AN ENGINEERING EVALUATION
OF THE STATUS OF UTILIZATION OF BERYLLIUM
FROM THE VIEWPOINT OF FRACTURE MECHANICS

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This paper was presented at the Beryllium Conference, sponsored by the National Materials Advisory Board, Arlington, Virginia, March 23-25, 1970, and will be published in the conference proceedings.
Abstract - The inherent brittleness of beryllium makes it a prime candidate for the utilization of fracture mechanics information for design purposes. One obstacle to the more widespread use of fracture mechanics in the design of beryllium structures is the scarcity of information on its fracture toughness and subcritical flaw growth characteristics. A summary of the available data is presented, which indicates that Be does not compare favorably with other modern structural alloys on the basis of its static fracture properties. However, the limited amount of fatigue crack growth information available indicates that beryllium may have certain advantages in fatigue applications, and that proof testing provides a straightforward means of ensuring the integrity of cyclically loaded beryllium structures. More information on the fatigue crack growth characteristics of beryllium is required to verify this. Additional fracture toughness test results are also required, and the effects of microalloying and thermal history on $K_C$ should be investigated in detail. Information on the stress corrosion characteristics of Be is presently not available. Such tests are certainly warranted, and are required in order to assess the integrity of beryllium structures in various environments. The combined effects of fatigue and
stress corrosion also need to be investigated so that fracture mechanics concepts can be applied to the prediction of safe operating stresses and expected lifetimes of beryllium structures operating under such service conditions.
INTRODUCTION

Linear elastic fracture mechanics has proven to be of considerable use in the design of structures fabricated from high-strength materials. These materials are prone to brittle-type failures, and considerable progress has been made in recent years in the fracture testing of structural metal alloys (1-3), and using these test results for design purposes (4,5). Fracture mechanics concepts have been rather extensively used in design with steel, aluminum, and titanium alloys, but have not found wide use in work with beryllium. The inherent brittleness of beryllium makes it a prime candidate for the use of fracture mechanics, but information on the fracture toughness and sub-critical flaw growth of beryllium is sparse. The purpose of this paper is to review and draw conclusions from the information presently available, and to point out the additional data that would assist in the design of fracture-resistant beryllium structures.
REVIEW OF FRACTURE MECHANICS

The classical theory of elasticity reveals that the stress distribution near a crack tip in an elastic solid is always of the following form (6)

\[
\sigma_y = \frac{K}{(2\pi r)^{1/2}} \cos \frac{\theta}{2} (1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2})
\]

\[
\sigma_x = \frac{K}{(2\pi r)^{1/2}} \cos \frac{\theta}{2} (1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2})
\]

\[
\tau_{xy} = \frac{K}{(2\pi r)^{1/2}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2}
\]

provided that the configuration and loading are at least locally symmetric with respect to the plane of the crack. The coordinate system in the vicinity of the crack tip is shown in Figure 1.

The factor \(K\), known as the stress intensity factor, completely controls the magnitude of the stress field surrounding the crack tip. This factor is a function of the geometry of the body, crack, and loading configurations, and can be determined by application of the classical theory of elasticity. Reference 6 provides a thorough review of results obtained prior to 1965, and later results abound in the literature.

Equations 1 predict infinite stresses at the crack tip, which is unrealistic, especially for materials capable of plastic deformation. A plastic zone will form at the crack tip in such materials, as shown schematically in Figure 2. The size of the plastic zone \(r_y\), can be estimated from equations 1 by solving for the \(r\) at which \(\sigma_y = \sigma_{yp}\) (yield stress) for \(\theta = 0\). The resulting expression is

\[
r_y = \frac{1}{2\pi} \left( \frac{K}{\sigma_{yp}} \right)^2
\]

The formation of this plastic zone will increase the effective crack length from \(a\) to \(a_e\) \((7, 8)\), where \(a_e = a + r_y\); as shown schematically in Figure 2.

Since the stress distribution near a crack tip in an elastic solid (and an elastic-plastic solid with highly localized plastic deformation at the crack tip) is completely controlled by the stress intensity factor, it may be expected that fracture occurs when the stress intensity reaches a critical value, i.e. \(K = K_c\). This critical value of the stress intensity factor is called the fracture toughness. Its value for a given material can
be determined experimentally.

