$$

where U_i and U_f are, respectively, the initial state and final state wave functions of the bound electron.

The use of the Coulomb-Born approximation alone is not enough, in general, to reduce the above expression for the scattering amplitude to an analytical form. Further approximations are necessary to accomplish this. Usually, one expands the wave functions and the interelectronic interaction in terms of the spherical harmonics and then integrates the resulting expression numerically. However, those who have pursued this way have confined themselves to the dipole term in the expansion of the interelectronic potential. According to an estimate by Burgess,² the dipole term, at moderate energies, accounts for about two-thirds of the total cross section. Using this procedure, with some variations in details, electron impact excitation cross sections have been calculated³ for hydrogenic ions by Tully (1960) and Burgess (1961), for He^+ by Tully (1960), for lithium-like ions (Be^+ , C^{3+} , O^{5+} , and Mg^{9+}) by Belly, Tully, and Van Regemorter (1963), for sodium-like ions (Mg^+ , Si^{3+} , and Fe^{15+}) by Belly, Tully, and Van Regemorter (1963); also by Kreueger and Czyzak (1965), and for potassium-like ions (Ca^+) by Van Regemorter (1960, 1961). Except for the case of $1s \rightarrow 2s$ transition in He^+ , there are no experimental data to compare with these calculations. In the case of He^+ where experimental data is available, the theoretical threshold behavior is different from the experimental one.

Another approximation frequently used in conjunction with the Coulomb-Born approximation is to assume that the main contribution to the cross section comes from optically allowed transitions. Furthermore, the potential V_{ji} is replaced by its asymptotic limit at large interelectron separations:

$$V_{ji}(\vec{r}) \xrightarrow{r \rightarrow \infty} \frac{\vec{r}}{r^3} \cdot \int U_j^*(\vec{r}') \vec{r}' U_i(\vec{r}') d^3r' .$$

In this case we get:

$$\sigma_{\text{tot}}(i \rightarrow j) = \frac{8\pi^2}{(3)^{\frac{1}{2}} k^2} \frac{f(j,i)}{E_j - E_i} g(k',k)$$

where we have introduced the oscillator strength $f(j,i)$ defined by:

$$\frac{1}{3} \left| \int d^3r' U_j^*(\vec{r}') \vec{r}' U_i(\vec{r}') \right|^2 \equiv \frac{f(j,i)}{E_j - E_i}$$

and the Krammers-Gaunt g -factor defined by:

$$\int d\Omega_{k'} \left| \int \frac{\vec{r}}{r^3} X_{k'}^{(-)*}(\vec{r}) X_k^{(+)}(\vec{r}) d^3r \right|^2 \equiv \frac{8\pi^2}{(3)^{\frac{1}{2}} k k'} g(k,k') .$$

This procedure leads to a crude estimate of the excitation cross section. One can improve it by introducing suitable cut-off factors in the relevant integrations.

Another procedure for the calculation of electron impact excitation cross section of ions is the so-called close coupling approximation. Here one expands the complete wave function of the system in terms of atomic wave functions and then tries to solve the resulting integrodifferential equations numerically. However, in the expansion of the total wave function one keeps only a few of the lowest lying atomic states. This leads to a small number of coupled integrodifferential equations. This approximation has been used in the

calculation of the excitation cross sections for He^+ and N^{4+} . In the case of He^+ where experimental data is available, there is a serious discrepancy between the experimental threshold behavior and the result of the close coupling approximation.

In this paper, in order to reduce the Coulomb-Born scattering amplitude to an analytical expression, we have followed Chen and Watson⁴ in using the combination of the eikonal and classical approximations. The simultaneous use of these two approximations has the advantage of reducing the expression for the scattering amplitude to a one dimensional path integral. The path is made up of points where two particular sequences of trajectories meet and are tangent. One sequence of trajectories has the initial momentum $\vec{k}$, the other has the final momentum $\vec{k}'$. The use of the eikonal approximation for the wave functions of the incident and scattered electrons requires:

$$k \gg 1, \quad k' \gg 1$$

where $\vec{k}$ and $\vec{k}'$ are the momenta of the incident and scattered electrons respectively. The validity of the classical theory of elastic scattering, as we shall see later, requires:

$$k \ll 4Z$$

and

$$k' \ll 4Z.$$

Thus, it is for highly ionized atoms that there is a region of overlap for the validity of these two approximations. This range is defined by:

$$1 \ll k \ll 4Z$$

and

$$1 \ll k' \ll 4Z.$$

To calculate the eikonal functions, we need to know the equations of the trajectories. However, it is shown by Chen and Watson⁵ that the eikonal is not very sensitive to the form of the trajectory chosen. Thus, for an elastic scattering with the incident momentum $\vec{k}$ and the scattering angle 2β we have chosen a trajectory which consists of two straight lines intersecting at an angle 2β . From now on we refer to this as a "triangular" trajectory.

In this calculation, we assume that all electrons are distinguishable; thus, neglecting the exchange amplitude. It is shown by Mott and Massey⁶ that at high incident energies, where the Born approximation is valid, the exchange amplitude is very small relative to the direct scattering amplitude, and if the transition can occur only through electron exchange, the cross section will be very small and drops rapidly with energy. For example, in the excitation of the 2^3P state of helium from the ground state, which occurs only through electron exchange, the total cross section drops by a factor of 270 as we go from $k = 1.9$ to $k = 4$. Since we are working at high energies ($k \gg 1$, $k' \gg 1$), we expect the exchange amplitude to be small. However, the exchange amplitude can easily be included in the Coulomb-Born approximation, and this will be the subject of a later paper.

