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TO: *E. R. Cohen 791-20 DS

COPIES TO: *M. S. Dunenfeld 791-21 DS *A. A. Jarrett 791-20 DS
*J. Flora 791-21 DS
*V. Swanson 790-13 SS
*E. U. Vaughan 791-20 DS
*A. R. Vernon 790-17 DS
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SUBJECT: Inertial Pressure and Void Formation: General Considerations

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I STATEMENT OF PROBLEM

A wave equation for the pressure in a mixture of water and gas bubbles (such as found in the KEWB Reactor) is derived. The driving force may be either the production of additional bubbles or the growth of existent bubbles.

Several illustrative solutions to the wave equation are found.

II INTRODUCTION:

In the fast transients (10 ms or faster) of the KEWB reactor, void formation is the predominant shut off mechanism. The bubbles that form the void are most certainly radiolytic gas bubbles rather than vapor bubbles, since the temperature remains below the solution boiling point. The nucleation process is not well understood. The relative importance of fission nucleation⁽¹⁾ and surface nucleation on impurities is not known.

This paper presents some general considerations on the buildup of inertial pressure associated with void formation. Some examples show how the inertial pressure depends on the driving term. Hopefully, analysis of the KEWB pressure traces will yield information on the nucleation process and the growth of nucleated bubbles.

(1) Experimental evidence for fission nucleation of vapor bubbles is found in Ghormley Journal of Nuclear Energy 6 May 1958.

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III DERIVATION OF WAVE EQUATION

It is assumed that the mixture of water and bubbles can be described by an average density ρ , velocity v , and pressure in water P .

The equation of continuity and motion are

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial z} (\rho v) = 0 \quad (1)$$

$$\rho \frac{dv}{dt} + \frac{\partial P}{\partial z} = 0 \quad (2)$$

The mixture is assumed to have an equation of state of the following form

$$\rho = \rho(P, x) \quad (3)$$

where x = molecules (in bubbles) per unit mass of mixture. The mixture is thus assumed to be isothermal. The use of the variable x rather than the void fraction was suggested by E. U. Vaughan since

$$c^2 = \frac{dP}{d\rho} \quad (4)$$

where c is the velocity of sound.

With the usual acoustic approximations, (1) and (2) become

$$\frac{\partial \rho}{\partial t} + \rho_0 \frac{\partial v}{\partial z} = 0 \quad (1')$$

$$\rho_0 \frac{\partial v}{\partial t} + \frac{\partial P}{\partial z} = 0 \quad (2')$$

The time derivative of ρ can be eliminated from (1') with (3)

$$\begin{aligned} \frac{\partial \rho}{\partial t} &= \left(\frac{\partial \rho}{\partial P} \right)_x \frac{\partial P}{\partial t} + \left(\frac{\partial \rho}{\partial x} \right)_P \frac{\partial x}{\partial t} \\ &= \frac{1}{c^2} \frac{\partial P}{\partial t} + \frac{\partial \rho}{\partial x} \frac{\partial x}{\partial t} \end{aligned} \quad (3')$$

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Finally, the wave equation is obtained by eliminating the velocity from (1') and (2')

$$\frac{\partial}{\partial t} \left(\frac{1}{c^2} \frac{\partial P}{\partial t} \right) - \frac{\partial^2 P}{\partial z^2} = - \frac{\partial}{\partial t} \left(\frac{\partial P}{\partial x} \right)_P \frac{\partial x}{\partial t} \quad (5)$$

The RHS of (5) describes the source of pressure. The LHS of (5) describes the propagation of pressure. The velocity of sound c depends on P and x and thus varies in space and time.

The wave equation (5) will be solved for one-dimensional motion with a fixed surface at $z=0$ and a free surface, $z = H(t)$. The boundary conditions are

$$P(z = H, t) = P_0$$

$$\frac{\partial P}{\partial z} (z=0, t) = 0$$

where P_0 equals the pressure of the atmosphere above the free surface.

