

**MASTER**

COMMENTS ON "SHAPES OF ANGULAR DISTRIBUTIONS  
OF ( $^3\text{He}, t$ ) AND ( $p, n$ ) TRANSITIONS TO  $0^+$  ANTI-  
ANALOG STATES"\*

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This comment on a recent Physical Review Letter by Noble points out that the assumptions used by Noble are not valid for the reaction considered. Hence, the anomaly in the angular distributions remains to be explained.

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\* Work supported in part by the U. S. Atomic Energy Commission

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## **DISCLAIMER**

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In a recent letter, Noble<sup>1</sup> purports to explain the recently observed<sup>2</sup> anomalous shapes of the angular distributions of tritons from the  $0^+ \rightarrow 0^+$   $^3\text{He}$ -induced charge-exchange transitions to anti-analog states of  $^{64,66}\text{Ga}$  and  $^{40}\text{K}$  on the basis of the structural relation between analog and anti-analog states. His derivation of the relation between the angular distributions exciting analog and anti-analog states may be reformulated in a simple and general way which clearly indicates the essential approximations. It will be seen that one of these approximations is grossly violated thereby invalidating the derivation.

We begin by assuming that the dominant mechanism of the ( $^3\text{He}, t$ ) reaction is a one-step direct process and is describable in the distorted wave Born approximation. Ignoring constant factors the reaction amplitude is<sup>3</sup>

$$M \propto \int \chi_f^{(-)*}(k_f, \underline{r}) \langle \phi_f | \Psi(t) | V_{\text{eff}} | \Psi(\text{He}) \phi_i \rangle \chi_i^{(+)}(k_i, \underline{r}) d\underline{r} \quad (1)$$

The functions  $\chi_i^{(+)}$  and  $\chi_f^{(-)}$  are distorted waves describing the initial and final states of relative motion; the intrinsic states of the projectiles and nuclei are denoted by  $\Psi$  and  $\phi$  in an obvious notation. The operator  $V_{\text{eff}}$  is taken to be an effective two-body interaction of which only the appropriate charge-exchange portion is needed.

It is convenient to take a definite reaction to spell out the nuclear states  $\phi$ . Thus for the reaction  $^{40}\text{Ar}(^3\text{He}, t)^{40}\text{K}$  we write<sup>4</sup>

$$|^{40}\text{Ar}\rangle = [d_{3/2}^{-2} f_{7/2}^2]_{T=2} \quad (2a)$$

$$|^{40}\text{K}\rangle_{\text{IAS}} = 2^{-1/2} [a_p^\dagger(d_{3/2}) a_n(d_{3/2}) + a_p^\dagger(f_{7/2}) a_n(f_{7/2})] |^{40}\text{Ar}\rangle \quad (2b)$$

$$|^{40}\text{K}\rangle_{\text{AAS}} = 2^{-1/2} [a_p^\dagger(d_{3/2}) a_n(d_{3/2}) - a_p^\dagger(f_{7/2}) a_n(f_{7/2})] |^{40}\text{Ar}\rangle, \quad (2c)$$

where IAS and AAS denote the isobaric analog state and anti-analog state respectively. Substituting into (1) and integrating over nuclear and projectile intrinsic coordinates yields

$$M_0 \propto \int \chi_f^{(-)*}(\underline{k}_f, \underline{r}) F_0(r) Y_0^0(\hat{r}) \chi_i^{(+)}(\underline{k}_i, \underline{r}) d\underline{r} \quad (3)$$

$$M_1 \propto \int \chi_f^{(-)*}(\underline{k}_f, \underline{r}) F_1(r) Y_0^0(\hat{r}) \chi_i^{(+)}(\underline{k}_i, \underline{r}) d\underline{r}$$

with

$$F_0 = I_d(r) + I_f(r)$$

$$F_1 = I_d(r) - I_f(r) \quad (4)$$

$$I_{ij}(r) = \int_0^\infty U_{ij}^{(p)}(r') U_{ij}^{(n)}(r') g_0(r', r) r'^2 dr',$$

where the IAS and AAS transitions are denoted by 0 and 1, respectively, and  $g_0(r', r)$  is the  $\lambda=0$  coefficient in the multipole expansion of the effective interaction. In Eqs. (3) and (4) the slight differences in Q value or orbital binding energies between the IAS and AAS have been ignored.

