RESONANCE CROSS SHIELDING
IN REACTOR ANALYSIS

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C. W. Lindenmeier
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July 1970

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RESONANCE CROSS SHIELDING IN REACTOR ANALYSIS

By

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ABSTRACT

Accidental overlap between the resonances of distinct isotopes is ignored in the present structure factor method. To ascertain the effect of accidental overlap and to develop possible modifications of the structure factor method, we have analyzed the specific overlap between Pu-239 and U-238.

The basic approach was to modify ETOX subroutines enabling them to generate "cross shielded" f-factors that account for self shielding and accidental overlap. A comparison between the cross shielded and the normal self-shielded f-factors then reveals any accidental overlap effects.

Significant overlap effects were not detected between resolved and unresolved resonances nor between unresolved and unresolved resonances. However, effects as large as 6% resulted from the overlap between the resolved resonances of U and Pu.

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I. INTRODUCTION

The structure factor method⁽¹⁾ provides a fast, reliable means for generating multi-group cross sections.⁽²⁾ Basic to the method are the assumptions that:

1. The slowing down flux as a function of lethargy is proportional to the reciprocal of the total cross section for the mixture of isotopes considered.
2. Over the lethargy range that the cross section of a particular isotope shows large variations, the cross sections of all other isotopes are slowly varying.

Since the second assumption is the weakest, particularly for mixtures of isotopes whose cross sections exhibit overlapping resonance structure, we have extended the structure factor method to include material structure factors where the material consists of a mixture of isotope and elements. While this approach permits the direct treatment of structural materials and coolants, there remains the need for treating mixtures of fissile and fertile isotopes, since the isotopic ratios of such isotopes will vary with burnup. We are led to attempt to evaluate the effect on a group cross section of an isotope by the presence of a second isotope when both have resonance structure. We will refer to this effect as cross shielding which we will treat as an additional effect to the self shielding that is already accounted for by the unmodified structure factor method.

To determine the effect of cross shielding we have calculated the structure factors for mixtures of plutonium and uranium by treating both cross sections in complete detail while evaluating the respective group constants and by using the usual structure factor approach where only one isotope is treated in detail with average cross sections used for the other. The Battelle-Northwest codes ETOX⁽³⁾ and 1DX⁽⁴⁾ were modified as required to perform the calculation. Cross shielding effects were found to be insignificant between resolved and unresolved cross sections. However, for the cases studied, effects as large as 6% were found due to the cross shielding between the resolved resonance regions of uranium and plutonium.

II. CROSS SHIELDING BETWEEN RESOLVED AND UNRESOLVED RESONANCES

A. MODEL

In ETOX, the calculations in the unresolved resonance region are based on the following equations for the infinitely dilute and self-shielded micro-group cross sections for any energy group E^* ,

$$\langle \sigma_x(E^*) \rangle = \frac{\int_{E_1}^{E_2} \frac{\sigma_x(E,T) dE}{E}}{\int_{E_1}^{E_2} \frac{dE}{E}} \quad (x = f, c, e,) \quad (2.1)$$

$$\langle \sigma_t(E^*) \rangle = \langle \sigma_f(E^*) \rangle + \langle \sigma_c(E^*) \rangle + \langle \sigma_e(E^*) \rangle \quad (2.2)$$

$$\overline{\sigma_x(E^*)} = \frac{\int_{E_1}^{E_2} \frac{\sigma_x(E,T) dE}{[\sigma_t(E,T) + \sigma_o] E}}{\int_{E_1}^{E_2} \frac{dE}{[\sigma_t(E,T) + \sigma_o] E}} \quad (2.3)$$

$$\overline{\sigma_t(E^*)} = \frac{\int_{E_1}^{E_2} \frac{dE}{[\sigma_t(E,T) + \sigma_o] E}}{\int_{E_1}^{E_2} \frac{dE}{[\sigma_t(E,T) + \sigma_o]^2 E}} - \sigma_o \quad (2.4)$$

where f represents fission, c capture, e elastic and t total.

The f-factors for any energy E^* are calculated by

$$f_x(E^*, T) = \frac{\overline{\sigma_x(E^*)}}{\langle \sigma_x(E^*) \rangle} \quad (x = f, c, e, t) \quad (2.5)$$

Collapsed group cross sections and f-factors are then obtained by averaging values calculated at equal lethargy spaced points (the E^* values) in the group.