IV SOLUTIONS OF WAVE EQUATION

$$1. \quad x = x_0 \epsilon(t)$$

The function $\epsilon(t) = 0 \quad t < 0$
 $\epsilon(t) = 1 \quad t > 0$

Integration of (5) over a small time t yields

$$\frac{\partial P}{\partial t}(t) = -c^2 \left(\frac{\partial P}{\partial x} \right)_P x_0 \delta(t) = \left(\frac{\partial P}{\partial x} \right)_P x_0 \delta(t)$$

Since

$$\left(\frac{\partial P}{\partial x} \right)_x \left(\frac{\partial P}{\partial x} \right)_P \left(\frac{\partial x}{\partial P} \right)_P = -1$$

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A second integration yields

$$P(t=0^+) - P_0 = P(\rho(0), x_0) - P(\rho(0), 0) \approx \left. \frac{\partial P}{\partial x} \right|_{\rho} (\rho(0), 0) x_0$$

which is the obviously correct pressure increment. Since the source (RHS of (5)) vanishes for positive time, the problem is equivalent to the sourceless wave equation with initial conditions

$$P(t = 0^+) = P(\rho(0), x_0)$$

$$\frac{\partial P}{\partial t} (t = 0^+) = 0$$

The solution to this problem is shown on the following page. The velocity of sound c is assumed to be a constant. The following facts were used in obtaining the solution

1. The "shock" interface moves with velocity c .
2. In acoustic approximation, reflection from a rigid surface results in a pressure at the surface which is double the incident pressure.
3. The free interface $H(t)$ is obtained by conserving matter. The result is

$$\frac{dH}{dt} = c \frac{\rho(P_0, 0) - \rho(P_0, x_0)}{\rho(P_0, x_0)} \quad \< \frac{2H}{c} + \frac{\Delta t}{2}$$

$\ll c$ if density change is small.

It can be shown that $\left| \frac{dH}{dt} \right|$ is constant in time.

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The arrows under the P_0 's indicate that these regions are moving with velocity dH/dt .

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This example is useful since the more general case $x=x(t)$ and $c=c(x)$ can be considered as a superposition of step functions or pulses. The above solution represents the Green's function for the wave equation. The Green's function method is discussed in the next section.

The figure on page 5 also suggests a means of stepwise numerical integration when $c=c(P,x)$.

The above example indicates that $H(t)$ can be set equal to H if the density change is small.

B. Green's Function Approach. $c=c(t)$.

The wave equation (5) can be rewritten

$$\frac{\partial^2 P}{\partial \tau^2} - \frac{\partial^2 P}{\partial z^2} = \frac{\partial}{\partial \tau} \left(\frac{\partial P}{\partial x} \cdot \frac{\partial x}{\partial \tau} \right) + \frac{\partial c}{\partial \tau} \cdot c \cdot \frac{\partial P}{\partial \tau} \quad (6)$$

with

$$c = c(t) \text{ and } \tau(t) = \int_0^t c(t') dt'$$

The Green's Function is defined by

$$\left(\frac{\partial^2}{\partial \tau^2} - \frac{\partial^2}{\partial z^2} \right) G(z, \tau - \tau') = \delta'(\tau - \tau') \quad (7)$$

with $G(z, \tau) = 0$ and $\frac{\partial G}{\partial \tau}(z, \tau) = 0$ $\tau < 0$

$$\frac{\partial G}{\partial z}(z=0, \tau) = 0 \text{ and } G(z=H, \tau) = 0$$

We are taking the free surface $H(t)=H$. The Green's function $G(z, \tau)$ is just the solution found in the preceding section with $P_0=0$, $P_1=1$, $H(t)=H$ and $\Delta t=0$.

The solution to (4') is then

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$$P(z, \tau) = \int_0^{\infty} G(z, \tau - \tau') d\tau' \left\{ \frac{\partial P}{\partial x} \bigg|_{\rho} \frac{\partial x}{\partial \tau'} + F(z, \tau') \right\} \quad (8)$$

$$F(z, \tau) = \int_0^{\tau} d\tau' \frac{\partial c}{\partial \tau'} c(\tau') \frac{\partial \rho}{\partial \tau}(z, \tau')$$

as can be seen by operation on (8) with $\left(\frac{\partial^2}{\partial \tau^2} - \frac{\partial^2}{\partial z^2} \right)$.

For times such that the expansion wave has not reached $z=0$,

$$G(z, \tau) = 1 - \epsilon(z - (H - \tau)) \quad 0 < \tau < H \quad (9)$$

$$\begin{aligned} \text{or} \quad G &= 1 & z < H - \tau \\ &= 0 & z > H - \tau \end{aligned}$$

and the solution is

$$P(z, \tau) = \int_{\tau-H+z}^{\tau} d\tau' \left\{ \frac{\partial P}{\partial x} \bigg|_{\rho} \frac{\partial x}{\partial \tau'} + F(z, \tau') \right\} \quad (10)$$

The "solution" (10) actually is an interval equation since the integrand depends on $P(z, \tau)$. This integral equation is considered to be solved either by iteration or stepwise integration.