The fracture toughness of metals has been found to vary with thickness; the toughness decreases with increasing thickness, asymptotically approaching a minimum value at large thicknesses (1,4). This variation with thickness is attributed to a plane-stress to plane-strain transition. The minimum toughness at large thicknesses is associated with plane-strain behavior, and this value of the toughness is denoted as $K_{IC}$. Sub-critical crack growth at stress intensities below $K_{IC}$ occurs to varying degrees in most materials due to stress corrosion cracking. Experimental investigations have revealed that the stress corrosion cracking rate is dependent on the stress intensity factor of the crack, and that crack growth does not occur at stress intensities below some threshold value, denoted by $K_{ICc}$ (10).

The equations for the stress intensity factor for the two important cases of a through crack, and a part-through crack (9) in an infinite plate are shown in Figure 3. The crack length has been plastically corrected in accordance with the above discussion. Also shown in this figure, are the equations for a "leak before break" criterion (11), which is based on the observation that good service performance is obtained if the critical crack length for an applied stress equal to the yield stress is greater than twice the plate thickness. This leads to $(K_c/\sigma_{yp})^2 = B$ (B is plate thickness). On the other hand, from experience with failures of aerospace and other structures, the worst tolerable situation in practice occurs when the critical crack length at an applied stress of 1/2 the yield stress is $B/4$. This leads to $(K_c/\sigma_{yp})^2 = 0.1 B$. Hence, the length parameter $(K_c/\sigma_{yp})^2$ is important in design considerations, and provides a basis for the comparison of various materials. Such a comparison of beryllium with other more conventional steel, aluminum and titanium alloys is presented in Figure 4. The toughness and yield stress data for beryllium indicated by the cross-hatched area will be presented later. On the basis of the above discussion and the data of Figure 4, beryllium structures up to about 0.1 inch thick would be expected to perform satisfactorily, (barring dynamic effects or other mitigating factors) whereas thicknesses greater than 2-3 inch would surely be potentially troublesome.

The data of Figure 4 shows that Be does not compare favorably with the newer more conventional materials, when compared on the basis of $K_{IC}$. 
That is, for a given $\sigma_0/\rho$, Be has a lower $(K_{IC}/\sigma_{yp})^2$ than some other available materials. However, when $K_{IC}$ of Be is compared with $K_{isc}$ of other materials, Be does compare favorably. No $K_{isc}$ data is available for Be, but if it was considerably below $K_{IC}$, then this comparison would not be valid. Additional information on the stress corrosion characteristics of beryllium is necessary before a conclusive comparison of beryllium with more conventional materials is possible. However, at this time, it appears that the fracture properties of beryllium developed to date do not come up to the latest steel, titanium and aluminum alloys. On the other hand, fracture properties are but one of many properties subject to "trade-offs" in selection of materials in design, and beryllium has at least reasonable fracture toughness, which combined with other advantageous properties could make it very attractive in special applications.
FRACTURE CHARACTERISTICS OF BERYLLIUM AND FRACTURE CONTROL

Several types of information are necessary when using fracture mechanics for the implementation of a full fracture control program.

**Initial flaw sizes** - Knowledge of the initial flaw sizes is very important in the estimation of the safe working stresses and operating lifetime of structures, and is often the weak link in the utilization of fracture mechanics in design applications.

**Static and dynamic final failure properties** - Knowledge of the fracture toughness of the material, and its variation with temperature, strain rate, and material thickness is also vital.

**Stress corrosion properties** - Knowledge of the flaw growth stress corrosion characteristics is necessary in order to estimate allowable stresses and safe operating lifetimes in various environments.

**Fatigue properties** - Knowledge of the fatigue crack growth characteristics of the material is necessary in order to estimate lifetimes under cyclic loading conditions. The combined effects of environment (corrosion) and fatigue must also be considered.

Each of these items, as it relates to beryllium, will be discussed in detail, and results for other materials will also be presented for purposes of comparison.

Design rules for efficient fracture-resistant design can be formulated once the information mentioned above is available. The application of such rules to actual design has been discussed at this conference by Mitsui (12). Tiffany and Masters (5) present a more thorough exposition of the underlying concepts.

