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Reactor Fuel Waste Disposal Project--

Development of Design Principle for Disposal
Into Underground Salt Cavities

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The University of Texas
Department of Civil Engineering
Sanitary Engineering Research Laboratory
Austin, Texas

Technical Report

REACTOR FUEL WASTE DISPOSAL PROJECT
DEVELOPMENT OF DESIGN PRINCIPLE FOR DISPOSAL OF REACTOR FUEL WASTE
INTO UNDERGROUND SALT CAVITIES

By

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Atomic Energy Commission

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January 1, 1959
Austin, Texas

SS

ABSTRACT

DEVELOPMENT OF DESIGN PRINCIPLE FOR DISPOSAL OF REACTOR FUEL WASTE INTO UNDERGROUND SALT CAVITIES

Publication No. _____

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The safe disposal of reactor fuel wastes is an important problem. One solution is placement of the wastes in underground salt cavities; this dissertation is an investigation of this solution. Upon preliminary study, the structural stability of the cavity appeared to be of prime importance. Consequently, investigations were made to evaluate the structural properties of cavities.

Theoretical investigations for spherical and cylindrical cavities included analysis of elastic stress, thermal stress and stress redistribution due to the development of a plastic zone around the cavity. The elastic stress distribution around various forms of cavity openings have been summarized in various diagrams. The problems of temperature distribution and accompanying thermal stress, due to heat emission from the waste, were also studied. The reduction of the cavity volume, the development of the plastic zone, and the resulting stress redistribution around the cavity have been presented as functions of cavity depth, internal pressure of cavity, strength of salt, and cavity temperature rise. Theoretical analyses have demonstrated that a salt cavity can be designed such that it is structurally stable as a storage container assuming a chemical equilibrium can be established between the liquid waste and salt.

Structural equilibrium equations as related to cavity temperature rise, loading condition, and cavity volume reduction have been developed for the purpose of designing the spherical and cylindrical cavities.

In the laboratory experiments, a compression test procedure for salt was developed. By using this procedure, fundamental properties of salt such as strength, Young's modulus, Poisson's ratio, strain-hardening and grain effect were determined. Experiments were also conducted for evaluating the effects of tri-axial compression and creep. Further, experiments were conducted to determine the effects of radiation, heat, and chemical solutions on the properties of salt. To investigate the comprehensive effect of heat, radiation, creep, tri-axial compression, and chemical solution, a testing device was constructed to simulate actual storage conditions.

Actual creep measurements were made in the Grand Saline salt mine in Texas, and the results appear to agree with the theoretical and experimental conclusions.

Based upon the results of these theoretical and experimental investigations, principles have been established for designing salt cavities which can be used for storage of radioactive wastes.

TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT	v
LIST OF FIGURES	x
LIST OF TABLES	xvi
NOTATION	xvii
 I. INTRODUCTION	 1
General Description	1
Object	6
Scope	6
PART ONE: THEORETICAL ANALYSIS	
 II. ELASTIC STRESS DISTRIBUTION AROUND VARIOUS FORMS OF CAVITY OPENINGS	 8
General Description	8
Initially Existing Stress in Salt Formations	10
Application of Theory of Elasticity and Photoelasticity	11
Effect of Depth of Overburden Layer	13
Effect of Size of Cavity Opening	15
Stress Concentration Around Tunnel Opening of Various Forms of Cross Sections	20
Effect of Multiple Tunnel Openings	25
Cavities of Three Dimensions	26
 III. PLASTICITY ANALYSIS	 30
Application of Theory of Plasticity	30
Development of Plastic Zone Around a Spherical Cavity	35
Development of Plastic Zone Around a Cylindrical Cavity	38
Practical Consideration	44
 IV. THERMAL STRESS ANALYSIS	 45
Thermal Stress	45
Flow of Heat	45
Thermal Stress Distribution Around a Spherical Cavity	50
Thermal Stress Distribution Around a Cylindrical Cavity	52
Practical Consideration	55

	<u>Page</u>
V. DEVELOPMENT OF STRUCTURAL EQUILIBRIUM EQUATIONS FOR DESIGNING CAVITIES	57
Stress Distribution Around Ideal Cavities with Elevated Temperature	57
Further Development of Plastic Front with Elevated Temperature	59
Structural Equilibrium Equations in Relation to Cavity Volume Reduction	63
Practical Consideration	70
PART TWO: EXPERIMENTAL INVESTIGATION	
VI. DEVELOPMENT OF STANDARD PROCEDURE FOR TESTING SALT	72
Difficulties in Compression Test of Salt	72
Standardized Procedure	75
Application of the End Friction Reducer	76
Dimensions of Test Specimen	78
VII. FUNDAMENTAL STRUCTURAL PROPERTY OF SALT	82
Strength	82
Young's Modulus	84
Poisson's Ratio	86
Strain-Hardening	88
VIII. STRENGTH THEORY OF SALT	91
Triaxial Compression	91
Mohr's Envelope	95
Effect of the Mean Principal Stress	100
Strength of a Cylindrical Cavity	101
Experimental Determination of the Equivalent Plastic Limit	108
IX. CREEP	114
Grain and Crystal	114
Test Procedure and Results	118
Analysis of Creep Curve	120
Practical Considerations	124