For example, setting $\frac{\partial P}{\partial x} = \frac{\partial P}{\partial x}(\tau=0)$ and $F(z, \tau) = F(z, 0) = 0$

$$P(z, \tau) \approx \frac{\partial P}{\partial x}(\tau=0) \{ x(\tau) - x(\tau - H + z) \} \quad (11)$$

which corresponds to the following picture (in discrete approximation).

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$$\tau = 4\Delta\tau$$

(cf Figure on p 5)

$$\text{with } \Delta P_0 = \left. \frac{\partial P}{\partial x} \right|_0 \frac{\partial x}{\partial \tau} \left(\tau = \frac{\Delta \tau}{2} \right) \Delta \tau$$

$$\Delta P_1 = \left. \frac{\partial P}{\partial x} \right|_0 \frac{\partial x}{\partial \tau} \left(\tau = \Delta \tau + \frac{\Delta \tau}{2} \right) \tau$$

$$P(z, \tau) = \Delta P_0 + \Delta P_1 + \dots + \Delta P_y \quad < H - 4\Delta\tau = H - \tau$$

$$= \Delta P_n + \dots + \Delta P_4 \quad z = H - n\Delta\tau$$

More accurately, set $\frac{\partial P}{\partial x}(\rho, x) = \frac{\partial P}{\partial x}(\rho_0, x)$ and $F = 0$ to obtain

$$P(z, \tau) = P(\rho_0, x(\tau) - P(\rho_0, x(\tau - H + z)) \quad \tau < H \quad (12)$$

with an obvious interpretation in terms of the preceding picture.

The significance of $F(z, \tau)$ will be discussed in a later section titled Secondary Waves.

$$C. \quad \frac{\partial x}{\partial t} = \beta \epsilon(t) \quad \frac{\partial p}{\partial x} = \text{const.}$$

In this example, a constant rate of bubble production is assumed from $t=0$ onwards. In particular, it is shown that the inertial pressure can reach its maximum value even though the driving term is constant in time and space. This result is contrary to the notion that the pressure would continue to increase indefinitely if the driving term is constant.

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This model has been used for the case in which the driving term is associated with the rapid nucleation of bubbles.

The wave equation (5) with the driving term

$$\frac{\partial x}{\partial t} = \beta \epsilon(t) \quad \frac{\partial p}{\partial x} = \text{const.} \quad (13)$$

is equivalent (by integration over small times) to the homogeneous wave equation

$$\frac{\partial}{\partial t} \left(\frac{1}{c^2} \frac{\partial p}{\partial t} \right) - \frac{\partial^2 p}{\partial z^2} = 0 \quad (14)$$

with the new initial conditions

$$\frac{\partial p}{\partial t}(z, t=0^+) = -c^2 \beta \frac{\partial p}{\partial x} \quad (15)$$

$$p(z, t=0^+) = p_0$$

The sound velocity c is considered to be a function of time. Equation (15) can be approximated by

$$\frac{1}{c} \frac{\partial}{\partial t} \left(\frac{1}{c} \frac{\partial p}{\partial t} \right) - \frac{\partial^2 p}{\partial z^2} = 0. \quad (16)$$

or

$$\frac{\partial^2 p}{\partial \tau^2} - \frac{\partial^2 p}{\partial z^2} = 0 \quad ; \quad \tau = \int_0^t c(t) dt \quad (16')$$

The neglected term appears to contribute little (at least in one particular calculation).

Equation (16') can be solved by usual methods (see Morse, Vibration and Sound p. 75) and the solution corresponds to the sum of two waves moving in opposite as shown below:

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The boundary conditions

$$\frac{\partial P}{\partial z}(z=0, t) = 0; \quad P(z=H, t) = P_0 = 0$$

are seen to be obeyed.

The pressure has the profile

and the inertial pressure measured at $z=0$

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Thus, the inertial pressure reaches its maximum value at $\tau=H$ (at which time the expansion wave reaches $z=0$.) even though the driving force is constant. It should be remembered that the velocity of sound c is decreasing in time.

Thus $P(z=0)vst$ can be obtained from $P(z=0)vs\tau$ and $\tau=\tau(t)^{(1)}$. In the KEWB spherical core, pressure relief by the expanding mixture is slowed down as the core fills. Qualitatively, this may account for the slow decay on the backside of the pressure traces. This behavior may be contrasted with that of the cylindrical core. The inertial pressure in the KEWB sphere never goes negative, since the mixture of water and bubbles will not sustain negative pressures.

