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Monte Carlo Simulation of Neutral Beam Injection into Fusion Reactors*

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Abstract

Motivations and techniques for the Monte Carlo computer simulation of energetic neutral beam injection for fusion reactors are described. The versatility of this approach allows a significantly more sophisticated treatment of charge transfer collision phenomena and consequent effects on engineering design than available from prior work. Exemplary results for a mirror Fusion Engineering Research Facility (FERF) are discussed.

Monte Carlo Simulation of Neutral Beam Injection into Fusion Reactors

Introduction

It is anticipated that fusion power reactors based on magnetic mirror plasma confinement will require the continuous injection of high-energy particle beams in order to maintain the plasma against population and energy losses.⁽¹⁻³⁾ The desire to increase the plasma confinement time is best served by particle injection nearly perpendicular to the confining magnetic field so that the particles start out as far away (in phase space) from the mirror loss-cones as possible. Cross-field penetration of the injected beam then requires that the beam be primarily composed of neutral atoms. The general arrangement of the neutral-beam injector system is illustrated schematically in Figure 1.

A fraction of the neutral-beam population injected into the target plasma will traverse the plasma without interaction, and this fraction will then impinge on the opposite wall or be recovered via a direct collection unit. The remainder of the original injected population will undergo competing electron-ionization, ion-ionization or ion-atom charge-transfer interactions with the plasma. Any ions formed in these collisions will generally be trapped in the plasma. Unless later generations of charge-transfer produced neutrals are reionized by subsequent collisions, they will escape the plasma and deposit their kinetic energy on the reactor first wall surface.

Both injection system efficiency estimates and reactor first wall design considerations are sensitive to the magnitude, as well as the spatial and velocity distributions, of the neutral particle currents originating from charge-transfer encounters, particularly for a small mirror-type

reactor such as envisioned for the Fusion Engineering Research Facility (FERF).⁽⁴⁻⁸⁾ Such a device is characterized by smaller plasma dimensions which increase the escape probability of a neutral particle. The FERF will also have a lower operating energy than expected for later power-producing mirror reactors, which increases the charge-transfer interaction rate. As a result, the neutral atom bombardment of the FERF first wall: a) represents the major heat load on the surface, which may be particularly critical due to its non-uniform distribution, b) results in a potentially serious sputtering problem, and c) could provide a source of cold neutrals returning to the plasma which, via charge transfer, may cause a severe plasma energy loss.

With these considerations in mind, a number of researchers have investigated various aspects of neutral-beam injection and have developed preliminary analytical or numerical models.⁽⁹⁻¹³⁾ In order to obtain tractable models, various crucial approximations, idealizations and averagings have had to have been made in the areas of a) the anisotropies induced by the magnetic geometry, b) the plasma spatial density profile, c) the treatment of the various interaction cross sections, and d) the characterization of the neutral beam. An improved modeling technique involving Monte Carlo (MC) computer simulation for studies of neutral-beam injection into steady-state, mirror-confined plasmas is being developed in our work. Such a technique is proving to be versatile enough to be applied to realistic reactor configurations and capable of sophisticated treatment of the neutral injection question.⁽¹⁴⁾ The results of such a study could be applied directly to the mirror FERF design effort and the technique can readily be extended and modified for application to other fusion systems involving neutral particle transport in a plasma environment.⁽¹⁵⁻¹⁷⁾

The remainder of this report describes the simulation technique involved in this work and compares some preliminary results with those obtained from simple neutral-beam injection models.

Simple Injection Models

Reference (2) presents a discussion of a straightforward analytical model for neutral-beam injection into a spherical target plasma. This model requires the following restrictive assumptions:

- a. The plasma density distribution is uniform.
- b. The neutral beam energy coincides with the characteristic plasma energy E_0 .
- c. The neutral beam diameter is much smaller than the size of the target plasma.
- d. One generation of charge-transfer neutral particles are considered.
- e. All charge-transfer neutrals are born at the center of the plasma and directed isotropically outward.