X.	CREEP MEASUREMENT IN THE GRAND SALINE SALT MINE	<u>Page</u> 127
	Test Procedure	127
	Experimental Results	132
	Significance of Mine Observations	132
XI.	VARIOUS FACTORS AFFECTING PROPERTIES OF SALT	138
	Development of Test Device	138
	Effect of Heat	144
	Effect of Solutions	149
	Effect of Gamma Irradiation	152
PART THREE: EVALUATION		
XII.	DESIGN PRINCIPLE	155
	Structural Equilibrium	155
	The Factor of Structural Loading	156
	Volume Reduction	157
	Cavity Temperature	158
	Other Considerations	158
XIII.	CONCLUSION	160
	Structural Equilibrium	160
	Structural Properties of Salt	163
APPENDIX		166
	Nature of Reactor Fuel Waste	166
BIBLIOGRAPHY		168

LIST OF FIGURES

<u>Figure</u>		<u>Page</u>
1-1	Primary Saline Deposits in the United States	5
2-1	Initial Stress Condition in Salt Formations	9
2-2	Stress Distribution Along the Horizontal Direction from the Boundary of Circular Tunnel Opening	9
2-3	Effect of Depth of Overburden Layer on Tangential Stress Concentration around the Boundary of Circular Tunnel Opening	9
2-4	Stress Distribution around Long Cylindrical Cavity	18
2-5	Stress Distribution around Spherical Cavity	18
2-6	Stress Concentration around Circular and Square Tunnel Opening	22
2-7	Effect of Orientation of Ovaloid Tunnel Opening on Stress Concentration around the Boundary	22
2-8	Higher Concentration of Critical Stress at Corner of Tunnel Opening	23
2-9	Critical Stress Concentration around Elliptical Tunnel Opening	23
2-10	Effect of Multiple Tunnel Openings on Concentration of Critical Stress	23
3-1	Octahedral Shearing Stress in Principal Stress Coordinate	32
3-2	Typical Stress-Strain Relation of Salt	32
3-3	Plastic Stress-Strain Relation of Salt	32
3-4	Dimensions and Stress Equilibrium of Volume Elements around Spherical and Cylindrical Cavities	32
3-5	Development of Plastic Zone around Spherical and Cylindrical Salt Cavities	39
3-6	Stress Distribution in Plastic and Elastic Zones around Spherical Salt Cavity at Various Depths	43
3-7	Stress Distribution in Plastic and Elastic Zones around Cylindrical Salt Cavity at Various Depths	43

LIST OF FIGURES (continued)

<u>Figure</u>		<u>Page</u>
4-1	Temperature Distribution around Spherical and Cylindrical Cavities with Steady State Heat Flow	48
4-2	Distribution of Unit Thermal Stress Increase around Spherical Salt Cavity per Unit Temperature Rise in Cavity	48
4-3	Distribution of Unit Thermal Stress Increase around Cylindrical Salt Cavity per Unit Temperature Rise in Cavity	48
5-1	Development of Plastic Zone around Spherical and Cylindrical Cavities with Various Temperature Rises	62
5-2	Relation between Ultimate Volume Reduction and Structural Loading with Various Temperature Rises in Spherical Cavity	68
5-3	Relation between Ultimate Volume Reduction and Structural Loading with Various Temperature Rises in Cylindrical Cavity	69
6-1	Mine Operation at 700 ft. below Surface in Grand Saline Salt Dome in East Texas	74
6-2	Saw and Drill Technique for Removing Test Specimen	74
6-3	Hand Cutting Preparatory to Machine Cutting	74
6-4	Failure Characteristics Resulting from Different End Friction	74
6-5	Impurity Strata Indicating Previous History of Deformation	74
6-6	Deviation of Impurity Strata from Normal Vertical Axis	74
6-7	Comparison of Strain Reading by SR-4 Gage and Dial Gage	77
6-8	Effect of End Friction Reducer on Stress-Strain Relation	77
6-9	Effect of Specimen Height On Stress-Strain Relation	79
6-10	Effect of Specimen Cross-section on Stress-Strain Relation	79
6-11	Effect of Size of Cube Specimen on Stress-Strain Relation	79

LIST OF FIGURES (continued)

