SC-DC-64-891

AUG 3 1964

UNIVERSITY OF NEW MEXICO
ALBUQUERQUE

MASTER

ENGINEERING EXPERIMENT STATION

TEMPERATURE DISTRIBUTION AND THERMAL STRESSES
IN RADIATION-COOLED CIRCULAR CYLINDERS

By
William E. Putman Jr.

EP

A Thesis
Submitted in Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Engineering

1964

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By

William E. Putman, Jr.

Facsimile Price \$ 11.50
Microfilm Price \$ 4.85

Available from the
Office of Technical Services
Department of Commerce
Washington 25, D. C.

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The University of New Mexico

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ABSTRACT

This study involves the determination of the transient temperature distribution and thermal stresses in a solid circular cylinder cooled by thermal radiation. The analytical solutions are presented in dimensionless form. Theoretical results are shown for temperature distribution and thermal stresses in the nondimensional form. Certain of these results are given in the dimensional form for the model cylinder used in this investigation.

The uranium-zirconium prototype element was thought to be undesirable for this investigation. Therefore, similitude was used to select a model which could be used for the experimental work. A solid, cylindrical, epoxy model was constructed, and thermocouples were embedded in the cylinder to determine the temperature distribution. Strain gages were to be embedded in the cylinder to determine the thermal stresses; however, a gage has not been found which is temperature compensated for the epoxy material.

Mechanical equipment was acquired or built for the experimental investigation, and included: a vacuum apparatus; a liquid-nitrogen-cooled heat sink; a collapsible cylindrical heater and shields; the framework to support the model and the heater and shields; and two mylar-coated, chromium radiation shields. This equipment was tested successfully where required.

Some of the thermocouples for the experiment were calibrated using two calibration points. The ionization gage in the vacuum system was calibrated at one point using a McCleod gage.

A test was set up to determine the emissivity and thermal conductivity of the epoxy material using a small test cylinder made of epoxy and instrumented with thermocouples. The results of this test appeared to be of doubtful use due to uncertainty in the thermocouple readings. The thermocouples require an accurate calibration to improve the experimental results for this test.

ACKNOWLEDGEMENTS

The author wishes to express his sincere appreciation for the advice and guidance of Dr. M. W. Wildin, Professor of Mechanical Engineering, who served as thesis advisor for this study.

A very special thanks is given to the author's wife Gloria, without whose devotion and understanding this thesis would not have been possible.

Thanks must also be given to the members of the Mechanical Engineering Department who contributed in some way to the completion of this work.

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LIST OF SYMBOLS

Symbol	Quantity	Dimensions*
$\bar{a}$	Outside Radius.	L
b	Slope of Curve Relating Thermal Conductivity to Temperature	$FT^{-1}\theta^{-2}$
C	Specific Heat of Solid.	$L^2T^{-2}\theta^{-1}$
e	Slope of Curve Relating Specific Heat to Temperature	$L^2T^{-2}\theta^{-2}$
E	Young's Modulus	FL^{-2}
F	Slope of Curve Relating Dimensionless Thermal Conductivity to Dimensionless Temperature	1
H	Slope of Curve Relating Dimensionless Heat Capacity to Dimensionless Temperature.	1
K	Thermal Conductivity.	$FT^{-1}\theta^{-1}$
N_R	Dimensionless Parameter Related to Surface Heat Flux	1
$\bar{r}$	Radial Position	L
r	Dimensionless Radial Coordinate	1
t	Time.	T
T	Absolute Temperature.	θ
α	Coefficient of Linear Thermal Expansion	θ^{-1}
ϵ	Total Hemispherical Emissivity.	1
E_r, E_θ, E_z	Dimensionless Principal Strain Components	1
$\epsilon_r, \epsilon_\theta, \epsilon_z$	Principal Strain Components	1

* L, T, F, θ denote dimensions of length, time, force, and temperature, respectively.

λ	Used with Superscript k, c, or α to Indicate a Dimensionless Property.	1
ν	Poisson's Ratio.	1
φ	Ratio of Temperature to Initial Temperature, T/T_i	1
ρ	Weight Density	FL^{-3}
σ	Stefan-Boltzmann Constant.	$L^{-1}FT^{-1}\theta^{-4}$
$\Sigma_r, \Sigma_\theta, \Sigma_z$	Dimensionless Principal Stress Components.	1
$\sigma_r, \sigma_\theta, \sigma_z$	Principal Stress Components.	FL^{-2}
τ	Dimensionless Time, $\frac{K_i t}{\rho C_i a^2}$	1

Superscripts:

$\left. \begin{matrix} k \\ c \\ \alpha \end{matrix} \right\}$	Used with λ to Indicate Dimensionless Thermal Conductivity, Specific Heat, and Coefficient of Linear Thermal Expansion, Respectively
--	--

Subscripts:

a	Denotes Arbitrary Value of a Quantity
i	Denotes Initial Value of a Quantity
m	Denotes a Quantity Pertaining to the Model
p	Denotes a Quantity Pertaining to the Prototype
r	Denotes Radial Component
R	Denotes a Reference Value
z	Denotes Axial Component
θ	Denotes Circumferential Component

1.0 INTRODUCTION

The problem considered here involves the determination of the temperature distribution and thermal stresses in a radiation-cooled, solid circular cylinder. A theoretical solution to the problem is developed in a report by Wildin and Putman (1) and includes: (1) the general equations and boundary conditions required to determine temperature and principal stresses as functions of position and time, (2) the nondimensionalization of the equations and boundary conditions to permit the use of similitude, and (3) the numerical results obtained through this analytical solution. The prototype element for which the analytical solution was developed is made of uranium-zirconium hydride. The main steps and the results of that report are summarized in Chapter 3 of this study.

The purpose of this investigation is to design and construct an apparatus to produce thermal stresses in a solid circular cylinder by radiation cooling, and to assemble equipment to measure the strains and temperatures associated with these stresses. Also included is the check-out of the experimental apparatus for proper operation, and the measurement of various thermal properties of the experimental model.

The uranium-zirconium element was thought to be undesirable for the experimental work; therefore, an experimental model made of aluminum-filled epoxy was used. The model and prototype

(uranium-zirconium element) were related by similitude, and the dimensions and initial conditions for the model were selected to satisfy the conditions required for similarity to exist between the model and prototype. Thermocouples were embedded in the model to determine the temperature distribution. Strain gages were to be mounted in the model to determine the stress distribution, but a strain gage has not been found which is temperature compensated for the aluminum-filled epoxy. The information on the design, construction, and instrumentation of the experimental model can be found in Chapter 4.

The mechanical apparatus acquired or developed for the investigation included a vacuum apparatus; a liquid-nitrogen-cooled heat sink; a collapsible, cylindrical heater surrounded by two radiation shields; the framework to support the model; and two mylar-coated chromium radiation shields. This equipment is used to simulate the conditions which would be experienced by the prototype element. Where required, these components were tested for proper operation. The development of the mechanical apparatus is presented in Chapter 5.

Before the thermal stress and temperature distribution tests could be run, information was required on the thermal properties of the epoxy material. A test was set up to determine these thermal properties using the experimental apparatus developed for the temperature distribution and thermal

stress tests. A special test cylinder instrumented with thermocouples was used for the test. The information on the test and the results obtained are given in Chapter 6.

2.0 LITERATURE REVIEW

2.1 Thermal Stresses

Kent (2) gives the solution to the problem of stress distribution in spheres and cylinders of elastic materials for cases in which the temperature is a function of both the radial coordinate and the time. Two types of temperature variation are included: that due to sudden change in surface temperature, as in quenching; and that due to linear surface heating, as in furnace heating. These are applied both to solid and hollow spheres and cylinders. Examples are included of the temperature and stress distribution in a long, solid forging of 30-inch diameter and in a cylindrical drum of 50-inch diameter when each is heated by raising the temperature of its convex surface at a constant rate. An example is also presented for the temperature and stress distribution in a solid steel cylinder of 10-inch radius when the surface is cooled 100°F below an originally uniform temperature. The general shape of the stress-versus-position curves presented by Kent for the 10-inch solid steel cylinder can be used to compare with the general shape of the stress-versus-position curves calculated in this investigation.

Thunborg (3) uses the existing equations for the thermal stresses in a thick-walled cylinder with free ends to determine the thermal stresses and strains when the thick-walled cylinder is subjected to a radial temperature gradient that

is invariant with time. An embedded strain gage technique was used in a plastic test specimen. The results obtained for the thermal stresses compared favorably with the theory. He therefore concluded that embedded strain gages could be used to determine thermal strains in a plastic model.

2.2 Strain Gages

Dove (4) presents a method for eliminating any signals from a mounted strain gage not induced by thermal strain when the specimen on which the strain gage is mounted is subjected to a change in temperature. This method involves the simultaneous measurement of strain and temperature at a point. A strain gage of the type used for the test is mounted on an auxiliary block of the same material as the test piece. The auxiliary model is subjected to free expansion for the temperature range of interest. A curve of strain versus temperature is plotted for the free expansion case.

The thermal strain is then determined by subtracting the strain reading obtained at some temperature for the free expansion case from the strain reading determined at the same temperature in the test piece at the location of interest. Dove mentions that a properly selected self-temperature-compensated strain gage will keep the correction factor (strain reading obtained for the free expansion case) small and thus improve the resolution of the method.

Theisen (5) used this method to measure the thermal strains in a moderately thick-walled cylinder. One point to note is the effect on the strain readings when a strain gage is mounted on a curved surface. A gage mounted on a curved surface will read lower than a gage mounted on a flat surface when both gages are exposed to the same temperatures.

2.3 Material Properties

The literature was searched for information about the material properties of the aluminum-filled epoxy used in this experiment. This was done to determine which property values were available so that experiments could be set up to determine those which were not. A report by Baker, Matthews, and Dove (6) contained some of the data of interest. One of these was the thermal conductivity of the material, for which several values were presented based on the percentage by volume of aluminum powder added to the epoxy.

The values of thermal conductivity for this report were determined for one temperature only. Also, there was some question as to how the percentage by volume of aluminum powder added to the epoxy was determined. The actual method for measuring the volume of aluminum powder was not presented in this report; however, it appears that it would be difficult to accurately measure this volume. There was no way of telling how accurate the results for this report were. Therefore, it appeared desirable to set up an experiment to check the values

of thermal conductivity presented and also obtain values for various temperatures.

2.4 Method for Determining Coefficient of Linear Thermal Expansion

In the initial survey of the properties available, information was missing for the coefficient of linear thermal expansion. The only data available was for the linear thermal expansion as a function of temperature. A method for determining the coefficient of linear thermal expansion using the linear thermal expansion versus temperature data was found in a report by J. C. Heap (7). Heap fitted straight lines to existing data for four stainless steels by the least squares method. First, Heap wrote the mathematical expression for the coefficient of thermal expansion

$$\alpha = \frac{1}{L} \frac{dL}{dt}$$

where L is the length of the test piece and $\frac{dL}{dt}$ is the differential change in length of the component with respect to a differential change in temperature. (This expression would be the value of α for the free expansion case. The general expression for α would be

$$\alpha = \frac{1}{L} \left(\frac{\partial L}{\partial t} \right)_{\sigma}$$

where σ is the stress.) Then he assumed the expression for the coefficient of linear expansion to be a linear function of temperature

$$\alpha = \alpha_0 + \alpha_c t$$

Heap combined the two expressions for α , rearranged the terms, and integrated from the reference temperature and length to an indicated temperature and length, with the resulting expression

$$\ln \left(\frac{L}{l_o} \right) = \alpha_o (t - t_o) + \frac{\alpha_c}{2} (t^2 - t_o^2)$$

where l_o is the reference length, t_o is the reference temperature, and t is the indicated temperature. If the length at an indicated temperature is expressed in terms of the reference length and change in length

$$L = l_o + \Delta l$$

then substituting into the previous equation

$$\ln \left(1 + \frac{\Delta l}{l_o} \right) = \alpha_o (t - t_o) + \frac{\alpha_c}{2} (t^2 - t_o^2)$$

where $\frac{\Delta l}{l_o}$ is the linear thermal expansion. Applying the least square method to the above equation, the expressions used for determining α_o and α_c are:

$$\alpha_o \sum_i (t_i - t_o)^2 + \frac{\alpha_c}{2} \sum_i (t_i - t_o)^2 (t_i + t_o) = \sum_i (t_i - t_o) \ln \left(1 + \frac{\Delta l_i}{l_o} \right)$$

$$\begin{aligned} \alpha_o \sum_i (t_i - t_o)^2 (t_i + t_o) + \frac{\alpha_c}{2} \sum_i (t_i - t_o)^2 (t_i + t_o)^2 \\ = \sum_i (t_i - t_o) (t_i + t_o) \ln \left(1 + \frac{\Delta l_i}{l_o} \right) \end{aligned}$$

2.5 Cryogenics and Vacuum Equipment

Scott (8) presented information on the handling of cryogenic liquids such as liquid nitrogen. He discussed types of insulation for storage and transporting of liquid nitrogen, the types of

metals that should be used for handling these fluids, and the types of vacuum seals that should be used with cryogenic fluids. He also discussed the type of materials to use for thermocouples for low temperature work, and the various methods for bringing thermocouple leads out of a vacuum system.

He presented a section on vacuum equipment including a discussion of vacuum pumps and the various vacuum measuring devices.

3.0 THEORETICAL ANALYSIS

In this chapter a solution to the problem of temperature distribution and transient thermal stresses is presented. To obtain more widely applicable solutions and to facilitate modeling of the thermal stresses, these equations are nondimensionalized. Then, in order to use a model for the actual tests, a similitude analysis is presented to show the relationships that must be satisfied in the nondimensional equations for similarity to exist between the temperature and stress distributions in the model and prototype. The methods used to satisfy these relationships are also presented. Last, theoretical results are presented in both the dimensional and nondimensional forms.

3.1 General Equations

To determine a theoretical solution for the temperature distribution and transient thermal stresses in a radiation-cooled, cylindrical element, several assumptions were established.

Radiative heat transfer was assumed to be the only means of heat transfer from the surface of the element with convective and conductive modes of transfer from the surface considered negligible. Therefore, if the absorption of incident radiation is assumed to be negligible, the net rate of heat transfer from the element is equal to the rate of emission of thermal radiation by the element. The heat transfer from the ends was neglected, which means the temperatures in the element were

considered constant in the axial direction. The heat transfer rate from the surface of the element was maximized by assuming the surroundings to which the element radiates to be at 0°K . The surroundings also were assumed to have the characteristics of a black body and consequently an emissivity of one.

The material used in the construction of the element was assumed to be homogeneous and isotropic and to have a uniform emissivity over the surface. The temperature distribution was assumed to be initially uniform throughout the cylinder. Therefore, the temperature distribution at any later instant is a function of radial position only. For the prototype, Woolsey (9) substantiated the assumption of homogeneity, but there was no substantiation for the assumption of isotropy since the manner of construction of the element was not known. The model material was homogeneous and isotropic since the materials used for model construction were very thoroughly mixed and placed in a high vacuum to remove all the air bubbles before hardening.

Another assumption was that the element material had a high absorption coefficient so that radiation was emitted only from the surface of the element.

Using the assumptions discussed, the heat transfer portion of the problem is stated in terms of the following differential equation, initial condition, and boundary conditions:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(rK \frac{\partial T}{\partial r} \right) = \rho C \frac{\partial T}{\partial t} \quad (1a)$$

$$T(\bar{r}, 0) = T_i$$

$$\frac{\partial T}{\partial r}(0, t) = 0, \quad t \geq 0 \quad (1b)$$

$$-K \frac{\partial T}{\partial r}(\bar{a}, t) = \sigma \epsilon T^4(\bar{a}, t), \quad t > 0$$

The differential equation (1a) for the temperature distribution is formulated in such a way that variable specific heat and thermal conductivity can be considered. The first condition (1a) is the initial condition, while the second condition (1b) specifies the symmetry of the temperature distribution about the axis of the rod. The last condition (1b) is obtained by equating the heat flux at the surface of the rod by conduction to that by radiation. The boundary and initial conditions follow directly from the assumptions stated previously. These conditions are discussed more fully in a report by Wildin and Putman (1).

The thermal stress equations were presented by Heap (7), treating Young's modulus and Poisson's ratio as constants and the coefficient of linear thermal expansion as a function of temperature. These equations are based on the assumption that plane sections normal to the axis of the cylinder remain plane, and they also depend on the boundary condition of no axial or radial constraints. Since no step changes in temperature occur when a body is cooled by emission of thermal radiation, the quasistatic theory of thermoelasticity can be employed. That is, inertia terms in the equation of motion can be neglected, as pointed out by McKinley and Ju (10).