Following that model, f_p denotes the fraction of the neutral-beam population that penetrates the plasma without interaction, f_t is the fraction that is trapped in the reactor plasma, and f_w is the fraction of the neutral beam that can be identified with neutral particles escaping the plasma and bombarding the first wall. Conservation of particles requires that

$$f_p + f_t + f_w = 1 \quad (1)$$

The neutral beam is attenuated exponentially as it traverses the plasma diameter such that

$$f_p = \exp \left\{ \frac{-n \langle \sigma v \rangle_T D}{v_o} \right\} \equiv \exp \{ \gamma \} \quad (2)$$

where D is the plasma diameter, n is the plasma number density, v_o is the neutral particle injection speed and $\langle \sigma v \rangle_T$ is the total interaction rate coefficient, which is the sum of contributions from electron ionization ($\langle \sigma v \rangle_{ei}$), ion ionization ($\langle \sigma v \rangle_{ii}$) and ion charge transfer ($\langle \sigma v \rangle_{cx}$). Following assumption (e), the fraction of those neutral-beam particles undergoing charge transfer and escaping to bombard the first wall without being reionized is taken to be

$$f_w = \frac{\langle \sigma v \rangle_{cx}}{\langle \sigma v \rangle_T} \left[1 - \exp \{ \gamma \} \right] \exp \left\{ \frac{1}{2\gamma} \left[\frac{\langle \sigma v \rangle_{ei} + \langle \sigma v \rangle_{ii}}{\langle \sigma v \rangle_T} \right] \right\} \quad (3)$$

Using Eq. (1), the trapping efficiency becomes

$$f_t = 1 - \left[\exp \{ \gamma \} \left\{ 1 - \frac{\langle \sigma v \rangle_{cx}}{\langle \sigma v \rangle_T} \exp \left\{ \frac{1}{2\gamma} \frac{\langle \sigma v \rangle_{ei} + \langle \sigma v \rangle_{ii}}{\langle \sigma v \rangle_T} \right\} \right] \right] \quad (4)$$

In Reference (7), the preceeding analytical model is extended to include the reionization of all later generations of charge-transfer neutrals, thus eliminating restriction(d). The corresponding neutral beam fractions become:

$$f_p' = \exp \{ \gamma \} \quad (5)$$

$$f_w' = \frac{\frac{\langle \sigma v \rangle}{\langle \sigma v \rangle_T} cx \left[1 - \exp \left\{ \gamma \right\} \right] \exp \left\{ \frac{1}{2} \gamma \right\}}{\left\{ 1 - \frac{\langle \sigma v \rangle}{\langle \sigma v \rangle_T} cx \left[1 - \exp \left\{ \frac{1}{2} \gamma \right\} \right] \right\}} \quad (6)$$

$$\text{and } f_t' = \frac{\left[1 - \exp \left\{ \gamma \right\} \right] \left\{ 1 - \frac{\langle \sigma v \rangle}{\langle \sigma v \rangle_T} cx \exp \left\{ \frac{1}{2} \gamma \right\} \right\}}{\left\{ 1 - \frac{\langle \sigma v \rangle}{\langle \sigma v \rangle_T} cx \left[1 - \exp \left\{ \frac{1}{2} \gamma \right\} \right] \right\}} \quad (7)$$

Figure (2) presents a comparison of the two analytic models by graphing the neutral beam fractions as a function of plasma density. The values of other system parameters are consistent with the FERF test case of Reference (6). The required interaction rate coefficient values are obtained from a generalized calculation which is described in a companion report.⁽¹⁸⁾ The parameters used for this preliminary calculation are summarized in TABLE I.

