THE EFFECT OF CORE POLARIZATION OF THE $^{209}$Bi$(p,p')$
$^{209}$Bi(1.61 MeV) REACTION AT 39.5 MeV

by

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ABSTRACT

Core polarization is important in the excitation of the 1.609 MeV state of $^{209}$Bi in the $(p,p')$ reaction at 39.5 MeV. This transition involves a single proton going from the $l_{9/2}$ to the $l_{13/2}$ level. It is shown that most of the observed cross section is due to admixtures of the state formed by coupling the valence proton of $^{209}$Bi to the highly collective $3^-$ state of $^{208}$Pb at 2.614 MeV. The macroscopic vibrational model is used to describe the core, and the admixture found is consistent with that implied by other recent experiments. A completely microscopic calculation, in which the treatment of core polarization is based on the work of Kuo and Brown, grossly underestimates the $L=3$ component of cross section.

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In $^{209}\text{Bi}$ there are two low lying $13/2^+$ states of very different character. The 1.609 MeV level lies near the expected position of a single proton outside a closed $^{208}\text{Pb}$ core and shows up strongly in $^{208}\text{Pb}(^3\text{He},d)^{209}\text{Bi}$). Some of $(^3\text{He},d)$ strength is observed in the $13/2^+$ state at 2.602 MeV. Both of these states decay to the ground state by strong E3 transitions$^2$). In the simplest picture the ground state is a $lh_{9/2}$ proton outside a closed $^{208}\text{Pb}$ core, and the 1.609 MeV state is a $l1_{13/2}$ proton outside the same core, whereas the 2.602 MeV state is a member of a weak coupling septet formed by coupling the $lh_{9/2}$ single particle to the strongly collective $3^-$ state of $^{208}\text{Pb}$ at 2.614 MeV. A perturbation calculation by Mottelson$^4$) has shown that the two $13/2^+$ states are mixed with the admixture of the 2.602 MeV state into the 1.609 MeV state as $\varepsilon^2=4.8\times10^{-2}$. In this calculation the coupling matrix element was estimated from the $\gamma$-decay of the $3^-$ state of $^{208}\text{Pb}$. The mixing of the states accounts for the observed $(^3\text{He},d)$ strengths.

In this letter new experimental data on the 1.609 MeV $13/2^+$ state in $^{209}\text{Bi}$ is presented. The differential cross section for the excitation of this level in the $(p,p')$ reaction was obtained using 39.5 MeV protons from the Michigan State University cyclotron and a self supporting Bi foil. The scattered particles were detected in a lithium-drifted germanium detector in a side entry configuration. The energy resolution was typically about 50 keV overall, and the peak to valley ratio was over 2000 to 1. Figure 1, a spectrum obtained at $35^\circ$, illustrates the difficulties of seeing the relatively weak single particle states, which are shown underlined.
Recent progress has been made in understanding the \((p,p')\) reaction in terms of "realistic forces". The main features of the differential cross sections for several transitions in the \(^{12}\text{C}(p,p')\) and \(^{40}\text{Ca}(p,p')\) reactions at incident energies from 25-55 MeV can be reproduced in local distorted wave calculations using the Kallio-Kolltveit (K-K) force\(^5\) as the projectile-target interaction with an approximate treatment of antisymmetrization\(^6\)\(^7\). The K-K force is a good approximation to the central part of the shell model reaction matrix. This same approach has been successful in describing the inelastic proton scattering from low-lying states in \(^{50}\text{Ti}\), \(^{89}\text{Y}\), and \(^{90}\text{Zr}\)\(^8\). Core polarization, which is important in these transitions, has been estimated by including 2p-lh or 3p-lh (whichever is appropriate) components in the target wave functions. These components are calculated using first order perturbation theory, and only those states formed by coupling the valence nucleon to particle-hole excitations of the core with energies up to \(2h\omega\) are considered. This is essentially the approach first used by Horie and Arima to calculate effective charges\(^9\). The same picture is used by Kuo and Brown in their work on the bound state problem\(^10\).