For a solid cylinder initially unstressed and at temperature (T_i), the principal thermal stresses are given by:

$$\begin{aligned}
 \sigma_z &= \frac{E}{1-\nu} \left[\frac{2}{a^2} \int_0^{\bar{a}} \left(\int_{T_i}^T \alpha dT \right) \bar{r} d\bar{r} - \int_{T_i}^T \alpha dT \right] \\
 \sigma_r &= \frac{E}{1-\nu} \left[\frac{1}{a^2} \int_0^{\bar{a}} \left(\int_{T_i}^T \alpha dT \right) \bar{r} d\bar{r} - \frac{1}{r^2} \int_0^{\bar{r}} \left(\int_{T_i}^T \alpha dT \right) \bar{r} d\bar{r} \right] \\
 \sigma_\theta &= \frac{E}{1-\nu} \left[\frac{1}{a^2} \int_0^{\bar{a}} \left(\int_{T_i}^T \alpha dT \right) \bar{r} d\bar{r} + \frac{1}{r^2} \int_0^{\bar{r}} \left(\int_{T_i}^T \alpha dT \right) \bar{r} d\bar{r} - \int_{T_i}^T \alpha dT \right]
 \end{aligned} \tag{2}$$

3.2 Nondimensionalized Equations

To obtain solutions for the stress and temperature distributions which are more widely applicable, as well as to facilitate modeling of the temperature distribution and thermal stresses, the equations for the stress and temperature distributions were nondimensionalized. The method for nondimensionalizing these equations can be seen in Reference (1). It should be noted that the subscript R is used to denote reference values. The nondimensionalized equations for the temperature distribution are:

$$\frac{1}{r} \frac{\partial}{\partial r} (r \lambda^k(\varphi) \frac{\partial \varphi}{\partial r}) = \lambda^c(\varphi) \frac{\partial \varphi}{\partial \tau} \tag{3a}$$

$$\varphi(r, 0) = T_i/T_R = \varphi_i$$

$$\frac{\partial \varphi}{\partial r}(0, \tau) = 0, \quad \tau \geq 0 \tag{3b}$$

$$\frac{\partial \varphi}{\partial r}(\bar{a}/r_R, \tau) = \frac{-N_R}{\lambda^k(\varphi)} \varphi^4(\bar{a}/r_R, \tau), \quad \tau > 0$$

The equations for the nondimensionalized thermal stresses are:

$$\begin{aligned}
 \Sigma_z &= \frac{2}{\bar{a}^2/\bar{r}_R^2} \int_0^{\bar{a}/\bar{r}_R} \left(\int_{\varphi_i}^{\varphi} \lambda^\alpha d\varphi \right) r dr - \int_{\varphi_i}^{\varphi} \lambda^\alpha d\varphi \\
 \Sigma_r &= \frac{1}{\bar{a}^2/\bar{r}_R^2} \int_0^{\bar{a}/\bar{r}_R} \left(\int_{\varphi_i}^{\varphi} \lambda^\alpha d\varphi \right) r dr - \frac{1}{r^2} \int_0^r \left(\int_{\varphi_i}^{\varphi} \lambda^\alpha d\varphi \right) r dr \\
 \Sigma_\theta &= \frac{1}{\bar{a}^2/\bar{r}_R^2} \int_0^{\bar{a}/\bar{r}_R} \left(\int_{\varphi_i}^{\varphi} \lambda^\alpha d\varphi \right) r dr + \frac{1}{r^2} \int_0^r \left(\int_{\varphi_i}^{\varphi} \lambda^\alpha d\varphi \right) r dr - \int_{\varphi_i}^{\varphi} \lambda^\alpha d\varphi
 \end{aligned} \tag{4}$$

3.3 Similitude Analysis

The model and prototype were related by similitude using the nondimensional equations (3a), (3b), and (4). If these equations have identical forms for prototype and model, then similarity is said to exist between the temperature and stress distributions in the model and prototype. It should be noted that several conditions must be satisfied in order for similarity to exist. Some of the conditions were arbitrarily satisfied for the prototype and model while others were not so easily satisfied since they were functions of the material properties. It is important to note also that the nondimensionalized equations with similarity established between prototype and model can be used to determine theoretical results for either the prototype or model.

First, looking at the differential equation and the initial and boundary conditions for the temperature distribution, several conditions had to be satisfied for similarity to exist as shown in Reference (1).

In the initial condition, similarity requires that the initial dimensionless temperature distribution be the same for the prototype and model. From this condition the initial temperature was selected as the reference temperature for both the prototype and model. This satisfies the condition without restricting the choice of initial temperature for either the model or prototype. If the initial temperature were not selected as the reference temperature, this would complicate the modeling process. A reference temperature would have to be selected for the prototype; and, to satisfy the similarity relationship, a reference temperature would be required for the model. This would restrict the selection of the initial temperatures for the model.

The first boundary condition,

$$\frac{\partial \varphi}{\partial r} (0, \tau) = 0 \quad (3b)$$

is satisfied by axisymmetric temperature distributions in both the model and prototype. In addition to a uniform initial temperature, this requires that the rate of heat transfer be uniform over the surface and that the material be homogeneous and isotropic. Hence, the emissivity of the surface of the model must be uniform and the surface temperature must be uniform. It is noted that satisfying this boundary condition does not aid in selecting reference values, but it gives some insight into the physical conditions which must be observed in

constructing the model and establishing the test conditions. These conditions will be discussed further in the chapters on the experimental model and experimental setup.

If the second boundary condition (3b) is to have the same form for model and prototype, it must be evaluated at the same value of dimensionless radial coordinate as shown in Reference (1). This condition was satisfied without restricting the outside radius of the model or prototype by letting the outside radius be the reference radius for both prototype and model. If the outside radius were not selected for the reference radius, then some other radius would have to be selected as the reference radius for the prototype; and, to satisfy the similitude requirement, a reference radius different from the outside radius would be required for the model. This would complicate the modeling process.

In order for the second boundary condition to have the same form for prototype and model, the following also must be satisfied:

$$\left[N_R \right]_p = \left[N_R \right]_m \quad (5)$$

Thus N_R is identified as an important dimensionless parameter in the phenomenon of cooling of a long cylinder by emission of thermal radiation. The equality of N_R for the model and prototype is ensured by selecting parameter values for N_R of the model which give the value of N_R to be duplicated. A complete analysis of how N_R was used in the design of the model and the

choice of initial conditions will be presented in the chapters dealing with the experimental model and experimental setup.

Another relationship that must be satisfied for the second boundary condition (3b) and also for the differential equation (3a) in order to have similarity between the model and prototype is that $\lambda^k(\varphi)$ have the same form for the model and prototype. The same must be true for $\lambda^c(\varphi)$ in the differential equation.

This type of relationship cannot be satisfied by arbitrarily selecting values, as was the case with the conditions previously discussed, since $\lambda^k(\varphi)$ and $\lambda^c(\varphi)$ are functions of the materials used for the model and prototype and therefore vary according to the composition of the materials.

The functional dependence of λ^k or λ^c on φ must be of the same type, i.e., linear, quadratic, exponential, etc., for the model and prototype throughout the temperature range of interest in the analysis. The functional dependence must not only be of the same type, quadratic for example; but also the functional constants must be the same for prototype and model. In other words, the curves of λ^k and λ^c as a function of temperature must have the same form and the same initial point.

It is known from the data presented in References (11) and (12) that both specific heat and thermal conductivity of the prototype can be approximated very closely by linear functions up to the maximum temperatures of interest. (The property values of the prototype material were measured in either a hydrogen or argon atmosphere, and therefore there is considerable doubt concerning the validity of using these values to predict what

will occur in the actual situation. Their use is justified only on the basis that they are the only available values.) From the data presented by Matthews (13), (6), it is known that specific heat for the model is approximated closely by a linear function throughout the range of temperatures of interest. Thermal conductivity for the model material is presented only for a temperature of 150°F which is in the range of temperatures of interest. However, since the thermal conductivity was only determined for one temperature, tests were set up to determine how the thermal conductivity varies with temperature as discussed in Chapter 6.

Although the functional dependence of λ^k on ϕ was unknown for the model material, the theoretical results predicted the temperature changes in the model tests to be small (1). Therefore, the assumption of thermal conductivity varying linearly with temperature for the model material was justified.

With λ^k a linear function of ϕ for both prototype and model, a relationship between thermal conductivity and temperature was written

$$K = K_a + b(T - T_a)$$

where K_a is the thermal conductivity corresponding to some arbitrary temperature, T_a as shown in Reference (1). The terms λ^k and ϕ were substituted for K and T with the following result:

$$\lambda^k(\phi) = \left(\frac{K_a}{K_R} - \frac{bT_a}{K_R} \right) + \frac{bT_i}{K_R} \phi$$

The initial thermal conductivity (K_i) was chosen for K_a and K_R and the initial temperature (T_i) was selected for T_a to simplify

the task of preserving identity of form between the prototype and model for the above relationship. The selection of K_i and T_i also insured that the initial values of λ^k were equal for model and prototype which was required if $\lambda^k(\phi)$ was to have the same form for prototype and model. Using the above selections, a simple form resulted for the relation between dimensionless thermal conductivity and temperature

$$\lambda^k(\phi) = (1-F) + F \phi \quad (6)$$

where

$$F = \frac{bT_i}{K_i}$$

and b is the slope of the curve relating thermal conductivity to temperature. The value F must be the same for prototype and model to insure similarity. Similar reasoning was used for the specific heat and led to the same type of simple relation between dimensionless specific heat and temperature

$$\lambda^c(\phi) = (1-H) + H\phi \quad (7)$$

where

$$H = \frac{eT_i}{K_i}$$

and e is the slope of the curve relating specific heat to temperature.

The function $\lambda^\alpha(\phi)$ must have the same form for model and prototype if the dimensionless stress equations (4) are to be identical for model and prototype. The initial value of α was chosen as the reference value so that:

$$\lambda^\alpha(\phi) = \frac{\alpha}{\alpha_i}$$

as shown in Reference (1).

At the beginning of the experiment, values for the coefficient of linear expansion were not available for either the prototype or the model. Information was available, however, for the linear thermal expansion of the model and prototype materials. The coefficient of thermal expansion for model and prototype was therefore determined using the linear thermal expansion curves and a curve fitting method presented by Heap (7). For the prototype α was calculated to be:

$$\alpha = 4.56 \times 10^{-6} + 3.86 \times 10^{-9} T \text{ in/in/}^{\circ}\text{F}$$

Using the same method for the model, the result for α was:

$$\alpha = 22.5 \times 10^{-6} + 50 \times 10^{-9} T \text{ in/in/}^{\circ}\text{F}$$

From the above results α is a weak function of temperature for the prototype and model in the temperature range of interest for the experiment. Also, the magnitude of the predicted strains for model and prototype is small. Hence $\lambda^{\alpha} = 1$ was used in all the stress equations.

The nondimensional stress equations (4) must be evaluated at the same value of the dimensionless radial coordinate in order to have the same form for the prototype and model. This condition was fixed previously in the second boundary condition (3b) by selecting the outer radius as the reference radius for the model and prototype.

The final form of the differential equation and the conditions which the temperature distribution must satisfy is:

$$\frac{1}{r} \frac{\partial}{\partial r} (r \lambda^k(\varphi) \frac{\partial \varphi}{\partial \tau}) = \lambda^c(\varphi) \frac{\partial \varphi}{\partial \tau} \quad (8a)$$

$$\varphi(r, 0) = 1$$

$$\frac{\partial \varphi}{\partial r}(0, \tau) = 0, \quad \tau \geq 0 \quad (8b)$$

$$\frac{\partial \varphi}{\partial r}(1, \tau) = \frac{-N_R}{\lambda^k(\varphi)} \varphi^4(1, \tau), \quad \tau > 0$$

The final form for the stress equations (4) using the selected reference values is:

$$\begin{aligned} \Sigma_z &= 2 \int_0^1 \varphi r dr - \varphi \\ \Sigma_r &= \int_0^1 \varphi r dr - \frac{1}{r^2} \int_0^r \varphi r dr \\ \Sigma_\theta &= \int_0^1 \varphi r dr + \frac{1}{r^2} \int_0^r \varphi r dr - \varphi \end{aligned} \quad (9)$$

3.4 Solution for the Temperature and Stress Distributions

The calculation of $\varphi(r, \tau)$ was the main problem since the dimensionless stress components can be computed from Equations (9) as soon as $\varphi(r, \tau)$ is known. To calculate $\varphi(r, \tau)$ Equation (8a) must be solved subject to the conditions (8b).

To investigate the possibility of an exact solution for some special case while retaining the important features

of the problem, thermal conductivity and specific heat were temporarily assumed to be constant so that

$$\lambda^k = 1$$

and

$$\lambda^c = 1$$

The nonlinear boundary condition still existed, however, preventing the use of product solutions or superposition. Various methods for solving for $\phi(r, \tau)$ were surveyed by Wildin (1) and a finite difference technique was finally used. A complete analysis of the technique and how it was used to calculate $\phi(r, \tau)$ is shown in Reference (1).

The finite difference equations were programmed for the CDC 1604 Digital Computer, and numerical results were obtained for the temperature distribution and thermal stresses using both constant and variable properties.

3.5 Theoretical Results

In the report by Wildin and Putman (1) numerical results were reported for both the stress and temperature distributions from the initial instant up to the value of τ at which the maximum¹ value of stress occurred for values of N_R ranging

¹The term "maximum" is employed herein to denote the greatest value of stress calculated for a given set of input parameters. Hence, it refers to the maximum with respect to both position and time.

from 0.0129 through 10. Additional calculations of the temperature distribution, but not the stress distributions, were carried out for $0 \leq \tau \leq 1$, for values of N_R ranging from 0.0129 to 5.0. Most of the calculations were made with constant thermal property values² ($\lambda^k = \lambda^c = 1$); however, calculations were carried out with various combinations of F and H to help determine the effect of variable properties on the results. The results showed that the variation of thermal conductivity and specific heat with temperature had little effect on the maximum value of stress for a given value of N_R . This is the reason why variable properties were not considered more extensively in the theoretical analysis.

A number of curves were presented by Wildin and Putman (1) to give a broader picture of the phenomenon of radiation cooling of a long, circular cylinder. Many plots were made with different values of N_R to show the effects of various values of N_R on the temperature distribution and thermal stresses. Also, some of the dimensionless results were converted into dimensional form for the prototype element.

To determine certain parameters for the experimental work, a value of N_R is required for the prototype element. As discussed in Reference (1) the maximum initial temperature used

²It will be recalled that the coefficient of linear expansion was treated as a constant as pointed out previously. Hence, $\lambda^\alpha = 1$ was used in all the calculated results.

to determine results for the prototype was 1500°F. The primary reason that higher temperatures were not used is because the prototype material acts in a ductile fashion above 1500°F. Also, the thermal properties are no longer linear functions of temperature above 1500°F. To obtain the maximum possible rate of heat transfer for this temperature, it was assumed that the body is black, i.e.,

$$\epsilon = 1$$

The equation for N_R from Reference (1) is:

$$N_R = \frac{\bar{a}\sigma\epsilon T_i^3}{K_i} \quad (10)$$

The other parameter values pertinent to the prototype material are:

$$\bar{a} = 0.625 \text{ inches}$$

$$K_i = 17 \text{ Btu/hr-ft-}^\circ\text{R}$$

$$E = 11 \times 10^6 \text{ lb/in}^2$$

$$\alpha = 4.56 \times 10^{-6} \text{ in/in/}^\circ\text{R}$$

$$\nu = 0.3$$

$$\frac{K_i}{\rho C_i} = 0.277 \text{ ft}^2/\text{hr}$$

This set of parameter values gives $N_R = 0.0395$ using an initial temperature of 1500°F. However, this is only an approximate value of N_R since the emissivity of the prototype is unknown. The use of this value of N_R in the design of the model and in

the choice of initial conditions will be presented in the chapters dealing with the experimental model and the experimental setup.

The important results for the case of $N_R = 0.0395$ are presented in Figures (1) through (10). Most of the results are presented in the nondimensional form so that dimensional values of temperature and thermal stress can be determined for either the prototype or model. However, some of the results are presented in the dimensional form for the model to aid in the experimental design.

Figure (1) shows the curves of dimensionless strain versus dimensionless radial position at the time of maximum stress ($\tau = 0.21$). The equations for determining the dimensionless strain curves from the dimensionless stresses are:

$$\begin{aligned} E_z &= \Sigma_z \\ E_r &= \frac{\Sigma_r}{1-\nu} - \frac{\nu}{1-\nu} (\Sigma_\theta + \Sigma_z) \\ E_\theta &= \frac{\Sigma_\theta}{1-\nu} - \frac{\nu}{1-\nu} (\Sigma_r + \Sigma_z) \end{aligned} \quad (11)$$

The expressions for calculating actual values of strain from the nondimensional strain data are:

$$\begin{aligned} \epsilon_z &= \alpha T_i E_z \\ \epsilon_r &= \alpha T_i E_r \\ \epsilon_\theta &= \alpha T_i E_\theta \end{aligned} \quad (12)$$

FIGURE 1

NORMAL DIMENSIONLESS STRAIN DISTRIBUTIONS AT TIME
OF MAXIMUM STRESS

Since αT_i is a constant, the strain is directly proportional to the dimensionless strain as shown in Equations (12). Therefore, the dimensionless strain curves, Figure (1), can be used as a guide for the positioning of the strain gages in the model. The gages must be embedded in the model at the radial positions where the largest magnitudes for all the components of strain can be obtained. This will insure greater accuracy for the thermal stresses since errors in the strain readings will be less significant percentage wise. The selection of the radial positions for the location of the strain gages in the model is discussed in Chapter 4.