As we have already indicated, the accuracy of our calculation increases as the ionic charge increases. After looking at Moore-Sitterley's table of atomic energy levels, we found four highly ionized atoms which have one electron in their external shell. These are the sodium-like ions Fe^{15+} , Co^{16+} , Ni^{17+} , and Cu^{18+} . The electron impact excitation cross section for the $3s \rightarrow 3p$ transition of Fe^{15+} has been calculated by Belly, Tully, and Van Regemorter³ (1963) using the Coulomb-Born approximation. Kreuger and Czyzak³ (1965) have calculated the cross section for the $3s \rightarrow 3p$ and $3p \rightarrow 3d$ transitions of Fe^{15+} in the Coulomb-Born approximation. These two calculations agree and are both at low energies ($k \approx 1$) where our procedure is not expected to be valid. However, if we extrapolate our calculation to such energies, our result differs considerably from theirs. When we compared our calculation with that of the Born approximation, we found that they agree (within a factor of one and a half) at high energies ($k \approx 24$).

EXCITATION OF ATOMIC IONS IN THE COULOMB-BORN APPROXIMATION

We consider the excitation of an ionized atom, which has one electron in its external shell, through electron impact. Since the mass of the electron is much smaller than that of the ion, we shall neglect the motion of the latter. Therefore, the center of mass of the system is essentially at the center of the ion. We shall also assume that the incident and atomic electrons are distinguishable. Throughout this paper we shall use atomic units. These are defined by putting $m = e = \hbar = 1$.

In our model, the Hamiltonian for the system of incident electron and ion is:

$$H = H_0 + U + V$$

where

$$H_0 = -\frac{1}{2} \nabla^2 + H_{\text{atom}}(\vec{r}')$$

$$U = -\frac{Z}{r}$$

$$V = \frac{1}{|\vec{r} - \vec{r}'|}$$

Here Z is the net charge of the ion; and $\vec{r}$ and $\vec{r}'$ denote the position coordinates of the incident and the outermost atomic electrons respectively. The origin of the coordinate system is taken to be at the center of the ion.

Let us represent the eigenfunctions of H_0 by χ , the eigenfunctions of $H_0 + U$ by $\phi^\pm$, and the eigenfunctions of $H_0 + U + V$ by $\psi^\pm$. Then, according to Lippmann-Schwinger formalism we have:⁷

$$\phi^\pm = \chi + \frac{1}{E - H_0 \pm i\epsilon} U \phi^\pm \quad (1)$$

$$\psi^\pm = \chi + \frac{1}{E - H_0 \pm i\epsilon} (U + V) \psi^\pm \quad (2)$$

the transition amplitude turns out to be⁸

$$T_{fi} = \langle \chi_f, (U + V) \psi_i^+ \rangle \quad (3)$$

Substituting for χ_f from (1), we get:

$$\begin{aligned} T_{fi} &= \langle \phi_f^- - \frac{1}{E - H_0 - i\epsilon} U \phi_f^-, (U + V) \psi_i^+ \rangle \\ &= \langle \phi_f^-, (U + V) \psi_i^+ \rangle - \langle \phi_f^-, U \frac{1}{E - H_0 + i\epsilon} (U + V) \psi_i^+ \rangle \\ &= \langle \phi_f^-, (U + V) \psi_i^+ \rangle - \langle \phi_f^-, U(\psi_i^+ - \chi_i) \rangle \\ &= \langle \phi_f^-, V \psi_i^+ \rangle + \langle \phi_f^-, U \chi_i \rangle \end{aligned} \quad (4)$$

In this problem the second term vanishes because the initial and final channels are not connected by U . Thus, we have:

$$T_{fi} = \langle \phi_f^-, V \psi_i^+ \rangle \quad (5)$$

If we approximate ψ_i^+ by ϕ_i^+ , we obtain the so-called Coulomb-Born approximation

$$T_{fi}^{C.B.} = \langle \phi_f^-, v \phi_i^+ \rangle \quad (6)$$

Thus, in order to calculate $T_{fi}^{C.B.}$ we need to know ϕ_i^+ . These wave functions, by definition, satisfy the following Schrodinger equation:

$$(H_{\text{atom}} - \frac{1}{2} \nabla_r^2 - \frac{Z}{r} - E_i) \phi_i^{\pm} = 0 \quad (7)$$

let

$$\phi_i^{\pm}(\vec{r}, \vec{r}') = X_{\vec{k}}^{\pm}(\vec{r}, \vec{r}') U_i(\vec{r}') \quad (8)$$

Substituting (8) into (7) and using the Schrodinger equation for the atom, we get:

$$(\nabla_r^2 + 2 \frac{Z}{r} + k^2) X_{\vec{k}}^{\pm}(\vec{r}) = 0 \quad (9)$$

where $E = \frac{1}{2} k^2$ represents the energy of the incident electron.

Equation (9) is known to have solutions⁹

$$\begin{aligned} X_{\vec{k}}^{(+)}(\vec{r}) &= (2\pi)^{-\frac{3}{2}} \exp(\frac{\pi}{2} \gamma) \Gamma(1 - i\gamma) F(i\gamma, 1, ikr - i\vec{k} \cdot \vec{r}) \exp(i\vec{k} \cdot \vec{r}) \\ X_{\vec{k}}^{(-)}(\vec{r}) &= (2\pi)^{-\frac{3}{2}} \exp(\frac{\pi}{2} \gamma) \Gamma^*(1 - i\gamma) F^*(i\gamma, 1, ikr + i\vec{k} \cdot \vec{r}) \exp(i\vec{k} \cdot \vec{r}) \\ &= (2\pi)^{-\frac{3}{2}} \exp(\frac{\pi}{2} \gamma) \Gamma(1 + i\gamma) F(-i\gamma, 1, -ikr - i\vec{k} \cdot \vec{r}) \exp(i\vec{k} \cdot \vec{r}) \end{aligned} \quad (10)$$

where $\gamma = \frac{Z}{k}$, and

$$F(a, b; z) = 1 + \frac{a}{b} \frac{z}{1!} + \frac{a(a+1)}{b(b+1)} \frac{z^2}{2!} + \dots \quad (11)$$

