ARGONNE NATIONAL LABORATORY
9700 South Cass Avenue
Argonne, Illinois

SYMPOSIUM ON PHYSICS AND NONDESTRUCTIVE TESTING

Held at Argonne National Laboratory
October 4 and 5, 1960

Program arranged by:

Nondestructive Testing Group
Metallurgy Division

Program Chairman:

W. J. McGonnagle

Operated by The University of Chicago
under
Contract W-31-109-eng-38

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WELCOMING REMARKS

W. J. McGonnagle: I wish to welcome you, on behalf of the Argonne National Laboratory, to this Symposium on Physics and Nondestructive Testing. For the purposes of this symposium we shall define nondestructive testing as the general name given to all methods for evaluating materials or measuring material properties without impairing the future usefulness of the material. In other words, the technology known as nondestructive testing is a branch of applied physics - with the restriction that the material cannot be destroyed or damaged during the physical measurement. The emphasis in this symposium will be on the basic physics underlying nondestructive methods for the evaluation of material or material properties. The emphasis will be on the fundamental principles rather than on the applications of the methods. We hope that this symposium will serve as a source of information to those of you who are responsible for the development of new and improved nondestructive testing methods and techniques and to those of you who have problems in this field of technology.

We'd like to begin our program today with a few welcoming remarks from various individuals. First, Dr. Norman Hilberry, the Director of Argonne National Laboratory. Dr. Hilberry.

Norman Hilberry: I just want to say welcome. We've been in this nondestructive testing business now for some considerable period of time. It dates way back to the Hanford days. At that time we used coatings which would melt so that one could get an idea of where the nonbonded areas were on relatively nonbondable materials. We had some really beautiful modernistic views. I've wondered sometimes why we didn't make some reproductions of them and hang them down in the Museum of Modern Art. I'm sure they would stand up very well in comparison with some of the things that one sees. We've come a long way since those days. We've got a long way to go. I've just been needling our people to point out that fuel elements aren't the only things that need nondestructive testing.

We had a project here at the Laboratory - one of the advantages of these labs is that you do get a rather maximal amount of interdivisional activity - to set up and do experiments on a population of some 200,000 mice. It was supposed to be a million mice, but the million (megamouse) part of the thing got to be a wee bit extreme. But at least 200,000 mice are a lot of animals. The "fool things" have a habit of dying and the point of the experiment is to understand why they die. Now most of them just die of old age and are obviously uninteresting. They die off something like 300 to 400 a day in such a sized population.

We have in the biological sciences a set of individuals called pathologists. Theoretically, each one of these dead mice should be cut up

and looked at in a microscope by the pathologists to determine just what was wrong with the mouse when he went to the hereafter. But, if you figured out how many pathologists you'd have to have to examine these many mice, you'd need more pathologists than there are.

Well, this seems like a peculiar situation. Since you have to go through all of this hocus pocus, I've just given the task to the nondestructive testing people of determining whether the mouse just died of natural old age or died of something interesting. They might build a setup, you see, consisting of a conveyor belt and other equipment. When the mouse dies, somebody throws it on the belt; nondestructive tests are made, and if the mouse died of old age, they throw him in the garbage can; if it died of something interesting, it is dumped into the proper can. Then the pathologists only have to look at perhaps 5 or 10 per cent of the dead mice which they can do with a good deal of detail. Needless to say, the comments I get from the pathologists are not printable.

On the other hand, this is the direction in which we must move in many of these things. The things that we are doing today in many of these testing techniques are things that were not only inconceivable forty years ago, when I started in this physics business, but they are things for which present day common techniques were not even available. Now we can do many things that before were completely impossible. I think one of the things we need to do is not allow ourselves to be so completely swamped by the day-in and day-out responsibilities of testing things, that we don't let our imaginations run a little bit riot after the things that are completely outside of our normal day-in and day-out responsibilities. New techniques may enable us to do that which before were believed impossible.

Consider the mouse experiment - well okay, it's an experiment - it's quite clear that if these techniques work with mice, they'll work with men. The result is that what one is doing is coming in through the back door and developing a whole new tabloid of diagnostic techniques which conceivably could make a tremendous difference in the overall welfare of mankind.

Clearly, our first responsibility is to try to prevent as many of these cleanup jobs as we've had around on occasion here and there with fuel elements. One should never lose sight of the fact that these same techniques have validity in many other areas, and part of the researcher's responsibility is to keep the broad view in mind as well as the specific. The attention which he showers upon a specific problem will not miss many and wide areas of applicability which perhaps in the terms of mankind's welfare might be more significant than the solution of the immediate problem with which he began.

As I look at this symposium's schedule, we're concerned with methods of approach and with ways of examining. The groundwork on which this has grown has been a very desperate need in the reactor business, at least it has here at Argonne. Let's keep in mind that these same techniques have a much wider applicability than this immediate area with which one is dealing though completely justified in terms of its area. There is that additional bonus, always just over the horizon, which makes the important job one of immeasurable importance.

The program looks extremely interesting to me. I wish I were going to be able to sit in and listen; but I've got to go chase budgets. This is a disease that one gets when one becomes a director - budgetitis. It's a chronic disease that has acute eruption periods. So I'm going to have to run away, but I hope you have an excellent meeting. More power to you.

W. J. McGonnagle: Thank you, Dr. Hilberry. The next person I would like to introduce is Dr. Frank Foote, Director of the Metallurgy Division. Dr. Foote.

F. G. Foote: Ladies and gentlemen. At Argonne the Nondestructive Testing Group is part of the Metallurgy Division. The Metallurgy Division also is responsible for the development, and in many cases the making, of a large variety of reactor components and particular fuel elements. We've been engaged in this business of making fuel elements ever since 1943, and I remember making fuel elements, and also nondestructively testing the same, or at least hoping we were nondestructively testing them. Everything dates in this task, more or less, since 1943. Even prior to the "Frost Test" that Dr. Hilberry mentioned. I remember the first nondestructive testing instrument which we had and which we used to test the Hanford slug production, which we were trying to develop. This instrument was a ouija board. The theory of this device was not too well known. We didn't know quite why it worked, or if it worked, I guess we might say. It seemed to work just about as well as any other device that we had, and we had such an instrument in the Laboratory which we used for some time. Not because we thought it was very good, but simply because we didn't have anything else, and we thought we ought to do some nondestructive testing. Well, we've come a long way since then. The devices that we now routinely use, are things that at that time were essentially impossible - were things that we did not even think of doing - the sort of things that we now routinely do. Our nondestructive testing group finds itself, actually, at the very center of a great deal of our reactor component and fuel element work. We've pretty much adopted a policy of essentially testing in one way or another practically everything that we make or buy. All of the parts, particularly the fuel element parts, that go into our experimental reactors are subjected to some type of nondestructive testing and evaluation. It means that our nondestructive testing group never lacks for problems. We always have plenty of things for them to test. Their main problem is trying to dream up a technique which will give us the sort of information we want. And my

opinion is that the Group has been extremely ingenious in devising ways and means of getting information regarding a complete variety of possible defects in materials which we buy or make. This symposium is designed, at least in part, to examine into the existence of some of the properties and behavior of phenomena which might be used, might be developed, into nondestructive testing techniques or methods. Presumably almost any physical property can be utilized in some way or other as a testing or evaluation technique. Well, of course, there are a great variety of these things, and part of the intent here is to examine into some of these properties - some of these phenomena - to get some understanding, some knowledge, as to what they are and perhaps to develop ideas as to how they might be applied to testing. I think, therefore, that this is a very important, and it certainly should be a very interesting Symposium. I'm sure things of interest and importance will come from it. The program covers a wide range, both in topics covered, and I suspect in the time that it's going to take to run through the program. I think, therefore, that I will stop at this point, and let the program proceed. I'm glad to see so many of you here. I hope that the physical arrangements are to your liking and that you will not only profit from the meeting, but also enjoy it.

W. J. McGonnagle: Thank you, Dr. Foote. The next speaker I would like to introduce is Mr. Jack Connor from the Atomic Energy Commission, the Division of Reactor Development. Mr. Connor.

J. G. Connor: First, on behalf of the Atomic Energy Commission, and more specifically the Division of Reactor Development, I wish to welcome you to this Symposium on Physics and Nondestructive Testing. Secondly, I wish to outline at least a few of the reasons, a few of the basic reasons as to why we in the Division of Reactor Development feel that a symposium of this kind is really important. The first reason centers about the general need, in fact a mandatory requirement, for disseminating information of the type we have here today. The second reason, and I think one which probably hits home to a greater extent, concerns the fundamental nature of the information which we have to be presented. As all of you are well aware, it seems as though the reactor business is today complex with many nondestructive testing problems that had to be solved yesterday. These problems, fortunately, are usually capable of successful solution. However, I, at least, oftentimes wonder if in the urgency involved in these problems we don't overlook a wealth of basic phenomena when the urgency is so persistent. In other words, I think we look at the outer rim of the wheel, and in many cases overlook the hub. Fortunately, today, we're going to examine the hub. The unequivocal center of that wheel. So, we in Reactor Development Division hope that symposia of this type will continue and hope this symposium is a success, because we feel that it will provide and give a very firm foundation on which the science of nondestructive testing can be based. Thank you.

W. J. McGonnagle: As Dr. Foote told you, the purpose of this symposium is to go behind the scenes, so to speak, and look at the fundamental physical principles involved in these many tests in the field of technology called nondestructive testing. We believe that, if we examine the fundamental principles and look at them in more detail, new and better tests will be able to come about so that we may then be able to handle such diverse problems as the mice, and then man, if the case presents itself.

For your information, a recording is being made of the information presented in the next two days so that we can publish a proceedings. One reason why our people are anxious to get your correct address, etc., is so that we can notify you of these proceedings when they are presented. You'll notice that at each of the sessions we have a technical secretary and the job of the technical secretary for each session is to see that the notes are properly edited. We'll also take pictures of the blackboard and discussions that occur on the blackboard so that these can be included in the proceedings, if they are pertinent.

The first paper this morning is entitled, "Neutron Diffraction Techniques and Their Application." This paper was to be given by Dr. Sidhu of Argonne National Laboratory; however, Dr. Sidhu is ill. Dr. LeRoy Heaton, a member of Dr. Sidhu's group, will present this paper. Dr. Heaton.

PROGRAM

SESSION I - PENETRATING RADIATION (9:30 AM - 1:00 PM)

Technical Secretary: N. S. Beyer

Neutron Diffraction Techniques and Their Applications

S. S. Sidhu - Argonne National Laboratory

Neutron Radiography

Harold Berger - Argonne National Laboratory

Examples of the Merits and Usage of Solid State Devices

J. Gilroy - Argonne National Laboratory

SESSION II - WAVE MOTION (2:00 PM - 5:00 PM)

Technical Secretary: H. Berger

Wave Velocities in Nonhomogeneous Elastic Media

W. S. Jardetzky - Manhattan College

Ultrasonic Wave Transmission at Plane Interfaces

H. E. VanValkenburg - Sperry Products, Inc.

Lamb Waves: Theory of Lamb Waves

Miss R. A. diNovi - Argonne National Laboratory

Lamb Waves: Experimental Confirmation of Lamb Waves

D. C. Worlton - Hanford Atomic Products Operation

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Reception - Guest Lounge, Building 617 (6:00 PM)

Dinner - Guest Lounge, Building 617 (6:45 PM)

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Japan - In Film and Slides

K. Ono - Hitachi Ltd., Hatachi, Japan

SESSION III - SOLID STATE PHYSICS (9:00 AM - 12:30 PM)

Technical Secretary: R. G. Peterson

Ultrasonic Attenuation and the Physics of Solids

Rohn Truell - Brown University

The Diffusion of Pulsed Current Fields in Good Conductors

C. J. Renken - Argonne National Laboratory

Magnetic Properties of Reactor Metals and Alloys

M. V. Nevitt - Argonne National Laboratory

SESSION IV - TECHNIQUES IN NONDESTRUCTIVE TESTING
(1:30 PM - 4:30 PM)

Technical Secretary: Miss R. A. diNovi

Mathematics in Nondestructive Testing

W. Miller - Argonne National Laboratory

Statistical Methods

J. W. Butler - Argonne National Laboratory

The Scientific Application of Particle Accelerators to Nondestructive
Testing

E. A. Burrill - High Voltage Engineering Corp.

NEUTRON DIFFRACTION TECHNIQUES AND THEIR APPLICATIONS

by

S. S. Sidhu

ABSTRACT

The nature of neutron radiation, production of thermal neutron beams, neutron diffraction apparatus together with neutron transmission and diffraction techniques and their applications to physical structure of materials, crystallography, magnetic scattering, nuclear physics and the liquid state are presented.

Note: Due to the illness of Dr. Sidhu, the paper was presented by Dr. L. R. Heaton. The material presented by Dr. Heaton can be found in the following reference.

S. S. Sidhu, L. R. Heaton and M. H. Mueller, Neutron Diffraction Techniques and Their Applications to Some Problems in Physics, J. Appl Phys 30, 1323-40 (1959).

NEUTRON RADIOGRAPHY

by

Harold Berger
Metallurgy Division
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Argonne, Illinois

ABSTRACT

Neutron radiography is considered with regard to the problems of the production of the neutron beam and neutron image, and the photographic detection of the neutron image. Neutron sources, the neutron absorption of materials, and an overall comparison of the various methods of detecting a neutron image are discussed.

Introduction

Webster defines a radiograph as "a picture produced upon a sensitive surface, as of a photographic plate, by some form of radiation other than light."⁽¹⁾ A neutron radiograph, then, is such a picture produced by the action of neutrons upon a sensitive surface. Several things are required in order to obtain a neutron radiograph; these include a source of neutrons, an absorbing object, and a surface sensitive to the image carrying radiation. In addition, of course, there should be some reason for employing neutron radiography instead of some other inspection method, such as X or gamma radiography, for example.

The reason for considering neutron radiography is of prime importance. This "raison d'être" for neutron radiography arises because the relative neutron absorption of materials is very different from the relative absorption of these materials for X rays. The mass absorption coefficient for neutrons is quite random if it is examined in terms of regularly increasing atomic number, whereas the mass absorption coefficient for X rays increases with some regularity if it is similarly examined. This fact is very well demonstrated graphically by Thewlis⁽²⁾ who plots mass absorption coefficients for X rays (0.098 Å) and neutrons (1.08 Å) versus atomic number. It is particularly interesting to find very high neutron mass absorption coefficients for several of the lighter elements, which can present rather difficult problems with X-ray techniques. Notably, light materials, such as hydrogen, lithium, and boron, absorb thermal neutrons to a high degree. Other materials having high neutron absorption coefficients include cadmium, samarium, europium, gadolinium and dysprosium.

It should be mentioned at this point that this discussion will be limited to radiography with thermal neutrons. The neutron energies involved, then, will be a fraction of an electron volt, and the corresponding wavelengths will be of the order of an Angstrom unit. The principal reason for not considering neutrons of higher energy is that the relative absorption differences which are prevalent for slow neutrons decrease appreciably for fast neutrons. To the fast neutron, then, most materials are almost equally absorbing. The increased scattering experienced with the use of fast neutrons and the decreased efficiency for fast neutron detection are also factors involved in limiting this discussion to thermal neutrons.

A second limitation on this discussion is concerned with the method of detecting the neutron image. For the most part, photographic detection methods will be discussed. One point which should be mentioned in this connection is that neutrons, fast or slow, have relatively little effect on normal photographic emulsions. The detection techniques which have been employed for the most part, then, make use of some intermediate material which is placed next to the photographic emulsion and which emits some photographically detectable radiation upon neutron irradiation. Materials which have been used for such neutron intensification screens include lithium or boron in conjunction with fluorescent materials (n, α , light reaction), cadmium (n, γ), and beta emitters such as silver, indium and gold. These methods will be more fully discussed in a later section.

Before discussing neutron sources and neutron image detectors in detail let us take a moment to briefly review some previous work in neutron radiography.

Historical Background

Successful neutron radiographic experiments were conducted more than twenty years ago in Germany by Kallman and Kuhn.⁽³⁾ The neutron source used by Kallman employed ion bombardment of a suitable target to produce a source of fast neutrons and subsequent slowing down, or moderation, of the neutron beam by water. The arrangement is pictured in Fig. 1. The water-free channel and the opening in the boron shield made an exit aperture for the slow neutron beam. The object and film pack were covered with lead to lessen the photographic effects of any gamma radiation in the imaging radiation beam. Kallman had greatest success with a detector consisting of photographic film sandwiched between two fluorescent layers. The fluorescent layers were coated with boron or preferably lithium, and the resultant alpha emission from the neutron-bombarded lithium resulted in light emission from the fluorescent layers. This light emission produced film blackening. Further improvements were obtained by Kallman by depositing a thin (<0.5 micron) aluminum reflecting layer between the lithium or boron and the fluorescent layer.

Fig. 1.

Neutron Source Used by Kallman
for Neutron Radiography.

Kallman also investigated several other materials which could be used in the photographic detection of neutron images. In particular, he mentioned the gamma emission from cadmium and the beta emission from gadolinium, silver and indium as being photographically useful for the detection of neutrons. Of these, gadolinium seemed to give the best results.

In this pioneering work in neutron radiography by Kallman, he also recognized the disturbing effects which result from the gamma radiation which is almost always associated with the neutron beam. Neutron image detection systems are usually also gamma sensitive, so that the gamma image is also recorded. Since the advantages of neutron radiography are allied with the fact that the relative absorption of X or gamma radiation is different from that of neutrons, the superimposed gamma radiation image will tend to obscure the sought-for neutron image. Kallman concluded that shielding of the neutron source did not completely solve this problem because the additional gamma radiation produced in the shielding materials tended to offset the shielding advantage they might offer. Increased distance of the object and film from the source was one solution to this problem, and since this solution also tended to increase resolution, it is apparently the one he found most satisfactory. He did point out that neutron pictures which were completely free of disturbing gamma radiation effects could be obtained by irradiating foils which became radioactive and then transferring these radioactive image-carrying foils to photographic film. Materials such as silver and indium were indicated as being especially suitable for this technique.

Peter⁽⁴⁾ extended the work of Kallman and Kuhn by employing a discharge tube of higher intensity and therefore obtaining a neutron source of higher intensity. The source used by Peter was essentially as pictured in Fig. 1 and was said to be equivalent to 13 kg radium. With this increased neutron intensity, Peter could obtain neutron radiographs by exposing X-ray film sandwiched between 0.004-in. silver screens and backed by 0.020-in. cadmium in one to three minutes. This compares to the four-hour exposures reported by Kallman using his lithium-phosphor-film

combination and a neutron source said to be equivalent to 2 to 3 grams radium. Peter also published a picture made by transferring a radioactive image-carrying silver screen to a piece of film out of the neutron beam. Details of exposure in this latter picture were not discussed.

More recently an investigation of neutron radiography was conducted in England by Thewlis and Derbyshire.^(2,5) A nuclear reactor (BEPO) was used to supply a source of thermal neutrons for these experiments and the detection method made use mainly of indium screens and X-ray film. The radiographs published by Thewlis were of comparable resolution quality to many gamma-ray radiographs. In addition to obtaining improved quality radiographs over the earlier work of Kallman and Peter, Thewlis also attempted to point the way toward applications of this inspection method. Among the application areas indicated as being attractive were the inspections of neutron shielding material and heavy metals, and the use of neutron radiography in biological studies. It is the high neutron absorption of hydrogen which makes this technique of potential value in this latter application. Neutron radiography is, of course, ideally suited to the inspection of neutron shielding materials, such as boral sheet. The method also seems to offer advantages in the inspection of heavy materials, such as uranium, because of the reduced exposure time which should result from lower neutron absorption coefficients. The greatest use, however, would seem to be in radiographing objects made of materials which do not lend themselves to other means of inspection. A neutron radiograph of a piece of waxed string in a two-inch lead block was used by Thewlis to illustrate this point very effectively.

Neutron Sources

With this background of previous work in this field, let us now take a look at some of the neutron sources which might be useful for this application. Although there are many source variations possible, the different types of neutron sources can be rather conveniently grouped as (1) radioactive sources, (2) accelerators, and (3) nuclear reactors. From experience with other imaging systems, some desirable characteristics for a neutron radiographic source can be indicated. In order to obtain good image resolution, the source should originate from a small area or have enough intensity so that objects can be radiographed with a considerable distance between source and film in a reasonable time. The source should also provide a uniform radiation intensity over the desired inspection area. A further requirement is that the radiation beam should not contain a masking radiation present in any significantly detectable intensity. In the case of a neutron radiographic source, the ratio of gamma to neutron radiation should be low.

Practically speaking, the first general requirement, that of a small originating area for the neutron source, is difficult to achieve for thermal neutrons. Thermal neutrons are usually obtained from neutron sources of

high energy by allowing the neutrons to diffuse through a moderating material.⁽⁶⁾ For thermal neutron sources, light materials such as hydrogen, deuterium, beryllium, or carbon are normally employed as the scattering or moderating material. The slowing down process in this case is predominantly the result of elastic collisions between the neutrons and these light nuclei. Appreciable thickness of moderating material is usually required to accomplish this slowing down process, and, as a result, thermal neutron sources are generally large in size compared to the focal spot of an X-ray tube, for example.

In addition to this comment on neutron sources in general, a few words about each type of neutron source might be in order.

Radioactive sources of neutrons of practical size usually have low neutron yields, making them rather unattractive for this type of application. The highest yield sources are of the Ra- α -Be type, which produce of the order of 10^7 n/second/curie. These neutrons, however, are fast neutrons, which include energies up to 13 Mev. After moderation and collimation, the thermal neutron yield for radiographic use would be appreciably less. Some idea of possible exposure times using such a source might be gained by a comparison with the work of Kallman, who used a neutron source said to be equivalent to 2 to 3 grams of radium. His exposures were of the order of four hours. An additional problem encountered with radioactive (α ,n) sources is that they have a relatively high ratio of gamma to neutron radiation.* This would pose a shielding problem in the use of such sources for neutron radiography, or would indicate the use of a method such as a radioactive foil to eliminate the masking effect of the gamma radiation.

Radioactive photoneutron (γ ,n) sources employ primarily beryllium or deuterium, since these have relatively low thresholds for the (γ ,n) reaction (1.67 and 2.23 Mev, respectively). The (γ ,n) sources combine a γ -emitting isotope (whose γ energy is above the threshold for the (γ ,n) reaction) with either beryllium or deuterium. These sources usually have a rather low neutron yield and offer no particular advantage over the (α ,n) sources for this application.

High-voltage accelerators are capable of producing much more intense neutron beams than are obtained from practical radioactive sources. As in the case of the radioactive source, however, moderation and collimation of the neutron beam is required in order to make it useful for most neutron radiography. Arrangements such as that used by Kallman and Peter (see Fig. 1) accomplish this purpose.

* One exception to this rule is the Po- α -Be source. The relatively short half-life (138 days) is a major disadvantage, however.

Many neutron-producing reactions are possible with accelerators, including the (γ, n) reaction discussed previously. In the latter case high-energy X rays provide the photon energy necessary for the reaction. Reactions used more generally for neutron production, however, involve the acceleration of positive ions toward various neutron-yielding targets. The neutron yields will be dependent upon the target material, the accelerated particle, the accelerating voltage, and the intensity of accelerated particles. In general, the intensities of fast neutrons available from normal accelerators will be in the range from 10^6 to 10^{11} neutrons/cm²/sec.

By comparison with these other sources, very high neutron intensities are available from nuclear reactors. For example, the CP-5 reactor at Argonne has a neutron flux of 3.5×10^{13} neutrons/cm²/second in the region of highest flux. Other reactors have appreciably higher neutron intensities. Although it is true that the reactor flux which is useful for neutron radiography will be of somewhat less intensity by the time it is collimated and brought out to a useful location, the nuclear reactor still yields a much higher useful flux of thermal neutrons than would be obtained from normally available radioactive sources or accelerators. For this reason, it can be concluded that a nuclear reactor should be the preferred source for use in this application.

However, from a practical standpoint, the accelerator seems to offer the best possibility for more generally available neutron sources for neutron radiography. Although the neutron intensities may be somewhat lower than those obtainable with a nuclear reactor, they still can be of the order of magnitude of those used in this investigation at Argonne (10^4 to 10^5 n/cm²/sec.) and can therefore produce neutron radiographs in a reasonable exposure time (10 minutes or less).*

Neutron Absorption

The absorption of neutrons in material is a combination of what might be called true absorption (absorption of a neutron by a nucleus) and neutron scattering. Both these processes remove neutrons from the incident neutron beam and therefore contribute to absorption. These factors are both considered in a total cross section for the material.

The absorption of neutrons is given by Eq. (1), which is in the form of the familiar absorption equation:

$$I/I_0 = e^{-N\sigma_t x}, \quad (1)$$

* Further work on a comparison of various types of neutron sources for use in neutron radiography is planned. In the meantime, the reader desiring more information is referred to several reviews in the literature.^(7,8)

where I/I_0 is the fraction of the neutron beam transmitted through an absorber of thickness x , e the Napierian logarithm base, N the number of nuclei per cm^3 in the absorbing material, σ_t the total cross section in units of cm^2 (1 barn = 10^{-24} cm^2), and x the absorber thickness in cm.

More extensive discussions of neutron absorption than are appropriate here are given in the literature,⁽⁹⁾ as are tables of neutron cross sections.⁽¹⁰⁾ It should again be emphasized that the fact that the relative neutron absorption of materials is different from that of X radiation is what makes neutron radiography a potentially useful inspection technique. The high neutron absorption of several light elements seems to be particularly advantageous as is the potential ability to discriminate between several materials which have similar X-ray absorption coefficients.

Photographic Neutron Image Detection

In addition to the work on neutron radiography mentioned previously, there have been several other investigations on the photographic detection of neutrons. Among these may be mentioned results in publications concerned with radiation dosimetry,⁽¹¹⁻¹⁴⁾ records of neutron diffraction patterns,^(15,16) and other applications.^(17,18) As has already been mentioned, neutrons have little influence on normal photographic emulsions so that intermediate materials, which emit some photographically detectable radiation, are normally used next to the film. The intermediate materials reported in these references include the alpha emitters boron or lithium combined with a phosphor, cadmium, which emits gamma radiation when bombarded with slow neutrons, and beta emitters (such as indium, silver, gold, gadolinium and rhodium). Photographic neutron detection has also been studied using boron or lithium-loaded film emulsions and standard emulsions.

The characteristics of several of these neutron photographic image intensification materials are shown in Table I. All of the columns shown in the table are of importance in choosing an intensifier material for use in neutron radiography. First of all, the relative abundance of the particular isotope involved should be high so that the naturally occurring material of that element will contain a large number of the desired nuclei. The reaction cross section should also be high in order that the probability for the reaction be large. The radiation emitted should be photographically effective and, finally, the half-life of the material (if it is a radioactive reaction) should be convenient. The relatively poor photographic response found with gold in this report, for example, stems from the long half-life (2.7 days) of the material, and our reluctance to consider exposure times which would be long enough to approach a saturation activity in the gold screens.

TABLE I
CHARACTERISTICS OF SEVERAL NEUTRON PHOTOGRAPHIC IMAGE INTENSIFIERS

MATERIAL	ISOTOPE INVOLVED IN REACTION	RELATIVE NATURAL ABUNDANCE (PER CENT)	CROSS SECTION FOR REACTION (THERMAL NEUTRONS VELOCITY = 2200 m/s) (BARNs)	REACTION	RADIATION PRODUCING PRINCIPAL PHOTOGRAPHIC EFFECT
LITHIUM	Li-6	7.52	910	Li-6(n, α)H-3	α EMISSION OR PREFERABLY LIGHT EMISSION EXCITED IN AN ASSOCIATED PHOSPHOR BY THE α .
BORON	B-10	18.8	3,770	B-10(n, α)Li-7	SAME AS LITHIUM
RHODIUM	Rh-103	100	$\left\{ \begin{array}{l} 12 \\ 140 \end{array} \right.$	Rh-103(n)Rh-104m	ISOMERIC TRANSITION TO Rh-104 OF HALF LIFE 4.5 MINUTES
				Rh-103(n)Rh-104	BETA EMISSION (2.5 Mev) OF HALF-LIFE 44 SECONDS
SILVER	Ag-107	51.35	44	Ag-107(n)Ag-108	BETA EMISSION (1.77 Mev) OF HALF-LIFE 2.3 MINUTES
	Ag-109	48.65	$\left\{ \begin{array}{l} 2.8 \\ 110 \end{array} \right.$	Ag-109(n)Ag-110m	BETA EMISSION OF HALF-LIFE 270 DAYS
				Ag-109(n)Ag-110	BETA EMISSION OF HALF-LIFE 24.2 SECONDS
CADMIUM	Cd-113	12.26	20,800	Cd-113(n, γ)Cd-114	GAMMA RADIATION PLUS SECONDARY RADIATION
INDIUM	In-115	95.77	$\left\{ \begin{array}{l} 145 \\ 52 \end{array} \right.$	In-115(n)In-116m	BETA EMISSION (ENERGY 1 Mev AND LESS) OF HALF-LIFE 54.1 MINUTES
				In-115(n)In-116	BETA EMISSION (ABOUT 3 Mev) OF HALF-LIFE 13 SECONDS
GOLD	Au-197	100	96	Au-197(n)Au-198	BETA EMISSION OF HALF-LIFE 2.7 DAYS

Note: Data for this table were obtained from references 6 and 10 and from Table of Isotopes, by D. Strominger, J. M. Hollander and G. T. Seaborg, Rev. Mod. Phys., 30, 585 (April 1958).

The data in the table indicate that, in some cases, several possible reactions are likely to produce photographically detectable radiation from the intensifier or converter material. For example, in the case of silver, the approximately equal division of naturally occurring material between the two isotopes Ag-107 and Ag-109 and the relatively high reaction cross sections for the 2.3-min Ag-108 and 24.2-sec Ag-110 activities means that both these isotopes will be responsible for much of the photographic effect obtained using silver screens for neutron radiography.

Relatively little work has been done to make a comparison of the relative photographic effect (speed) of these various materials, nor has much been reported on comparing image resolution qualities for these materials. These questions have been the subjects of an investigation of

neutron radiography here at Argonne, and some preliminary results can now be reported. All of the data reported herein have been taken using a monochromatic neutron beam from the Argonne Neutron Spectrometer.⁽¹⁹⁾ In this neutron diffraction equipment, a collimated neutron beam from the reactor (Argonne reactor CP-5) is brought out to a monochromatizing crystal. The monochromatic neutron beam from the crystal emerges through a set of defining slits and a beam monitor to the neutron diffraction sample. The diffracted beams are detected by a BF_3 counter on the movable spectrometer arm. Most of the monochromatic neutron beam is transmitted through, or around, the sample and is caught by a beam catcher at a distance of about 5 to 6 ft from the sample. This beam at the beam catcher, having a maximum intensity of the order of 10^5 thermal neutrons/cm²/sec, has been employed for these tests reported below. A schematic drawing of this arrangement is shown in Fig. 2.

Fig. 2. Schematic Drawing of Neutron Spectrometer Showing Arrangement Used for Neutron Radiography.

This neutron beam has a rather small area of uniform maximum intensity, covering approximately a $1\frac{1}{2} \times 3$ -in. oval at the beam catcher. Scatter increases the overall coverage to about 3×5 in. In spite of this small coverage area, many radiographs have been taken using this beam and, in fact, the beam was particularly useful in obtaining film density data for the various neutron intensification materials. The ratio of gamma to neutron intensity was quite low as far as film-blackening effects are concerned,* and the fact that the neutron beam was monochromatic made absorption calculations relatively simple. The de Broglie wavelength of the neutron beam used for the majority of these measurements was $1.05 \text{ \AA} \pm 0.01 \text{ \AA}$, which corresponds to an energy of 0.055 ev.

* A sheet of No Screen film exposed to the neutron beam without screens for a period three times as long as that used for normal neutron exposures (18-min versus 6-min normal exposure) showed no detectable image.

The first efforts of this current research program attempted to determine experimentally the thicknesses of the various intensifying materials which yielded the greatest film density for a comparable neutron exposure. For the beta emitters, an estimate for the optimum thickness can be made on the basis of the range of the beta-particle energy. However, the intensifier thickness which produces the optimum photographic density can be difficult to determine strictly on a theoretical basis, because the decay schemes usually include several beta and gamma-emission energies. For the gamma emitters, such as cadmium, the optimum thickness should essentially absorb the neutron beam, assuming the resultant screen thickness does not appreciably absorb the gamma radiation.

The optimum screen thickness data presented were obtained by sandwiching X-ray film between known thicknesses of intensifier screen material and exposing this combination to the neutron beam for a known total neutron exposure. In the case of the screens which became radioactive upon neutron exposure, the screens were permitted to remain in contact with the film for a period of at least three half-lives after the neutron exposure was completed. Films and screens were exposed to the neutron beam contained in an aluminum-front, spring-loaded X-ray cassette. In so far as possible, films exposed in a particular set of experiments were processed together to eliminate discrepancies due to developing differences. Films were developed in Kodak Liquid X-ray Developer at $68^{\circ} \pm \frac{1}{2}^{\circ} \text{F}$ for 5 min without agitation. Density measurements were taken using a Macbeth-Ansco densitometer (model 12A) having a density range from 0 to 6.0.

Figure 3 is a graph of the data used to determine the optimum screen thickness for cadmium intensifiers. In the upper portion of Fig. 3, the net density produced on KK X-ray film is plotted against the thickness of the back intensifying screen (the screen farthest from the neutron source).

Fig. 3

Determination of Optimum Cadmium Screen Thickness for Constant Neutron Exposure.

For each curve shown, the front screen was held constant and the thickness is indicated on the curve. It can be seen that the curves all tend to level off for a back screen thickness of 0.020 in. and that the highest density curve is obtained for a front screen thickness of 0.010 in. The lower curve is a plot of film density versus front screen thickness for a back screen thickness of 0.020 in. This curve shows a broad peak in density for a front screen of 0.010 to 0.015 in. From this data, it has been concluded that the optimum film density for 0.05-ev neutron exposure of KK film and cadmium screens is produced by a combination of a 0.010-in. front screen and a 0.020-in. back screen. The shorthand designation used in this article to indicate such an arrangement is 10-20 Cd, where the first number refers to the front screen thickness in mils, the second number to the back screen thickness in mils, and the chemical symbol identifies the material.

It is interesting to note that 0.010-in. cadmium absorbs about 95% of a 0.05-ev neutron beam. The additional density produced by the back screen seems to be the result of electron emission and secondary radiation generated in the cadmium by the hard gamma radiation from the cadmium (n, γ) reaction. This is somewhat confirmed by the fact that there is little film density change produced by replacing the cadmium back screen by a 0.010-in. lead screen, and by the fact that some increase in density is obtained by using the normal cadmium front and back screens but sandwiching the film between 0.005-in. lead screens.

The reported thicknesses of cadmium screens used for photographic neutron detection by other workers varied from 0.010 to 0.040 in. The results of neutron dosimetry investigations reported by Dessauer^(20,21) indicated that the optimum cadmium screen thicknesses for use in mixed radiation fields should be 0.020 in. for front and back screens. Since Dessauer applied his screens by folding a single cadmium sheet around the film pack, however, he may not have investigated front and back screens of different thicknesses.

It should be mentioned that the optimum screen thicknesses determined in this report take into account only relative photographic speed. It is recognized that thinner screens might very well yield improved resolution.

Optimum screen thicknesses for silver and indium have also been determined in a similar manner, and the data are shown graphically in Figs. 4 and 5. Since both these materials become radioactive upon exposure to neutrons, it is necessary to control both the actual neutron exposure and the time that the radioactive screens are in contact with the film after the exposure. For the data presented in Figs. 4 and 5, the period of radioactive transfer was, in both cases, greater than three half-lives. For silver the two isotopes which are most likely to yield a photographic effect are Ag-108 and Ag-110, and the half-lives involved are 2.3 min and 24.2 sec, respectively. The 10-min radioactive transfer time used for these silver data should be sufficient to achieve more than 90% saturation.

Fig. 4. Determination of Optimum Silver Screen Thickness for Constant Neutron Exposure.

Fig. 5. Determination of Optimum Indium Screen Thickness for Constant Neutron Exposure.

In the case of indium, the long half-life of In-116m (54.1 min) means that several hours of radioactive transfer will be required to achieve 90% saturation. The data shown in Fig. 5 were taken using an overnight transfer period.