The values of strain for the radial positions where strain gages are to be embedded in the model also can be determined from Figure (1) using Equations (12). The magnitudes of the strains will be used as a guide in the selection of recording equipment for the experiment as discussed in Chapter 5.

Another important feature to note in Figure (1) is the magnitude of the radial and circumferential strains at the center of the model. The small magnitude of these strains will play an important part in the embedding of the strain gages since they appear to be insignificant compared to the magnitude of the axial strain.

The dimensionless principal thermal stresses are shown in Figure (2) as a function of radial position at the time of maximum stress ($\tau = 0.21$). The principal dimensional stresses are

FIGURE 2

NORMAL DIMENSIONLESS STRESS DISTRIBUTIONS
AT APPROXIMATE INSTANT OF MAXIMUM STRESS

directly proportional to the principal dimensionless stresses. Therefore, according to Figure (2), the maximum dimensional stresses are the axial and circumferential stress components at the surface of the model which have the same magnitude. The radial component of stress vanishes at the surface since there are no radial constraints applied. It should be noted that the sum of the radial and circumferential components of stress equals the axial stress as shown in Equations (9), which means the radial stress must vanish at the surface since the axial stress equals the circumferential stress. It should also be noted that the stresses are compressive at the center of the cylinder and tensile at the surface.

The next five figures (3 through 7) show the dimensionless stress as a function of dimensionless time at the radial positions in the cylinder where gages are to be embedded. It can be seen that the stresses at the outside surface of the cylinder reach a maximum before the stresses at the center by looking at Figures (3) and (7).

The expressions for determining the dimensional stresses from the dimensionless stresses are:

$$\sigma_{r,\theta,z} = \frac{\alpha E T_i}{(1-\nu)} \Sigma_{r,\theta,z} \quad (13)$$

where r denotes the radial component of stress, θ the circumferential component, and z the axial component. Using the stresses determined from these expressions, the experimentally

FIGURE 3

VARIATION OF DIMENSIONLESS STRESS WITH DIMENSIONLESS TIME AT THE CENTER

FIGURE 4

VARIATION OF DIMENSIONLESS STRESS WITH DIMENSIONLESS
 TIME AT .583 OF THE OUTER RADIUS

FIGURE 5

VARIATION OF DIMENSIONLESS STRESS WITH DIMENSIONLESS
TIME AT .583 OF THE OUTER RADIUS

FIGURE 6

VARIATION OF DIMENSIONLESS STRESS WITH DIMENSIONLESS
TIME AT NINE-TENTHS OF THE OUTER RADIUS

FIGURE 7

VARIATION OF DIMENSIONLESS STRESS WITH DIMENSIONLESS TIME AT .98 OF THE OUTER RADIUS

determined stresses can be compared to the theoretical results. Figure (8) shows the dimensional stress distributions for the model at approximate instant of maximum stress.

When the experimental work is carried out, the stresses must be determined from the strain data recorded during the tests. Therefore, using the equations for the thermal strains presented in Timoshenko (14), the following expressions were derived for the experimental thermal stresses in a three-dimensional stress field:

$$\begin{aligned}\sigma_r &= \frac{E}{(1-2\nu)(1+\nu)} \left[\epsilon_r(1-\nu) + \nu(\epsilon_\theta + \epsilon_z) \right] \\ \sigma_\theta &= \frac{E}{(1-2\nu)(1+\nu)} \left[\epsilon_\theta(1-\nu) + \nu(\epsilon_r + \epsilon_z) \right] \\ \sigma_z &= \frac{E}{(1-2\nu)(1+\nu)} \left[\epsilon_z(1-\nu) + \nu(\epsilon_r + \epsilon_\theta) \right]\end{aligned}\tag{14}$$

At the surface of the model, a biaxial stress field exists since the radial stress component is zero. To determine the experimental stresses in a biaxial stress field, the equations are:

$$\begin{aligned}\sigma_\theta &= \frac{E}{1-\nu^2} (\epsilon_\theta + \nu\epsilon_z) \\ \sigma_z &= \frac{E}{1-\nu^2} (\epsilon_z + \nu\epsilon_\theta)\end{aligned}\tag{15}$$

The following two figures show the dimensionless temperature curves which were plotted from the theoretical results.

FIGURE 8

DIMENSIONAL STRESS DISTRIBUTIONS AT APPROXIMATE
 INSTANT OF MAXIMUM STRESS FOR THE MODEL

Figure (9) shows the ratio of temperature to the initial temperature as a function of radial position at the time of maximum principal stress ($\tau = 0.21$). From this curve the largest and smallest temperature changes up to the time of maximum stress can be determined. These values will be required to determine the type of equipment to be used for recording temperatures during the test.

Figure (10) presents the ratio of temperature to initial temperature as a function of dimensionless time for various positions in the model. These curves show how the temperature varies at the points in the model where strain gages and thermocouples are embedded. It should be noted that the temperatures dropped at different rates early in the test; i.e., more rapidly at the surface than at the center of the cylinder, but after approximately $\tau = 0.15$ the temperatures dropped at the same rate at all radial positions.

To determine the dimensional time for the prototype or model, the following expression is used:

$$t = \frac{\frac{a^2}{K_i}}{\left(\frac{1}{\rho C_i}\right)} \tau \quad (16)$$

The dimensional time required for a test is important in the selection of recording equipment. If the signal from a measuring device, a strain gage for example, was varying rapidly during the test, then recording equipment having a fast response time would be needed.

FIGURE 9

VARIATION OF DIMENSIONLESS TEMPERATURE
WITH DIMENSIONLESS RADIUS AT TIME
OF MAXIMUM STRESS

FIGURE 10

VARIATION OF DIMENSIONLESS TEMPERATURE
WITH DIMENSIONLESS TIME

4.0 EXPERIMENTAL MODEL

4.1 Model Selection and Construction

The prototype element which is made of hydrided uranium-zirconium was not acceptable for the experimental work since it presented formidable safety and security problems. Also, temperature and strain could be measured only on the surface of the prototype due to the small diameter of the cylinder and the brittleness of the material which would make the embedding of strain gages and thermocouples very difficult. Therefore, the process of modeling the prototype was selected for this investigation so a material could be used in which strain gages and thermocouples could be embedded. It is important to note how the assumptions established in the theory affected the design and instrumentation of the experimental model.

The material used for the model was an aluminum-filled epoxy having the same composition as models used by Cone (15), Baker and Dove (16), and Thunborg (3). The mechanical and thermal properties of the model, and details on construction are discussed in Appendix B. The material selected has been used successfully in the Mechanical Engineering Department of the University of New Mexico for problems requiring the use of embedded strain gages and thermocouples.

The model material was assumed to be homogeneous and isotropic. In the construction of the model, the mold containing the liquid material was placed in a vacuum to draw out the air

bubbles. This process was used to obtain a smooth model which satisfied the assumptions of homogeneity and isotropy.

The model material was assumed to have a high absorption coefficient so that radiation was emitted only from the surface of the cylinder.

The emissivity and reflectivity at the surface of the model were assumed to be uniform. To help satisfy this condition, the surface of the model was smoothed to a uniform finish.

The specific heat, thermal conductivity, and coefficient of linear expansion of the model material were assumed to be weak functions of temperature in the temperature range of interest as discussed in the theoretical analysis. This appeared to be a good assumption according to the available data (6) on these thermal properties.

To determine the radius for the model and the initial temperature for the experimental work, Equation (10) was used. The value calculated for N_R using the prototype parameters corresponding to a temperature of 1500°F was 0.0395 as shown in Chapter 3. For this case the emissivity of the prototype was maximized to obtain the largest value of N_R so that the largest possible value of initial temperature could be calculated for the experimental work. This value of N_R can be used in Equation (10) for the model study since N_R for the prototype and model must be equal according to Equation (5). With N_R known and the thermal properties of the model assumed constant, $\bar{a}$ and T_i can be calculated for the model. The value of thermal conductivity

used for this calculation was $0.52 \text{ Btu/hr-ft-F}^\circ$ from a report by Matthews(6). An emissivity of 0.45 was selected for this computation although there was no information available on the emissivity at the time of this analysis. The initial temperature for the model was calculated to be 138°F using a 3-inch diameter cylinder. This computation can be seen in Appendix A. The 3-inch diameter was selected because the space requirements in the experimental apparatus restricted the diameter of the model that could be used. This problem will be more fully discussed in Chapter 5. Also, the diameter of the model could not be smaller than 3 inches since space was required on the radial cross section to embed strain gages, as will be shown later in this chapter. The initial temperature of 138° was in the temperature range where information was known about the thermal properties of the epoxy material (6).

The model was constructed with a length to diameter ratio of 5 to 1. Since the diameter was 3 inches the model was made 15 inches long. This was done to ensure a constant stress distribution for a short length on either side of the axial center cross section of the model. The stresses will vary axially near the ends of a cylinder, but this axial variation should diminish at one diameter from the ends of the cylinder.

4.2 Model Instrumentation

The instrumentation of the model with thermocouples and strain gages was to be completed in two phases. In the first

phase, thermocouples were to be embedded in the model. The model was then to be mounted in the test apparatus by the method to be discussed in Chapter 5, and experiments were to be conducted to determine the temperature distribution in the model. These experiments would also serve to completely check out the operation of the experimental apparatus which includes the heat sink, heater and shields, and the vacuum equipment. This equipment is discussed in Chapter 5.

In the second phase, the model was to be removed from the test apparatus and strain gages embedded in the model. The model, instrumented with both thermocouples and strain gages, would then be remounted in the test setup; and another series of tests would be run to determine the thermal stresses in the model.

For the first phase, the model was sectioned into two cylindrical pieces 7 1/2-inches long to prepare for instrumentation by the thermocouples. The thermocouples were then mounted on the machined surface at the end of one of the pieces, Figure (12).

Copper-constantan thermocouples made of 36 gage wire were selected for the experiment since copper wire and carefully selected constantan wire are more homogeneous than most of the other wire materials (8). This is important since inhomogeneities in the wire produce spurious emfs when the wire traverses areas of varying temperature. Copper-constantan thermocouples are also good for low temperature applications. The small wire

diameter was selected to reduce the possibility of re-enforcing the model material when the wires were embedded. Also, the small wire was used to reduce the amount of heat that could be conducted along the wire. However, as a result of the work done in Chapter 6, the heat conducted along the wires was found to be significant where the lead wires were not brought out of the model along the same isotherm that the thermocouple bead was mounted on. Selected 24 gage copper and constantan wire was used for the lead wires from the model to the recording equipment.

The thermocouples were located in the model at the same radial positions selected to mount the strain gages for a purpose to be discussed later in this chapter. The radial positions for mounting the strain gages were determined from the maximum strain distribution curves shown in Figure (1); and the positions selected were: the radial center of the model ($\bar{r}/a = 0$), a point just beneath the outer surface of the model ($\bar{r}/a = 0.98$), and two intermediate points ($\bar{r}/a = 0.583$ and $\bar{r}/a = 0.90$).

Six thermocouples were mounted at thermocouple mounting section (A) shown in Figure (11). Thermocouples (1) through (4) were located at the radial positions shown in Figure (12). Thermocouples (5) and (6) were located at the same radial positions as thermocouples (2) and (3), but on the other side of the model to check circumferential symmetry of the temperature in the model. Thermocouples (7), (8), and (9) were located at thermocouple mounting sections (B), (C), and (D), respectively

FIG.II, EXPERIMENTAL MODEL.

FIG.12, MULTIPLE THERMOCOUPLE SECTION (A).

FIG.13, SINGLE THERMOCOUPLE SECTIONS (B), (C), AND (D).

as shown in Figure (11). At each of these sections (B), (C), and (D), one thermocouple was mounted 0.03 inches below the surface, Figure (13). Thermocouples (7), (8), and (9) were used to check the assumption of uniform temperature in the axial direction in the model.

The method for embedding the thermocouple bead was to drill a small hole slightly larger than the bead at the position where it was to be mounted. The bead was placed in the hole, and a small amount of liquid model material was poured on top of the bead. The material was allowed to dry, and the model was placed in the oven for 24 hours at 200°F to cure the material. This same procedure was used for all the thermocouples. The lead wires for the six thermocouples mounted at section (A) were brought out of the model according to the pattern shown in Figure (14). Several of the pairs of leads were brought along isotherms in the circumferential direction for some distance from the bead before bringing them radially out of the model, Figure (14). The reason was to move the position where the leads leave the model away from the thermocouple bead. This was done to reduce the effects on the thermocouple bead caused by heat flowing along the leads where they leave the model. The center thermocouple leads, however, were brought straight out of the model, so there may be some error introduced into the readings from this thermocouple due to heat flowing down the wires. The lead wires were brought to the surface of the model two inches axially from thermocouple mounting section (A) along the radially cut

SECTION A-A

FIG. 14, METHOD OF MOUNTING THERMOCOUPLES IN THE MODEL.

grooves shown in Figure (14). The grooves were filled with epoxy after the leads were in place. The epoxy was then allowed to harden and was cured for 24 hours at 200°F.

After the lead wires were brought to the surface, the two 7 1/2-inch cylinders were rejoined with epoxy model material; and the air bubbles were removed from the material by forcing them out with heat before the epoxy hardened. Once the two parts were rejoined, there was no way to be absolutely sure that all the bubbles were out of the epoxy material used to rejoin the parts. After the epoxy hardened and was cured, the entire model was polished on the lathe to attain a high finish. Care was taken to prevent breakage of the lead wires protruding from the surface of the cylinder. The lead wires were then brought axially along the surface of the model to the end of the model. To attach the leads to the surface of the model, small drops of liquid epoxy were placed over the wire and allowed to harden. At the end of the model, the lead wires were attached to small tabs. Larger lead wires were attached to the tabs to carry the signal from the thermocouple to the recorder. This completed the thermocouple instrumentation of the model.

In the second phase the plan was to make another radial cut in the model as close to the center section where the thermocouples were mounted as possible. Strain gages then would be mounted on the machined surface of the 7 1/2-inch cylinder which did not contain the thermocouples. The strain gages would be mounted at the same radial positions as thermocouples

(1) through (4) shown in Figure (12). At position (1), a biaxial stress field, Equations (15), was assumed since this position was close to the surface of the model. The radial stress was assumed to be zero at this position since this position is very close to the surface, and the radial stress is zero at the surface. This means that one strain gage would be mounted in the circumferential direction and one gage in the axial direction. It would be impossible to mount a strain gage in the radial direction at this position. Since the radial stress at position (1) had a finite value, an analysis was made to determine the error introduced into the axial and circumferential stresses by assuming this radial stress to be zero. The maximum error in these stresses was 0.7 percent, and therefore the assumption of a biaxial stress field was a good one.

At positions (2) and (3), a three-dimensional stress field exists; and, therefore, one strain gage would have to be mounted on each of the principal axes at both positions to determine the stresses. At position (4) the predicted radial and circumferential strains were very small compared to the axial strain as shown in Figure (1). A calculation was made to determine the error which would be introduced into the stresses if the strains in the radial and circumferential directions were assumed to be zero. The maximum error in the stresses was 2 percent. Therefore, at position (4) only one strain gage would be mounted in the axial direction.

Two strain gages were to be mounted axially along the length of the model at 0.03 inches below the surface. These gages would be mounted at the same axial positions as thermocouples (7) and (8) shown in Figure (11). The purpose of these gages would be to check the assumption of a uniform stress distribution along the length of the model. This assumption excludes the ends of the model up to at least one diameter from each end.

A three-lead wire system was selected to be used with each strain gage. The reason for using a three-lead wire system was to eliminate the error introduced into the strain readings due to changes in resistance in the lead wires caused by changes in temperature along the wires. A discussion of the three-lead wire system for strain gages can be seen in the book by Dove (4).

For the particular situation under investigation, an acceptable strain gage has not been found. The problem is to find a strain gage which is temperature compensated for $22.5 \text{ in/in/}^{\circ}\text{F}$ and which can be embedded in the model without re-enforcing the epoxy material.

The reason the temperature compensation problem is important can be seen by noting the small magnitude of the strains shown in Figure (15).

To compensate for the difference in coefficient of thermal expansion between the strain gage and the epoxy material in a test of this kind, a free expansion strain curve must be plotted,

FIGURE 15

DIMENSIONAL STRAIN DISTRIBUTIONS AT THE TIME
OF MAXIMUM STRESS FOR THE MODEL

as discussed by Dove (4). A strain gage of the type to be used in the actual testing is embedded in a block of the same epoxy material used for the experimental model, and a thermocouple is embedded next to the strain gage. The block is then subjected to the same temperature range that will be experienced by the model. The block is stabilized at various temperatures in the range of interest, and the strain and temperature readings are recorded. With this data a curve of strain versus temperature for the free expansion case is plotted.

