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ATOMIC INTERNATIONAL A Division of North American Aviation, Inc.		NAA-SR TDR NO. 7485	APPROVALS <i>[Signature]</i> <i>[Signature]</i>
TECHNICAL DATA RECORD		PAGE 1 OF 37	
AUTHOR <i>Douglas Logan</i> DOUGLAS LOGAN		DEPT. & GROUP NO. 717-70	DATE 6-22-62
TITLE Stress Analysis of SCTI Mixing Tees		GO NO. 7593	RECOMMENDED FOR OUTSIDE DISTRIBUTION ☐ YES ☐ NO
		S/A NO 4486	
		TWR 32333	
PROGRAM SCTI	SUBACCOUNT TITLE Sodium System - Process Equipment		SIGNATURE
DISTRIBUTION USE AN ASTERISK (*) TO INDICATE THOSE WHO ARE TO RECEIVE COMPLETE COPIES	STATEMENT OF PROBLEM		
W. F. Anderson D. F. Casey C. P. Craig R. Cygan K. W. Foster R. M. Galantine G. Landler J. E. Mahlmeister J. S. McDonald N. M. Mishizaki H. C. Parris H. L. Sletten J. Susnir F. J. Waters A. C. Williams J. B. Williams L. Zimmer R. L. [unclear]	<p>I. Design and analyze a mixing tee capable of operating under the conditions of the SCTI test installation.</p> <p>ABSTRACT. II. Summary of Results. Figure 1, Appendix A, shows a specially designed tee which is varified by stress analysis to be operable under the design conditions of the SCTI test installation which are:</p> <ol style="list-style-type: none"> 1. Main line sodium flow is changed from primary sodium flow at 1300°F to secondary sodium flow at 700°F. This is done at such a rate as to cause the downstream temperature to change 35°F/sec. There are 25 of these cycles. 2. Startup causes a temperature change from ambient to 350°F on both lines at a rate of 100°F/hr. There are 5 of these cycles. 3. The main line flow temperature is changed from 1200°F to 1300°F in 9 seconds. There are 25 of these cycles. 4. From the hold temperature of 350°F each line is raised at the rate of 100°F/hr. The secondary line is raised to 700°F and the main line to 1200°F. There are 300 of these cycles. 5. At any temperature the mainline flow temperature drops 110°F in 7.5 minutes. There are 7000 of these cycles. 6. Primary sodium at 1300°F mixes with secondary sodium at 1100°F to give a main line flow temperature of 1200°F. There are 25 of these cycles. 7. The design pressure is 100 psi. <p>NOTE: For ease of fabrication and increased mixing of fluids, the design drawing uses a modified version of Figure 1, Appendix A. The spun head is replaced by a cone as on the outlet end and the incoming side branch is turned to direct the side branch sodium flow into the incoming sodium flow on the through run. Temperature distribution in the main structure has not been significantly changed due to these modifications, therefore, the structural integrity of the modified tee is essentially unaltered.</p>		

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III. Method Used and Sample Calculations.

The SCTI mixing tees must have an accumulative damage factor of less than 0.8 (Ref. 2) after subjection to the following design conditions (Ref. 7).

1. Main line sodium flow is changed from primary sodium flow at 1300°F to secondary sodium flow at 700°F. This is done at such a rate as to cause the downstream temperature to change 35°F/sec. There are 25 of these cycles.
2. Startup causes a temperature change from ambient to 350°F on both lines at a rate of 100°F/hr. There are 5 of these cycles.
3. The main line flow temperature is changed from 1200°F to 1300°F in 9 seconds. There are 25 of these cycles.
4. From the hold temperature of 350°F each line is raised at the rate of 100°F/hr. The secondary line is raised to 700°F and the main line to 1200°F. There are 300 of these cycles.
5. At any temperature, the mainline flow temperature drops 110°F in 7.5 minutes. There are 7000 of these cycles.
6. Primary sodium at 1300°F mixes with secondary sodium at 1100°F to give a main line flow temperature of 1200°F. There are 25 of these cycles.
7. The design pressure is 100 psi.

A. Analysis of Standard 10" Tee.

The possibility of using a standard tee was eliminated when:

1. A comparative analysis of the standard tee and a tee using a thermal liner yielded for cyclic condition number one, calculated damage factors of approximately 0.3 and 0.04 respectively.
2. A standard tee used in a LASL test installation (Ref. 9) was badly cracked after being subjected to less severe mixing conditions than will exist for the SCTI loop.

The damage to the LASL tee was believed to have been caused by the fluctuation inherent in the mixing of two fluids at different temperatures. At the present state of knowledge, there is no method by which mixing fluctuation effects can be predicted. However, the proper use of a thermal liner will prevent this type of damage to the main structural body.

The presentation of the analysis of the 10" standard tee is not for the numerical solution but to show the method used.

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The Assumptions made for this analysis, referring to Figure 1A, are:

1. Temperature gradients through the walls are neglected. Since the walls are thin, the temperature differences would not be large.
2. The junction of the side ~~profile~~ branch with the straight through run is assumed to act as a ring. From a side profile of the tee, the side branch appears to be a cylinder, to ring to cylinder attachment. This portion which appears like a ring is idealized to represent a fictitious ring connecting the side branch to the through branch.

The tee temperature distribution caused by the mixing of different temperature fluids was conservatively estimated from probable flow patterns. The temperature distribution in the ring on the side where the two flows first meet points c and d, in Figure 1A, was assumed to have a linear gradient of the whole temperature difference. Directly opposite this at points a and b, the ring will be uniform in temperature and equal to the temperature of the side branch sodium. Thus the lower portion of the ring circumference must feel the entire temperature difference. In the cylinder immediately after the two fluids are brought together at plane b-e, the cylinder will have a circumferential temperature distribution, which is assumed sinusoidal, being at the side branch sodium temperature on top, point b, and at the straight through sodium temperature on the bottom, point e. This type of temperature distribution causes a two dimensional effect at the branch connecting region on the side where the two fluids first meet and a three dimensional effect on the opposite side. For cyclic condition, number 1, the distribution that results when the primary sodium flow is at 1300°F and the secondary sodium flow is at 700°F is shown in Figure 1A.

The analysis of the tee under these conditions will consist of the combination of three separate effects. They are:

1. The ring is assumed to have a symmetric temperature distribution as shown in Figure 1-B and a discontinuity analysis is made at the junctions a, c and b, d.
2. Point b is then brought back to 700°F and the free-body rotation of a line segment through point o to point d, due to a 700°F to 1300°F temperature distribution is calculated. This rotation is then said to be caused by an "equivalent" external moment acting on the system. These effects are analyzed from the methods outlined by P. P. Bijlaard in Reference 6.
3. The sinusoidal circumferential temperature from 1300°F at point e to 700° at point b is analyzed next by methods outlined in Reference 8.

Therefore, the total stress across section e-b is the summation of the individual stresses calculated in steps 1, 2 and 3; and the stress at section d-f is the summation of the individual stresses calculated in steps 1 and 2.

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FIGURE 1 A

FIGURE 1 B

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Step 1. Discontinuity Analysis.