Substituting (8) into (6), we get:

$$\begin{aligned} T_{fi}^{\text{C.B.}} &= \langle \phi_f^-, v \phi_i^+ \rangle = \langle U_f(\vec{r}), X_{\vec{k}}^{(-)}(\vec{r}), \frac{1}{|\vec{r} - \vec{r}'|} U_i(\vec{r}') X_{\vec{k}}^{(+)}(\vec{r}') \rangle \\ &= \iiint d^3r d^3r' U_f^*(\vec{r}) \frac{1}{|\vec{r} - \vec{r}'|} U_i(\vec{r}') X_{\vec{k}}^{(-)*}(\vec{r}) X_{\vec{k}}^{(+)}(\vec{r}') \\ &= \int d^3r X_{\vec{k}}^{(-)*}(\vec{r}) V_{fi}(\vec{r}) X_{\vec{k}}^{(+)}(\vec{r}) \end{aligned} \quad (12)$$

where

$$V_{fi} = \int U_f^*(\vec{r}) \frac{1}{|\vec{r} - \vec{r}'|} U_i(\vec{r}') d^3r' \quad (13)$$

In the barycentric coordinate system, the differential cross section for the excitation of the ion has the form:¹⁰

$$\frac{d\sigma}{d\Omega} = (2\pi)^4 \left(\frac{k'}{v'} \right)^2 \frac{v'}{v} |T_{fi}^{\text{C.B.}}|^2$$

where v and v' are the barycentric velocities of the electrons in the initial and final states respectively. For non-relativistic velocities $\frac{k'}{v'} \approx m = 1$ (in atomic units). Thus, for non-relativistic velocities we have:

$$\frac{d\sigma}{d\Omega} = (2\pi)^4 \frac{k'}{k} |T_{fi}^{C.B.}|^2. \quad (14)$$

The total cross section is:

$$\sigma = (2\pi)^4 \frac{k'}{k} \int d\Omega |T_{fi}^{C.B.}|^2.$$

Usually, we are interested in transitions of the type $n\ell \rightarrow n'\ell'$.

Therefore we sum over final states (m') and average over initial states (m):

$$\sigma = (2\pi)^4 \frac{k'}{k} \frac{1}{2\ell + 1} \sum_m \sum_{m'} \int d\Omega |T_{fi}^{C.B.}|^2. \quad (15)$$

THE EIKONAL APPROXIMATION

Now, in order to calculate $T_{fi}^{C.B.}$ we use the so-called first eikonal approximation⁴ to the wave functions $X^\pm$:

$$X_k^\pm(\vec{r}) = \frac{1}{(2\pi)^{\frac{3}{2}}} e^{iS_k^{(\pm)}(\vec{r})} \quad (16)$$

where the eikonal function S is defined by a path integral along the classical trajectory:

$$S_k(\vec{r}) = \int^r \chi_k(s) ds$$

with

$$\chi_k = (k^2 + 2 \frac{Z}{r})^{\frac{1}{2}}$$

the condition for the validity of the eikonal approximation is¹¹

$$k \gg 1 \quad (17)$$

Using the first eikonal approximation for both $X_k^{(+)}$ and $X_k^{(-)}$,

Eq. (12) takes the form:

$$T_{fi}^{C.B.} = (2\pi)^{-3} \int \exp(-i S_k^{(-)}(\vec{r})) v_{fi}(\vec{r}) \exp(i S_k^{(+)}(\vec{r})) d^3r. \quad (18)$$

According to (17), this equation is valid provided

$$k \gg 1$$

and

(19)

$$k' \gg 1$$

Two trajectories pass through each point $\vec{r}$ of the integrand in (18). One corresponds to an elastic scattering with the initial momentum $\vec{k}$ and the other corresponds to an elastic scattering with the final momentum $\vec{k}'$. Chen and Watson have shown⁴ that if the conditions for the validity of both the eikonal approximation and the classical theory of elastic scattering are satisfied, then, the main contribution to the integral comes from points $\vec{r}_0$ at which the two trajectories are tangent. Furthermore, in this case, the integral reduces to a one dimensional path integral. A rough estimate for the validity of the classical theory of elastic scattering can be obtained from the requirement

$$\frac{\delta k}{\Delta k} \ll 1$$

where δk is the uncertainty in momentum and Δk is the momentum transfer. For the case $F = -\frac{Z}{r^2}$ this leads to¹²

$$k \ll 4Z \quad . \quad (20)$$

Hence, for high Z there is a region where the criteria for the validity of both the classical and the eikonal approximations are satisfied. It is in this region that the results of our calculation

are applicable. Let (R_1, R_2) and (R'_1, R'_2) be, respectively, the radii of curvature of the two constant eikonal surfaces $S_k^{(+)}(\vec{r}_0) = \text{const.}$ and $S_{k'}^{(-)}(\vec{r}_0) = \text{const.}$ which pass through $\vec{r}_0$. Then, following Chen-Watson treatment,¹³ we get:

$$T_{fi}^{\text{C.B.}} \approx \int V_{fi}(\vec{r}_0) \delta(\alpha_{11}; \alpha_{22}) \pi(|\alpha_{11}| |\alpha_{22}|)^{-\frac{1}{2}} \exp(i \phi(s)) ds \quad (21)$$

where the path integral is along the points of tangency of a sequence of trajectory pairs, and

$$\phi(s) = S_k^{(+)}(\vec{r}_0) - S_{k'}^{(-)}(\vec{r}_0)$$

$$\alpha_{11} = \frac{1}{2} \left(\frac{\chi_k}{R_1(k)} - \frac{\chi_{k'}}{R'_1(k')} \right)$$

$$\alpha_{22} = \frac{1}{2} \left(\frac{\chi_k}{R_2(k)} - \frac{\chi_{k'}}{R'_2(k')} \right)$$

$$D \equiv \frac{\alpha_{11}}{|\alpha_{11}|} + \frac{\alpha_{22}}{|\alpha_{22}|} ; \quad \delta(\alpha_{11}; \alpha_{22}) = \begin{cases} +i & \text{if } D = +2 \\ -i & \text{if } D = -2 \\ +1 & \text{if } D = 0 \end{cases}$$