The strain data is then recorded for the actual tests and also the temperatures by a thermocouple mounted at the same radial position as each strain gage. To determine the true thermal strain, the value of strain determined from the free expansion case must be subtracted from the strain readings determined from the actual test runs for a given temperature.

If the difference between the coefficient of thermal expansion of the model and the strain gage material is large, then the correction factor from the free expansion curve will be correspondingly large. Since the thermal strains will be small in the tests, the correction factor could be much larger than the thermal strains, thus diminishing the resolution of the method. This could seriously affect the results for the thermal strains. Therefore, it is imperative that a strain gage compensated for the coefficient of thermal expansion of the model material be found.

The best strain gage found to date which can be embedded in the model without seriously re-enforcing the epoxy material is an FAB 12-12-S-15 strain gage manufactured by Baldwin-Lima-Hamilton. This gage, which has been used by several experimenters (3, 6, 15), is temperature compensated for a coefficient of thermal expansion of 15×10^{-6} in/in/ $^{\circ}$ F.

Another gage which has been tested for embedding purposes is a C-18-121 gage manufactured by Tatnall Measuring System Company. This gage is compensated for a coefficient of thermal expansion of 18×10^{-6} in/in/ $^{\circ}$ F. However, the top side of the gage will not bond to the epoxy material when the gage is embedded as discussed by Baker (6). To see if this problem could be resolved a tensile specimen was sectioned at the center, and a C-18-121 gage was mounted on the cut surface. The top of the strain gage was then coated with Eastman 910 adhesive, and the gage was embedded by rejoining the two halves of the test specimen. The top of the gage failed to bond to the epoxy material with Eastman 910 adhesive between the top of the gage and the epoxy material. Therefore, this gage is still not usable for embedding in epoxy at the present time where a three dimensional stress field is involved.

An effort is still being made to find a properly compensated gage which is thin enough to keep from re-enforcing the model when embedded. The ideal gage would be an encapsulated gage which is temperature compensated for 22.5×10^{-6} in/in/ $^{\circ}$ F.

However, until a better temperature compensated strain gage is found, the accuracy of the strain results is doubtful.

4.3 Thermocouple Calibration

A calibration was conducted on six of the copper-constantan thermocouples to determine any deviations of the indicated thermocouple temperatures from the temperatures on the International Temperature Scale. This was accomplished by determining the emf output of each thermocouple at two fixed temperature calibration points.

The two fixed points used were the melting point of ice (32°F) and the melting point of benzoic acid (252.3°F) taken from Reference (17). The apparatus used for the calibration is shown in Figure (16).

The reference junction was an ice bath filled with ice made of distilled water. Distilled water ice was used since all the impurities are removed, and it will hold the reference junction closer to 32° than tap water ice. Baker, Ryder, and Baker (18) recommended that all junctions between two dissimilar metals in the circuit be made by fusing the metals. Therefore, the reference junctions were made by fusing the metals in a mercury bath covered with oil (18). This was done to avoid inhomogeneities which might result from using solder or other brazing materials in the junction. The ice bath reference junction is similar to the one shown on page 60 in Baker (18).

- A. VARIABLE TRANSFORMER
- B. LEEDS AND NORTHRUP PORTABLE
PRECISION POTENTIOMETERS (2)
- C. SWITCHING BOARD
- D. ICE BATH REFERENCE JUNCTION
BOTTLES (2)
- E. HEATER FOR CALIBRATION WITH
BENZOIC ACID

FIG.16 THERMOCOUPLE CALIBRATION
APPARATUS

A Leeds and Northrup Portable Potentiometer was used to record the thermocouple signals through a switching board.

For the ice point calibration, the thermocouples were placed in an ice bath similar to the reference junction ice bath. All six of the thermocouples showed a zero emf on the potentiometer.

The calibration of the thermocouples at the melting point of benzoic acid was done with a unique apparatus. A heater was build which consisted of a pyrex test tube surrounded by a coil of nichrome wire and embedded in a sauerisen mold, Figure (16). The sauerisen is used to contain the heat generated in the nichrome coil. Power for the nichrome coil is supplied by a variable transformer. For a calibration run, the powdered benzoic acid was placed in the test tube. Then a thermocouple encased in a very small tube, was placed in the powdered benzoic acid. The heat was applied, and temperatures were recorded on the potentiometer. The calibration point for each thermocouple was determined when the temperature stopped climbing and leveled out for a short length of time as shown in Table 1.

There was a deviation from the International Scale for all six of the thermocouples. Most of the thermocouples read about 0.7° higher than the melting point of benzoic acid, as shown in Table 2. However, one of the thermocouples read 1° higher, and one read $.8^{\circ}$ lower than the melting temperature of benzoic acid. Therefore, the maximum deviation of the thermocouples was 1.8° .

TABLE 1

THERMOCOUPLE CALIBRATION DATA*

TIME (Seconds)	Thermocouples (Millivolts)					
	1	2	3	4	5	6
0	5.02	5.20	5.02	5.16	5.32	5.30
10	5.08	5.24	5.06	5.18	5.33	5.30
20	5.16	5.26	5.12	5.20	5.34	5.30
30	5.22	5.28	5.20	5.22	5.36	5.30
40	5.26	5.32	5.28	5.26	5.36	5.32
50	5.28	4.32	5.34	5.30	5.36	5.34
60	5.30	5.32	5.36	5.31	5.36	5.35
70	5.32	5.32	5.36	5.33	5.36	5.36
80	5.34	5.34	5.36	5.34	5.36	5.36
90	5.36	5.36	5.42	5.36	5.36	5.36
100	5.36	5.40	5.50	5.36	5.36	5.36
110	5.37	5.44		5.36	5.37	5.36
120	5.37			5.36	5.40	5.36
130	5.37			5.36	5.41	5.36
140	5.37			5.36	5.42	5.36
150	5.37			5.36	5.43	5.36
160	5.37			5.36	5.44	5.36
170	5.37			5.36		5.36
180	5.37			5.38		5.36
190	5.37			5.40		5.36
200	5.38					5.36
210	5.38					5.38
220	5.39					5.40
230	5.40					5.44
240	5.41					5.46
250						5.48

* The melting point of benzoic acid is 252.3°F or 5.342 millivolts.

TABLE 2

THERMOCOUPLE CALIBRATION RESULTS

Thermo- Couple Number	Temperature at Calibration Point of Benzoic Acid (°F)	Deviation From Melting Point of Benzoic Acid* (°F)	Temperature at Calibration Point of Ice (°F)	Deviation From Actual Freezing Point of Ice (°F)
1	253.3	1.0	32	0
2	251.5	- .8	32	0
3	253.0	.7	32	0
4	253.0	.7	32	0
5	253.0	.7	32	0
6	253.0	.7	32	0

* Melting point of benzoic acid is 252.3 °F.

At operating temperatures for the tests with the experimental model (125° to 150°F), the temperature variation for the thermocouples would be about 0.5° assuming that there is a linear variation between the zero variation at the ice point and the 1.8° variation at the melting point of benzoic acid.

5.0 EXPERIMENTAL SETUP

5.1 Mechanical Equipment

The mechanical equipment used for this study included a vacuum apparatus; a liquid-nitrogen-cooled heat sink; a collapsible, cylindrical heater surrounded by two collapsible, cylindrical radiation shields; the framework to support the model; and two mylar-coated, chromium radiation shields. It should be noted that the design and construction of this equipment was governed by the various assumptions and other conditions which were defined in the theoretical analysis.

To simulate an environment where radiative heat transfer was the dominant means of transfer of heat from the model, a vacuum apparatus which would develop pressures in the range of 10^{-4} to 10^{-6} millimeters of mercury, Figures (17) and (18), was obtained on loan from Sandia Corporation.

A Welch mechanical pump rated at 13 cfm and driven by a 3/4 horsepower motor, and a CVC 4-inch mercury diffusion pump provided pumping for the vacuum system. To measure the pressures in the vacuum system, a thermocouple vacuum gage was provided for low vacuum readings and a hot-filament ionization gage for high vacuum readings. For prevention of back flow between the working chamber and the diffusion pump, two cooling systems were provided. One was a freon cold trap; and the other, a large, dry ice or liquid nitrogen cold trap. They

- A. VACUUM EQUIPMENT
- B. BELL JAR
- C. COLD TRAP (DRY ICE OR LIQUID NITROGEN)
- D. HEAT SINK AND SUPPORT FRAME
- E. INSULATED TUBE (CONNECTS 110 LITER BOTTLE
OF LIQUID NITROGEN TO HEAT SINK)

FIG. 17, VACUUM SYSTEM

FIG.18, VACUUM SYSTEM FLOW DIAGRAM

were both located between the working chamber and the diffusion pump. The system also included several large and small valves and various tubing and connections. The entire apparatus was mounted in a console, Figure (17).

A steel base plate mounted on top of the console was connected to a vacuum system, and a bell jar resting on the base plate provided the working chamber. The mechanical equipment associated with the experiment was mounted either on top of the base plate or through the base plate.

To simulate the prototype surroundings, a cylindrical heat sink concentric with the experimental model was constructed.

The model surroundings for the experiment were required to have the characteristics of a black body; i.e., an emissivity of one. To satisfy this condition, a thin sheet of copper 20-inches wide, 70-inches long, and 1/16-inch thick was bent into a corrugated pattern and shaped into a cylinder as shown in Figure (19). The diameter of the cylinder was fixed by space requirements inside the bell jar. The purpose of the corrugation was to increase the emissivity of the inside wall of the heat sink. The corrugated pattern and black paint that was added later to the inside wall of the heat sink combined to approximate an emissivity of one. The paint is a commercial product called Cat-a-lac flat black paint³ which is good for cryogenics work and should produce an emissivity of 0.97 according to the

³Manufactured by Finch Paint and Chemical Company

SCALE: 1" = 4"

FIG. 19

LIQUID NITROGEN COOLED HEAT SINK.

manufacturer. The value of 0.97 is a lower limit for all wave lengths.

The surroundings to which the model radiates were assumed to be at 0°K to maximize the heat transfer rate from the surface of the model. If the temperature of the model is at 300°K (room temperature), then the heat sink must be at 90°K or lower since the fourth power of 90°K is small compared to the fourth power of 300°K . Therefore, the temperature of the heat sink, if at or below 90°K , can be assumed to be 0°K for the purposes of simulation. Hence, a method for cooling the heat sink to 90°K or lower was required.

The method selected for cooling the heat sink was to build a reservoir of some sort around the outside of the heat sink. The reservoir would be required to carry a liquid which would provide temperatures of 90°K or lower. The reservoir would be filled from the bottom under pressure and vented at the top. The following setup was developed to satisfy this requirement.

To provide a reservoir for the coolant, a 1/4-inch copper tube was placed in each groove on the outside surface of the heat sink, Figure (19), and was soldered in place with silver solder. At each end of the heat sink a flow channel was required to carry the coolant to the copper tubes. This was accomplished by machining a 3/16-inch deep groove 1/2-inch wide on the same radius as the copper tubes in an 11-inch diameter beryllium-bronze flange 3/8-inch thick, Figure (20).

FIG. 20, HEAT SINK FLANGE ASSEMBLY (TYPICAL).

The tubes were brought through the flange and into the groove where they were soldered in place with silver solder. The flange was heated uniformly to a high temperature before the silver solder would flow between the flange and the copper tubes. The high temperatures caused slight warping in the flange. A similar groove was machined in another 11-inch diameter beryllium-bronze flange. This flange was bolted to the flange attached to the copper tubes as shown in Figure (20) so the grooves in each of the flanges fitted together to form the flow channel. To prevent the coolant from leaking out of the flow channel, O-ring seals were required between the flanges. Copper O-rings were used with thin sheets of mylar between the O-ring and the flanges as shown in Figure (20). The flanges were machined on the lathe to remove any warping before bolting them together. Figure (21) shows the dimensions of the flange.

The corrugated grooves in which the copper tubes are located were filled with soft solder to provide better heat transfer between the copper tubes and the corrugated copper sheet.

The heat sink was mounted vertically two inches above the base plate on hollow epoxy legs. Epoxy was used for the legs since it has a low thermal conductivity and would make the heat transfer between the base plate and the heat sink as small as possible. Figure (22) shows the heat sink mounted on the base plate.

The coolant used for the heat sink is liquid nitrogen, which has a boiling point of 77°K . The temperature on the

HALF SCALE

FIG. 21, HEAT SINK FLANGE DIMENSIONS.

FIG. 22, HEAT SINK MOUNTED ON THE BASE PLATE

inside surface of the heat sink was measured by two thermocouples, one mounted at the top and one at the bottom of the heat sink on the corrugated surface. Any difference between the boiling temperature of the liquid nitrogen and the inside wall of the heat sink was due to heat transfer through the copper wall of the heat sink by conduction. Some temperature difference might have been due to boiled-off vapor in the copper tubes.

The liquid nitrogen was stored under 25 pounds of pressure in a 110 liter bottle. This liquid nitrogen was fed through an insulated tube to the base plate, through the base plate in a vacuum-jacketed tube, and into the bottom of the heat sink. Liquid filled the lower flow channel, flowed through the copper tubes, and filled the upper flow channel. This system constituted the reservoir around the heat sink. To vent the boiled-off vapors from the top of the reservoir, a stainless steel tube carried the vapors to the outside through another vacuum-jacketed tube in the base plate. To maintain the liquid level in the reservoir, the rate of flow of liquid nitrogen into the bottom of the heat sink was controlled by a valve on the storage bottle.

To suspend the model inside the heat sink, a support frame was constructed as shown in Figure (23). The aluminum frame having four legs was mounted on the base plate and was designed to stand above the heat sink. The four legs were connected to an aluminum ring centered above the heat sink. The model was lowered through the ring and attached by four wires to a metal cross bar which rests on the aluminum ring. A single

FIG. 23, SUPPORT FRAME AND HEATER PULLEY SYSTEM.

wire in the bottom of the model was attached to the base plate to help align the model in the center of the heat sink. The aluminum support frame was also used to support a collapsible heater and shields.

In the theoretical analysis, the model was initially assumed to be at a uniform temperature throughout. To satisfy this requirement experimentally, a cylindrical heater which would heat the model uniformly was built. Since this was a thermal shock study, the heater had to be pulled away from the uniformly heated model to expose the model to the heat sink. Space requirements in the working chamber made it necessary to pull the heater above the model. However, the space above the model was such that a heater the length of the model could not be removed from the model unless the heater was collapsible.

A collapsible heater was developed which consisted of a coil of nichrome wire wrapped with fiberglass cloth and aluminum foil as shown in Figure (24).

To build the heater, a 4 1/2-inch diameter steel mandrel was used. The mandrel was split down both sides for easy removal after the heater was assembled.

The mandrel was mounted on the lathe, and 18-gage nichrome wire was fed onto the mandrel at the rate of two turns per inch. Fiberglass cloth was wrapped tightly over the nichrome wire. Around the fiberglass cloth a double layer of heavy aluminum foil was wrapped. The nichrome wires were sewn to

FIG. 24, CROSS SECTION OF CYLINDRICAL HEATER
AND RADIATION SHIELDS.

the fiberglass cloth to keep the wires in their respective grooves.

The mandrel was removed, and the foil and fiberglass cloth were crimped between the wires to make the heater collapsible. Rings made of asbestos pipe were attached to each end of the heater for support, and sauerisen cement was used to bond the ends of the heater to the asbestos rings.

To shield the heater from the heat sink while the model was being heated to an initially uniform temperature, two collapsible, cylindrical radiation shields were placed around the outside of the heater. Figure (24) shows a cross section of the heater and shields. They were connected with four blocks or spacers on each end of the cylinder by the method shown in Figure (24). These blocks were made of epoxy so the heat transfer between the heater and the shields would be as small as possible. The radiation shields, each consisting of a coil of wire wrapped with aluminum foil, were cut from a long piece of ducting material called Thermaflex A. The two characteristics that made this ducting useful were the shiny surfaces on both sides and the fact that it was collapsible.

To pull the heater and shields above the model, a pulley system was developed. Another system was developed to pull the heater and shields back down over the model. Since the whole test was to take place inside a vacuum system, a remote method of mechanically raising and lowering the heater and

shields was required. Figure (23) shows the pulley system that was developed to pull the heater and shields above the model. A drive shaft was mounted in the base plate and attached under the base plate to a hand crank through bevel gears. On top of the base plate the drive shaft was connected to a set of pulleys through another set of bevel gears. Pulleys were mounted on the support frame as shown in Figure (23) to carry the wire cables over the top of the support frame. From the top of the support frame, the wires dropped down inside the heat sink and were attached to the heater and shields.

A similar set of pulleys, connected through the base plate to a hand crank, was mounted on the other side of the base plate behind the two pulleys shown in the figure. These two pulleys were used to pull the heater and shields down over the model and were connected to them with cables which passed through two small pulleys mounted on the base plate directly below the heater and shields.

When the heater and shields were pulled upward, they contacted the support frame and collapsed into half their stretched length. When pulled down, they stretch out to full length, thus covering the model. Two cables were connected to the top of the heater and shields to suspend them when pulled down. Two wires were attached to the support frame and the base plate to guide the heater and shields when they were moved up or down.