Free Body Diagram

FIGURE 2

M = discontinuity moment, IN-lb/IN.

V = discontinuity shear force, lb/IN.

δ = deflection, IN

θ = angle of rotation, radians.

At point 1, compatibility relationships require that

$$\theta_{11} = \theta_{R1}$$

$$\delta_{11} = \delta_{R1}$$

For

$$\theta_{11} = \theta_{R1}$$

$$1) \frac{M_1}{R_1 D_1} - \frac{V_1}{2 R_1^2 D_1} = \frac{R_{RC}}{E I} \left[M_2 + V_2 l_2 - M_1 - V_1 l_1 \right] R_R + R_{RC} \alpha_R \left. \frac{dT}{dL} \right|_{R1}$$

— (REF. 4)

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For $\delta_{11} = \delta_{R1}$

$$(2) \frac{M_1}{2\beta_1^2 D_1} - \frac{V_1}{2\beta_1^3 D_1} + R_1 \epsilon_{11} t_{11} = - \frac{R_{RC} \ell_1}{EI} [M_2 + V_2 \ell_2 - M_1 - V_1 \ell_1] R_R$$

$$- \ell_1 R_{RC} \epsilon_R \left. \frac{dT}{dL} \right|_{R1} + R_{RC} \epsilon_R T_R \quad \text{--- (REF. 4)}$$

At point 2 compatibility relations again require that

$$\theta_{22} = \theta_{R2}$$

$$\delta_{22} = \delta_{R2}$$

For $\theta_{22} = \theta_{R2}$

$$(3) \frac{-M_2}{\beta_2 D_2} + \frac{V_2}{2\beta_2^2 D_2} = \frac{R_{RC}}{EI} [M_2 + V_2 \ell_2 - M_1 - V_1 \ell_1] R_R + R_{RC} \epsilon_R \left. \frac{dT}{dL} \right|_{R2}$$

$$\text{--- (REF. 4)}$$

For $\delta_{22} = \delta_{R2}$

$$(4) \frac{M_2}{2\beta_2^2 D_2} - \frac{V_2}{2\beta_2^3 D_2} + R_2 \epsilon_{22} t_{22} = \frac{R_{RC} \ell_2}{EI} [M_2 + V_2 \ell_2 - M_1 - V_1 \ell_1] R_R$$

$$+ \ell_2 R_{RC} \epsilon_R \left. \frac{dT}{dL} \right|_{R2} + R_{RC} \epsilon_R T_R \quad \text{--- (REF. 4)}$$

WHERE:

$$D = \frac{E h^3}{12(1-\nu^2)}, \text{ lb-IN}$$

E = modulus of elasticity, lb/IN²

h = thickness of shell wall, IN

I = moment of inertia of the ring, psi

l = perpendicular distance from the center of gravity of the ring to the edge where shear forces act ($\ell_1 = \ell_2$), IN

R = radius, IN

$\frac{dT}{dL}$ = temperature difference between the discontinuity edge and the average temperature of the piece divided by the perpendicular distance from the center of gravity to the discontinuity edge (the positive or absolute value is used), °F/IN.

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α = coefficient of thermal expansion, $\text{IN}/^\circ\text{F} - \text{IN}$.

R_{RC} = Radius to centroid of ring, IN

β = cylinder characteristic = $\sqrt[4]{3(1-\nu^2)/R^2 h^2}$, IN^{-1}

ν = Poisson's ratio.

Subscripts:

The first numeral refers to the piece. If the letter R is used, it refers to the ring. The second numeral refers to the discontinuity point in question. If the letter C is used, it refers to the center of gravity of the ring.

For cyclic condition number 1, the numerical value of the constants are:

$$D_1 = D_2 = 9.36 \times 10^4 \text{ lb-IN}$$

$$E = 21 \times 10^6 \text{ psi}$$

$$h = 0.365 \text{ IN}$$

$$I = 0.46 \text{ IN}^4$$

$$l_1 = l_2 = 1.27 \text{ IN}$$

$$R_1 = R_2 = 5.193 \text{ IN}$$

$$R_{RC} = 5.938 \text{ IN}$$

$$\frac{dT}{dL} = 236.2 \text{ } ^\circ\text{F/IN}$$

$$\alpha_{11} = 0.992 \times 10^{-5}$$

$$\alpha_{12} = 1.031 \times 10^{-5}$$

$$\alpha_{22} = 1.05 \times 10^{-5}$$

$$\beta_1 = \beta_2 = 0.934 \text{ IN}^{-1}$$

$$\nu = 0.3$$

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Simultaneous solution of these equations result in:

$$M_1 = -356.6 \text{ IN-lb/IN}$$

$$V_1 = -2222 \text{ lb/IN}$$

$$M_2 = -1987 \text{ IN-lb/IN}$$

$$V_2 = -2155 \text{ lb/IN}$$

The negative signs indicate the directions shown should be reversed.

Step 2. Bijlaard Analysis, Ref. 6.

FIGURE 3

Because of the temperature difference between A & B, the plane at the intersection of the side branch into the straight run through points A, O, and B rotates around the axis going through O perpendicular to the paper in a clockwise manner. To determine the stresses resulting from this temperature distribution, it will be assumed that an "equivalent" external moment is applied such that the same free rotation of the AB plane around O exists as with the temperature differential. The stresses induced by this moment can be determined by a Bijlaard analysis.

The angle of rotation of AOB is given by

$$\theta = \alpha \Delta T$$

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where

α = coefficient of thermal expansion, IN/IN^{°F}

$\overline{\Delta T} = \frac{\Delta T}{2}$ = one-half of the total temperature difference around the circumference of the intersection point, °F.

For stainless steel at these temperatures:

$$\alpha = 1.031 \times 10^{-5} \text{ IN/IN-}^{\circ}\text{F}$$

$$\overline{\Delta T} = \frac{\Delta T}{2} = 300^{\circ}\text{F}$$

Therefore:

$$\theta = 0.003093 \text{ radians.}$$

The information in the referenced Bijlaard paper is presented in the form of nondimensional parameters in tables and graphs. The two parameters of importance in this analysis are:

$$\beta = 0.875 \frac{r}{a}$$

$$\gamma = \frac{a}{t}$$

where

a = radius of the shell

r = radius of cylindrical attachment to the shell

t = shell wall thickness

For the 10 inch tee used;

$$a = 5.193 \text{ IN}$$

$$r = 5.193 \text{ IN}$$

$$t = 0.365 \text{ IN}$$

thus

$$\beta = 0.875$$

$$\gamma = 14.23$$

With these values of β and γ it was found from the referenced paper that,

$$\frac{\theta}{M/a^3 \beta^2 E} = 55$$

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$$\frac{M_x}{M/a\beta} = 0.0149$$

$$\frac{M_\phi}{M/a\beta} = 0.01165$$

$$\frac{N_x}{M/a^2\beta} = -0.562$$

$$\frac{N_\phi}{M/a^2\beta} = -1.095$$

Substitution of $E = 21 \times 10^6$ psi and $\Theta = 0.003093$ into the equations for cyclic condition number 1, yields the following discontinuity values:

$$M_x = 416 \text{ IN-lb/IN}$$

$$M_\phi = 326 \text{ IN-lb/IN}$$

$$N_x = -3020 \text{ lb/IN}$$

$$N_\phi = -5890 \text{ lb/IN}$$

These values were obtained by extrapolation beyond the limits presented in the referenced paper. However, in all cases the extrapolation was made to maximize the curve trend and therefore, it is believed that the order of magnitude of the results is reasonable.