The reaction amplitudes can be cast in a convenient form by integrating over the angular coordinates  $\hat{r}$  so that

$$M_\nu(q) \propto \int_0^\infty J(r, q) F_\nu(r) dr, \quad \nu = 0, 1$$

$$J(r, q) = \int \chi_f^{(-)*}(\underline{k}_f, \underline{r}) \chi_i^{(+)}(\underline{k}_i, \underline{r}) Y_0^0(\hat{r}) d\hat{r}, \quad (5)$$

where  $q$  is the magnitude of the momentum transfer  $q = |\underline{k}_f - \underline{k}_i|$ . For plane waves one obtains  $J(r, q) \propto r^2 j_0(qr)$  while for diffraction models one gets  $J(r, q) \propto r J_0(qr)$  for  $r > R$  and zero for  $r < R$ . Noble employs a mixture of plane wave and diffraction models with a  $qr$  functional dependence throughout. (The  $r$  or  $r^2$  factor may be replaced by an average value near the nuclear surface since  $qR \gg 1$  for scattering angles of interest.) However, both plane wave and diffraction models are rather extreme approximations for the ( ${}^3\text{He}, t$ ) reactions we are considering.

The IAS and AAS form factors,  $F_0(r)$  and  $F_1(r)$  are rather simply

related since they are sums or differences of functions with similar asymptotic behavior. For the  $^{40}\text{Ar}(^3\text{He,t})^{40}\text{K}$  case  $I_d(r)$  and  $I_f(r)$  are nodeless with roughly exponential falloff behavior in the region of the nuclear surface. Thus  $F_1(r)$  is roughly proportional to the radial derivative of  $F_0(r)$

$$F_1(r) \propto \frac{d}{dr} F_0(r). \quad (6)$$

This approximation is tested in Fig. 1 and is seen to be quite accurate. Noble<sup>1</sup> employs a more complicated (and less accurate) approximation relating  $F_0(r)$  and  $F_1(r)$  which leads to equivalent results.

Using the above approximation in Eq. (5) one obtains

$$\begin{aligned} M_1(q) &\propto \int_0^\infty f(r,q) \frac{d}{dr} F_0(r) dr \\ &\propto \int_0^\infty \frac{d}{dr} f(r,q) F_0(r) dr \end{aligned} \quad (7)$$

by partial integration. Finally, if  $f(r,q)$  is a function of the product  $qr$  only, i.e.  $f(r,q) = f(qr)$  then by switching derivatives we have

$$M_1(q) \propto \frac{d}{dq} M_0(q) \quad (8)$$

which gives the observed interlacing of the IAS and AAS angular distributions. This is the essence of Noble's explanation<sup>1</sup> of the anomaly reported by Hinrichs et al.<sup>2</sup>

However, the above "proof" hinges on the assumption that  $f(r,q)$  is a function of  $qr$  only. Fortunately, it is easy to test this assumption using distorted wave functions which have been generated<sup>5</sup> from optical potentials which fit  $^3\text{He}$  elastic scattering in this region of energy and target nucleus. Figure 2 shows the modulus of  $f$  as a function of  $qr$  and  $r$ . In the important surface region between 4F and 6F (c.f., the form factors in Fig. 1) the function  $f$  increases by an order of magnitude and also changes shape and phase (not shown). In this region the assumption leading to Eq. (8) is not valid. The functional form is so as

to preferentially weigh contributions from large  $r$  thus explaining the insensitivity of the calculations of Hinrichs et al.<sup>2</sup> to the form factor details<sup>6</sup>.

The ( ${}^3\text{He}, t$ ) angular distribution anomaly is reminiscent of a similar anomaly a decade ago where ( $\alpha, \alpha'$ ) excitations of  $4^+$  levels of even nuclei were found<sup>7</sup> to exhibit an anomalous angular distribution. The anomaly was explained in a plane wave theory<sup>8</sup> although later work<sup>9</sup> showed that the plane wave result was fortuitous and, more significantly, obscured the understanding of the competing reaction mechanisms. Perhaps an additional mechanism (such as second-order excitation of the AAS via the IAS) is needed to explain the current angular distribution anomaly in ( ${}^3\text{He}, t$ ) reactions.