The results shown in Fig. 4 indicate that optimum screen thicknesses for silver will be a 20-30 combination. In the case of indium, the optimum screen thicknesses indicated in Fig. 5 are also a 0.020-in. front screen and 0.030-in. back screen.

Relatively little data have been found in the literature concerning the use of silver screens of specific thickness for neutron photographic detection. One such reference is that of Peter,⁽⁴⁾ who specified the use of

0.004-in. silver front and back screens in his work on neutron radiography. A consideration of the range of the emitted beta particle results in approximate thicknesses of 30 and 50 mils of silver for Ag-108 (1.77-Mev beta) and Ag-110 (2.84 and 2.24-Mev beta energies), respectively. The two silver isotopes appear to be the most likely ones to contribute to film blackening for the use of silver screens.*

Indium screens used in the neutron photographic investigations of Thewlis⁽²⁾ were 0.005 in. thick. The work of Tittle⁽²²⁾ would indicate that this thickness would be the optimum one to use for a radioactive foil transfer technique. Tittle subjected indium foils to neutron bombardment and found that, among the different thicknesses used, he obtained the highest counting rate from 100-mg/cm² foils. His foils were permitted to decay for 3 min before counting, however, in order to eliminate activities from shorter half-lives. The 20-30 In screen combination reported here does make use of these shorter-lived indium isotopes (particularly of In-116, 13 sec) which have beta energies up to about 3 Mev. These data, then, do not appear to be inconsistent with that of Tittle. On the basis of beta range, also, the optimum thickness would appear to be close to that found in this film work, since the range of a 1-Mev beta (In-116m, 51% of the beta decay has an energy of 1.00 Mev) in indium is about 20 mils. Wollan *et al.*⁽¹⁶⁾ used indium of approximately this thickness (0.5 mm) in their investigation of photographic recording of neutron diffraction images.

Optimum thicknesses for gold and rhodium have not yet been investigated experimentally. The screen thicknesses used for the experiments with rhodium and gold have been of the order of the beta ranges, at least for the front screens.

The relative photographic effect (relative photographic speed) of several of these materials has been investigated for a particular exposure (6 min) to a particular neutron flux (9.4×10^4 n/cm²/sec)** and the data are summarized in Table II (silver has arbitrarily been rated 1.0). Although one cannot put too much reliance on these photographic speed figures, they do indicate the order of magnitude of decrease in exposure times which can be obtained by using the materials with high relative photographic effect. In the case of cadmium and the scintillator,[†] no radioactivity is involved, so the relative speed between these two materials should hold for any exposure situation. In the data for this table, silver has almost reached saturation activity (2.3-min half-life, 6-min exposure) while longer exposures would probably tend to improve the relative speed ratings for rhodium, indium, and gold by allowing these materials to reach a greater percentage of saturation activity.

* See Table I.

** Flux measurements were made by gold foil activation.

† The scintillator used was a commercial adaptation of the B-10-loaded ZnS phosphor described by Sun *et al.*⁽¹⁵⁾

TABLE II

RELATIVE NEUTRON PHOTOGRAPHIC EFFECT^a OBTAINED
WITH VARIOUS INTENSIFYING MATERIALS FOR
CONSTANT NEUTRON EXPOSURE^b

Intensifying Material	Relative Neutron Photographic Effect ^a
B ¹⁰ Loaded Scintillator ^c	6.61
Rhodium ^d (10-30 Screens) ^e	1.93
Indium ^d (20-30 Screens)	1.46
Cadmium (10-20 Screens)	1.32
Silver ^d (20-20 Screens)	1.00
Gold ^f (6-10 Screens)	.37

^a Relative photographic speed obtained by comparing film density obtained on No Screen film and obtaining relative exposure from characteristic curve Silver is arbitrarily set equal to 1.

^b All exposures 6 min in a neutron flux of approximately 9.36×10^4 n/cm²/sec, unless otherwise indicated.

^c Type NE402 neutron detector obtained from Nuclear Enterprises, Ltd The relative exposure for this material was obtained with a 2-min neutron exposure and compared to a similar exposure using Cd screens.

^d Neutron exposures of radioactive screens and film were followed by a three half-life, radioactive decay transfer period. For rhodium this was 13.2 min; the indium radioactive decay period was 2.7 hr, the silver was permitted to decay for a period of 6.9 min

^e Screen thicknesses shown are the front (toward the neutron source) and back screen thicknesses in mils.

^f Neutron exposure of gold screens and film was followed by a radioactive decay period of one half-life (2.7 d).

This information on relative photographic effects confirms comparisons of intensifier materials by other workers Rausa⁽¹¹⁾ has indicated that rhodium yields greater film density than cadmium for the same neutron exposure. Dessauer⁽²¹⁾ further indicated that the order of relative photographic effectiveness was rhodium, indium, and cadmium. Kallman's work pointed out that the effectiveness of cadmium and silver was about the same and that gadolinium was superior to either indium or silver.⁽³⁾

In the matter of comparing these various intensifier materials for image resolution qualities, there appears to be little opportunity to make comparisons with previous work. At this point in the present investigation at Argonne, it appears that rhodium, indium, silver, and gold will all yield useful image resolution qualities, whereas cadmium and the scintillator material presently being used have yielded rather poor resolution thus far.* Figures 6 and 7 demonstrate the improved resolution which is obtained using the beta-emitting screens instead of cadmium. The neutron radiographs shown are positive prints, so white areas are areas of high neutron transmission and black areas indicate neutron absorption. In both Figs. 6 and 7 the same objects have been radiographed with cadmium screens and, with silver screens in Fig. 6, and with indium screens in Fig. 7. The improved definition obtained with the beta-emitting screens is obvious. Screen thickness does not appear to be an important factor in this particular comparison because in both cases the cadmium screens used were thinner than the beta-emitting screens.

Fig. 6. Neutron Radiographs of a Pocket Flashlight Taken With No Screen Film, Neutron Exposure Time in Both Cases Was 6 Min. The Picture on The Left Was Taken Using 10-20 Cadmium Screens. The One on The Right (Sharp One) Was Taken Using 18-18 Silver Screens

* Screen thicknesses used in these resolution tests are those indicated in Table II. An investigation of resolution using different screen thicknesses is planned.

Fig. 7. A Neutron Radiograph of Some Miscellaneous Brass Gears on No Screen Film; The Exposure in Both Cases Was 6 Min. The Picture on The Right Was Taken Using 10-20 Cadmium Screens. The Picture on The Left Employed 20-30 Indium Screens.

In the case of cadmium, the poor resolution obtained can be explained by the fact that the $\text{Cd-113} (n, \gamma) \text{Cd-114}$ reaction yields gamma radiation of very high energy. Several gamma energies are found from this reaction, including energies up to about 9 Mev.⁽²³⁾ The inherent image unsharpness obtained with X or gamma radiography using radiation of such energies is known to be high (of the order of one-half millimeter).⁽²⁴⁾

The poor resolution obtained with the presently used scintillator probably is caused by scatter from the hydrogenous backing material of the scintillator package, and the fact that the scintillator package prevents close contact of the film and scintillator.* Further tests are planned on a scintillator intensifier screen designed to eliminate these objections.

* The scintillator used for these tests was an NE402 neutron detector obtained from Nuclear Enterprises in Winnipeg, Canada.

It should be pointed out that the resolution achieved with a neutron imaging system may be appreciably influenced by the materials contained in the object being inspected and by the materials in the vicinity of the detector, as well as by the detector itself. As was indicated in the discussion on neutron absorption, scatter of the neutrons by the absorbing object may be a major portion of the neutron absorption for a particular material. For example, this is true of hydrogen, which has a total mass absorption coefficient for 1.08-Å neutrons of $48.5 \text{ cm}^2/\text{gm}$, of which $48.4 \text{ cm}^2/\text{gm}$ is due to scattering.⁽²⁾ This is the most extreme example, but, in general, scattering accounts for much slow neutron absorption, particularly among the light nuclei.⁽⁶⁾ This scattered radiation, whether it originates in the inspection sample, the detector, or the material in the vicinity, can contribute to over-all fog and to decreased image resolution.

Another property of these various photographic neutron-detection methods which is of some interest is the relative efficiency for recording gamma radiation. The neutron beam used in this investigation at Argonne is somewhat unusual in that neutron beams normally contain a relatively high gamma-radiation flux. Since the primary advantage in employing neutron radiography is the difference in relative absorption of materials to neutrons and to X or gamma radiation, it is desirable that the detector used have a low response to gamma radiation, in order that the desired neutron image not be masked by the gamma image. Using this as a basis for judgment, the scintillator and rhodium screens appear very good for use in such mixed radiation beams. Silver, cadmium, and indium are grouped next, with gold being the least favorable. These preliminary comparison tests were conducted using 0.05-ev neutrons and cobalt-60 gamma radiation.* Since secondary radiation plays an appreciable part in the photographic detection of hard gamma radiation and, since the secondary radiation generally increases with atomic number, this result was not entirely unexpected.

An imaging method which can be used to eliminate completely the effects of any gamma radiation in the neutron beam is the radioactive foil technique discussed by Kallman and used by Peter. One problem with this technique is that the neutron intensity should be relatively high in order to produce readily usable foil activities. In the neutron flux available for this investigation ($9.4 \times 10^4 \text{ neutrons/cm}^2/\text{sec}$), rhodium and silver do not yield useful film densities even if irradiated and allowed to remain on film for periods greater than three half-lives for each operation. Indium does appear useful, but requires exposure and transfer times of the order of hours. Long times would also be required in the case of gold foils.

* This comparison was made using intensifier material thicknesses as given in Table II and using No Screen film.

To follow up this phase of the investigation and to determine what advantage might be gained by allowing radioactive screens to remain in contact with the film for the case in which the film and screen were exposed to the neutron beam together, some tests were conducted in the following manner. Intensifier screens and film were exposed to the neutron beam for a given exposure, and the intensifier materials were removed from the cassettes at various times after the neutron exposure was completed. In the case of silver, the neutron exposure time was three half-lives (6.9 min). The neutron exposure for rhodium was one half-life (4.5 min) and for indium the exposure was one-tenth half-life (5.4 min). These neutron exposures were chosen as typical of the ones being used in these tests.

For the silver screens, most of the film density was produced during the combined screen and film neutron exposure, since the exposure plus only a one-min radioactive transfer period yielded a density equivalent to 80 to 90% of the relative film exposure which could be obtained by a similar neutron exposure plus a three-half-life (6.9 min) transfer period. This could be increased to 90% to 95% if the radioactive transfer was permitted to continue for a total time of two minutes.

Using the same basis of comparison for rhodium the film density was such that 95% of the possible film exposure was accomplished in the neutron exposure plus a one-min transfer period. In the case of the longer half-life material, indium, approximately 70% of the ultimate relative film exposure was produced by the neutron exposure plus a 3-min transfer. This could be increased to 75% in a 10-min transfer and to 85% in a one half-life transfer period (54 min). This fact is illustrated for indium screens by Fig. 8, which shows a comparison of neutron radiographs of a photographic interval timer taken with the same exposure times but different radioactive transfer periods. In both cases the indium screens and the film were exposed together to the neutron image for a period of 6 min. Figure 8a shows the result obtained with a 10-min transfer period, Fig. 8b that for an overnight transfer period. The results are comparable, indicating that there is no particular advantage to be gained by long transfer times.

These figures certainly tended to confirm the previous results found upon neutron irradiation of only the foil and subsequent transfer to film. It also indicates that an appreciable portion of the photographic exposure is contributed by some of the very short half-life activities associated with these materials. The 24.2-sec beta activity of Ag-110 is one of these short half-life activities which must account for much of the photographic effect produced by the use of silver intensifier screens. The high reaction cross section for this isotope (110 barns) compared to that for the 2.3-min activity Ag-108 (44 barns) backs up this statement.* In the case of rhodium too the high reaction cross section for the 44-sec half-life Rh-104 (140 barns)

* See Table I for a summary of these reaction cross-section values.

versus that (12 barns) for the 4.5-min activity Rh-104m confirms the result obtained. With regard to indium, the reaction cross sections indicate that the 54.1-min half-life activity, indium 116m, contributes the greater share of the photographic exposure. Nevertheless, the high reaction cross section for the 13-sec half-life indium-116 (52 barns) helps account for the fact that a large percentage of the photographic exposure with indium screens is obtained during the neutron exposure.

(a)

(b)

Fig. 8. Neutron Radiographs of an Interval Timer Are Shown. Both Radiographs Were Taken Using No Screen Film and 20-30 Indium Screens and a Neutron Exposure of 6 Min. The Picture at The Left (8a) Employed a 10-Min Transfer Period and The Picture at The Right (8b) Employed an Overnight Transfer Period for The Radioactive Decay of The Indium Screens.

Results such as those indicated above point out the difficulties involved in the radioactive transfer, neutron imaging method and emphasize the need for higher intensity neutron sources and fast transfers, if such techniques are to be used with these materials.

Other Neutron Imaging Methods

Although no successful neutron radiography other than by photographic methods is known to the author, several other possible techniques do appear potentially useful and should be mentioned. One such technique would employ an array of small neutron detectors which could then control an image signal to a display device, such as an electroluminescent panel or a television presentation. Such a neutron image detector has been suggested for use with a silicon p-n junction detector, which can be made quite small and which can be made to be neutron sensitive.⁽²⁵⁾ A second detector which might be useful in such a device is a CdS crystal, which could be quite small and has been shown to have some response to neutrons.⁽²⁶⁾

Although this method does appear theoretically feasible for neutron image presentation, the practical difficulties of fitting small detectors close together to obtain usable image resolution and the problems involved in building an array of many detectors having very similar detection characteristics do appear to be great. These objections might be overcome by some sort of a photoelectric imaging tube, such as those used for X-ray imaging systems.⁽²⁷⁾ Such devices have been proposed for neutron detection, particularly in the patent literature.⁽²⁸⁾ Apparently in this case, too, many practical difficulties would have to be overcome to make such nonphotographic neutron imaging methods useful.

Discussion of Experimental Results

As a result of this investigation some information has been gained concerning the relative merits of several possible intensifier materials for use in neutron image photographic detection. Comparing the materials used in this study, it appears that rhodium is among the best for such use. This comparison has taken into account photographic speed, resolution qualities, and relative neutron-gamma response. In all these cases, rhodium was at or near the top of the comparisons. The B-10-loaded scintillator, which gave the best speed results, cannot be rated very well on an overall basis because of the poor resolution obtained using it.

The other important result of the present experimental investigation concerns the appreciable photographic effect contributed by many of the short-lived activities associated with these intensifier materials. In particular, this result points out the difficulty in using such materials for radioactive foil transfer techniques and the fact that there is little to be gained by permitting these radioactive foils to remain in film contact for very long when the film and intensifier were exposed to the neutron beam together. Of the materials used in this study, indium appears to be the most useful for a radioactive foil transfer imaging technique in a neutron flux of the order of 10^5 n/cm²/sec. Even though the short-lived isotope In-116 (13-sec half-life) contributes appreciable photographic effect, the longer-lived

isotope In-116m (54.1-min half-life) still renders such transfer methods usable, even with the neutron flux used in this study ($0.94 \times 10^5 \text{ n/cm}^2/\text{sec}$).

Conclusions

The neutron radiographs presently being produced can be obtained in reasonable exposure times and yield resolution qualities generally equivalent to those obtained with gamma radiography. The most promising potential applications for neutron radiography are naturally associated with the relative absorption differences for neutrons versus those obtained with X radiation. It is in performing inspections which take advantage of these differences that neutron radiography should be most useful.

Another area of application which may be useful is in biological studies, in which the high neutron absorption of hydrogen is the contributing factor. The radiograph shown in Fig. 9 illustrates this possible area of application.

Fig. 9. This is a Neutron Radiograph of a Grasshopper, Showing Primarily The Areas of High Hydrogen Content Within The Grasshopper. The Exposure Was 40 Min Using 20-30 Indium Screens and AA Film.

It is difficult to predict, at present, whether or not neutron radiography will ever be widely used as an inspection method. It does, however, seem to offer certain advantages for some types of inspection problems, and, as the techniques improve, one might reasonably expect it to come into at least limited use as one more inspection method available for use with such problems.

Acknowledgement

It is a pleasure for the author to acknowledge the assistance and cooperation of many Argonne National Laboratory people in this investigation. In particular, the author would like to mention Dr. Warren J. McGonnagle for his encouragement and helpful discussions; Drs. S. S. Sidhu, L. R. Heaton and M. H. Mueller for their cooperation in making neutrons available for this study, and Mr. I. R. Kraska for his assistance throughout this project.

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Discussion NEUTRON RADIOGRAPHY

Dragsdorf, Kansas State University: I wonder if using a particular type of sample for your screens, say a loaded sample, wouldn't increase the electron output or electron efficiency which the film would seek. In other words, I notice your boron-loaded simulator here had quite a bit greater efficiency than the others. Wouldn't this be also true of, say, silver or indium?

Berger: Are you suggesting that we use boron and lithium-loaded emulsions?

Dragsdorf: No, I'm suggesting that maybe instead of using foils, that you use a screen which contains small particles of that foil material, so that you would increase the electron efficiency output from this screen.

Berger: Sounds like a good idea. I certainly don't see any objection to it. Actually from the resolution standpoint, I think we're going to have to use screens somewhat thinner than we used, so that this may not be an appreciable factor in the end result. But I think your point is well taken.

D. S. Eppelsheimer, Missouri School of Mines: I wonder if you've ever tried this scheme in trying to resolve the location of hydrogen in metal structures where embrittlement has been reported as a concentration of hydrogen in certain regions.

Berger: No, we haven't actually tried it, but it's something we'd like to take a look at in the near future.

H. M. Cleare, Eastman Kodak: The scintillator that you used in this case was the phosphor zinc sulphide or silver-activated zinc sulphide? Is there a blue or a yellow-green emitter?

Berger: The emission is mainly in the blue. The phosphor is silver-activated zinc sulphide.

Cleare: And the film material you used with this?

Berger: We have used several types of X-ray film, KK down to M.

Cleare: Did you use any of the fluorescent screen type film materials - - blue brand, royal blue . . . ?

Berger: In some of our very early work we tried the normal screens used for X-ray work, and these did not work too well, however; we should probably try them again, now that we've improved our technique somewhat.

Cleare: Thank you.

J. Kastner, Picker X-Ray: I just want to make a comment about that scintillator. I think that's a plastic scintillator from UK Enterprises and not zinc sulphide at all. I believe it's an organic like anthracene or something like that.

Berger: I wrote to Nuclear Enterprises and mentioned the work of Sun in discussing their scintillators with them. Sun used a B-10-loaded plastic, a boron compound in which was embedded zinc sulphide. I'm assuming that they furnished me the same type of scintillator.

L. Goodman, Atomics International: You didn't mention it specifically, but I assume from the information you did give that, whatever you're going to use as your sample, or whatever it is you're looking at in the neutron radiographic technique, you're not going to use a material which in itself becomes active as a result of neutron irradiation.

Berger: This is probably true. If you are using a material which contains silver, for example, it would become radioactive. However, the betas would probably not get through the aluminum cassette front, so I don't think they would affect the resultant picture. You may see an example of the type of thing you're talking about in one of the pictures on the viewer. These pictures show the image of a cadmium strip taken using these various intensifier materials. Appreciable darkening within the shadow of the cadmium strip can be seen for gold-intensifying materials, and it is the secondary radiation from the gamma emission from the cadmium which seems to produce the darkening in that picture.

Clarke Williams, The U. S. Bureau of Public Roads: From this question you'll understand that I'm a highway engineer, and not a physicist. Is there any hope of this being ever developed to the point that it can make a neutron examination in the field? Right now I've followed the geometry of the thing and it's strictly adjacent to a reactor or an accelerator or something of that nature. We are interested in, for instance, discovering whether or not reinforcement in concrete has deteriorated by corrosion or cable ravelling in pre-stress concrete. Do you see any hope of this becoming a field instrument?

Berger: The problem, of course, is connected with the neutron source, and frankly I don't think I can answer your question. You could use a radioactive source, which might be portable and might not, depending upon how strong you are, but the neutron yields from these sources are relatively feeble.

Williams: You mentioned the possibility of radiography using a radioisotope, but it would probably take 4-hr exposure or something of that nature.

Berger: This was based on the work of Kallman, who had a source said to be equivalent to 2 or 3 grams of radium. That's right.

Williams: Thank you.

A Participant: I'd like to ask a question. Most of us are familiar with microradiography. Have you ever made any attempts along these lines using neutrons? Now, I can realize that the resolution of the photographic emulsion of the plate would be very important, so if you have tried it, have you tried any of the spectrographic plate emulsions as a photographic material for this application?

Berger: No, we haven't. And I think probably the present limitation on our resolution is not the film, though many of our pictures are quite grainy. The limitation seems to be in the thickness of the intensifier screens we've used. I see no reason why we couldn't appreciably improve our resolution by using thinner screens and increasing exposure time. We haven't yet done too much with improving resolution, but this is something we hope to do.

EXAMPLES OF THE MERITS AND USAGE OF SOLID STATE DEVICES

by

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We could not in a reasonable time discuss many of the solid state devices in sufficient detail to be valuable, so let us consider here only a few particular examples, and emphasize developing a critical viewpoint concerning their advantage, their limitations, and the type of precautions necessary in their use.

Our first example is thermocouples. Here one of the basic limitations of most solid state devices has long been recognized: the need for extreme purity or reproducibility of composition, not just during manufacture of the basic materials, but also during fabrication and use. That this need is not yet fully satisfied, even for "conventional" materials, is shown by a recent article citing apparent temperature errors of over 10% in molybdenum wires, and an apparent shift of over 2% in a certified platinum/platinum-rhodium junction. How many of us would consider it necessary to calibrate successive pieces of wire individually, or to recalibrate a certified junction frequently?

In the thermocouple art we can also illustrate one of the current assets of solid state devices: new materials, or old materials in new purity or reproducibility, may at any moment offer entirely new prospects in instrumentation. These may make yesterday's judgments of proper test methods or equipment quite erroneous. The first reference exemplifies this, indicating that tungsten/rhenium offers both high signal and high stability to over 4000°F.

Until recent years, most efforts for new thermoelectric materials were directed toward higher junction voltages, better linearity, or more stability. Now the goal is efficiency in conversion of heat to electricity or electrical heat "pumping," with startling success in experimental materials. This may necessitate analyzing some circuits by viewing thermocouples as current rather than as voltage sources. In any event it will make possible effective local cooling by reverse current. It is the author's belief that this will bring about a revolution in instrumentation, because in many instances measurements are not limited or made difficult by the use of electric heaters, and mechanical cooling is relatively too complicated for extensive use.

Finally, present thermocouple usage probably illustrates the worst pitfall possible: use of the wrong device. In the author's opinion, more than half of the applications for which thermocouples are being placed in service today could use other devices advantageously. The reasons for the use of thermocouples are obvious - availability of standard materials and associated measuring equipment, and familiarity with their use. But platinum resistance thermometers typically provide more than 10 times as much change of voltage with temperature. This makes possible $\frac{1}{10}$ the "pickup" problems, relative freedom from contact and switching difficulties, and better stability and reproducibility. Their technology is summarized in the second reference.

Where sensitivity is important, Thermistors or Sensitors generally offer 100 times the signal voltage of thermocouples. Their relative natures are discussed in the third reference. These devices are too new for their stability and reproducibility to be well known, but their temperature coefficients appear to be very stable, so that they are probably the best device for most differential measurements.

For our next example, let us consider two solid state devices for radioactivity measurements, the thallium-activated sodium iodide crystal and the semiconductor junction. Again the purity problem is foremost. A sodium iodide crystal reacts irreversibly with atmospheric water vapor to become opaque to its scintillant radiation, so that meticulous packaging is required. Experience indicates that this problem has been adequately solved. For most quantitative analysis work, this is the device of choice, when coupled with a photomultiplier tube. Silicon devices are now being studied as direct solid state transducers, both barrier type formed by an added gold layer and junction type from impurity "doping." The edges of these devices react with the atmosphere to develop deleterious surface conductivity, and it is probably not yet certain that the present techniques of sealing in plastic or glass are adequate for long-term use.

We also see once more the impact of current technology. Search for more efficient scintillators than thallium-activated sodium iodide, NaI(Tl), does not look promising, and in it only about 1 photon is produced per 40 electron volts dissipated, with corresponding inefficiency of electron production at the photocathode. This is in extreme contrast with the production of about 1 electron-hole pair per 3 electron volts dissipated in a typical silicon device. On the other hand, attempts to produce large volumes of sensitive silicon appear fruitless at the present time.

Based on these limitations, the choice between these detectors will usually be fairly clear for a considerable time to come. Where small sensitive volume or area is critical - as in arrays whose elements are smaller than photomultiplier tubes, or when heavily ionizing particles are to be observed in a strong background of other activity - silicon or similar detectors will probably take precedence, and these are likely to offer better resolution at low energies. But for typical gamma spectrometry, the stability and reproducibility of large NaI(Tl) crystals will probably be dominant.

In both the examples discussed above, the question of choice of solid state device was dismissed with generalities, but this is not proper in a critical evaluation of testing methods or devices. This can only be obtained through a detailed analysis of each of the various systems proposed, with a most suspicious viewpoint. For instance, in contrast with the 2% measured difference in subsequent readings on platinum/platinum-rhodium thermocouples mentioned above, a system using this device, without provision for recalibration was recently claimed to have a constancy of about 0.013% (reference 4).

To illustrate the use of a detailed analysis to provide a critical evaluation, let us concentrate on the AC Differential Transformer Bridge, and compare it with a typical Wheatstone Bridge. A secondary purpose of this discussion is to call attention to a bridge system which has surprising advantages in some applications.

Figure 1 shows the schematic diagram, and the expression for output current which we have derived using fairly reasonable assumptions.

Assumptions:

R_1, R_2, R_3 are constant and purely resistive.

Transformer coupling coefficients are unity.

Pure sine wave generator, angular frequency ω .

Magnetic permeability is constant, $G = 1.256 \times 10^{-6} \frac{\mu A}{l}$

Then:

$$I_3 = \frac{I_T G \omega (n_1 R_2 - n_2 R_1) n_3}{R_3 (R_1 + R_2) \sqrt{1 + \left[\frac{G \omega n_3^2}{R_3} + \frac{G \omega (n_1 + n_2)^2}{R_1 + R_2} \right]^2}}$$

Fig. 1. Differential Transformer AC Bridge

With more realistic assumptions (inconstant permeability, non-unity coupling, meter or transducer impedance, or distributed capacity in the transformer secondary), this derivation appears to be hopelessly complex.

Since this equation seems surprisingly elaborate, the partial circuits of Figs. 2 and 3 and their equations are presented. These indicate the contribution of induced currents in the secondary and primary loops to the general equation of Fig. 1.

Fig. 2.

"Reduced" Differential Transformer with "Range Switching" Primary Only

$$G = 1.256 \times 10^{-8} \frac{\mu A}{L}$$

μ = Permeability ratio, $\frac{\text{core}}{\text{air}}$

A = Core cross-section, cm^2

L = Core length, cm

ω = Angular frequency

$$I_3 = \frac{I_0 G \omega n_0 n_3}{R_3 \sqrt{1 + \left[\frac{G \omega n_3^2}{R_3} + \frac{G \omega (n_1 + n_2)^2}{R_1 + R_2} \right]^2}}$$

Fig. 3. "Reduced" Differential Transformer with Loading by Bridge Primaries

For simplicity, let us consider the general equation for the case $n_1 = n_2$, shown in Fig. 4. If we assume a fixed number for n_1 and consider n_3 as a variable, there is a value n_3^1 for which the bridge output current is a maximum for a small difference in sensing and reference resistors. Similarly, if we assume a fixed n_3^1 , there is an optimum value for n_1 , which is not equal to the number assumed for the first calculation. Considering both n_1 and n_3 as variables, the situation is that the output current is a maximum only when n_1 and n_3 are both infinite, and both approach infinity such that n_3^2/R_3 is approximately equal to $(2n_1)^2/(R_1 + R_2)$.

When $n_1 = n_2$, $Q_1 \cong Q_3$, $Q_1 > 1$, $R_1 \cong R_2$

$$I_3 \cong I_T \frac{(R_2 - R_1)}{4\sqrt{2R_2R_3}}$$

$$\left(\text{Comparable D.C. Bridge Equation, } I_G \cong I_T \frac{(R_2 - R_1)}{4(R_2 + R_3)} \right)$$

If maximum power in sensing resistor R_1 , and $R_2 - R_1 = \Delta R$

$$I_3 \cong \frac{\sqrt{P_1}}{2\sqrt{2}} \frac{\Delta R}{R} \frac{1}{\sqrt{R_3}} \left(\text{D.C. Bridge, } I_G \cong \frac{\sqrt{P_1}}{2} \frac{\Delta R}{R} \frac{\sqrt{R_2}}{(R_2 + R_3)} \right)$$

$$E_3 \cong \frac{\sqrt{P_1}}{2\sqrt{2}} \frac{\Delta R}{R} \sqrt{R_3} \left(\text{D.C. Bridge, } E_G \cong \frac{\sqrt{P_1}}{2} \frac{\Delta R}{R} \frac{R_3 \sqrt{R_2}}{(R_2 + R_3)} \right)$$

$$\text{When } n_1 = n_2, I_3 = I_T \frac{(R_2 - R_1)}{(R_2 + R_1)} \frac{G\omega n_1 n_3}{R_3 \sqrt{1 + \left[\frac{G\omega n_3^2}{R_3} + \frac{4G\omega n_1^2}{R_1 + R_2} \right]^2}}$$

$$\text{Let } \frac{G\omega n_3^2}{R_3} = \frac{L_3 \omega}{R_3} = Q_3 \quad \frac{G\omega (2n_1)^2}{R_1 + R_2} = \frac{L_{1+2} \omega}{R_1 + R_2} = Q_1$$

$$I_3 = I_T (R_2 - R_1) \sqrt{\frac{Q_1 Q_3}{4(R_1 + R_2) R_3 [1 + (Q_1 + Q_3)^2]}}$$

$$\text{Partial Maximum when } Q_3^2 = 1 + Q_1^2 \text{ or } Q_1^2 = 1 + Q_3^2$$

Fig. 4. Differential Transformer
AC Bridge

Also, because the equations for partial maxima involve the fourth power of the numbers of turns, we see that a moderate fractional mismatch of n_1 and n_3 , as related to R_1 , R_2 , R_3 and $G\omega$, results in a drastic loss of output current. However, for reasonable values of the circuit constants, the output is close to its ultimate when $Q_1 \approx Q_3$ and each is greater than unity. Also, with a good transformer core (high G value), this occurs for very moderate numbers of turns if R_1 and R_3 are reasonable.

The practical result of this is that n_1 , n_2 and n_3 must be carefully chosen to be consistent with R_1 , R_2 and R_3 , and (in combination with G and ω) to give Q values of about 2. Otherwise, the bridge may be extremely insensitive. Although the equations indicate that large values of n are preferable, the loss of power in the distributed capacity of the transformer secondary may then be excessive.

G is difficult to compute, since the permeability value needed is that for very weak fields. Also, R_3 is typically not known from the manufacturer's literature. The circuit of Fig. 2 is very useful for determining G and R_3 , by measuring I_0 , etc., and connecting a high-impedance voltmeter across the load resistor R_3 .

In a typical measuring system, bridge sensitivity is limited by the power which may be developed in the sensing resistor. Also, a typical transducer, such as a resistance thermometer or strain gage, produces a particular fractional change in resistance in response to a physical change, regardless of its actual resistance value. So, it is advisable to consider P_1 , the power developed in the sensing resistor, and $\Delta R/R$, the fractional resistance change, as the two fixed constants of the transducer. The final approximations of Fig. 4 then become particularly worthwhile for evaluation.

We see that the bridge output current is then limited solely by the meter resistance, and independent of the value of the sensing resistor. Another way of viewing this is to say that proper choice of turns enables us to make a "perfect" impedance match between any reasonable values of transducer and meter resistance.

We see that this is also true if the bridge is feeding a voltage amplifier rather than a current meter - and that now the higher the amplifier input impedance, the larger the voltage input signal. (Practically, however, this is limited by distributed capacity in the secondary winding.) This ability to use low transducer resistance without penalty is especially valuable in avoiding "pickup." For instance, a bridge with sufficient sensitivity to respond to a fractional change in resistance of about 10^{-5} operated satisfactorily with no shielding of the transducer leads. It was insensitive to touching the terminals, using a sensing resistance of about 0.5 ohm. (The transformer core and secondary leads were of course shielded.)

Figure 4 also gives the comparative equations for a constant current Wheatstone-type DC Bridge with approximately equal arms. (In these equations, R_3 is the galvanometer resistance.) We see that for fixed transducer power and percentage resistance change, both the current and voltage output fall off badly with increasing transducer resistance. Also, the output current falls off more rapidly with increased meter resistance than in the Differential Transformer Bridge, and the output voltage does not continue to rise with amplifier input impedance.

Thus, with a transducer resistance of 1 ohm, and an amplifier impedance of 1 megohm, the AC Bridge looks, by comparison with the DC, like a magnetic amplifier with gain of 707. We see also that this is an extremely stable gain, since the permeability and frequency do not show up at all in this approximation.

However, while this theory looks promising, we must look carefully at the effect of our inadequate assumptions. Figure 5 shows a typical set of experimental values (heavy lines) compared with computations from the theory. The discrepancies are drastic. The waveform was that from a Sola transformer, and the theoretical curves were computed for circuit values as measured at about 50 microamperes. Correcting them by using measured values at the actual currents still left considerable discrepancy. In this case the core permeability increased with current about 30%, while the meter resistance decreased over a decade.

Using this meter from zero balance results in very low sensitivity and poor linearity. On the other hand, if used always at half-scale deflection or more (either as an out-of-balance bridge or by adding an appropriate range-switching current), the system has several times the theoretical sensitivity and is especially sensitive for small changes in resistance. (Note the wide relative separation of the top experimental curves of Fig. 5.) Under these circumstances, breaking the overall transducer variation into several ranges can give high linearity within each range, as well as large equivalent total deflection. Operating entirely out of balance may also avoid transducer "sign" uncertainty.

If a high-gain amplifier is used, so little flux need be produced in the core that the permeability change may be negligible. In one experiment with an additional amplifier gain of over 1000, operating from zero balance, there appeared to be less than 2% departure from linearity in three decades.

The predicted independence of frequency seems experimentally correct, provided that Q values of about 2 are used, and neither core loss nor distributed capacity becomes excessive. Sixty to 3000 cps is a typical range.

Thus, we see that an AC Bridge system is available that offers considerable attraction in simplicity and sensitivity for use with low-resistance transducers, and is particularly suitable for feeding high-impedance amplifiers.

This system can provide a large change in output voltage for a small change in bridge voltage, so we may view this device as a solid state magnetic amplifier. We see that the details of this analysis have pointed out that its advantages are strictly dependent on the conditions of its use, and can only be critically evaluated by careful comparison with conventional instruments.

We should note that many of the advantages of the Differential Transformer Bridge were pointed out long ago by Professor Trowbridge (reference 5), and are recognized in its use with a movable core as a linear motion transducer. Also, we are very grateful to Messrs. C. J. Davis, A. J. MacKay, and D. N. Eggenberger for help in this study.

Fig. 5. Comparison of Theory and Experiment for a Differential Transformer AC Bridge

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WAVE VELOCITIES IN NONHOMOGENEOUS ELASTIC MEDIA

by

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ABSTRACT

The velocity of wave propagation is easily defined for homogeneous elastic media. More precision in the definition is necessary in the case of heterogeneous media. Different approaches are used in the American and Soviet literature. Wave fronts are derived from Fourier transforms in the case of one homogeneous medium or in layered homogeneous media. There are no wave equations for potentials in the case of heterogeneous media since the dilation and rotation are coupled. The equations of dynamics for small displacements must be considered simultaneously with the equations of geometric optics. The latter can be interpreted as a certain kind of kinematical constraints. Then the velocity of propagation can be defined from the Malus Law.

An exact relationship between the phase velocity and the factor $\beta = (\mu/\rho)^{1/2}$ for the two-dimensional case can be easily derived. It is then shown that the phase velocity depends on the period and variations in the amplitude in addition to rigidity and density.