Step 3. Circumferential Temperature Distribution on the Outlet Pipe.

The temperature distribution on the outlet pipe is assumed to be given by;

$$t = 1000 + 300 \sin \theta$$

with θ measured as shown in figure 4

FIGURE 4

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Defining the average temperature through the cross-section of the pipe as;

$$\bar{T} = \frac{1}{h} \int_{-\frac{h}{2}}^{+\frac{h}{2}} T dz \quad (\text{Ref. 8})$$

where h = wall thickness

z = variable measured from the center plane of the wall.

yields $\bar{T} = 1000 + 300 \sin \theta$

Defining the equivalent linear temperature drop through the wall thickness as;

$$\Delta T = \frac{12}{h^2} \int_{-\frac{h}{2}}^{+\frac{h}{2}} T z dz \quad (\text{Ref. 8})$$

which by inspection of the temperature distribution

yields $\Delta T = 0$

To determine the circumferential moment from this temperature distribution $\bar{T}$ and ΔT must be put into the following form:

$$\left[\frac{\alpha}{a} \bar{T} + \frac{\alpha}{h} \Delta T \right] = A_n \cos n\theta + B_n \sin n\theta \quad (\text{Ref. 8})$$

thus

$$\frac{\alpha}{a} \bar{T} = \frac{\alpha}{a} 1000 + 300 \frac{\alpha}{a} \sin \theta = A_n \cos n\theta + B_n \sin n\theta$$

showing that $A_0 = \frac{\alpha}{a} 1000$, $B_1 = 300 \frac{\alpha}{a}$

and all other constants in the Fourier expansion are zero.

M_ϕ is given by

$$M_\phi = -EI [C_1 + C_2 \cos \theta + C_3 \sin \theta] \quad (\text{Ref. 8})$$

where

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E = modulus of elasticity, lb/IN²

I = moment of inertia, IN⁴

$$C_1 = A_0 - \frac{\alpha}{a} \bar{T}_0$$

$$C_2 = A_1$$

$$C_3 = B_1$$

$\bar{T}_0$ = the constant term of the Fourier expression for $\bar{T}$

Therefore:

$$C_1 = 0$$

$$C_2 = 0$$

$$C_3 = 300 \frac{\alpha}{a}$$

and finally

$$M_\phi = -EI 300 \frac{\alpha}{a} \sin \theta$$

Step 4. Stresses from Cyclic Condition Number 1.

The stresses induced from cyclic condition number 1 are shown in Tables 1 and 2.

Using the distortion energy concept the equivalent uniaxial stresses at points b and d may be computed from

$$\sigma_i = \sqrt{\sigma_\phi^2 + \sigma_y^2} - \sigma_\phi \sigma_y$$

— (REF. 2)

thus at

1. Point d, inner surface

$$\sigma_i = 190,900 \text{ psi}$$

2. Point d, outer surface

$$\sigma_i = 211,100 \text{ psi}$$

3. Point b, inner surface

$$\sigma_i = 153,000 \text{ psi}$$

4. Point b, outer surface

$$\sigma_i = 136,300 \text{ psi}$$

TABLE 1

Stresses at Point d (Refer to Figure 1A)

Cause	Circumferential Stress	Value	Remarks	Longitudinal Stress	Value	Remarks
V_2 from Step 1	$\sigma_\phi = \frac{-2V_2 a \beta}{h}$	83,700 psi	Tension			
M_2 from Step 1 Plus M_x from Step 2	$\sigma_\phi' = \pm \frac{\nu 6(M_2 + M_x)}{h^2}$ $\sigma_\phi' = \frac{2(M_2 + M_x)}{h} a \beta^2$	69,200 psi	Tension outside compression inside	$\sigma_x = \pm \frac{6(M_2 + M_x)}{h^2}$	230,700 psi	Tension outside compression inside
M_ϕ from Step 2	$\sigma_\phi = \pm \frac{6M_\phi}{h^2}$	87,090 psi	compression throughout			
N_ϕ from Step 2	$\sigma_\phi = + \frac{6M_\phi}{h^2}$	31,300 psi	Tension outside compression inside			
N_ϕ from Step 2	$\sigma_\phi = + \frac{N_\phi}{h}$	23,550 psi	Tension throughout			
N_x from Step 2				$\sigma_x = \frac{N_x}{h}$	12,100 psi	Tension throughout
Design Pressure	$\sigma_\phi = \frac{Pa}{h}$	2,080 psi	Tension throughout	$\sigma_x = \frac{Pa}{2h}$	1,040 psi	Tension throughout
Σ		78,260 psi	Compression inside		217,560 psi	Compression inside
		122,740 psi	Tension Outside		243,840 psi	Tension outside

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TABLE 2

Stresses at Point b (Refer to Figure 1A)

Cause	Circumferential Stress	Value	Remarks	Longitudinal Stress	Value	Remarks
V_2 from Step 1	$\sigma_\phi = -\frac{2 V_2}{h} a \beta$	83,700 psi	Tension throughout			
M_2 from Step 1 Plus M_x from Step 2	$\sigma_\phi' = \pm \frac{6 V (M_2 + M_x)}{h^2}$ $\sigma_\phi = \frac{2 (M_2 + M_x)}{h} a \beta^2$	45,240 psi	Tension outside compression inside	$\sigma_x = \pm \frac{6 (M_2 + M_x)}{h^2}$	150,800 psi	Tension outside compression inside
M_ϕ from Step 2	$\sigma_\phi = \pm \frac{6 M_\phi}{h^2}$	31,300 psi	Compression outside Tension inside			
N_ϕ from Step 2	$\sigma_\phi = \frac{N_\phi}{h}$	23,550 psi	Compression throughout			
N_x from Step 2				$\sigma_x = \frac{N_x}{h}$	12,080 psi	Compression throughout
M_ϕ from Step 3	$\sigma_\phi = \pm \frac{M_\phi}{I} \frac{h}{2}$	1,590 psi	Compression outside tension inside	$\sigma_x = \pm \frac{V M_\phi}{I} \frac{h}{2}$	477 psi	Compression outside tension inside
Design Pressure	$\sigma_\phi = \frac{P a}{h}$	2,080 psi	Tension throughout	$\sigma_x = \frac{P a}{2 h}$	1,040 psi	Tension throughout
Σ	Inside	18,420psi	Compression	Inside	161,360psi	Compression
	Outside	6,280psi	Tension	Outside	139,280psi	Tension

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It is assumed that the temperature cycle involved occurs in going from ambient temperature to the temperature conditions just analyzed. Thus the worst stress occurs at the point d, outer surface with a value of

$$\sigma_c = 211,100 \text{ psi}$$

or an alternating stress of

$$\sigma_{alt} = 105,600 \text{ psi}$$

Using the design criteria presented in reference 2, the effective alternating stress is equal to the alternating stress since $\sigma_{alt} > \sigma_y$ (yield stress). From the thermal fatigue curve for 304SS, effective alternating stress of 105,600 psi, a total of 84 cycles would be allowed. Therefore, the damage factor for cyclic condition number 1 is:

$$D = \frac{25}{84}$$

$$D = 0.298$$

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B. Analysis of a Special Design Tee.