INTEGRATION PATH

To identify the integration path, we look for points where pairs of trajectories meet and are tangent. However, to simplify this calculation, we replace the classical trajectory by a "triangular" trajectory. This requires the energy to be high and the scattering angle to be small. At high energies, however, the scattering amplitude is sharply peaked in the forward direction.¹⁴ The restriction on the scattering angle is, therefore, satisfied whenever the energy is high. The "triangular" trajectories corresponding to an elastic scattering with initial momentum $\vec{k}$ and an elastic scattering with final momentum $\vec{k}'$ are shown in Figs. (1a) and (1b) respectively.

Fig. 1a.

Fig. 1b.

In these figures the angles 2β and $2\beta'$ are the classical elastic scattering angles for momenta $\vec{k}$ and $\vec{k}'$ respectively.

Now, to find the integration path, we write the equations of a "triangular" trajectory with the initial momentum $\vec{k}$ and a "triangular" trajectory with the final momentum $\vec{k}'$. Then, we find points where they meet and are tangent. It turns out that the integration path consists of the two dashed lines shown in Figs. (2a) and (2b).

Fig. 2a.

Fig. 2b.

In Fig. (2a) we have trajectories I and II which correspond to elastic scattering with the scattering angles $2\beta_1$ and $2\beta_2$ respectively. Similarly, in Fig. (2b) we have trajectories III and IV with the scattering angles $2\beta_3$ and $2\beta_4$ respectively. At each point of these trajectories we have a surface of constant eikonal which is normal to the trajectory at that point. The two principal radii of curvature of the eikonal surface corresponding to trajectory "i" will be denoted by R_{1i} and R_{2i} . Thus, for example, R_{13} and R_{23} are the principal radii of curvature of the eikonal surface corresponding to trajectory III. In Fig. (2a) we have:

$$d \simeq d' = \frac{Z}{E'\theta} \quad (22)$$

and in Fig. (2b) we have:

$$d' \simeq d = \frac{Z}{E\theta} \quad (23)$$

where θ is the scattering angle.

Having identified the integration path, we can write Eq. (21)

in the form:

$$T_{fi}^{C.B.} = \int_{-\infty}^0 dz V_{fi}(\vec{r}_0) \exp(i\phi(z)) \delta(\alpha_{11}; \alpha_{22}) \pi[|\alpha_{11}| |\alpha_{22}|]^{-\frac{1}{2}}$$

$$\times (1 + \frac{1}{2} \beta_2^2) + \int_0^{\infty} dz' V'_{fi}(r'_0) \exp(i\phi(z')) \delta(\alpha'_{11}; \alpha'_{22})$$

$$\times \pi[|\alpha'_{11}| |\alpha'_{22}|]^{-\frac{1}{2}} (1 + \frac{1}{2} \beta_3^2) \quad (24)$$

where

$$\alpha_{11} = \frac{1}{2} \left(\frac{\chi_k}{R_{12}(k)} - \frac{\chi_{k'}}{R_{11}(k')} \right) ; \quad \alpha_{22} = \frac{1}{2} \left(\frac{\chi_k}{R_{22}(k)} - \frac{\chi_{k'}}{R_{21}(k')} \right)$$

$$\delta(\alpha_{11}; \alpha_{22}) = \begin{cases} +i & \text{for } D = \frac{\alpha_{11}}{|\alpha_{11}|} + \frac{\alpha_{22}}{|\alpha_{22}|} = +2 \\ -i & \text{for } D = -2 \\ +1 & \text{for } D = 0 \end{cases}$$

$$\alpha'_{11} = \frac{1}{2} \left(\frac{\chi'_k}{R_{13}(k)} - \frac{\chi'_{k'}}{R_{14}(k')} \right) ; \quad \alpha'_{22} = \left(\frac{\chi'_k}{R_{23}(k)} - \frac{\chi'_{k'}}{R_{24}(k')} \right)$$

$$\delta(\alpha'_{11}; \alpha'_{22}) = \begin{cases} +i & \text{for } D' = \frac{\alpha'_{11}}{|\alpha'_{11}|} + \frac{\alpha'_{22}}{|\alpha'_{22}|} = +2 \\ -i & \text{for } D' = -2 \\ +1 & \text{for } D' = 0 \end{cases}$$

$$r_0 = [z^2 + (d' - \beta_2 z)^2]^{\frac{1}{2}} ; \quad r'_0 = [z'^2 + (d + \beta_3 z')^2]^{\frac{1}{2}}$$

$$\beta_1 = \frac{Z}{2E'd'} ; \quad \beta_2 \approx -\frac{Z}{2Ed'} ; \quad \beta_3 = -\frac{Z}{2Ed} ; \quad \beta_4 \approx -\frac{Z}{2E'd}$$

$$\phi(z) = S_k^{(+)}(z) - S_{k'}^{(-)}(z)$$

$$\phi'(z') = S_k^{(+)}(z') - S_{k'}^{(-)}(z') \quad .$$

CALCULATION OF THE EIKONAL

Here, we want to calculate the eikonal functions $S_k^{(+)}(\vec{r}_0)$ and $S_{k'}^{(-)}(\vec{r}_0)$. If we substitute

$$\psi = (2\pi)^{-\frac{3}{2}} e^{iS_q(\vec{r}_0)}$$

into the Schrodinger equation, we get:

$$\chi_q \equiv |\vec{\nabla} S_q| = [q^2 + 2 \frac{Z}{r_0}]^{\frac{1}{2}} \approx q + \frac{r}{r_0} \quad (\text{at high energies}) .$$