The velocity of wave propagation is easily defined for homogeneous elastic media. If a displacement $\vec{s}(u,v,w)$ of a particle of a solid body is given in terms of a scalar potential ϕ and of components ψ_i of a vector potential, these four functions will satisfy wave equations of the form

$$\nabla^2 \phi = \frac{1}{\alpha^2} \frac{\partial^2 \phi}{\partial t^2} \quad ; \quad \nabla^2 \psi_i = \frac{1}{\beta^2} \frac{\partial^2 \psi_i}{\partial t^2} \quad ; \quad i = 1, 2, 3, \quad (1)$$

where

$$\alpha^2 = (\lambda + 2\mu)/\rho \quad ; \quad \beta^2 = \mu/\rho \quad ; \quad (2)$$

and λ, μ are Lamé constants, with ρ the constant density of the medium.

Similar equations exist for the dilatation (θ) and rotation ($\vec{\Omega}$):

$$\nabla^2 \theta = \frac{1}{\alpha^2} \frac{\partial^2 \theta}{\partial t^2} \quad , \quad \nabla^2 \vec{\Omega} = \frac{1}{\beta^2} \frac{\partial^2 \vec{\Omega}}{\partial t^2} \quad . \quad (3)$$

Then, a trial substitution for plane waves, for example,

$$u = A_1 \exp \left[i \frac{2\pi}{\ell} (r_1 x + r_2 y + r_3 z + ct) \right], \quad v = \dots, \quad w = \dots, \quad A_1 = \text{const}, \quad (4)$$

in the equations of motion in a solid body, when the body forces are omitted, yields a cubic equation for c^2 with the first root equal to α^2 and a double root equal to β^2 . Thus we obtain two velocities of propagation. The interpretation is maybe even better seen if we solve the first equation in (1) or (3) in spherical coordinates. The general solution is

$$\phi = \frac{1}{R} [f_1(R - \alpha t) + f_2(R + \alpha t)] \quad , \quad (5)$$

and just for waves propagating from a point source

$$\phi = (1/R) F(R - \alpha t) \quad . \quad (6)$$

Now α is interpreted as the velocity of compressional (longitudinal) waves, and from solutions similar to (5) or (6) for remaining equations in (1) and (2), β is defined as the velocity of shear waves.

The importance of conditions (2) is evident for two kinds of problems. In order to be able to study a wave propagation in a medium, we must know its physical parameters: either the Lamé constants or Young's modulus and Poisson's ratio, in addition to the density of the medium. On the other hand, a measurement of the velocities of the two kinds of waves makes it possible to determine the physical parameters. This idea was widely used in seismology, and led to important results about the composition and structure of the Earth. Recently the number of investigations dealing with the properties of rocks under laboratory conditions and in situ is growing rapidly.

It is well known that there are only two kinds of waves in an infinite isotropic homogeneous solid medium. If there are boundaries, new kinds of waves are generated through reflection, refraction, and interference, and some of them propagate with velocities different from α and β . An example is given in the case of a half-space, i.e., a semi-infinite solid medium, along the surface of which Rayleigh waves are propagating. Their importance in seismology cannot be overemphasized. If we use the notations v_p , v_s , and v_R for velocities of compressional, shear, and Rayleigh waves, respectively, we have $v_R = 0.9194 v_s$.

If along a profile the travel time curves for longitudinal and Rayleigh (or transverse) waves and the corresponding velocities can be found, the Knopoff's diagram yields Poisson's ratio. Then, we can also determine the Young's modulus and the shear modulus, as has been done, for example, by Myachkin and Solov'eva⁽¹¹⁾ for wave propagation at a frequency of 50 kc in a potassium mine and on a short base, i.e., 30-100-cm-long profiles (with an accuracy in velocity determination of 1-3%). Some new methods for the determination of the v_p/v_s ratio in rocks are suggested in the paper by Molotova and Vassil'ev⁽¹⁰⁾ in which an extensive list of references is also given. In the same issue, Kovalev and Molotova⁽⁷⁾ describe a new device for generating shear waves in a bore hole. More and more attention is given by Soviet investigators to factors other than velocities of propagation, e.g., to amplitudes, wave polarization as recorded on seismograms, and predominant periods of vibration. For example, Kogan, Passechnik, and Sultanov⁽⁶⁾ pointed out the difference in periods of surface waves (and other kinds) generated by earthquakes and explosions.

If we consider a medium with more individual homogeneous layers, new types of waves are known to propagate in it: Love waves, Stoneley waves, and many others. With increasing number of layers, more theoretical difficulties arise, as compared with the simpler rules of geometric optics. But even in the case of solutions for a single medium we are still confronted with certain problems. Love,⁽⁹⁾ for example, pointed out that the classical Stokes analysis "is not adequate to express the effects of an arbitrary initial disturbance confined to a finite portion of a medium." Moreover, the Poisson's formula cannot represent any quantity which does not vanish at the front of an advancing wave. Even the more general Kirchhoff's formula needs, according to Love,⁽⁸⁾ an extension. These are solutions to wave equations. However, Baker and Copson⁽¹⁾ noted that sometimes we should consider the Maxwell equations rather than the wave equations, since the components of an electric and magnetic vector representing some secondary disturbances derived from Kirchhoff's solution do not satisfy the Maxwell equations themselves. Pendse⁽¹²⁾ arrived at the conclusion that even the use of scalar and vector potentials is not always free from ambiguity.

The last remark about this kind of theoretical difficulty which I would like to make is that of Love⁽⁹⁾ for the case of an initial disturbance within a certain limited region. Love found, namely, that there are effects which are propagated with velocities of compressional and distortional waves (α and β in a homogeneous medium) as well as others which are propagated with velocities ranging from α to β .

Let us now review the situation in the case of an isotropic heterogeneous medium. The vector equation for small displacements in a heterogeneous medium has the form

$$\rho \frac{\partial^2 \vec{s}}{\partial t^2} = \rho \vec{F} + \nabla(\lambda \theta) + \mu \nabla \theta + \mu \nabla^2 \vec{s} + 2(\nabla \mu \cdot \vec{s}) \quad , \quad (7)$$

where the symmetric strain tensor Φ is defined in the well-known way. If we apply the operation ∇ , taking the scalar and vector products, we obtain from (7) two equations with terms $\nabla \cdot \vec{s} = \text{div} \vec{s} = \theta$ and $\nabla \times \vec{s} = \text{curl} \vec{s} = 2\vec{\Omega}$ which cannot be separated. Thus the dilatation and rotation are coupled and there are no individual wave equations for various potentials in the case of a heterogeneous medium.

Nevertheless, there are wave fronts and rays, and the question arises as to what is now meant by the velocity of propagation.

In order to arrive at a conclusion, we shall consider a simple case. We have three scalar equations equivalent to Eq. (7):

$$\rho \frac{\partial^2 u}{\partial t^2} = \rho X + \frac{\partial}{\partial x} (\lambda \theta) + \mu \nabla^2 u + \mu \frac{\partial \theta}{\partial x} + 2 \frac{\partial \mu}{\partial x} \frac{\partial u}{\partial x} + \frac{\partial \mu}{\partial y} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \frac{\partial \mu}{\partial z} \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right), \dots \quad (8)$$

and one can easily see where there are the coupling terms. Under certain conditions some of them vanish, and only the components u and w remain coupled. In one of the simplest cases we have one component in a medium where no external forces are acting ($y=0$), and all characteristic parameters depend solely on the coordinate z . Then the problem is reduced to one equation:

$$\rho \frac{\partial^2 v}{\partial t^2} = \mu \nabla^2 v + \frac{\partial \mu}{\partial z} \frac{\partial v}{\partial z} \quad (9)$$

This equation has been investigated for several particular functions $\mu = \mu(z)$ and $\rho = \rho(z)$. The quantity $\beta = (\mu/\rho)^{1/2}$ is referred to in these investigations as the velocity of propagation of the disturbance, obviously by analogy with the case of a homogeneous medium.

L. deBroglie in his famous paper⁽²⁾ marking the origin of wave mechanics suggested that the term "wave propagation" be used if there is a function $\chi(x, y, z, t)$ describing a phenomenon and satisfying the wave equation

$$\Delta \chi = \frac{1}{V^2} \frac{\partial^2 \chi}{\partial t^2} \quad (10)$$

where the velocity of the wave propagation V can vary in the medium but does not depend on time. Sokolev⁽¹⁴⁾ generalized Kirchhoff's formula for such a case.

The harmonic solution of Eq. (10) is usually written in the form

$$\chi(x,y,z,t) = A(x,y,z) e^{i\omega[t-S(x,y,z)]} \quad (11)$$

The expression in brackets is the phase (or $[ct - S^*]$). The quantity V is called the velocity of wave propagation by analogy with the case of a homogeneous medium, where A and V are constant, and V therefore coincides with the phase velocity. In a heterogeneous medium the amplitude will not remain constant, at least for the reason that if a refraction occurs in a layered medium the refracted waves have different amplitudes from those of incident waves.

The equation

$$S(x,y,z) = \text{constant} \quad (12)$$

represents wave surfaces (or fronts), and the family of curves orthogonal to the surfaces (12) is the family of rays.

The phase velocity is by definition

$$V = 1/(dS/dr) = [(\partial S/\partial x)^2 + (\partial S/\partial y)^2 + (\partial S/\partial z)^2]^{-1/2}, \quad (13)$$

if dr is a linear element along a ray.

According to de Broglie, we have to see whether there is a simple relationship between the two "velocities" V and V .

Assuming that

$$\chi = Ce^{i\omega t + i\phi} ; C = \text{const} ; \phi = a + ib ; S = -\frac{a}{\omega} ; A = Ce^{-b}, \quad (14)$$

de Broglie arrived at the conclusion that, if the second derivatives of ϕ are very small as compared to $(\text{grad } \phi)^2$, one can use $V = \phi$ and the methods of geometric optics can be applied. The condition introduced by de Broglie seems to be rather a formal one, being of a purely mathematical nature.

If there are doubts about the interpretation of the quantity V as a velocity of a wave propagation in case of the wave equation for a heterogeneous medium, they can only increase in case of Eq. (9). Why should $\beta = (\mu/\rho)^{1/2}$ represent the wave velocity? Why, for example, are no derivatives of μ or ρ involved in the definition? Is it possible that some restrictions for the application of geometric optics just result from the fact that the phase velocity V is identified with V ? Among the attempts to amend the expression for β should be mentioned the study of Sezawa and Kanai,⁽¹³⁾ who assumed that the vibration of a particle in an elastic medium is resisted by a force proportional to its displacement $(-a'v)$ in addition to the classical elastic forces. They also link β to the wave length. However, no justification for such an assumption was given.

As is well known, the results of geometric optics can be applied to the propagation of various kinds of physical phenomena due to the fact that some basic statements of geometric optics remain true for any kind of corresponding disturbances. The Malus law is one of those statements. The rays in an isotropic medium are defined by the set of normals to the surfaces $S = \text{const.}$, i.e.,

$$\text{grad } S = n \vec{a}_0, \quad (15)$$

where $\vec{a}_0$ is a unit vector in the direction of the normal and n is the index of refraction. Hence,

$$(\text{grad } S)^2 = (\partial S / \partial x)^2 + (\partial S / \partial y)^2 + (\partial S / \partial z)^2 = n^2. \quad (16)$$

If

$$\phi = t - S \quad (17)$$

denotes the phase, for a constant value, $\phi = \phi_0$, we have

$$dt - dS = 0. \quad (18)$$

The displacement along a ray is dr and the phase velocity

$$V = \frac{dr}{dt} = 1 / (dS/dr) = 1/n \quad (19)$$

by (18) and (16).

Let us now consider Eq. (9), which must be satisfied by the component v of a disturbance propagating in a heterogeneous medium. If we write

$$v = A(x, y, z) e^{i\omega[t - S(x, y, z)]}, \quad (20)$$

this function must be a solution of Eq. (9). We note that for a harmonic oscillation, $\exp(i\omega t)$, the quantity v is replaced by an expression depending on the amplitude $A(x, y, z)$ and the phase or function $S(x, y, z)$.

Besides the above equation, originating from dynamical theory, there is an equation, from geometric optics, that characterizes the propagation of the phase. This is

$$(\partial S / \partial x)^2 + (\partial S / \partial y)^2 + (\partial S / \partial z)^2 = 1/V^2 = 1/\hat{\beta}^2, \quad (21)$$

where the symbol $\hat{\beta}$ is introduced to adjust the notations:

$$\hat{\beta} = \mathcal{V} ; \quad \beta = (\mu/\rho)^{1/2} . \quad (22)$$

If $\hat{\beta}$ is identified with β , Eq. (21) will determine the family of surfaces $S(x,y,z) = S_0 = \text{const}$, since μ and ρ are given for a medium. However, we can draw a conclusion in a different way. We defined the phase velocity β (or $\mathcal{V}$) by Eq. (13) or as an expression $(dS/dr)^{-1}$. It will be known, therefore, after the function itself is found. Thus, in Eq. (21) we have two unknown functions: S and $\hat{\beta}$ (or $\mathcal{V}$), which will determine the shape of wave fronts and how fast a disturbance (displacement) is transferred along the rays.

In a paper presented at the Annual Meeting of the American Geophysical Union in Washington (May, 1958), I suggested that Eq. (21) be interpreted as a condition of the type of kinematical conditions imposed by constraints in the theory of motion of rigid bodies. If we consider Eq. (21) as such a supplementary condition (of a kinematical nature) which must be satisfied simultaneously with dynamic equations, the phase velocity $\hat{\beta}$ becomes a third unknown function, joining the amplitude A and phase S .

In order to get more insight in the problem, we consider again our simple case, i.e., Eq. (9). We have

$$v = A(x,z) \exp i\omega[t-S(x,z)] = v_0 \exp \left[i \frac{2\pi}{T} g \right] \quad (23)$$

$$\frac{1}{\beta^2} \frac{\partial^2 v}{\partial t^2} = \nabla^2 v + \frac{1}{\mu} \frac{\partial \mu}{\partial z} \frac{\partial v}{\partial z} \quad (24)$$

$$(\partial S / \partial x)^2 + (\partial S / \partial z)^2 = 1 / \hat{\beta}^2 , \quad (25)$$

with $v_0 = \text{const}$, $\omega = 2\pi/T$, if we now put

$$A(x,z) = v_0 \exp[-2\pi \xi], \quad \xi(x,z) = \xi, \quad g = t-S + i\xi T . \quad (26)$$

If we insert v from Eq. (23) into Eq. (24), we obtain first

$$\frac{2\pi i}{T} \left[\nabla^2 g - \frac{1}{\beta^2} \frac{\partial^2 g}{\partial t^2} + \frac{1}{\mu} \frac{\partial \mu}{\partial z} \frac{\partial g}{\partial z} \right] + \left(\frac{2\pi i}{T} \right)^2 + \left[\left(\frac{\partial g}{\partial x} \right)^2 + \left(\frac{\partial g}{\partial z} \right)^2 - \frac{1}{\beta^2} \left(\frac{\partial g}{\partial t} \right)^2 \right] = 0 . \quad (27)$$

Separating real and imaginary parts, we obtain

$$\nabla^2 S + \frac{1}{\mu} \frac{\partial \mu}{\partial z} \frac{\partial S}{\partial z} - 4\pi (\nabla S \cdot \nabla \xi) = 0 \quad (28)$$

$$T^2 \left[\nabla^2 \xi + \frac{1}{\mu} \frac{\partial \mu}{\partial z} \frac{\partial \xi}{\partial z} \right] + 2\pi \left[(\nabla S)^2 - T^2 (\nabla \xi)^2 - \frac{1}{\beta^2} \right] = 0 . \quad (29)$$

For given ρ and μ , i.e., β , and a given period, Eqs. (28) and (29) are two simultaneous differential equations. If they are solved for S and ξ , i.e., A , Eq. (25) yields the third unknown, $\hat{\beta}$.

The exact relationship between the phase velocity $\hat{\beta}$ and the factor β is expressed then by the equation

$$\frac{1}{\hat{\beta}^2} = \frac{1}{\beta^2} - \frac{T^2}{2\pi} \left[\nabla^2 \xi + \frac{1}{\mu} \frac{\partial \mu}{\partial z} \frac{\partial \xi}{\partial z} - 2\pi (\nabla \xi)^2 \right] , \quad (30)$$

which we derive from Eqs. (29) and (25).

It shows that the true phase velocity $\hat{\beta}$ is determined in addition to rigidity and density also by the period and variations in the amplitude. From (28) and (29) one might expect that S and ξ also depend on the period T , i.e., on the frequency ω .

Certainly great mathematical difficulties will be met in the solution of the problem even for simple density and rigidity distributions.

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Discussion

WAVE VELOCITIES IN NONHOMOGENEOUS ELASTIC MEDIA

by

W. S. Jardetzky

C. J. Renken, Argonne National Laboratory: I'd like to ask you about an equation - I believe it's equation 12. Wouldn't the surface of the constant phase be given by the expression t minus $S(x,y,z)$ equals a constant instead of just $S(x,y,z)$ equals a constant?

Jardetzky: Well, these surfaces, x , y , and z , are defined at a given moment for a given moment t . For every moment, for every instant, you have a family of surfaces. You see, if you keep t , everything is moving. The disturbance is moving from one surface to another, so if you take a momentum picture, then this is a surface.

Renken: All right, but take the time derivative of S and you come out with S as a derivative of t equals zero, which doesn't make sense. You have to take t minus $S(x,y,z)$ equal to a constant; then take you derivative; then you will obtain your phase velocity.

Jardetzky: That's here in equations 17 and 18. When you differentiate, you get dt equals dS .

Renken: Well, I don't quite see what that has to do with equation 12. Another question, what would you do if A was a function of x,y,z , and also t ?

Jardetzky: This is an old story. I will tell you something. About 15 or 16 years ago I was asked - that was in Austria - to clarify your question. There was a very interesting specialist in wave propagation who wrote thousands of papers until they closed to him the entrance to all magazines and journals. The person who asked me to clarify this question found that they did something unjust. He was merely considering, not only the phases to these functions, but also the amplitude depending on x . So this was really from the mathematical viewpoint a most general approach. He obtained then two families of surfaces, equal phase and equal amplitude for every instant. Well, it was a very hard task because I couldn't restore several links which he had omitted, so I could get through only a certain number of his papers, which were absolutely correct and certainly I do not object to anything like this. Certainly you can look for a solution, a mathematical solution, by considering the amplitude, also the function of not only coordinate and time, too. What comes out, I don't know, because I told you he used awful mathematics, and because he just omitted many of the transformations he used. I couldn't get through these. This was exactly what you are asking. I am writing the solution de Broglie was using. But this is not the only way. I mean, to use harmonic oscillations as a start of a discussion is a matter of convenience, not necessity. You can use many other approaches, if you like.

Leonard Goodman, Atomics International: I want to know whether or not there are any good references on this material.

Jardetzky: I have them all here; I will give you a list of references.

Ed Fisher, Argonne National Laboratory: Your conclusion, the equation used for your conclusion (equation 30), is strikingly similar to the one for the effects of diffraction in a dispersive medium, where you have a plane wave which is being diffracted by radio waves. I myself can't see the connection between the two; that is how they both come to the same conclusion. Do you have any remarks on this?

Jardetzky: No, I don't. It may be that somebody before me came to the same conclusion. You see, so many people are working on this, that it is no wonder.

Peter Morris, National Lead Company of Ohio: The boxed equation on the center board (equation 16), did you imply that the ray is in all cases normal to the wave front, where you wrote down the gradient of S ? I think you made a statement here that you're considering that the ray is in all cases normal to the wave front.

Jardetzky: Well, that is Malus Law when the ray is normal to the surface. If it is not normal, you cannot use this. So this is only for isotropic media.

Morris: If you're talking about a heterogeneous material - - -

Jardetzky: This doesn't mean it's an isotropic material. Heterogeneous means that, for example, that the density is a continuous function of coordinates. An isotropic means that special properties in different directions are different.

Morris: But if the density is a function of direction, then isn't the velocity of propagation a function of direction?

Jardetzky: No, no, that's a more general case which you want. Now, if you have an isotropic medium, that means certain properties (which are not the density; it can be rigidity, for example) rigidity can be the same in all directions, or rigidity can be distributed as in an isotropic medium. It is distributed over an ellipsoid, you know, with three principal directions. So for rigidity, for μ for example, you can pick an isotropic. But the point is that you use the word homogeneous in two different senses usually. The simplest is you mean density is the same; but what about a mixture involving water? Consider a mixture of alcohol and water and you take a drop of oil; the density is the same at the same layer, but the other properties are not. On the other hand, if you use a word homogeneous in the first sense, you have that medium homogeneous all the way.

Morris: This is isotropic?

Jardetzky: Yes, it is. Both are. Because these are liquids, they are both isotropic, but in the chemical sense, these two things are nonhomogeneous.

Morris: You're treating here the isotropic cases?

Jardetzky: Isotropic, yes. That's what I mentioned from the very beginning. I didn't touch on isotropic. That is more difficult.

ULTRASONIC WAVE TRANSMISSION AT PLANE INTERFACES

by

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Nondestructive testing has a long way to go to approach perfection. One of the missing links is certainly a better understanding of nondestructive testing fundamentals, not only by those who make and use nondestructive testing equipment, but also by the people who apply the results that it furnishes them. Therefore, a symposium like this can be of great value to everyone, either directly or indirectly concerned with this technology. Papers like Dr. Jardetzky's help to direct us toward an understanding of the physical principles involved. The job of the instrument people is to try to bridge the gap between these more sophisticated aspects of the basic theory and things that the engineers can get their hands on to use.

One of the physical phenomena involved in ultrasonic testing, one that's encountered in every phase of it, involves wave reflection at interfaces. This is a basic aspect of mechanical and ultrasonic testing. It's involved in the generation of acoustical waves, in their coupling into specimens, and in their reflection and detection. We make use of it for generation of various wave modes and we try to avoid it where it causes spurious or unwanted signals. The general theory is available from several sources in the literature on the technology of sound, wave mechanics, and seismology, a portion of which we were introduced to this morning.

Unfortunately, the theory in many cases is not easily applied to ultrasonic testing problems by the engineer, perhaps because of inability to take equations in one language and translate them quickly into his own technology. Moreover, this information is widely scattered in a number of diverse publications, ranging from the Nondestructive Testing Handbook to journals as far afield as optics, seismology, mechanics and aerodynamics. So what we tried to do in organizing the present SNT Handbook* was to make available in one place a collection of data relating to all the fields of nondestructive testing, including the ultrasonic phenomena. Much of the information I will present today can be found in the SNT Handbook.** However, it is admittedly presented in a form in the Handbook that takes considerable analysis and searching of the literature in order to understand it completely. So this review of some of the simpler aspects of interface phenomena as related to mode conversion and refraction is given to make the data more available to you and to make it more readily usable. This theory is limited

*Editor's Note: Nondestructive Testing Handbook, edited by R. C. McMaster, Ronald Press, New York, 1959, in two volumes.

**See Nondestructive Testing Handbook, Vol. 2, Section 43, "Ultrasonic Test Principles."

to isotropic, homogeneous, nondispersive materials having flat surfaces, and to plane wave propagation. For example, in the far field of a typical transducer, we can assume essentially nondiverging waves - plane wave fronts. These are not impractical requirements to impose on a typical problem of flaw detection. You are going to hear a paper a little later on the subject of Lamb waves, and I think some of the same conditions will be applied, at least to the elemental theory. From a good understanding of these phenomena and the equations, and the data that are available, I think one can come close to predicting effects observed in typical test problems, and in understanding the results that are obtained. I have a series of figures which I'll proceed through reasonably rapidly, and we can go back to any of them later if further explanation seems desirable.

Figure 1 shows the basic velocity equations, the simplest forms that Dr. Jardetzky showed for the isotropic, homogeneous case. In general, we can generate and detect, in typical solids, waves having the three well-known velocities: the thin rod, the longitudinal, and the pure polarized transverse waves. The transverse velocity V_t is sometimes called V_s in seismology and the waves "S-waves" (S_h waves for those polarized in the horizontal plane and S_v waves for polarization in the vertical). The longitudinal velocity (V_L) is sometimes called the compressional wave, or the longitudinal wave, or the bulk velocity, or the P wave in seismology. In all these equations we have essentially an elastic component given as a modulus of some kind (in this particular case Young's modulus E is carried through all the first three equations) and the density ρ (this is the density

SOUND VELOCITY IN SOLIDS

GENERAL	$v = \sqrt{\frac{\text{ELASTICITY}}{\text{DENSITY}}}$
LONGITUDINAL	
a) THIN ROD	$v_R = \sqrt{\frac{E}{\rho}}$
b) BULK	$v_L = \sqrt{\frac{E(1-\sigma)}{\rho(1+\sigma)(1-2\sigma)}} = \sqrt{\frac{K + \frac{4}{3}E_s}{\rho}}$
TRANSVERSE	$v_T = \sqrt{\frac{E}{\rho} \frac{1}{2(1+\sigma)}} = \sqrt{\frac{E_s}{\rho}}$

Fig. 1

of the homogeneous material). For a composite material, ρ is the apparent density seen by the traveling wave. Also appearing is a function of Poisson's ratio in each equation. The shear wave equation reduces to the simpler form of the shear modulus divided by the density. The longitudinal wave equation in a bulk extended solid can also be shown as a function of the bulk modulus and the shear modulus. Now if one divides V_L by V_t , all the parameters except Poisson's ratio can be eliminated, yielding a simple function of the ratio V_L/V_t . The same can be done with V_0 and either of the other two velocities. The surface wave velocity being a function of the shear modulus can also be used.

Figure 2 shows a summary of these results. Plotted here are the ratios V_t/V_L , V_R/V_L , V_0/V_L , and V_R/V_t . All are shown as functions of Poisson's ratio for all possible real values, that is, for a realizable homogeneous material having a Poisson ratio between zero and a half. For most of the solids of concern in typical flaw-detection work, the Poisson ratio tends to be somewhere between 0.2 and 0.35, so that the shear wave

Fig. 2

has roughly half the velocity of the longitudinal wave, and the surface wave has approximately 0.9 the velocity of the shear wave. This turns out to be very useful, of course, because, as Dr. Jardetzky points out, if we measure these velocities, we can compute the elastic moduli. However, since the velocity of the shear wave is about half that of the longitudinal wave, it becomes possible to isolate each by electronic time separation methods of one kind or another, and to take advantage of the significant time difference. There are a few exceptional materials having a Poisson's ratio in the lower range; fused quartz is an example, and a few in the very high range, plastics for example.

Remembering that the velocity of the shear wave has about half the velocity of the longitudinal wave, consider the impinging of a shear wave polarized in a plane

perpendicular to a plane of incidence. If one takes a simple case where there's a large acoustic impedance mismatch between, say, the medium in which the wave is starting and the second medium, as shown in Fig. 3 (for example, metal on one side and air on the other side), one would expect even from simple physical conditions that, if a wave were generated which reflects off in the other direction, it would have at least some component of the original shear motion. This is the condition that generates the shear waves that are encountered in seismology, in which the impinging wave may come at an angle to a discontinuity. I've made no attempt here to define any angles. It's obvious from Snell's Law that if the velocities of the waves are different, then the angle of incidence and the angle of reflection must be different.

MODE CONVERSION IN SOLID

Fig. 3

Fig. 4

Figure 4 shows a typical case that would be encountered in practical testing. We have a piezoelectric generator generating a pure longitudinal wave, that is, with the particles traveling vertically. If the wave encounters a solid interface, for instance a block of steel against air, at a fairly shallow angle, one would expect that a longitudinal wave would be reflected with the angle of reflection equal to the angle of incidence. However, a shear wave having a lower velocity

would have a smaller angle of reflection (by Snell's Law), and it would go off at an angle more nearly equal to the perpendicular. Now this is a typical condition that exists say in a long shaft, as for example, if one sends a longitudinal wave down a long solid shaft, such as a railroad axle. As expected, not only longitudinal waves, but also shear waves are encountered.

Figure 5 shows a condensation of these angular effects as a function of Poisson's ratio. Since by dividing the equations for longitudinal and shear waves one gets a function of only Poisson's ratio, it is possible to give the angles also as unique functions of Poisson's ratio. For a Poisson ratio of 0.28 for steel, an incident angle of the longitudinal waves of 60° , a shear wave at 30° is generated.

Fig. 5

Up to this point, then, we've defined only the angular relationships, and these were simply given by Snell's Law and the related velocities. In Fig. 6, amplitude is plotted. So far we've talked about longitudinal waves incident, for instance, at an interface between a solid and air. All the energy must be preserved. The acoustic coupling allows no transmission into the air. There are now two emergent beams: one longitudinal, and one transverse. The amplitude relationships are determined by another set of equations, due to Knott, a seismologist. In the figure are shown a number of interesting cases. Represented is the ratio of the reflected longitudinal amplitude to the incident longitudinal amplitude, that is, the ratio of beam A_1 to A . They are both longitudinal waves.

Fig. 6

Now for the 60° angle of incidence mentioned before and for a Poisson ratio of 0.25, the reflected amplitude is in fact zero.. In other words, there is no longitudinal wave reflected at that angle, so all of the energy must have been transferred into shear.

This suggests several things which might be encountered in practical testing. First of all, we have a way of generating useful shear waves polarized in the vertical plane simply by bouncing the beam at a very shallow angle to the surface. Secondly, we have the problem that we cannot reflect a longitudinal wave at 60° in material of Poisson's ratio of 0.25. This summarizes the amplitude relationships. Remember, in general, that pulse echo equipment using piezoelectric crystals, measures amplitude, not energy. Therefore, in switching back and forth between, say, energy equations for transmission through plane surfaces, as is commonly given in these angular equations, it's well to be very careful always to remember whether you're looking at amplitude information or energy information. This is particularly important if we consider the extra effect of beam spread or the condensation of the beam due to the refraction effect. These curves are due to Arenberg and appeared in the Journal of the Acoustical Society under the subject of ultrasonic delay lines for electronic work. You can see the divergent sources that one has to use in order to bring together useful information in this case.

Figure 7 relates the incident shear wave and the reflected shear wave. This should double check. If one has an incident angle of 30° and a Poisson ratio of 0.25, notice that no shear wave is reflected. Therefore, the energy all would go back into longitudinal, so that these curves do double check themselves. Both of these appear in the SNT Handbook. With this kind of an explanation, which unfortunately cannot be effectively covered in the Handbook, one can make better use of the information. The interesting area is for Poisson's ratios of 0.22 to 0.35. Notice in many cases that it's impossible to get complete conversion until you get up to a Poisson's ratio of about 0.29; there will always be some shear left over after the first bounce.

Figure 8 summarizes that information (also due to a paper by Arenberg). This tells what angles will give complete conversions, say from one mode to another; it tells what angle will give the maximum conversion; it tells the included angle between the two. Take the case of a Poisson ratio of about 0.25 and 60° for the angle for complete conversion. The included angle between the two is shown here, so that when we have a complete conversion with a Poisson ratio of 0.28, you'll notice the included angle is about 90° , that is, the shear wave was incident 30° and the longitudinal incident 60° . Notice that this information now also crosschecks with all the previous data.

Fig. 7

Fig. 8

In Fig. 9 is shown a little different aspect of the mode conversion problem. If the velocities are different in two media, we would expect by Snell's Law to have refraction. In order to have a continuity of conditions at the interfaces, it may be that we not only have a refracted longitudinal wave, but a refracted shear wave. There also, in general, will be a reflected

Fig. 9

angles before the longitudinal wave would be beyond critical. This is shown in Fig. 10. For the case where the incident angle is such that the longitudinal angle (refracted angle) is beyond critical, one would have left only the shear wave with an angle given by Snell's Law. This is a practical case, for instance, for the generation of shear waves by the wedge-search unit approach.

Figure 11 shows a way of analyzing this effect experimentally. This device is the equivalent of the optical refractometer. We have a semicircle of material corresponding to a typical test specimen and another semicylindrical sample of the material which is going to be studied for the search unit wedge. A typical wedge material is Lucite. A transducer is mounted on the top of this semicircular assembly. The ultrasonic beam is caused to pass from the first material into the second. The latter might be an aluminum block, for example, if one were trying to determine what angles were optimum for generating a 65° angle in the aluminum. Snell's Law gives only one condition required for the emerging wave - the refracted angle; it doesn't tell whether such a wave will exist or what its amplitude will be. By tilting this refractometer, one can experimentally get both the angles and the amplitudes of the emerging wave. If a second crystal is used on the far side of the lower semicircle to pick up the signal, one gets information about the transmitted signal. If a pulse echo system is used, the wave

wave at the same angle and the same mode as the incident wave, and probably a second wave, not shown, which would be, as an example, the shear wave in the case of the incident longitudinal wave.

A practical condition under which one encounters this is immersed testing, in which it is desired to pass a beam from a liquid couplant, water as an example, into a piece of steel or aluminum. By looking at Snell's Law one would predict that, with a velocity ratio of about 4 to 1, it wouldn't take large

Fig. 10

would transmit through the first medium, into the second, and be reflected back. This very simple device can be used either with an echo system or as a transmission system.

Fig. 11

If one were going to look at the amplitude of the transmitted shear wave, one would have to use a shear sensitive crystal to pick up the shear wave in the second block. Typical results are shown in Fig. 12. If you

Fig. 12

want first of all to look only at the pattern of the emerging beam, the receiver crystal can be swung through the arc shown; the side loads of the beam and the beam divergence can be seen. If the angle and the amplitude are both measured, then a plot such as shown in Fig. 13 can be obtained. This presents in one place a lot of interesting information. For a 36° or 37° Lucite wedge one notices that the longitudinal wave cannot

exist, since the entry angle is beyond critical. The shear wave will emerge at about 45°. This is a convenient angle for a great number of practical ultrasonic tests. However, the amplitude of the beam is given by the upper set of curves. As the incident angle is varied from zero toward seventy,

Fig. 13

discussed previously.

The practical example I mentioned is shown in Fig. 14, in which a pipe testing technique using a Lucite wedge of about 36° is used to transform the longitudinal wave from an X-cut thickness vibrator into a shear mode by refraction and mode conversion. The shear mode at 45° has an interesting property which could be obtained if one goes back to Fig. 13 and notes at this angle, which is beyond critical for the longitudinal wave, no energy is lost to the longitudinal. This is really what makes this type of a test practical; that is, except for a certain amount of beam spread which may eventually, by spurious reflections, generate a longitudinal mode, the wave now remains in pure shear. Therefore, one has a wave traveling with half the velocity of a longitudinal wave and staying in pure shear for innumerable bounces between parallel faces as in a piece of heavy wall tubing, for example.

The problem of determining the amplitude theoretically for this refraction mode-conversion case, from medium one to medium two, has been worked out theoretically for a number of specific cases. Dr. F. A. Firestone did this for transmission from water to steel. These curves are also presented in the Handbook of Nondestructive Testing, and, I admit, in a way that is very difficult to decode unless someone has given you a clue as

the longitudinal wave drops rapidly, approaching zero where it should, where the critical angle is reached. The shear wave is building up in amplitude and conveniently peaks at about 45° . This is desirable because for a great many tests that's a good angle, for example, in steel or aluminum plate or pipe. The shear wave then begins to decrease rapidly, going to zero at critical for shear, which is about 64° for Lucite and aluminum. Then the surface wave suddenly appears for a very narrow range of incident angles, about 65° - 67° with a Lucite wedge. This, incidentally, is a simple, useful way of generating surface or Rayleigh waves for practical testing. Again we have here the experimental version of the half-space or extended surface

Fig. 14

Fig. 15

(represented by the emerging S and L curves) at, say, 10° , one finds the 10° index mark on the S and L curves. From the L curve we find that the longitudinal wave has an energy of about 0.1 unit (the incident wave has 1.0 unit) and emerges at an angle of around 50° . Simultaneously, there will be a shear wave which will appear at an angle of about 20° and has a relatively low energy around 0.03 unit. This suggests one of several things. First, if it's necessary to angulate at all (in other words, in order to pick up a defect in an angle), one must do one of two things - either work fairly close to the normal, using the longitudinal wave before it begins to drop off, or go far enough into shear to get a fairly strong shear wave amplitude. The upper right quadrant shows the reflected energy which goes out into the water; with 100% incident, we have roughly 90% reflected into the water, and somewhere around 10% transmitted into the solid.