The tee to be analyzed in this section is shown in Figure 1, Appendix A.

By visual inspection of the tee, it is observed that the flow paths of the two fluids result in large temperature gradients to the main structure of the tee. The region between the outlet cone and the inlet spun head to the incoming pipe of the side branch will bear the entire temperature difference. The region between the outlet cone and the incoming pipe of the side branch will have the maximum stresses due to these temperature differences. In analyzing this region of high stress the following assumptions are made:

1. The sodium outside the thermal liner is assumed stagnant. This should be a very good assumption as the region is to be packed with steel wool where convection would alter the temperature gradients as used in the analysis.
2. The side branch of the 20" standard tee is assumed to act as a ring with a cross section in the shape of 1/4 of a circle with a 5.25" mean radius.
3. Gradients through the walls are neglected. Because of the thermal liner and the thin walls, this assumption is warranted.

The assumed direction of moments and shear forces and positive direction for rotation and deflection at the points of discontinuity are shown in the Free Body Diagram.

M = discontinuity moment, IN-lb/IN

Q = discontinuity shear force, lb/IN

δ = deflection, IN

Θ = angle of rotation, radius

Piece 1 is the incoming pipe on the side branch of the tee.

Piece 2 is the cone between the incoming pipe and the side branch of the 20" standard tee.

Piece 3 is the side branch of the 20" standard tee assumed as a ring as discussed in assumption number 2.

Piece 4 is the cone on the outlet end of the tee.

Piece 5 is the thermally lined outlet pipe of the tee.

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Outward deflections are positive.
Counterclockwise rotations are positive.

FREE BODY DIAGRAM

1. Discontinuity Analysis.

At point 1, compatibility relationships require that

$$\theta_{11} = \theta_{21}$$

$$\delta_{11} = \delta_{21}$$

For $\theta_{11} = \theta_{21}$

$$(5) \frac{-M_1}{\rho_1 D_1} + \frac{Q_1}{2 \rho_1^2 D_1} = -\alpha_{21} \gamma_{21} \tan \phi \left. \frac{dt}{dy} \right|_{21} + W_{1M1} M_1$$

$$+ W_{1Q1} Q_1 + W_{1P} P \text{ --- (REF. 1)}$$

For $\delta_{11} = \delta_{21}$

$$(6) \frac{M_1}{2 \rho_1^2 D_1} - \frac{Q_1}{2 \rho_1^3 D_1} + \frac{P R_{11}^2}{(E h)_{11}} \left[1 - \frac{\nu}{2} \right] + R_{11} \alpha_{11} t_{11} =$$

$$\alpha_{21} \gamma_{21} \sin \phi t_{21} + \delta_{1M1} M_1 + \delta_{1Q1} Q_1 + \delta_{1P} P \text{ --- (REF. 1)}$$

At point 2

$$\theta_{22} = \theta_{32}$$

$$\delta_{22} = \delta_{32}$$

For $\theta_{22} = \theta_{32}$

$$(7) -\alpha_{22} \gamma_{22} \tan \phi \left. \frac{dt}{dy} \right|_{22} + W_{2M2} M_2 + W_{2Q2} Q_2 + W_{2P} P =$$

$$\frac{R_{3C}^2}{(E I)_{3C}} \left[M_2 + Q_2 l_2 - M_3 - Q_3 l_3 \right] - R_{3C} \alpha_{3C} \left. \frac{dT}{dL} \right|_{32} \text{ --- (REF. 1)}$$

For $\delta_{22} = \delta_{32}$

$$(8) \alpha_{22} \gamma_{22} \sin \phi T_{22} + \delta_{2M2} M_2 + \delta_{2Q2} Q_2 + \delta_{2P} P = \frac{R_{3C}^2}{(E I)_{3C}} \left[M_2 + Q_2 l_2 \right.$$

$$\left. - M_3 - Q_3 l_3 \right] l_2 - l_2 R_{3C} \alpha_{3C} \left. \frac{dT}{dL} \right|_{32} + R_{3C} \alpha_{3C} T_3 \text{ --- (REF. 1)}$$

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At point 3

$$\theta_{33} = \theta_{43}$$

$$\delta_{33} = \delta_{43}$$

For $\theta_{33} = \theta_{43}$

$$(9) \frac{R_{3c}^2}{(EI)_{3c}} \left[M_2 + Q_2 l_2 - M_3 - Q_3 l_3 \right] - R_{3c} \alpha_{3c} \frac{dT}{dL} \Big|_{33} =$$

$$-W_{3M3} M_3 - W_{3Q3} Q_3 - W_{3P} P \quad \text{--- (REF. 1)}$$

For $\delta_{33} = \delta_{43}$

$$(10) -\frac{R_{3c}^2}{(EI)_{3c}} \left[M_2 + Q_2 l_2 - M_3 - Q_3 l_3 \right] l_3 + l_3 R_{3c} \alpha_{3c} \frac{dT}{dL} \Big|_{33}$$

$$+ R_{3c} \alpha_{3c} T_3 = \delta_{3M3} M_3 + \delta_{3Q3} Q_3 + \delta_{3P} P + \alpha_{43} Y_{43} \sin \phi t_{43}$$

$$\text{--- (REF. 1)}$$

At point 4

$$\theta_{44} = \theta_{54}$$

$$\delta_{44} = \delta_{54}$$

For $\theta_{44} = \theta_{54}$

$$(11) -W_{4M4} M_4 - W_{4Q4} Q_4 - W_{4P} P = \frac{M_4}{\beta_5 D_5} - \frac{Q_4}{2\beta_5^2 D_5} \quad \text{--- (REF. 1)}$$

For $\delta_{44} = \delta_{54}$

$$(12) \delta_{4M4} M_4 + \delta_{4Q4} Q_4 + \delta_{4P} P + \alpha_{44} Y_{44} \sin \phi t_{44} =$$

$$(13) \frac{M_4}{2\beta_5^2 D_5} - \frac{Q_4}{2\beta_5^3 D_5} + \frac{P R_{54}^2}{(EI)_{54}} \left[1 - \frac{\nu}{2} \right] + R_{54} \alpha_{54} t_{54} \quad \text{--- (REF. 1)}$$

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In the preceding conical shell equations, the terms: $W_1 M_1$, $W_1 Q_1$, $W_1 P$, etc., and S_{1M} , S_{1Q} , S_{1P} , etc., are influence coefficients which were theoretically determined and presented in Reference (1). See Appendix A for the definitions. The other constants are:

$$D = \frac{E h^3}{12(1-\nu^2)} \text{ , lb-IN}$$

E = modulus of elasticity, lb/IN²

h = thickness of shell wall, IN

I = centroidal moment of inertia of the ring, IN⁴

ℓ = perpendicular distance from the center of gravity of the ring to the edge where the discontinuity shear forces act ($\ell_2 = \ell_3$), IN

P = pressure, lb/IN²

R = radius, IN

T = average temperature, °F

$\frac{dT}{dL}$ = temperature difference between the discontinuity edge and the average temperature of the piece divided by the perpendicular distance from the center of gravity to that edge (the positive or absolute value is used), °F/IN

y = distance along meridian from apex of cone, IN.