<u>Figure</u>		<u>Page</u>
6-12	Effect of Dimension and End Friction on Maximum Stress	81
7-1	Poisson's Ratio for Crystal Grain in Aggregate and Aggregate as a Whole Determined by SR-4 Strain Gages	87
7-2	Strain-Hardening in Crystal and Aggregate as Determined by SR-4 Strain Gages	87
7-3	Orientalional Analysis of Strain Hardening by SR-4 Strain Gages	87
8-1	Plastic Deformation of Specimen by Uniaxial Compression at 770°F.	92
8-2	400,000 Pound Test Apparatus for Measuring Creep Rates in Cylindrical Cavity	92
8-3	Cylindrical Cavity Test Apparatus	92
8-4	Closure Resulting from Creep in Cylindrical Salt Cavity	92
8-5	Partially Closed Cylindrical Cavity	92
8-6	The 98% Cavity Closure	92
8-7	Tri-axial Compression of Aggregate Salt Under Various Confining Pressures	94
8-8	Tri-axial Extension of Aggregate Salt Under Various Confining Pressures	94
8-9	Tri-axial Compression of Aggregate Salt Exposed to Confining Liquid; i.e., Kerosene	94
8-10	Mohr's Envelopes for 2% Offset Yield Under Compression and Extension at Various Temperatures	96
8-11	Principle of Reducing Maximum Shearing Stress by Adjustment of Internal Pressure of Cavity under Non-Hydrostatic Initial Stress Condition	96
8-12	Effect of Mean Principal Stress on Stress-Strain Curve	102
8-13	Closure Rate of Model Salt Cavity	102

LIST OF FIGURES (continued)

<u>Figure</u>		<u>Page</u>
8-14	Cross-Section of Test Cylinder	104
8-15	Stress Distribution and Reduction of Radius in Process of Creep Closure of Cylindrical Cavity	106
9-1	Hydraulic Testing Machine and SR-4 Strain Indicating Devices	115
9-2	Test Specimen Used for Studying Poisson's Ratio, Transient Creep, and Inter-Granular Movement	115
9-3	Typical Failure of Single Crystal Compressed Parallel to Crystal Plane	115
9-4	SR-4 Strain Gages; Type AR-1 for Strain in Aggregate and Type A-8 for Strain in Grains	115
9-5	Self-Regulating Pressure System	115
9-6	Furnace Attached to Creep Testing Stand	115
9-7	Comparison of Transient Creep in Single Crystal, Grain Crystal in Aggregate, and Aggregate as a Whole	117
9-8	Transient Creep Under Various Loads	117
9-9	Schematic and Circuit Diagrams of Pressure Self-Regulator for Creep Test	119
9-10	Effect of End Friction on Creep of Standard Specimens Under Compression of 3,000 psi	119
9-11	Effect of Mean Principal Stress on Steady State Creep	119
9-12	Ideal Normal Creep Curve Compared on Linear and Semi-Logarithmic Scales	121
9-13	Determination of Coefficient of Transient Creep	121
10-1	Station B; Measuring Creep Closure Rate of Mine Opening by Using Dial Gage Device	128
10-2	Typical Superficial Scaling with Sharp Angle of Fracture Developed at Vertical Corner of Pillar	128
10-3	Point No. 1; SR-4 Strain Gage Measurement on Wall of Mine	128

LIST OF FIGURES (continued)

<u>Figures</u>		<u>Page</u>
10-4	Point No. 4; Measuring Strain on Floor of Mine Opening with SR-4 Strain Gages	128
10-5	Broken Timber Frames Resulting from Creep	128
10-6	Typical Fracture of Supporting Timbers	128
10-7	Plan of the Grand Saline Salt Mine Opening	129
10-8	Profile of Grand Saline Salt Dome	129
10-9	Schematic Diagram of Stations and Measuring Devices for Determination of Closure Rate of Mine Opening	131
10-10	Creep Closure Rate of Tunnel Openings in Grand Saline Salt Dome at Depth of 700 feet	134
11-1	Darkening Effect of Gamma Irradiation upon Natural and Synthetic Salt Crystal	139
11-2	Removal of Dark Color from Irradiated Salt by Heat or Recrystallization	139
11-3	Specimen Tested by Uniaxial Compression Showing Increase of Ductility with Increase of Temperature	139
11-4	Crystal from Grand Saline Salt Mine Tested with Confining Pressure of 14,500 psi at 300°F. and 570°F.	139
11-5	Creep Test and Heating Apparatus	139
11-6	Creep Test of Standard Specimen Submerged in Reactor Fuel Waste Substitute (Aluminum Nitrate Solution)	139
11-7	High Pressure Cell Assembly	140
11-8	Arrangement for Permeability Measurement and Comprehensive Test of Salt	141
11-9	Assembly of Comprehensive Test Device	142
11-10	Effect of Elevated Temperature upon Stress-Strain Relation of Standardized Specimen with Friction Reducer	146
11-11	Tri-axial Compression of Natural Salt Crystal at Various Temperatures	147

LIST OF FIGURES (continued)

<u>Figure</u>		<u>Page</u>
11-12	Tri-axial Compression of Artificial Salt Crystal at Various Temperatures	147
11-13	Tri-axial Extension of Artificial Salt Crystal at Various Temperatures	147
11-14	Effect of Elevated Temperature on Creep under Constant Pressure at 770 psi	150
11-15	Effect of Submersion in Various Solutions for One Month on Stress-Strain Relation	150
11-16	Effect of Submersion in Solutions on Creep	150
11-17	Effect of Gamma Irradiation on Stress-Strain Relation	154
11-18	Effect of Gamma Irradiation on Transient Creep of Aggregate Salt	154
11-19	Effect of Gamma Irradiation on Dissolving Rate of Granular Salt	154
A.	Total Accumulation of Amounts of Reactor Fuel Waste and Selected Fission Products Estimated by General Electric Research Group (Hanford Plant, Richland, Washington)	168