To get S_q we integrate along the prescribed path with z-axis being taken along $\vec{q}$. $S_q^{\pm}$ are obtained after imposing the following asymptotic conditions¹⁵

$$S_q^{(+)}(\vec{r}_0) \xrightarrow{z \rightarrow -\infty} qz - r \ln(qr_0 - qz)$$

$$S_q^{(-)}(\vec{r}_0) \xrightarrow{z \rightarrow \infty} qz + r \ln(qr_0 + qz) .$$

Thus, in the case of Fig. (2a) we have:

$$S_k^{(+)} = kz_1 - r \ln(kr_0 - kz_1) \quad z < 0$$

$$S_{k'}^{(-)} = k'z'_1 + r' \ln(k'r_0 + k'z'_1) \quad z_2 > 0 \quad (25)$$

$$S_{k'}^{(-)} = k'z_1 + r' \ln(k'r_0 - k'j_1) + C_1 \quad z_2 < 0 .$$

The constant C_1 can be obtained by matching Eqs. (26) and (27) at $z_2 = 0$. It turns out to be

$$C_1 = 2\gamma'.$$

Here

$$z_1 = z_2 \cos \beta_1 + x_2 \sin \beta_1; \quad z'_1 = z_2 \cos \beta_1 - x_2 \sin \beta_1$$

where

$$x_2 = d - \beta_1 z_2.$$

Similarly, in the case of Fig. (2b) we have:

$$\begin{aligned} S_{k'}^{(-)} &= k' z_4 + \gamma' \ln(k' r'_0 + k' z_4) & z_3 > 0 \\ S_k^{(+)} &= k z_4 - \gamma \ln(k r'_0 + k z_4) + C_2 & z' > 0 \\ S_k^{(+)} &= k z'_3 - \gamma \ln(k r'_0 - k z'_3) & z' < 0 \end{aligned} \quad (26)$$

where

$$z'_3 = z' \cos \beta_3 + x' \sin \beta_3$$

$$z_4 = z' \cos \beta_3 - x' \sin \beta_3$$

$$C_2 = -2\gamma$$

with

$$x' = d + \beta_3 z'.$$

Having calculated $S_k^{(+)}$ and $S_{k'}^{(-)}$ for Figs. (2a) and (2b), we can write $\phi(z)$ and $\phi'(z')$ explicitly:

$$\begin{aligned}\phi(z) &= (k - k')z_1 - \gamma \ln(kr_0 - kz_1) - \gamma' \ln(k'r_0 - k'z_1) - 2\gamma' \\ \phi'(z') &= (k - k')z_4 - \gamma \ln(kr'_0 + kz_4) - \gamma' \ln(k'r'_0 + k'z_4) - 2\gamma' .\end{aligned}\tag{27}$$

Here z_1 can be expressed in terms of z and β_2 ; and z_4 can be written in terms of z' and β_3 .

CALCULATION OF THE RADII OF CURVATURE

First, we consider trajectory II, of Fig. (2a), corresponding to $X_k^{(+)}$. This trajectory, which is symmetric about the x-axis, is initially along the z_1 -axis. The calculation of the radii of curvature is most easily done in the (x_1, z_1) coordinate system where $S_k^{(+)}$ has azimuthal symmetry. These two coordinate systems are related by a rotation about the origin through an angle β_2

$$z_1 = z \cos \beta_2 + x \sin \beta_2$$

$$x_1 = -z \sin \beta_2 + x \cos \beta_2$$

According to Chen-Watson calculation, the two principal radii of curvature pertaining to this case are

$$\begin{aligned} \frac{1}{R_{12}(k)} &= \frac{[\partial \theta_c(z, d)] / \partial d}{[\partial |x_1|] / \partial d} \\ \frac{1}{R_{22}(k)} &= \frac{\theta_c(z, d)}{|x_1|} \end{aligned} \tag{28}$$

where $\tan \theta_c(z, d)$ is the slope of the trajectory, at a point (x, z) , with respect to the z-axis. For trajectory II it is given by

$$\theta_c(z, d) = \beta_2 + \beta_2(z)$$

where

$$\beta_2(z) = -d \int_0^z \frac{dU}{dR_0} \frac{dz''}{R_0} ; \quad R_0 = (d^2 + z''^2)^{\frac{1}{2}}$$

$$\beta_2 = \lim_{z \rightarrow \infty} \beta_2(z) .$$

Here

$$U = 1 - (1 - V/E)^{\frac{1}{2}} \approx \frac{V}{2E} \quad (\text{at high energies}) .$$

Next, we consider trajectory I, corresponding to $X_{k'}^{(-)}$, of Fig. (2a). This time, we rotate the coordinate system (x_2, z_2) about the origin through an angle β_1 to obtain the (x'_1, z'_1) coordinate system whose z'_1 -axis is parallel to $\vec{k}'$. In this case, according to Chen-Watson calculation, we have:

$$\begin{aligned} \frac{1}{R_{11}(k')} &= - \frac{[\partial \theta'_c(z_2, d')]/\partial d'}{[o|x'_1|]/\partial d'} \\ \frac{1}{R_{21}(k')} &= - \frac{\theta'_c(z_2, d')}{|x'_1|} \end{aligned} \quad (29)$$

where

$$\theta'_c(z_2, d') = \beta_1 - \beta_1(z_2) = \beta_1 [1 - z_2(z_2^2 + d'^2)^{-\frac{1}{2}}]$$

$$\beta_1 = \lim_{z_2 \rightarrow \infty} \beta_1(z_2) .$$

The radii of curvature of the surfaces of constant eikonal corresponding to the trajectories of Fig. (2b) can be calculated in a similar way. Explicit expressions for all of these radii of curvature are given in Appendix A.