α = coefficient of thermal expansion, IN/°F - IN

β = Cylinder characteristics = $\sqrt[4]{3(1-\nu^2)/R^2 h^2}$, IN⁻¹

ν = Poisson's ratio

ϕ = half-angle of cone, radians

Subscripts.

The first numeral refers to the piece and the second to the discontinuity point in question. Sometimes the letter c is used as the second subscript, this refers to the center of gravity of the ring.

When sodium flows of different temperatures are mixed, a temperature gradients exists on the main structure of the tee along the members shown in the Free Body Diagram. These gradients were determined by a Teledeltos analog study. Figure 2, Appendix A shows the gradients as found by the study.

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2. Specific Analysis of Cyclic Condition Number 1.

Cyclic condition number 1 imposes the worst thermal gradients of all the cyclic conditions. This gradient occurs when 1300°F sodium flows in the main line and 700°F sodium flows in the secondary line. Under these conditions, the values of the data from the Teledeltos study are such that 1.00 volt = 60°F.

For the preceding conditions, the following constants apply to the compatibility equations at the four discontinuity points.

$D_1 = 7.03 \times 10^{-4} \text{ lb-IN}$	$\tan \phi = 0.5774$
$D_5 = 9.18 \times 10^{-4} \text{ lb-IN}$	$\left. \frac{dt}{dy} \right _{21} = 0.4 \text{ } ^\circ\text{F/IN}$
$E_{11} = 23 \times 10^6 \text{ lb/IN}^2$	$\left. \frac{dt}{dy} \right _{22} = 33.6 \text{ } ^\circ\text{F/IN}$
$E_{3c} = 20 \times 10^6 \text{ lb/IN}^2$	$\left. \frac{dT}{dL} \right _{32} = 73.4 \text{ } ^\circ\text{F/IN}$
$E_{54} = 19 \times 10^6 \text{ lb/IN}^2$	$\left. \frac{dT}{dL} \right _{33} = 52.7 \text{ } ^\circ\text{F/IN}$
$h_{11} = 0.322 \text{ IN}$	$W_{1M1} = 4.67 \times 10^{-6} \text{ lb}^{-1}$
$h_{54} = 0.375 \text{ IN}$	$W_{1Q1} = 2.47 \times 10^{-6} \text{ IN/lb}$
$I_{3c} = 10.96 \text{ IN}^4$	$W_{1P} = 2.63 \times 10^{-6} \text{ IN}^2/\text{lb}$
$l_2 = l_3 = 3.34 \text{ IN}$	$W_{2M2} = -7.25 \times 10^{-6} \text{ lb}^{-1}$
$P = 100 \text{ lb/IN}^2$	$W_{2Q2} = 5.80 \times 10^{-6} \text{ IN/lb}$
$R_{11} = 4.152 \text{ IN}$	$W_{2P} = 15.55 \times 10^{-6} \text{ IN}^2/\text{lb}$
$R_{3c} = 11.66 \text{ IN}$	$W_{3M3} = -8.40 \times 10^{-6} \text{ lb}^{-1}$
$R_{54} = 6.188 \text{ IN}$	$W_{3Q3} = 6.72 \times 10^{-6} \text{ IN/lb}$
$\sin \phi = 0.5 = \sin 30^\circ$	$W_{3P} = 18 \times 10^{-6} \text{ IN}^2/\text{lb}$
$t_{11} = 700 \text{ } ^\circ\text{F}$	$W_{4M4} = 6.81 \times 10^{-6} \text{ lb}^{-1}$
$t_{21} = 700 \text{ } ^\circ\text{F}$	$W_{4Q4} = 4.35 \times 10^{-6} \text{ IN/lb}$
$t_{22} = 871 \text{ } ^\circ\text{F}$	$W_{4P} = 7.09 \times 10^{-6} \text{ IN}^2/\text{lb}$
$t_{43} = 1292 \text{ } ^\circ\text{F}$	$y_{21} = 8.63 \text{ IN}$
$t_{44} = 1300 \text{ } ^\circ\text{F}$	$y_{22} = 19.50 \text{ IN}$
$t_{54} = 1300 \text{ } ^\circ\text{F}$	$y_{43} = 19.50 \text{ IN}$
$T_3 = 1116 \text{ } ^\circ\text{F}$	$y_{44} = 12.50 \text{ IN}$

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$$\begin{aligned}\alpha_{11} &= 0.991 \times 10^{-5} \text{ IN}^{\circ}\text{F-IN} \\ \alpha_{21} &= 0.991 \times 10^{-5} \text{ IN}^{\circ}\text{F-IN} \\ \alpha_{22} &= 1.015 \times 10^{-5} \text{ IN}^{\circ}\text{F-IN} \\ \alpha_{3c} &= 1.042 \times 10^{-5} \text{ IN}^{\circ}\text{F-IN} \\ \alpha_{43} &= 1.05 \times 10^{-5} \text{ IN}^{\circ}\text{F-IN} \\ \alpha_{44} &= 1.05 \times 10^{-5} \text{ IN}^{\circ}\text{F-IN} \\ \alpha_{54} &= 1.05 \times 10^{-5} \text{ IN}^{\circ}\text{F-IN} \\ \beta_1 &= 1.111 \text{ IN}^{-1} \\ \beta_5 &= 0.844 \text{ IN}^{-1} \\ \delta_{1M1} &= 2.47 \times 10^{-6} \text{ IN/lb} \\ \delta_{1Q1} &= 2.73 \times 10^{-6} \text{ IN}^2/\text{lb}\end{aligned}$$

$$\begin{aligned}\delta_{1P} &= 4.93 \times 10^{-6} \text{ IN}^3/\text{lb} \\ \delta_{2M2} &= 5.80 \times 10^{-6} \text{ IN/lb} \\ \delta_{2Q2} &= -9.08 \times 10^{-6} \text{ IN}^2/\text{lb} \\ \delta_{2P} &= -17.21 \times 10^{-6} \text{ IN}^3/\text{lb} \\ \delta_{3M3} &= 6.72 \times 10^{-6} \text{ IN/lb} \\ \delta_{3Q3} &= -10.51 \times 10^{-6} \text{ IN}^2/\text{lb} \\ \delta_{3P} &= -19.92 \times 10^{-6} \text{ IN}^3/\text{lb} \\ \delta_{4M4} &= 4.35 \times 10^{-6} \text{ IN/lb} \\ \delta_{4Q4} &= 5.75 \times 10^{-6} \text{ IN}^2/\text{lb} \\ \delta_{4P} &= 14.36 \times 10^{-6} \text{ IN}^3/\text{lb} \\ v &= 0.3\end{aligned}$$