**Initial flaw sizes** - Estimates of the initial flaw size are vital in the determination of allowable stresses and safe operating lifetimes of structures. Conventional NDT techniques, such as radiography and ultrasonic inspection, are useful, but often erratic, for obtaining such estimates. Another promising technique is acoustic emission, (13-16) which relies on the detection of the low level sounds emitted by a material when it is plastically deformed. References 13-16 provide a description of typical test results and the instrumentation required to obtain them. In short, the acoustic emission from a flawed part is different from that of a flaw-free one. These differences can be used not only to detect the presence of flaws, but also to estimate their severity. Acoustic emission characteristics
of beryllium have been studied (14), and applied to the non-destructive evaluation of beryllium parts - as discussed at this conference by Cruff (17).

Proof testing is another technique for guarding against unexpectedly large initial flaws. The proofing of a structure to loads greater than anticipated in service is often a valuable means of assuring structural integrity. This technique may be especially useful in assuring the integrity of beryllium structures subjected to fatigue loading, for reasons which will be discussed later. The utilization of acoustic emission in conjunction with the proof testing further enhances the usefulness of such testing.

**Static failure properties** - In principle, the fracture toughness of a material exhibiting an elastic range can be measured knowing the crack length and load at failure for any of the many geometries for which the stress intensity factor equation is known. In practice, certain geometries are more advantageous, in that they require less material, or a lower load to break the specimen. References 1-3 provide a discussion of most useful specimen geometries and standard test procedures for fracture toughness testing of conventional high-strength metals. The procedures developed for conventional materials have been difficult to apply to testing of beryllium, and an ASTM E-24 sub-committee on this subject has been recently formed, under the chairmanship of Professor Hans Conrad of the University of Kentucky.

The introduction of a controlled crack in beryllium has been one obstacle in the measurement of fracture toughness. Some investigators have succeeded in introducing fatigue cracks at room temperature (12,18) and others have used a precracking technique that utilizes transverse compression zones (19-21) to arrest a crack introduced by wedge loading a machined notch. No data is presently available on the effect of the technique of introducing the crack on the resulting fracture toughness. Tests to assess these effects should be conducted, since if for no other reason, the precrack techniques required do not conform to ASTM recommended practice.

As mentioned earlier, the fracture toughness of metals varies with thickness. Figure 5 shows this variation for N50A beryllium (19), which is a grade of at least 99% purity. This figure shows the characteristic decrease of $K_c$ with increasing thickness, and the minimum attained at large thicknesses. $K_c$ for this material appears to be about 13 ksi-in$^{1/2}$, a rather low value for a structural metal.
Figure 6 provides a summary of data available in the literature, as compiled by J. Hurd of the Brush Beryllium Company for the ASTM sub-committee on fracture toughness testing of beryllium. These results were obtained at several laboratories, on materials with different purities and fabrication histories, and with different types of precracks. Hence, the large variation in results is not surprising. It is seen that most of the data is between 10 and 20 ksi-in$^{1/2}$, with some as high as 40 ksi-in$^{1/2}$.

Figure 7 provides the results of tests conducted at the Boeing Company on various grades of beryllium (22) which fall in the range of 10-20 ksi-in$^{1/2}$. Several interesting conclusions can be drawn from these results.

1. Mixtures of beryllium with other metals, such as Lockalloy and Metax, do not appear to have any advantage over beryllium from a fracture toughness standpoint.

2. Brake grade block can have a higher $K_c$ than generally observed for S200 beryllium. This is in line with the results reported by Terry and Ely (23) on the relative performance of aircraft brakes constructed from these two grades.

3. Aging can effect the fracture toughness. This is an expected result, considering the effects of aging on the tensile properties reported by Foos (24). The effects of various thermal histories may account for scatter in fracture toughness that has been observed between different batches of the same grade of beryllium.

Figure 8 presents the variation of the fracture toughness of S200 hot pressed block with temperature (25). The increase in $K_c$ with temperature is an expected result, and has been observed in a wide variety of materials. This data shows two sets of values on nominally the same material; one set with a consistently higher toughness than the other. Comparable scatter from batch to batch of S200 has also been observed by Harris and Dunegan (19) and Ledman (26), and may be due to microalloying and thermal history variations. Further investigations to isolate the causes of this scatter may lead to the possibility of producing material with consistently higher toughness.