CALCULATION OF THE INTERACTION MATRIX ELEMENT

In order to calculate

$$V_{fi}(\vec{r}) = \int \psi_{n', \ell', m'}^*(\vec{r}') \frac{1}{|\vec{r} - \vec{r}'|} \psi_{n \ell m}(\vec{r}') d^3 r'$$

we approximate the wavefunctions $\psi_{n', \ell', m'}$ and $\psi_{n \ell m}$ by hydrogenic-type wave functions. Thus, in spherical coordinates we have:

$$\psi_{n \ell m}(\vec{r}) = A_{n \ell} \exp\left(-\frac{1}{2} a_n r\right) r^\ell L_{n+\ell}^{2\ell+1}(a_n r) Y_\ell^m(\hat{r})$$

where

$$A_{n \ell} = \left[a_n^3 \frac{(n - \ell - 1)!}{2n[(n + \ell)!]^3} \right]^{\frac{1}{2}} (a_n)^\ell$$

and

$$a_n = 2 \frac{Z'}{na_0}; \quad Z' = Z + 1.$$

Using the expansions

$$L_{n+\ell}^{2\ell+1}(a_n r) = \sum_{s=0}^{n-\ell-1} (-a_n)^s \frac{[(n + \ell)!]^2}{(n - \ell - 1 - s)! (2\ell + 1 + s)! s!} r^s$$

$$\frac{1}{|\vec{r} - \vec{r}'|} = \sum_{\lambda, \mu} \frac{4\pi}{2\lambda + 1} \frac{r^{\lambda <}}{r^{\lambda + 1 >}} Y_\ell^{\mu*}(r) Y_\ell^\mu(\hat{r}')$$

and the integral

$$\int d\Omega' Y_{\ell', m'}^*(\hat{r}') Y_{\lambda, \mu}(\hat{r}') Y_{\ell, m}(\hat{r}') \\ = (-)^{m'} \left[\frac{1}{4\pi} (2\ell' + 1)(2\lambda + 1)(2\ell + 1) \right]^{\frac{1}{2}} \begin{pmatrix} \ell' & \lambda & \ell \\ -m' & \mu & m \end{pmatrix} \begin{pmatrix} \ell' & \lambda & \ell \\ 0 & 0 & 0 \end{pmatrix}$$

we can write

$$V_{fi}(\hat{r}) = \sum_{\lambda=|\ell-\ell'|}^{\ell+\ell'} \sum_{s=0}^{n-\ell-1} \sum_{t=0}^{n'-\ell'-1} B(n' \ell' m'; n \ell m; \lambda) P_{\lambda}^{|m'-m|}(\cos \theta) \\ \times \left[r^{-(\lambda+1)} \int_0^r \exp(-cr') r'^{\nu} dr' + r^{\lambda} \int_r^{\infty} \exp(-cr') r'^{\nu} dr' \right] \quad (30)$$

where

$$B(n' \ell' m'; n \ell m; \lambda) = (-)^{\epsilon_{\mu}} \begin{pmatrix} \ell' & \lambda & \ell \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell' & \lambda & \ell \\ -m' & \mu & m \end{pmatrix}$$

$$\frac{[(n' + \ell')!]^2}{t!(n' - \ell' - t - 1)! (2\ell' + 1 + t)!} \frac{[(n + \ell)!]^2}{s!(n - \ell - s - 1)! (2\ell + 1 + s)!}$$

$$A_{n', \ell'} A_{n \ell} (-a_n)^s (-a_{n'})^t \left[\frac{1}{(\lambda + |\mu|)!} (\lambda - |\mu|)! (2\ell + 1)(2\ell' + 1) \right]^{\frac{1}{2}}$$

and

$$\nu = \ell + \ell' + s + t + \lambda + 2 ; \quad \nu' = \ell + \ell' + s + t - \lambda + 1$$

$$C = \frac{1}{2}(a_n + a_{n'}) ; \quad \cos \theta = z/r_0$$

$$\epsilon_\mu = \begin{cases} m & \text{for } \mu \geq 0 \\ m' & \text{for } \mu < 0 \end{cases} .$$

Here use has been made of the fact that the $3j$ symbol

$$\begin{pmatrix} \ell' & \lambda & \ell \\ -m' & \mu & m \end{pmatrix}$$

vanishes unless

$$|\ell - \ell'| \leq \lambda \leq \ell + \ell'$$

and

$$-m' + \mu + m = 0$$

or

$$\mu = m' - m .$$

Finally, after performing the remaining integrals, we get

$$\begin{aligned} V_{fi}(r_0) &= \sum_{\lambda=|\ell-\ell'|}^{\ell+\ell'} \sum_{s=0}^{n-\ell-1} \sum_{t=0}^{n'-\ell'-1} B(n'\ell'm'; n\ell m; \lambda) P_\lambda^{|\mu|}(\cos \theta) \\ &\times \left\{ r_0^{-(\lambda+1)} \left[\frac{\nu!}{c^{\nu+1}} - e^{-Cr_0} \sum_{p=0}^{\nu} \frac{\nu!}{p!} \frac{r_0^p}{c^{\nu-p+1}} \right] + r_0^\lambda \left[e^{-Cr_0} \sum_{q=0}^{\nu'} \frac{\nu'!}{q!} \frac{r_0^q}{c^{\nu'-q+1}} \right] \right\} . \end{aligned} \quad (31)$$