Two radiation shields were located between the heat sink and the bell jar. These highly reflective shields made of

chromium foil coated with mylar were used to shield the heat sink from the bell jar and surroundings and thereby reduce the heat transferred to the heat sink from the surroundings.

A system was required to pass the thermocouple lead wires through the base plate without introducing leaks in the vacuum system. A thermocouple lead-through was developed for this purpose, Figure (25). This system consisted of a copper tube which was mounted in the base plate at one end and was connected to a copper cup filled with apiezon wax at the other end. The thermocouple lead wires were brought through the copper tube and out of the vacuum system through the apiezon wax as shown in Figure (25). This type of lead-through was selected since the number of thermocouples that could be used inside the vacuum system was limited only by the size of the copper tube used to carry the thermocouples out of the vacuum system. An important advantage of this system was the fact that extra thermocouple lead wires could be inserted in the lead-through after it had been sealed. This could be done by heating the apiezon wax in the copper cup and removing the spun aluminum cap. The added leads would then be inserted in the copper tube and pressed into the hot wax. Another important advantage of this system was the fact that the thermocouple leads could be passed through the base plate without being cut, which would reduce the chances of introducing spurious emf's into the leads.

Before sealing the lead wires in the wax, they were stripped of their insulation for 1/8-inch to 1/4-inch to prevent air

FIG. 25, THERMOCOUPLE LEAD—THROUGH.

leakage between the wire and the insulation. It was also important to electrically insulate the bottom of the copper cup and the rim of the aluminum cap to protect the bared thermocouple leads from shorting. This was accomplished with pieces of thin mylar sheet placed on the bottom of the copper cup and on the rim of the aluminum cap.

To carry the strain gage signals out of the vacuum system and also the power leads for the collapsible heater, two electrical lead-throughs were obtained from Consolidated Vacuum Corporation. The ceramic header mounted in each lead-through contained 14 pins, which meant there was a total of 28 channels available. Two of the pins were required for the heater, which left enough pins for eight strain gages since each gage required three leads.

The power for the collapsible heater was supplied by a variable transformer.

5.2 Recording Equipment

Equipment was required to record both the temperature and the strains. Special emphasis was placed on the magnitude of the theoretically determined temperatures and strains and their use in the selection of the recording equipment.

To select the recording equipment, the temperature changes experienced by the model during the test were required. Figure (10) shows the variations of temperature with time for the theoretical results. At $\tau = 0.4$ the maximum temperature

change was 24°F or 0.521 millivolts as calculated in Appendix A. The minimum temperature change would be 0°F . Therefore, a recorder is required which will record readings in the range of 0 to 0.521 millivolts accurately.

Using Equation (16), the dimensional time required for the strain to reach a maximum in the model can be determined. The dimensionless time where the maximum stresses occurred was $\tau = 0.21$, Figure (7). Using this value, the dimensional time was calculated to be 11.4 minutes as shown in Appendix A. It was decided that since it took considerable time to reach maximum stress, a single-channel recording instrument with a switching circuit could be used to measure the temperatures.

The recording device that was selected was a Honeywell Elektronik 15 Recorder. This recorder is a single-channel ink recorder having a 10-inch-wide tape with a full-scale deflection of one millivolt, which is sufficient to handle the signals from all the thermocouples in the model. A switching board capable of handling up to ten thermocouples was built to connect the thermocouples to the recording equipment. When the tests are conducted, in order to zero the Honeywell recorder at an initial temperature in the model, a Leeds and Northrup Portable Precision Potentiometer will be connected between the switching board and the Honeywell recorder. Therefore, with the ink recorder zeroed at the initial temperature, only changes in temperature would be recorded on the tape during the test.

An ice reference bath using ice made from distilled water will be used for the reference junction.

To monitor the temperatures at the top and bottom of the heat sink, another Leeds and Northrup Portable Potentiometer will be used.

To select recording equipment for the strain gage signals, the strain values from the theoretical results were required, Figure (15). The maximum strains from Figure (15) are the axial and circumferential strains at the surface of the model which have a value of $121 \mu\text{in/in}$. A calculation of the maximum strain is shown in Appendix A. The minimum strain will be $0 \mu\text{in/in}$. Therefore, the range of strains to be measured is from 0 to $121 \mu\text{in/in}$.

To eliminate the problems inherent in switching devices, an 18-channel oscillograph (CEC Recorder) was selected to record the strains. The galvanometers in this recorder are not sensitive enough to record the small signals from the strain gages in this test, and therefore an AC coupled amplifier system, Reference (4), was acquired. The system consists of 12 type C-3 amplifiers and is manufactured by William Miller Corporation. A bridge circuit board with 10 wheatstone bridge circuits is placed in the system between the strain gages and the C-3 amplifiers. The AC power supply for the wheatstone bridges is contained in the same console with the C-3 amplifiers.

Using the bridge circuit, amplifier system, and oscillograph, the small strain signals can be recorded. At the present time there are 10 available channels for strain gages since the bridge circuit board contains only 10 wheatstone bridges.

A flow diagram of the recording equipment is shown in Figure (26).

The pressure in the vacuum chamber was measured with a hot-filament ionization gage which is located between the chamber and the diffusion pump. Since this gage was not mounted directly into the working chamber, the vacuum at the gage would be different from the vacuum in the chamber. How large the discrepancy would be is unknown, but there would be an error in the reading.

When the ionization gage was received from the manufacturer, it was not calibrated. Therefore, a McCleod gage was obtained on loan from Sandia Corporation so that the ionization gage could be accurately calibrated. The ionization gage and McCleod gage were connected into the vacuum system at the same location so they would both be at the same pressure. Using the McCleod gage as the standard, the ionization gage was found to read a higher pressure than the McCleod gage by a factor of ten. In other words, when the McCleod gage read 10^{-5} millimeters of mercury, the ionization gage read 10^{-4} millimeters of mercury, etc.

FIG. 26, RECORDING EQUIPMENT FLOW DIAGRAM

6.0 PROPERTY TESTS

At the beginning of the project, a search of the literature was made to obtain the property values of aluminum-filled Epocast 10 required for the thermal stress and temperature distribution tests. As a result of the search, nothing was found for the emissivity of this material, and very little information was available for the thermal conductivity. Therefore, an experiment was set up to determine the emissivity and thermal conductivity of aluminum-filled Epocast 10. A discussion of the model material is contained in Appendix B.

Since equipment was being constructed for the thermal stress and temperature distribution experiments, a test was sought which could utilize the same apparatus. A thick-walled test cylinder, Figure (27), made of aluminum-filled Epocast 10 was developed for the experiment. In the center of the thick-walled cylinder, a series of heaters were located to produce heat flow radially through the cylinder. Thermocouples were embedded at various points in the test cylinder to measure temperatures during the tests. The cylinder and heaters were placed concentrically inside the liquid-nitrogen-cooled heat sink, and the bell jar was placed around the entire apparatus. The heat sink is shown in Figures (19) and (20), and the vacuum system is shown in Figures (17) and (18).

FIG. 27, MATERIAL PROPERTIES MODEL
THERMOCOUPLE WIRING DIAGRAM.

With this experimental setup, tests were conducted to determine the emissivity and thermal conductivity of aluminum-filled Epocast 10. The experiment and subsequent results are presented in this chapter.

6.1 Theory

Several assumptions are required to determine the emissivity and thermal conductivity of aluminum-filled Epocast 10.

Some of these assumptions are needed to satisfy the conditions required for determining the radiant heat transfer from the surface of the test cylinder when the convective and conductive modes of transfer of heat are negligible. The surroundings for the test cylinder are assumed to be at a low enough pressure to eliminate convective and conductive heat transfer from the surface of the cylinder. The surroundings the test cylinder sees are assumed to be at 0°K and have an emissivity of one. The surface temperature of the test cylinder is assumed to be uniform in the test section. The location of the test section is shown in Figure (27).

From the Stefan-Boltzmann equation the expression for the radiant heat transfer from the surface of the test cylinder is:

$$\dot{q} = 2\pi r_o \ell \sigma \epsilon T_o^4$$

where r_o is the outside radius of the test cylinder, ℓ the length of the test section and T_o the surface temperature of

the test cylinder in the test section. Rearranging the above equation

$$\epsilon = \frac{\dot{q}}{2\pi r_o l \sigma T_o^4} \quad (17)$$

The total hemispherical emissivity of the aluminum-filled epoxy can be determined using Equation (17). The heat flow ($\dot{q}$) from the surface of the test cylinder in the test section is determined by measuring the power supplied to the center heater. The surface area of the test cylinder is $2\pi r_o l$. The surface temperature (T_o) is determined by measuring the radial temperature gradient through the test cylinder at the axial center of the test section.

Other assumptions are needed to satisfy the conditions required for determining the heat conduction radially through the test section. Axial and circumferential symmetry are assumed in the test cylinder. Thermal conductivity is assumed independent of temperature and therefore constant in the temperature range between the thermocouples at the axial center of the test section. See Figure (28) for the location of these thermocouples. Steady state conditions are assumed to exist through the test section, and there are no heat sinks in the test cylinder material. The heat supplied to the inner surface of the test cylinder in the test section is assumed to flow radially to the outer surface with no losses out the ends of the test section.

FIG. 28, MATERIAL PROPERTIES MODEL

THERMOCOUPLE LOCATIONS.

Using these assumptions, the expression from Fourier's equation for the heat conduction radially through the test section is:

$$\dot{q} = 2\pi r \ell K \frac{dT}{dr}$$

where r is any radial location in the test cylinder between the inner and outer radii, and $\frac{dT}{dr}$ is the temperature gradient through the material in the radial direction. Separating variables and integrating both sides of the above equation

$$K = \frac{\dot{q} \ln(r_1/r_2)}{2\pi \ell (T_2 - T_1)} \quad (18)$$

where the thermocouples used to measure the radial temperature gradient are located at the radii (r_1) and (r_2) , and the values (T_1) and (T_2) are the temperatures at these points. The thermal conductivity of the aluminum-filled epoxy is determined using Equation (18). The radii (r_1) and (r_2) and the length of the test section (ℓ) are known. The rate of heat flow $(\dot{q})$ is determined the same way as for emissivity, and the temperatures (T_1) and (T_2) are obtained from the thermocouple data.

6.2 Test Cylinder and Heater Construction

The aluminum-filled epoxy material for the test cylinder was obtained from the same 3-inch diameter cylinder used for the thermal stress and temperature distribution tests.

The first step in the development of the test cylinder was to drill a cylindrical hole through the center of the 3-inch

cylinder parallel to the longitudinal axis. The hole diameter was machined slightly larger than the outside diameter of a thin-walled copper sheath containing the heaters used to supply heat to the test cylinder. The copper sheath and heaters will be described later. The tolerances between the inside diameter of the test cylinder and the outside diameter of the copper sheath were held close so the copper sheath would fit tightly inside the test cylinder. The inside diameter of the test cylinder was 0.8440 inches while the outside diameter of the copper sheath was 0.8435 inches.

The cylinder was sectioned into two halves in the axial direction, and thermocouples were mounted on the machined surface of the section, Figure (27). The thermocouples were made of 36 gage copper and constantan wire, and the leads were matched sets of 24 gage copper and constantan wire. The thermocouples and lead wire came from specially graded stock guaranteed to be accurate within $\pm 1 \frac{1}{2}^{\circ}$ in the temperature range of interest.

Four thermocouples were mounted at the axial midpoint of the test section as shown in Figure (28). These thermocouples were used to determine the radial temperature gradient in the test cylinder and the surface temperature by extrapolation. Also, circumferential symmetry was checked by observing thermocouples (1) and (3) or thermocouples (2) and (4) which are at the same radial position on either side of the test cylinder.

The remainder of the thermocouples were used to measure the axial gradient at the ends of the test section and were located as shown in Figure (27).

Thermocouples (1) through (8) were referenced to an ice bath while each pair of thermocouples (9) and (10) located near the inside surface of the test cylinder across the end of the test section was referenced to its mate. That is, the constantan leads were connected between each pair, and the output was taken through the copper leads. These thermocouple pairs (9) and (10) will hereafter be called guard sets (9) and (10).

The thermocouples were located by accurate radial measurement from radial locations on the inside surface of the test cylinder as shown in Figure (28). Location of the thermocouples in the radial direction is critical since the temperature varies radially in the test section. A thermocouple mounted out of position in the radial direction would introduce errors into Equations (17) and (18) since the values for temperature and radial position would be inconsistent.

The thermocouple leads (small diameter wire) were brought out the ends of the test cylinder along the machined surface on which the thermocouple beads were mounted, as shown in Figure (27). For each thermocouple, one of the lead wires was brought out along the same isotherm the thermocouple bead was mounted on; and the other lead wire was brought out approximately 0.04 inches from this isotherm. This was done so the lead wires

would be as close as possible to the same temperature as the thermocouple bead. The lead wires from thermocouples (5) and (8), however, were brought out on entirely different isotherms than the isotherms where the thermocouple beads were mounted, as seen in Figure (27).

Small tabs were bonded to the ends of the test cylinder as shown in Figure (27) to permit electrical connection between the large and small thermocouple leads without putting force on the small leads.

Since a high degree of accuracy was required to maintain close tolerances for measurements of thermocouple locations, a split mandrel was fabricated, Figure (29). When the test cylinder was reassembled with the split mandrel in place, the surface of the test cylinder was very accurately determined by the outer diameter of the mandrel. With the inner surface of the test cylinder established, accurate measurements could be made from radial points on the inner surface.

The 3-inch diameter cylinder was reassembled with the mandrel in place, thus embedding the thermocouples. The aluminum-filled epoxy used to join the halves together was cured. Then, using the mandrel as a center, the outside diameter of the test cylinder was machined to the size shown in Figure (28). During the machining of the outside diameter, the mandrel was used to keep the outside surface concentric with the inside surface of the test cylinder. The mandrel was removed, and the heater system was inserted in the hole.

FIG. 29, TEST CYLINDER MANDREL.

The heater core consisted of three separate coils of nichrome wire wrapped on a thick-walled cylinder of ceramic tubing. Each coil was located as accurately as possible in the axial direction on the ceramic tubing since the position of these coils fixed the location of the test section in the test cylinder. This means the center heater coil and the test section in the test cylinder were the same length in the axial direction. The leads for the coils of wire were brought to the top of the core through the inside of the ceramic tube where they were connected to the power leads through an electrical socket. The heater core was inserted in a thin-walled copper sheath, and a copper plug was soldered to the bottom of the sheath. To prevent shorting between the heater wire and the copper sheath, fiberglass tape was wrapped around the heater coils to insulate them from the copper. The two end heaters, hereafter called guard heaters, were used to adjust the heat flow across the ends of the test section while the center heater was used to supply heat to the center section. This is how the assumptions mentioned earlier of a uniform surface temperature for the test cylinder, a uniform radial flow of heat through the test section with no losses out the ends of the test section, and steady state conditions through the test section were satisfied.

6.3 Test Equipment

The equipment for the test included the power supply for the heaters, measuring equipment for the center heater and

thermocouples, and mechanical equipment required to simulate the test conditions.

AC power was supplied to each of the guard heaters by a variable transformer, and DC power was supplied to the center heater by two 12-volt batteries.

The circuit for the center heater was arranged so the power to the center heater could be measured by determining the voltage and current through the heater. Figure (30) shows a circuit diagram of the center heater. The voltage drop across the center heater was measured with a digital DC voltmeter. The current through the center heater was determined by placing a one ohm precision resistor in series with the heater and measuring the voltage across the resistor with the digital voltmeter. The one ohm resistor is accurate to within ± 1 percent of one ohm. The digital voltmeter is accurate to within ± 0.1 percent of full scale for the scale being used.

The equipment to record the thermocouple signals consisted of a switching board for 10 thermocouples and a Leeds and Northrup Portable Precision Potentiometer. Since steady state conditions were necessary during the test, the output from all the thermocouples was easily handled by one potentiometer through the switching board.

The vacuum chamber, liquid-nitrogen-cooled heat sink, and radiation shields were constructed for the thermal stress and temperature distribution tests and were not altered in any way

FIG. 30, CENTER HEATER POWER CIRCUIT

for the property tests. A description of these items can be found in Chapter 5 of this report. This mechanical equipment was used to satisfy the assumptions mentioned earlier of a pressure low enough to eliminate convective and conductive modes of heat transfer from the surface of the model, and of the model surroundings being at 0°K and having an emissivity of one.

6.4 Experimental Procedure

The test cylinder and heaters were suspended concentrically inside the heat sink. The radiation shields were wrapped around the heat sink, and the bell jar was placed over the entire test apparatus. The vacuum system was started and allowed to pump at least overnight to attain the required vacuum. A vacuum of 10^{-4} millimeters of mercury was desired to eliminate the effects of convective and conductive heat transfer from the outer surface of the model. A full bottle of liquid nitrogen (110 liters) was connected to the heat sink.