Knowledge of the effect of loading rate on the fracture toughness is also desirable if not essential in assessing the suitability of a material for service conditions. A limited amount of data for beryllium is available (22,25), which indicates that beryllium is moderately strain rate sensitive.
However, results on fatigue cracked beryllium panels at McDonnell (27,28) have shown very extensive branching of running cracks, which is usually indicative of either high strain rate sensitivity or very low toughness. The poor performance of beryllium under impact conditions also indicates that this material behaves poorly at high loading rates. Additional work on the effect of loading rate on the fracture toughness of beryllium certainly appears to be warranted.

**Fatigue properties** - Data on the fatigue performance of beryllium is sparse. Some results have recently been obtained at Lawrence Radiation Laboratory on the fatigue behavior of specimens precracked by the side loading technique (19-21). The specimens were the same as those used for the work presented in reference 19, and the test results are presented in Figure 9. $K_{II}$ is the initial maximum stress intensity factor applied during the fatigue cycling. Data for 7075-T6 aluminum, and 17-7PH steel (5) are also included for comparison. These results show that the strength of beryllium is degraded only a relatively small amount by fatigue loading, so that this material is well suited for fatigue applications. In fact, the allowable cyclic stresses for Be can be higher than those for materials which are much superior in static loading applications. For example, the allowable cyclic stress for a lifetime of $10^4$ cycles and an initial flaw size of 0.2 in. is shown in Table 1 for several materials. These results indicate that allowable stresses for beryllium are comparable to those for 7075-T6 Al and 17-7PH steel in fatigue applications.

The data on Figure 9 also indicates the usefulness of proof testing in guaranteeing the structural integrity of Be parts subjected to fatigue loading. As an example, consider a structure that is proofed to 1 1/4 times its working load. Making the conservative assumption that failure is impending at the proof load, then $K_{II}/K_{IC}$ at the working load will be $1/(1.25) = 0.8$. Figure 9 shows that the S200 Be can be guaranteed to have a lifetime of nearly $10^6$ cycles, whereas 7075-T6 and 17-7PH parts subjected to the same proof to working load ratio could be guaranteed for only 1000 and 100 cycles, respectively. These techniques are especially appropriate in the lower cyclic applications ($<10^5$ cycles), but for higher cyclic lives other more detailed considerations are relevant. It has been found (10,29) that fatigue crack growth rates depend only on the stress intensity factor at the crack tip. As a first approximation,
crack growth rates depend only on the cyclic stress intensity, $\Delta K$. Measurements of $da/dn$ vs. $\Delta K$ provide more fundamental information on fatigue behavior than the $K_{1i}/K_{1c}$ vs. cycles to failure procedure. The only known data of this kind is reported by Finn, et.al. (27), and is presented in Figure 10. Typical results for steel and aluminum alloys (10) are also indicated, along with an approximate data point for hot pressed S200 obtained at LRL. This figure shows that the fatigue crack growth rate of Be for a given $\Delta K$ is considerably less than that of aluminum, but slightly greater than steel. Further it should be pointed out that on a stress or strength over density basis, $\Delta K/p$, beryllium is superior to all other metal alloys for which the (in air) fatigue crack growth properties are known. However on a strength over modulus basis the fatigue crack growth characteristics of beryllium are not especially attractive and environmental influences might further degrade beryllium on a comparative basis.

An interesting feature of the Be results is the suggestion of a threshold stress intensity factor, $K_{th}$, below which fatigue crack propagation does not occur. This threshold for various reasons is thought to be about 7 ksi-in$^{1/2}$ for Be, but additional tests should be conducted to verify this. A threshold value of $K$ has also been observed in aluminum and steel (30,31) and typical values are indicated in Figure 10, as are also typical values of $K_c$ and $K_{th}/K_c$ for steel and aluminum is typically 1/10, and fatigue crack growth rates are observed to cover several decades. However, for Be, $K_{th}/K_c = 7/12 = 0.58$, and growth rates cover only about 2 decades. The closeness of $K_{th}$ to $K_c$ accounts for the difficulty of observing fatigue cracks in Be, because stable growth occurs only in a relatively narrow range of $\Delta K$. This closeness also allows proof testing to be used to good advantage in assessing the structural integrity of cyclically loaded Be parts. The good fatigue resistance of Be was indicated at this conference by Ingels (32), who mentioned that the magnesium supports on Agena components failed during fluctuating load tests, whereas the beryllium skins did not.