A similar expression can be written for $V'_{fi}(r'_0)$. To get this expression, we make the following substitutions in $V_{fi}(r_0)$:

$$r_0 \longrightarrow r'_0$$

$$\cos \theta \longrightarrow \cos \theta' = \frac{z'}{r'_0} .$$

APPLICATION TO SODIUM-LIKE IONS

As we have already seen, under certain conditions, the scattering amplitude reduces to a path integral:

$$T_{fr}^{C.B.} = \int_{-\infty}^0 dz v_{fi}(\vec{r}_0) e^{i\phi(z)} \delta(\alpha_{11}; \alpha_{22}) \frac{\pi}{(|\alpha_{11}| |\alpha_{22}|)^{\frac{1}{2}}} (1 + \frac{1}{2} \beta_2^2) \\ + \int_0^{\infty} dz' v_{fi}'(\vec{r}') e^{i\phi(z')} \delta(\alpha'_{11}; \alpha'_{22}) \frac{\pi}{(|\alpha'_{11}| |\alpha'_{22}|)^{\frac{1}{2}}} (1 + \frac{1}{2} \beta_3^2) .$$

A numerical evaluation of these integrals showed that with the exception of a phase factor all other factors in each integrand change very slowly. Thus, we can break each integral to a sum over integrals which have the rapidly varying phase factor as their integrand. Each of these integrals is over a small interval. The interval is chosen such that the rest of the factors in the original integrand can be taken effectively constant. By increasing the number of integrals in each sum, we can get any desired accuracy. In this calculation, we have chosen a limit of one per cent accuracy.

In looking for highly ionized atoms with one external electron, we found four candidates for which the energy levels are given in Moore-Sitterley's table of atomic energy levels. These are the sodium-like ions Fe^{15+} , Co^{16+} , Ni^{17+} , and Cu^{18+} . For these ions we have calculated the total cross section for the $3s \rightarrow 3p$ and $3p \rightarrow 3d$ transitions. The result is shown in Figs. 3 through 6. In these figures we have plotted total cross section against ϵ which

is the energy of the incident electron in units of the excitation energy. In Fig. 7, we have plotted a typical differential cross section. One general feature of these figures is that for any ion the total cross section for $3p \rightarrow 3d$ transition is smaller than that of $3s \rightarrow 3p$ transition. Furthermore, the total cross section for $3p \rightarrow 3d$ transition varies slower than that of $3s \rightarrow 3p$ transition. Another general feature is that the total cross section decreases as the ionic charge increases.

Of these four ions that we have considered, only Fe^{15+} has been dealt with in the literature. There are two independent calculations of the total cross section for the $3s \rightarrow 3p$ transition of Fe^{15+} . One calculation is by Belly, Tully, and Van Regemorter;³ the other calculation is by Dreuger and Czyzak.³ The latter have also calculated the total cross section for the $3p \rightarrow 3d$ transition of Fe^{15+} . These two calculations are done in the Coulomb-Born approximation and include energies up to $\epsilon = 4$. For these relatively low energies, our procedure is not expected to be valid. However, if we extrapolate our result down to $\epsilon = 4$, we find that the total cross section for the $3s \rightarrow 3p$ excitation of Fe^{15+} as calculated by these two groups, is about fourteen times larger than our result (at $\epsilon = 4$). However, the ratio

$$\frac{\sigma_{\text{tot}}(3s \rightarrow 3p)}{\sigma_{\text{tot}}(3p \rightarrow 3d)}$$

in our calculation agrees with the same ratio in the calculation of Kreuger and Czyzak. We have also compared our result with that of the

Born approximation. At $\epsilon = 4$, the Born approximation cross section for the $3s \rightarrow 3p$ excitation of Fe^{15+} is about six times larger than our result; but, as the energy of the incident electron increases, our result approaches that of the Born approximation. At $\epsilon = 225$ ($k \approx 24$) the Born approximation cross section is about one and a half times larger than our result. At higher energies, the two calculations depart again. However, at very high energies, our result is not expected to be valid due to the restrictions:

$$k \ll 4Z$$

and

$$k' \ll 4Z .$$

APPENDIX A

THE PRINCIPAL RADII OF CURVATURE OF THE SURFACES OF CONSTANT EIKONAL

If we denote by R_{1i} and R_{2i} the two principal radii of curvature of a surface of constant eikonal corresponding to trajectory "i", then, following Chen's and Watson's treatment,⁵ we get

$$\frac{1}{R_{21}} = \frac{-e_1 \beta_2 \left[\frac{1}{d'} + \frac{z}{d'} \frac{1}{(z^2 + d'^2)^{1/2}} + \frac{zd'}{(z^2 + d'^2)^{3/2}} \right]}{1 + \frac{1}{2} \beta_2^2 + 2\beta_2 \frac{z}{d'}}$$

$$\frac{1}{R_{22}} = \frac{e_1 \beta_2 \left[1 + \frac{z}{(z^2 + d'^2)^{1/2}} \right]}{d' \left(1 - \frac{1}{2} \beta_2^2 \right) - 2\beta_2 z}$$

$$\frac{1}{R_{11}} = \frac{e_2 \beta_1 \left[\frac{1}{d'} - \frac{z_2}{d'} \frac{1}{(z_2^2 + d'^2)^{1/2}} - \frac{z_2 d'}{(z_2^2 + d'^2)^{3/2}} \right]}{1 + \frac{1}{2} \beta_1^2}$$

$$\frac{1}{R_{21}} = \frac{-e_2 \beta_1 \left[1 - \frac{z_2}{(z_2^2 + d'^2)^{1/2}} \right]}{d' \left(1 - \frac{1}{2} \beta_1^2 \right)}$$