TABLE I

FERF Parameters for Preliminary Injection Models

Average plasma number density, $\bar{n}$	$1.5 \times 10^{14} \text{ \#/cm}^3$
Characteristic plasma ion (D^+) energy, E_o	70 keV
Characteristic plasma electron energy, E_e	7 keV
Spherical plasma diameter, D_p	50 cm
$\langle \sigma v \rangle_{ei}$	$\sim 1.075 \times 10^{-8} \text{ cm}^3/\text{sec}$
$\langle \sigma v \rangle_{ii}$	$\sim 3.40 \times 10^{-8} \text{ cm}^3/\text{sec}$
$\langle \sigma v \rangle_{cx}$	$\sim 2.06 \times 10^{-8} \text{ cm}^3/\text{sec}$
Injection atom (D) energy, E_i	65 keV
Injection atom speed	$2.5 \times 10^8 \text{ cm/sec}$
γ	1.964
Effective mirror ratio, R'	3.38
$n\tau_{ion}$	$6.44 \times 10^{12} \text{ cm}^{-3}\text{sec}$

A preliminary version of the Monte Carlo simulation code, called NUBIN2, which relaxes assumption (e) and allows charge-transfer neutrals

to be born along the beam path, consistent with the exponential attenuation of the beam, but retains the (unrealistic) assumption of isotopic production of neutral particles has produced results which are compared with the corresponding analytic values in TABLE II. In the limit of infinitely many test particles, the Monte Carlo approach would be expected to return the same f_p value as obtained by the analytic approach. The disparity here is a consequence of the statistical fluctuations associated with a finite (hence insufficient) number of test particles. While these considerations also influence the f_w and f_t values, these later differences also result from the modifications to the underlying physical model. Still, the basic agreement is a commentary on the utility of the simple models for preliminary calculations.

TABLE II
Comparison of Preliminary FERF Injection Model Global Results

	1-Generation (analytic)	∞ -Generation (analytic)	NUBIN2* (Monte Carlo)
f_p	0.011	0.011	0.014
f_w	0.110	0.071	0.197
f_t	0.879	0.918	0.789

*40,000 test particles

The Monte Carlo code NUBIN2 was developed primarily to demonstrate the feasibility of the technique and to provide "ballpark" comparison with the analytic models. The current operating version, NUBIN3, does not require the same restrictive assumptions as the simple models, and hence the results can be expected to be somewhat different. It is this refined version of the code that will be considered in the remainder of this report.

Monte Carlo Techniques

A. Geometrical Considerations

For current purposes, the target plasma is modeled by a sphere of radius R_p , centered at Cartesian coordinates $P(a,b,c)$. In order to estimate the radial dependence of neutral beam trapping, the plasma may be subdivided into $n < 20$ concentric, spherical subregions of radius $R_k = R_p/k$, where $k=1, \dots, n$. It would be possible to assign distinct plasma properties to each such subregion to provide a stepwise approximation to a radial temperature profile. The plasma density profile is taken to be a continuously varying function of distance r from the plasma center and is modeled by the following representation:

$$n(r) = n_o \left[1 - \chi \left(\frac{r}{R_p} \right)^j \right] \quad , \quad \begin{array}{l} 0 \leq r \leq R_p \\ 0 \leq \chi \leq 1 \\ j = 1, 2, 3, \dots, \infty \end{array} \quad (8)$$

where n_o denotes the central plasma density. χ and j are fitting parameters, which if taken to be 1.0 and 3 respectively, correspond to the "cubic" profile used in Reference (8) to represent the mirror-FERF plasma. Equation (8) is graphed for several values of j in Figure (3). Equation (8) may be used to relate the average plasma density, $\bar{n}$, and the central plasma density, n_o , in the following manner:

$$\bar{n} = \frac{\int n(r) dV}{\int dV} = n_o \left[1 - \frac{3\chi}{(j+3)} \right] \quad (9)$$