**Stress corrosion properties** - No data presently exists regarding the stress corrosion cracking properties of beryllium, so some representative data for other materials will be presented. As in fatigue crack growth testing, there are two ways of presenting stress corrosion cracking data.
The first is to plot the initial applied stress intensity factor vs the time to failure under sustained load. Such a plot for 4340 steel in water is presented in Figure 11 (33). This material has a fracture toughness of 80 ksi-in$^{1/2}$, but a $K_{\text{ISCC}}$ of about 15 ksi-in$^{1/2}$. Hence if a stressed structure contains a crack where $K$ is above 15 ksi-in$^{1/2}$, it will have a finite lifetime in the presence of water, including high atmospheric humidity. Thus it is seen that a high fracture toughness is certainly not sufficient to guarantee a safe structure when environmental effects are present.

The second means of obtaining stress corrosion data is to measure the crack growth rate as a function of the applied stress intensity factor. This is a more fundamental approach. Typical data for a titanium alloy in salt water is presented in Figure 12 (34), which shows that very rapid crack growth rates occur at stress intensity levels considerably below the static fracture toughness value of 72 ksi-in$^{1/2}$. Data similar to this is required for beryllium in order to assess the integrity of beryllium structures in various environments.

Environmentally influenced fatigue crack growth - Many structures are used under service conditions in which fatigue and stress corrosion must be considered simultaneously. Wei (35,36) has developed a theoretical treatment of this problem, which assumes that the crack growth rate will be the simple superposition of the growth due to fatigue and stress corrosion operating independently. He presents experimental data that indicates the validity of this theory for high-strength steel and titanium. Figure 13 illustrates the dramatic environmental influence above $K_{\text{ISCC}}$ on fatigue crack growth in 8-1-1 titanium alloy where the shift with frequency is calculated using superposition and the data on Figure 12. The theory is not applicable to aluminum alloys in water, because of synergistic effects of corrosion and fatigue (37), see Figure 14. Aluminum alloys have been found to be immune (except in the short transverse direction) to stress corrosion cracking in water under most conditions. However, the presence of water accelerates fatigue crack growth, which indicates that in this case the corrosion and fatigue contributions are not simply additive. In steel the data takes on much the same trends as in aluminum alloys (e.g. Figure 14), however, the environmental shift appears to be predictable by superposition above $K_{\text{ISCC}}$ and synergistic below $K_{\text{ISCC}}$. 
Data on the combined effects of corrosion and fatigue on beryllium should be obtained to determine if the additive approach of the superposition method is applicable, or if synergistic effects appear. Such information must be available before fracture mechanics concepts can be applied to the prediction of safe operating stresses and expected lifetimes of beryllium structures operating under typical service conditions.
The following conclusions can be drawn regarding the status of the utilization of beryllium from a fracture mechanics standpoint:

1. Beryllium does not compare favorably with the most modern structural alloys on the basis of its static fracture properties.

2. Scatter in the fracture toughness from batch to batch of nominally identical material has been observed, which is probably due to variations in microalloying and thermal history. A thorough study of the effects of these parameters should be conducted, and may lead to the production of consistently tougher material.

3. Standard fracture toughness testing procedures for beryllium should be developed, and effects of material texture and method of introducing the precrack should be investigated.

4. More data on the effect of loading rate on the fracture toughness of beryllium should be obtained.

5. Beryllium appears to be relatively resistant to fatigue crack growth. It appears to have a high threshold level for fatigue crack growth, which would account for its good fatigue characteristics.

6. Beryllium compares favorably from a fatigue standpoint with some conventional structural metals.

7. The closeness of the apparent threshold stress intensity for fatigue crack growth to $K_c$ might allow proof testing to be used to good advantage in guaranteeing the integrity of cyclically loaded beryllium structures.

8. More fatigue crack growth data needs to be obtained on beryllium, and the presence of the fatigue threshold should be investigated in detail.

9. Mixtures of beryllium with other metals, such as Lockalloy and Matex, do not have any advantage over beryllium from a fracture toughness standpoint.

10. No fracture mechanics data on the stress corrosion cracking characteristics of beryllium presently exists. Such data is required in order to assess the integrity of beryllium structures operating in various environments.