LIST OF TABLES

<u>Table</u>		<u>Page</u>
2-1	Critical Stress in Spherical Salt Cavity	28
6-1	Laboratory Procedures for Testing Salt	75
7-1	Maximum Stress of Salt as Determined by Various Investigators	83
7-2	Young's Modulus by Different Test Procedures	85
8-1	Determination of σ_0 from Compressive Strength Test Data	110
9-1	Summary of the Coefficient of Creep	125
10-1	Objectives of Seven Creep Measuring Points by Using SR-4 Strain Gages	130
10-2	Summary of Measurement and Computation of Closure Rate of Mine Opening	133
11-1	Summary of Comprehensive Test Device Capacity	144

NOTATION

- C = Specific Heat
 E = Young's Modulus
 K = Thermal Conductivity
 K_e = Elastic Limit of Octahedral Shearing Stress
 K_p = Plastic Limit of Octahedral Shearing Stress
 P_i = Internal Pressure of Cavity
 P_o = External Pressure of Cavity (Overburden Load of Cavity)
 S_r, S_t = Radial and Tangential Stresses around Cavity in Infinite Elastic Body
 S_x, S_y, S_z = Principal Stresses Initially Existing in Underground Formation before Cavity is Created
 T = Temperature
 T_1 = Temperature at Boundary of Cavity
 T_2 = Temperature at Outside of Thick-walled Shell
 T_c = Temperature Rise in Cavity above Ambient Temperature
 T_r = Temperature Rise at Radial Distance r
 X = Factor of Structural Loading
 Y = Factor of Plastic Front Radius
 Z = Factor of Temperature Effect
 a = Radius of Spherical or Cylindrical Cavity
 a' = Reduced Radius of Model Cylindrical Salt Cavity
 b = Outside Radius of Spherical or Cylindrical Thick-walled shell
 c = Reduced Cavity Radius due to Plastic Closure
 r = Radius

t = time

w = Ratio of Outside Radius to Inside Radius of a Thick-walled Cylindrical Shell

α = Coefficient of Linear thermal expansion or Instantaneous Creep at time δ

β = Coefficient of Transient Creep

γ = Coefficient of Steady State Creep

δ = Time at Instantaneous Creep Reading

$\epsilon_1'', \epsilon_2'', \epsilon_3''$ = Principal Plastic Strains

$\epsilon_x, \epsilon_y, \epsilon_z$ = Strains in Three Mutually Perpendicular Directions

ζ = Ratio of Reduced Cavity Volume to Original Volume

λ = Thermometric Conductivity

μ = Density

ν = Poisson's Ratio

ρ = Radius of Plastic Front

$\sigma_1, \sigma_2, \sigma_3$ = Principal Stresses

σ_e = Elastic Limit in Compression Test

$\sigma_m = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$ = The Mean of the Principal Stresses

σ_o = Equivalent Plastic Limit of Salt

σ_p = Plastic Limit in Compression Test

$\sigma_r, \sigma_t, \sigma_z$ = Stresses in Radial, Tangential and Axial Directions

$\sigma_x, \sigma_y, \sigma_z$ = Stresses in Mutually Perpendicular Directions

τ_o = Octahedral Shearing Stress

CHAPTER I

INTRODUCTION

General Description

The ultimate disposal of radioactive waste in quantity has become one of the most important problems of this age. The accumulation of radioactive waste has been progressing at an accelerated rate, as shown in Fig. A in the Appendix. However, there has been no concomitant development of methods of disposing of this waste. Yet, without adequate means for safe, permanent storage of this waste, public health remains in jeopardy, and the development of nuclear energy is arrested. This paper is an attempt to contribute to the solution of this problem: to provide a means whereby large quantities of radioactive waste can be ultimately disposed of, safely and economically.

Reactor fuel waste, created by reactor fuel element reprocessing plants, is responsible for most of the present supply of radioactive waste. It is characterized by a high concentration of radioactive isotopes. Although the primary concern of this paper is reactor fuel waste, other high-level radioactive wastes may be similarly considered and treated because of their common nature.

The reactor fuel waste produced since the introduction of the first nuclear reactor has been stored in artificial containers to date. Because the lifetime of these containers is much shorter than the radioactive part of the waste, this means of storage is necessarily a temporary one. The temporary nature of these containers poses further problems of safety, the necessity of continual replacement, and the consequent expense. For example, radioactive waste has been stored in stainless steel containers which have an expected lifetime of about 50 years, whereas the radioactivity of the waste remains above the allowable level for possibly 500 years.

The particular problems which arise in the actual disposal of the waste are high radioactivity which requires thick shielding and remote handling, heat generation from radioactive isotope disintegration requiring an adequate cooling system, and the highly corrosive nature of the waste. Because of these inherent difficulties no safe, efficient and economical method of disposing of radioactive waste has been devised.