B. Neutral Test Particle Tracking

For purposes of Monte Carlo simulation, the neutral test particles are advanced through the plasma along a sequence of connected line segments from Cartesian point $P(x_i, y_i, z_i)$ to point $P(x_{i+1}, y_{i+1}, z_{i+1})$ until the particle escapes to bombard the reactor first wall. The new particle position is related to the old particle position by the equations

$$x_{i+1} = x_i + dl \quad (10a)$$

$$y_{i+1} = y_i + dm \quad (10b)$$

$$z_{i+1} = z_i + dn \quad (10c)$$

where d is the shorter path length to either a) interaction with a plasma particle, or b) interception of a plasma boundary surface. The quantities l, m, and n are the direction cosines of the test particle velocity vector, which, in terms of the spherical coordinates, θ and ϕ , are given by

$$l = \sin\theta\cos\phi \quad (11a)$$

$$m = \sin\theta\sin\phi \quad (11b)$$

$$n = \cos\theta \quad (11c)$$

The geometrical configuration is illustrated in Figure (4) along with a typical neutral test particle simulation history. Here, three charge-transfer collisions (the closed circles) occur in the plasma (at points 1, 2, 3) and the particle escapes to impact the first wall (at point 4). Boundary crossings occur at the open circles along the trajectory.

In the concentric spherical geometry used in NUBIN3, the distance to the interception of a plasma boundary surface is obtained by substitution of Equations (10a-c) into the general equation of a sphere of radius, R_k , centered at point P(a,b,c), yielding

$$[dl + x_i - a]^2 + [dm + y_i - b]^2 + [dn + z_i - c]^2 = R_k^2 \quad (12)$$

An analogous procedure would be followed in nonspherical applications, but it may prove more difficult to solve for d. The Cartesian coordinates of the test particle position in the alternative reference frame whose origin is taken to be the plasma center of course are:

$$r_x = x_i - a \quad (13a)$$

$$r_y = y_i - b \quad (13b)$$

$$r_z = z_i - c \quad (13c)$$

such that the distance of the test particle from the plasma center, r_i ,

is given by

$$r_i^2 = [r_x^2 + r_y^2 + r_z^2] \quad (14)$$

Rearrangement of Equation (12) allows solution for the distance, d , which is found to be the smaller, non-negative, real root of the standard quadratic expression

$$d = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \quad (15a)$$

$$\text{where } A \equiv l^2 + m^2 + n^2 \quad (15b)$$

$$B \equiv 2(lr_x + mr_y + nr_z) \quad (15c)$$

$$\text{and } C \equiv r_i^2 - R_k^2 \quad (15d)$$

The scheme employed to sample the distance, d , to interaction with the plasma is a modification of a flexible, relatively new, neutronics technique.⁽¹⁹⁾ Use of this method allows treatment of the non-uniform plasma density indicated by Equation (8) without resorting to inconvenient numerical integration of

$$I(S) = \int_0^S \Sigma_T(s') ds'$$

such that $p(S) = \exp(-I(S))$ is the probability that a test particle can move through a distance S without interacting with the background plasma, where $\Sigma_T(s')$ is the macroscopic total interaction cross section.

This sampling scheme, illustrated in Figure (5), involves: a) sampling the distance to a tentative interaction point using a constant total macroscopic cross section, $C(S)\Sigma_T(S)$, where S is the particle position prior to this sampling of a tentative interaction point, b) advancing the test particle to the tentative interaction point, c) possibly altering the particle weight estimator, W , and d) either accepting the tentative point as a "real" interaction point or rejecting the tentative point and returning to a). For this application, the sampling multipliers $C_k(S)$

are assigned to each

are assigned to each of the concentric spherical plasma subregions as initialization parameters. * Reference (19) provides a more thorough discussion of this sampling technique.