Kuo has suggested\(^11\) that the particle-hole treatment of core polarization may not be adequate when there is the possibility of contributions from highly collective phonons of the core. This appears to be the case for this transition. Because of this we calculate the cross section in two ways:

1. Including only 2p-lh components in the wave functions.
2. Replacing the components with p-h coupled to core angular momentum \(J_c=3\) by components which contain the 3\(^-\) state of \(^{208}\text{Pb}\).
In the latter calculation we use the macroscopic vibrational model to describe the core. The procedure is essentially that described by Mottleson in Ref. 4. The calculation could be kept completely microscopic by using the R.P.A. vector of Gillet to describe the 3\(^{-}\) state in \(^{208}\)Pb.

The wave functions which contain only the 2p-1h admixtures will be designated Set I. They are defined as follows:

\[
|\text{j}_\text{m}\rangle = |\text{j}_\text{m}\rangle + \sum_j A_j (j' (ph) J_c) |[j' (ph) J_c]| \text{j}_\text{m}\rangle
\]

(1)

\[
A_j (j' (ph) J_c) = - (\Delta E)^{-1} <[j' (ph) J_c]| G |j\rangle
\]

(2)

In these relations \(|\text{j}_\text{m}\rangle\) specifies the state of the valence proton in the presence of a closed core. The quantity \((ph) J_c\) refers to a particle-hole state of the core with angular momentum \(J_c\) and excitation energy \(\Delta E\). This is coupled to a valence proton in the state \(j'\) to give \(j\). \(G\) denotes the coupling interaction which is taken to be the K-K force. Particle-hole pairs are formed from the following shells.

Particles: 2g, 1\(^{1}\)l\(^{1}\)/2, 1\(^{1}\)j\(^{1}\)/2, 2h\(^{1}\)l\(^{1}\)/2, 3d, 4s, 2f, 3p, 1h\(^{2}\)/2, 1\(^{1}\)l\(^{1}\)/2 for protons

2g, 1\(^{1}\)l\(^{1}\)/2, 1\(^{1}\)j\(^{1}\)/2, 2h\(^{1}\)l\(^{1}\)/2, 3d, 4s, 1j\(^{1}\)/2, 2h\(^{2}\)/2, 3f\(^{7}\)/2 for neutrons

Holes: 1f, 2p, 1g, 2d, 3s, 1h\(^{1}\)/2, 1d, 2s for protons

1f, 2p, 1g, 2d, 3s, 1h\(^{1}\)/2, 1h\(^{2}\)/2, 2f, 3p, 1l\(^{1}\)/2 for neutrons.

Harmonic oscillator wave functions have been used, and the energy denominators were taken in part from experiment\(^{13}\)) and in part from the Nilsson scheme at zero deformation. The size parameters \(\hbar \omega\) is 6.8 MeV.

The wave function used in the second calculation will be designated Set II. They are the same as Set I except for the replacement:
\[
\sum_{\text{ph}} A_j (j' (ph) J_c = 3) |j' (ph) J_c = 3, J_m \rangle + B_j (j') |j' \otimes 3^-, J_m \rangle
\]

\[
B_j (j') = (E_j - E_j, -\hbar \omega_3)^{-1} \langle j' \otimes 3^-; j | V | j' \rangle
\]

\[
\langle j' \otimes 3^-; j | V | j \rangle \propto \langle k \rangle (\frac{\hbar \omega_3}{2C_3})^{1/2} \langle j' || Y_3 || j \rangle
\]

\[
V = -k(r) \sum_{LM} \alpha_{LM} v_{LM}(r)
\]

Eq. (3) - (6) are the usual expressions encountered when the macroscopic vibrational model is used in the treatment of particle-vibrational coupling. The quantity \(k(r) = R dU(r)/dr\) where \(U(r)\) is the single particle potential seen by the extra core proton, \(R\) specifies the nuclear radius, \(\langle k \rangle\) denotes a radial integral, \(\hbar \omega_3\) is the excitation energy of the \(3^-\) phonon of \(^{208}\text{Pb}\), and \(C_3\) gives a measure of the core stiffness to this lowest octupole vibration. Ref. 4 gives \(\langle k \rangle = 60\) MeV and \(C_3 = 649\) MeV. Analyses of the \(^{208}\text{Pb}(p,n')^{208}\text{Pb}\) reaction give a \(\beta_3 = 0.13\) for this state\(^{14}\) which is the only state with a large value of \(\beta\) in \(^{208}\text{Pb}\). The relation \(\beta_3 = (7)^{1/2} (\hbar \omega_3/2C_3)^{1/2}\) implies \(C_3 = 543\) MeV which is smaller than the value from Ref. 4 and corresponds to an admixture \(\epsilon^2 = 5.5 \times 10^{-2}\) of the \(2.602\) MeV \(13/2^+\) state in the \(1.609\) MeV \(13/2^+\) state. The smaller value of \(C_3\) was used in this work.