In 1955, the Committee on Waste Disposal of the Division of Earth Sciences, National Academy of Science, responsive to the urgent need for solution of this problem, suggested that disposal of radioactive waste in underground cavities mined in salt beds and salt domes offered the most promising practical solution⁶³. This conclusion of the Committee was drawn after extensive exploratory work was carried out by leading scientists in interrelated fields, such as Sanitary Engineering, Chemistry, Physics, Geology, Geophysics, Oceanography, and Economics, under the guidance of the U.S. Atomic Energy Commission. The Committee concluded that the principle of disposing of radioactive wastes by concentration and separation from our environment was more desirable than the principle of diluting them in our environment to a certain allowable level of radioactivity. Certainly, for high-level radioactive waste there is no place on this earth for such dilution, either in the ocean or in the air. The Committee therefore made a specific recommendation on disposal of radioactive waste, saying that:

Even though storage in tanks is at present the safest and possibly the most economical method of containing waste, disposal of radioactive waste in salt is the most promising method for the near future; research should be pushed immediately on the structural problem of stability versus size of cavities at a given depth; on the thermal problem - getting rid of the heat or keeping it down to acceptable levels - and the economics of such disposal.

Along with this recommendation, the following approaches to the solution of the problem were considered by the Committee as of secondary importance:

1. Stabilization of the waste in a soil or ceramic material forming a relatively insoluble product, which could be placed in dry mines, surface sheds or large cavities in salt.
2. Disposal of waste in porous beds interstratified with impermeable beds in a synclinal structure, which would be of particular interest for disposal of the large volumes of medium and low-level wastes.
3. The removal of long life radioactive isotopes, particularly Cs^{137} and Sr^{90} from the wastes, which would make disposal somewhat easier.

The following is a summary of the major advantages of disposal in salt:

1. Strength - Salt has considerable compressive strength but little tensile strength. The lack of tensile strength, however, is not crucial since tensile stress is not usually exerted around common forms of salt cavities.
2. Plasticity - Salt is plastic and produces plastic flow without cracking when it is subjected to a certain shearing stress. Therefore, salt will act as a self-sealer of the storage cavity as well as an absorber of higher stress concentration around the cavity opening.
3. Imperviousness - Salt under compressive stress is impervious to the passage of gases and liquids. The mined-out area stays dry, and if the space is filled with any kind of fluid material, no seepage is allowed.
4. High Solubility - Salt is highly soluble in water. The solubility of NaCl in water in grams per 100 ml. is 35.7 and 39.12 at 0°C. and 100°C., respectively. After reaching saturation, no further dissolution occurs. This makes possible the creation of a huge cavity by simply washing the salt out, then storing the salt-saturated liquid waste without affecting the storage cavity.

5. High Thermal Conductivity and Melting Point - The comparatively high thermal conductivity of salt and its sufficiently high melting point are favorable for the storage of high-level radioactive waste.

6. Availability - Salt basins and domes occur in abundance throughout the country⁴¹, in an area of more than half a million miles square, extending through eighteen states, as shown in Fig. 1-1. These deposits, which are located along the south side of the Great Lakes and along the Gulf Coast, have been considered most seriously for the disposal of radioactive waste because of their convenient location for industries. The Great Lakes deposits range in thickness from 200 to 2,000 feet, at depths of 500 to 10,000 feet. The depth of many Gulf Coast salt domes is less than a few thousand feet, making it feasible to construct underground storage cavities economically.

7. Storage Capacity and Economy - A large cavity in a salt deposit can be constructed with much less expense than any other type of storage tank. The present rate of expense for storage tanks is \$0.25 to \$2.00 per gallon of constructed cavity space⁵⁰. Many salt cavities of 10 to 100 million gallon capacity have been constructed with a capital investment of a few cents per stored gallon. If the mined-out cavity in a salt dome can be used successfully, the unit storage expense will be further reduced.⁵⁰ The total space created by mining rock salt during a period of twenty years, 1934 to 1953, in the nation, was calculated to be about seven billion gallons⁶³.

8. Previous Experience of Storage in Salt - In the last decade, hundreds of millions of gallons of liquid petrochemical products have been successfully stored in salt cavities by various oil companies^{50, 54, 66}.

FIG. 1-1. PRIMARY SALINE DEPOSITS IN THE UNITED STATES.

Although extensive experience has been acquired in the mining of salt, very little study has been made on the structural stability of salt cavities. This is because salt cavities and tunnels at the present mining depth of a few thousand feet are strong enough without need for additional support. Hence, no need has arisen for structural investigation of a salt cavity. However, studies on the structural behavior of salt have been conducted, by many investigators^{22-25, 32,35,39,60,67}. Aside from these reports, most of the previous publications about salt have been geological surveys and tectonic explanations rather than structural studies. No reference has been found on stress analysis of a salt cavity, or on the strength of salt structures.

Object

The final purpose of the study is to develop a design principle for the practical use of salt cavities as disposal containers. This study was begun with the theoretical analysis of stress distribution around various forms of cavities regarding elasticity of salt, thermal stress, and plasticity of salt. Experiments were conducted in the laboratory and at the Grand Saline salt mine in Texas, to investigate the strength of a salt cavity.