Now where there's appreciable refraction these may not add up, since they represent intensity of the beam and do not take into account the beam widening or narrowing due to refraction. Figure 16 shows what happens when the sound, starting now in the steel, is reflected back to the water interface and transmitted into the water. Find the incident point that one's concerned with, for instance 45° , count the index marks, and then go back along curve L to determine what the amplitude of the transmitted wave is into the water. Notice that, as in the previous curve, the first 10° is rather flat. This is, I think, interesting. In fact, the whole specification

to how the original information was derived. In Fig. 15 the long arrows represent the incident waves. The angle of incidence is given counter-clockwise. The incident wave must be a longitudinal wave, since it originates in a liquid (that is, there can be no appreciable shear transmitted through the liquid). The small scale marks show as index marks on the lower curves and on the curves to the right. The long arrows show as long index marks. If one wants to compute the power (these are energy equations) that is transmitted into a piece of steel in both the longitudinal and shear modes

Fig. 16

strong transmission into the water, as one would expect, now occurs at greater angles, just as the strong shear mode was initially generated at greater angles of incidence.

for the air frame industry, based on angulating in aluminum, hinges around this simple theoretical fact, which was later discovered experimentally. One can angulate up to about 10° from water to steel or aluminum without getting appreciable drop off due to mode conversion or refraction, so that this part of the curve was quite flat. In the lower left quadrant are represented the reflected energies. You'll notice that now there are both L and S waves reflected if one has a merging L wave. Figure 17 shows the same thing for the shear wave. Here we have an incident shear in the steel at a number of angles. The

Fig. 17

This has been a very rapid summary of considerable data. However, it's all based on straightforward, plane wave theory and most of it is now available for the first time, in an integrated presentation in the Handbook of Nondestructive Testing. The implications, I think, are well worth bearing in mind. These effects cannot only be useful for the generation of waves of various modes for the study and investigation of the interior of solid materials, but they also have to be understood to avoid fallacious, misleading results because of unexpected mode conversions.

Discussion

ULTRASONIC WAVE TRANSMISSION AT PLANE INTERFACES

Goodman, Atomics International: In two or three of those curves in which you had plotted the amplitude ratio versus, I think, angles of incidence and various Poisson's ratios, you indicated you had negative amplitude ratios. I'm not sure just what you mean by a minus amplitude ratio.

Van Valkenburg: Well, this simplest approach, I think, would be to consider that the wave consists of condensations and rarefactions, depending on the relative impedance of the first and second medium; a condensation may be reflected either as a condensation or a rarefaction. In other words, when one is talking amplitude, one does have to talk about polarity.

Goodman: In other words, it's almost as though you were talking about a phase change.

Van Valkenburg: If it's sinusoidal or if the waves were unidirectional impulse waves, a complete reversal would be noted. There is a section in the SNT Handbook, I think it's 44 or 45, which discusses possible uses of phase reversal for identifying types of discontinuities. In other words, an air gap would give reflection of one polarity, whereas an inclusion of higher density would give a reverse polarity. There are pretty good bibliographies in the new handbook, also, to which I refer you to for references to Knott's equations, as well as the work of Kalsky, Arenberg, and others.

THEORY OF LAMB WAVES

by

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ABSTRACT

The nature and properties of Lamb waves, that type of stress wave in elastic plates theoretically discovered by Horace Lamb in 1916, will be discussed. Emphasis will be placed on those properties which make Lamb waves unique when they are compared with other types of elastic wave motion.

The list of the several types of elastic waves that may be generated in an elastic plate usually includes just shear, longitudinal, and surface waves. In recent years, however, another type of elastic wave motion, namely, Lamb waves, has received increasing recognition. Lamb waves designate that type of stress waves in elastic plates theoretically discovered by Horace Lamb and made public in 1916 at a meeting of the Royal Society of London. Nothing further was done with the subject, at least as far as literature references indicate, until 1945, when Floyd A. Firestone and Daniel S. Ling, who were then at the University of Michigan, undertook an extensive theoretical and experimental study. They were the first to produce such waves in plates, verifying, after 26 years, Lamb's theory, and they were the first to bring out those properties of Lamb waves which make them unique when contrasted with other elastic stress waves. As will be made evident later, these properties of Lamb waves account not only for their uniqueness but also for their usefulness. This discussion will be concerned with the nature and properties of Lamb waves from the theoretical viewpoint.

In developing the theory, it is assumed that the medium is perfectly elastic, homogeneous, and isotropic. The motion of the particle takes place in two dimensions, with u and v denoting the horizontal and vertical displacements. The origin of the coordinate system is taken in the medial plane of the plate, with x the coordinate along the plate in the direction along which the wave travels, and the y axis normal to this. The plate has a thickness d . We define the following quantities:

P_{yy} = stress acting on a plane normal to the y axis in the y direction

P_{xy} = stress acting on a plane normal to the x axis in the y direction

λ = Lamé constant

μ = rigidity modulus

u = displacement of the particle along the x axis

v = displacement of the particle along the y axis

ρ = density of medium .

The stress strain relations are:

$$\begin{aligned} P_{yy} &= \lambda \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + 2\mu \frac{\partial u}{\partial y} \\ P_{xy} &= \mu \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \end{aligned} \quad (1)$$

Let ϕ and ψ be potential functions such that

$$\begin{aligned} u &= \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y} \\ v &= \frac{\partial \phi}{\partial y} - \frac{\partial \psi}{\partial x} \end{aligned} \quad (2)$$

where

$$\begin{aligned} \frac{\partial^2 \phi}{\partial t^2} &= \frac{\lambda + 2\mu}{\rho} \nabla^2 \phi \\ \frac{\partial^2 \psi}{\partial t^2} &= \frac{\mu}{\rho} \nabla^2 \psi \end{aligned} \quad (3)$$

We see that ϕ is the potential function associated with longitudinal waves and ψ that associated with shear waves. We assume plane wave motion so that the particle motion is independent of z and also that ϕ and ψ are periodic with respect to time and distance. The latter dependence will be taken care of by the factors $e^{i\sigma t}$ and $e^{i\epsilon x}$ where

$$\epsilon = 2\pi/\lambda \quad .$$

The time factor $e^{i\sigma t}$ will be omitted in the discussion.

Define

$$h^2 \equiv \frac{\rho \sigma^2}{\lambda + 2\mu} \quad ; \quad k^2 = \frac{\rho \sigma^2}{\mu} \quad , \quad (4)$$

so that

$$\begin{aligned} (\nabla^2 + h^2)\phi &= 0 \\ (\nabla^2 + k^2)\psi &= 0 \end{aligned} \quad . \quad (5)$$

Also define

$$\alpha^2 = \epsilon^2 - h^2 \quad (6)$$

$$\beta^2 = \epsilon^2 - k^2 \quad . \quad (7)$$

Then (5) can be written as

$$\begin{aligned} \frac{\partial^2 \phi}{\partial y^2} &= \alpha^2 \phi \\ \frac{\partial^2 \psi}{\partial y^2} &= \beta^2 \psi \end{aligned} \quad . \quad (8)$$

The solutions of (8) are

$$\phi = (A \cosh \alpha y + B \sinh \alpha y) e^{i\epsilon x}$$

and

$$\psi = (D \cosh \beta y + C \sinh \beta y) e^{i\epsilon x} \quad ,$$

but we should also note that

$$\begin{aligned} \phi &= (A \cosh \alpha y) e^{i\epsilon x} \\ \psi &= (B \sinh \beta y) e^{i\epsilon x} \end{aligned} \quad (9)$$

and

$$\begin{aligned} \phi &= (C \sinh \alpha y) e^{i\epsilon x} \\ \psi &= (D \cosh \beta y) e^{i\epsilon x} \end{aligned} \quad (10)$$

are also solutions.

Lamb designated solutions of type (9) as the symmetric wave motion and of type (10) as the antisymmetric wave motion. Using (2) and (8) and substituting into the stress-strain equations, we get the stress-strain equations in terms of quantities that have been defined:

$$\frac{P_{yy}}{\mu} = (\epsilon^2 + \beta^2)\phi - 2i\epsilon \frac{\partial \psi}{\partial y}$$

$$\frac{P_{xy}}{\mu} = 2i\epsilon \frac{\partial \phi}{\partial y} + (\epsilon^2 + \beta^2)\psi$$

We shall treat only the symmetric case; the procedure is identical for the antisymmetric case. Using (9), that is, the symmetric-type solution and substituting into the last expression for the stress-strain equations, we get

$$\frac{P_{yy}}{\mu} = \left[A(\epsilon^2 + \beta^2) \cosh \alpha \frac{d}{2} - B\beta 2i\epsilon \cosh \beta \frac{d}{2} \right] e^{i\epsilon x}$$

and

$$\frac{P_{xy}}{\mu} = \pm \left[2iA\alpha \sinh \alpha \frac{d}{2} + B(\epsilon^2 + \beta^2) \sinh \beta \frac{d}{2} \right] e^{i\epsilon x}$$

Now, with the boundary condition that at the surface $y = d/2$, the stress must be zero, that is,

$$P_{yy} = P_{xy} = 0$$

After elimination of the constants A and B, there is obtained

$$\frac{2i\epsilon \beta \cosh \beta \frac{d}{2}}{(\epsilon^2 + \beta^2) \cosh \alpha \frac{d}{2}} = \pm \frac{(\epsilon^2 + \beta^2) \sinh \beta \frac{d}{2}}{2i\epsilon \alpha \sinh \alpha \frac{d}{2}}$$

or, finally, the frequency equation

$$\frac{\tanh \beta \frac{d}{2}}{\tanh \alpha \frac{d}{2}} = \frac{4\epsilon^2 \alpha \beta}{(\epsilon^2 + \beta^2)^2}$$

Replacing the values for α , ϵ , and β in the above equation, defining longitudinal velocity V_L and shear velocity V_s by

$$V_L = \sqrt{\frac{\lambda + 2\mu}{\rho}} ; \quad V_s = \sqrt{\frac{\mu}{\rho}}$$

and recalling that by definition the phase velocity V is $V = \sigma/\epsilon$, we can write

$$\frac{\tan \pi f d \sqrt{\frac{V_s^2 - V^2}{V_s^2 V^2}}}{\tan \pi f d \sqrt{\frac{V_L^2 - V^2}{V_L^2 V^2}}} = \frac{4 \sqrt{\left(1 - \frac{V^2}{V_L^2}\right) \left(1 - \frac{V^2}{V_s^2}\right)}}{\left(2 - \frac{V^2}{V_s^2}\right)^2}$$

This is the frequency for the symmetric-type motion in terms of frequency, plate thickness, shear velocity, and longitudinal velocity.

The equation as it stands now is single-valued and will give just one mode of vibration having a phase velocity less than both longitudinal and shear velocity. The equation, however, can become multivalued in two ways. First, the wave velocity can exceed the shear velocity. Then $V_s < V < V_L$, and

$$\frac{\tan \pi f d \sqrt{\frac{V^2 - V_s^2}{V_s^2 V^2}}}{\tan \pi f d \sqrt{\frac{V_L^2 - V^2}{V_L^2 V^2}}} = \frac{4 \sqrt{\left(1 - \frac{V^2}{V_L^2}\right) \left(\frac{V^2}{V_s^2} - 1\right)}}{\left(2 - \frac{V^2}{V_s^2}\right)^2} ;$$

secondly, the wave velocity can exceed both the shear and longitudinal velocity. Then $V_s < V_L < V$, and

$$\frac{\tan \pi f d \sqrt{\frac{V^2 - V_s^2}{V_s^2 V^2}}}{\tan \pi f d \sqrt{\frac{V^2 - V_L^2}{V_L^2 V^2}}} = \frac{-4 \sqrt{\left(\frac{V^2}{V_L^2} - 1\right) \left(\frac{V^2}{V_s^2} - 1\right)}}{\left(2 - \frac{V^2}{V_s^2}\right)^2} .$$

This set of equations gives an infinite number of modes of vibration: the higher the root of the equation, the higher the mode of the vibration. These equations are readily evaluated for any material for which the shear and longitudinal velocities are known. One chooses a value of V , the wave velocity, and then looks for the values of the frequency-thickness product (fd) that will satisfy the equation. It is this dependence of the phase velocity on the fd product which make Lamb waves so unique, for there is no such dependence for shear, longitudinal, or surface waves.

Figure 1 illustrates the plotting of this relationship (that is, of phase velocity versus frequency-thickness product) for aluminum and for the symmetric and antisymmetric type motion. The essential features of these graphs remain the same for different metals. The phase velocity of

the Lamb wave mode for a given metal depends only on the frequency-thickness product. For any frequency-thickness product, there are a finite number of modes that can be generated, but the number of possible modes increases as the fd product increases. For example, in aluminum at a frequency-thickness product of 2.0×10^5 in./sec, three asymmetric and three symmetric modes can be generated, whereas at a frequency-thickness product of 5.0×10^5 in./sec, six asymmetric and six symmetric modes can be generated.

Fig. 1a Waves in Aluminum Plate Symmetrical Type Phase Velocity Plotted Against Product of Frequency and Plate Thickness

Fig. 1b Waves in Aluminum Plate Asymmetrical Type Phase Velocity Plotted Against Product of Frequency and Plate Thickness

Let us now go into further detail concerning the particle and plate vibrations themselves. Lamb's grouping of these infinite number of modes into symmetric and antisymmetric is a designation which depends on the direction of the particle displacement. This is illustrated in Fig. 2. With the symmetric type, there is mirror symmetry with respect to the median plane through the plate. When a particle in the upper half of the plate is moving up and to the right, the particle occupying the equivalent position in the lower half is moving down and to the right; the corresponding components parallel to the plane of the plate move in the same direction, the corresponding perpendicular components move in the opposite sense. The situation is reversed for the asymmetric type. When a particle in the upper half of the plate is going up and to the right, the corresponding particle in the lower half is going up and to the left. The corresponding components parallel to the plane of the plate move in the opposite direction, the corresponding perpendicular components move in the same direction. The two components - parallel and perpendicular to the plate - combine to give a particle orbit which is elliptical. The particles in the lower and upper halves of the plate trace orbits with the same major and minor axes, but the direction of traversing the ellipses are different. Whether the major axis of the ellipse is perpendicular to the surface or to the direction of propagation depends on the ratio of thickness to the wavelength of the vibration.

Fig. 2 Particle Displacements and Orbits for Symmetric and Antisymmetric Types

Let us now consider the motion of the plate itself (Fig. 3). First, as u and v are functions of x , y , and t , the time, and as the disturbance is moving in the x direction, they are functions of the form

$$u = U(y)\cos(\epsilon x - \sigma t)$$

$$v = V(y)\sin(\epsilon x - \sigma t) \quad .$$

Figure 3 shows a section of the plate on which has been drawn the direction of the motion of the particles. This diagram is a picture at one instant of time, as this is, in reality, a progressive wave motion, and we must realize that this drawing moves to the right with the phase velocity of the particular mode. The curved lines drawn in the compartments are "lines

of motion," that is, a tangent to these curves gives the direction of the motion of a particle located at the point of tangency. As the diagram and the lines of motion move to the right, a particle located at a given point in the plate will have different lines - or different directions of motion at different times. It is to be emphasized that the particles are not traveling around these curved lines or "lines of motion." The motion of each particle itself is a very small ellipse about its equilibrium position, and the "lines of motion" just tell us in what direction the particle is moving.

Fig. 3 Vibration Pattern of a Symmetrical Wave

Although the "lines of motion" do not give us any information concerning the magnitude of the displacements, they are related to u and v in the following way. If u is large and v is small at a given point, then the line of motion should be practically horizontal at that point. If v is large and u is small at a given point, then the line of motion should be nearly vertical.

The plate is divided into compartments bounded by vertical and horizontal lines. In each compartment we have shown only one "line of motion," but really each point should have a line of motion passing through it. They have been left out for the sake of clarity. It should now be evident from the discussion of the "lines of motion" and their relation to u and v , the displacements, that the horizontal lines bounding the compartments are those lines along which $v = 0$, that is, there is no vertical displacement of the particles on these lines. The vertical lines bounding the compartments are lines on which $u = 0$, that is, there is no horizontal displacement of the particles on these lines. Thus, sections of the plate are divided into boxes along whose boundaries the motion is entirely horizontal or entirely vertical.

Now, u and v can vanish in two ways: $U(y)$ or $V(y)$ can be zero, or the cosine and sine term can vanish. The cosine equally zero and making u vanish, and $V(y)$ equally zero making v vanish give the lines marking out the compartments. As the vertical lines $v = 0$ are due to the cosine vanishing, they move along the plate with the wave, that is, with time. This is also true for those lines on which $v = 0$ because the sine vanishes; these are located halfway between the vertical lines along which $u = 0$, and bisect the boxes vertically.

In summary, then, both the horizontal and vertical displacements are sinsoidal in form along any given horizontal line in the plate with an amplitude dependent on the depth.

There are also some conditions under which the plate surfaces will not vibrate in either the vertical or the horizontal direction. These conditions will not be derived but simply stated. Any symmetric or antisymmetric mode that travels in a plate with a phase velocity equal to the shear velocity will not vibrate the plate surface in the vertical direction. Any symmetric or antisymmetric mode that travels in a plate with a phase velocity equal to $\sqrt{2}$ shear velocity will not vibrate the plate surface in the horizontal direction.

There has been some tendency to confuse Lamb waves and Rayleigh surface waves. They are not the same. By a surface wave we mean a wave which travels along surfaces with their fields confined to the surface; they are exponentially attenuated in a direction normal to the surface; the disturbance exists for only a few wavelengths below the surface. This is not characteristic of Lamb waves. They exist throughout the body of the plate and not just at the surface; they are not exponentially attenuated normal to the plate surfaces and, hence, are not surface waves. Nevertheless, it is true that there is some definite relationship between Lamb waves and Rayleigh waves. We have derived the period equation for the first modes:

$$\frac{\tanh \beta f}{\tanh \alpha f} = \frac{4\epsilon^2\alpha\beta}{(\epsilon^2 + \beta^2)^2} \quad .$$

Now, for waves which are very short compared with the thickness, ϵf , αf , and βf are large and the equation will tend to the form

$$(2\epsilon^2 - k^2)^2 - 4\epsilon^2\alpha\beta = 0 \quad ,$$

which is recognized as the equation to determine the period of Rayleigh waves. In other words, in the limit $d/\lambda \rightarrow \infty$ the phase velocity of the first Lamb wave modes will approach Rayleigh wave velocity. This is evident in Fig. 1, the plot of phase velocity versus frequency-thickness. Here we can see that the first modes are already starting to approach the surface wave velocity.

In recent years, Mr. Daniel Worlton of the Hanford Atomic Products Operation has undertaken an investigation of Lamb waves. He will, as the next speaker, discuss the experimental aspects and verification of Lamb waves.

Discussion
LAMB WAVES

Peter Morris, National Lead: You indicate phase velocities in excess of the dilatational wave velocity. Have you been able to demonstrate clearly that energy is propagated with this velocity, and that this is actually the velocity of propagation of the wave front, and that you're not measuring at some angle to the propagation of the wave front?

Worlton: I'm reasonably certain that we are propagating energy at phase velocities in excess of the longitudinal wave velocity.

Morris: This isn't like the wave that comes into the shore at an angle.

Worlton: The experimental data would indicate that we definitely are. See, this is the pulse I showed you in which the velocity of the dispersion effect occurred at a wave velocity higher than the longitudinal wave velocity. Well, this was aluminum. I think that was up around 3 or $3\frac{1}{2}$ times 10^5 .

Morris: Then you can get indicated velocities which are in excess of the velocity of the wave front if you measure it at an angle to the wave front.

Worlton: These are the data right here. That's the interpretation I put on it. Roberta, I think, has made some measurements of phase velocity in this region. Have you found that the phase velocity actually did occur up around 3 or $3\frac{1}{2}$? This is where she's measuring a time rather than angle.

R. Peterson, Argonne: I have a question for both of them. First, Roberta. Would you repeat where in the development of the solution of the ordinary wave equations did you obtain, on the one hand, the Lamb wave and, on the other, the ordinary longitudinal and shear wave solutions. Could you show that again?

Roberta di Novi: When you're deriving the equation for Lamb waves, you're applying a boundary condition. This will give you a new solution, whereas when you're just deriving the equation for shear longitudinal, you do not have a boundary condition. This is the essential difference.

Peterson: (to Worlton): I'm not quite sure of the nomenclature on one of the last slides that you showed. You have them labeled Lamb wave shear and Lamb wave longitudinal.

Worlton: This designation simply was to indicate that in one case the Lamb wave mode was used that would excite the particles predominately in the vertical direction. This I called longitudinal, the other I called shear. That's all that the indication was.

Clarke Williams, Public Roads: You both spoke of particle displacement, both vertical and horizontal. When you speak of particle displacement, due to the vibration, what magnitude are you speaking of in particles - size of molecules or smaller?

Worlton: They're larger than molecules. I don't know as I have a good definition of a particle that I could quote. It's a region small enough in area that all the stresses, etc., may be considered a constant throughout that area. Practically speaking, this is a very, very small volume of a plate, larger than a molecule.

Williams: Of a lesser size than the molecule, probably ...

Worlton: No, I would think larger than a molecule, considerably larger than a molecule, but considerably smaller than a millimeter, say.

L. Goodman, Atomics International: I think that what you can say about the definition of a particle is .. first of all, the definition is strictly arbitrary and say "region wearing." If you set the limits that the total velocity, acceleration, etc., vectors of a given percentage of all the grains or crystals in a particular region will all add up, then you can say this region is all moving in the same direction at the same time, at the same velocity, etc. It's strictly an arbitrary thing.

H. Van Valkenburg, Sperry Products: Because of the great interest in this subject, I'd like to point out that some of Dr. Firestone's early reports are available in printed form. If someone will write to me directly, we'll be glad to supply them. I'm still interested in the questions of phase velocity versus group velocity and how energy gets to be transmitted at a speed equal to infinity, but maybe someone had better explain it to me after the meeting.

P. Edwards, Los Alamos: You did mention that someone had worked on detail on liquid-loaded plates in these modes. Could you reference that, please ?

Worlton: All the data that I presented were obtained with liquid loaded plates, that is, submerged in water.

Edwards: The theoretical treatment ?

Worlton: In the theoretical treatment, Lamb assumes plate surfaces free of stress.

Edwards: Yes, but you mentioned some two co-authors had made a study of this.

Worlton: Osborne and Hart made a detailed theoretical study of the effect of water loading. See Dr. Jardetzky's book, reference about 1953. It's in the Acoustic Journal, just a few years back. I don't recall the date.

Goodman: This is one slightly practical question. In one of the slides, I think it was No. 15, you showed a plate with a hole drilled through the underside - a half-inch-thick plate - and that was the one where you showed the first symmetrical - asymmetrical mode field, which was level all across; then you got the big peak for the rest of the modes. The only thing I'm wondering about was that hole. What was its diameter?

Worlton: We used holes of different diameters.

Goodman: How small a hole could you detect in this way?

Worlton: We have used all eighth of an inch with focused crystals and careful alignment of the transducers. We can go below an eighth of an inch, but of that order of magnitude, I'd say.

Goodman: In other words, you couldn't expect anything better than that if you were running, say, production-line testing purposes. Not with the kind of line you apparently were talking about.

Worlton: It could be of that order of magnitude.

EXPERIMENTAL CONFIRMATION OF LAMB WAVES AT MEGACYCLE FREQUENCIES

By

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ABSTRACT

A theory formulated in 1916 by Horace Lamb, predicting that plates may vibrate up to an infinite number of modes, is confirmed by a method that is described. Measurements made on a finite plate and curved objects reveal that in each case the phase velocity of the wave agrees closely with predicted values.

The theory is extended to correlate experimental observations. Equations are developed relating phase velocity to frequency and plate thickness in terms of longitudinal and shear wave velocities. Curves are obtained for aluminum, zirconium, stainless steel, brass, and uranium.

The distinguishing characteristics of the various modes are discussed in the light of potential nondestructive testing applications. It is shown that interior particles are displaced in elliptical orbits, with vertical motions existing at the surfaces when the wave velocity is $[2]^{1/2}$ times the shear wave velocity, and with horizontal surface motions existing for wave velocities equal to longitudinal wave velocity.

I. Introduction

The rapid growth of industrial ultrasonic testing applications in recent years has stimulated increasing interest^{1,2} in the behavior of elastic waves in thin sections. Nearly all of the reported investigations of the

¹ Firestone, F. A. and Ling, D. S., Report on the Propagation of Waves in Plates - Lamb and Rayleigh Waves, Sperry Products, Inc., Technical Report 50-6001, Danbury, Conn.

² Yamamoto, Miaki and Kazuyuki, Ita, Ultrasonic Testing of Sheets and Small Diameter Bars, 3rd International Conference on Nondestructive Testing, Tokyo, Japan.

subject appear to be based upon a classic theoretical paper by Horace Lamb, On Waves in Elastic Plates,³ which he published in 1916.

There seems to be a definite lack of experimental data to support the theoretical studies⁴ made to date. It would appear, in fact, that the fundamental tenets of the theory have been left without full experimental confirmation. The present work was undertaken with the objectives of confirming experimentally the validity of the theory at megacycle frequencies, and to extend it, where possible, to correlate anomalous ultrasonic testing data.

Since the incentive for this work was largely a result of the potential usefulness of the theory to ultrasonic testing, the results are interpreted and discussed in this light.

II. Theory

A. Frequency Equations

Lamb analyzed the behavior of two-dimensional stress waves in infinite plates whose surfaces are free of stress. The cornerstone of his theory rests upon two frequency equations, which he expressed as follows:

$$\frac{\tanh \beta d}{\tanh \alpha d} = \frac{4 \xi^2 \alpha \beta}{(\xi^2 + \beta^2)^2} \quad (1a)$$

and

$$\frac{\tanh \beta d}{\tanh \alpha d} = \frac{(\xi^2 + \beta^2)^2}{4 \xi^2 \alpha \beta} \quad (1b)$$

where ξ is the wavelength constant, $2\pi/\xi$ the wavelength, d the one-half plate thickness, and β and α are defined in terms of the elastic properties of the plate material.

For reasons that will be discussed presently, waves satisfying Eq. (1a) are referred to as symmetrical waves, and those corresponding to Eq. (1b) as asymmetrical waves.

³ Lamb, Horace, On Waves in Elastic Plates, Proc. Roy. Soc. (London), 90A, 114 (1916).

⁴ Firestone and Ling have, in particular, done much to elucidate the theory from an ultrasonic testing standpoint. Firestone coined the phrase Lamb Waves to describe the wave motion theoretically postulated by Lamb. The terminology is retained here.

These equations can be put in a form more suitable for our purposes by means of the substitutions

$$\beta = \xi \left[1 - \frac{V^2}{V_S^2} \right]^{1/2} = \xi R_S \quad (2a)$$

and

$$\alpha = \xi \left[1 - \frac{V^2}{V_L^2} \right]^{1/2} = \xi R_L \quad , \quad (2b)$$

Where

$$\xi = 2\pi f/V$$

f = frequency

V = wave phase velocity

V_L = longitudinal wave velocity

V_S = shear wave velocity.

It follows from the above definitions that the dimensionless quantity R_S becomes imaginary for values of V greater than V_S , and that R_L becomes imaginary as V exceeds V_L . Writing

$$R_S = ir_S \quad \text{for} \quad V_S < V < V_L \quad (3a)$$

and

$$R_L = ir_L \quad \text{for} \quad V_S < V_L < V \quad , \quad (3b)$$

the frequency equations may be expressed as follows:

Symmetrical

(4a)

$$\frac{\tanh \pi f D \frac{R_S}{V}}{\tanh \pi f D \frac{R_L}{V}} = \frac{4R_L R_S}{(1 + R_S^2)^2}$$

Asymmetrical

(4b)

$$\frac{\tanh \pi f D \frac{R_S}{V}}{\tanh \pi f D \frac{R_L}{V}} = \frac{(1 + R_S^2)^2}{4R_S R_L}$$

for $V < V_S < V_L$

Symmetrical

(5a)

$$\frac{\tan \pi f D \frac{r_S}{V}}{\tanh \pi f D \frac{R_L}{V}} = \frac{4 R_L r_S}{(1 - r_S^2)^2}$$

Asymmetrical

(5b)

$$\frac{\tan \pi f D \frac{r_S}{V}}{\tanh \pi f D \frac{R_L}{V}} = - \frac{(1 - r_S^2)^2}{4 r_S R_L}$$

for $V_S < V < V_L$

(6a)

(6b)

$$\frac{\tan \pi f D \frac{r_S}{V}}{\tan \pi f D \frac{r_L}{V}} = - \frac{4 r_L r_S}{(1 - r_S^2)^2}$$

$$\frac{\tan \pi f D \frac{r_S}{V}}{\tan \pi f D \frac{r_L}{V}} = - \frac{(1 - r_S^2)^2}{4 r_L r_S}$$

for $V_S < V_L < V$

where D denotes total plate thickness.

Written this way, the variables: phase velocity, frequency, and plate thickness, are expressed in terms of the constants: longitudinal wave velocity and shear wave velocity. The expressions are more easily handled numerically (and visualized physically) if frequency and thickness are considered as a single factor. Thus, it is convenient to look upon the equations as expressions relating V to fD, that is, phase velocity of the wave to the product of frequency times plate thickness. Equations 5 and 6, being multivalued, allow for an infinite number of fD products for any given value of V larger than V_S . This means, physically, that a number of waves of different velocity may exist in a plate at a fixed frequency. In other words, the higher roots of these equations correspond to higher modes of vibration.

Longitudinal wave velocity and shear wave velocity have been accurately measured for most metals. Curves plotting V versus fD may therefore be readily obtained by selecting arbitrary values of V and solving Eqs. 4 - 6 for the corresponding fD values. This procedure was followed in the derivations of the curves for aluminum, zirconium, stainless steel, brass, and uranium as shown in Figs. 1 through 5. The points of discontinuity existing at $V = V_S$ and $V = V_L$ were bridged by drawing the curves through points evaluated at neighboring values of V. Values for V_S and V_L were taken from Reynolds.⁵

⁵ Reynolds, M. B., New Elastic Constants of Some Metals, KAPL 576 (May 1951).

Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.

Fig. 5.

The curves predict that up to an infinite number of modes are allowed in any given plate. Each travels with a unique phase velocity that depends upon the fD value in question, and may vary from infinity to near shear wave velocity. If the frequency is known, the modes allowed in a given plate, and their respective phase velocities, may be directly ascertained.

The curves embody the essential points of the theory. Before their experimental confirmation is discussed, it will be helpful to point out characteristics of the 1st modes which bear on data to be presented later.

The 1st modes are given by Eq. 4 and the first root of Eq. 5. At large fD values the hyperbolic tangents of Eq. 4 approach unity.

Thus,

$$4RLR_S = (1 + R_S^2) \quad , \quad (7)$$

which, when expanded, becomes

$$\left[\frac{V^2}{V_S^2}\right]^3 - 8\left[\frac{V^2}{V_S^2}\right]^2 + 24\left[\frac{V^2}{V_S^2}\right] - 16\left[\frac{V^2}{V_L^2}\right] + 16\frac{V_S^2}{V_L^2} - 16 = 0 \quad (8)$$

This is Rayleigh's equation, which describes wave motions that travel near the surface of elastic bodies. Interestingly, the experimental data suggest that the 1st modes are indistinguishable from Rayleigh waves in aluminum at fD products larger than about 2×10^5 (inch)(cycles)/sec.

At small fD values, where the wavelength is large in relation to the plate thickness, the hyperbolic functions of Eqs. 4 may be replaced by their arguments. Therefore,

$$(1 + R_S^2)^2 = 4R_L^2 \quad (9)$$

at small fD values. This expands to:

$$V^2 = 4V_S^2 (V_2 - V_S^2)/V_L^2 \quad , \quad (10)$$

which is identical to the velocity equation derived from elementary theory where the thickness is assumed infinite in relation to the wavelength. It follows, therefore, that at small fD values the 1st symmetrical modes are equivalent to ordinary compressional waves.

B. Particle Displacements

Particle displacements associated with the flow of energy are related to the potential functions ϕ , and ψ by the equations

$$u = \frac{\partial \phi}{\partial \chi} + \frac{\partial \chi}{\partial y} \quad (11a)$$

$$v = \frac{\partial \phi}{\partial y} - \frac{\partial \psi}{\partial \chi} \quad , \quad (11b)$$

where u is the displacement in a direction parallel to the plate surface (the x direction) and v is the particle displacement normal to the plate surface (the y direction). Lamb takes

$$\phi = \left[A \cosh R_L \frac{\omega}{V} \right] e^{i\xi\chi} \quad (12a)$$

$$\psi = \left[B \sinh R_S \frac{\omega}{V} \right] e^{i\xi\chi} \quad (12b)$$

for symmetrical waves, and

$$\phi = \left[A \sinh R_L \frac{\omega}{V} \right] e^{i \xi \chi} \quad (13a)$$

$$\psi = \left[B \cosh R_S \frac{\omega}{V} \right] e^{i \xi \chi} \quad (13b)$$

for asymmetrical waves, where A and B are constants which are related by plate surface boundary conditions (where the stress is zero), and ω is 2π times the frequency.

Substituting Eqs. (12a) and (12b) into (11a) and (11b) there results that

$$u = -BR_S \frac{\omega}{V} \left[\frac{2 \cosh \pi f D \frac{R_S}{V}}{(1 + R_S^2) \cosh \pi f D \frac{R_L}{V}} \cosh R_L \frac{\omega}{V} y - \cosh R_S \frac{\omega}{V} y \right] \cos \frac{\omega}{V} \chi \quad (14a)$$

$$v = B \frac{\omega}{V} \left[\frac{(1 + R_S^2) \sinh \pi f D \frac{R_S}{V}}{2 \sinh \pi f D \frac{R_L}{V}} \sinh R_L \frac{\omega}{V} y - \sinh R_S \frac{\omega}{V} y \right] \sin \frac{\omega}{V} \chi \quad (14b)$$

for the symmetrical waves. Similarly, use of Eqs. (9a) and (9b) leads to

$$u = -BR_S \frac{\omega}{V} \left[\frac{2 \sinh \pi f D \frac{R_S}{V}}{(1 + R_S^2) \sinh \pi f D \frac{R_L}{V}} \sinh R_L \frac{\omega}{V} y - \sinh R_S \frac{\omega}{V} y \right] \cos \frac{\omega}{V} \chi \quad (15a)$$

$$v = B \frac{\omega}{V} \left[\frac{(1 + R_S^2) \cosh \pi f D \frac{R_S}{V}}{2 \cosh \pi f D \frac{R_L}{V}} \cosh R_L \frac{\omega}{V} y - \cosh R_S \frac{\omega}{V} y \right] \sin \frac{\omega}{V} \chi \quad (15b)$$

for asymmetrical waves.

Modes of higher order are more interesting from an ultrasonic testing standpoint than are the 1st modes. For $V_S < V < V_L$ these equations become

(16a)

$$u = Br_S \frac{\omega}{V} \left[\frac{2 \cos \pi f D \frac{r_S}{V}}{(1 - r_S^2) \cosh \frac{R_L}{V}} \cosh R_L \frac{\omega}{V} y - \cos r_S \frac{\omega}{V} y \right] \sin \frac{\omega}{V} x \quad (16b)$$

$$v = B \frac{\omega}{V} \left[\frac{(1 - r_S^2) \sin \pi f D \frac{r_S}{V}}{2 \sinh \pi f D \frac{R_L}{V}} \sinh R_L \frac{\omega}{V} y - \sin r_S \frac{\omega}{V} y \right] \cos \frac{\omega}{V} x$$

for symmetrical, and

(17a)

$$u = Br_S \frac{\omega}{V} \left[\frac{2 \sin \pi f D \frac{r_S}{V}}{(1 - r_S^2) \sinh \pi f D \frac{R_L}{V}} \sinh R_L \frac{\omega}{V} y - \sin r_S \frac{\omega}{V} y \right] \cos \frac{\omega}{V} x \quad (17b)$$

$$v = B \frac{\omega}{V} \left[\frac{(1 - r_S^2) \cos \pi f D \frac{r_S}{V}}{2 \cosh \pi f D \frac{R_L}{V}} \cosh R_L \frac{\omega}{V} y - \cos r_S \frac{\omega}{V} y \right] \sin \frac{\omega}{V} x$$

for antisymmetrical types.

With these equations one may gain insight into the behavior of the individual particles as energy flows along the plate. At a fixed instant of time, for example, it is apparent that there are periodic intervals of x at which there is neither vertical nor horizontal motion. The zero-motion points for u and v are displaced from each other along x by one-quarter wavelength. Since either u or v is zero, for a particular value of x , for all values of y , these planes of inactivity may be shown by vertical lines extending through the plate (Fig. 6). The solid lines designate planes for which v equals zero, and the dashed lines those for which u is zero.