Once the neutral

Once the neutral test particle has been advanced to its interaction point, it becomes necessary to select a suitable charge transfer interaction partner from the background ion population. This is accomplished using a multidimensional rejection sampling⁽²⁰⁾ from the probability density function

$$f(v, \mu, \phi) = \frac{K f(v^2/v_p^2) M(\mu) \sigma_{cx}(v_R) v_R \left[v/v_p^2 \right]}{\langle \sigma v \rangle_{cx}} \quad (16)$$

where ϕ = normal

where K = normalization constant

$f(v^2/v_p^2) =$

$f(v^2/v_p^2)$ = plasma ion speed distribution function⁽¹⁰⁾

$M(\mu)$ = plasma

$M(\mu)$ = plasma ion angular ($\mu = \cos \theta$) distribution function⁽²¹⁾ ($M(\phi) = 1$)

v_R = relative

v_R = relative speed between neutral test particle and plasma ion

$\sigma_{cx}(v_R)$ = microscopic

$\sigma_{cx}(v_R)$ = microscopic charge transfer interaction cross section⁽¹⁰⁾

v_p = characteristic

v_p = characteristic plasma speed

The ion selected by

The ion selected by this technique becomes the new neutral test particle having its distinct

having its distinct speed and new velocity vector direction cosines, thus updating Equations

updating Equations (11 a-c). Under the assumption that little energy is exchanged in a charge

exchanged in a charge-transfer collision, it is unnecessary to modify the speed or orientation of the

orientation of the selected plasma ion as it becomes neutralized.

The overall

The overall Monte Carlo sampling scheme is illustrated in Figure (6).

* The choices of $C_k(S)$

* The choices of $C_k(S)$ are optimally made to resolve the tradeoff between increasing variances with small C_k values and increased computation time at large C_k values. For now, the choice is made on the interval $[4 \leq 6]$ with the larger values occurring in the outer, optically -thin plasma region.

Except to remark that the modular structure of this code allows for convenient, later substitution of increasingly sophisticated techniques as our experience develops, no detailed description of the code will be attempted in this report.

C. Characterization of the Neutral Beam

As currently implemented, the geometric model of convenience for the neutral beam assumes a point source of test particles located at some arbitrary point, $P(x_0, y_0, z_0)$, outside the plasma target. The injection orientation $\vec{I}(\theta, \phi)$ is selected to intercept the plasma target and can be varied so as to parametrically study the effects of a) different attenuation chord lengths and b) different injection angles with respect to the magnetic axis, $\vec{B}(\theta, \phi)$. The beam is taken to be a right circular cone of half-angle width, α . Three alternate beam modes may be specified, namely

- MODE #1. Pencil beam injection ($\alpha = 0.0$ radians)
2. Uniform transverse intensity ($\alpha > 0.0$ radians)
3. Gaussian transverse intensity ($0.0 \leq \beta \leq \alpha$ radians)

In Mode 3, β represents the angle at which the neutral beam intensity has fallen by a factor of $1/e$ of its centerline value.

The beam is taken to be monoenergetic but the Monte Carlo procedure, of course, does not preclude later inclusion of an energy-distributed source. Finally, it may be noted that the use of the point source is consistent with the FERF design which involves the focusing of the beam from several sources through a small aperture prior to entering the plasma chamber. (γ)

D. Convergence of the Monte Carlo Procedure

In Monte Carlo work, it is imperative to acknowledge and estimate the inevitable statistical uncertainty in the simulation results. For those problems which are amenable to analytic solutions, a direct comparison of the analytic and MC solutions, and hence a direct measure of the convergence of the MC simulation, is possible. However, it is for precisely those problems beyond the reach of analytic solution that MC techniques are attractive! Still, some aspects of the problem (e.g. the penetrating fraction, f_p , of Equation (2)) may provide opportunities for straightforward verification of the MC solution.