Normally, σ_0 is assumed to be constant over the group interval. In other words, any cross section variation with energy of any isotope included in the σ_0 will not be reflected in the $f_x(E^*, T)$.

Suppose we are interested in calculating Pu-239 f-factors in the portion of the unresolved region of Pu-239 which is still in the resolved region of U-238. We propose to make U-238's contribution to σ_0 energy dependent and compare the resulting Pu-239 f-factors with the case when σ_0 is constant.

The URES subroutines were extracted from ETOX and modified to operate alone and to accept an energy dependent σ_0 . Briefly, the latter was accomplished by allowing the unresolved calculations to be done at a hundred equal lethargy spaced points within the coarse group with each energy point capable of having a different σ_0 .

Since we are interested in demonstrating whether or not cross shielding produces any significant effects, the above objective will be limited to investigating only group 20 (see Table II.1) by using a simplified (but flexible) model for the "U-238 resonances" in group 20. This model is shown in Figure 2.1.

Figure 2.1 Hypothetical U-238 total cross section for group 20 (454 ev to 749 ev). The dashed resonance is an example for the case of wide resonances (see test).

TABLE II.1

Group Structure

<u>Group</u>	<u>Lower Energy</u>	<u>Upper Energy</u>
1	6.065 Mev	10.00 Mev
2	3.679	6.065
3	2.231	3.679
4	1.353	2.231
5.	.8208	1.353
6	.4979	.8208
7	.3020	.4979
8	.1832	.3020
9	.1111	.1832
10	67.38 Kev	.1111
11	40.87	67.38 Kev
12	24.79	40.87
13	15.03	24.79
14	9.119	15.03
15	5.531	9.119
16	3.355	5.531
17	2.035	3.355
18	1.234	2.035
19	748.5 ev	1.234
20	454.0	748.5 ev
21	275.4	454.0
22	167.0	275.4
23	101.3	167.0
24	61.44	101.3
25	37.27	61.44
26	22.60	37.27

In order to make this study representative, ZPR III Assembly 52 was analyzed with 1DX to obtain the following for group 20:

$$\sigma_o^{239} = 225 \text{ barns}$$

$$N^{239} = .001799 \text{ atoms/b-cm}$$

$$N^{238} = .007208 \text{ atoms/b-cm} \quad (2.6)$$

$$\langle \sigma_t^{238} \rangle = 20 \text{ barns}$$

$$\sigma_o^{238} = 50 \text{ barns}$$

As will be seen these numbers are required to obtain typical examples of the σ_o variation with energy.

The Pu-239 f-factors were calculated by the modified subroutines for three cases: (1) Constant Case (2) Narrow Resonance Case (3) Wide Resonance Case.

The Constant Case consists of running the modified subroutines with $\sigma_o = 225$ at every point in group 20. This is the control case and it represents the usual ETOX-1DX results.

The Narrow Resonance Case consists of removing "U-238's" contribution from σ_o and making the contribution energy dependent according to Figure 1. This was done in the following manner:

By definition,

$$\sigma_o^{239} = \frac{\sum_{i=1}^M N^i F_t^i \langle \sigma_t^i \rangle + N^{238} F_t^{238} \langle \sigma_t^{238} \rangle}{N^{239}} \quad (2.7)$$

where

$$F_t \langle \sigma_t \rangle \equiv f_f \langle \sigma_f \rangle + f_c \langle \sigma_c \rangle + f_e \langle \sigma_e \rangle$$

In order to obtain U-238's contribution, one must calculate F_t^{238} from Figure 1 by using the following formula

$$F_t^{238} = \frac{1}{\langle \sigma_t^{238} \rangle} \frac{\int \frac{\sigma_t^{238}}{\sigma_t^{238} + \sigma_o^{238}} du}{\int \frac{du}{\sigma_t^{238} + \sigma_o^{238}}} \quad (2.8)$$

With $\sigma_o^{238} = 50$ and σ_t^{238} given by Figure 1 (note that Figure 1 was constructed so that $\langle \sigma_t^{238} \rangle = 20$), it was found that $F_t^{238} = .769$. Therefore, we can write

$$\begin{aligned} \sum_{i=1}^M N^i F_t^i \langle \sigma_t^i \rangle &= (.001799)(225) - (.007208)(.769)(20) \\ &= .294 \end{aligned} \quad (2.9)$$

We are now prepared to calculate $\sigma_o(E)$ which is simply

$$\sigma_o(E) = \frac{(.294) + (.007208) \sigma_t^{238}}{.001799} \quad (2.10)$$

with σ_t^{238} as given by Figure 1. This is the energy dependent σ_o used for the Narrow Resonance Case.