Fig. 6. Particle Motions Corresponding to Symmetrical and Asymmetrical Waves.

It follows that, at any plane for which u motion is zero, the v displacement is maximum, and vice versa. As time progresses, the particular particle motion under observation travels along the plate to the right with phase velocity V .

For values of x which do not lie upon these vertical boundaries, both u and v also vanish, in general, at discrete values of y . For the symmetrical waves, v motion is zero at the medial plane for all values of x and at all times. For the asymmetrical waves the same is true for u displacements. The horizontal zero-motion lines are likewise represented in Fig. 6 as solid and dashed lines. Thus, sections of the plate are divided into compartments along whose boundaries the motion is either entirely horizontal or entirely vertical. A particle situated within a compartment traverses an elliptical orbit. The curved lines drawn within the boxes represent directions of motions for particles situated along the curve. Thus, a tangent to these curves gives the direction of motion of a particle located at the point of tangency.

Equation (16a) shows that for symmetrical waves the vertical (v) deflection changes sign, at a given value of x , for positive and negative values of y . This means physically that two particles situated in opposite halves of the plate, equidistant from the medial plane, are displaced in opposite directions. Thus, as the upper portion moves up, the lower portion moves down, and vice versa. Conversely, the u component remains unchanged for positive and negative values of y . Hence, particles situated at equal distances above and below the center of the plate are deflected in the same horizontal direction at the same time.

It follows from Eqs. (17a) and (17b) that the reverse is true for asymmetrical waves; consequently, these waves displace the plate surfaces in the same vertical directions at any one value of x and one instant of time. However, in this case horizontal motions are of opposite sign in the lower and upper portions of the plate. The lower sketches in Fig. 6 depict the manner in which each wave type distorts the plate.

In many ultrasonic testing applications particle motions existing at the plate surfaces are of particular interest. Surface motions may be ascertained by letting $y = d$ in the particle displacement equations and taking the ratio of v/u . The results are

$$\frac{v}{u} = \frac{2R_L}{1 + R_S^2} \tanh \pi f D \frac{R_L}{V} \quad (18a)$$

for symmetrical waves, and

$$\frac{v}{u} = \frac{2R_L}{1 + R_S^2} \coth \pi f D \frac{R_L}{V} \quad (18b)$$

for asymmetrical waves. The factor on the left of the above equations is the ratio of maximum displacements. Equations 18a and 18b are plotted in Figs. 7 and 8 for the first few modes of each type in aluminum. It may be seen by inspection that this ratio goes to infinity for both types of waves for $R_S^2 = 1$, or when $V = [2]^{1/2} V_S$. Thus, modes traveling at this critical phase velocity displace surface particles in a direction normal to the plates surfaces only. It likewise follows that modes traveling at phase velocity $V = V_L$ displace surface particles predominately in the horizontal direction.

Fig. 7.

Fig. 8.

III. Experimental Results

The experimental data were obtained by submerging aluminum plates under water with a send-and-receive ultrasonic transducer in an arrangement illustrated schematically in Fig. 9. A beam generated by the sender excites Lamb waves within the plate if it impinges upon its surface under the proper conditions. Consider, for example, the wave front AB at the instant of contact of the plane wave with the plate at A, and at a subsequent instant Δt at which the point B reaches the plate at C. The velocity with which the front travels along the plate is thus

$$V = V_w / \sin \theta \quad , \quad (19)$$

where V_w is the velocity of sound in water and θ is the angle of incidence. If θ is chosen to make V equal to the phase velocity of a particular mode for the plate thickness and frequency in question, the mode will be excited in the plate. As the excited wave travels in the direction of the arrow (see Fig. 9), it emits energy at an exit angle equal to the angle of incidence. This radiated energy may be detected with appropriately placed receivers to permit the behavior of waves within the plate to be studied. Since the velocity given by Eq. 19 is equal to the phase velocity of modes within the metal, the angle corresponding to any given phase velocity can be readily determined.

Fig. 9. Experimental Arrangement For Generating and Detecting Lamb Waves in Plates.

It should be pointed out that submerging the plates in water violates Lamb's basic assumption that the plate surfaces be free of stress. However, Osborne and Hart⁶ have generalized Lamb's equations to account for water loading. Their results show that, except for allowing for additional modes near $V = V_W$ (which are not of interest here), the effect of water does not appreciably affect either the number of modes allowed in a given plate, nor their phase velocities.

The experimental data are in excellent agreement with Lamb's equations. In Fig. 10 for example, a 0.051-in. thick aluminum plate is inspected with 5-Mc ultrasonic pulses. The energy reaching the receive crystal is plotted versus angle (computed from Eq. 19, taking V_W as 0.57×10^5 in./sec) as the incident and exit angles are kept equal and swept through an approximate thirty-degree arc. Modes are observed at very near their predicted values. Since the 1st symmetrical and 1st asymmetrical modes travel in aluminum with the same phase velocity at fD values larger than about 2×10^5 (in.)(cycles)/sec, they appear in this example at the same angular value. Since they are indistinguishable, they are referred to simply as "1st modes."

⁶ Osborne, M. F. M., and S. D. Hart, Transmission, and Reflection, and Guiding of an Exponentiated Pulse by a Steel Plate in Water. J. Acoust. Soc. Amer. 17 1-18 (1945).

Fig. 10. Theoretical and Measured Lamb Wave in an Aluminum Plate.

To confirm the theory further, plates ranging in thickness from 0.030 to 0.102 in. were tested at selected frequencies in order to obtain fD products within the range for 0.5×10^5 to 4×10^5 (in.)(cycle)/sec. The results of these tests, when placed directly upon the theoretical curves (see Figs. 11 and 12), show that the theory is indeed valid for these experimental conditions.

It is interesting that, with this technique, the curves derived from the frequency equations are valid for curved surfaces as well as for the flat surface which Lamb considered. Figure 13 shows the results of the same experimental procedure applied to the outside of a 1.5-in.-diameter aluminum pipe. The various modes appear as predicted; moreover, results obtained from testing convex surfaces correlate equally well. Thus, it appears that Lamb waves may be excited in curved and flat surfaces with equal facility.

Figs. 11 and 12. Measured Values Obtained at Various Frequencies and Plate Thickness Plotted on Theoretical Curves.

Fig. 13. Comparison of Theoretical and Measured Lamb Waves in an Aluminum Cylinder.

Another interesting experimental effect lends supporting evidence to the theory in an unusual manner. In Fig. 14 a 0.051-in. plate is inspected with 5-Mc pulses. The waveforms show that the shape of the received 4th symmetrical mode is noticeably different from the received 1st modes after each has traveled an equal distance in the plate. Apparently, a velocity dispersion of the 4th symmetrical mode causes the slightly different pulse frequencies to travel with unequal velocities, lengthening the pulse time width. The effect is not present in the case of the 1st modes for which, according to the curves, velocity is independent of frequency.

Fig. 14.

Finite Plates

The experimental studies reported thus far concern plates of relatively large surface area, in conformity with an assumption of the Lamb theory. It is of interest now to determine how the theory holds for small plates.

Figure 15 shows the results of testing rectangular 0.041-in. strips of $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$ -in. widths at a frequency of 10 megacycles/sec. The samples were cemented to a mounting block and placed under the crystals as before. The upper left diagram shows results of fixing the angles to generate the 3rd symmetrical mode, leaving the spacing between crystals fixed, and translating both crystals as a unit across the strip. All predicted modes are observed in the samples, although their amplitudes are diminished in the smaller sections. Good correlation was also obtained with circular samples, as illustrated in the lower portion of Fig. 15.

The next "finite" samples studied were obtained by drilling flat bottomed holes of $\frac{1}{4}$, $\frac{3}{8}$, and $\frac{1}{2}$ -in. diameter, in aluminum blocks to within different depths of the opposite surface. This arrangement (see Fig. 16) provided, in effect, circularly shaped plates whose upper surfaces extended to infinity, but whose lower surfaces were fixed by the hole surfaces.

As with other samples, all modes possible for the given thickness and frequency were observed in regions of the "plate" at the theoretically predicted values of θ . The curves shown in Fig. 16 illustrate the signal response as a sample with a one-half-in. hole, 0.045 in. from the surface, is scanned. The various modes are clearly excited in the metal over the hole. Of course, the large change in signal response as the test head passes over the hole makes its presence and location clearly evident.

Fig. 15. Lamb Waves in Finite Plates.

Fig. 16. Lamb Waves in Sample with Simulated Defect.

The figure shows, however, that the 1st modes are excited everywhere in the block and are not altered in amplitude as they flow in the metal over the hole. At this particular fD product the 1st modes travel with a phase velocity equal to Rayleigh wave velocity. These results imply that under these conditions their energy is contained close to the surface and the waves become indistinguishable from Rayleigh waves.

Attenuation Measurements

Equations 18 and 19 provide another means of checking the theory experimentally. One would expect, for example, Lamb waves to attenuate in plates submerged in water at a rate markedly dependent upon the ratio v/u . A mode that causes the surfaces to move predominately in the horizontal direction - with very little vertical motion - should impart energy to the water at a relatively slow rate and, therefore, attenuate rather slowly. Conversely, modes of large v/u ratios, which displace the surface predominately in the vertical direction, would lose their energy to the liquid at a relatively rapid rate and die out rather quickly.

These arguments agree with experimental observations. From Figs. 7 and 8, for example, it would be assumed that modes propagating at velocities near $V = [2]^{1/2} V_S$ would travel with reduced attenuation. From Fig. 1 it can be seen that a 5-Mc 2nd symmetrical pulse travels in a 0.030-in.-thick plate at a velocity near $V = [2]^{1/2} V_S$, as does a 3rd symmetrical wave in a 0.051-in. thick plate. Figure 17 shows that these modes are, in fact, able to propagate greater distances in plates of these particular thicknesses.

These considerations have interesting implications. If the given explanations for the data presented in Fig. 17 are correct, they imply that this technique provides a method of vibrating an inaccessible plate surface in any direction desired by driving the opposite surface with a plane wave suitably.

The concept appears to be of practical importance, as Lamb waves are applied to the problem of inspecting the integrity with which "plates" are bonded to base metals. The upper sample of Fig. 18, for example, was prepared by bonding an aluminum plate to an aluminum base with an adhesive bonding resin. A hole was first drilled into the block to simulate an unbond defect. As the surface is scanned in the same manner as before, a marked difference in behavior of the various modes is observed in the vicinity of the simulated defect. The 1st modes do not respond to the defect since they behave as Rayleigh waves at this particular fD product. The 2nd symmetrical mode, whose surface displacements are predominately in the horizontal direction at the required velocity, are excited everywhere in the plate and are not appreciably influenced by the hole. However, the 2nd asymmetrical mode - which in this sample produces predominately vertical particle displacements - is established only in proximity to the defect and, consequently, makes the defect readily apparent.

Consider now the nuclear fuel element illustrated in the lower portion of Fig. 18. A thin aluminum jacket is bonded with an aluminum-silicon alloy to the uranium target material. Our previous argument suggests that one could stress the bonding layer with forces oriented either normal to, or parallel with, the bonding layer by choosing modes at the appropriate values of V in accordance with Figs. 7 and 8.

Fig. 17. Attenuation of Lamb Waves with Different μ/v Ratios

Fig. 18. Lamb Waves in Samples with Simulated Bonding Defects.

Experimentally, it is indeed observed that markedly different results are obtained depending upon the type of motion employed.

Normal particle displacements, which arise from forces which stress the bonding layer in tension, reveal unbond defects in instances where the bond is actually fractured. These types of waves are therefore useful for detecting broken bonds. Significantly, however, waves which produce shear force at the bonding layer are observed to reveal bonds that are mechanically weak, even though they are not actually broken.

IV. Summary

Lamb's theoretical predictions that an infinite number of symmetrical and asymmetrical waves may exist in plates is experimentally confirmed. The experimental studies show that the theory holds for finite

plates submerged in water. Moreover, Lamb waves are observed in curved objects, as, for example, pipes and tubing, according to theoretical predictions. Solid objects clad with a thin covering metal, such as nuclear fuel elements, can support Lamb waves in the cladding, depending upon the integrity of the bond to the base metal.

Each mode propagates with a unique phase velocity dependent upon the order of the mode, metal thickness, and frequency. Surface particles motions are displaced with the flow of energy in elliptical orbits. For particles situated at the plate surfaces, the displacements become predominately normal to the surfaces at phase velocities near $[2]^{1/2}$ times the shear wave velocity, and become predominately parallel with the surface at phase velocities near longitudinal velocities. Waves producing horizontal surface motions travel in water loaded plates with reduced attenuation, and are able to reveal certain bonding discontinuities that are not revealed by other ultrasonic waves.

These facts are certain to make Lamb waves increasingly important to nondestructive testing.

ULTRASONIC ATTENUATION AND THE PHYSICS OF SOLIDS

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ABSTRACT

An outline will be given of those interactions between ultrasonic stress waves in solids and the various types of absorbing or scattering elements which can be studied by observing the propagation behavior of the stress waves. A discussion will be given of what can be learned about solids and defects in solids from the physical point of view. Specific examples related to changes in mechanical properties and changes induced by radiation effects will be given.

I will assume that it does not require any explanation of reasons for talking about Solid State Physics methods to a group interested in non-destructive testing. If one is concerned only with detecting voids, cavities, and gross flaws, and if this is all that one wants to detect and study, excluding other more subtle properties of the material, then what I have to say will be of no interest. If, however, one is concerned with the physical properties of the materials (which from my point of view are considerably more important than finding the ordinary flaws), then what I have to say may be of interest.

What I intend to do is to outline all of the known interactions between ultrasonic or elastic stress waves propagating through a material, and the various kinds of losses, or loss effects, that one can expect to encounter. There are about ten such effects. The first half of my talk is going to be devoted to telling you that there are such things, and telling you what they are, and in an extremely brief way, to tell you somewhat about how these interactions come about. In the last half of this talk I intend to try to give you a couple of examples of how one goes about learning something from one of these interactions.

The reason for putting the talk into this form is simply that one asks very often, "What about ultrasonics and ultrasonic methods? What can one hope to learn by using it? Where can it best be used?" A thorough answer to such questions would require a good sized book. Nevertheless I will try to deal with some aspects of these questions. The effect

that one encounters in this work cuts right through the whole section of Solid State Physics and, in fact, cuts into Nuclear Physics as well.

The following list is intended to outline the pattern of this talk.

Ultrasonic Methods in Solid State Physics

Use

Propagation of Elastic Waves in Solids

Examine or Study

- (a) Interaction of elastic waves with the various loss mechanisms by means of measurement of propagation behavior and changes therein.
- (b) "Effects" produced by elastic waves but observed by other detection methods.

Elastic Waves

- (1) Equations of Motion of Elastic Waves and Stress-Strain Relations
- (2) Strain

$$e = e_{\text{elastic}} + e_{\text{interaction}} \quad (1)$$

- (3) Propagation behavior given by α and V

$$\sigma = \sigma_0 e^{-\alpha x} \sin w \left(t - \frac{x}{V} \right) \quad (2)$$

α (attenuation)

V (phase velocity of wave)

Referring now to the preceding outline, we are studying the effects of Solid State Physics that one can study by means of the propagation of elastic stress waves in solids. We are going to use the elastic stress waves to examine the interaction of these elastic waves with various loss mechanisms. In order to study the elastic waves, we study the propagation characteristics. Studying the propagation characteristics of a stress wave means measuring the loss factor and the velocity. Measured changes in the attenuation and the velocity determine changes in the propagation behavior in the solid.

Equation (2) shows the simplest form that a stress wave may take, a plane wave propagating through the material with an attenuation factor α and a phase velocity V . The velocity is just as important as the attenuation.

The theory for any interaction, if it is complete, must account for any changes in the velocity as well as any changes in the attenuation. According to (b) above, there are effects produced by elastic waves and observed by other methods, that is, one can produce changes in a material and observe it by some other scheme. An example of this is paramagnetic resonance. One looks at a paramagnetic resonance line, as Jacobsen¹ did in quartz, and one alters the paramagnetic resonance line by putting elastic stress waves in the material. This is an example of observing an interaction by some other method than an ultrasonic one, but producing an effect nevertheless. It's an interaction that one can study by either method really: either by the ultrasonic method, or by the paramagnetic resonance method.

The elastic waves are governed by the equations of motion and the stress-strain relations. These are the equations of motion for the sort of thing we're talking about. The strains are defined in terms of the displacement components in this fashion:

$$\sigma_{ij,j} = \rho \ddot{s}_i \quad (3)$$

$$\sigma_{ij} = C_{ijkl} e_{kl} \quad (4)$$

$$\sigma_{ij,j} = \rho \ddot{s}_i = C_{ijkl} e_{kl,j} \quad (5)$$

$$e_{ij} = (1/2)(s_{j,i} + s_{i,j}) \quad (6)$$

The double dot means second time derivative, while the comma notation means spatial derivative. Equation (3) is the derivative of the σ_{ij} with respect to x_j for any component, which is equal to the density times the second time derivative of the displacement component s_i . In the crystal-line medium one has a stress-strain relation of the type indicated by equation (4). The C_{ijkl} are the elastic moduli. One has to be careful in talking about strain because the strain is seldom or ever purely the elastic strain.

I have indicated in equation (1) the elastic strain plus what we call an interaction strain. The interaction strain is always a nonlinear one and ultrasonics sees only nonlinear loss effects, although there may be purely elastic velocity changes. The interaction strain simply means that in the stress-strain relation one has a loss, whereas if one has a purely elastic strain, as far as mechanical things are concerned, one would have no loss. There would be no attenuation.

¹E. H. Jacobsen, N. S. Shiren and E. B. Tucker, Effects of 9.2 kMc/sec Ultrasonics on Electron Spin Resonances in Quartz, Physical Review Letters, 3, 81 (1959).

Purely elastic strain may have electrical interactions. Elastic strain can produce a displacement of the lattice in such a way as to produce electric fields by displacing ions, or charged regions, that are coupled to the lattice. One forms electric fields, or one changes the lattice in such a way as to have electric fields formed and the electric fields are coupled to anything that is charged. One might have dislocation strain, which is a strain that arises as the result of causing dislocations to be displaced or to move or to oscillate in the lattice. One can have a variety of forms of strain.

As mentioned earlier, any complete physical explanation of an attenuation or energy loss must account for the corresponding velocity behavior. This fact is important in testing any theory for a particular interaction in which one happens to be interested. There are many interactions between elastic waves and some physical characteristic of the solid. In interactions which cause a change in the particular physical characteristic at which one happens to want to look, such as deformation, the change in the characteristic can be observed by a change in the propagation behavior as given by a change in α and in V , that is, in the attenuation and velocity. In all these changes, or in all these effects or interactions, the physical quantity is somewhat coupled to the lattice, that is, the quantity which is changing as a result of whatever one is doing, and which is being observed ultrasonically, has got to be coupled to the lattice of the solid, because the elastic wave only sees the lattice in which it is propagating. The elastic wave sees only the lattice, and if the effect of interest is not coupled to the lattice the elastic wave does not see it. The physical properties that are sufficiently closely coupled to the lattice can be observed by measuring this change in the propagation behavior. With these ideas in mind, I believe that you can see that if one can determine that some effect in which one may be interested is in some way closely coupled to the crystal lattice, then one can expect to see it ultrasonically.

The following list shows ten of the interactions that I am talking about. As far as I know, these are all that are known at the present time.

Interactions

- (1) Direct Scattering by Defects
- (2) Thermoelastic or Heating Effects
- (3) Dislocation Damping
- (4) Magnetoelastic Loss Effects in Ferromagnetic Materials
 - (a) Magnetic Domain Wall Motion
 - (b) Spin Wave-Phonon Effects
- (5) Conduction Electron Interaction with Elastic Waves

- (6) Elastic Waves (Ultrasonic) Interaction with Thermal Elastic Waves (Lattice Vibrations)
- (7) Elastic Wave Interaction with Nuclear Spin Systems
- (8) Elastic Wave Interaction with Electron Spins in Paramagnetic Centers
- (9) Elastic Wave Interaction with Electrons in Piezoelectric Materials
- (10) Acoustoelectric Effect in Semiconductors

I will run down this list and tell you as rapidly as I can, just a little about each one. Any one of these effects requires considerable discussion if it is to be done thoroughly. In any case I hope that it is clear that the ultrasonic business cuts through the whole of Solid State Physics as I said in the beginning. To discuss all of these topics thoroughly and reasonably completely is quite a job.

The interaction arising from the direct scattering by defects, either very small defects or very large defects, depends on frequency. This means a loss of energy from an ultrasonic beam brought about by the removal of energy from the beam, that is, removal of the energy from the beam and not returned to it. The effect arises from a kind of refraction and reflection produced by gradients in the elastic properties. Gradients in the elastic properties may be very sharp; for instance, a discontinuity such as a grain boundary is a gradient in the elastic properties. It may also be a small gradient, that is, the transition in the elastic properties may take place over an appreciable fraction of the wavelength, in which case the smaller gradient produces weaker scattering. In any case, it is the gradients in the elastic properties in the medium that are producing the effect. Now, gradients in the elastic properties mean gradients in the moduli, which in this case must be functions of the space coordinates. In other words, if we are talking about an isotropic material, it would be λ and μ , the Lamé constants, and ρ , the density. If, for example, in one region the values are λ_1 , μ_1 and ρ_1 and there is a transition to another medium with values λ_2 , μ_2 and ρ_2 , the transition may be sharp or it may be a gradual one. In any case, if one analyzes this problem in terms of elasticity, one finds that one can calculate scattering cross sections for most situations. The calculation of a cross section for scattering, even for the simplest case in elasticity, is not a trivial problem; it can become complicated, especially for shear waves and configurations where one loses certain symmetries. In any case one can express the attenuation in terms of the cross section. If many scatterers are present, and they interact with one another, one can also express the velocity in terms of the scattering effects. This last problem is a much more difficult one because it involves multiple scattering instead of simple scattering, simple scattering being defined as that by individual scatterers

that scatter independently of neighbors. This brings up the point that there are different kinds of scattering that must be considered: Rayleigh scattering and non-Rayleigh scattering. Rayleigh scattering is, for our purposes, defined as something that gives an effect that changes with the fourth power of the frequency. Hardly ever in ultrasonic work do you encounter Rayleigh scattering. It is usually a non-Rayleigh effect; it is usually, therefore, an extremely complicated thing if one really wants to understand it in detail. We have done only a very few problems of this type. These usually demand large amounts of numerical calculation.

The next thing on the list is the thermoelastic or heating effect. The thermoelastic effects are those brought about only from compressional waves; transverse waves do not produce thermoelastic effects. A pure shear wave cannot heat the lattice, but compressional motion in a lattice distorts it, compresses it, and generates heat which flows from the compressed to the rarified regions. In the high-frequency case this is a path of the order of half a wavelength for the heat flow, and when the wavelengths become as small as a micron this is a very short heat path and is very much smaller than the size of the sample. In other words, the heat flow path is very much less than one encounters in kilocycle frequencies, for which the heat path may be comparable with the sample size. One can calculate everything needed (both in anisotropic and in isotropic media) regarding the heat flow between the regions of compression and rarefaction. The attenuation turns out to depend on the first power of the coefficient of linear expansion in the particular crystal direction. It depends linearly on the thermal conductivity, it also depends on the square of the frequency, and it is inversely proportional to the specific heat squared.

Dislocation damping is one of the interactions of strong interest to us in our own laboratory; there are many situations in which it is an important effect. To understand dislocation effects thoroughly is a long process, but one can imagine, as a model, a string driven by some oscillating force, such as a stress wave passing through the solid. An edge-type dislocation can oscillate under the influence of such a force; this is a forced oscillating string with damping. The differential equation has a damping term that accounts for the loss in this vibrating string. This damping coefficient is the thing of interest in the stress wave. It is a specific damping coefficient, but one of the difficulties is that no one knows precisely what the origin of the damping in dislocations really is; it may be a mechanical loss, a kind of friction or viscosity loss. It may be electrical in nature. It has not been demonstrated what type of loss dislocations have, and it is still of considerable interest. There is also the dislocation strain, the interaction strain indicated in equation (1), and the total strain is equal to the elastic strain plus the dislocation strain. If one evaluates that dislocation strain term, which one can, it turns out to depend on dislocation density and the mean displacement of the dislocation. There is some question about just how far dislocations do move under vibrating forces. Despite

the unsatisfactory state of some of the basic details of the model, the dislocation theory does seem to work, that is, the dislocation theory as it is worked out does account for various kinds of effects, and it does seem to apply in all cases for which it can properly be expected to apply. Another point is that it seems to work not only in one kind of experiment, but in several different kinds of experiments. I believe that the dislocation theory, at the present time, cannot be said to account quantitatively for things we would like to see it account for, but it comes very close in a number of cases, and it certainly seems as though the basic ideas are the correct ones. If they are not, then there have certainly been an interesting series of coincidences in several kinds of experiments where the theory fits.

The magnetoelastic loss effects occur in ferromagnetic materials. There are a couple of effects, and the theory of these is in a more unsatisfactory state than the theory for some of the other effects. The magnetic domain wall motion is only qualitatively understood. When one sends a stress wave through a solid (a compressional wave) and an interaction occurs with the domain walls by means of changing the lattice strains, this has the effect of changing the magnetic field, and, of course, a changing magnetic field produces an electric field. The resulting electric field produces a current, and one has micro-eddy currents. The current oscillation occurs on either side of this domain wall as it oscillates. This is the general appearance of the effect and I believe that it is right, but it has not been proved. This picture does account for the effects we see, certainly qualitatively and perhaps quantitatively.

Next we have the spin echo or the spin wave-phonon effects. Incidentally, phonon is a word with which you may not be familiar. It merely means you have an elastic wave for which one imagines that the energy is quantized in terms of Planck's Constant, h , times the frequency. It is the same as with light waves, where one speaks of photons having energy $h\nu$. In our case the phonons are quanta of elastic energy. The term "phonon interaction" simply means elastic wave interactions. The interaction of the spin waves with phonons or elastic waves is something that has been worked out. Not very much experimental work has been done to confirm these predictions, but the ideas seem to be in a satisfactory state. I am sorry I am not prepared to go into details on this topic; it is a topic involving considerable special background and not one that will contribute much for the purposes of this group. So I am skipping over this, and asking to be forgiven on the grounds that it's complicated and requires much more time than we have. You can see that, if spins in an atomic system are coupled to the lattice (since the atom itself is coupled to the lattice), a disturbance of the lattice should be connected with the spins. The effect generally is to cause the spins to precess in a magnetic field. This causes losses in the ultrasound beam.

Conduction electron interaction with the elastic waves is something that one observes only at low temperatures. It can be observed below 50°K , and is usually studied around 4°K or lower. These effects come about primarily because stress waves propagating through the lattice produce electric fields, or changing electric fields. Free electrons can interact with this electric field and so are accelerated. If one regards the energies of these electrons as lying on the surface of a sphere in momentum space (Fermi surface), then the compression of the lattice causes a distortion of such a sphere in the sense that in the direction of the compressional wave one accelerates the electrons. The Fermi energy surface is deformed into an ellipse, which means that the electron energies are changed in the direction in which the strain can interact through the mechanism of the electric field produced. The result is that some electrons are put into a higher energy state. The only way they can return to their former energy states is by collision with the lattice, i.e., by normal electrical conductivity collisions of the type that bring the deformed Fermi surface back into a normal spherical Fermi surface. There is a relaxation time associated with this, and the relaxation time is the same sort of relaxation time that is involved in conductivity problems. I should perhaps explain the reason why the interaction occurs only at low temperatures. It has to do with the relative sizes of the electron mean free path and the ultrasonic wavelength. In other words, it has to do with the region over which the strain extends, and the relative mean free path of the electron. In a way, this expresses the time it can interact with that particular stress wave, or that part of the stress wave. Details of this type of interaction depend on the value of k , the propagation factor, which depends on the frequency and the velocity, or, in other words, the wavelength and the mean free path of the electrons. The situation is such that when L , the mean free path of the electrons, is much less than λ , the wavelength, the ultrasonic attenuation is negligible. In other words, when the electron mean free path is very much shorter than the wavelength of the ultrasound, there is no attenuation. On the other hand, when L is considerably greater than λ , that is, when the electron mean free path is larger than the ultrasonic wavelength, the attenuation becomes appreciable. That roughly is the picture. This only comes about at low temperatures. Why it comes about at low temperatures has to do with what happens to the electron mean free path. Under normal conditions at room temperature, the electron mean free path is very much less than the ultrasonic wavelength. And lowering the temperature changes the electron path so that it approaches the same size as the ultrasonic wavelength.

Next we will discuss thermal vibrations of the lattice and the interaction with ultrasonic waves. You are aware that the temperature of a solid, the temperature of its lattice, produces what are known as lattice vibrations, the thermal vibrations that are always present until absolute zero is reached, where in principle, the lattice vibrations should vanish. These vibrations are produced by the temperature agitation of the lattice. The specific heat and the thermal conductivity theories are all concerned with these lattice

vibrations. There are circumstances under which, apparently, the artificially introduced ultrasonic lattice vibrations can interact with the thermal lattice vibrations. The ultrasonic waves are introduced at one frequency, whereas the thermal lattice vibrations extend over a frequency spectrum extending from 10^9 or 10^{10} cycles per second, up to about 10^{13} or 10^{14} cycles per second. At present, ultrasonic waves cannot be produced much above 10^{10} cycles per second; 25,000 megacycles is the highest I'm aware of, perhaps 40,000 megacycles. Waves have been produced at that frequency in quartz. It is not necessary, however, to have waves at lattice vibration frequencies in order to get interaction with the lattice vibration waves. This is true because there are nonlinear or anharmonic effects present. When an ultrasonic wave of a given frequency is put artificially into the lattice, there may be more energy in that mode than there is in any one of the lattice modes. The ultrasonic wave is a source of "hot phonons" of a particular frequency.

It turns out that an interaction is present between the thermal modes and this hot phonon source; in other words one has a cross coupling that causes the energy to be removed from this ultrasonic mode and dissipated in other modes. There is a coupling, a nonlinearity in the lattice behavior, that allows the artificially introduced ultrasonic waves to dissipate themselves among, or to interact with, or to affect the distribution of, the much higher frequency thermal lattice waves. This is called phonon-phonon interaction, meaning that it's an interaction of thermal elastic waves with ultrasonic elastic waves where there is a considerable difference in the frequencies.

There is another type of interaction where the elastic waves interact with nuclear spin systems. One looks at nuclear magnetic resonance and the changes that occur when one introduces elastic waves. The elastic waves distort the lattice to produce an effect on the quadrupole moment. There is a quadrupole coupling between the nuclear spin system and the lattice; when the lattice is deformed, an electric field is produced, which interacts with the quadrupole moment of the nucleus. Hence the nucleus feels this lattice deformation, with the result that there is a change in energy levels of the nuclear spins. This gives rise to an energy loss because it is a nonlinear effect. This has been investigated in a number of materials. First of all, I believe it was done in NaCl by Tantal. At any rate, there is such an interaction, and it produces a change of population in nuclear spin levels.

There is another effect where there is an elastic wave interaction with electron spins in paramagnetic centers. This is a situation in which one observes the resonance of electron spins in a paramagnetic center; that is, if one has an energy level difference arising from two different electron spins, the spin resonance arising from the energy level transition can be observed in a spectrometer, and by introducing elastic waves one

can cause the population of the energy levels to be altered. The effect arises as shown in Fig. 1. With a single energy level a magnetic field is applied which splits the levels. Now one has a separation of those levels $h\nu$. Then either electromagnetically or ultrasonically one can put in energy of the right frequency to cause transitions from the lower level to the higher level and so change the population of spins in the upper level. One can in this way study the transition of electrons from one energy level with spin of plus a half to the other energy level with spin minus a half. Each level has a certain population, N_1 and N_2 , and the difference ΔN depends on the magnetic field, the g factor, and frequency.

Fig. 1. Energy Level Splitting

One can make these transitions come about either ultrasonically or electromagnetically. If we are studying the transitions electromagnetically, and we superimpose elastic waves of the right frequency, we can decrease the effect by causing the population of the levels to become nearly equal, thereby changing the whole transition probability. In this way one can observe an interaction between elastic waves and the spin systems. This has been done by Jacobsen¹ at General Electric. Such experiments are done at high frequencies and low temperatures. This experiment was carried out at 10,000 megacycles/second.

The elastic wave interaction with electrons in piezoelectric materials is an effect that is not really new, but it has been rediscovered and restudied with newer and better methods, particularly in cadmium sulfide crystals. If in cadmium sulfide crystals one introduces elastic stress waves and then allows light of wavelength between about 5000 and 9000 Å to fall on the crystal, one finds a strong change in attenuation. The attenuation may increase by a factor of 5 or 10. This effect is very large. The only embarrassing feature is that there are at least two different kinds of crystals, and in one type the attenuation goes up and in the other it goes down when the incident light is present. This has caused some difficulty in trying to understand the effect, but there seems to be a fairly satisfactory explanation of the matter. In any case, it seems quite clear that this is a piezoelectric field effect, in which there occurs an interaction with the conduction electrons that are available when one shines light on the sample. Magnetic fields cause no effect, but there is a very interesting kind of structure that is observed if one measures attenuation as a function of temperature down to helium temperature. As the temperature is lowered from room temperature to, roughly, liquid nitrogen temperature, a structure is found in the attenuation versus temperature curve. There are peaks in this curve of attenuation as a function of temperature. Correlating these peaks with results of certain conductivity experiments, one finds that attenuation peaks and conductivity peaks fall very close together in temperature.

I will not attempt to discuss this last feature except to repeat that there are interactions, and they come about from a stress field producing an electric field, which in turn interacts with free electrons put there by shining light on the sample and obtaining photoelectrons.

The acoustoelectric effect is a bunching of electrons that comes about as a result of having stress waves present. It is a simultaneous bunching of electrons and holes. The experiment must be done in a semiconductor in which electrons and holes are available and can be bunched much like the charge in a reflex oscillator. This is done under the action of a deformation potential in the lattice, i.e., the strain produces this deformation. The result is a DC field across the ends of the semiconductor as long as the acoustic wave is traveling in the semiconductor. This was discussed by Weinrich and White.²

If one wanted to discuss these ten topics fully, you can see from this sketch that I have tried to give, that it would be rather a long story, and if there was any doubt in your mind to begin with, there shouldn't be now that such a discussion would cut through the whole field of Solid State Physics.

In passing, I'd like to make some comments about the physical optics of ultrasound. I don't know that it is correct to call it optics, but there is a counterpart in sound waves of about everything that one finds in physical optics. One can produce interference, refraction, double refraction, and internal conical refraction. All these things have been done, and the last two are particularly interesting. One can use these effects to obtain information about properties of a material. They have both been used for such purposes.

Furthermore there is a field that has been growing extremely rapidly in the last five or six years, perhaps longer, but certainly more rapidly as time goes on, and that's the field of the high-frequency ultrasonics. No one knows quite how far this is going, but frequencies as high as 25,000 mc/sec and higher are being produced. Microwave ultrasonics lies in the same frequency range as did microwave devices used during the war. Sound velocity in a solid may range from 2×10^5 cm/sec to about 2×10^6 cm/sec depending on the particular solid. With a velocity of about 5×10^5 cm/sec and a frequency such as 25,000 mc/sec, one has a wavelength that is well below half the wavelength of visible light - a couple thousand angstroms, and there is no reason why one has to stop there. Actually this is about four hundred times the five-angstrom lattice spacing. I mention this only because presumably the lattice spacing is the limiting feature, with wavelengths that are shorter than a lattice spacing, the lattice probably will not support the propagation of the wave. So one expects an

²Weinrich and White, Observation of The Acoustoelectric Effect, Phys Rev., 106, 1104 (1957).

upper limit here. However, there are other things that probably will set a limit before such wavelengths are reached - things such as the attenuation of the waves. At present the upper frequency available is set by technique difficulties and not by fundamental questions.