Following the discussion of Reference (22), a general, but indirect, measure of the convergence of the MC procedure used in NUBIN3 is employed. If, out of a total of N test particles used in a MC simulation, M test particles produce some definable outcome (e.g. ionizational trapping in a plasma subregion), then the fraction, M/N , is an estimate of the probability, p , that that outcome, out of all possible alternative outcomes, will occur. A measure of the probability, P , that the estimate is within a specified tolerance, ϵ , of the probability, p , is given by

$$P[|(M/N) - p| < \epsilon] = \text{erf}(t/\sqrt{2}) + R \quad (17)$$

$$\text{where } \text{erf}(x) \equiv \frac{2}{\sqrt{\pi}} \int_0^x e^{-x^2} dx, \text{ the error function}$$

$$t = \epsilon \sqrt{\frac{N}{\frac{M}{N} \left(1 - \frac{M}{N}\right)}}$$

and R = a small error term that vanishes as $N \rightarrow \infty$

For non-analogue MC simulation, N becomes the total weight estimator used in the simulation and M becomes the weight tabulated for the particular outcome. As the simulation converges, the measure, P, approaches unity asymptotically.

E. Tabulation of the First Wall Surface Bombardment

The Monte Carlo technique can provide information regarding the non-uniform first wall surface bombardment by later-generation, charge-transfer-produced neutral particles. This contrasts with the earlier analytic models which assume a uniform surface flux. In order to tabulate the bombardment of neutral test particles, the surface area of the first wall, which is assumed to be a spherical shell of radius, R_w ($R_w > R_p$), concentric with the target plasma, is partitioned into an array of "detector" subregions. As neutral test particles impact in a given subregion, an accumulation of test particle weight estimators is made which is finally tabulated as a census of the spatially distributed first wall bombardment flux. In addition, an overall bombardment energy distribution function is constructed. Information regarding the angle of incidence of impacting particles is also available which could later be used in surface sputtering studies. At this stage, tabulation of a separate bombardment energy distribution for each surface subregion is prohibitive but it is recognized that the spatial and energy distribution are, at least, weakly coupled.

A scheme for partitioning the first wall surface area such that each detector subregion has equal area, permitting direct comparison of the bombardment flux has been devised.

The total surface area of the first wall surface, A, is given by

$$A = R_w^2 \int_0^{2\pi} d\phi \int_0^{\pi} \sin \theta d\theta = 4\pi R_w^2 \quad (18a)$$

$$= R_w^2 [\phi] \Big|_0^{2\pi} [-\cos\theta] \Big|_0^{\pi} \quad (18b)$$

In order to partition A into n subregions of equal area and at the same time maintain certain symmetries, two circular subregions, each of area A/n , are taken to be centered at the poles ($\theta = 0.0$ and $\theta = \pi/2$). The remainder of the first wall surface is partitioned into spherical rectangles by lines of "latitude" and "longitude." The resolution of the detector grid, which scales inversely with subregion area, is a function of the parameter, m, such that

$$n = 2(1 + 4^m) \quad (19)$$

The angular boundaries of the circular end regions (θ_1 and θ_{2m+1}) are obtained from Equation (18b) by solving

$$4\pi R_w^2/n = 2\pi R_w^2 [1 - \cos \theta_1] \quad (20a)$$

to yield

$$\theta_1 = \arccos (1 - 2/n) = \arccos \left(1 - \frac{1}{1+4^m} \right) \quad (20b)$$

$$\theta_{2m+1} = \pi - \theta_1 \quad (20c)$$

The angular boundaries of the rectangular subregions are obtained from Equation (18b) by solving

$$4\pi R_w^2 = R_w^2 [\phi_{j+1} - \phi_j] [\cos \theta_k - \cos \theta_{k+1}] \quad (21)$$

$$\text{where } j = 1, 2, \dots, 2^{m+1}, \quad \phi_1 = 0$$

$$k = 1, 2, \dots, 2^m,$$

to obtain the recursion relations

$$\phi_{j+1} = \phi_j + \pi/2^m \quad (22a)$$

$$\cos \theta_{k+1} = \cos \theta_k - [2^{m+1}/(1+4^m)] \quad (22b)$$

The resolution obtained by setting $m=4$ is considered adequate for present purposes.