11. Investigations of the combined effects of fatigue loading and stress corrosion cracking must be conducted before fracture mechanics concepts can be applied to the prediction of safe operating stresses and expected lifetimes of beryllium structures operating under service conditions.
REFERENCES


(18) W. F. Brown, NASA Lewis Research Center, Cleveland, Ohio, private communication


(20) D. O. Harris, "Crack Extension in Transversely Loaded Elastic Plates", PhD Dissertation, Department of Applied Mechanics, Stanford University, Stanford, Ca. (May 1969)


(26) J. L. Ledmen, "Fracture Toughness of Beryllium", Beryllium Sub WOG-JWOG Sandia Corporation, Albuquerque, New Mexico (May 1968)


(30) B. M. Linder, "Extremely Slow Crack Growth Rates in Aluminum Alloy 7075-T6", MS Thesis, Lehigh University (1965), (see alternatively Ref. 10)


TABLE 1. Estimation of allowable stress for fatigue loading.

Arbitrarily select lifetime of $10^6$ cycles, and initial flaw size of 0.2 inch ($a_i = 0.1$ in)

$$K_{II} = \sigma \left(\pi a_i\right)^{1/2} = 0.544\sigma$$

$$\sigma = \frac{K_{II}}{0.544} = 1.78K_{II}$$

let $k = K_{II}/K_{IC}$ for $10^6$ cycles

then

$$\sigma = 1.78kK_{IC}$$

<table>
<thead>
<tr>
<th>Material</th>
<th>$k$</th>
<th>$K_{IC}$ (ksi-in$^{1/2}$)</th>
<th>$\sigma$ (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7075-T6 Al</td>
<td>.5</td>
<td>25</td>
<td>22</td>
</tr>
<tr>
<td>17-7PH Steel (long.)</td>
<td>.36</td>
<td>55</td>
<td>35</td>
</tr>
<tr>
<td>(weld)</td>
<td>.36</td>
<td>31</td>
<td>20</td>
</tr>
<tr>
<td>S200 Hot Pressed Block</td>
<td>.9</td>
<td>12</td>
<td>19</td>
</tr>
<tr>
<td>McDonnell Hot Rolled Sheet (.038&quot; thick, Ref. 27)</td>
<td>—</td>
<td>-35</td>
<td>36 (K_{II} = 20)</td>
</tr>
</tbody>
</table>
APPENDIX

Legend for Symbols of Figure 6

Sample legends

- Notch as-machined
- Wedge crack, Dunegan technique
- Notch fatigued

- Hot Pressed
- Forged
- Sheet

○ Harris and Dunegan (UCRL)

△ Harrod, et al (Westinghouse)

○ Ledman (Sandia)

▽ NASA Cleveland

□ Akron-Brush
also

Brush

◆ Finn, et al (McDonnell)

◇ Greene (Lockheed)

SEN-tension test max. load data
WOL-tension test max. load data
SEN-3 point bend intercept data only
SEN-3 point bend max. load data only; intercept calculation data available
SEN-tension test max. load data
SEN-tension test max. load data
CENTER-NOTCH-tension test max. load data
WOL-tension test max. load data

SEN-single edge notched specimen
WOL-Wedge opening loading

All data represented in Figure 6 were obtained from room temperature testing. No heat treatment data were included. Test rates are unspecified in Figure 6.
FIGURE CAPTIONS

FIGURE 1. Coordinate system in the vicinity of a crack tip.

FIGURE 2. Development of plastic zone and increase in effective crack length.

FIGURE 3. Stress intensity for through and part-through cracks, and equations for leak-before-break.

FIGURE 4. Fracture toughness comparison of beryllium with other structural metals.

FIGURE 5. Variation with thickness of the fracture toughness of N50A beryllium, from ref. 19.

FIGURE 6. Summary of fracture toughness data available on beryllium, prepared by J. Hurd for ASTM subcommittee on fracture testing of beryllium. See appendix for key to symbols.

FIGURE 7. Fracture toughness of various grades of beryllium and associated materials, from ref. 22.

FIGURE 8. Variation with temperature of the fracture toughness of hot pressed S200 beryllium block, from ref. 25.

FIGURE 9. $K_{\text{c}}/K_{\text{c_s}}$ vs. cycles to failure for various materials. Specimen numbers indicated by data points.