The next thing I want to do is to present a couple of examples. People continually ask, "What can you hope to learn by using ultrasonic methods?" It depends entirely upon which of the various types of interaction one is using and it depends entirely upon the particular problem. One of the more interesting uses of ultrasonic measurements, one in which we have been particularly interested, has been the study of radiation effects in solids, particularly the cobalt gamma irradiation of NaCl. We do the following experiment: With a sample of NaCl in the form of a flat cylinder with a quartz transducer on it, a room full of ultrasonic equipment and a room full of cobalt gamma ray source equipment, irradiate this sample with Co^{60} gamma rays (Fig. 2). There is a velocity increase and an at-

Fig. 2. Schematic of Experiment Arrangement

tenuation decrease. If one measures attenuation it will be found that it will decrease as a function of time of irradiation. The rate depends on the strength of the gamma source. With a relatively low irradiation rate, such as about 3500 roentgens per hour, we measure attenuation and the time of irradiation (see Fig. 3). We now know that the size of the change depends on the dislocation density and the loop lengths. The dislocation density depends usually on how much the sample has been deformed. Sometimes we have had crystals that have been deformed and we didn't know it. They had been deformed somewhere in the preparation. Sometimes we have had crystals where there was no effect at all; the attenuation did not change appreciably. In other cases there might be a 95 per cent change. We normalize this attenuation, that is, we subtract from all measure values the attenuation value that we get at infinite time, at which no further change takes place. Presumably the residual value has nothing to do with what the gamma rays are doing. We take the maximum value minus the value at time t , divide it by the total difference between the maximum value α_m and the minimum value, let's call that α_∞ ; this we call normalized attenuation α^* :

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Fig. 3. Decrease in Attenuation with Increasing Time of Irradiation

$$\alpha^* = \frac{\alpha - \alpha_{\infty}}{\alpha_{\max} - \alpha_{\infty}} \quad . \quad (7)$$

These measurements were first made at room temperature. The question was, of course, what was the mechanism of these changes? In other words, why does the loss go down when we irradiate? Irradiating with gamma rays presumably produces a lot of ionization. The attenuation decreases. We were rather sure we were not putting dislocations into the NaCl by gamma rays. We know pretty well, both empirically and otherwise, that attenuation depends on dislocation density, Λ , and loop length, L , to the fourth power:

$$\alpha \sim \Lambda L^4 \quad , \quad (8)$$

and velocity changes, or fractional velocity changes depend, among other things, on this to the second power:

$$\Delta V/V \sim \Lambda L^2 \quad . \quad (9)$$

Since we can measure both of these things, and since we are rather sure that the dislocation density is not changing, then Λ is a constant. We are fairly certain that we are observing the loop length changes as the irradiation proceeds. If we are studying just the loop length changes it means the loop lengths are in some way shortening. With a string that is vibrating one may pin it at some additional point and thereby cut down the amplitude, and so cut down the vibrational energy. If one pins the dislocation completely, it can't vibrate at all. If it cannot vibrate, there is obviously no loss from vibrational motion. If this is the effect, if we're pinning dislocations (which we are pretty certain that we are), we suppose that the pinning agents, the point defects interacting with the dislocations, have diffused into the dislocations after being freed to move or after being in some way produced by the gamma rays. We first believed the point defects were vacancies which diffused into the dislocation and pinned them. This is a reasonable thing. We found out however that when we did such experiments at lower temperatures it did not appear that this explanation was correct. We found out that at liquid nitrogen temperature our data coincided with that taken at room temperature. There was no difference between the liquid nitrogen attenuation change and the room temperature attenuation change. The argument is then that if there is no temperature effect, there is no thermal activation for the motion of whatever it is that's doing the pinning. So at liquid nitrogen temperature, at least, and at room temperature, the effect is temperature independent. If this is so, then the effect is certainly not thermal, and one can argue that it is an electrical effect, but the matter is not easily proved.

Next I will indicate the effect of deformation on the irradiation (gamma) of NaCl. There is a large difference among attenuation-irradiation curves for deformed and undeformed samples even at room temperature. Varying amounts of deformation produce the effect shown in Fig. 4. For Curve (a) there is no deformation or at least no deformation deliberately introduced. We will call this a sample with zero deformation. It doesn't mean that it has no dislocations. Curve (d) is for a

Fig. 4. Attenuation as a Function of Time of Irradiation and Deformation

sample which has been deformed 1.3 or 1.4 per cent, and this makes a large difference in the irradiation results. The attenuation decreases very much more rapidly to a given value than it does for a sample that has not been deformed. Every one of the curves that we have falls exactly in a pattern of increasing deformation. They're very close together, we have about ten such curves which all lie in order of increasing deformation; there is certainly no question about the effect.

So, there are a number of effects that we can study here and such effects allow us to study what mechanisms operate in the solid under certain conditions. What is happening with increasing deformation? If the effect is temperature independent, is the deformation effect then really an electrical effect or is

there something else? Deformation is believed to put jogs into dislocations. There is evidence for it. And if we add jogs, there is a charge associated with each jog. We're running up the charge by deforming; we are therefore, running up the effect of the force interaction between our point defects, which are, incidentally, probably a sheath of vacancies positively charged sitting outside the dislocation. If this is the case the problem is then a space charge problem. The force can then be increased by deforming because of the jog situation. There are, however, other possible answers to this question of what the effect is, but I have used this example to show how one uses ultrasonic methods and tools to examine such questions in Solid State Physics.

Discussion

ULTRASONIC ATTENUATION AND THE PHYSICS OF SOLIDS

Leonard Goodman, Atomics International: Refer to your set of equations on the board directly behind you (equations 4 and 5). In your C_{ijkl} , is that a subscript or direction cosine?

Truell: These are subscripts. these are the elastic coefficients.

Goodman: All four of them ?

Truell: All of them. This is standard notation, except possibly the comma notation. The C_{ijkl} is just crystal elasticity. This may be found in any book, such as those of Nye³ or Voigt⁴ or Kolsky,⁵ that discusses crystal plasticity.

Goodman: The second question: what is your coupling medium, at these high frequencies and low temperatures, between the crystal and your sample ?

Truell: It's a long story if I try to tell the whole thing. We put transducers on with a thin film of grease or glycerine. It's very touchy business, and you don't always succeed. It depends strongly on the material to which one is bonding the quartz transducer. The bond will freeze at the low temperatures, but you hope it freezes with good contact, or with good bond. One very satisfactory thing we've been able to do is to use quartz, both as sample and transducer. We can also do this with cadmium sulfide, since it is also piezoelectric. We can go down to helium temperature without using transducers at all. We simply put electrodes on and use the sample as its own transducer. Obviously there are only a few things with which one can do that. Generally, with samples such as germanium or metals (in particular, with pure aluminum), one can use bonded transducers but silicon is very difficult. Henry, do you have any comments ?

Henry Bohm, Wayne State University: One of the common ones is Nonaq grease or propyl alcohol.

Truell: There are several possibilities for improving the situation. We have thought for a long time that if we could make the bond at low temperatures we would be all right. And this has been done. In fact, this was done with the NaCl even to go to nitrogen temperature. There is, however another possibility. One can make the bond at low temperatures with something that doesn't freeze until you are at very low temperatures, for example with certain gases. Take the sample to a low temperature, play a jet of the gas on the sample so that it condenses, and then bond the crystal to it. This is obviously quite a business, technique-wise.

Charles Vance, Portland Cement Association. Where you talked about the interaction between the thermal elastic waves and your ultrasonic

³Nye, Physical Properties of Crystals

⁴W. Voigt, Lehrbuch der Kristallphysik

⁵Kolsky, Stress Waves in Solids

elastic waves: Do the studies which you have done indicate that there are in this coupling effect, about which you talked, resonance effects which are usable for amplification, for example, of either the thermal waves or the ultrasonic waves?

Truell: In principle it certainly could. It's a nonlinear effect. I don't believe that we know enough about the effects, but the idea is interesting. I am afraid that I can't really answer your question without more time to think about it. There seems to be no question about the phonon-phonon effect, or interaction. The nature of the effect is like that in thermal conductivity. Ultrasonic work at high frequencies is getting to look very much like thermal conductivity studies, except that instead of dealing entirely with the lattice waves one is dealing also with the ultrasonic waves. The big difference is that one has, with ultrasonic waves, a single frequency. One has a monochromatic wave which one may use to explore or study a spectrum of thermal vibration waves. This is a tremendous advantage over thermal conductivity work. I've wandered from what you asked, but I think it's an interesting point. How useful this will be, I don't know. I believe that it will be tremendously useful to study materials. As an amplification device, I frankly hadn't thought of it.

Vance: My thinking was that so long as one can vary the ultrasonic frequency, why then you can fairly readily explore this type of action.

Truell: That's right, in one way that is what we are doing.

Vance: What brought it up to my thinking was an item which I read recently, which some of you gentlemen have undoubtedly seen, the fact that for the first time in history as far as anyone knows, they have been able to amplify light.

Truell: The only thing I am sure of is that there will be an interaction between the two.

Goodman, Atomics International: In one of the items of the ten, you indicated dislocation damping. In other words, interaction between the elastic wave and dislocations in the crystal. Then we're talking here about mechanical change. Have you any reason to believe that as a result of some of the testing you have done on these crystals, you have produced any change in the mechanical properties of the crystal, as a result, say, of getting dislocations to pile up, or diffuse, or -

Truell: Very definitely. This is one of the things of which we can be quite certain. We have done many types of deformation experiments. Particularly, we have studied attenuation as a function of strain or deformation while increasing the strain. The attenuation change depends very strongly on strain rate, it depends very sensitively on temperature, and it

depends on material purity. High-purity Al is quite different from commercial Al. There is a breakaway of the dislocations that, I believe, one can actually see. Having deformed a material, one can then watch it recover. One has in high-purity Al or even Al of commercial purity, a total recovery. Starting at a certain attenuation value, let's make a plot of strain versus attenuation up to a certain point, shown in the left side of Fig. 5. This may change during deformation by a factor of two, three, or

Fig. 5. Variation of α Under Specific Conditions

much more. Now, we will watch the attenuation after deformation is stopped. We will either unload it or leave it loaded, it doesn't matter. You will see a recovery curve like the right-hand part of Fig. 5. The attenuation drops very rapidly in Al. This recovery curve yields a great deal of information about how material is going to behave under cycling conditions such as in fatigue or any type of further deformation. Of course, I'm talking entirely in terms of dislocations, I'm not talking about stress-strain behavior here. But your specific question, if I understood it was, what changes have been made in the material.

Goodman: Permanent changes.

Truell: Permanent changes, right. If you cycle a piece of material slowly, and watch this kind of thing over 1000 to 1500 cycles, depending on the strain, and of course many other things, you will produce permanent changes after something like (under conditions I happen to know), 500 to 1000 cycles. The material is then permanently changed if you use our methods of looking at these changes. It will not recover completely to where it was in any interval of time you are going to be there to watch it. The moduli are changed. How much depends on many things. If you continue to cycle, the slip planes come into play. We may have introduced slip in the first ten cycles, and if so this will help to pin dislocations in an irreversible way. This can cause the attenuation to decrease. If you continue to cycle, the attenuation will increase. These are permanent changes. The slip planes that have started to operate will pin the dislocations because there is no recovery whatever, after a sufficient amount of cycling, and the number need not be large. We do experiments in which we measure attenuation as a function of the number of cycles (see Fig. 6) and we have a permanent recording device that records every cycle, attenuation-wise, as we go. We cycle slowly, so we can follow each cycle completely. The results of these experiments are to be found in two reports.

Fig. 6. Attenuation Versus Number of Cycles

C. J. Renken, Argonne: On item 6, again, you speak of ultrasonic elastic waves, thermal elastic waves or lattice vibrations and the interaction between the two. I would like to ask how two sets of elastic waves interact with each other?

Truell: Such effects can exist only because they are not elastic in the strict sense - there are non-linear or anharmonic effects present. The experimental evidence is rather clear. This seems to be an Umklapp type of effect or the phonon-phonon type of interaction that you'll find, for example, discussed in Peierls book on Solid State Physics, the Quantum Theory of Solids.

THE DIFFUSION OF PULSED CURRENT FIELDS IN GOOD CONDUCTORS

by

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ABSTRACT

The use of pulsed fields in electromagnetic testing is a fairly recent development. This technique offers a number of theoretical and practical advantages over methods which use steady-state sinusoidal fields. It is possible to utilize fields of small cross-sectional area to produce electromagnetic test systems of improved resolution. The relatively slow rate of diffusion of current fields in good conductors makes possible the time separation of reflections from the surface of the metal and from internal discontinuities.

Recent development in the field of electromagnetic testing⁽¹⁻⁴⁾ indicate that techniques which use pulsed electromagnetic fields offer some significant advantages over the older, steady-state methods. To understand the possible advantages of this technique, it will be useful to examine the manner in which transient fields travel in metals.

The Concept of Phase, Group, and Signal Velocities

Consider the various velocities of propagation associated with the transmission of electromagnetic energy through various media. The concept of phase velocity applies only to those waves which are periodic in space and time. If such a wave is represented by the function

$$f(z,t) = A \exp j(kz - \omega t) \quad ,$$

then the surfaces of constant phase are given by

$$kz - \omega t = \text{constant},$$

and the velocity by

$$v_p = \omega/k \quad .$$

This is the definition of phase velocity. A wave train of finite length cannot be represented by a function of the form given above, so the term "phase velocity" loses its meaning in this case.

The concept of group velocity can be obtained by considering two harmonic waves which differ slightly in frequency and wave number:

$$f_1 = \cos (kz - \omega t)$$

$$f_2 = \cos [(k + \delta k)z - (\omega + \delta \omega)t] \quad .$$

By adding these, it is easy to determine that the resultant field has an effective amplitude

$$A = 2 \cos \frac{1}{2} (z \delta k - t \delta \omega) \quad .$$

As might be expected, this is the well-known expression for the phenomenon of beats. The surfaces over which the group amplitude is constant is given by

$$z \delta k - t \delta \omega = \text{constant} \quad ,$$

and the group velocity is

$$v_g = \delta \omega / \delta k \quad .$$

The concept of group velocity applies only to those functions which can be represented by a Fourier series extending over a narrow part of the spectrum. A pulse is not such a function, since its frequency distribution is continuous. In a nondispersive medium, the phase velocity is a constant, so $\delta \omega = v_p \delta k$ and the group velocity equals the phase velocity. But in a dispersive medium, as the interval δk is increased the respective phase velocities of the components of the signal diverge in value; then the concept of group velocity as a velocity of the traveling field loses its significance. In a metal, the group velocity may exceed the velocity c . At one time this caused consternation among scientists because it was thought that the group velocity was necessarily associated with the velocity of energy propagation.

The most important thing to be gained from this discussion is that these velocities are not directly associated with velocity of a pulsed current field diffusing in a metal. We will see that the velocity of such a field must be defined in a somewhat arbitrary manner.

Let us assume that a field is propagating in a metal and station ourselves at a point in the metal in time to watch its arrival. According to Brillouin,⁽⁵⁾ the imminent arrival of the main body of the energy is

signaled by the arrival of the first precursor wave traveling with a velocity $v = c$. It grows slowly in amplitude and period; then the amplitude drops while the period approaches the natural period of the electrons. Next to appear is the second precursor, which is traveling with a velocity

$$\frac{\bar{\omega}_0 c}{(\bar{\omega}_0^2 + a^2)^{1/2}},$$

where $\bar{\omega}_0$ is the atomic resonance frequency. It increases in amplitude and then dies away in somewhat the same manner as does the first precursor. Suddenly the amplitude of the signal rises, indicating the arrival of the main body of the disturbance traveling with a velocity which Brillouin terms the signal velocity. If the signal is a pulse, it is more or less distorted by its passage through the metal. Its velocity depends upon which part of the waveform we choose to define its arrival. The waveform changes as it passes through the metal; moreover, the more sensitive the instruments, the higher the velocity measurable. Perhaps this all sounds arbitrary and inexact. However, in our experimental work the meaning of the term signal velocity is quite clear. It refers to the arrival of the main bundle of energy and is unmistakable, as will be seen later. The signal velocity may be surprisingly low; in copper it is on the order of eleven times the thermal diffusion rate for that same metal.⁽⁶⁾ For certain thicknesses of austenitic stainless steel, velocities around 300,000 cm/sec have been measured.

Since the wave number of a sinusoidal field traveling in a metal can be approximated by

$$k = (\mu \epsilon \omega^2 + j\mu \sigma \omega)^{1/2},$$

one is tempted to compare the metal to a poorly designed, lossy electronic delay line.

The Case of the Plane Wave Impinging on a Good Conductor

The theoretical analyses of the pulsed-current diffusion problem by Miller,⁽⁶⁾ Waidelich,⁽⁷⁾ Vallese,⁽⁸⁾ Simms,⁽⁹⁾ and others have provided useful information for the experimental work. We will examine one of these analyses⁽⁷⁾ to note the assumptions made and to enable us to better understand the experimental results which will follow. The situation is shown in Fig. 1. Assume a plane wave traveling in the x direction with $\bar{E}$ parallel to the y axis and $\bar{H}$ parallel to the z axis. Maxwell's equations for the wave inside the conductor are given as

$$\begin{aligned} \nabla \cdot \bar{E} &= 0 & \nabla \times \bar{H} &= \sigma \bar{E} \\ \nabla \cdot \bar{H} &= 0 & \nabla \times \bar{E} &= -\mu S \bar{H} \end{aligned} \quad (1)$$

where S is the complex variable of the LaPlace transform method. It can be shown that

$$\frac{d^2 H_Z}{dx^2} = s\mu\sigma H_Z \quad . \quad (2)$$

Note we are neglecting displacement current. From Eq. (2) and its counterpart involving E_y , we find that in the metal

$$H_Z = A \exp [-(\mu\sigma s)^{1/2} x] + B \exp [(\mu\sigma s)^{1/2} x] \quad (3)$$

and

$$E_y = \frac{(s\mu)^{1/2}}{\sigma} [A \exp -(\mu\sigma s)^{1/2} x - B \exp (\mu\sigma s)^{1/2} x] \quad . \quad (4)$$

These are the expressions for the positive and negative traveling waves in the metal.

Fig. 1

Geometry Associated with the Problem of a Plane Wave Parallel to the Metal Surface

In air,

$$H_Z = A \exp \left(\frac{-sx}{c} \right) + B \exp \left(\frac{sx}{c} \right) \quad (5)$$

and

$$E_y = Z_0 \left[A \exp \left(\frac{-sx}{c} \right) - B \exp \left(\frac{sx}{c} \right) \right] \quad , \quad (6)$$

where $c = (\mu\epsilon)^{-1/2}$ and $Z_0 = (\mu/\epsilon)^{1/2}$, and μ and ϵ are the permeability and permittivity of the air. The boundary conditions may now be applied to Eqs. 3,4,5, and 6. This has been done for the case of a metal clad with another metal, a familiar situation to those working in the field of nondestructive testing. The solution for the reflection from the metal-metal

boundary is shown in Fig. 2, where an impulse-type input function has been assumed. Since the physical evidence of these fields is likely to appear as a voltage across a pickup coil, Fig. 2 shows the shape of the derivative of the magnetic intensity component of the reflected field, H_z . Note also in the expression for dH_z/dt the occurrence of the factor

$$\frac{\left(\frac{\mu_1 \sigma_2}{\sigma_1 \mu_2}\right)^{1/2} - 1}{\left(\frac{\mu_1 \sigma_2}{\sigma_1 \mu_2}\right)^{1/2} + 1}.$$

This is the reflection at the metal-to-metal boundary, which is zero if the cladding and base metal have the same conductivity and permeability. The reflection coefficient at the surface is nearly unity, the reflection coefficient at the metal-to-metal boundary is a small fraction, and the attenuation of the field in the metal is high. All this means that the reflection from the interface as it appears at the surface will be very weak compared to the surface reflection, perhaps of the order of 10^{-4} as intense.

Fig. 2 The Reflection from the Metal-to-Metal Boundary

Figure 3 shows the ratio of the voltage reflection from the metal-to-metal boundary to the voltage reflection from the air-to-metal boundary, again assuming an impulse input. It should be mentioned that the surface reflection in this case shows up as an impulse with a long tail attached to it. This same problem has also been solved for the assumed condition of a step input function, a half sinusoid, and a number of other waveforms. The results are much more complicated, and won't be taken up here. Once the problem has been solved using the step input function, the convolution integral can be used to derive solutions using other input functions.

Fig. 3 The Voltage Ratio of the Reflection from the Metal-to-Metal Boundary to the Reflection from the Air-to-Metal Boundary

Practical Uses of the Pulsed Field Technique

Practical techniques which use pulsed current fields can be classified in two ways: the through transmission and the reflection techniques. The through transmission technique⁽⁴⁾ is, simply as its name implies: a field is sent completely through the metal and is detected on the other side by appropriate means, as is shown in Fig. 4. The amplitude of the received pulse can be measured to obtain information about conditions within the metal, or the amount the pulse is delayed in its passage can be measured to accomplish the same thing. The latter is perhaps the more attractive

of the two methods because the amount of delay is relatively insensitive to the location of the metal in the space between the transmitting and receiving coils. Even though the space is maintained constant, variations of the distance between transmitting coil and the surface of the metal do

Fig. 4 The Geometry of the Through Transmission Technique

affect the amplitude of the received pulse and, in a small degree, the pulse delay. It is not difficult to guess at the reasons for this phenomenon, but the available time is not sufficient for a discussion of them. If the source produces plane waves, the pulse delay should be completely independent of the spacing for a metal with constant permeability and conductivity. Figure 5 shows a plot of magnetic field intensity at the location

of the receiving coil before and after a metal plate is inserted between it and the source of the field. Note how the pulse traveling through the metal is lengthened and delayed. If the signal velocity is measured in accordance with the apparent beginning of each pulse, we calculate a velocity of 2.8×10^5 cm/sec. In a better conductor, the velocity would be lower.

Fig. 5 A Plot of Magnetic Field Intensity at the Location of the Receiving Coil Before and After a Metal Plate is Inserted Between It and the Source of the Field

The reflection technique allows operation from one side of the metal only and detects possible internal abnormalities by means of the reflections they create. This method was first used by Waidehlich.⁽¹⁾

Related work has also been done by Libby⁽³⁾ using a slightly different approach. Our latest work in this field can be classified under the heading of reflection techniques. It should be mentioned that the source of the fields for our work has been in most cases a small aperture in a specially designed mask which emits a field of relatively small cross-sectional area. The apertures themselves range from 0.060 in. in diameter down to about 0.020 in. Although a current field diffusing in a metal is never completely localized either in space or time, our experimental results show that for many test situations the use of these masks results in greatly improved resolution over conventional eddy current techniques. Some experimental results produced while using fields of small cross-sectional area are shown in Figs. 6 through 9. No attempt will be made to analyze these curves, but they all illustrate the relatively slow rate of diffusion of current fields in good conductors. Figs. 8 and 9 result from a cladding situation especially selected to show the metal-to-metal interface reflection. Most practical situations will result in a much weaker reflection. Obviously, the aperture is not a source of plane waves with the aperture-to-metal spacings ordinarily used, so the results of the analysis given previously are useful only in a qualitative way.

It seems that the use of pulsed fields of limited cross-sectional area offers at least a slightly different approach to the problems of electromagnetic testing. Electronic equipment must now be designed to convert these theoretical and experimental concepts into practical test systems.

Figs. 6 and 7 Voltage Reflections from the Near Surface and Opposite Surface

Figs. 8 and 9 Voltage Reflections from the Surface and Metal-to-Metal Interface

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Discussion

THE DIFFUSION OF PULSED CURRENT FIELDS IN GOOD CONDUCTORS

Paul Edwards, Los Alamos: Are you currently doing, or contemplating, any work on the interaction between these fields and ultrasonic fields in the domain in which they are appropriately pretty equal in velocity?

Renken: No.

Charles Vance, Portland Cement Association: Just a real quick one. I'm not sure I heard you correctly. In what was originally your last slide there, did you say that's a good conductor clad to a poor conductor, or vice versa?

Renken: Are you referring to the case of stainless steel clad on aluminum?

Vance: I just want to determine which was which. You said one was a good conductor on a poor conductor, and the other was poor conductor on good conductor.

Renken: As long as we are speaking about metals, stainless steel is a poor conductor compared to aluminum.

Vance: Relatively speaking?

Renken: Yes.

Peter Morris, National Lead: You've indicated two separate curves, one for the reflection from the forward face, and one for the reflection from the inner face. Now you plotted these separately. Did you, in effect, have to separate this out of a common oscilloscope trace?

Renken: That's right. According to the analysis, one can obtain the shape of the surface reflection by using a very thick material. This can be subtracted from the reflection obtained from the clad material to obtain the metal-to-metal interface reflection. The big electronic problem, of course, is separating these two signals. I think we have ways of doing this now, mainly due to the fact that for some period of time in most of these situations, there is a definite delay before the interface reflection returns. At least, the interface reflection is delayed appreciably behind the surface reflection, unless the cladding is too thin. And, of course, this gives us a magnitude of the surface reflection without interference from the interface reflection. And so, if the shape of the interface, or the surface reflection is stored, and the proper coefficient is put in front of it, one can obtain the surface reflection at any particular spacing. This has been done, at least, on a trial basis.

Henry Bohm, Wayne State University: Would you say something about pulse magnitude shape?

Renken: Well, I could say that the pulses usually have a peak power of, say, perhaps 50,000 watts, but it's very difficult to say just what the peak magnetic intensity is, mainly because the aperture complicates the problem. But I don't know what the peak magnetic field intensity might be at the surface of the metal. The peak currents in the test coil are probably around 40 or 50 amp.

R. S. Paul, Hanford: Could you sketch your experimental wave form and indicate how you get from that to the curves you show?

Renken: Well, the curves I've shown were not actually time derivatives of the magnetic field but rather they are approximations. I've shown the actual shapes and time functions picked up by the receiving coil.

Paul: Well, aren't these surface reflections, the back surface reflections actually combined in the one signal that you see?

Renken: Yes. They are occurring at the same time.

Paul: The curves you have: one of them is a calculated curve, the other is the difference between that and the experiment?

Renken: Right. I don't know if you ever can see them separately. One reason is that when the metal is thick enough the internal energy storage term in the metal has the shape of a very long, low-frequency pulse. And the surface, or the interface reflection, is always going to be occurring sometime during the duration of this long pulse.

MAGNETIC PROPERTIES OF REACTOR METALS AND ALLOYS

by

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ABSTRACT

A survey is made of some of the magnetic properties of metals and alloys of reactor significance, involving principally the thoride elements, thorium, uranium and plutonium, the rare earth elements and the transition elements, titanium, zirconium and hafnium.

The fundamental concepts of diamagnetism, paramagnetism, ferromagnetism, ferrimagnetism and antiferromagnetism are given a short preliminary review and some of the important experimental techniques used in magnetic measurements are described briefly. Principal attention is given to paramagnetic susceptibility, saturation and spontaneous magnetizations of ferromagnets, Curie temperature, crystalline anisotropy of magnetization, and the Hall effect.

The similarities in the electronic structures of the thorides, the rare earths, and the transition metals are pointed out, and are used as a framework for the interpretation of some of the magnetic properties.

Introduction

The unique materials requirements of nuclear energy and the increasing complexity of nuclear reactors have resulted in the use of an extensive list of metals and alloys. Even when our discussion is confined to the rather narrow limits of magnetic properties it is not possible in a brief discussion to deal with all of the metallic materials that have been used or have potential use in nuclear reactors. Instead, we must choose a representative number of them and acknowledge that some significant omissions may necessarily result.

We may logically pick three groups of elements and their alloys. The first, and most important, is comprised of the thoride elements thorium, uranium and plutonium. These are the elements whose isotopes are either fissionable or can be transmuted to fissionable isotopes.

Needless to say, one or more of these elements or their alloys must be used in every nuclear reactor.

The lanthanides or rare earths make up the second group. These elements have a variety of useful nuclear properties. Yttrium is nearly as light as aluminum and has a low neutron cross section. Gadolinium and europium can be used as thin, lightweight nuclear shielding for aircraft and, because of their high neutron cross sections, they can be used as control rods. Samarium can also be considered as a shielding material.

The elements of the titanium group, titanium, zirconium and hafnium, constitute the third set. Titanium has potential usefulness as a cladding material in fast reactors. Zirconium has a low thermal neutron cross section and has found considerable use in thermal reactors. It has good resistance to aqueous corrosion. Hafnium has a high neutron cross section and can be an important control rod material.

Basic Concepts

Before pursuing our examination of the magnetic behavior of these metals and their alloys, it may be helpful to undertake a brief discussion of some of the elementary concepts of magnetism. The theory of magnetism has shown that it is of electronic origin. When an atom or ion has an electronic structure such that one or more of its subshells is incompletely filled, it possesses a permanent magnetic moment. When placed in the presence of a magnetic field, the atom or ion behaves as a tiny bar magnet. If there is no external field and the interactions between the unit magnets are small compared to the disorienting forces of thermal agitation, the atomic moments are randomly oriented and the material has no net external magnetic moment. In certain circumstances, however, the lowest free energy obtains for the situation in which all of the moments are aligned in a parallel fashion and the sample as a whole does possess a net moment. The occurrence of such a situation is known as ferromagnetism. Under other conditions the moments can be aligned in an antiparallel fashion on several magnetic sublattices. Sometimes the sublattices are nonequivalent and the bulk moment of the material has a nonzero value. Such a material is known as a ferrimagnet. Other materials have geometrically identical sublattices having moments equal in magnitude but aligned antiparallel. The net moment is zero in this case and we have what is known as an antiferromagnet.

Below a critical temperature a ferromagnetic material exhibits large magnetic saturation, even in small fields. The fact is that any microscopic region is saturated even at zero field, with spins aligned parallel, but the overall magnetization is small because the regions of parallel spins, called domains, are randomly oriented in the bulk sample.

The disruptive tendencies of thermal agitation oppose all of the cooperative phenomena just mentioned and with increasing temperature a point is reached at which thermal agitation overcomes the orderly arrangement of magnetic moments. This is the Curie temperature.

Figure 1, taken from Selwood,⁽¹⁾ shows the variation of saturation magnetization of a ferromagnet from absolute zero to the Curie temperature. Figure 2, taken from the same source, shows how the magnetic susceptibility of an antiferromagnet reaches a peak value at its Néel temperature.

Fig. 1. Saturation Magnetization of a Ferromagnet⁽¹⁾

Fig. 2. Susceptibility of an Antiferromagnet⁽¹⁾

Ferromagnets above the Curie temperatures, and many other metals and alloys as well, possess magnetic behavior known as paramagnetism. Simple theory suggests that the materials obey the Curie-Weiss law for the susceptibility χ :

$$\chi = C/(T - \theta)$$

where C is the Curie constant, T is temperature, and θ should be equal to the Curie temperature of a ferromagnetic, but is negative for an antiferromagnetic.

A simpler Curie law can be obeyed if the atomic moments are free of interactions with their neighboring atoms, other than thermal interactions:

$$\chi = C/T$$

This behavior is to be expected if the electron spins responsible for magnetism are deep in the core of the atom and shielded by the outer electrons. We shall see that such behavior in metallic solids is observed in some of the rare earths.

In order to complete our discussion we should point out that metals and alloys also possess a weak form of paramagnetism associated with the interaction of their conduction electrons with a magnetic field. Furthermore, there is imposed on all of the forms of magnetic behavior yet discussed a negative diamagnetic contribution to the magnetic behavior that results from the orbital motion of electrons. These last two forms of magnetism will not figure significantly in the balance of our discussion.

Terms and Experimental Methods

We now turn briefly to a discussion of terms and experimental methods. The most significant property is the magnetic susceptibility. The gram susceptibility, χ , is defined as follows:

$$\chi = \sigma / H \quad ,$$

where σ is the magnetization in emu per gram and H is the field in oersteds. The volume susceptibility, κ , is defined as follows:

$$\kappa = I / H \quad ,$$

where I is the magnetization per unit volume. For paramagnets χ or κ is positive, whereas for diamagnetic materials negative values occur. In both cases the numerical values are small, being commonly of the order of 1×10^{-6} . For the remainder of this paper the term susceptibility will denote χ unless otherwise stated. Ferromagnetic susceptibilities are positive and very large, reaching values as high as 10^5 . It is more common to describe a ferromagnet in terms of its saturation magnetization and its spontaneous magnetization. When expressed per unit of mass, the symbols $\sigma_{\infty, T}$ and σ_0, T are used to designate these quantities. The magnetic moment per atom is more commonly expressed in Bohr magnetons per atom (μ_B) and the symbol n_0 is used.

Experimental Methods

Paramagnetic susceptibilities are usually measured with a magnetic balance. This measurement depends on the fact that, when a body having a magnetic moment is placed in a nonuniform field, the body will experience a linear displacing force in the direction of the field gradient.

The Gouy method, shown schematically in Fig. 3, involves suspending a long sample, usually cylindrical, between the poles of a magnet so that one end is in a region of large field while the other is in a region of negligible field. The force, which is proportional to the volume susceptibility, the area, and the square of the field, is measured by observing the apparent change in weight of the sample.

Fig. 3. The Gouy Method⁽¹⁾

For ferromagnetic materials an experimental method involving magnetic induction is often employed. A sample of the test material, in a uniform magnetic field, is withdrawn from an induction coil which is in series with a fluxmeter whose deflection has been calibrated in terms of magnetic moment. A device of this kind which is in use at the Argonne National Laboratory has been described in the literature.⁽²⁾ It employs a large electromagnet and a helium cryostat (see Fig. 5). Two induction coils, each containing 10,000 turns of copper wire, are connected in series opposition and are placed in series with a ballistic galvanometer of high sensitivity. A sample of pure nickel can be used for calibration.

Induction methods can also incorporate AC circuitry and there is a wide variety of specialized instruments employing this concept. Space limitations prevent a discussion of these.

The Hall effect is an interesting property which is peripheral to magnetic measurements. When a conductor is placed in a magnetic field which is perpendicular to the direction of current flow, a voltage is developed in the direction which is normal to the plane of the field and the current. This is called the Hall effect. The Hall coefficient, which relates the voltage generated to the applied current and

The Faraday method (see Fig. 4) is similar, but employs a small specimen which is placed in a region in which there is a large field gradient. A force which is proportional to the mass susceptibility, the field, and the field gradient is exerted on the specimen. In a method developed by Sucksmith the balance is replaced by a phosphor bronze ring attached by a rigid member to the specimen. The force on the specimen is determined by the distortion of the ring, which is greatly amplified by a system of mirrors. A calibration is required.

Fig. 4. The Faraday Method⁽¹⁾

field, is negative for some metals, in which electrons are the charge carriers. It is positive for semiconductors and other metals whose conduction is by positive holes.

Fig. 5. Apparatus for Saturation Magnetization Measurements(2)

Some Experimental Results

The data and interpretations which are embodied in the following discussion have been selected from the unclassified literature. The elements and their alloys are grouped in the manner suggested in the Introduction: the thoride elements, the lanthanide elements and the titanium group elements.

The Thoride Elements

Thorium. For this metal χ is temperature independent and equal to 0.410×10^{-6} emu/gram.⁽³⁾ The absence of Curie-Weiss dependency indicates that in the metallic state the thorium atom is not characterized by a significant localized moment.

Uranium. In the isolated atom uranium has unpaired electrons in the 6d subshell. In the metallic state, uranium is a paramagnet characterized by a rise in χ with temperature, and with abrupt changes in χ associated with $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$ transformations. The data of Bates and Hughes⁽⁴⁾ is shown in Fig. 6; the considerable hysteresis that accompanies the allotropic changes is demonstrated by the use of open circles to denote data taken during heating and crosses to indicate data taken during cooling. The room-temperature value of χ is believed to lie between 1.67×10^{-6} and 1.72×10^{-6} emu/gram. A high density of states is indicated by the high value of χ . Friedel⁽⁵⁾ has considered the band structure of alpha uranium and has suggested that this structure can be considered as belonging to a metal having about one electron in a narrow and almost empty band, 3 or 4 electrons completely filling another band, and possibly about one electron in a broad conductivity band having normal mass. However, uranium has a positive Hall coefficient ($+3.4 \times 10^{-5}$ cm³/coulomb at 20°C) that does change significantly up to 300°C.⁽⁶⁾ This fact does not seem to support the concept of a broad conductivity band.

Fig. 6. Susceptibility of Metallic Uranium⁽⁴⁾

Uranium Alloys. We shall limit our discussion to several alloys that are representative of various types of magnetic behavior.