Discussion of NUBIN3 Results

The current operating version of the Monte Carlo code, NUBIN3, has been applied to a reference configuration consistent with the mirror FERF design study.⁽⁸⁾ This effort, in addition to producing more detailed information for the neutral-beam aspects of the FERF design, also served as an opportunity to debug, shakedown and optimize the Monte Carlo approach. The basic system parameters are identical to those used to obtain the preliminary results presented in TABLE II, thus providing a verification check of the results. The inclusion of the cubic plasma radial density profile and the incorporation of the full angular dependence of the collisional interaction rate coefficients are expected to provide the most rigorous treatment to date of this problem. In addition to the primary neutral beam energy component at E_0 , the important parasitic beam components at $E_0/2$ and $E_0/3$ are also included.⁽²³⁻²⁴⁾ TABLE III summarizes the system parameters used in this study. It may be remarked that the neutral beam has finite transverse dimensions but is essentially comparable to the pencil beam used in the earlier work. Also, injection is nearly perpendicular to the magnetic axis and the beam is directed through the center of the target plasma so as to maximize the trapping of the beam.

TABLE III

Mirror FERF Parameters Used in NUBIN3 Benchmark Calculations

Coordinates of plasma center, $P(a,b,c)$	(0,0,0)	cm
Plasma radius, R_p	25.0	cm
First wall radius, R_w	35.0	cm
Average plasma number density, $\bar{n}$	2.25×10^{14}	$\text{\#}/\text{cm}^3$
Central plasma number density, n_0	2.25×10^{14}	$\text{\#}/\text{cm}^3$
Plasma density profile parameters, χ	0.0	
j	3.0	
Characteristic plasma ion (D^+) energy, E_0^i	70.0	keV
Characteristic plasma electron energy, E_e	7.2	keV
Magnetic axis orientations, $\vec{B}(\theta, \phi)$	(0,0)	radians
Effective mirror ratio, R'	3.38	
Coordinates of neutral beam point source, $P(x_0, y_0, z_0)$	(-50,0,0)	cm
Injection orientation, $\vec{I}(\theta, \phi)$	$(\pi/2, 0)$	radians
Angular divergence of neutral beam, α	0.002	radians
Gaussian neutral beam parameter, β	0.001	radians
Neutral beam component energies, E_0	65.0	keV
$E_0/2$	32.5	keV
$E_0/3$	21.7	keV
$\gamma_0 = \gamma(E_0)$	4.54	
$n\tau_{ion}$	6.44×10^{12}	$\text{cm}^{-3} \text{sec}$

The global results of this study are presented in TABLE IV.

TABLE IV

NUBIN3 FERF Injection Model Global Results.*

Beam component:	E_o	$E_o/2$	$E_o/3$
Fraction of beam population, $^{(24)}f_b$:	~ 0.51	~ 0.22	~ 0.24
Number of test particles:	1.75×10^4	2×10^4	2×10^4
f_p	0.008	0.001	<0.001
f_w	0.100	0.143	0.206
f_t	0.892	0.856	0.794

* totals may not equal 1.0 due to round-off. $P > 0.90$ for $\epsilon = 0.005$

The contributions of later-generation, charge-transfer produced neutrals to these global results is indicated by TABLE V. The O-generation f_w value corresponds to the f_p values in TABLE IV. It is possible to construct the normalized energy distribution functions of the later ($n \geq 1$) generation neutral. These distributions, for each of the primary neutral beam components are illustrated in Figure 7. The target plasma energy distribution function is plotted as a solid line for comparison. Using the calculated average energies, $\langle E \rangle$, of the bombardment distributions, it is possible to estimate the power loss from the plasma for each of the beam components. Following the notation of Reference (23), $\tilde{E}_p$ denotes the characteristic target plasma energy and $E_o = \tilde{E}_p$. P_T is the total power escaping from the plasma due to charge-transfer wall bombardment and O-generation penetration.