On the day of the test the first step was to turn on the digital DC voltmeter and warm it up for at least 15 minutes. Next, the ice bath reference bottle was filled with finely crushed ice made from distilled water. Distilled water ice was used since it contains no mineral impurities and will hold the reference junction closer to 32°F than tap water ice. The large cold trap above the diffusion pump in the vacuum system, Figure (17), was filled with dry ice.

The two thermocouples located at the top and bottom of the heat sink were connected to a Leeds and Northrup Portable Potentiometer, and the flow of liquid nitrogen to the heat sink was started. As the heat sink cooled down, the temperatures of the two thermocouples at the top and bottom of the heat sink were monitored continuously since large temperature differences had developed between the top and bottom of the heat sink during the preliminary tests. The flow of liquid nitrogen was therefore regulated to hold the temperature difference below 80°F. To do this the flow was stopped several times to allow the temperatures at the top and bottom of the heat sink to equalize. One eighth of the 110 liter bottle was used to cool the heat sink to liquid nitrogen temperature, and the entire cooling process required 45 minutes.

While the heat sink was cooling down, power was supplied to the test cylinder heaters. The DC power supply was connected to the center heater; and, to adjust the power to the heater, a predetermined voltage was set in the digital voltmeter by two variable resistors in the circuit. One variable resistor was used for coarse voltage adjustments and the other for fine adjustments. Each guard heater was set at a predetermined power by adjusting the voltage in the variable transformer. This voltage was selected on the basis of the voltages used in preliminary tests. It was decided for future tests that initially each variable transformer should be set at a voltage

which would make the power to the guard heaters the same as the power to the center heater.

The precision potentiometer was carefully balanced, and preliminary readings were taken for each thermocouple in the test cylinder to check the performance of the potentiometer and the thermocouples. When the heat sink reached a temperature of -313°F (the boiling temperature of liquid nitrogen) at the top and bottom, the recording of data was begun.

The test cylinder heated slowly when power was applied to the heaters, and the temperatures in the cylinder were slow stabilizing. Also, each time power was adjusted during the test, the response of the temperatures in the test cylinder was slow. Therefore, the data were taken at time intervals varying from 10 to 30 minutes. The procedure for taking data at any given time interval was to record the time, record the voltage and current to the center heater, and record the voltage to each of the guard heaters. Then readings were taken for each thermocouple in the test cylinder by switching through each position on the switching board and reading the potentiometer.

When the temperatures in the test cylinder stabilized, the data were analyzed to determine what adjustments in the power were required. The guard thermocouples at the top and bottom of the test section, Figure (27), were observed to see if axial temperature gradients existed across the ends of the test section. To reduce any existing gradients, small power adjustments were made in the guard heaters.

The experimental procedure to this point was essentially the same for all test runs. However, a problem developed during the initial runs which rendered the data for the early runs unreliable. Thermocouples (5) and (6), Figure (27), were originally referenced to each other as a guard set across the lower end of the test section near the outer surface of the test cylinder. Thermocouples (7) and (8) were similarly connected at the upper end of the test section. These two guard sets could not be balanced during the early tests. From the data it was suspected that thermocouples (5) and (8) were reading higher temperatures than they should have been. The reason for this is discussed in the results.

A decision was made to reference thermocouples (5) and (8) to an ice bath so the temperature of each thermocouple could be determined. Also, shielding was added at both ends of the test cylinder and heaters to reduce heat losses out the ends. Two concentric, cylindrical shields were placed around the power leads at the top, and three flat discs were placed at the bottom. The material used for the shields was a thin chromium sheet coated with thin layers of mylar. This was the same material that was used around the outside of the heat sink.

Since thermocouples (5) and (8) were in error, another method was required to measure the axial temperature gradients across the ends of the test section near the outer surface of the test cylinder. The problem was solved by using thermocouples

(1) and (6), which have the same radial position, to measure the axial gradient at the bottom of the test section. Thermocouples (3) and (7) were used for the same purpose at the top of the test section. Thermocouple guard sets (9) and (10) were still used to measure the axial temperature gradients across the ends of the test section near the inner surface of the test cylinder.

The method just discussed was used to determine the axial temperature gradients for the last two runs. These two runs, which showed considerable promise, are described in the procedure that follows. After stabilizing the temperatures in the test cylinder, small adjustments were made in the guard heaters to reduce the axial gradients across the ends of the test section. The two thermocouples near the outside surface of the test cylinder, (1) and (6), balanced for Run 1. The two outside thermocouples in the upper set, (3) and (7), showed heat flow out the top of the test section. An adjustment had just been made in the guard heaters to reduce the gradient between thermocouples (3) and (7) when a transformer in the vacuum system burned out, cutting off all the vacuum measuring equipment. The bell jar was isolated from the vacuum system while the trouble was assessed. The vacuum system appeared undamaged except for the loss of the measuring equipment. Therefore, the bell jar was rejoined to the vacuum system; and a second run was made.

A temperature decrease was noted in the test cylinder, Tables 3 and 4, during the time the bell jar was isolated from

the vacuum system. This can be seen by observing the 9:30 PM data in Table 3 and the 11:00 PM data in Table 4. However, the temperatures recovered slowly after the bell jar was returned to the vacuum system.

Run 2 was continued and attempts were made to reduce the inner and outer gradients in the upper and lower sets to zero. A zero gradient was obtained on the outside thermocouples in the lower set, (1) and (6). The outside axial gradient at the top was reduced considerably, but could not be reduced to zero. The inside guard thermocouples in the lower set (9) showed a small axial gradient out of the test section while the inside guard thermocouples in the upper set (10) showed an axial gradient of the same magnitude into the center test section. Several hours were spent attempting to reduce these gradients with little success. Therefore, Run 2 was ended so the data could be analyzed and a value of emissivity and thermal conductivity calculated.

6.5 Data Reduction

Several test runs were made with the epoxy test cylinder and heater system, but the early test runs were unreliable as discussed in the experimental procedure. Therefore, the only data reduced for this experiment was the data from the last two runs, Tables 3 and 4, termed Runs 1 and 2. An analysis of these two runs shows the data to be more promising than the early test runs.

TABLE 3
Test Data, Run No. 1

TIME	Thermocouples (Millivolts)										Center Heater		Guard Heaters (Volts)	
	1	2	3	4	5	6	7	8*	9*	10*				
	Meas.	Meas.	Meas.	Meas.	Meas.	Meas.	Meas.	Meas.	Guard	Guard	Volts	Amps	Top	Bot.
11:10	.702	.737	.744	.765	.772	.743	.697	.755	.004	-.008	2.926	.2066	10	10
11:31	.871	.903	.908	.927	.938	.910	.845	.903	.000	-.008	2.926	.2068	10	10
11:54	.970	1.008	1.008	1.028	1.039	1.012	.947	.999	.011	.000	2.922	.2067	10	10
12:15	1.054	1.091	1.090	1.111	1.122	1.095	1.022	1.081	.011	.000	2.924	.2067	10	10
12:40	1.145	1.179	1.179	1.201	1.218	1.185	1.113	1.173	.002	.001	2.924	.2067	10	10
1:25	1.164	1.203	1.200	1.219	1.230	1.197	1.124	1.185	.004	-.002	2.924	.2067	10	10
1:40	1.172	1.207	1.206	1.225	1.237	1.205	1.135	1.195	.002	.002	2.926	.2067	10	10
1:50	1.184	1.220	1.220	1.239	1.252	1.220	1.149	1.208	.004	.002	2.926	.2067	10	10
2:00	1.190	1.227	1.224	1.242	1.251	1.223	1.150	1.212	.003	.003	2.926	.2067	10	10
2:10	1.196	1.235	1.232	1.252	1.263	1.232	1.162	1.219	.003	.003	2.926	.2067	10	10
2:20	1.204	1.241	1.237	1.258	1.268	1.238	1.165	1.223	.004	.004	2.926	.2067	10	10
2:35	1.204	1.241	1.237	1.257	1.270	1.236	1.166	1.228	.005	-.002	2.920	.2067	10	10
2:55	1.221	1.255	1.253	1.273	1.278	1.242	1.193	1.250	.010	-.008	2.920	.2067	10.5	9.5
3:20	1.252	1.288	1.281	1.303	1.308	1.269	1.227	1.282	.012	-.012	2.922	.2067	10.5	9.5
3:30	1.261	1.296	1.291	1.313	1.315	1.281	1.239	1.291	.015	-.011	2.924	.2067	10.5	9.5
3:40	1.279	1.315	1.310	1.337	1.335	1.300	1.255	1.312	.012	-.012	2.924	.2067	10.5	9.5
4:00	1.303	1.338	1.333	1.357	1.360	1.319	1.277	1.335	.015	-.013	2.924	.2067	10.5	9.5
4:20	1.315	1.352	1.347	1.368	1.373	1.332	1.288	1.349	.012	-.012	2.924	.2067	10.5	9.5

* Positive gradient means heat flow out of the center test section

TABLE 3
Test Data, Run No. 1 (Cont.)

TIME	Thermocouples (Millivolts)										Center Heater		Guard Heaters (Volts)	
	1	2	3	4	5	6	7	8	9	10				
	Meas.	Meas.	Meas.	Meas.	Meas.	Meas.	Meas.	Meas.	Guard	Guard	Volts	Amps	Top	Bot.
4:45	1.325	1.362	1.354	1.376	1.379	1.341	1.298	1.354	.012	-.012	2.924	.2067	10.5	9.5
5:00	1.325	1.368	1.359	1.380	1.382	1.342	1.302	1.360	.012	-.012	2.924	.2067	11.0	9.5
5:15	1.335	1.370	1.364	1.384	1.384	1.349	1.303	1.362	.012	-.012	2.924	.2067	11.0	9.5
5:35	1.335	1.373	1.367	1.386	1.389	1.352	1.309	1.368	.018	-.012	2.924	.2067	10.5	9.5
5:50	1.341	1.376	1.369	1.390	1.395	1.354	1.313	1.373	.015	-.014	2.923	.2066	10.5	9.5
6:30	1.385	1.435	1.422	1.444	1.448	1.409	1.373	1.427	.03	0	2.925	.2066	11	9.5
7:00	1.435	1.428	1.468	1.495	1.494	1.455	1.415	1.474	.006	-.017	2.916	.2066	11	9.5
7:30	1.425	1.467	1.460	1.480	1.480	1.440	1.400	1.460	.012	-.015	2.914	.2065	11	9.5
8:00	1.425	1.465	1.460	1.484	1.485	1.443	1.404	1.463	.013	-.017	2.914	.2065	11	9.5
8:30	1.366	1.405	1.393	1.415	1.408	1.372	1.348	1.400	.020	-.020	2.915	.2066	11	9
9:00	1.273	1.305	1.300	1.318	1.315	1.274	1.248	1.300	.020	-.017	2.915	.2066	11	9
9:30	1.273	1.305	1.300	1.323	1.310	1.273	1.248	1.306	.020	-.015	2.915	.2066	11.5	9

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Vacuum: 2×10^{-4} mm Hg.

Room Temperature: 74°F

Liq. Nitrogen Temp: -313°F

TABLE 4
Test Data, Run No. 2

TIME	Thermocouples (Millivolts)										Center Heater		Guard Heaters (Volts)	
	1	2	3	4	5	6	7	8	9	10				
	Meas.	Meas.	Meas.	Meas.	Meas.	Meas.	Meas.	Meas.	Guard	Guard	Volts	Amps	Top	Bot.
11:00	0.990	1.024	1.019	1.040	1.035	0.996	0.977	1.032	.017	-.015	2.913	.2065	11.5	9
11:20	1.073	1.113	1.133	1.124	1.118	1.083	1.065	1.120	.030	-.016	2.913	.2065	11.5	9
11:30	1.100	1.134	1.130	1.146	1.142	1.106	1.085	1.139	.020	-.016	2.914	.2064	11.5	9
12:00	1.178	1.220	1.205	1.230	1.227	1.236	1.167	1.222	.020	-.015	2.914	.2065	11.5	9
12:30	1.231	1.252	1.251	1.265	1.261	1.218	1.200	1.255	.016	-.016	2.914	.2065	11.5	9
12:50	1.253	1.295	1.289	1.310	1.305	1.263	1.242	1.292	.018	-.016	2.914	.2065	11.5	9
1:15	1.270	1.312	1.309	1.326	1.318	1.281	1.260	1.310	.022	-.016	2.914	.2065	11.5	9
1:40	1.300	1.340	1.336	1.355	1.352	1.305	1.286	1.338	.020	-.017	2.913	.2065	11.5	9
2:00	1.335	1.372	1.363	1.382	1.377	1.336	1.318	1.372	.020	-.015	2.913	.2065	11.5 ⁺	9
2:20	1.345	1.390	1.379	1.400	1.392	1.354	1.334	1.385	.020	-.020	2.913	.2065	11.5 ⁺	9
2:40	1.374	1.411	1.410	1.429	1.422	1.380	1.364	1.412	.018	-.020	2.913	.2065	11.5 ⁺⁺	9 ⁻
3:20	1.410	1.454	1.440	1.465	1.460	1.415	1.398	1.453	.016	-.019	2.913	.2064	11.5 ⁺⁺	9 ⁻
3:45	1.425	1.468	1.459	1.480	1.474	1.427	1.410	1.464	.020	-.020	2.913	.2065	11.5 ⁺⁺	9 ⁻
4:00	1.425	1.470	1.460	1.480	1.475	1.425	1.414	1.465	.018	-.018	2.913	.2064	11.5 ⁺⁺	9 ⁻
At 4:00 AM Shut off guard heaters														
4:30	.641	.656	.649	.648	.639	.594	.572	.602	.018	-.013	4.290	.3041	0	0
5:00	-.243	-.232	-.236	-.240	-.244	-.278	-.295	-.269	.016	-.005	4.287	.3041	0	0
5:30	.288	.362	.364	.373	.366	.331	.290	.340	.025	0	7.188	.5067	0	0
5:50	.225	.250	.241	.261	.247	.212	.184	.233	.022	0	7.190	.5067	0	0
6:15	.236	.265	.265	.277	.266	.234	.205	.257	.023	.010	8.310	.5830	0	0
6:30	.286	.325	.320	.335	.325	.287	.255	.312	.025	.011	8.309	.5830	0	0

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Vacuum: 2×10^{-4} mm Hg.

Room Temperature: 74°F

Liq. Nitrogen Temp: -313°F

For calculating the emissivity and thermal conductivity, the data from two particular time intervals were used. For Run 1 the data from the 9:30 PM interval were used, while the 4:00 AM data were used for Run 2. These two time intervals were used because steady state conditions existed in the test cylinder and the axial gradients across the ends of the test section were at their lowest magnitudes.

The equations necessary to calculate emissivity and thermal conductivity for aluminum-filled Epocast 10 were developed in the theory section of this chapter.

To calculate both emissivity and thermal conductivity, the heat supplied to the test section at the inside surface of the test cylinder was required. The heat supplied was determined by measuring the power to the center heater. A sample calculation is shown in Appendix A.

The surface temperature of the test section was required to determine emissivity. This value was obtained by extrapolating to the surface the temperatures of the two thermocouples at the axial center of the test cylinder as shown in Figure (31). The total hemispherical emissivity was calculated as shown in Appendix A using the surface temperature and computed power.

To calculate the thermal conductivity, the temperature difference between the two thermocouples at the center of the test section (1 and 2 or 3 and 4) was required. These temperature differences determined from the data are shown in Figure (31).

FIGURE 31

VARIATION OF TEMPERATURE WITH
RADIAL POSITION IN THE TEST CYLINDER

Using the temperature difference and power, the thermal conductivity was calculated as shown in Appendix A.

6.6 Results

The property values of emissivity and thermal conductivity for aluminum-filled Epocast 10 (50 percent aluminum by weight) calculated from the test data (Runs 1 and 2) are presented in Table 5.

The emissivity was computed for Run 1 and Run 2 using both pairs of thermocouples at the axial center of the test section (pair 1 and 2 and pair 3 and 4). For both runs the temperature gradient between thermocouples (1) and (2) was unlike the gradient between thermocouples (3) and (4), Figure (31), which made the surface temperatures obtained from these gradients different. However, the small variations in surface temperature had little effect on the emissivity. The temperatures in the test cylinder were higher for Run 1 than for Run 2 which also had little effect on the emissivity. The emissivity values were slightly lower for Run 2 than for Run 1. There were no values of emissivity available for this aluminum-filled epoxy so a comparison of emissivities was not possible.