For the Wide Resonance Case, the resonances of Figure 1 were made twice as wide and half as high (see the dashed profile in Figure 1). For this case F_t^{238} was calculated to be .867 so that

$$\sum_{i=1}^M N^i F_t^i \langle \sigma_t^i \rangle = .280 \quad (2.11)$$

and

$$\sigma_o(E) = \frac{.280 + (.007208) \sigma_t^{238}}{.001799} \quad (2.12)$$

This is the energy dependent σ_o used for the Wide Resonance Case.

B. RESULTS

The Pu²³⁹ f-factors for the three cases are shown in Table II.2.

TABLE II.2

<u>Pu-239 f-factors</u>				
<u>Case</u>	<u>f_f</u>	<u>f_c</u>	<u>f_e</u>	<u>f_t</u>
Constant	.879	.814	.945	.861
Narrow Resonances	.878	.812	.944	.860
Wide Resonances	.879	.813	.945	.861

It's obvious that making the U-238 contribution to σ_0 energy dependent has no significant effect on the Pu-239 f-factors for group 20. In other words, for this reactor composition, the U-238 resonances have no significant shielding effect on the unresolved resonances of Pu-239.

It is assumed that the resolved-unresolved cross shielding effects of this study should be about as large as, or larger than those which could be obtained from any other reasonable combination of group and composition. Group 20 exhibits a relatively uniform distribution of a relatively large number of well-developed U-238 resonances. The composition of Assembly 52 has a relatively large amount of U-238 which possibly affects the Pu-239 f-factors (U-238/Pu-239 = 4).

One could proceed to refine the calculations to better simulate the actual U-238 resonances but we believe that the models used cover a range that brackets the actual physical situation.

For the above reasons it is believed that the "negative" results with group 20 are sufficient to conclude that the resolved-unresolved cross shielding is a small effect which can usually be ignored.

C. UNRESOLVED-UNRESOLVED

The above results lead us to believe that no significant effects can be expected from unresolved-unresolved cross shielding.

If one wishes to account for accidental overlap in the unresolved regions, he will have to start at the point where he constructs the statistical description of an isotope and construct a statistical description for the particular mix of isotopes in which he is interested. Such a consideration is beyond the scope of the present study.

III. CROSS SHIELDING BETWEEN RESOLVED RESONANCES

A. MODEL

We now investigate what effect resolved U-238 resonances may have on the Pu-239 f-factors in the resolved region (and vice versa).

The ETOX resolved resonance calculations are based on the following equations:

$$f_x = \frac{\int \frac{dE}{E}}{\int \frac{\sigma_x(E,T) dE}{E}} \cdot \frac{\int \frac{\sigma_x(E,T) dE}{[\sigma_t(E,T) + \sigma_o] E}}{\int \frac{dE}{[\sigma_t(E,T) + \sigma_o] E}} \quad (x = f, c, e,) \quad (3.1)$$

$$f_t = \frac{\int \frac{dE}{E}}{\int \frac{\sigma_t(E,T) dE}{E}} \cdot \left\{ \frac{\int \frac{dE}{[\sigma_t(E,T) + \sigma_o] E}}{\int \frac{dE}{[\sigma_t(E,T) + \sigma_o]^2 E}} - \sigma_o \right\} \quad (3.2)$$

where

$$\sigma_o = (N^{238} \sigma_t^{238} + \sum N' \sigma_t') / N^{239} \quad (\text{sum does not include Pu-239 nor U-238}) \quad (3.3)$$

and where the integrals are over the energies of a group. As before, ETOX considers σ_o as constant over the group.