$$\phi = 30^\circ$$

Solving the two equations at point 1, it is found that

$$M_1 = -25.7 \text{ IN-lb/IN}$$

$$Q_1 = -49.8 \text{ lb/IN}$$

Solving the four equations at points 2 and 3, it is found that

$$M_2 = -3053 \text{ IN-lb/IN}$$

$$Q_2 = -4744 \text{ lb/IN}$$

$$M_3 = -4356 \text{ IN-lb/IN}$$

$$Q_3 = -4940 \text{ lb/IN}$$

Solving the two equations at point 4, it is found that

$$M_4 = -58.9 \text{ IN-lb/IN}$$

$$Q_4 = -136.9 \text{ lb/IN}$$

The minus sign on all quantities indicates that the assumed directions of the moments and shear forces should be reversed.

It is obvious that M_2 , Q_2 , M_3 , and Q_3 will cause the highest stresses.

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For the ring, the maximum stress is given by

$$(13) \quad S = \frac{M_T R_{3C}}{I/l_2} \quad \text{--- (REF. 4)}$$

where $M_T = M_3 + Q_3 l_3 - M_1 - Q_2 l_2$

$$= 4356 + 4940 \times 3.34 - 3053 - 4744 \times 3.34$$

$$M_T = 1958 \text{ IN-lb/IN}$$

$$R_{3C} = 11.66 \text{ IN}$$

$$I = 10.96 \text{ IN}^4$$

$$l_2 = l_3 = 3.34 \text{ IN}$$

This gives a stress of $S = 6957 \text{ psi}$

This is very small and can be applied an infinite number of times.

For the cone, the stresses at the large end are given by:

$$(14) \quad \sigma_y = \frac{y \sin \phi \cos \phi}{2h} P - \frac{\sin \phi}{h} Q \pm \frac{6}{h^2} M \quad \text{--- (REF. 1)}$$

and

$$(15) \quad \sigma_\theta = \frac{3}{4} \frac{\tan^2 \phi}{1-\nu} \left[\frac{(m^2 A/6) \mp \nu C \mp 2G}{C + 2\nu G} \pm 1 \right] P$$

$$+ \frac{y \tan \phi}{h} \left[\frac{1 + \frac{\sin^2 \phi}{2} \pm 6 \frac{1-\nu^2}{m^2} A - 5 \frac{\nu^2 B}{2} - \nu C}{C + 2\nu G} \right] P$$

$$+ \frac{\sin \phi}{h} \left[\frac{\pm 6 \frac{1-\nu^2}{m^2} A - 5 \frac{\nu^2 B}{2} - \nu C}{C + 2\nu G} \right] Q + \frac{6}{h^2} \left[\frac{-\frac{m^2 A}{6} \pm \nu C \pm 2G}{C + 2\nu G} \right] M$$

--- (REF. 1)

In the equations where there are $\pm$ or $\mp$ the upper sign in each case refers to the inner surface and the lower sign to the outer surface. The new terms in these equations are:

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$$A = \xi \left[\text{ber}_2' \xi \text{bei}_2 \xi - \text{bei}_2' \xi \text{ber}_2 \xi \right]$$

$$B = \left[\text{ber}_2' \xi \right]^2 + \left[\text{bei}_2' \xi \right]^2$$

$$C = \xi \left[\text{ber}_2 \xi \text{ber}_2' \xi + \text{bei}_2 \xi \text{bei}_2' \xi \right]$$

$$G = \left[\text{ber}_2 \xi \right]^2 + \left[\text{bei}_2 \xi \right]^2$$

$$\cos \phi = \cos 30^\circ = 0.866$$

$$m = \sqrt[4]{12(1-v^2)} = 1.818$$

$$\xi = 2\mu\sqrt{y}$$

$$\mu = \sqrt[4]{12(1-v^2)/h^2 \tan^2 \phi} \quad , \quad \text{IN}^{-1/2}$$

Since the large diameters of both conical shells are equal, ξ is the same for both pieces and therefore, the values of A, B, C and G will be the same for both cones.

Evaluation gives:

$$\xi = 2 \sqrt[4]{12(1-0.3^2)/0.5^2 \times 0.5774^2} \sqrt{19.5}$$

$$\xi = 29.9$$

Using the recurrence formulas from Reference (5) and the values in the tables of Reference (3), it is found that:

$$\text{bei}_2 29.9 = -0.98772 \times 10^8$$

$$\text{bei}_2' 29.9 = -0.40918 \times 10^8$$

$$\text{ber}_2 29.9 = 0.32503 \times 10^8$$

$$\text{ber}_2' 29.9 = 0.96577 \times 10^8$$

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These constants give values of

$$A = -24.545 \times 10^{16}$$

$$B = 1.100 \times 10^{16}$$

$$C = 21.47 \times 10^{16}$$

$$G = 1.08123 \times 10^{16}$$

Substituting into the stress equations for the cones yields;

a) At piece 2, point 2, inner surface

$$\sigma_y = -67,684 \text{ psi}$$

$$\sigma_\theta = +37,103 \text{ psi}$$

b) At piece 2, point 2, outer surface

$$\sigma_y = +78,860 \text{ psi}$$

$$\sigma_\theta = +111,342 \text{ psi}$$

c) At piece 4, point 3, inner surface

$$\sigma_y = -98,760 \text{ psi}$$

$$\sigma_\theta = +10,580 \text{ psi}$$

d) At piece 4, point 3, outer surface

$$\sigma_y = +110,328 \text{ psi}$$

$$\sigma_\theta = +122,363 \text{ psi}$$

Using the distortion energy concept the equivalent uniaxial stress can be calculated by

$$\sigma_i = \sqrt{\sigma_y^2 + \sigma_\theta^2 - \sigma_y \sigma_\theta} \quad \text{--- (REF. 2)}$$

thus at;

e) Piece 2, point 2, inner surface

$$\sigma_i = 92,040 \text{ psi}$$

f) Piece 2, point 2, outer surface

$$\sigma_i = 99,150 \text{ psi}$$

g) Piece 4, point 3, inner surface

$$\sigma_i = 104,500 \text{ psi}$$

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h) Piece 4, point 3, outer surface

$$\sigma_c = 116,820 \text{ psi}$$

A conservative approach will be used here, in that the cyclic conditions will be assumed to run from ambient temperature (zero thermal stress in all numbers) to the stress conditions just analyzed. Thus, the maximum stress occurs in piece 4 at point 3, on the outer surface with a value of

$$\sigma_c = 116,820 \text{ psi}$$

This value gives an alternating stress of:

$$\sigma_{alt} = 58,410 \text{ psi}$$

Using the design criteria presented in Reference (2), the effective alternative stress is equal to the alternating stress since $\sigma_{alt} > \sigma_y$.