In the uranium-hydrogen system a compound, UH₃, exists whose beta allotrope exhibits an interesting ferromagnetic behavior. Its properties have been investigated by numerous workers.⁽⁷⁻¹¹⁾ It shows high hysteresis and a saturation magnetization of 1.18 Bohr magnetons/atom of uranium. This value is much lower than that expected for trivalent or quadravalent uranium ions:

Fig. 7. Magnetization Curves for UH_3 , UD_3 and UO_2 ⁽¹¹⁾

All three alloy systems show an initial decrease in χ up to about 0.5 a/o solute followed by an increase up to 2 a/o in alloys quenched from the gamma-phase temperature. Figure 8 shows the data of Bates and Loasby.⁽¹²⁾ There is no obvious explanation for the initial decrease. The subsequent increase is probably due to an increase in valence electron concentration per atom. Valences of the solutes are: Zr = 4, Nb = 5, and Mo = 6. The 2 a/o solute seems to be maximum to be held in supersaturated solid solution.

U-Fe, U-Co, and U-Ni. Although alloys in these systems are interesting magnetically because the alloying elements are ferromagnetic metals, not much data is available other than for a few intermetallic compounds. The compounds U_6Fe , U_6Co , and U_6Ni , all of which are isostructural, have susceptibilities within 5% of the value 2.00×10^{-6} . The value of χ increases slowly with temperature in a manner similar to alpha uranium. However, the temperature dependence is moderate, so bonding is primarily metallic. The compound UFe_2 , a MgCu_2 -type Laves phase, is ferromagnetic. Its Curie temperature

Therefore $\beta(\text{UH}_3)$ must exhibit a quasimetallic behavior leading to the nonintegral value for σ . Figure 7 shows the magnetic isotherm at 1.3°K for UH_3 and also similar data for the deuteride UD_3 and the oxide UO_2 .⁽¹¹⁾ The presence of $\beta(\text{UH}_3)$ as a contaminant probably led earlier investigators to the erroneous conclusion that pure uranium was ferromagnetic or at least had high χ at low temperatures

U-Mo, U-Nb, and U-Zr.

Dilute alloys of molybdenum, niobium, and zirconium in uranium exhibit similar magnetic behavior.

Fig. 8. Susceptibility of U-Mo, U-Nb, and U-Zr Alloys⁽¹²⁾

is given as 151°K and its saturation magnetization is 19.9 emu/gram, about $\frac{1}{6}$ that of Fe. Above the Curie temperature, UFe_2 obeys a Curie-Weiss law. The compound UNi_2 also shows Curie-Weiss behavior down to lowest temperature tested, but is not known to become ferromagnetic. The data on these compounds were determined by Gordon.⁽¹³⁾

Plutonium. The susceptibilities of the five allotropic forms of Pu are given below:⁽¹⁴⁾

<u>Allotrope</u>	<u>$\chi \times 10^6$, emu/gm</u>	<u>Temperature Dependence</u>
Alpha	2.20	Independent of Temperature
Beta	2.31	.
Gamma	2.22	Decreases Slowly
Epsilon	2.17	with Temperature
Delta	2.16	

Plutonium Alloys. Magnetic data on plutonium alloys are very limited. One interesting feature is that the compound PuFe_2 , which is isostructural with UFe_2 , is also ferromagnetic. Its Curie temperature is 633°K.⁽¹⁵⁾

The Lanthanide Elements

These elements have electronic structures for their isolated atoms that are characterized by the progressive filling of the 4f subshell.

Lanthanum. For lanthanum the value of χ is small, rising from 0.73×10^{-6} emu/gram at room temperature to 1.5×10^{-6} at 6°K.⁽¹⁶⁾ In the metallic state as well as in the free atom the 4f shell is empty, and the feeble paramagnetism is probably due to the conduction band of electrons.

Cerium. Lock's data,⁽¹⁶⁾ shown in Fig. 9, shows Curie-Weiss behavior down to about 100°K for Ce with an effective moment of $2.51\mu_B$. This value is in good agreement with a theoretical value calculated on basis of the ground state of the atom: $\chi_{RT} = 1.67 \times 10^{-3}$ emu/gram. At lower temperatures there is an anomaly associated with a change in crystal structure.

Praseodymium obeys the Curie law down to 100°K.⁽¹⁶⁾ At 25°C its susceptibility is 3.775×10^{-5} emu/gram. The effective moment deduced from Curie law, $3.56\mu_B$, is in good agreement with theoretical value for the free Pr^{+++} ion. Its susceptibility curve flattens out below about 4°K, at a value of 1.55×10^{-3} emu/gram.

Neodymium. The susceptibility of this element follows the Curie-Weiss law from about 20°K to 230°K with calculated moment equal to $3.3\mu_B$.^(16,17)

Fig. 9. Inverse Susceptibility of Ce(16)

Two sharp anomalies occur below 20°K which can be observed in Fig. 10. The anomaly at 7.5°K is due to the onset of antiferromagnetism. There is no explanation for the anomaly at 20°K.

Promethium. No information is available on this unstable element.

Samarium. There is some divalent tendency in the chemical behavior of samarium, and it is possible that a proportion of the atoms have 6 rather than 5 electrons in 4f states. The result is a lower susceptibility at low temperatures above the antiferromagnetic transition. The susceptibility increases with decreasing temperature, reaching a maximum at 14.8°K and dropping off very rapidly below this temperature. It rises again below 7°K. No Curie or Curie-Weiss behavior is observed at any temperature. The maximum at 14.8°K is due to the onset of antiferromagnetism. At 25°C, χ is 8.480×10^{-6} emu/gram.⁽¹⁶⁾

Europium exhibits Curie-Weiss behavior with a calculated atomic moment of $7.12\mu_B$.⁽¹⁸⁾ This value is difficult to interpret theoretically

since it is considerably smaller than the theoretical value, $7.94\mu_B$. A transition to ferromagnetism reported earlier⁽¹⁸⁾ has not been substantiated in a more recent study.⁽¹⁹⁾

Fig. 10. Inverse Susceptibility of Nd⁽¹⁶⁾

Gadolinium. Below 290°K gadolinium is ferromagnetic. Its saturation moment closely approximates theoretical value of $7\mu_B$.⁽²⁰⁻²³⁾ Above the Curie temperature it obeys the Curie-Weiss law with a derived magnetic moment $7.95\mu_B$. Lock⁽²⁴⁾ has pointed out that the large saturation moment is not of great practical value because the high atomic weight of gadolinium reduces the actual saturation intensity to a value not much higher than that of iron.

Terbium to Thulium. Table I taken from Lock's paper⁽²⁴⁾ summarizes the magnetic data for terbium,⁽²⁵⁾ dysprosium,⁽²⁶⁻²⁸⁾ holmium,⁽²⁹⁾ erbium,^(21,30-32) and thulium.^(29,33,34) All possess enormous paramagnetic susceptibilities, greater than $7\mu_B$, and have magnetic transitions at comparatively high temperatures. All become ferromagnetic, and some have an intervening antiferromagnetic state between the paramagnetic and the ferromagnetic states. Large susceptibilities result from the large effective moments.

Table I

MAGNETIC DATA FOR THE ELEMENTS

Gd, Tb, Dy, Ho, Er and Tm⁽¹⁶⁾

Element		Gd	Tb	Dy		Ho		Er		Tm
Effective atomic moment above magnetic anomalies (μ_B)	Theor.	7.94	9.7	10.6		10.6		9.6		7.6
	Expt.	7.95	9.0	10.2		10.9		10.0		7.6
Magnetic order		F	F	AF	F	AF	F	AF	F	AF
Transition temp, °K		289	230	174	83	132	20	78	20	51

Ytterbium and Lutetium. These elements are only weakly paramagnetic. At room temperature χ is 1×10^{-6} emu/gram or less.^(35,36) In ytterbium metal the majority of the atoms have a filled 4f shell, with only about 1 in 260 having 13 electrons in the 4f shell. Lutetium has a filled 4f shell plus 3 conduction electrons, and there is no possibility of a magnetic contribution from the 4f electrons in this case.

A more complete summary of the magnetic properties of the pure lanthanide metals can be found in the recent compilation by Gschneidner.⁽³⁷⁾

Alloys of the Rare Earth Elements. Information in the literature on the magnetic properties of these elements is not extensive. Several recent studies have provided magnetization data on intermetallic compounds of rare earths with transition metals.^(38,39)

The Titanium Group Elements

Titanium. The value of χ is small and positive for titanium and increases with increasing temperature.^(40,41) Its room temperature value is $\chi = 3.2 \times 10^{-6}$ emu/gm. This behavior is inconsistent with either the Curie or the Curie-Weiss law, and it probably indicates that the paramagnetic susceptibility does not arise from local moments on the titanium atoms. The paramagnetism of titanium has been considered to arise from the Pauli paramagnetism of the conduction electrons.

The Hall coefficient of titanium at room temperature appears to vary from -4.5×10^{-5} cm³/coulomb⁽⁴²⁾ to $+3.3 \times 10^{-5}$ cm³/coulomb,⁽⁴³⁾ with impurity content being a principal variable.

Zirconium has a lower gram susceptibility at room temperature (1.34×10^{-6} emu/gram) than titanium, but its temperature dependence is of the same kind.^(40,41) In both titanium and zirconium there are anomalies

at high temperature, which are probably associated with the change in crystal structure. All measurements of Hall coefficients at room tempera-

ture appear to have yielded positive values, but there are numerical variations from $1.30 \times 10^{-5} \text{ cm}^3/\text{coulomb}$ (42) to $12.6 \times 10^{-5} \text{ cm}^3/\text{coulomb}$.(44) This variation may be ascribable to impurities.

Fig. 11. Susceptibility of Ti, Zr and Hf(43)

The susceptibility of hafnium has a room temperature value of $4.2 \times 10^{-6} \text{ emu/gram}$, and it increases with increasing temperature, resembling the behavior of titanium and zirconium.(45) The temperature dependence of the susceptibility of the three Ti group elements is shown in Fig. 11.(46)

The magnetic behavior of the alloys of titanium, zirconium, and hafnium will not be discussed because of space limitations. Very limited data exist.

Summary

We may say in conclusion that the metals and alloys discussed contain atoms with similar electronic structures and that there is therefore a general kind of similarity in the magnetic properties of all of them. However, there are important differences. The titanium group elements are transition elements having partially filled d-shells. These electrons are not well shielded by outer shells; as a result, the magnetic behavior of these elements in the metallic state differs very much from what one would predict from the electronic structure of the isolated atoms. In contrast, the 4f magnetic electrons of the rare earths are well shielded from the crystalline field, and the electronic structures in the solid metals are not radically different from those of the isolated atoms. The thoride elements represent something of a compromise between these limits, but their magnetic behavior resembles more closely the transition metals than the rare earths.

Acknowledgments

The author wishes to thank the various authors and journals who granted permission to reproduce the graphs and tables presented herein.

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MATHEMATICS IN NONDESTRUCTIVE TESTING

by

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ABSTRACT

A survey of the role of mathematics and computers in the problems of nondestructive testing is presented. A discussion is given of those problems which reduce to classical problems of applied mathematics, and a summary of the states of progress on these problems is presented. Included are estimates of typical computer capacities and the times needed to handle many of these typical problems.

I. Introduction

Many of the mathematical problems of physics and nondestructive testing can be reduced to classical problems of applied mathematics. With the advent of automatic electronic computers in 1946 (ENJAC, University of Pennsylvania), a new tool was presented to the applied mathematician for solving these classical problems. A great deal of progress has been made in the last fifteen years, and many problems, formerly quite difficult, can now be handled rather routinely. To be sure, computers are no panacea for all the difficulties of research and development, and if computers and computer techniques are not well understood one can be led astray with the results of the computer effort. In this respect, however, computers are no different from other devices or techniques employed by research scientists; one must know and understand the tools of his trade.

I shall de-limit the boundaries of this discussion by confining myself to numerical applications of computers, making only the following brief remarks about other applications. Heretofore, the greatest use of computers (in scientific work) has been in numerical calculations, and the development and standardization of techniques has progressed furthest in this field. It may well be, however, that computers will make their greatest contribution in their nonarithmetic applications. Nonarithmetic applications, wherein computers are already making considerable contributions, are simple handling and sorting great volumes of data, simulation of the design logic of complex electronic systems, and processing and controlling other engineering systems. In these applications the data are represented numerically and

some arithmetic is performed, but the logical features of the computer are the most important features for this work. Recently, considerable progress has been made toward mechanization of such logical problems as theorem proving, problem solving, and pattern recognition. Although the progress in these areas looks very promising, the utility of these techniques (commonly called inferential analysis) in realistic problems is yet to be demonstrated.

Furthermore, I shall not discuss statistical methods, since they will be covered in the talk that follows.

After summarizing the current status of the work on the classical numerical problems of applied mathematics, I shall discuss in some detail an application of the Monte Carlo method, a numerical method of particular practical significance with the advent of the high-speed automatic electronic computer.

II. Classical Problems of Applied Mathematics

The classical problems of applied mathematics may be classified in the following categories:

1) Interpolation and Approximation

$$f^*(\vec{x}_i) \equiv f(\vec{x}) \quad i = 1, 2, \dots, n$$

2) Quadrature

$$\int f(\vec{x}) \, d\vec{x}$$

3) Matrices and Linear Equations

$$Ax = b$$

$$Ax = \lambda x$$

4) Systems of Ordinary Differential Equations

$$\frac{dy_i}{dx} = f_i(x, y) \quad i = 1, 2, \dots, n$$

5) Partial Differential Equations

$$\Delta u = 0, \frac{\partial u}{\partial t}, \frac{\partial^2 u}{\partial t^2}$$

$$u(x_1, x_2, \dots, x_n, t)$$

$$\Delta u = \sum_{i=1}^n \frac{\partial^2 u}{\partial x_i^2}$$

potential equation - elliptic

heat equation - parabolic

wave equation - hyperbolic

6) Integral Equations

$$a(x)f(x) = F(x) + \lambda \int_a^b K(x, \theta)f(\theta)d\theta$$

$$\left. \begin{array}{l} a(x) \\ F(x) \\ K(x, \theta) \end{array} \right\} \text{ given}$$

$a \equiv 0$ 1st kind

$a \equiv 1$ 2nd kind

$a \equiv a(x)$ 3rd kind

7) Statistics

$$P(\xi \leq x) = F(x)$$

8) Stochastic Techniques

$$P\{A\} = \int_A dF(\theta)$$

As a reference and practical manual for the numerical treatment of these problems, I would recommend the book Mathematical Methods for Digital Computers, edited by Antony Ralston and Herbert S. Wilf. Each chapter deals with one of the above topics. In each chapter the process of handling a problem takes the problem from its formulation up to, but not including, the actual coding for a digital computer.

The format of the chapters is:

- a) Mathematical Discussion
- b) Summary of the Calculation Procedure
- c) Flow Chart
- d) Standard Subroutines
- e) Sample Problem
- f) Memory Requirements
- g) Estimation of Running Time
- h) References

In particular, one may find items b), c), and d) extremely useful. In many texts one finds a mathematical discussion interlaced with the derivation of a method for solution in such a manner that the reader may find it difficult to decide what is the precise sequence of steps required to solve a problem. In item b) the reader will find the method derived in item a) stated in "recipe" form, that is, do this, then this, then this, etc.

Item g) is also quite helpful in deciding whether a given computer is appropriate for a given problem. Approximate running times for a problem are given in terms of the parameters of a problem and the addition and multiplication times of whatever computer is being considered.

III. Examples of Typical Numerical Problems

In the time allotted it is clearly not possible to discuss many of the above problems. However, it is possible to obtain a reasonably good idea of the utility of numerical techniques by examining the matrix and linear equation problems. I would choose the matrix problems as being particularly illustrative, because matrix inversion and matrix eigenvalue computations are the heart of many of the techniques for differential equations, integral equations, and the statistical methods.

If one were to require a matrix inversion in his problem and would choose the rank annihilation method presented in the above text, the time required for an $N \times N$ matrix is approximately

$$T \cong 5 N^3(\mu + \nu),$$

where μ and ν are the multiply and add times, respectively, for the computer. For typical computers for 20×20 matrices one has the following times:

IBM 650 : 400 sec ≥ 6 min
 IBM 704 : 10 sec
 IBM 7030: 0.24 sec

If one required the eigenvalues and eigenvectors of an $N \times N$ matrix and were to choose the Jacobi method for their computation, the time required

would be approximately

$$T \cong 10 N^3 (\mu + 2 \nu).$$

Again, for a 20 x 20 matrix the times would be

IBM 650 : 800 sec $\geq$ 13 min
 IBM 704 : 20 sec
 IBM 7030 : 0.6 sec

IV. The Monte Carlo Method and Applications

The Monte Carlo Method consists in formulating a game of chance or a stochastic process which produces a random variable whose expected value is the solution of the problem at hand. An approximation to the expected value is then obtained by means of sampling from the appropriate probability distribution. Clearly only an approximation to the correct answer is obtained. In such stochastic techniques the primary source of error is due to the fact that only a finite sample size is taken instead of due to numerical round off as in other methods.

The Monte Carlo method is utilized in two ways, namely, (1) Monte Carlo in analog, and (2) Monte Carlo in simulation. In the former case the physical problem has been clearly formulated as a precise mathematical problem and the Monte Carlo technique is just another numerical technique. An example of the Monte Carlo analog method is the evaluation of a definite integral via Monte Carlo.

$$I = \int_0^a f(x) dx$$

$$A = a \cdot b$$

The geometrical interpretation is that the integral above represents the area under the curve $f(x)$. This application, though illustrative, is indeed simple. One simply generates points uniformly distributed over the rectangle enclosing the curve, counts the fraction F which fall on or below the curve $f(x)$, and determines the area by

$$I = \int_0^a f(x) dx \cong F a \cdot b$$

The outcome of the individual trials obey the binomial distribution. For N trials the expected value of the number of successes is

$$S = Np,$$

where $p = I/A$. The standard deviation of S , namely, $\sigma_S = \sqrt{Np(1-p)}$. Thus

$$I^* = \frac{S^*}{N} A$$

$$\frac{\sigma_I}{I^*} \approx \frac{1}{A} \sqrt{\frac{\frac{I^*}{A} \left(1 - \frac{I^*}{A}\right)}{N \left(\frac{I^*}{A}\right)^2}} = \frac{1}{A} \sqrt{\frac{\left(1 - \frac{I^*}{A}\right)}{N \frac{I^*}{A}}}$$

I do not feel that use of Monte Carlo in analog is in the spirit of Monte Carlo and do not like to emphasize this use. When the problem has been clearly formulated in such precise mathematical form, I would generally recommend seeking more deterministic numerical techniques.

It would appear to me that in the case of Monte Carlo in simulation, one is invoking the true spirit of Monte Carlo. We have found the Monte Carlo method particularly useful in scattering problems involving neutral particles. Scattering of charged particles in matter generally involves such a large number of events that these problems do not lend themselves to Monte Carlo techniques. I should like to describe one of an array of problems with which we have been concerned.

In this particular application we were presented the problem of calculating response spectra of crystals of NaI and CsI for gamma rays. We did not calculate the resolution effects of light collecting and photo-multiplier response, but actually calculated the energy loss spectrum of the gammas and then smeared out the energy loss spectrum with an experimentally determined resolution function. The principal part of the calculation was then the calculation of the energy loss spectrum.

The computer program constructed for this investigation yields the energy loss spectrum in the form of a histogram of the number of gamma rays experiencing any energy loss in a given energy interval; that is, the energy range is divided into intervals by the set of energy points $\epsilon_0, \epsilon_1, \epsilon_2, \dots, \epsilon_T$, where $\epsilon_0 = 0$ and $\epsilon_T = E_0$ (the energy of the incident gamma ray), and the calculations determine I_i , the number of gammas that lose, in the crystal, an amount of energy in the interval $(\epsilon_{i-1}, \epsilon_i)$. Thus a histogram is recorded in a manner similar to the record of an experiment. In all the calculations presented herein, the energy intervals selected were of equal size and 128 in number.

Let N be the total number of gamma rays emitted by the source into the hemisphere including the crystal, G be the fraction of these which strike the crystal face, I_0 be the number of gammas which pass through the crystal

without interacting, and $I = \left(\sum_{i=1}^T I_i \right)$ be the number of interacting gammas. Then

$$NG = I_0 + I$$

For a broad parallel beam G is unity; for a point source on the crystal axis G is $\frac{1}{2}\pi$ times the mean solid angle subtended by the crystal face for point sources uniformly distributed across the disk. The quantity herein called the interaction ratio is given by

$$F = [I/(I + I_0)]$$

and the absolute detection efficiency would be

$$GF = (I/N)$$

In the present investigation the quantities which are calculated for a given crystal, initial energy, and given value of N are G , I_0 , and I_i for $i = 1, 2, \dots, 128$. The set of I_i gives the energy loss spectrum itself and from these other quantities the efficiency and interaction ratio may be determined. The calculations were carried out on one of Argonne's high-speed computers, GEORGE. The computer program was set up to permit calculations for right circular cylinders of NaI of any size up to 64 in. in height by 64 in. in diameter, and for initial gamma-ray energies from 0.01 to 15 Mev. The types of sources which may be employed are (1) broad parallel beam, (2) narrowly collimated parallel beam, (3) point source on the crystal axis, and (4) collimated point source on the crystal axis, where the disk radius may be any size up to the radius of the crystal.

The primary processes which gamma rays may undergo are Rayleigh and Compton scattering, the photoelectric effect, and pair production. During these processes secondary gammas and secondary electrons and positrons are generated. Moreover, in the slowing down of the electrons and positrons, bremsstrahlung photons are produced, and, in the annihilation of positrons, photons are created. The secondary photons which must be considered, therefore, are the scattered gammas in Rayleigh and Compton scattering, the fluorescent radiation subsequent to the photoelectric effect, gammas created in the annihilation of positrons subsequent to pair production, and bremsstrahlung photons. The charged particles which must be accounted for are Compton scattered electrons, photoelectrons, and pair-produced electrons and positrons. All other processes have been neglected since they occur with very small frequency at the energies considered herein.

The complete history of an incident gamma ray and all secondary, tertiary, etc., radiations is simulated by the Monte Carlo program. For each incident gamma the total amount of energy absorbed by the crystal,

as a result of the various processes mentioned above, is recorded, and a count is placed in the appropriate channel of the energy loss histogram.

The simulation process proceeds in the following way. The energy of the initial gamma ray is determined by the source energy, and its direction is determined by random sampling from the appropriate source distribution. For example, for a point source on the crystal axis, a vector is selected from vectors isotropically distributed throughout the solid angle subtended by the crystal face. Having determined the direction of the incident gamma, its path length through the crystal is selected by randomly sampling the exponential distribution employing the total attenuation coefficient. If the path length sampled exceeds the maximum length of the ray in the crystal (for a vector in the chosen direction), the photon is considered to have escaped and the history is terminated. For those photons not escaping, the position of the first collision is determined by the path length selected. The nature of the first collision is determined by sampling the relative distribution of allowed events. Having selected a type event, the direction of scattering of the particles is determined by sampling the appropriate angular distributions which are discussed below. The history of all products of a collision are traced through the crystal. For example, in a Compton collision the scattered photon and the scattered electron are traced.

If the continuous energy loss spectrum for gamma rays of initial energy E_0 is given by $I(\epsilon)d\epsilon$ and the resolution function is given by

$$(2\pi\alpha\epsilon)^{-\frac{1}{2}} \exp \left[-\frac{(\epsilon - \epsilon')^2}{2\alpha\epsilon} \right] ,$$

then the experimental response function for gammas of initial energy E_0 is given by

$$R(\epsilon') = \int_0^{E_0} \frac{1}{(2\pi\alpha\epsilon)^{\frac{1}{2}}} \exp \left[-\frac{(\epsilon - \epsilon')^2}{2\alpha\epsilon} \right] I(\epsilon) d\epsilon .$$

Since the loss spectrum is given in the form of a histogram, $I(\epsilon_i)$, the discrete response function $N(\epsilon_j)$ is given by

$$N(\epsilon'_j) = \sum_{i=0}^T \omega_i \frac{\exp \left\{ -[(\epsilon_i - \epsilon'_j)^2]/2\alpha\epsilon_i \right\}}{(2\pi\alpha\epsilon_i)^{\frac{1}{2}}} I(\epsilon_i) ,$$

where $\omega_0 = \omega_T = \frac{1}{2}$ and $\omega_i = 1$, $i \neq 0$ or T (trapezoidal rule). The constant α is chosen empirically to give a selected line width at maximum energy.

Whereas the resolution function

$$\frac{1}{(2\pi\alpha\epsilon)^{\frac{1}{2}}} \exp \left[-\frac{(\epsilon - \epsilon')^2}{2\alpha\epsilon} \right]$$

is known to agree with experiment at low gamma-ray energies, it deviates from experiment at large energies. Nonetheless, for all comparisons with experiment this resolution function was used.

In such a brief summary as this one cannot hope to cover very much of the topic proposed. I only hoped to show that computers and numerical methods do have a role in helping solve your theoretical problems.

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Discussion

MATHEMATICS IN NONDESTRUCTIVE TESTING

Clement Tatro, Michigan State University: You mention here that this only good for particles that aren't charged. Is the problem too difficult for those energies of particles in scattering where you can essentially talk about a nucleon as opposed to a neutron?

Miller: Yes. It really is a matter of the macroscopic cross section compared to the material that you're dealing with. It's just a question of whether you have too many events or not. In the case you described, you don't notice any difference between proton and neutron cross sections. Clearly, you could do the calculation for either case, for example, the scattering of protons through nuclear matter. In many cases you might be able to handle it by Monte Carlo, but if you're going to consider their atomic scattering events, then the proton would experience a huge number of events in traversing the small amount of matter. You just couldn't hope to trace all of these events.

Tatro: You do however, have to describe mathematically, something connected with the shape?

Miller: Yes. The sampling procedure depends very much on the particular type of scattering that's taking place. You must know the scattering law for the individual scattering events. The thing that you don't know is how to fit all these things together into the macroscopic problem. For example, you must know the probability for a photoelectric event in a gamma-ray problem, and the angular distribution for ejected photoelectrons. Then you sample from this to determine the future course for the electron that was ejected in this case.

STATISTICAL METHODS

by

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ABSTRACT

Nondestructive testing often involves quantitative measurements which must be processed by statistical methods. There is then a choice to be made between point estimation, interval estimation, and hypothesis testing. Factors involved in this choice are discussed, and all three techniques are applied to an idealized radioactivity thickness gage of the transmission type. The possible role of automatic computing machines is also examined.

1. Introduction

Nondestructive testing often leads to quantitative measurements which must be processed by statistical methods. There is frequently a choice to be made between the techniques of point estimation, interval estimation, and hypothesis testing. In the following discussion, these three techniques are illustrated by applying them in turn to an idealized radioactivity thickness gage of the transmission type. Since many complicating features, such as scattering and counter dead time, are ignored, this treatment is not to be regarded as a serious contribution to the theory of these gages, but rather as an illustration of the use of various statistical methods.

2. Probability Background

Consider the simple situation shown in Fig. 1.

Fig. 1

Suppose that a particle once emitted from the source in the direction of the counter has a probability $p = e^{-\sigma b}$ of penetrating the plate and being registered by the counter. This implies that the efficiency of the counter is being taken as 100%. The objective of the setup is to produce an estimate of the quantity p and thus of the thickness b , assuming that the cross section σ is known.

Now the waiting time T between successive particle emissions by a radioactive source has the probability density

$$S(t) = ae^{-at} \quad (0 < t < \infty) \quad ,$$

where a is a positive constant associated with the particular source. Let

$$T_k = t_1 + t_2 + t_3 + \dots + t_k \quad ,$$

where the t 's have the density S , then T_k is the waiting time from any given time to the emission of k particles. To find the probability density of T_k , we have⁽¹⁾

$$S_k(t) = \int_0^t du S(t-u)S_{k-1}(u) = \int_0^t du S(t-u)S_{k-1}(u) \quad ,$$

$$S_0(t) = S(t) \quad .$$

Using the Laplace transforms

$$\sigma_n(s) = \int_0^\infty dt e^{-st} S_n(t) \quad (n = 1, 2, 3, \dots) \quad ,$$

we obtain

$$\sigma_k(s) = \sigma_1(s) \sigma_{k-1}(s) = [\sigma_1(s)]^k$$

But

$$\sigma_1(s) = \frac{a}{s+a} \quad ,$$

so that

$$\sigma_k(s) = a^k / (s+a)^k$$

and

$$S_k(t) = \frac{a^k}{(k-1)!} t^{k-1} e^{-at}$$

To derive the more familiar Poisson distribution for the number N of counts in a fixed time interval t , observe that

$$\begin{aligned} P_r [T_k < t] &= P_r [N \geq k] = \sum_{\ell=k}^{\infty} P_r [N=\ell] = P_r [N=k] + \sum_{\ell=k+1}^{\infty} P_r [N=\ell] \\ &= P_r [N=k] + P_r [T_{k+1} < t] \quad , \end{aligned}$$

so that

$$\begin{aligned} P_r [N=k] &= P_r [T_k < t] - P_r [T_{k+1} < t] = \int_0^t du S_k(u) - \int_0^t du S_{k+1}(u) \\ &= \frac{(at)^k}{k!} e^{-at} \quad , \end{aligned}$$

the last line resulting by direct evaluation of the two integrals.

All of these calculations refer only to a source and counter with no absorbing plate between. If an absorbing plate is interposed between the source and counter, each emitted particle has a probability $1-p$ of being absorbed before reaching the counter. Under these conditions, one may have to wait for ℓ particles to be emitted before registering one (the ℓ th). This will happen with probability $(1-p)^{\ell-1}p$, since the loss of each particle is an independent event. The density of the waiting time T must therefore be modified to

$$S'(t) = \sum_{\ell=1}^{\infty} (1-p)^{\ell-1}p S_{\ell}(t) = pae^{-at} \sum_{\ell=1}^{\infty} \frac{[(1-p)at]^{\ell-1}}{(\ell-1)!} = pae^{-pat} \quad ,$$

hardly a surprising result on physical grounds. In the previous formulas, the presence of the plate is therefore taken into account by replacing a by $q = pa$; the problem is then to estimate q and hence b .

There are essentially three ways of sampling from such a process:

(a) A fixed time interval of length t is established and the number N of counts in this interval is registered.

(b) A fixed number k of counts is chosen and the waiting time T_k to register this number of counts is recorded.

(c) The times t_k between successive counts may also be recorded as data. Usually, sampling scheme a) is used and the result is quoted as $q = (1/t)(N \pm \sqrt{N})$. However, a more systematic procedure is desirable in order to clarify the nature of the statistical assertion being made.

3. Point Estimation

A quite general method of constructing point estimators is afforded by R. A. Fisher's principle of maximum likelihood.⁽²⁾ For estimating one parameter, the process consists in maximizing the likelihood function

$$L(x_1, x_2, x_3, \dots, x_n \mid \theta)$$

with respect to the parameter θ to be estimated, where L is simply the joint density function of the sample with the observed values $x_1, x_2, x_3, \dots, x_n$ substituted. Roughly speaking, one finds the value of θ which renders the observed sample least remarkable or surprising. We proceed to apply this technique to the three sampling schemes described above. It will be seen that it is usually more convenient to maximize the logarithm of L rather than L itself.

Scheme (a)

Here

$$L = \frac{(qt)^N}{N!} e^{-qt} \quad ,$$

and the variation of L with q is somewhat as shown in Fig. 2.

Fig. 2

To find the maximum, write

$$\log L = N \log q + N \log t - \log N! - qt \quad ,$$

$$\frac{\partial}{\partial q} \log L = \frac{N}{q} - t = 0 \Rightarrow q^* = \frac{N}{t} \quad .$$

The expected value of the estimator q^* is

$$E(q^*) = \frac{1}{t} E(N) = e^{-qt} \sum_{k=0}^{\infty} k \frac{(qt)^k}{k!} = q \quad ,$$

showing that q^* is an unbiased estimate of q . The variance is

$$E[(q^* - q)^2] = \frac{1}{t^2} E(N^2) - 2q^2 + q^2 = \frac{q}{t} ,$$

and if the unknown quantity q is replaced by the estimate q^* the standard deviation ($\sqrt{\text{variance}}$) becomes $\sqrt{N}/t$, thus justifying to some extent the usual treatment mentioned earlier.

Scheme (b)

In this case

$$L = S_k(T_k) = \frac{q^k}{(k-1)!} T_k^{k-1} e^{-qT_k} ,$$

$$\log L = k \log q - \log (k-1)! + (k-1) \log T_k - qT_k ,$$

and

$$\frac{\partial}{\partial q} \log L = \frac{k}{q} - T_k = 0 \Rightarrow q^* = \frac{k}{T_k} .$$

The expected value of q^* is now

$$E(q^*) = kE\left(\frac{1}{T_k}\right) = k \frac{q^k}{(k-1)!} \int_0^\infty dt t^{k-2} e^{-qt} = \frac{k}{k-1} q (k \geq 2) ,$$

showing that in this case q^* is a biased estimate of q . It is probably a good idea to correct for bias by using, instead, the estimator

$$q^* = (k-1)/T_k .$$

The variance of this last estimator is then

$$E[(q-q^*)^2] = (k-1)^2 E\left(\frac{1}{T_k^2}\right) - q^2 = \frac{q^2}{k-2} (k \geq 3) .$$

These calculations illustrate the point that maximum likelihood estimators are often biased; in simple problems it is sometimes possible to remove bias by the method used in the last example.

Scheme (c)

Application of the maximum likelihood method here leads to the same results as in Scheme (b), although the likelihood function is slightly different. This sampling plan is, however, used below to construct a sequential test.

4. Confidence Intervals⁽³⁾

The method of confidence intervals aims at the construction of two statistics, θ_1 and θ_2 , say, in such a way that a statement of the form

$$P_r(\theta_1 < \theta < \theta_2) = c$$

can be made concerning the unknown parameter θ . Here c is the confidence level, usually taken as 0.95, 0.99, etc., depending to a large extent on the available sample size. Notice that θ_1 and θ_2 are random variables and will vary from one experiment to the next.

Confidence intervals for the first two of the above sampling plans may be derived as follows:

Scheme (a)

Since t is a fixed number, the distribution of the estimator q^* is derived from the Poisson distribution of N :

$$P_r(N=k) = \frac{(qt)^k}{k!} e^{-qt} = P_r q^* = \frac{k}{t}$$

Now imagine that two numbers k_0 and k_1 are defined as, respectively, the largest and smallest integers satisfying the inequalities

$$e^{-qt} \sum_{k=0}^{k_0} \frac{(qt)^k}{k!} \leq \frac{1-c}{2} \text{ and } e^{-qt} \sum_{k=k_1}^{\infty} \frac{(qt)^k}{k!} \leq \frac{1-c}{2}$$

When c is given, these numbers may be shown to be nondecreasing functions of qt . From the defining equations, one then has

$$P_r\left(\frac{k_0}{t} \leq q^* \leq \frac{k_1}{t}\right) \geq c,$$

this being a statement of inequality rather than equality because of the integral character of k_0 and k_1 . The inequality on the left side may be transformed by

$$k_0(qt) \leq q^*t \implies qt \leq k_0^{-1}(q^*t),$$

since k_0 is nondecreasing. Treating the right-hand side similarly, the confidence interval assertion

$$P_r\left[\frac{1}{t} k_1^{-1}(N) \leq q \leq \frac{1}{t} k_0^{-1}(N)\right] \geq c$$

results, in which the relation $q^*t = N$ has been used.

An easier way of presenting the actual calculations is as follows. A value of N having been observed, solve the equations

$$e^{-\lambda} \sum_{k=0}^N \frac{\lambda^k}{k!} = \frac{1-c}{2} \text{ and } e^{-\lambda} \sum_{k=N}^{\infty} \frac{\lambda^k}{k!} = \frac{1-c}{2}$$

for roots λ_0 and λ_1 , respectively. The desired confidence interval is then

$$\frac{\lambda_1}{t} \leq q \leq \frac{\lambda_0}{t}.$$

As a simple numerical example, suppose that in a particular experiment 21 counts were observed in a time of 0.1 sec, incidentally yielding a point estimate $q^* = 210$. A 95% confidence interval is obtained by solving the equations

$$e^{-\lambda} \sum_{k=0}^{21} \frac{\lambda^k}{k!} = 0.025 \text{ and } e^{-\lambda} \sum_{k=21}^{\infty} \frac{\lambda^k}{k!} = 0.025.$$

Reference to tables⁽⁴⁾ gives $\lambda_0 = 32.1$ and $\lambda_1 = 13.0$, resulting in the confidence statement $130 \leq q \leq 321$. This is a central confidence interval based on equal tail areas; it corresponds roughly to the usual $10(21 \pm 2\sqrt{21})$. Noncentral confidence intervals may also be used if circumstances warrant.