The cross section for this transition has 20 components each designated by (LSJ) referring to orbital, spin, and total angular momentum transfer. Details for calculating the cross section from the wave functions being considered in this work are given in Ref. 8. In this work, as a matter of convenience, we have used a pseudopotential for the projectile target interaction. This is known to give results consistent with those obtained using the K-K force and treating antisymmetrization approximately. The \(2p-1h\) components of the cross section have
been included only in the $S=0$ terms in the cross section because it is only in these components that they add coherently. In using wave function Set II the components of the wave functions defined by Eq. (3), (4), (5), and (6) contribute only to the $(LSJ) = 303$ component of the cross section. The remaining nineteen components are the same in sets I and II.

Fig. 2 shows the total differential cross sections obtained with wave function Set I and Set II. The $(303)$ components are also shown for both cases. The differential cross section (II) gives a good fit to the experimental data. The $(303)$ (II) component is dominant at forward angles. The enhancement due to core polarization, i.e. ratio of integrated cross sections with and without core polarization, of $(303)$ (II) is about 200. Because of the large enhancement the magnitude of $(303)$ (II) is proportional to the admixture, $\epsilon^2$. Neglecting all other components the data places an upper limit on $\epsilon^2 = 10^{-1}$. Wave function Set II gives $B(E3) = 2.4 \times 10^{-2} e^2 b^3$ which is slightly larger than the experimental values $1.3 - 2.0 \times 10^{-2} e^2 b^3$.

The particle-hole model fails to reproduce the effect of the $3^-$ phonon of $^{208}\text{Pb}$. The enhancement of $(303)$ (I) is about 13 which is an order of magnitude smaller than the value obtained for $(303)$ (II). This model predicts that many components make important contributions to the total differential cross section. In particular, $(303)$ (I) is comparable in magnitude to (112) which involves the lowest allowed $L$- and $J$- transfers. As the lowest $J$-transfer is highly favored in $\gamma$-transitions, the particle-hole model predicts that the 1.609 MeV $13/2^+$ state will decay to the ground state predominantly by an $M2$ transition which is in contradiction to experiment. In a previous analysis of the
$^{89}$Y(p,p')$^{89}$Y(0.908 MeV) reaction\(^8\)), which involves a single proton going from the $2p_{1/2}$ to the $1g_{9/2}$ level, the particle-hole model gave a good fit to the angular distribution and predicted appreciable contributions from both the (314) and (505) components. In this case there are no strongly collective core states contributing because \(^{88}\)Sr has no strong low lying $5^-$ state.

It is concluded that highly collective core phonons can play an extremely important part in the core polarization process, and that care must be exercised in applying the particle-hole model.

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REFERENCES


8. F. Petrovich, H. McManus, and J. R. Borysowicz (to be published).


REFERENCES (cont.)

FIGURE CAPTIONS

Figure 1. Spectrum of protons from the $^{209}\text{Bi}(p,p')^{209}\text{Bi}$ reaction taken at 35° in the lab. The resolution is about 45 keV. The peaks shown underlined are single particle states in $^{209}\text{Bi}$.

Figure 2. The experimental data is compared with the theoretical results obtained with both sets of wave functions. The total differential cross sections and the (303) component are shown for both cases.
$^{209}_{\text{Bi}} (p, p') ^{209}_{\text{Bi}}$

$E_p = 395 \text{MeV}$

$\theta_{\text{LAB}} = 35^\circ$
$^{209}\text{Bi} + p$

$E = 39.5$ MeV

$\frac{13}{2}^+ ; Q = -1.61$ MeV

Total (Set I)

Total (Set II)

$303$ (Set I)

$303$ (Set II)