As a result of these analyses, suggestions for the safe disposal of radioactive waste are presented regarding the most suitable form, dimension and depth for a salt cavity.

Scope

The major source of the salt used in the laboratory was the salt dome located at Grand Saline, Texas, 50 miles east of Dallas, where the Morton Salt Company has been mining salt since the early 1930's. This salt was

used as the test specimen, and salt from other sources was studied for comparative purposes. The two types of salt cavities considered are mined-out tunnels and washed-out cavities. Liquid high-level radioactive waste, which comes directly from nuclear fuel reprocessing plants was the type of waste considered for the purposes of this investigation. The disposal of other types of waste, such as the solid form or low-level radioactive waste, may be carried out by applying the same principles and methods.

The scope of this study is limited to investigation of the structural stability of a storage cavity of salt under various conditions. No consideration was given to technical problems such as the economical evaluation of disposal sites, the construction method for a cavity, transportation of the waste, installation of cooling systems, monitoring systems, and other details.

The word "salt" as used in this paper, will mean rock salt of natural salt deposits.

PART ONE: THEORETICAL ANALYSIS

CHAPTER II

ELASTIC STRESS DISTRIBUTION ANALYSIS AROUND VARIOUS FORMS OF CAVITY OPENINGS

General Description

Stress acting around a cavity opening in a solid medium is the principal cause of either plastic deformation or collapse of the cavity. Since stress is not a property, but a state of material, the stress around a cavity can be mathematically determined, regardless of the nature of the material of which the cavity is made. This chapter is intended to obtain a concept of natural stress of underground salt formations due to overburden load and lateral pressures, and to determine the effects of the depth, size, shape, number, and geometry of cavity openings upon the elastic stress distribution.

The theories of elasticity and photoelasticity can be applied to stress analysis of underground salt cavities within a certain depth, depending upon cavity geometry. For theoretical analysis of the elastic stress, salt formations (both salt domes and salt beds) can be reasonably assumed to be homogeneous and isotropic. The size of the salt formation is assumed to have infinite dimensions compared with the size of the cavity opening.

Any additional lateral pressure acting on the salt formation has been further assumed to be static and less significant than the pressure due to overburden load at the depth of cavity opening. In general, regions in which salt deposits occur are effectively at rest in terms of hundreds of years.

Thus, the major stress existing in a salt formation to be considered is that due to the overburden load. However, if any additional lateral pressure is known to be acting on the salt formation, the stress distribution around the cavity due to this lateral pressure can be obtained by the same analysis as that of the overburden load, and then the one superimposed over the other, as illustrated in Fig. 2-1.

FIG. 2-1 INITIAL STRESS CONDITIONS IN SALT FORMATION.

FIG. 2-2 STRESS DISTRIBUTION ALONG THE HORIZONTAL DIRECTION FROM THE BOUNDARY OF CIRCULAR TUNNEL OPENING. ^{10, 14, 21, 65}

FIG. 2-3. EFFECT OF DEPTH OF OVERBURDEN LAYER ON TANGENTIAL STRESS CONCENTRATION AROUND THE BOUNDARY OF CIRCULAR TUNNEL OPENING. ^{14, 44, 69}

Initially Existing Stress in Salt Formations

The initial stress naturally existing in any underground formation before an opening is made, is primarily determined by two factors: the overburden load and the Poisson's ratio of the formation.

The relation between the principal stresses and the Poisson's ratio can be mathematically derived from the fundamental stress-strain formulas in three dimensions. The strains in an element in underground formation on three mutually perpendicular directions (x, y, and z) can be described as:

$$\begin{aligned}\epsilon_x &= \frac{1}{E} [S_x - \nu(S_y + S_z)] \\ \epsilon_y &= \frac{1}{E} [S_y - \nu(S_z + S_x)] \\ \epsilon_z &= \frac{1}{E} [S_z - \nu(S_x + S_y)]\end{aligned}\tag{2-1}$$

where ϵ_x, ϵ_y = strains in horizontal directions

ϵ_z = strain in vertical direction

S_x, S_y = stress in horizontal direction

S_z = stress in vertical direction

E = Young's modulus

ν = Poisson's ratio

Since no lateral expansion of the element due to the overburden load is possible in an underground formation, and the vertical stress is equal to the overburden load, the following relations can be substituted in Eq. 2-1:

S_z = vertical stress = overburden load

$S_x = S_y$ = lateral pressure

$\epsilon_x = \epsilon_y = 0$

The result is:

$$S_y = \frac{\nu}{1-\nu} S_z\tag{2-2}$$

Therefore, this equation means that the vertical stress, S_z , in an underground formation is always equal to its overburden pressure, and the lateral stress, S_x and S_y have values greater than zero and equal to or less than the overburden load, depending upon the Poisson's ratio of the formation.

To determine the initially existing stress distributions in salt, experimental determination of the Poisson's ratio is necessary. As discussed in Chapter VII, results of the laboratory experiments as well as calculations on seismic data agree that the value of the Poisson's ratio is approximately 0.5 at depths 300 feet or more, which makes the stress distribution in the salt hydrostatic.

