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Abstract

This is the ion depletion model used in HYDRA's LZR package. It derives from a formulation due in large part to Dr. Manoj Prasad (circa 2002).

1 Model Equations

The algorithm used for ion energy depletion in HYDRA is used within the context of the LZR package with *ray_trace_order* = 1, ie. straight line trajectories.

We solve the following depletion equation for E_b, the energy/nucleon (KeV), along these trajectories:

$$\frac{\partial E_b}{\partial s} = -C_{n_e}\frac{Z_b^2}{\beta_b^2}(R_{BF}\Lambda_B + C_G\Lambda_F)$$

The terms in this equation for the energy loss $\frac{\partial E_b}{\partial s}$ are given by

$$C_{ne} = 5.1X10^{-22}\frac{ne}{A_0}$$

with A_0 the energetic ion atomic weight.

The ionization state of energetic ions is computed as

$$Z_b = Z_{b0}(1 - \exp^{-\mathrm{Betz}})$$

where Z_{b0} is the energetic ion atomic number,

$$\text{Betz} = C_{\text{Betz}}\sqrt{\beta_b^2 + \beta_e^2}$$

and

$$C_{\text{Betz}} = 1/\alpha Z_{b0}^{.69}$$

and α is the fine structure constant.

β_b and β_e are the light speed normed velocities of the energetic ion and thermal electrons given by $\beta_b^2 = 1 - 1/\gamma_b^2$ and $\beta_e^2 = 1 - 1/\gamma_e^2$ with $\gamma_b = 1 + E_b/M_p$ and $\gamma_e = 1 + T_e/M_e$.

The quantity R_{BF}, the ratio of bound to free electrons is

$$R_{BF} = \overline{Z}/Z^* - 1$$

where $\overline{Z}$ and Z^* are the background (target) effective and average charge state averaged over the mass fractions of the ionic (target) mix. Note that at full ionization $Z^* \rightarrow \overline{Z}$ so $R_{BF} \rightarrow 0$ and the bound electron contribution vanishes, while for vanishing ionization, since $n_e \sim Z^*$ the bound electron contribution varies as $n_e R_{BF} \sim \overline{Z} - Z^*$.

Defining the (small) ratio $R_\beta = \beta_e/\beta_b$ we write the coefficient of the ion energy loss due to free electrons C_G as

$$C_G = 1/(1 - R_\beta(.1263R_\beta(.1195 - 1.5075R_\beta))).$$

This derives from an expansion of $G(x) = \text{erf}(x) - x\ \text{erf}'(x)$ for $x >> 1$. The remaining terms Λ_B and Λ_F, the Coulomb logarithms for bound and free collisions, are given as

$$\Lambda_B = \ln(1 + 2M_e c^2\beta_b^2/\overline{I}) + \ln\gamma_b^2 - \beta_b^2 = \ln(1 + 102200\beta_b^2/\overline{Z}) + \ln\gamma_b^2 - \beta_b^2$$

and

$$\Lambda_F = \ln(1 + \beta_b^2\frac{1.376X10^{16}}{\sqrt{n_e}}) + .5(\ln\gamma_b^2 - \beta_b^2).$$

where $\overline{I} = \overline{Z}/102200$ is the average ionization potential for bound electrons and $\ln\gamma_b^2 - \beta_b^2$ is a relativistic correction factor in both Coulomb logarithms.

The depletion equation is initialized at $E_b = E_{b0}$ for each energetic ion *beamlet* and tracked until $E_b < E_{bfloor}$, at which point all the remaining energy is deposited (ie. in the Cell where this condition first occurs).