Scheme (b)

In this situation, the count number k is given, so that the distribution of the estimator $q^* = k/T_k$ is derived from the distribution of the waiting time T_k . It is not necessary to correct for bias.

Since the probability density of T_k is

$$S_k(t) = \frac{q^k}{(k-1)!} t^{k-1} e^{-qt},$$

we have

$$\begin{aligned} P_r[x < q^* < x + dx] &= P_r\left[x < \frac{k}{T_k} < x + dx\right] = P_r\left[\frac{k}{x} - \frac{kdx}{x^2} < T_k < \frac{k}{x}\right] \\ &= \int_{\frac{k}{x} - \frac{kdx}{x^2}}^{\frac{k}{x}} dt \frac{q^k}{(k-1)!} t^{k-1} e^{-qt} = \frac{(qk)^k}{(k-1)!} \frac{dx}{x^{k+1}} e^{-\frac{qk}{x}}. \end{aligned}$$

Here a central confidence interval is found by observing a value of T_k and solving the equations

$$\frac{\lambda^k}{(k-1)!} \int_{T_k}^{\infty} du u^{k-1} e^{-\lambda u} = \frac{1-c}{2} \quad \text{and} \quad \frac{\lambda^k}{(k-1)!} \int_0^{T_k} du u^{k-1} e^{-\lambda u} = \frac{1-c}{2}$$

for λ_0 and λ_1 , respectively. The confidence interval assertion is then to the effect that

$$P_r [\lambda_1 < q < \lambda_0] = c \quad .$$

These equations may be solved by reference to tables of the incomplete gamma function⁽⁵⁾ or to tables of χ^2 . Slightly more elegant results are obtained with this sampling plan than by use of scheme (a); the fact that T_k is a continuous variable produces an exact confidence interval.

5. Hypothesis Testing⁽⁶⁾

The general ideas involved in statistical hypotheses testing can be described very simply. Referred to a particular statistical model of an experimental procedure, a hypothesis H is any statement or assertion which permits calculation of sampling distributions conditional on H .

Suppose that we wish to formulate a test of the hypothesis H_0 against an alternative hypothesis H_1 . The general procedure is to partition the space of all permissible sample values $x_1, x_2, x_3, \dots, x_n$ (the sample space) into two disjoint regions W_0 and W_1 which together exhaust the space. Denote the joint density function of the sample values if H_0 is true by

$$\sigma(x_1, x_2, x_3, \dots, x_n | H_0) \equiv \sigma(x | H_0)$$

and likewise denote by $\sigma(x | H_1)$ the sample density conditional on H_1 . Before a particular sample x is drawn, we agree to accept H_0 if $x \in W_1$ and reject H_0 if $x \in W_0$. Here W_0 is called the critical region.

Now the quantity

$$\alpha = \int_{W_0} dx \sigma(x | H_0)$$

is the conditional probability, given H_0 , that the sample point will fall in the critical region; it is therefore the probability of rejecting H_0 when it is

in fact true. This is called an error of the first kind or a type I error. Similarly,

$$\beta = \int_{W_1} dx \sigma(x | H_1)$$

is the probability of accepting H_0 when H_1 is in fact true; this is known as a type II error.

The Neyman-Pearson theory of hypothesis testing is concerned with the selection of appropriate critical regions. If the sample size is fixed, a reasonable approach is to hold α constant and determine W_0 so as to minimize β . The quantity $1 - \beta$ is known as the power of the test. If an indefinitely large sample is available, it is possible to control both types of errors to prescribed levels. The question then becomes one of choosing W_0 so as to minimize the sample size. This is quite a complicated problem in general and research is still active in this field.

The foregoing theory will be illustrated by applying it to the problem under discussion in several ways.

Sampling Scheme (b)

A test of the hypothesis

$$H_0: q_1 < q < q_2 \quad (q_1 < q_2)$$

against the alternative

$$H_1: q < q_1 \quad \text{or} \quad q > q_2$$

can be constructed as follows:⁽⁷⁾

$$\text{Reject } H_0 \text{ if } T_k < C_1 \text{ or } T_k > C_2$$

$$\text{Accept } H_0 \text{ if } C_1 < T_k < C_2 \quad ,$$

where the numbers C_1 and C_2 are determined by solution of the equations

$$\int_0^{C_1} dt S_k(t | q_1) + \int_{C_2}^{\infty} dt S_k(t | q_1) = \alpha$$

and

$$\int_0^{C_1} dt S_k(t | q_2) + \int_{C_2}^{\infty} dt S_k(t | q_2) = \alpha \quad .$$

As before, α is the desired probability for errors of the first kind. The integrals can be calculated by reference to tables of the incomplete gamma function;⁽⁵⁾ the notation $S_k(t|q)$ simply indicates explicitly the dependence of the density $S_k(t)$ on the parameter q .

This test is best for controlling errors of type II in a certain sense which is not described here for lack of space. In practice, one would decide on the significance level α and adjust the sample size k to secure satisfactory control of type II errors. In this example, the actual probability level for type II errors depends on statistical characteristics of the population of objects being tested, and can be calculated if this information is known or if a model population can be assumed.

Sampling Scheme (c)

With this sampling plan, the last described test can be applied in a sequential way by ceasing to sample if at any stage $T_j > C_2$, where $j \leq k$. If $T_j > C_2$, it is then certain that $T_k > C_2$ and the hypothesis H_0 may be rejected without further sampling. This procedure evidently results in some saving of sampling time.

For testing the simple hypothesis $H_0: q = q_0$ against a simple alternative $H_1: q = q_1$, A. Wald's famous sequential probability ratio test⁽⁸⁾ may be applied. Each t_k has probability density $S(t|q) = q^n \exp(-qt)$ and they are statistically independent; consequently the joint density of a sample of n is

$$\sigma(t_1, t_2, t_3, \dots, t_n | q) = q^n e^{-q_n^T},$$

where

$$T_n = \sum_{k=1}^n t_k.$$

The test is carried out according to the following procedure:

Using the sample values t_k as they are obtained, calculate successively the ratios

$$\gamma_n = \frac{\sigma(t_1, t_2, t_3, \dots, t_n | q_1)}{\sigma(t_1, t_2, t_3, \dots, t_n | q_0)}.$$

At any stage, follow the decision rule: If

$$r_n > \frac{1-\beta}{\alpha}$$

reject H_0 , if

$$\frac{\beta}{1-\alpha} < r_n < \frac{1-\beta}{\alpha}$$

take another observation, and if

$$r_n < \frac{\beta}{1-\alpha}$$

accept H_0 . Here α and β are respectively the desired probabilities for errors of the first and second kinds. Taking logarithms of the above inequalities and rearranging slightly, the work can be presented in a more convenient form: If

$$T_n > \frac{1}{q_0 - q_1} \left(\log \frac{1-\beta}{\alpha} + n \log \frac{q_0}{q_1} \right)$$

reject H_0 , if

$$\frac{1}{q_0 - q_1} \left(\log \frac{\beta}{1-\alpha} + n \log \frac{q_0}{q_1} \right) < T_n < \frac{1}{q_0 - q_1} \left(\log \frac{1-\beta}{\alpha} + n \log \frac{q_0}{q_1} \right)$$

take another observation, and if

$$T_n < \frac{1}{q_0 - q_1} \left(\log \frac{\beta}{1-\alpha} + n \log \frac{q_0}{q_1} \right)$$

accept H_0 .

Sequential tests have the two pronounced advantages that α and β can be prescribed in advance and that no complicated probability distributions need be evaluated. These advantages are secured at the price of allowing the sample to be a random variable; however, in many cases it can be shown that the expected value of the sample size is considerably smaller than the required sample size for a comparable nonsequential technique.

6. Use of Computing Machines

Both digital and analogue computing machines can be used very effectively to facilitate statistical work. Tables can be easily computed to fit specific situations in the field. More interestingly, model experiments can be done which would be economically impossible without the computer. For example, in more complex sequential tests than the ones described above it is often very difficult to compute the expected sample size. With a computer this can be done directly by simulating the test on a model population and actually observing the average sample size. Again,

if a sequential procedure is to be used for process control, a special purpose computer can be designed to perform the necessary calculations and make the decision to accept, reject, or continue before the next sample value is obtained.

Very often, application of advanced statistical techniques is stymied because of difficulties in evaluating integrals over complicated probability distributions. Here again, computing machines are capable of opening up large new areas of statistical application.

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THE SCIENTIFIC APPLICATION OF PARTICLE ACCELERATORS TO NONDESTRUCTIVE TESTING

by

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ABSTRACT

The high-energy particles and radiations obtainable from particle accelerators are broadly useful in the field of nondestructive testing. The applications of supervoltage and high-energy X rays for radiography are numerous and well explored. The possible use of positive ions and neutrons, on the other hand, has not been investigated to any appreciable extent. This paper describes the physical characteristics of phenomena involving positive ions and neutrons, and their interaction with matter. Nondestructive testing methods arising from understanding of these phenomena are suggested, including continuous activation analysis of production flow, neutron radiography, and the analysis of surfaces and thin coatings. A discussion is included of some of the problems involved in reducing these suggested methods to practice.

Introduction

Particle accelerators have been used in nondestructive testing since the early days of World War II. Their use has been predominantly in the field of industrial radiography, with penetrating X rays in the range from 1 to 22 Mev, for the inspection of heavy and bulky objects. Accelerators such as the resonant transformer, Van de Graaff machines, betatrons, and electron linear accelerators were impressed into vital service during the war or became practical developments out of wartime technical progress.

The radiations available from particle accelerators are not exclusively penetrating X rays. High-energy electrons and positive ions are directly accelerated by some of these electronuclear machines, and X rays and neutrons are produced as secondaries by the interaction of the accelerated charged particles with special target materials. It is of great value to examine periodically the behavior and fundamental characteristics of these nuclear particles and radiations, and to determine their possible application to nondestructive testing.

The practicality of such an examination arises from the fact that particle accelerators of many types and ratings are now available commercially, a circumstance that has developed only within the past decade. Prior to that time, the particle accelerator was the physicist's instrument almost exclusively. Today, chemists, metallurgists, biologists, engineers - to name but a few - are exploiting the fundamental characteristics of high-energy particles and radiations for practical applications never envisioned by nuclear physicists.

Characteristics of Accelerator-produced Radiations

This paper confines its scope to a discussion of nuclear particles and radiations produced by accelerators in the energy range from 1 to 10 Mev. The applications to be described may be of possible interest in nondestructive testing or in allied fields. The data presented are exemplary of the wealth of available information on the subject.

The particles and radiations of possible interest include:

- Electrons: easily absorbed, intensely ionizing;
- Positive ions: very easily absorbed, very intensely ionizing, producers of radioactivity;
- X rays: penetrating in nature, weakly ionizing;
- Neutrons: very penetrating in nature, very weakly ionizing, producers of radioactivity.

The radiation characteristics of greatest importance are:

- Production: intensity, energy (by acceleration or by nuclear-reaction kinetics), angular distribution in energy and intensity;
- Interaction: preferential absorption in materials, scattering processes, nuclear-reaction products, induced radioactivity;
- Detection: differential systems (electronic counting by scintillators, ion chambers, etc., nuclear reactions), integral systems (photosensitive materials, fluorescent techniques, nuclear reactions).

Knowledge within these areas is extensive and has been obtained from research results in many fields, predominantly from nuclear physics. Figure 1 indicates many of the uses to which particle accelerators are being applied today in basic research, engineering, and production. In each of these fields, techniques have been developed that are of possible

value in nondestructive testing. This paper attempts to suggest a few of the more important characteristics of nuclear particles and radiations, and to describe a selection of research applications that could possibly lead to new methods for nondestructive testing.

Fig. 1 The Uses of Particle Accelerators

1. Electrons: Although electrons are apparently not broadly useful for nondestructive testing, a review of their characteristics is included for completeness of presentation. From a heated cathode, electrons can be produced for acceleration in the range above 1 Mev in continuous (dc) or pulsed mode. Continuous beams of electrons are measured in terms of milliamperes or kilowatts. Pulsed electron beams, with pulse lengths from 0.01 to 1000 microseconds, are available with peak intensities as high as several amperes, depending on the capacitance of the accelerating system.

High-energy electrons produce X rays through extranuclear interactions, preferentially with heavy atomic species. With sufficient energy, these X rays can engage in photodisintegration reactions with nuclei to produce energetic neutrons. These secondary products are discussed in detail later.

The penetration of high-energy electrons through matter is limited. Figure 2a illustrates, as a function of the absorber thickness, the distribution of ionization resulting from electron absorption. In the super-voltage range, the penetrated thickness is approximately proportional to the electron energy and to the specific gravity of the material.

Because electrons are charged particles, they can be deflected by electrostatic or magnetic fields. To provide a uniform irradiation of electrons across a surface, a magnetic scanning system is often utilized, with scanned widths up to 30 inches or so. Figure 2b shows the range of a linearly scanned electron beam. In general, high-energy electrons are used for the irradiation of plastics or chemicals to produce polymeric changes or to catalyze reactions in materials, or for the sterilization of surgical supplies, pharmaceuticals, or foods.

Fig. 2 Some Characteristics of High-energy Electrons

2. X Rays: As mentioned earlier, supervoltage X rays are produced by the bombardment of materials with high-energy electrons. The efficiency of converting electron energy to X-ray energy increases exponentially as the bombarding energy is increased, and linearly with the atomic number of the target material. Figure 3 shows the pronounced trend in X-ray intensity as the bombarding electron energy is increased.

These data were obtained from gold, tungsten, and similar heavy-metal targets. X-ray intensity is usually measured in terms of roentgens per minute at a meter's distance from the target or (for energies much above 3 Mev) rads/min at a meter.

Fig. 3 X-ray Intensity as a Function of Electron Energy

Fig. 4 Angular Distribution of X-ray Intensity as a Function of Electron Energy.

As the bombarding electron energy is increased, the distribution of the X radiation becomes more pronounced in the forward direction, as shown in Fig. 4. For radiographic purposes, in which a significant area of X-ray coverage is needed, electron energies should be kept as low as practical. Where differential counting or detecting schemes are utilized, however, a sharply pronounced beam of X rays at very high energies may be preferred. In some types of radiographic examinations, it is desirable to utilize the sideward-directed X-ray beam for panoramic inspection. This technique can be applied only at lower X-

ray energies.

There is a decreasing trend in X ray energy with respect to the forward beam energy as the angle with respect to the electron bombardment is increased. This diminution in effective energy is relatively small, of the order of one-third reduction from the energy at 0° to the energy at 90° .

High-energy X rays produce neutrons by interaction with special nuclei. These secondary reactions are discussed later.

X-ray absorption in matter is exponential in nature, and is smoothly related to the specific gravity, thickness, and atomic character of the absorber. Three absorption processes control the interaction of X rays in materials: photoelectric absorption (to a very small degree in the 1 to 10-Mev range), Compton absorption and scattering (the prevalent process except at very high energies and with very heavy materials), and pair-production absorption (preferentially at very high energies and with materials of very high atomic number). One method of describing the absorption characteristics of materials is to consider the thickness required to reduce the X-ray intensity by a factor of two. This thickness is referred to as the "half-value layer." Figures 5a, 5b, and 5c indicate the trend of half-value layers of light, medium, and heavy materials as a function of bombarding electron energy producing the corresponding X rays.

Fig. 5 Half-value Layers as a Function of Electron Energy

In the energy range of interest, X-ray scattering tends to confuse the absorption picture by apparently reducing the total effective absorption coefficient or by virtually increasing the half-value layer. In considering a high-energy X-ray technique, it is important to understand

whether the X rays are to be used in a narrow-beam configuration (relatively low scattering condition) or in a broad-beam configuration (where the scattering is in equilibrium with the direct radiation). Either of these geometries leads to consistent results, but confusion concerning the beam characteristics sometimes occurs in the use of an intermediate situation in which some, but not all, of the scattered radiation is removed.

High-energy X rays are normally detected, in a differential manner, by electronic counting systems, including Geiger counters, scintillators, etc. For an integrated record or observation of the effect of X rays, photosensitive film or fluorescent and scintillating screens are often used. To amplify the effects of radiations on photosensitive film, fluorescent systems are sometimes used to convert the X-ray energy to visible light energy. As the X-ray energy is increased, however, the amplification factor becomes less, and the effect of scattered radiation (lower in energy) becomes preferentially more pronounced.

High-energy X rays are used predominantly in radiography and radioscopy with continuous beams. Photosensitive films are in general the recorder of choice. The exposure times normally encountered are indicated in Fig. 6, which is a group of exposure-thickness technique charts for various types of high-energy particle accelerator sources of X rays. Particle accelerators are in industrial use today for the production radiographic inspection of heavy and bulky materials. For the most part, steel and solid rocket propellant are the materials most often inspected with these high-energy devices. To some extent, very heavy materials, such as uranium or plutonium, are also inspected.

Particle accelerators can produce controllably pulsed beams of X rays with little adaptation to standardized equipment. In certain super-voltage installations, these bursts of X rays are being used for "flash radiography." There is sufficient energy in an individual burst from some types of accelerators to produce a detailed radiograph with a combination of fluorescent material and photosensitive film. Pulses as short as 0.1 microsecond are possible.

For example, a 42-Mev microwave electron linear accelerator is currently being used for consecutive-flash radiography. This machine delivers 10 pulse trains per second, in which 10 pulses of 0.3-microsecond duration are produced. Each pulse yields approximately 1 roentgen of penetrating X rays, sufficient for the radiography of several inches of steel. It is feasible to consider a stroboscopic radiographic technique in which repeated flashes of X rays are superimposed in synchronism with the motion of some rotating or reciprocating mechanism. The advantage of such a radiographic technique would be in differentiating between the sharp image of objects whose motion is completely reproducible from cycle to cycle, and the diffused images of objects not accurately aligned or balanced.

Fig. 6 Required Exposure for Various X-ray Energies in the Case of Steel

3. Positive Ions: Many types of particle accelerators can produce beams of high-velocity positive ions almost as readily as high-energy electrons. For the most part, hydrogen and helium isotopes (protons, deuterons, alphas, etc.) are ionized for acceleration in these machines. There is some growing interest in producing ions of heavier elements for specialized activities, especially in nuclear research. Ions are produced in a particle accelerator by means of a gas discharge, usually initiated by a radio-frequency field. These ions are then extracted by an electrostatic lens from the plasma for acceleration by the electric fields existing in accelerators. Positive ions in the energy range from 1 to 10 Mev can be produced with intensities in the milliamperere and microampere region, either continuously or pulsed. Ions produced from direct accelerators are in the form of well-collimated parallel beams of small cross section. This configuration

permits the use of magnetic or electrostatic fields to guide the positive ion beam in different directions to the particular target of interest.

Positive ions are used to produce intense quantities of neutrons and typical reactions are discussed below. In the energy range from 1 to 10 Mev, positive ions are easily absorbed; their reactions are thereby virtually confined to the surface of the materials bombarded. Among the nuclear reactions of interest are those which produce radioactivity. The concept of converting the surface of a material to a controlled radioactive nature presents some interesting application possibilities.

Fig. 7 Schematic Drawing for the Use of a Van de Graaff Positive-ion Accelerator

An interesting non-destructive testing application for positive ions is in the analysis of surfaces and tenuous gases. Figure 7 shows a Van de Graaff positive-ion accelerator, whose output ion beam is carefully analyzed to enhance its homogeneity and to remove spurious ion species. Directly after analysis, the beam is allowed to impinge upon the target sample whose surface is to be analyzed. The protons are elastically scattered by the nuclei on the surface of the material. The scattered protons are analyzed by a spectrometer. These ions lose a small amount of energy, depending upon the atomic species bombarded; for example, more kinetic energy is imparted to a light nucleus than to a very heavy nucleus. Figure 8 indicates a typical spectrum obtained from a surface analysis of a steel plate. The position of the peaks indicates the atomic character of the substance creating the

peak, and the height of the resonance with respect to the background is a measure of the quantity of that isotope present. With this analytical technique, surfaces of a few microns in thickness can be investigated qualitatively as well as quantitatively, and surface films less than 10^{-2} micron in thickness can be evaluated.

Fig. 8 Spectrum Obtained by Analysis of a Steel Plate

Fig. 9 Decay of Radioactivities of Tool Bits

discontinuity in the decay curve is a direct indication of the amount of radioactivity (and hence the total material) worn off. This method produces radioactivity for a distance of 10 microns in steel, and can be used to detect a wear of at least 1 micron.

4. Neutrons: Neutrons of many energies can be produced by particle accelerators, depending upon the nuclear reaction chosen and on the environment of the neutron-producing target. For the purpose of this discussion, neutrons can be divided into three broad groups: thermal (with energies well below 1 electron volt), slow or resonance (in the range between 1 electron volt and 1 kev), and fast (above several kev in energy).

Another interesting application for positive-ion bombardment is in the study of the wear of machined parts. The surface of a gear tooth, lobe of a cam, or edge of a tool bit can be made temporarily radioactive through positive-ion bombardment, usually with high-energy deuterons. The decay of this radioactivity can be measured progressively during the test. Figure 9 shows the results of an experiment in which several tool bits were bombarded with 2-Mev deuterons and were subsequently used in a lathe. The

Neutron energies depend more on the type of nuclear reaction than on the energy of the bombarding particle, contrary to the case of X rays produced by electron bombardment.

By bombarding materials of small atomic number with positive ions, prolific quantities of neutrons can be produced, as shown in Fig. 10. For some reactions, the efficiency of neutron production rises sharply with bombarding energy. In other cases, there is some saturation in efficiency, caused by target limitations.

Fig. 10 Production of Neutrons in Various Processes

Fig. 11 Neutron Yields by X rays
Produced from High-energy
Electrons

from particle accelerators can be thermalized by surrounding the neutron-producing target with hydrogenous material such as paraffin or polyethylene. Figure 12 shows a typical distribution of thermal neutron flux in such a moderating material for several initial neutron energies. For most applications, the sample to be bombarded should be placed at the region of maximum flux, fairly close to the neutron-producing target.

The spatial distribution of neutrons produced from the target of a particle accelerator is in general approximately isotropic. As the bombarding energy of positive ions increases, however, the distribution becomes more pronounced in the forward direction.

A wide range of monoenergetic neutron energies can be obtained from a selection of positive-ion neutron-producing reactions. Figure 13 shows that neutron energies from 30 keV to 30 MeV can be obtained, depending upon the reaction, the bombarding energy, and the angle of observation.

An alternative method of producing neutrons from particle accelerators is by means of electron bombardment of targets of large atomic-weight to produce penetrating X rays, which in turn are used to irradiate special nuclear materials, thereby producing neutrons through photodisintegration reactions. X rays above 1.63 MeV in energy can produce neutrons from beryllium or deuterium. Above 9 MeV in energy, the photodisintegration and photofission of uranium is a most prolific source of neutrons. Figure 11 shows the trend of neutron production from these high-energy electron X-ray sources.

Thermal neutrons have a broad usefulness in activation analysis and radioisotope production, through neutron capture by isotopes. Neutrons

Fig. 12 Thermalization of Neutrons

Fig. 13 Neutron Energy as a Function of the Energy of Bombarding Particles

Neutrons interact with matter by absorption and scattering processes. In the case of absorption, nuclear reactions often take place, leading to the production of a radioactive end product. Unlike X rays, neutron-absorption curves display a complex resonance structure rather than a smooth trend, exemplified by Fig. 14 which is a curve of total neutron cross-section (measure of absorption) as a function of neutron energy in aluminum. The resonance behavior of neutron absorption is often useful in specific applications. The shape of the absorption curves varies widely from one isotope to another.

Fig. 14 Neutron Resonances in Aluminum

Neutrons are absorbed exponentially in material, resembling high-energy X rays in this behavior. A selection of half-value layer trends with neutron energy is given in Fig. 15. Materials of small atomic weight tend to absorb neutrons more readily than materials of large atomic weight.

Fig. 15 Half-value Layers for Neutrons

The scattering characteristic of neutrons, especially in the slow or resonance energy range, is prolific for materials of small atomic number, such as hydrogen. In fact, a technique for analyzing the quantity of hydrogenous material can be developed from this scattering propensity.

The detection of neutrons is more difficult than is the case for the other radiations and particles being considered in this paper. Because neutrons are not charged in nature, they produce but small ionization in materials. Consequently, normal electronic counting schemes cannot be applied. It is necessary, therefore, to interpose a material that is particularly sensitive to neutrons by means of the production of a charged particle for detection through a nuclear reaction. Similarly, photosensitive materials can be made sensitive to neutron bombardment by an intermediary in the form of a "nuclear-reaction intensifying screen."

Fig. 16 Schematic Arrangement for Carrying out a Neutron-activation Analysis

Neutrons from particle accelerators are proving to be very useful for chemical analysis in industry. The method is nondestructive and can be adapted in many cases to monitor the quality of products or materials by statistical sampling or by continuous evaluation. Both thermal and fast neutrons are utilized to irradiate the sample. The constituents of the material are made preferentially radioactive, in accordance with their neutron-absorbing characteristics. The resulting radioactivity is detected by electronic counting schemes to measure the intensity, radioactive decay, and energy of the emitted radiations. Figure 16 is a schematic diagram of the neutron-activation analytical method.

The sensitivity of this technique makes it possible to analyze for trace elements at concentrations of a few parts per million. Alternatively, activation analysis of relatively gross amounts of an element can be a routine process. The induced radioactivity is short-lived and relatively weak in intensity; the inspected sample can generally be returned to the production line after a short delay during which the radioactivity decays to insignificance.

Activation analysis is rapid, usually taking only a few minutes for complete analysis. Virtually no sample preparation is needed, and the

state of the material does not affect the effectiveness of the technique. Powders, liquids, gases, crystals - all can be analyzed. Reagent contamination is not a problem.

Figure 17a shows a typical curve obtained from a relatively simple chemical system, in which one element is particularly sensitive to neutron-activation analysis. The counting system in this case is a simple Geiger counter or scintillator. In more complex systems, it is sometimes desirable to utilize a multichannel analyzer, with a scintillating counter, to obtain the spectra of the emitted gamma rays, as shown in Fig. 17b. A radioactive isotope can be uniquely identified through its decay half-life and its emitted radiation energy.

Fig. 17 Representative Decays in Neutron-activation Analysis

Of possible interest to industry is the concept of radiography with neutrons. Because neutron absorption depends upon individual nuclear properties which are not smoothly related to isotopic or atomic number, neutron radiography could be useful for the evaluation of special materials in which the neutron absorption exaggerates the relative proportion of components present. With thermal neutrons, the absorption differences are considerably greater than those observed with fast neutrons. Unfortunately, it is difficult to conceive of a point source of thermal neutrons without considerable sacrifice in available neutron flux. Resonance neutrons, in the region below 50 kev, can be obtained from particle accelerators. By careful selection of these neutron energies, exaggerated absorption differences can also be exploited.

The difficulty of detecting neutrons tends to limit the effectiveness of this radiographic technique for the present. Photosensitive films are not normally sensitive to neutrons, and it is necessary to consider an intermediate, in the form of a neutron-sensitive foil which produces ionizing radiations as the result of neutron bombardment. The development of such neutron-sensitive combinations will inevitably come as the demand for this inspection method increases.

5. Areas of Possible Interest in Nondestructive Testing: Although nondestructive testing techniques are practical applications of fundamental physical principles, many phenomena have been overlooked in a search for specialized evaluation schemes. The engineering and inspection requirements in today's exotic technologies make it necessary to survey the field of physics continually for new ramifications of existing methods or for the practical adaptation of untried phenomena. It is difficult to organize a program of investigation for the development of new nondestructive testing methods, because individual organizations do not necessarily have a full justification for an extensive research and development program toward this end. As testing requirements become more stringent, the required apparatus tends to become more complex and expensive. Budgetary limitations and lack of available scientific manpower contribute to this dilemma. A periodic review of possible applications is therefore desirable to spark individual interest in exploring new methods to solve existing nondestructive testing problems. The following examples may be of some value.

X rays for radiography have been explored and utilized extensively. One limitation of this technique is the effect of scattered X radiation on the detecting systems. Ordinarily, radiographic sensitivities in the range from 0.5 to 2% of the total thickness can be observed. By eliminating scattered radiation, it is possible to enhance the sensitivity of the method by an order of magnitude. The control of scattered radiation from particle accelerators is easier to contemplate than at lower energies for several reasons:

The energy of scattered radiation is considerably lower than that of the direct X rays, which have different absorption characteristics.

The available intensity from supervoltage X-ray sources is sufficiently great to permit the use of techniques which reduce scattered radiation by preferential attenuation of scattered and direct radiations.

The use of nuclear reaction thresholds in detection systems can be contemplated with radiations in the Mev range.

Penetrating X rays from particle accelerators can be effectively used for the inspection of "black boxes," in which the interrelationship of components after assembly can be evaluated only with great difficulty by other methods. Stereoscopic techniques could be used to check condition and alignment. Projection radiography, with X rays from an extremely small focal spot, could magnify the objects of interest for greater resolution of detection. Electronic tubes, missile control systems, and reactor fuel elements are but a few types of objects that could be effectively inspected after assembly.

Flash radiography with supervoltage X rays has not been exploited except in a few widely scattered cases. Pulsing techniques are available, and X-ray energies and intensities within the pulse are presently adequate for observing the effects of rapidly moving objects. Electronic sweep systems for translating the image across the film during the motion of the object are certainly feasible. Flash-radiographic systems can be adapted to stroboscopic inspection, particularly with the use of very small X-ray focal spots.

Continuous X-ray inspection methods for hot steel blooms, solid propellants, pressure-vessel welds, etc., could be effectively developed out of today's knowledge of X-ray-image conversion systems and the effects of supervoltage X rays on them. The results that are currently obtainable are marginal, but the room for improvement is theoretically considerable.

Positive ions from particle accelerators have only recently become available for general use in the laboratory and in industry. The application of these particles for the surface analysis of materials can be considerably enhanced by employing very heavy ions, such as argon, to improve the resolution of the method.

Neutron radiography is a tantalizing technique with great potential, but requires considerable development to explore its possible usefulness to industry. An entirely new concept of radiography can grow out of such an investigation

The adaptation of the neutron-activation analytical techniques to production is a matter of time. With the development of compact and intense sources of neutrons, apparatus considerations will become minimized. The interrelation of accelerator and instrumentation for detecting the neutrons must be perfected to produce an industrially acceptable method. The basic characteristics of the process are well known; the engineering details can readily be solved with concentrated attention.

6. Concluding Remarks: Particle accelerators can be broadly useful in research applications for nondestructive tests. Although some of the methods described are primarily for analytical purposes, their nondestructive nature may enable them to be adapted into quality control and final inspection systems. There is a great need for basic research in nondestructive testing in this era of specialized technologies. Valuable techniques are often neglected or not exploited to their fullest, and empiricism tends to control the situation. Acceleration equipment now available from many manufacturers removes many of the restrictions in such research, although the present costs may be considered to be high. As practical uses develop, the equipment costs will inevitably become lower, and the operation of such devices will become relatively simple procedure.

Discussion

THE SCIENTIFIC APPLICATION OF PARTICLE ACCELERATORS TO NONDESTRUCTIVE TESTING

H. M. Cleare; Eastman Kodak Co.: Can you reference your activities, that is, the thermal neutrons from your sources, to that described yesterday from the pile? Is there any information on intensity that is, the number of neutrons per square centimeter per second available?

Burrill: We can provide you with this information. I don't know the curve of which you speak. I was not available yesterday, and unfortunately missed the talk on radiography.

Cleare. The radiographs here were made at 10^5 neutrons per square centimeter per second. Is this anywhere in the range that you could produce?

Burrill: Yes. The thermal neutron or the fast neutron fluxes that are available from even the more moderate particle accelerators are in the 10^7 to 10^{10} neutrons per second per square centimeter range. So that the intensities that were reported yesterday were certainly in the particle accelerator realm of things.

Peter Morris, National Lead Co.: The 10^5 that was reported yesterday was a monochromated neutron beam. Yet the beam described by you has a wide range.

Burrill: In the thermal flux sense they would range over the Maxwellian distribution that you would expect in the thermalized neutron flux. As far as the fast flux, or the intermediate flux, is concerned, this would be a monochromatic flux measurement, because the neutrons are monochromatic as they emerge from the nuclear reaction in the particle accelerator case. This is for fast and intermediate neutrons.

Morris: Do you have a source available of 2-100 Kev neutrons that are essentially monochromatic?

Burrill: Yes. The lithium p-n reaction gives these to us.

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Mr. Harold Berger is an Associate Physicist with the Metallurgy Division of Argonne National Laboratory. He received his B.S. and M.S. degrees in Physics from Syracuse University (1949 and 1951). Previously he was associated with Battelle Memorial Institute and the General Electric Company. Mr. Berger has been active in the field of radiation detection research and development for the past nine years. He has authored several publications in this field and holds a half dozen patents. He is a member of the American Physical Society, the Society for Nondestructive Testing and the Transducer Committee of the Instrument Society of America.

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Mr. E. Alfred Burrill is Vice President and Director of Marketing at the High Voltage Engineering Corporation. He received his B.S. in Physics from the Massachusetts Institute of Technology in 1943. Since graduation from the Massachusetts Institute of Technology he has held several other positions with the High Voltage Engineering Corporation. He is a member of the following societies: American Physical Society; American Nuclear Society, American Society for Testing Materials; Society for Nondestructive Testing; Atomic Industrial Forum; and the American Association for the Advancement of Science.

Dr. John Gilroy

Dr. John Gilroy is Acting Director of the Electronics Division, Argonne National Laboratory. He received his A.B. from Harvard University in 1936 and Ph.D. from Harvard University in 1942. He was a Research Assistant, Instructor in Physics, Optics, and Communication Engineering at Harvard University from 1936-1942. From 1945-1946 he was a Physicist with the Air Reduction Company of Jersey City, New Jersey, and from 1946-1956 was Director of Research at Ohio Chemical & Surgical Equipment Company, Madison, Wisconsin.

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Dr. W. F. Miller is Director of the Applied Mathematics Division at the Argonne National Laboratory. He received his B.S. (with Highest Distinction) (1949); M.S. (Experimental Physics) (1951); and Ph.D. (Mathematical Physics) (1956) from Purdue University. He has been associated with the Argonne National Laboratory since 1955. Fields of research: Computer Applications in Atomic and Nuclear Physics, Stochastic Processes, Methods of Mathematical Physics.

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Mr. C. John Renken is an Assistant Electrical Engineer with the Metallurgy Division of Argonne National Laboratory. He was educated at the Columbia Division of the University of Missouri, receiving the degree of M.S. in Electrical Engineering in 1956. His work since that time has been devoted mainly to the practical and theoretical aspects of electromagnetic testing.

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Dr. S. S. Sidhu, a Senior Physicist with the Metallurgy Division of Argonne National Laboratory, received his B.S., M.S., and Ph.D. degrees from the University of Pittsburgh. Before joining the Laboratory, he was Professor of Physics and Director of X-Ray Laboratory at the University of Pittsburgh. His major field of

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Dr. R. Truell

Dr. R. S. Truell is a member of the Division of Applied Mathematics, Brown University, Providence, Rhode Island. He received his B.S. from Lehigh University in 1935 and his Ph.D. in Experimental Physics from Cornell University in 1942. His industrial experience includes positions with the Radio Corporation of America and Stromberg-Carlson. Since 1946 he has been on the staff of Brown University. He is now engaged in research in solid state physics and the use of ultrasonics for measuring properties of solids.

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Mr. H. E. VanValkenburg is Manager of Research, Sperry Products, Inc., Danbury, Connecticut. He received his B.S. in Electrical Engineering from Union College in 1942. After graduating he joined the Engineering Laboratories of the General Electric Company at Schenectady. For the past 15 years he has been responsible for projects related to ultrasonic inspection. He has had a number of papers in the field of ultrasonic testing published. He is active in technical societies concerned with calibration and standardization problems, serving on committees of ASTM, SNT, ASTE, and Acoustical Society of America.

Mr. D. C. Worlton

Mr. D. C. Worlton is Manager of Testing Methods Operation at the Hanford Atomic Products Operation. He was graduated from the University of Utah in 1948. He has been actively engaged in the development and application of ultrasonic testing equipment. He has worked extensively, both in the research and development of new testing techniques and their application to routine production line testing.

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