We are again interested in making a part of σ_o energy dependent. The resolved resonance subroutines of ETOX were extensively modified to operate alone and to calculate f-factors according to the following equations,

$$f_x^{239} = \frac{\int \frac{dE}{E}}{\int \frac{\sigma_x^{239}(E,T) dE}{E}} \cdot \frac{\int \frac{\sigma_x^{239}(E,T) dE}{[\sigma_t^{239}(E,T) + \frac{N^{238}}{N^{239}} \sigma_t^{238}(E,T) + \sigma_o^p] E}}{\int \frac{dE}{[\sigma_t^{239}(E,T) + \frac{N^{238}}{N^{239}} \sigma_t^{238}(E,T) + \sigma_o^p] E}} \quad (3.4)$$

$$f_t^{239} = \frac{\int \frac{dE}{E}}{\int \frac{[\sigma_f^{239}(E,T) + \sigma_c^{239}(E,T) + \sigma_e^{239}(E,T)] dE}{E}} \quad (3.5)$$

$$\left\{ \frac{\int \frac{dE}{[\sigma_t^{239}(E,T) + \frac{N^{238}}{N^{239}} \sigma_t^{238}(E,T) + \sigma_o^p] E}}{\int \frac{dE}{[\sigma_t^{239}(E,T) + \frac{N^{238}}{N^{239}} \sigma_t^{238}(E,T) + \sigma_o^p]^2 E}} - \sigma_o^p \right\}$$

where

$$\sigma_o^p = \frac{\sum N^i \sigma_t^i}{N^{239}} \quad (\text{sum does not include U-238 nor Pu-239}). \quad (3.6)$$

Equation (3.5) incidentally resulted when equation (3.4) was programmed. When comparing with equation (3.2), we find that equation (3.5) is wrong because U-238's contribution is not included in the subtraction. No effort was made to correct this (it's not a trivial task), and the total f-factors were simply ignored.

With equation (3.4) programmed, we are now prepared to generate "U-238 shielded" (or cross shielded) Pu-239 f-factors for any particular composition characterized by a "background" of σ_o^p and the ratio $R = N^{238}/N^{239}$.

LDX can also calculate Pu-239 f-factors for the above compositions. However, it does so via an iteration on Pu-239 and U-238 f-factors which were calculated using equations (3.1) and (3.2) with an arbitrary set of σ_o values. We wish to show how these self-shielded LDX f-factors differ from the cross shielded f-factors obtained above. This comparison proved

to be somewhat difficult. The problem is forcing the σ_o of equation (3.1) and the σ_o^P of equation (3.4) to reflect the same composition. The following scheme was adopted.

Let the composition be characterized by the following two quantities

$$R = \frac{N^{238}}{N^{239}} \text{ and } \sigma_o^m = \frac{\sum N^i \sigma_t^i}{N^{238} + N^{239}} \quad (3.7)$$

where σ_o^m is meant to represent the "background" of all other materials in the composition except Pu-239 and U-238. With these definitions the σ_o^P of equation (3.4) is given by

$$\sigma_o^P = (R + 1) \sigma_o^m \quad (3.8)$$

This equation was also programmed into the modified subroutines. A typical run to obtain cross shielded f-factors is to assume a particular U-238/Pu-239 ratio (R) and give σ_o^m a set of arbitrary values. This represents a range of compositions with differing "backgrounds" but having the same relative amounts of U-238 and Pu-239. Of course, additional runs are made to obtain similar results for different U-238/Pu-239 ratios.

In order to obtain the self-shielded F-factors for these same compositions, a special 1DX setup was utilized. First, three isotopes were generated, U-238, Pu-239 and Fake.

The modified subroutines were used to generate self-shielded U-238 and Pu-239 f-factors for an adequate set of σ_o values (equation (3.4) becomes equation (3.1) by simply setting the appropriate atom density to zero). This was necessary because we wanted a common base for both methods. That is, the actual integrals in equation (3.4) are broken up in a manner depending on the location of resonances and since we now have both U-238 and

Pu-239 resonances, the cross sections and self-shielded f-factors are slightly different than when U-238 and Pu-239 resonances were considered separately.

The Fake isotope was created as having no f-factors but having a constant capture cross section equal to 1 barn in every group. Its function will become evident shortly.

Now, (in setting up the LDX runs) if we let the "background" $\Sigma N' \bar{\sigma}_t' = N^f \sigma_t^f = N^f$ and if we limit $N^{238} + N^{239} = 1$, we can easily vary N^{238} , N^{239} , and N^f to obtain the self-shielded f-factors for any desired N^{238}/N^{239} ratio and $\sigma_o^m = N^f$ value. In other words by inputting the N^{238} , N^{239} , and N^f , which describe composition (3.7), LDX can now compute the proper σ_o with which it will find the appropriate self-shielded f-factors.

