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# High Energy Ion Energy Depletion Model in HYDRA

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## Abstract

This is the ion depletion model used in HYDRA's LZR package. It derives from a formulation due in large part to Dr. Manoj Prasad (circa 2002).

## 1 Model Equations

The algorithm used for ion energy depletion in HYDRA is used within the context of the LZR package with *ray\_trace\_order* = 1, ie. straight line trajectories.

We solve the following depletion equation for  $E_b$ , the energy/nucleon (KeV), along these trajectories:

$$\frac{\partial E_b}{\partial s} = -C_{ne} \frac{Z_b^2}{\beta_b^2} (R_{BF} \Lambda_B + C_G \Lambda_F)$$

The terms in this equation for the energy loss  $\frac{\partial E_b}{\partial s}$  are given by

$$C_{ne} = 5.1 \times 10^{-22} \frac{ne}{A_0}$$

with  $A_0$  the energetic ion atomic weight.

The ionization state of energetic ions is computed as

$$Z_b = Z_{b0} (1 - \exp^{-\text{Betz}})$$

where  $Z_{b0}$  is the energetic ion atomic number,

$$\text{Betz} = C_{\text{Betz}} \sqrt{\beta_b^2 + \beta_e^2}$$

and

$$C_{\text{Betz}} = 1/\alpha Z_{b0}^{.69}$$

and  $\alpha$  is the fine structure constant.

$\beta_b$  and  $\beta_e$  are the light speed normed velocities of the energetic ion and thermal electrons given by  $\beta_b^2 = 1 - 1/\gamma_b^2$  and  $\beta_e^2 = 1 - 1/\gamma_e^2$  with  $\gamma_b = 1 + E_b/M_p$  and  $\gamma_e = 1 + T_e/M_e$ .

The quantity  $R_{BF}$ , the ratio of bound to free electrons is

$$R_{BF} = \bar{Z}/Z^* - 1$$

where  $\bar{Z}$  and  $Z^*$  are the background (target) effective and average charge state averaged over the mass fractions of the ionic (target) mix. Note that at full ionization  $Z^* \rightarrow \bar{Z}$  so  $R_{BF} \rightarrow 0$  and the bound electron contribution vanishes, while for vanishing ionization, since  $n_e \sim Z^*$  the bound electron contribution varies as  $n_e R_{BF} \sim \bar{Z} - Z^*$ .

Defining the (small) ratio  $R_\beta = \beta_e/\beta_b$  we write the coefficient of the ion energy loss due to free electrons  $C_G$  as

$$C_G = 1/(1 - R_\beta(.1263R_\beta(.1195 - 1.5075R_\beta))).$$

This derives from an expansion of  $G(x) = \text{erf}(x) - x \text{erf}'(x)$  for  $x \gg 1$ . The remaining terms  $\Lambda_B$  and  $\Lambda_F$ , the Coulomb logarithms for bound and free collisions, are given as

$$\Lambda_B = \ln(1 + 2M_e c^2 \beta_b^2 / \bar{I}) + \ln \gamma_b^2 - \beta_b^2 = \ln(1 + 102200 \beta_b^2 / \bar{Z}) + \ln \gamma_b^2 - \beta_b^2$$

and

$$\Lambda_F = \ln(1 + \beta_b^2 \frac{1.376 \times 10^{16}}{\sqrt{n_e}}) + .5(\ln \gamma_b^2 - \beta_b^2).$$

where  $\bar{I} = \bar{Z}/102200$  is the average ionization potential for bound electrons and  $\ln \gamma_b^2 - \beta_b^2$  is a relativistic correction factor in both Coulomb logarithms.

The depletion equation is initialized at  $E_b = E_{b0}$  for each energetic ion *beamlet* and tracked until  $E_b < E_{b\text{floor}}$ , at which point all the remaining energy is deposited (ie. in the Cell where this condition first occurs).