FIGURE 10. Fatigue crack growth rates in beryllium, along with typical results for steel and aluminum alloys. Be data from ref. 27.

FIGURE 11. Time to failure under sustained load for 4340 steel in water showing the plateau at $K_{\text{isc}}$, from ref. 33.

FIGURE 12. Stress corrosion cracking rates for mill annealed 8Al-1 Mo-1 V titanium alloy sheet in 3 1/2% aqueous NaCl solution, from ref. 34. $K_c = 72$ ksi-in$^{1/2}$

FIGURE 13. Environmental effects on fatigue crack growth in 8-1-1 titanium alloy, from ref. 36.

FIGURE 14. Environmental effects on fatigue crack growth in 7075-T6 aluminum alloy, from ref. 37.
Crack
Real crack

Equivalent elastic crack

\[ K = \text{applied crack tip (surrounding) stress field intensity} \]

\[ a_E = a_{\text{effective}} \]

\[ r_y = \frac{K^2}{2\pi a_{\text{yp}}} \]

\[ \sigma = \frac{K}{\sqrt{2\pi r}} \]
At fracture

\[ K = \sigma \sqrt{\pi \sigma_E} \]

\[ \sigma_E = \sigma + \frac{K^2}{2 \pi \sigma_{yp}} \]

At fracture

\[ K = \sigma \sqrt{\pi \left( \sigma + \frac{K^2}{2 \pi \sigma_{yp}} \right)} \Rightarrow K_{CR} \]

or

\[ K_{CR}^2 = \frac{\left( \sigma_{yp} \right)^2}{\left[ 1 - \frac{1}{2} \left( \frac{\sigma}{\sigma_{yp}} \right)^2 \right]} \cdot \alpha \]

"Leak before break" \[ \sigma = \sigma_{yp}, \alpha = B \Rightarrow \]

Worst tolerable situation \[ \sigma = \frac{\sigma_{yp}}{2}, \alpha = \frac{B}{4} \Rightarrow \]

\[ \left( \frac{K_{CR}}{\sigma_{yp}} \right) \approx B \]

\[ \left( \frac{K_{CR}}{\sigma_{yp}} \right) \approx 0.1B \]
Paris Figure 4
Paris Figure 5
Figure 6

The graph shows a scatter plot with the thickness in inches on the x-axis and the K (kelvin) value on the y-axis. The data points are distributed across the range of thickness from 0.1 to 1.0 inches and K values from 0 to 50 kelvins.
Fracture toughness — $K_{IC}$, ksi-in. $^{1/2}$

- IS ingot sheet (.05 - .09)
- Older PR-20 (.04)
- Newer PR-20 (.07)
- Extrusion (.025)
- Lockalloy (.05)
- Matex (.065)
- Brake grade block QMC 5117
- Brake grade block RR 233
- Block RR 233 after aging

Boeing
\begin{itemize}
\item WOL, 1 in. thick
\item DCB, 1/4 in. thick
\end{itemize}

\text{Hot pressed S200, Westinghouse}
17-7PH Steel, various strength levels, data of Tiffany and Masters, ASTM STP 381

-12
-11
-S200 Beryllium LRL data
-7075-T6 aluminum LRL data

Paris Figure 9
Paris Figure 10
\( K_{IC} \) (ksi√in) vs. Time to fracture (min)

- \( K_{IC} = 15 \) at 0.5 min
- Time to fracture: 0 to 1000 min

Paris Figure 11
Specimen No. 1
Specimen No. 2:
  First test
  Second test
Specimen No. 3:
  First test
  Second test

Crack growth rate, $\Delta 2a/\Delta t$ (\text{\textmu in./sec})

Stress-intensity factor, $K$ (ksi-Vin.)

Paris Figure 12


Ti-8Al-1Mo-1V alloy
salt water at RT

Estimated baseline value

$K_{\text{max}}$ ksi/$\text{in.}$

d$/dN$ (in./cycle)

0.5 cps

30 cps

Paris Figure 13
\[ \frac{d}{d\varepsilon} \{ (1-R) K_c - \Delta K \} \text{ Kg. mm}^{-1/2} \text{ kilocycles}^{-1} \]

\[ \Delta K - \text{kg. mm}^{-3/2} \]