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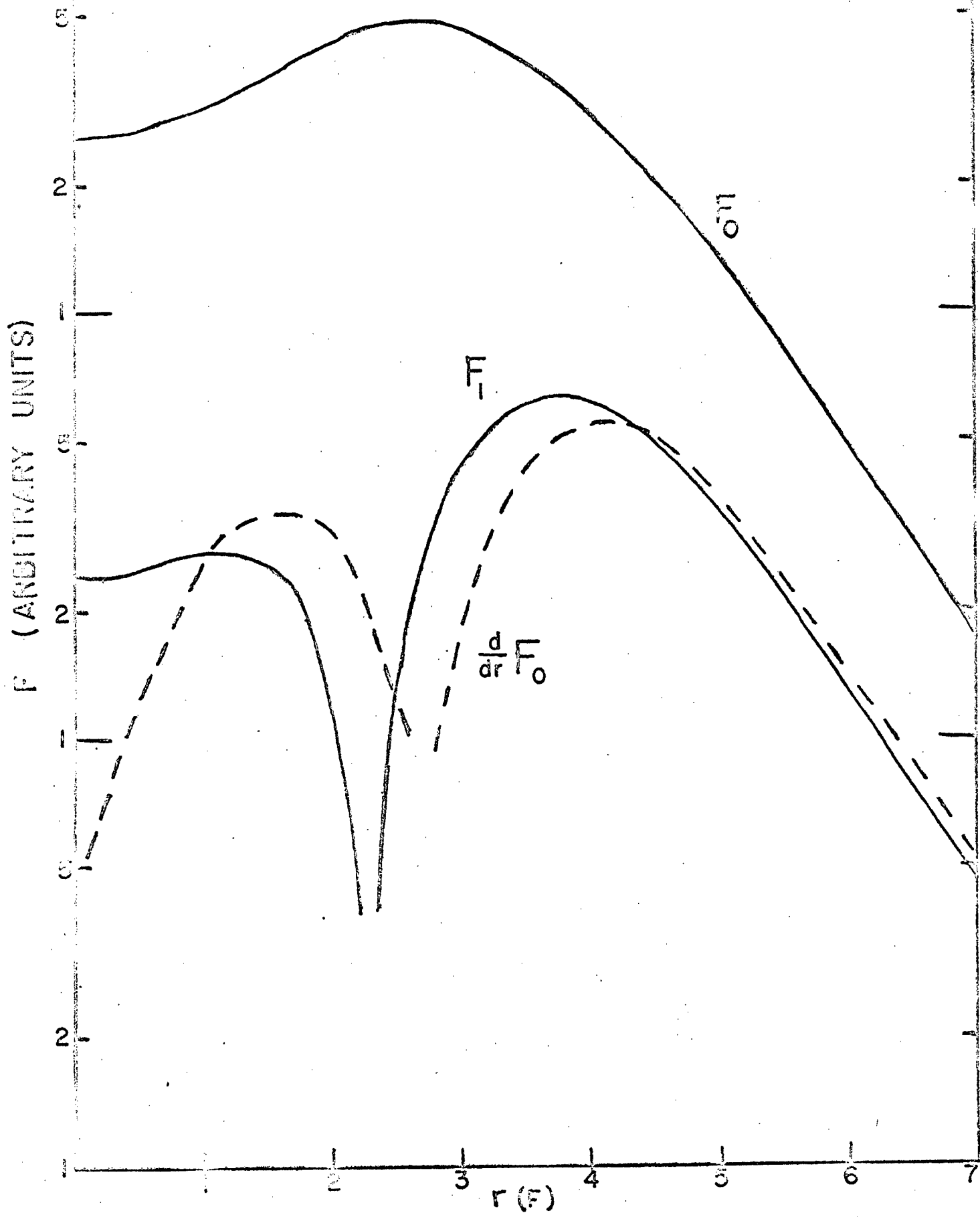


Figure Captions

Fig. 1 Form factors for the transitions to the IAS ( $F_0$ ) and ASS ( $F_1$ ) states in  $^{40}\text{K}$  via the ( $^3\text{He},t$ ) reaction. The curves are computed in a microscopic model formulation (see ref. 3) with orbitals generated in a Woods-Saxon well of parameters  $r_0=1.25\text{ F}$ ,  $a=0.65\text{ F}$  and spin-orbit strength of 25 with Coulomb repulsion from a uniform sphere of radius  $1.25\text{ A}^{1/3}$  included for proton orbitals. The well depth was adjusted to give the neutron and proton separation energies, 9.875 and 5.930 MeV, respectively, appropriate for the AAS state in  $^{40}\text{K}$  (taken to be the 1.65 MeV level reported in ref. 2). A microscopic interaction of Yukawa form with range parameter  $1\text{F}$  was used. The radial derivative of  $F_0$  is compared with  $F_1$ .

Fig. 2 Modulus of the function  $\mathcal{J}(r,q)$  defined by Eq. (5) in the text. The distorted waves used are specified in the caption to fig. 1.

FORM FACTORS FOR  $^{40}\text{Ar}(^3\text{He}, t)^{40}\text{K}$



RADIAL DEPENDENCE OF THE  
DISTORTED WAVES FOR  $^{40}\text{Ar}(^3\text{He},t)^{40}\text{K}$