Application of Theory of Elasticity and Photoelasticity

A solution of the problem of stress distribution around a cavity opening using the theory of elasticity requires a basic assumption that the cavity opening approximates a definite geometrical figure of extreme simplicity. For two dimensional stress analysis, mathematical determination of stress distribution could be applicable only to such simple forms of cavity opening as tunnels with circular or elliptical cross-sections. In the case of these ideal forms of opening, the theoretical approach is perhaps the best, as it provides an exact solution to the problem. Most salt cavities, however, do not have simple forms. If the washing-out method were used, probably the cavity form would be so irregular that a mathematical analysis would be impossible. Yet, approximation of critical stresses around irregular cavities can be made for practical purposes, using the mathematical solutions of ideal cavity forms.

It is in the solution of the highly complicated form of cavity that photoelasticity has its best application. The theory of photoelasticity depends

upon the fact that a plane polarized ray, within certain transparent media, is split into two components if the media are birefringent. The two parts vibrate in the principal planes of stress with different velocities of propagation. The lag of one component behind the other, so-called relative retardation, is proportional to the difference between the two principal stresses.

In general, a solution by the photoelastic method requires (1), selection of a certain transparent material which possesses a Poisson's ratio close to that of salt, (2), making a model cavity opening of specific form, (3), loading the model similarly to the pressure conditions naturally existing in the salt formation, (4), freezing stresses in the model and then slicing the model, and (5), translation of the optical measurement into stress distribution in the prototype salt cavity. Young's modulus of the material for a model is not related to this method, and the difference between prototype and model in Poisson's ratio has a minor influence on the result. The scale ratio has no significance and can be chosen to any value of convenience.

In model and in prototype, the stress distribution depends only upon the form of cavity opening and its orientation in respect to the given stress. Thus, the model stresses are, for practical purposes, directly proportional to the corresponding prototype-stress.

In the three dimensional analysis of stress distribution, both the mathematical determination and the method of photoelasticity can be applied under any external pressure conditions. Mathematical solutions are available only for extremely simple forms of cavity openings, such as a sphere, an infinitely long cylinder, and an ellipsoid. A three dimensional photoelastic technique may be possible by using the "freezing" method of stress analysis, in which

the fringes (bands of constant intensity of the polarized ray representing a contour of constant shear stress) are locked or frozen into the model as a result of heat treatment. After heat treatment, the required slices are obtained from the model by cutting with a saw^{27,28}.

Two or more conditions of stress distribution around a single cavity can be superimposed in order to obtain the final distribution of the multiple external pressures. This principle of superimposition, using the freezing method, makes it possible to determine stress distribution around any irregular form of cavity under any complex external pressure conditions. The error that may be involved in the photoelastic model study has been proven by various workers to be negligible.^{17,29} Apparatus required for the photoelastic model study is relatively easy to build since the principal elements of the apparatus are a polariscope and camera. Extensive works of stress analysis of various types of machine parts have been published and some of them can be utilized for the salt cavity problem^{16,21,52}. Stress distribution around the most common and simple forms of cavity is summarized from the above mentioned publications in the following sections.

Effect of Depth of the Overburden Layer

The effect of depth of the overburden layer above a cavity opening upon the stress concentration around the opening has attracted much attention from civil engineers because it is a most important structural problem in the construction of tunnel openings. This problem has been solved through the introduction of a bipolar coordinate system by those pioneers, Yamaguchi⁶⁹, Mindlin⁴⁵, and Panek⁵¹. The maximum stress at ceiling, floor and rib of a circular horizontal tunnel of infinite length are shown in Fig. 2-3 as a function of the ratio of overburden depth to opening diameter. In the diagram,

the stresses are expressed in terms of the stress concentration factors, that is, the value of stress expressed by overburden pressure as a measuring unit. A plus sign of the factor indicates tensile stress and a minus sign indicates compressive stress. It is apparent that the effect of the overburden depth on the stress concentration factor diminishes very rapidly with an increase in depth, and the overburden effect becomes insignificant at a depth of more than three times the diameter of the opening; beyond which the stress concentration factor comes close to the constant value. A comparison of the three groups of different initial stress conditions indicates that the initial stress condition of a formation is a controlling factor of the stress distribution around an opening at any depth.

In the hydrostatic condition, probably in most salt formations at a depth of more than 300 feet, no tensile stress occurs around the opening, and the stress on the floor (at C) first becomes critical at a shallow depth. At a depth less than the opening diameter, the stress concentration factor rapidly increases on both ceiling and floor; and the cavity becomes very unstable. Thus, the larger the size of cavity, the larger the depth of the overburden layer should be in order to have the same stress concentration factor.

There should be, however, a depth at which stress becomes minimal because the deeper the cavity, the greater the overburden pressure, and the less the stress concentration factor. Therefore, the depth should be a few times that of the opening diameter.

It should be understood that the effect of depth of the overburden layer on the circular tunnel is similar to that on other cavity forms. The effect of depth on the stress concentration around an opening decreases as depth increases, and then the initial stress condition in a formation determines the stress concentration.