At this point, it was discovered that the LDX σ_o and temperature interpolation schemes could be improved. See Appendix A.

B. RESULTS

Resolved-resolved cross shielding effects are possible in groups 22 to 26 but for brevity we will discuss only some of the results for group 24.

In Figures 3.1 through 3.6, the dashed curves are the self-shielded f-factors as calculated by LDX while the solid curves are the cross shielded f-factors as calculated by the modified RRES subroutines. Since the reader may not be so familiar with σ_o^m , when possible, the corresponding LDX σ_o values are included in parenthesis on the figures.

Figure 3.1 is a plot of Pu-239 f-factors versus σ_o^m . The temperature is 300°K and the U-238/Pu-239 ratio is 1. The greatest difference between the self- and cross-shielded f-factors approaches 6% and occurs around $\sigma_o^m = 500$ barns.

Figure 3.2 is a plot of Pu-239 f-factors versus a/o U (the percent of U + Pu atoms that are U atoms). The temperature is 300°K and the "background" is $\sigma_o^m = 10$ barns. The differences between the self- and cross-shielded f-factors generally increases as the percentage of U increases.

Figure 3.1: Pu-239 f-factors vs σ_O^m .
 (1DX σ_O values are in parentheses)

Figure 3.2: Pu-239 f-factors vs a/o U.
 (1DX σ_o values are in parentheses)

Figure 3.3 is a plot of Pu-239 f-factors versus temperature. The "background" is $\sigma_0^m = 10$ barns and the U-238/Pu-239 ratio is 1. It appears that the differences increase with temperature.

Of course, while U-238 is affecting Pu-239 f-factors, Pu-239 is affecting U-238 f-factors. The modified RRES subroutines are easily changed so that they will calculate "Pu-239 shielded" U-238 f-factors. The U-238 complements to Figure 3.1 through 3.3 are shown in Figures 3.4 through 3.6.

We note an unexpected result with f_e^{238} . The self-shielded value can, at times, be smaller than the cross shielded value. Appendix B contains a simple numerical example illustrating that this behavior is possible. This behavior may also result from the interference portion of the U-238 elastic resonances. Such behavior will definitely complicate any scheme to modify 1DX to account for cross-shielding.

Figure 3.5 reveals that U-238 self- and cross-shielded f-factor differences become large as the percentage of Pu-239 increases.

The large discrepancies on the right hand sides of Figures 3.2 and 3.5 are of minor significance since they occur for mixtures where there is the least percentage of the isotope. This emphasis can be avoided by constructing and comparing the self- and cross-shielded f-factors for each mix of U-238 and Pu-239. Additional changes to the modified RRES subroutines allowed calculations of the cross-shielded mix f-factors. These results are solid curves of Figure 3.7. The self-shielded mix f-factors were constructed from the 1DX U-238 and Pu-239 f-factors in the following manner.

$$f_x^{\text{mix}} = \frac{\alpha f_x^{239} \langle \sigma_x^{239} \rangle + \beta f_x^{238} \langle \sigma_x^{238} \rangle}{\alpha \langle \sigma_x^{239} \rangle + \beta \langle \sigma_x^{238} \rangle} \quad (x = f, c, e,) \quad (3.9)$$

Figure 3.3: Pu-239 f-factors vs temperature.
 (1DX σ_0 values are in parentheses)

Figure 3.4: U-238 f-factors vs σ_O^m .
 (1DX σ_O values are in parentheses)

Figure 3.5: U-238 f-factors vs a/o U.
 (1DX σ_o values in parentheses)

Figure 3.6: U-238 f-factors vs temperature.
 (1DX σ_0 values are in parentheses)

Figure 3.7: Mix f-factors vs N^{238}/N^{239} ratio.

where α and β are the respective fractions of plutonium and uranium, and the angle brackets designate infinitely dilute values. These results are shown in the dashed curves of Figure 3.7.

Figure 3.7, which is essentially a combination of figures 3.2 and 3.5, shows that differences of the order of 6% can exist for certain mixes of U and Pu. Note that on the left, the f-factors terminate on the plutonium f-factors while on the right they terminate on the uranium f-factors (except for the fission f-factors which approach 1 on the right, since the fission cross section for uranium is zero in this group).