At an effective alternating stress of 58,410 psi, a total of 580 cycles would be allowed. Therefore, the damage factor for cyclic condition number 1 is:

$$D = \frac{25}{580}$$

$$D = 0.0431$$

3. Cyclic Condition 5.

Under cyclic condition number 5, the case that would cause the largest stresses would occur when the secondary line was and remained at 700°F while the main line went from 1300°F to 1190°F.

The stresses have already been evaluated at the condition where the secondary line is 700°F and the primary line is 1300°F under cyclic condition number 1. Thus, what remains here is to evaluate the stresses where the secondary line is 700°F and the primary line is 1190°F. Under these conditions, the values of the data from the Teledeltos study are such that 1.00 volt = 49°F.

The constants that are applicable to the compatibility equations for the four discontinuity points are:

$$D_1 = 7.03 \times 10^{-4} \text{ lb-IN}$$

$$D_5 = 9.67 \times 10^{-4} \text{ lb-IN}$$

$$E_{11} = 23 \times 10^6 \text{ lb/IN}^2$$

$$E_{3c} = 21 \times 10^6 \text{ lb/IN}^2$$

$$E_{54} = 20 \times 10^6 \text{ lb/IN}^2$$

$$h_{11} = 0.322 \text{ IN}$$

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$$\begin{aligned} h_{54} &= 0.375 \text{ IN} \\ I_{30} &= 10.96 \text{ IN}^4 \\ l_2 = l_3 &= 3.34 \text{ IN} \\ P &= 100 \text{ lb/IN}^2 \end{aligned}$$

$$\begin{aligned} R_{11} &= 4.152 \text{ IN} \\ R_{30} &= 11.66 \text{ IN} \\ R_{54} &= 6.188 \text{ IN} \end{aligned}$$

$$\sin \phi = 0.5$$

$$t_{11} = 700^\circ\text{F}$$

$$t_{21} = 700^\circ\text{F}$$

$$t_{22} = 840^\circ\text{F}$$

$$t_{43} = 1183^\circ\text{F}$$

$$t_{44} = 1190^\circ\text{F}$$

$$t_{54} = 1190^\circ\text{F}$$

$$T_3 = 1040^\circ\text{F}$$

$$\tan \phi = 0.5774$$

$$\left. \frac{dt}{dy} \right|_{21} = 0.33^\circ\text{F/IN}$$

$$\left. \frac{dt}{dy} \right|_{22} = 27.4^\circ\text{F/IN}$$

$$\left. \frac{dT}{dL} \right|_{32} = 60^\circ\text{F/IN}$$

$$\left. \frac{dT}{dL} \right|_{33} = 42.8^\circ\text{F/IN}$$

$$W_{1M1} = 4.67 \times 10^{-6} \text{ lb}^{-1}$$

$$W_{1Q1} = 2.47 \times 10^{-6} \text{ IN/lb}$$

$$W_{1P} = 2.63 \times 10^{-6} \text{ IN}^2/\text{lb}$$

$$W_{2M2} = -7.25 \times 10^{-6} \text{ lb}^{-1}$$

$$W_{2Q2} = 5.80 \times 10^{-6} \text{ IN/lb}$$

$$W_{2P} = 15.55 \times 10^{-6} \text{ IN}^2/\text{lb}$$

$$W_{3M3} = -7.98 \times 10^{-6} \text{ lb}^{-1}$$

$$W_{3Q3} = 6.38 \times 10^{-6} \text{ IN/lb}$$

$$W_{3P} = 17.1 \times 10^{-6} \text{ IN}^2/\text{lb}$$

$$W_{4M4} = 6.47 \times 10^{-6} \text{ lb}^{-1}$$

$$W_{4Q4} = 4.13 \times 10^{-6} \text{ IN/lb}$$

$$W_{4P} = 6.74 \times 10^{-6} \text{ IN}^2/\text{lb}$$

$$y_{21} = 8.63 \text{ IN}$$

$$y_{22} = 19.50 \text{ IN}$$

$$y_{43} = 19.50 \text{ IN}$$

$$y_{44} = 12.50 \text{ IN}$$

$$\alpha_{11} = 0.991 \times 10^{-5} \text{ IN}/^\circ\text{F-IN}$$

$$\alpha_{21} = 0.991 \times 10^{-5} \text{ IN}/^\circ\text{F-IN}$$

$$\alpha_{22} = 1.009 \times 10^{-5} \text{ IN}/^\circ\text{F-IN}$$

$$\alpha_{30} = 1.035 \times 10^{-5} \text{ IN}/^\circ\text{F-IN}$$

$$\alpha_{43} = 1.047 \times 10^{-5} \text{ IN}/^\circ\text{F-IN}$$

$$\alpha_{44} = 1.047 \times 10^{-5} \text{ IN}/^\circ\text{F-IN}$$

$$\alpha_{54} = 1.047 \times 10^{-5} \text{ IN}/^\circ\text{F-IN}$$

$$\beta_1 = 1.111 \text{ IN}^{-1}$$

$$\beta_5 = 0.844 \text{ IN}^{-1}$$

$$\delta_{1M1} = 2.47 \times 10^{-6} \text{ IN/lb}$$

$$\delta_{1Q1} = 2.73 \times 10^{-6} \text{ IN}^2/\text{lb}$$

$$\delta_{1P} = 4.93 \times 10^{-6} \text{ IN}^3/\text{lb}$$

$$\delta_{2M2} = 5.80 \times 10^{-6} \text{ IN/lb}$$

$$\delta_{2Q2} = -9.08 \times 10^{-6} \text{ IN}^2/\text{lb}$$

$$\delta_{2P} = -17.21 \times 10^{-6} \text{ IN}^3/\text{lb}$$

$$\delta_{3M3} = 6.38 \times 10^{-6} \text{ IN/lb}$$

$$\delta_{3Q3} = -10.00 \times 10^{-6} \text{ IN}^2/\text{lb}$$

$$\delta_{3P} = -18.92 \times 10^{-6} \text{ IN}^3/\text{lb}$$

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$$\begin{aligned} \delta \quad I_{M1} &= 4.13 \times 10^{-6} \text{ IN}^4/\text{lb} & \nu &= 0.3 \\ \delta \quad I_{Q1} &= 5.47 \times 10^{-6} \text{ IN}^2/\text{lb} & \phi &= 30^\circ \\ \delta \quad I_{P} &= 13.63 \times 10^{-6} \text{ IN}^3/\text{lb} \end{aligned}$$

Since the stresses will be less than in the case of cyclic condition number 1, there is no need to calculate the stresses for point 1 and point 4.

Solution of the four equations at points 2 and 3 yields;

$$M_2 = -2932 \text{ IN-lb/IN}$$

$$Q_2 = -4476 \text{ lb/IN}$$

$$M_3 = -4023 \text{ IN-lb/IN}$$

$$Q_3 = -4641 \text{ lb/IN}$$

The minus sign on all quantities indicates that the assumed directions of the moments and shear forces should be reversed.

There is no need to calculate the ring stress as the total stress in cyclic condition number 1 could be tolerated and infinite number of times.