The mixture of bubbles and water at any point is seen in the above example to be motionless until an expansion wave from the free surface arrives at that point. At such a point, the pressure is given by $P(\rho(0),x(t))$

D. Secondary Waves

Consider the wave equation (S) with the source term deleted

$$\frac{\partial}{\partial t} \left(\frac{1}{c^2} \frac{\partial P}{\partial t} \right) - \frac{\partial^2 P}{\partial z^2} = 0 \quad (17)$$

Let $c=c(t)$. Just as secondary waves result in reflected waves if the index of refraction varies in space, so will reflected waves arise if the sound velocity c is time dependent. This point was made by E. U. Vaughan.

For an example, let an expansion wave be propagating inwards as in our first example (p.5).

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System destroyed at $t=t_2$.

Assume the following time-dependence

$$c = c_0 \quad 0 < t < t_1$$

$$= c_1 \quad t_1 < t$$

Integration of (17) across $t=t_1$ implies that

$$\frac{1}{c^2} \frac{\partial P}{\partial t} \text{ is continuous} \quad (18)$$

P is continuous

Effective initial conditions can thus be obtained for $t=t_1+\epsilon$.

$$\text{Since } P=P_0 + (P_1-P_0)\epsilon(-\lfloor z-(H-c_0 t) \rfloor) \quad t < t_1$$

$$\frac{\partial P}{\partial t} = -c_0(P_1-P_0)\delta(z-(H-c_0 t)) \quad t < t_1$$

The initial conditions are

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$$P = P_0 + (P_1 - P_0) \epsilon(-\sqrt{z - (H - c_0 t_1)}) \quad t = t_1 + \epsilon$$

$$\frac{\partial P}{\partial t} = -\frac{c_1^2}{c_0} (P_1 - P_0) \delta(z - (H - c_0 t_1)) \quad t = t_1 + \epsilon$$

Since the waves must propagate with velocity c_1 for $t > t_1$, we look for solutions of the form

$$P(z, t) = P_0 + a \epsilon(-\sqrt{z - (H - c_1 t)}) + b \epsilon(-\sqrt{z + c_1 t - H - c_1(t - t_1)})$$

We neglect contribution of expansion wave originating at free surface. Then, the initial conditions imply

$$a + b = P_1 - P_0$$

$$ac_1 - bc_1 = (P_1 - P_0)c_1^2/c_0$$

$$\text{or } a = \frac{1}{2}(P_1 - P_0)\left(1 + \frac{c_1}{c_0}\right) = (P_1 - P_0)\left[1 - \frac{1}{2}\left(1 - \frac{c_1}{c_0}\right)\right] \quad (19)$$

$$b = \frac{1}{2}(P_1 - P_0)\left(1 - \frac{c_1}{c_0}\right)$$

Thus for $c_1 < c_0$, the pressure will have the following form at $t = t_1 + \epsilon$

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In some sense, the reflected wave arises due to the continuity of

$\frac{1}{c^2} \frac{\partial P}{\partial t}$ in (18). In the general case (Eq 5), it is continuity of

$$\frac{1}{c^2} \frac{\partial P}{\partial t} - \frac{1}{c^2} \frac{\partial P}{\partial x} \frac{\partial x}{\partial t} = \frac{\partial P}{\partial t} = -\rho_0 \frac{\partial v}{\partial x} \quad (20)$$

i.e., $\frac{\partial v}{\partial x}$ is continuous in time as expected from continuity of v in time from (2').

The above example indicates the physical significance of the $F(x, \tau')$ contribution in (10). That is, the picture following (H) should include reflected waves.

V. Summary

The wave equation (5) for the pressure is derived. This equation is solved for a step increment in the pressure. The solution is shown on page 6. This solution is important since it is basically the Green's function.

The Green's function method is described. The figure on page 9 shows how the solutions for single increments in pressure (page 6) can be combined to obtain the pressure profile for an arbitrary pressure source.

A second example of constant driving term is solved. The results are shown on page 13. This example was solved by the superposition of forward and backward running waves (see page 12), but is also easily solved by the Green's function method graphically as shown in the figure on page 9.

It should be emphasized from the second example that if the driving term is spatially uniform, then the pressure at the bottom of the core is given by

$$P(\rho(t=0), x(t))$$

until the expansion wave from the free surface arrives. This point has been the basis of some calculations based on fission nucleation.

Finally, the neglect of the term $F(x, \tau)$ in the Green's function method (e.g., see equation 8) is discussed in the section titled "Secondary waves."

The author acknowledges many helpful conversations with E. U. Vaughan.