TABLE V
TERMINAL PARTICLE FRACTION BY GENERATION

GENERATION #	E_o		$E_o/2$		$E_o/3$	
	f_t	f_w	f_t	f_w	f_t	f_w
0	0.537	0.008	0.478	0.001	0.452	<0.001
1	0.218	0.040	0.226	0.072	0.209	0.130
2	0.088	0.028	0.096	0.035	0.085	0.043
3	0.034	0.016	0.037	0.018	0.033	0.017
4	0.012	0.007	0.014	0.008	0.012	0.007
5	0.004	0.003	0.005	0.003	0.004	0.003
6	0.002	0.001	0.002	0.001	0.002	0.001
7	0.001	~0.0	0.001	~0.0	0.001	~0.0
8	~0.0	~0.0	~0.0	~0.0	~0.0	~0.0
9	~0.0	~0.0	~0.0	~0.0	~0.0	~0.0
10	~0.0	~0.0	~0.0	~0.0	~0.0	~0.0

of each component and P_{INJ} is the injected power of that component. The normalized power loss ratio for each component is given by

$$P_T/P_{INJ} = f_p + f_w \langle E \rangle / E_{INJ} \quad (23)$$

These losses are tabulated in TABLE IV. These results have been compared with a series of 1-generation calculations performed at the Lawrence Livermore Laboratory⁽²⁵⁾ using parameters consistent with those of TABLE III. Such differences as were found in these benchmark calculations can be ascribed to a) slight differences in the interaction cross section used, b) the additional trapping of $n > 2$ generation charge-transfer neutrals and c) a somewhat different technique for obtaining P_T values.

TABLE VI

Power Loss Summary for FERF Neutral Beam Components, $E_o = 65$ keV

	E_o	$E_o/2$	$E_o/3$
f_p	0.008	0.001	<0.001
f_w	0.100	0.143	0.206
$\langle E \rangle$ of f_w (keV)	34.6	33.4	33.4
P_T/P_{INJ} @ $\gamma_o = 5$ ⁽²⁴⁾	0.10	0.23	0.41
P_T/P_{INJ} from NUBIN3	0.103	0.249	0.543

Weighting the component power loss ratios by the respective beam component fractions from TABLE IV, 23.8% of the total neutral beam power is found to be lost to the first wall through neutral particle impact.

Conclusion

The preliminary application of Monte Carlo simulation to the modeling of neutral beam injection for fusion reactors suggests the desirability of

further effort, both for reason of verification of the models used previously in this work and for the additional information and insight provided by this flexible and powerful technique. Ultimately, this approach should prove valuable for parameter studies for fusion device design projects.

SCHEMATIC OF NEUTRAL-BEAM INJECTOR SYSTEM

Figure 1: Overall schematic of the neutral-beam injection system for a mirror fusion reactor.

TERMINAL NEUTRAL-BEAM FRACTIONS

Figure 2: Comparison of analytic 1-generation and ∞ -generation injection models as a function of target plasma density following Equations (2.7) of this report.

Figure 3: Radial plasma density profile, following Equation (8) of this report, for typical parameters. A uniform plasma density is obtained by setting $\chi = 0.0$.

Figure 4: Overall schematic of the geometrical configuration used in Monte Carlo simulation of neutral-beam injection, indicating the optional parameters. A typical neutral test particle history is also shown.

Figure 5: Flow chart of the distance to interaction estimator used in the Monte Carlo code. (cf. Reference 19.)

MONTE CARLO SIMULATION SCHEME

Figure 6: Overview flow chart of the Monte Carlo neutral-beam injection simulation code NUBIN3.

Figure 7: Normalized energy distributions of ($n \geq 1$ generation) charge-transfer produced neutral particles that escape to bombard the reactor first wall. The background plasma energy distribution is plotted as a solid line for comparison.

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