IV. CONCLUSIONS

This study indicates that as anticipated the effects of cross shielding between two isotopes within a mixture is not fully compensated for by the unmodified structure factor method. However, these effects are noticeable only in the cases where respective resolved resonances overlap. Where accuracies in group cross sections of better than ten percent are required the groups spanning the resolved resonance region should be modified to accommodate for cross shielding effects. While for structural materials this may be accomplished by calculating the f-factors for the appropriate mixture of isotopes directly from ENDF/B, a more sophisticated approach is desirable for the fissile and fertile isotopes. Design and evaluation of appropriate approaches constitute the next phases of this study.

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APPENDIX A

LDX INTERPOLATION SCHEMES

Equations (3.1) and (3.2) were used to calculate self-shielded Pu-239 f-factors for group 24. Many values of σ_0 were used so that we could produce accurate curves representing the variation of Pu-239 f-factors with σ_0 . These curves are the solid lines of Figure A-1.

When LDX is used, f-factors are read in for a maximum of six σ_0 's and three temperatures. Thus, LDX must interpolate on σ_0 and temperature to obtain intermediate values. When f-factors for $\sigma_0 = .002, 10, 100, 200,$ and 1000 and $T = 300, 900,$ and 2100 are read in, LDX generated the circled points of Figure A-1. It is obvious the LDX interpolated values depart from the actual f-factor values.

This was not acceptable and LDX was modified to accept Lagrange interpolation schemes for both the temperature and σ_0 interpolations. The new LDX values are the triangles of Figure A-1.

The results appear to be satisfactory so the new interpolation schemes were used in this study.

Figure A-1: A Check of LDX Interpolation Schemes.

APPENDIX B

CROSS SHIELDING EFFECTS

We want to show, with a simple numerical example, that the relative positions of the resonances of two isotopes can cause the cross shielded f-factors to be greater than or less than the self-shielded f-factors.

Consider the following representation of the resonances of two hypothetical isotopes in some energy group.

The self-shielded f-factor for isotope 1 is (assume a background of 10 and equal densities)

$$f_1^S = \frac{1}{\langle \sigma_1 \rangle} \frac{\int \frac{\sigma_1 du}{\sigma_1 + \bar{\sigma}_2 + \sigma_0}}{\int \frac{du}{\sigma_1 + \bar{\sigma}_2 + \sigma_0}} \approx \frac{1}{40} \frac{(4) \left(\frac{\Delta u}{8} \right) \left(\frac{80}{80 + 15.6 + 10} \right)}{(4) \left(\frac{\Delta u}{8} \right) \left(\frac{1}{80 + 15.6 + 10} \right) + (4) \left(\frac{\Delta u}{8} \right) (15.6 + 10)} \approx .39$$

The cross shielded f-factor for isotope 1 is

$$f_1^D = \frac{1}{\langle \sigma_1 \rangle} \frac{\int \frac{\sigma_1 du}{\sigma_1 + \sigma_2 + \sigma_o}}{\int \frac{du}{\sigma_1 + \sigma_2 + \sigma_o}} = \frac{1}{40} \frac{(4) \left(\frac{\Delta u}{8}\right) \left(\frac{80}{80 + 10}\right)}{(\Delta u) \left(\frac{1}{80 + 10}\right)} = 1$$

Suppose now the resonances of isotope 2 were moved to the left until they coincided with those of isotope 1. The self-shielded f-factor for isotope 1 is still .39. However, the cross shielded f-factor for isotope 1 becomes

$$f_1^{D'} = \frac{1}{\langle \sigma_1 \rangle} \frac{\int \frac{\sigma_1 du}{\sigma_1 + \sigma_2 + \sigma_o}}{\int \frac{du}{\sigma_1 + \sigma_2 + \sigma_o}} = \frac{1}{40} \frac{(4) \left(\frac{\Delta u}{8}\right) \left(\frac{80}{80 + 80 + 10}\right)}{(4) \left(\frac{\Delta u}{8}\right) \left(\frac{1}{80 + 80 + 10}\right) + (4) \left(\frac{\Delta u}{8}\right) \left(\frac{1}{10}\right)} \approx .11$$

Thus

$$f_1^D > f_1^S > f_1^{D'} .$$

From these two examples we see that cross shielding effects may be either negative or positive.

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