for the case of Fig. (2a), and

$$\frac{1}{R_{13}} = \frac{-e_3 \beta_3 \left[\frac{1}{d} + \frac{z'}{d} \frac{1}{(z'^2 + d^2)^{1/2}} + \frac{z'd}{(z'^2 + d^2)^{3/2}} \right]}{1 + \frac{1}{2} \beta_3^2}$$

$$\frac{1}{R_{23}} = \frac{e_3 \beta_3 \left[1 + \frac{z'}{(z'^2 + d^2)^{1/2}} \right]}{d \left(1 - \frac{1}{2} \beta_3^2 \right)}$$

$$\frac{1}{R_{14}} = \frac{e_4 \beta_4 \left[\frac{1}{d} - \frac{z_3}{d} \frac{1}{(z_3^2 + d^2)^{1/2}} - \frac{z_3 d}{(z_3^2 + d^2)^{3/2}} \right]}{1 + \frac{1}{2} \beta_4^2 - 2\beta_4 \frac{z_3}{d}}$$

$$\frac{1}{R_{24}} = \frac{-e_4 \beta_4 \left[1 - \frac{z_3}{(z_3^2 + d^2)^{1/2}} \right]}{d \left(1 - \frac{1}{2} \beta_4^2 \right) + 2\beta_4 z_3}$$

for the case of Fig. (2b). Here we have defined:

$$e_1 = \frac{x_1}{|x_1|}$$

$$e_2 = \frac{x'_1}{|x'_1|}$$

$$e_3 = \frac{x'_3}{|x'_3|}$$

$$e_4 = \frac{x_4}{|x_4|}$$

where

$$x_1 = -2\beta_2 z + d' \left(1 - \frac{1}{2} \beta_2^2\right)$$

$$x'_1 = d' \left(1 - \frac{1}{2} \beta_1^2\right)$$

$$x'_3 = d \left(1 - \frac{1}{2} \beta_3^2\right)$$

$$x_4 = 2\beta_4 z_3 + d \left(1 - \frac{1}{2} \beta_4^2\right).$$

Here, z_2 is related to z by a rotation about the origin through an angle $\beta_2 - \beta_1$. Similarly, z_3 is related to z' by a rotation through an angle $\beta_3 - \beta_4$:

$$z_2 = z - (\beta_2 - \beta_1)d'$$

$$z_3 = z' + (\beta_3 - \beta_4)d$$

where

$$d' = \frac{Z}{E'\theta}$$

$$d = \frac{Z}{E\theta}$$

$$\beta_1 = -\frac{\theta}{2}$$

$$\beta_2 = -\frac{E'}{E} \frac{\theta}{2}$$

$$\beta_3 = -\frac{\theta}{2}$$

$$\beta_4 = -\frac{E}{E'} \frac{\theta}{2}.$$

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REFERENCES

1. B. L. Moisewitsch and S. J. Smith, Rev. Mod. Phys. 40, 238 (1968).
2. A. Burgess, Astrophys. J. 132, 503 (1960).
3. A. Burgess, Mem. Soc. Roy. Sci. Liege 4, 299 (1961); J. Tully, M.Sc. dissertation (1960), London; O. Belly, J. Tully, and H. Van Regemorter, Ann. Phys. (Paris) 8, 303 (1963); T. K. Krueger and S. J. Czyzak, Mem. Roy. Astron. Soc. 69, 145 (1965); H. Van Regemorter, Ann. Astrophys. 23, 817 (1960), Monthly Notices Roy. Astron. Soc. 121, 213 (1960), and Compt. Rend. 252, 2667 (1961).
4. J. C. Y. Chen and K. M. Watson, Phys. Rev. 174, 152 (1968).
5. J. C. Y. Chen and K. M. Watson, Phys. Rev. 188, 236 (1969).
6. N. F. Mott and H. S. W. Massey, The Theory of Atomic Collisions (Oxford University Press, London, 1965), p. 507.
7. M. L. Goldberger and K. M. Watson, Collision Theory (John Wiley and Sons, Inc., New York, 1965).
8. See, for example, p. 203 of Ref. 7.
9. L. D. Landau and E. M. Lifshitz, Quantum Mechanics (Pergamon Press, 1965).
10. See, for example, p. 90 of Ref. 7.
11. K. M. Watson, Properties of Matter under Unusual Conditions, edited by Hans Mark (John Wiley and Sons, Inc., New York, 1968), p. 19.
12. David Bohm, Quantum Theory (Prentice Hall, Inc., New Jersey, 1959), p. 554.
13. See, for example, p. 162. of Ref. 4.

14. R. Glauber, Lectures in Theoretical Physics, (Interscience Publishers, Inc., New York, 1959), vol. 1, p. 328.
15. See, for example, pp. 518 and 521 of Ref. 9.

Fig. 3. Total cross sections for the $3s \rightarrow 3p$ (curve 1) and $3p \rightarrow 3d$ (curve 2) excitations of Fe^{15+} .

Fig. 4. Total cross sections for the $3s \rightarrow 3p$ (curve 1) and $3p \rightarrow 3d$ (curve 2) excitations of Co^{16+} .

Fig. 5. Total cross sections for the $3s \rightarrow 3p$ (curve 1) and $3p \rightarrow 3d$ (curve 2) excitations of Ni^{17+} .

Fig. 6. Total cross sections for the $3s \rightarrow 3p$ (curve 1) and $3p \rightarrow 3d$ (curve 2) excitations of Cu^{18+} .

Fig. 7. Differential cross sections for the $3s \rightarrow 3p$ (curve 1) and $3p \rightarrow 3d$ (curve 2) excitations of Fe^{15+} .

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