For the cones, the stresses are given by the same equations and constants as in cyclic condition number 1. Solving the cone stress equations, it is found that

a) At piece 2, point 2, inner surface

$$\sigma_r = -65,048 \text{ psi}$$

$$\sigma_\theta = +33,513 \text{ psi}$$

b) At piece 2, point 2, outer surface

$$\sigma_r = +75,688 \text{ psi}$$

$$\sigma_\theta = +105,121 \text{ psi}$$

c) At piece 4, point 3, inner surface

$$\sigma_r = -91,067 \text{ psi}$$

$$\sigma_\theta = +11,327 \text{ psi}$$

d) At piece 4, point 3, outer surface

$$\sigma_r = +102,037 \text{ psi}$$

$$\sigma_\theta = +114,373 \text{ psi}$$

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Using the distortion energy concept again, the equivalent stresses are:

- e) At piece 2, point 2, inner surface

$$\sigma_i = 86,747 \text{ psi}$$

- f) At piece 2, point 2, outer surface

$$\sigma_i = 93,915 \text{ psi}$$

- g) At piece 4, point 3, inner surface

$$\sigma_i = 97,242 \text{ psi}$$

- h) At piece 4, point 3, outer surface

$$\sigma_i = 108,729 \text{ psi}$$

The yield stress for 304 SS at 1190°F is 12,000 psi.

The mean stress for cycling between 1300°F and 1190°F is calculated from:

$$\sigma_m' = \frac{\sigma_{1300} + \sigma_{1190}}{2} \quad \text{--- (Ref. 2)}$$

and the alt stress from;

$$\sigma_{alt} = \frac{\sigma_{1300} - \sigma_{1190}}{2} \quad \text{--- (Ref. 2)}$$

Using these equations, it is found that

- i) At piece 2, point 2, inner surface

$$\sigma_m' = 89,394 \text{ psi}$$

$$\sigma_{alt} = 2,642 \text{ psi}$$

- j) At piece 2, point 2, outer surface

$$\sigma_m' = 96,533 \text{ psi}$$

$$\sigma_{alt} = 2,618 \text{ psi}$$

- k) At piece 4, point 3, inner surface

$$\sigma_m' = 100,871 \text{ psi}$$

$$\sigma_{alt} = 3,629 \text{ psi}$$

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- 1) At piece 4, point 3, outer surface

$$\sigma_m' = 112,775 \text{ psi}$$

$$\sigma_{alt} = 4,046 \text{ psi}$$

Since $\sigma_m' + \sigma_{alt} > \sigma_y$ and $\sigma_{alt} < \sigma_y$ the effective mean stress is;

$$\sigma_m = \sigma_y - \sigma_{alt} \quad \text{--- (Ref. 2)}$$

Only piece 4, point 3, inner and outer surfaces will be investigated, since they have the maximum alternating stresses.

Thus at:

- m) At piece 4, point 3, inner surface

$$\sigma_m = 8,371 \text{ psi}$$

- n) At piece 4, point 3, outer surface

$$\sigma_m = 7,954 \text{ psi}$$

A fatigue diagram as shown in Figure 3, Appendix A, is used to determine the effective alternating stress ($\sigma_{alt \text{ eff.}}$).

The $\sigma_{alt \text{ eff}}$ is 12,000 psi for both surfaces on piece 4 at point 3. The number of allowable cycles at this value of $\sigma_{alt \text{ eff}}$ is infinite. Therefore, the damage factor resulting from cyclic condition number 5 is 0.

4. Cyclic Conditions 1, 2, 3, 4 and 6.

Since cyclic condition number 5 with its 7000 cycles resulted in a zero damage factor, the other cyclic conditions will not have to be analyzed. This is so, because the very conservative approach can be taken which assumes that all the cycles of conditions 1, 2, 3, 4 and 6 are exactly equal. Since the total number of design cycles for all 5 of these conditions is 380, the total damage factor from these sources can not exceed;

$$D_T = \frac{380}{580}$$

$$D_T = 0.66$$

Therefore, the design is satisfactory for the SCTI test installation.

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IV. References and Appendix

A. References

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$$W_{IH1} = \frac{m^2 r_1}{E h^2} \Omega_2$$

$$W_{IP} = \frac{m^2 r_1^2 \tan \alpha}{2 E h^2} \Omega_2 + \frac{3(1+\nu) \tan^2 \alpha}{m E} \sqrt{\frac{r_1}{2 h \cos \alpha}} \Omega_1$$

$$- \frac{3 r_1 \tan \alpha}{2 E h \cos \alpha}$$

$$\delta_{IM1} = \frac{m^2 r_1}{E h^2} \Omega_2$$

$$\delta_{IH1} = - \frac{m r_1}{E h} \sqrt{\frac{2 r_1 \cos \alpha}{h}} \Omega_3$$

$$\delta_{IP} = - \frac{m r_1^2 \sin \alpha}{E h} \sqrt{\frac{r_1}{2 h \cos \alpha}} \Omega_3 - \frac{m^2 r_1 \tan^2 \alpha}{8 E (1-\nu)} \Omega_2$$

$$+ \frac{(1-\frac{\nu}{2}) r_1^2}{E h \cos \alpha}$$

$$W_{2M2} = \frac{m^3}{E h^2} \sqrt{\frac{2 r_2}{h \cos \alpha}} \Omega_4$$

$$W_{2H2} = \frac{m^2 r_2}{E h^2} \Omega_5$$

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$$W_{2P} = \frac{m^2 r_2^2 \tan \alpha}{2 E h^2} \Omega_5 - \frac{3(1+\nu) \tan^2 \alpha}{m E} \sqrt{\frac{r_2}{2 h \cos \alpha}} \Omega_4 - \frac{3 r_2 \tan \alpha}{2 E h \cos \alpha}$$

$$\delta_{2M2} = \frac{m^2 r_2}{E h^2} \Omega_5$$

$$\delta_{2H2} = \frac{m r_2}{E h} \sqrt{\frac{2 r_2 \cos \alpha}{h}} \Omega_6$$

$$\delta_{2P} = \frac{m r_2^2 \sin \alpha}{E h} \sqrt{\frac{r_2}{2 h \cos \alpha}} \Omega_6 - \frac{m^2 r_2 \tan^2 \alpha}{8(1-\nu) E} \Omega_5 + \frac{(1-\frac{\nu}{2}) r_2^2}{E h \cos \alpha}$$

where:

E = modulus of elasticity, lb/IN²

h = thickness of shell wall, IN

$m = \sqrt[4]{12(1-\nu^2)}$

r = radius of parallel circle, IN

α = half-angle of cone, radians

ν = Poisson's ratio

Ω = special functions presented in graphical form in Reference 1.

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Appendix A

MODEL NO.

FIGURE 2
GRADIENTS AS DETERMINED BY
AN ANALOG STUDY

FIGURE 3 APPENDIX A
FATIGUE DIAGRAM FOR COMBINED STEADY
STATE AND TRANSIENT CONDITIONS FOR
POINT 3