The thermal conductivity was determined for Runs 1 and 2 using the temperature difference between thermocouples (1) and (2) and the temperature difference between thermocouples (3) and (4). There was considerable variation in the calculated values of thermal conductivity. For example, in Run 2 the thermal

TABLE 5

SUMMARY OF RESULTS

	Run 1	Run 2
Center Heater Power (Btu/hr)	2.055	2.050
Emissivity--Using T_o Determined From Thermocouples (1) and (2)	0.470	0.448
Emissivity--Using T_o Determined From Thermocouples (3) and (4)	0.466	0.443
Thermal Conductivity--Using the Temperature Difference From Thermocouples (1) and (2). (Btu/hr-ft-°F)	0.655	0.473
Thermal Conductivity--Using the Temperature Difference From Thermocouples (3) and (4). (Btu/hr-ft-°F)	0.910	1.040
Measuring Thermocouple T_1 (°F)	89.44	96.04
Measuring Thermocouple T_2 (°F)	90.83	97.96
Measuring Thermocouple T_3 (°F)	90.62	97.52
Measuring Thermocouple T_4 (°F)	91.61	98.39
Measuring Thermocouple T_5 (°F)	91.04	98.17
Measuring Thermocouple T_6 (°F)	89.44	96.04
Measuring Thermocouple T_7 (°F)	88.35	95.57
Measuring Thermocouple T_8 (°F)	90.87	97.74
Guard Thermocouple set T_9 (°F)	0.952	0.858
Guard Thermocouple set T_{10} (°F)	0.714	0.858

conductivity was 0.473 Btu/hr-ft-°F using thermocouples (1) and (2), and 1.04 Btu/hr-ft-°F using thermocouples (3) and (4). Comparing Run 1 with Run 2, the results did not appear to follow any pattern.

The thermal conductivity computed using the temperature difference for thermocouples (1) and (2) was lower for Run 2 than for Run 1 while the opposite situation occurred for the values determined from thermocouples (3) and (4).

The value of thermal conductivity taken from the report by Matthews (6) was available for comparison. This value of 0.52 Btu/hr-ft-°F was determined using a model containing 35 percent of aluminum powder by volume. The material in the present test cylinder contained about 31 percent by volume of aluminum powder. Matthews' model contained many small air bubbles which were not present in the test cylinder used here since the test cylinder material was molded in a higher vacuum. Comparing the test cylinder to Matthews' model, the decrease in aluminum powder (35 percent by volume to 31 percent by volume) would reduce the thermal conductivity, while the reduction in porosity (no small air bubbles) would increase the thermal conductivity. It would be very difficult to quantitatively determine the effects of these variations. However, since no other values were available, the value of 0.52 Btu/hr-ft-°F was used for comparison of results.

The thermal conductivity computed using the temperature difference for thermocouples (1) and (2) was higher than Matthews'

(6) value for Run 1 and lower than Matthews' value for Run 2. The average of the two values was $0.56 \text{ Btu/hr-ft-}^{\circ}\text{F}$ compared to Matthews' value of $0.52 \text{ Btu/hr-ft-}^{\circ}\text{F}$.

The values of thermal conductivity computed using the temperature difference for thermocouples (3) and (4) for Runs 1 and 2 were considerably different than Matthews' value. The average value for the two runs was $0.975 \text{ Btu/hr-ft-}^{\circ}\text{F}$ compared to $0.52 \text{ Btu/hr-ft-}^{\circ}\text{F}$ for Matthews. The average value of thermal conductivity does, however, compare with a value of $1.03 \text{ Btu/hr-ft-}^{\circ}\text{F}$ determined by Matthews using a model containing 50 percent aluminum powder by volume. The above results suggest that the temperature differences determined from thermocouples (3) and (4) are in error.

The zero temperature gradient required across the ends of the test section to eliminate heat losses from the test section was never attained in any of the tests. The measured gradients were from about 0.65° to 2.25°F for both runs. The only balance that was obtained was between thermocouples (1) and (6) in the lower set.

Thermocouples (5) and (8) were reading higher than they should have been according to the test data. This is explained by observing how the leads for thermocouples (5) and (8) were brought out of the test cylinder. The leads were brought out on an isotherm at a different radial position than the isotherm where the thermocouple bead was located as shown in Figure (27).

The leads for thermocouple (5) were brought out of the model on approximately the same isotherm on which thermocouple (2) was located, and the leads for thermocouple (8) were brought out on the isotherm where thermocouple (4) was located. The temperatures from thermocouples (4) and (8) were compared to determine the effect on thermocouple temperature when the leads are brought out of the model on a different radial isotherm than the one on which the bead is located.

The temperature of thermocouple (8) was about the same as the temperature of thermocouple (4) for all runs even though thermocouple (8) was located farther away from the heater than thermocouple (4). The explanation is that heat was being conducted down the thermocouple leads from the isotherm at temperature (T_4) to the thermocouple bead at temperature (T_8). Once the heat reached the thermocouple bead, the temperature of the junction was increased until the flow of heat into and out of the thermocouple bead was equalized. The same phenomenon was apparent between thermocouples (2) and (5). However, thermocouple (5) was actually at a higher temperature than thermocouple (2). It appears that there is a small uncertainty (defined in the error analysis) involved in one of these thermocouples or possibly both of them. It seems unlikely that thermocouple (5) would read higher temperatures than thermocouple (2) since thermocouple (5) is located farther away from the inner surface than thermocouple (2).

The temperatures of thermocouples (3) and (4) consistently ran higher than the temperatures of thermocouples (1) and (2) respectively as shown in Tables 3 and 4. This phenomenon was observed in all the preliminary runs as well as the data runs. One possible reason for the temperatures being higher on one side than on the other could have been unsymmetrical heating in the center heater coil. Another reason could have been an impurity or some other substance on the surface of the test cylinder. A very fine white coating of unknown composition was discovered on parts of the surface of the test cylinder after the tests were run. There was yet another reason, namely, the uncertainty in the signals from the thermocouples. The temperature uncertainty may well be the primary reason for the difference in these two pairs of thermocouples. A discussion of the temperature uncertainty can be found in the error analysis.

An analysis was made to determine the power supplied to the guard heaters for the 4 AM data in Run 2. The top heater received 18 Btu/hr of heat while 8.4 Btu/hr was supplied to the bottom heater. Each of the guard heaters received considerably more heat than the 2.05 Btu/hr supplied to the center heater. To account for the large difference between the heat supplied to the center heater and that supplied to each of the guard heaters, an analysis was made of possible sources of heat loss from the test cylinder and heater system. A rough calculation showed the heat loss from the thermocouple leads at the ends

of the test cylinder to be significant. The heat loss by radiation was also estimated for the bottom end of the test cylinder and the copper plate on the bottom of the heaters. The combination of all these heat losses accounted for a considerable amount of the difference between the heat supplied to the bottom heater and the heat supplied to the center heater.

6.7 Error Analysis

An error analysis was conducted to determine possible sources of error in the methods used to determine the emissivity and thermal conductivity of the aluminum-filled epoxy material and thereby gain knowledge of the accuracy of the experimental results. In an analysis of this type every phase of the experiment must be explored to determine where errors might exist. From the analysis, the following errors were found.

A source of error was discovered during the calibration of a series of thermocouples at 252.3°F , the melting point of benzoid acid (see Chapter 4). Some of the thermocouples read higher than the calibration point while others read lower, as shown in Table 2. The difference between the thermocouple that read the highest and the one that read the lowest was defined as the temperature uncertainty. The calibration was done on the six thermocouples that were mounted in the experimental model as discussed in Chapter 4. The three thermocouples that were mounted on the surface of the experimental model and all the thermocouples mounted in the test cylinder for property

measurements were not calibrated. The calibrated thermocouples and those that were not calibrated came from the same batch of wire. Therefore the temperature uncertainty was considered to apply for the thermocouples used in the property test. The maximum temperature uncertainty determined at the melting point of benzoic acid was 1.8° , Table 2.

The test runs were conducted in the range of 88°F to 98°F , and therefore the temperature uncertainty of 1.8° was considered high for the runs. The six thermocouples showed no temperature uncertainty when calibrated at 32°F . To determine a temperature uncertainty in the range of temperatures for the test runs, a linear variation was assumed between the zero uncertainty at 32°F , and the 1.8° uncertainty at 252.3°F as shown in Figure (32). From the figure a temperature uncertainty of 0.5° was obtained for a temperature of 94°F .

To determine the errors in the system, Run 2 was used since the axial heat losses across the ends of the test section appeared to be lower in this run.

A calculation of the heat lost axially out the ends of the test section was made. The analysis was based on the axial temperature gradients existing across the ends of the test section. To obtain an estimate for the heat loss, the axial temperature gradient was assumed to vary linearly from the inner surface to the outside of the test cylinder along the end of the test section. The actual radial distribution of

FIGURE 32

LINEAR VARIATION OF TEMPERATURE UNCERTAINTY
AS A FUNCTION OF THE THERMOCOUPLE CALIBRATION
TEMPERATURE

the temperature gradient would be difficult to ascertain, but a linear distribution seemed reasonable for this computation. The temperature gradients were extrapolated to the inner and outer surfaces of the test cylinder using the linear relationship. An expression was derived for the heat loss in terms of the temperature gradient and the thermal conductivity of the plastic. A sample calculation of the heat loss is found in Appendix A. The thermal conductivity of $0.52 \text{ Btu/hr-ft-}^{\circ}\text{F}$ determined by Matthews (6) was used in the computations.

First, the heat loss out the ends of the test section was calculated using the data from Run 2. The loss was 7 percent of the power supplied to the center heater. Next, the temperature uncertainty of 1.8° was applied to the temperature gradients in such a fashion as to produce the greatest heat loss, and the value for the loss was increased to 41 percent of the power to the center heater. The uncertainty of 0.5° was applied to the temperature gradients, and the heat loss amounted to 16 percent of the center heater power. These values can be seen in Table 6.

Another source of error was in the measuring circuit used to determine the power to the center heater. The digital voltmeter was accurate to within ± 0.1 of 1 percent of full scale deflection on the meter. Also, the one ohm resistor used to determine the current through the center heater was accurate to within one percent of one ohm. The error from this source was

TABLE 6

ERRORS IN PROPERTY VALUES*

	Error in Power (Btu/hr)	Error in Emissivity (%)	Error in Thermal Conduc- tivity(%)
Error due to heat loss out the ends of the test section.	.144	7	7
Error due to heat loss out the ends of test section using a temperature uncertainty of 1.8°	.831	41	41
Error due to heat loss out the ends of test section using a temperature uncertainty of 0.5°	.319	16	16
Error in the power measuring circuit for the center heater	.040	2	2
Thermocouple errors using a temperature uncertainty of 0.5° where (T_1) increased and (T_2) decreased	--	0.27	37
Thermocouple errors using a temperature uncertainty of 0.5° where (T_1) decreased and (T_2) increased.	--	0.40	21
Thermocouple errors using a temperature uncertainty of 0.5° where (T_3) increased and (T_4) decreased.	--	0.30	148
Thermocouple errors using a temperature uncertainty of 0.5° where (T_3) decreased and (T_4) increased.	--	0.52	37

* All error calculations were made using the 4:00 AM data for Run 2.

calculated to be 2 percent of the power to the center heater. This calculation can be seen in Appendix A.

Assuming the major sources of error in the power were the ones calculated, the total error was about 18 percent of the power to the center heater. The effect of this error on the emissivity and thermal conductivity would be direct, and therefore the results for these properties could be in error by 18 percent due to errors in power. The emissivity and thermal conductivity, however, could be in error by as much as 43 percent if the maximum temperature uncertainty were considered.

The temperature uncertainty would directly affect the temperature output indicated by the thermocouples at the axial center of the test section (pair 1 and 2 and pair 3 and 4). This would introduce additional errors into the results for the emissivity and thermal conductivity.

The surface temperature of the test cylinder was affected by the temperature uncertainty in thermocouples (1) through (4). This would affect the results for the emissivity. However, when the uncertainty of 0.5° was applied to thermocouples (1) through (4), the maximum uncertainty in the emissivity was 0.5 percent.

For the thermal conductivity, the temperature difference between thermocouples (1) and (2) or (3) and (4) was affected by the temperature uncertainty. Using an uncertainty of 0.5° , the uncertainty in the thermal conductivity was calculated to be as high as 148 percent of the thermal conductivity computed from the data.

An error is presented in the temperatures indicated by the thermocouples whose leads are not brought out of the test cylinder on the same isotherm as the thermocouple bead. The leads could conduct heat into the bead or away from the bead depending on whether the leads are on a hotter or cooler isotherm than the bead. This error, though difficult to calculate, could be significant as seen in the discussion of thermocouples (5) and (8) in the results. The error introduced into thermocouples (5) and (8) did not affect the calculated results since these thermocouples were not used. However, since the leads for the other thermocouples were not brought out exactly on the same isotherm as the thermocouple bead as discussed in the section on the test cylinder and heater construction, there could be small errors in the calculated results.

6.8 Conclusions

The emissivity was about the same for both runs and for both sets of thermocouples (1 and 2 or 3 and 4). The major sources of error in the emissivity were caused by the heat loss through the ends of the test section and the temperature uncertainty in the thermocouples. The temperature uncertainty was used to determine the maximum heat loss through the ends of the test section, and this value combined with errors in the measuring circuit for the power constituted the largest sources of error in the emissivity. The effects on the emissivity due to variations in surface temperature caused by the temperature uncertainty were small. The asymmetry in the temperature distribution caused very little variation in the values of

emissivity. An average value for the emissivity from the test runs is 0.45 which is considered accurate within ± 45 percent.

The values of thermal conductivity determined from the tests varied considerably, both between the two runs and between the two sets of thermocouples on either side of the test cylinder (1 and 2 or 3 and 4). The major sources of error in the thermal conductivity were the heat loss through the ends of the test section and the temperature uncertainty in the thermocouples. The heat loss determined using the temperature uncertainty introduced errors into the thermal conductivity as high as 41 percent of the heat supplied. The direct effect of the temperature uncertainty on the temperatures used in the expression for thermal conductivity was to introduce errors as high as 148 percent of the calculated thermal conductivity.

The values of thermal conductivity calculated for both runs using thermocouples (3) and (4) appeared to be in error as discussed in the section on results. The thermal conductivity from thermocouples (1) and (2) for both runs was close to Matthews' value of $0.52 \text{ Btu/hr-ft-}^{\circ}\text{F}$, with an average value being $0.56 \text{ Btu/hr-ft-}^{\circ}\text{F}$. This value is considered accurate within ± 200 percent. The results for thermal conductivity appear to be of doubtful use due to the uncertainty in the thermocouples. It was probably chance that the values of thermal conductivity for both runs for thermocouples (1) and (2) were close to Matthews' value.

The top guard heater was supplied with considerably more power than the bottom heater during the test runs. The difference between the power supplied to the bottom guard heater and that supplied to the center heater was partly accounted for by the heat flow out the thermocouple leads. However, a large amount of heat supplied from the top heater was still unaccounted for. For the test runs, generally none of the heat is lost through the test section as shown by the data, Tables 3 and 4, for the guard thermocouples across the ends of the test section. As previously discussed, a positive sign means heat flow out of the test section and a minus sign means heat flow into the test section for the data for guard sets (9) and (10). These two sets, for any particular time interval, generally show a net heat flow out of the test section or no heat flow at all. The guard sets near the outer surface of the model generally showed heat flow out of the test section. However, looking further at the data for guard sets (9) and (10), there were times during the test when some of the heat from the top heater may have flowed out through the test section. Most of the heat from the top heater that was unaccounted for must have been lost through the electrical plug and the power leads.

The problem of erroneous readings for thermocouples (5) and (8) due to placing the lead wires on different isotherms cannot be solved satisfactorily. The errors introduced into these two thermocouples are significant enough to render them

unreliable for future testing unless the effects of the lead wires on the temperature at the thermocouple junction can be determined.

The uncertainty in the thermocouple readings was a very large error in the test runs. An accurate calibration of each of the thermocouples used in the property measurement tests was not performed because the thermocouples used in these tests came from the same batch of wire as six other thermocouples which were calibrated as shown in Chapter 4. The calibrated thermocouples were found to have a maximum deviation from the International Temperature Scale of 1° . The thermocouples were therefore considered accurate enough for the property tests. However, after evaluating the data and the results from the tests, the temperature uncertainty appeared to have a significant effect on the values determined. This uncertainty must be eliminated to obtain reasonable results.

6.9 Recommendations

To eliminate the temperature uncertainty in the thermocouples, a very accurate calibration should be made of all the thermocouples in the test cylinder. This should increase the accuracy of the results for emissivity and thermal conductivity.

Thermocouples (5) and (8) should not be relied on for future experiments with the test cylinder. In view of what happened in this case, the effect of not embedding the

thermocouple leads on the same isotherm as the thermocouple bead will introduce errors into the thermocouple readings. In future work involving the embedding of thermocouples, the leads, where possible, should be embedded on isotherms at the same temperature as the thermocouple bead. One way to reduce the flow of heat along the leads is to use the smallest thermocouple wire available.

The thermocouples in the guard sets (9) and (10) should be referenced to an ice bath instead of each other as they were for the test runs. This way, individual temperatures can be determined for each thermocouple. These thermocouples can then be used to obtain a better picture of the radial heat flow in the test section as well as being used for guard sets at the ends of the test cylinder.

More test runs should be made at higher temperatures to determine how the emissivity and thermal conductivity vary with temperature. These property values should be determined in the range of temperatures to be used in the thermal stress and temperature distribution tests. The emissivity and thermal conductivity were assumed to be weak functions of temperature for the thermal stress and temperature distribution tests and therefore constant. This, however, would still be a check to determine how these property values vary with temperature.

Radiation shields should be added to the ends of the test cylinder and heater system to reduce the heat losses out the

ends. Shields should be placed around the thermocouple leads, and extra shields should be placed around the power leads since these appear to be the two major sources of heat loss out the ends.

The thermal conductivity values determined from these tests were very scattered and inconsistent. Another method should be used to determine the thermal conductivity of this material so that the conductivity values for different tests can be compared.

7.0 CONCLUSIONS AND RECOMMENDATIONS

7.1 Conclusions

The heat sink has been operated successfully for many hours at liquid nitrogen temperature (-313°F) without leaking. The soft copper O-rings and thin sheets of mylar provide an excellent seal between the bronze flanges. Care must be taken to prevent large temperature gradients from developing in the heat sink during the cooling process.

The vacuum system has been operated successfully while maintaining pressures of approximately 2×10^{-4} millimeters of mercury in the working chamber. The heater and shields developed for the thermal stress and temperature distribution tests have been tested successfully both for mechanical operation and for providing heat.

A strain gage which can be used for embedding purposes and which is temperature compensated for the coefficient of thermal expansion of the epoxy has not been found to date.

The results for the thermal property tests appear to be of doubtful use for further work. The uncertainty in the thermocouple readings was the main source of error for the results.

The temperatures obtained from embedded thermocouples whose leads were brought out of the model on a different isotherm than the thermocouple bead were in error due to heat flow along the wires. The results were affected when the leads were off the isotherm by only a small amount.

The ionization gage in the vacuum system was found to read a higher pressure than the actual value by a factor of ten. In other words, when the ionization gage read 10^{-4} millimeters of mercury the pressure was actually 10^{-5} millimeters of mercury using a McCleod gage as a standard.

Until completely cured, the epoxy material becomes soft when heated above 150°F .

When rejoining two parts with epoxy it is very difficult to tell whether all the air bubbles have been removed from the material used to rejoin the parts. This could have a large effect on the results for this experiment.

7.2 Recommendations

Further work should be done to obtain a properly compensated strain gage.

A very accurate calibration should be conducted for the thermocouples to be used in the property tests to reduce the uncertainty in the thermocouple readings. This should increase the accuracy of the results for emissivity and thermal conductivity. More property tests should be run after the uncertainty is eliminated from the thermocouple readings, to determine useful values of emissivity and thermal conductivity for the epoxy. The tests should be conducted at various temperatures throughout the range of temperatures of interest to determine the variation of emissivity and thermal conductivity with temperature.

When bolting together the flanges on the heat sink a thin layer of vacuum grease should be spread on the flanges and the O-rings to provide a better seal. The bolts should be tightened down on alternate sides to obtain uniform loading on the O-rings.

To prevent the epoxy from resoftening during the curing process, hold the temperature of the oven below 150°F for several hours. Then move the temperature up to 200° for 24 hours.

To insure the removal of bubbles from the epoxy material when rejoining parts a vacuum should be applied to draw out the air bubbles where possible. If it is not possible to use the vacuum system, then heating the epoxy carefully while it is still liquid will help to drive off the bubbles. The heating should be done after the epoxy has been applied to the parts.

When embedding thermocouples in epoxy, the lead wires should be brought out of the test model on the same isotherm on which the thermocouple bead is mounted.

APPENDIX A

SAMPLE CALCULATIONS

1. To determine the initial temperature (T_i) for the experimental model where a 3-inch diameter is selected for the model

$$(N_R)_p = (N_R)_m = 0.0395$$

The model parameters used for this analysis are:

$$\bar{a} = 0.125 \text{ ft}$$

$$\epsilon = 0.45$$

$$K_i = 0.52 \text{ Btu/hr-ft-}^\circ\text{F}$$

Therefore, using Equation (10) and $N_R = 0.0395$

$$N_R = \frac{\bar{a}\epsilon\sigma T_i^3}{K_i}$$

or

$$T_i^3 = \frac{N_R K_i}{\bar{a}\epsilon\sigma}$$

$$= \frac{(0.0395)(0.52)}{(0.125)(0.45)(0.1714)(10^{-8})}$$

$$T_i = 598^\circ\text{R}$$

$$= 138^\circ\text{F}$$

2. To calculate the largest change in temperature ($\Delta T_{\max}$) for the experimental tests using Figure (10) the expression is:

$$\frac{T}{T_i} = \text{Dimensionless Temperature } (\phi)$$

$$T = \phi T_i$$

Using the value of ϕ at $\tau = 0.4$ which is the largest value for the data shown

$$\begin{aligned} T &= (0.96)(598) \\ &= 574^{\circ}\text{R} \end{aligned}$$

Therefore:

$$\begin{aligned} \Delta T_{\max} &= T_i - T \\ &= 24^{\circ}\text{F} \end{aligned}$$

3. To calculate the dimensional time at the instant of maximum stress ($\tau = 0.21$)

$$\begin{aligned} t &= \frac{a^2}{\left(\frac{K_i}{\rho C_i} \right)} \tau \\ &= \frac{(0.125)^2 (0.21) (60)}{(1.77)(10^{-2})} \\ &= 11.4 \text{ minutes} \end{aligned}$$

4. To determine the maximum strain value (ϵ_z) in the model at $\tau = 0.21$ using Figure (2) the expression is:

$$\begin{aligned}\epsilon_z &= \alpha T_i [\Sigma_z] \\ &= (22.5)(10^{-6})(598)(0.8997)(10^{-2}) \\ &= 121 \mu\text{in/in}\end{aligned}$$

5. To determine the power input to the center heater ($\dot{q}$) for Run 2

$$\begin{aligned}\dot{q} &= EI \\ &= (2.913)(0.2064) \\ &= 0.601 \text{ watts}\end{aligned}$$

where E is the voltage drop across the center heater and I is the current through the center heater.

To convert the power to rate of heat transfer

$$\begin{aligned}\dot{q} &= (0.601)(3.412) \\ &= 2.05 \text{ Btu/hr}\end{aligned}$$

6. To determine the emissivity (ϵ) for the epoxy material using the surface temperature determined from thermocouples (1) and (2) in Run 2

$$\begin{aligned}\epsilon &= \frac{\dot{q}}{2\pi r_o \epsilon \sigma T_o^4} \\ &= \frac{(2.05)}{(2)(\pi)(0.0635)(0.0703)(0.1714)(10^{-8})(555.6)^4} \\ &= 0.448\end{aligned}$$

7. To determine the thermal conductivity (K) for the epoxy material using the temperature difference between thermocouples (1) and (2) in Run 2

$$K = \frac{\dot{q} \ln(r_1/r_2)}{2\pi l (T_2 - T_1)}$$

$$= \frac{(2.05) \ln(0.732/0.602)(12)}{(2)(\pi)(0.8437)(1.92)}$$

$$= 0.47 \text{ Btu/hr-ft-}^\circ\text{F}$$

8. Determination of the axial heat loss ($\dot{q}_L$) from the ends of the test section. (The temperature gradients at the inside and outside surfaces of the test cylinder at both ends of the test section from Run 2 were used for this analysis.)

a. Heat loss from the top of the test section ($\dot{q}_L$):

(1) To determine the temperature gradients at positions (1) and (2)

$$(a) \quad \frac{\Delta T_1}{\Delta X_1} = \frac{0.858^\circ \text{F}}{0.0156 \text{ ft}}$$

$$= 54.96^\circ \text{F/ft}$$

where ΔT_1 is the difference in temperature between the thermocouples in guard set (10) at position (1) and ΔX_1 is the distance between the two guard thermocouples in guard set (10).

$$(b) \quad \frac{\Delta T_2}{\Delta X_2} = \frac{1.95^\circ \text{F}}{0.043 \text{ ft}}$$

$$= 45.35^\circ \text{F/ft}$$

where ΔT_2 is the difference in temperature between the two guard thermocouples (3) and (7) at position (2) and ΔX_2 is the distance between thermocouples (3) and (7).

(2) To determine length A from the diagram by similar triangles

$$A = \frac{(0.26)(\Delta T_1/\Delta X_1)}{(\Delta T_2/\Delta X_2 + \Delta T_1/\Delta X_1)}$$

$$= \frac{(0.26)(54.96)}{100.3}$$

$$= 0.1425 \text{ in}$$

(3) To determine the temperature gradient at the inside surface of the test cylinder by similar triangles

$$(\Delta T_i/\Delta X_i)$$

$$\frac{\Delta T_i}{\Delta X_i} = \frac{(\Delta T_1/\Delta X_1)(0.1925)}{(0.1425)}$$

$$= 74.25^\circ \text{F/ft}$$

(4) To determine the temperature gradient at the outside surface of the test cylinder by similar triangles ($\Delta T_o / \Delta X_o$)

$$\begin{aligned}\frac{\Delta T_o}{\Delta X_o} &= \frac{(\Delta T_2 / \Delta X_2)(0.148)}{0.1175} \\ &= 57.12^\circ \text{F/ft}\end{aligned}$$

(5) To determine the heat loss across the top of the test section ($\dot{q}_L$)

$$\begin{aligned}\dot{q}_L &= K_i 2\pi r dr \frac{dT}{dX} \\ &= 2\pi K_i \int_{r_i}^{r_o} \frac{dT}{dX} r dr\end{aligned}$$

and

$$\frac{dT}{dX} = C(r - r^*)$$

where C is the linear slope of the radial temperature gradient through the test cylinder.

(6) To determine the slope (C)

$$\begin{aligned}C &= \frac{\Delta T_o / \Delta X_o - \Delta T_i / \Delta X_i}{r_o - r_i} \\ &= \frac{57.12 - (-74.25)}{0.3405} \\ &= 4631^\circ \text{F/ft}^2\end{aligned}$$

Therefore

$$\frac{dT}{dX} = 4631 (r - r^*)$$

and

$$\dot{q}_L = (2\pi)(4631)K_i \int_{r_i}^{r_o} r(r - r^*) dr$$

With $r^* = 0.0512$ ft from the figure, the equation for the heat loss is integrated and the heat loss is determined to be:

$$\dot{q}_L = -0.0202 K_i \text{ Btu/hr}$$

where the minus sign means heat flow into the test section.

b. To determine the heat loss from the bottom of the test section the same approach was used as for the top and the result is:

$$\dot{q}_L = + 0.2967 K_i \text{ Btu/hr}$$

c. Therefore the total heat loss from the test section is:

$$\begin{aligned} \dot{q}_L &= (-0.0202 + 0.2967) K_i \\ &= 0.2765 K_i \text{ Btu/hr} \end{aligned}$$

out the bottom of the test section.

d. To determine the heat loss using a thermal conductivity of $0.52 \text{ Btu/hr-ft-}^\circ\text{F}$ from Matthews (6)

$$\begin{aligned} \dot{q}_L &= (0.2765)(0.52) \\ &= 0.1438 \text{ Btu/hr} \end{aligned}$$

e. To determine what percentage of the power to the center heater the heat loss is:

$$\begin{aligned} \text{Percent Loss} &= \frac{0.1438}{2.05} \\ &= 0.07 \\ &= 7\% \end{aligned}$$

9. Determination of the error in the measuring circuit.

a. The maximum uncertainty in the digital DC voltmeter is 0.1 percent of full scale for the scale being used (e).

(1) For reading the voltage through the center heater the 10 volt scale was used. Therefore, the uncertainty (e_1) is:

$$\begin{aligned} e_1 &= (0.001)(10) \\ &= 0.01 \text{ Volts} \end{aligned}$$

(2) For reading the voltage across the one ohm resistor the 1 volt scale on the DC voltmeter was used. Therefore, the uncertainty (e_2) is:

$$\begin{aligned} e_2 &= (0.001)(1) \\ &= 0.001 \text{ Volts} \end{aligned}$$

(3) For the one ohm resistor the error is one percent of one ohm. Therefore, the uncertainty (e_3) in the resistor is:

$$\begin{aligned} e_3 &= (1)(.01) \\ &= .01 \text{ ohms} \end{aligned}$$

b. Applying these uncertainties to the 4:00 AM data in Run 2:

(1) The voltage to the heater (v) would be:

$$\begin{aligned} v &= 2.913 + .01 \\ &= 2.923 \text{ volts.} \end{aligned}$$

(2) The voltage across the 1 ohm resistor (v_R) would be:

$$\begin{aligned} V_R &= 0.2064 + .001 \\ &= 0.2074 \text{ volts} \end{aligned}$$

(3) The resistance in the 1 ohm resistor (R) would be:

$$\begin{aligned} R &= 1.000 - .01 \\ &= 0.99 \text{ ohms} \end{aligned}$$

(4) Therefore the current (i) through the heater is:

$$\begin{aligned} i &= \frac{V_R}{R} \\ &= \frac{0.2074}{0.99} \\ &= 0.2095 \text{ amps} \end{aligned}$$

(5) The power (P) would be:

$$\begin{aligned} P &= (0.2095)(2.923) \\ &= 0.6124 \text{ watts} \\ P &= (0.6124)(3.412) \\ &= 2.09 \text{ Btu/hr} \end{aligned}$$

(6) The uncertainty in the power compared to the power supplied to the center heater would be:

$$\text{Percent Uncertainty} = \frac{2.09 - 2.05}{2.05}$$

$$= 0.020$$

$$= 2\%$$

APPENDIX B

MODEL MATERIAL

The composition of the model material was:

<u>Compound</u>	<u>Percent by Weight</u>
Epocast No. 10 ⁴	43
Hardener No. 927 ⁴	6
Cabosil ⁵	1
120 Mesh Aluminum Powder ⁶	50

After proper curing, this combination of materials resulted in a shrink-free casting which was easily machined. The material, when cut apart, rejoined, and cured, developed the same strength at the joint as the original part (6).

The modulus of elasticity, Poisson's ratio, and coefficient of thermal expansion of the model material showed little dependence on the temperature in the range of interest in this test. The material was also electrically nonconductive.

A detailed study of the physical properties of the material is given in a UNM report (6).

⁴Manufactured by Furane Plastic Company.

⁵Manufactured by Cabot Corporation.

⁶Manufactured by Reynolds Aluminum Company.

Some of the mechanical and thermal properties of interest in this test are:

<u>Property</u>	<u>Magnitude</u>
Young's Modulus ⁷	$1.11 \times 10^6 \text{ lb/in}^2$
Coefficient of Thermal Expansion ⁸	$22.5 \times 10^{-6} \text{ in/in/}^\circ\text{R}$
Poisson's Ratio ⁹	0.33
Thermal Diffusivity	$1.77 \times 10^{-2} \text{ ft}^2/\text{hr}$
Emissivity ¹⁰	0.45
Thermal Conductivity ⁸	$0.52 \text{ Btu/hr-ft-}^\circ\text{R}$
Density ⁹	105.2 lb/ft^3
Specific Heat ⁸	$0.28 \text{ Btu/lb-}^\circ\text{R}$

The following procedure was used to mold the model for this test.

The aluminum powder and Cabosil were carefully measured and thoroughly mixed. Then the correct amount of Epocast 10 was added to the aluminum powder and Cabosil mixture. When these components were thoroughly mixed, they were heated to 150°F . and the proper amount of hardener 927 added. The mixture was then stirred vigorously with a rotating blade to insure complete homogeneity of the material.

⁷This value was taken from Reference (16).

⁸These values were determined from the data obtained in Reference (6) for 150°F .

⁹These values were obtained from tests conducted in the ME Department at the University of New Mexico.

¹⁰This value was estimated for the experimental work since no information was available.

After adding the hardener 927 at 150°F, the pot life of the material was approximately 45 minutes. The material was poured into a mold and then placed in a vacuum chamber. The pressure was slowly reduced to 100 microns in the chamber. As the pressure was decreasing the material foamed vigorously and expanded considerably until all the trapped air in the material was released. The material continued to boil vigorously after the trapped air was released due to the low pressure. A thermometer placed in the material being evacuated showed that the temperature of the material increased as the pressure was lowered.

When 100 microns was reached the vacuum was released, and the model was allowed to set for 24 hours at room temperature and pressure. Then the model and mold were placed in a commercial oven and cured at 200°F for 24 hours. Before curing, the material was brittle and easily cracked. During cure, the material has a tendency to soften and therefore must be restrained until fully cured. After curing, the material was removed from the mold and machined to the proper size.

This method of using a high vacuum to remove the air bubbles from the material resulted in a very smooth, homogeneous model.

APPENDIX C

EQUIPMENT LIST

1. Honeywell, Electronik 15 Chromatography Recorder:
Model Y15307856-01-05-0-000-120-07(009)
Serial Number Q3643215001
2. Leeds and Northrup Portable Precision Potentiometer:
Model 8662
Serial Number 1106217
3. Type C-3 Amplifier and AC Power Supply System:
Model CA12
Serial Number 3
4. Recording Oscillograph (18 Channel):
Model 5-124
Serial Number 1081
5. Leeds and Northrup Portable Potentiometer:
Model 8690
Serial Number 1606967
6. Variable Transformers - 3
7. Vacuum System - by Scientific Engineering Laboratory
8. Twelve Volt Storage Batteries - 2

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