Effect of Size of Cavity Opening

Both mathematical analysis of stress and experimental observation by photoelasticity have proven that, in an initially stressed medium, the extent of the influence of cavity creation upon stress distribution is directly proportional to the size of the opening. The maximum stress always appears on the boundary of the cavity in a tangential direction, the minimum stress appearing on the boundary in a radial direction. However, the stresses are independent of the size of the cavity and constant at a certain depth of the cavity.

The influence of opening a tunnel in underground formations on different initial stress conditions is shown in Fig. 2-2¹⁰. As seen in the diagram, the maximum and minimum values of the stress around the boundary tend to approach the initially existing stress as the distance from the boundary is increased. The stress concentration factor can be determined by the distance from the boundary in terms of the opening radius. The influence of opening the tunnel, therefore, is insignificant if the distance from the boundary is greater than one diameter.

For a circular tunnel or long cylindrical cavity in salt deeper than 300 feet, where the initial stress is hydrostatic, the mathematical calculation of stress distribution around the tunnel can be derived from the stress equations for a thick-walled cylinder⁶⁵:

$$\begin{aligned}\sigma_r &= \frac{-1}{r^2(b^2 - a^2)} \left[b^2(r^2 - a^2) P_o + a^2(b^2 - r^2) P_i \right] \\ \sigma_t &= \frac{-1}{r^2(b^2 - a^2)} \left[b^2(r^2 + a^2) P_o - a^2(b^2 + r^2) P_i \right]\end{aligned}\tag{2-3}$$

where σ_r = stress in radial direction at distance r from the axis of cylinder

σ_t = stress in tangential direction at distance r from axis of cylinder

a = radius of internal opening of the cylinder

b = external radius of the cylinder

P_i = internal pressure

P_o = external pressure

By assuming hydrostatic stress in a salt formation, the stress around a salt cavity can be derived by the following substitutions in this equation:

P_o = overburden pressure

P_i = pressure inside of the cavity

b = infinity

Therefore:

$$\begin{aligned} S_r &= \lim_{b \rightarrow \infty} \sigma_r = \frac{-1}{r^2} \left[(r^2 - a^2) P_o + a^2 P_i \right] \\ S_t &= \lim_{b \rightarrow \infty} \sigma_t = \frac{-1}{r^2} \left[(r^2 + a^2) P_o - a^2 P_i \right] \end{aligned} \quad (2-4)$$

These are the stress equations for the cylindrical cavity. It is clear in the equation that the effect of cavity opening on the stress distribution becomes insignificant when the radius, r , increases; and at the infinite distance, by substituting $r = \infty$ in Eq. 2-4, the stresses become:

$$S_r = -P_o$$

$$S_t = -P_o$$

The stresses around the cavity boundary are obtained by substituting $r = a$ in Eq. 2-4.

$$S_r = -P_i$$

$$S_t = -2P_o + P_i \quad (2-5)$$

The maximum shearing stress, $Q_{\max}$, which is the direct cause of failure and creep of the material is:

$$Q_{\max} = \lim_{b \rightarrow \infty} \frac{\sigma_t - \sigma_r}{2} = - (P_o - P_i) \frac{a^2}{r^2} \quad (2-6)$$

Therefore, the maximum shearing stress is also greatest on the boundary as shown in Fig. 2-4. Since materials are not subject to failure or creep when no shearing stress exists, a structurally stable condition around a cavity can be accomplished by making $P_i = P_o$ in Eq. 2-6. This means that the inside pressure of a cavity is made equal to the overburden load. Another interpretation of Eq. 2-6 is that the shearing stress rapidly approaches zero as the distance from the cavity increases.

In Fig. 2-4, the stress distribution occurring a distance from the cavity wall of a long cylindrical cavity is shown as a special example where the internal pressure, P_i , is zero. This is the case where a cavity is created and left empty.

An identical discussion would be applicable for a spherical salt cavity, if the stress equation of a thick-walled sphere (Eq. 2-7)⁶⁵ is adapted to the salt cavity.

$$\begin{aligned} \sigma_r &= \frac{-1}{r^3(b^3 - a^3)} \left[b^3(r^3 - a^3)P_o + a^3(b^3 - r^3)P_i \right] \\ \sigma_t &= \frac{-1}{2r^3(b^3 - a^3)} \left[b^3(2r^3 + a^3)P_o + a^3(b^3 + 2r^3)P_i \right] \end{aligned} \quad (2-7)$$

By taking an identical approach and using the same nomenclature, the stress around a spherical cavity in radial and tangential directions is obtained as:

$$\begin{aligned} s_r &= \lim_{b \rightarrow \infty} \sigma_r = \frac{-1}{r^3} \left[(r^3 - a^3)P_o + a^3P_i \right] \\ s_t &= \lim_{b \rightarrow \infty} \sigma_t = \frac{-1}{2r^3} \left[(2r^3 + a^3)P_o - a^3P_i \right] \end{aligned} \quad (2-8)$$

FIG. 2-4. STRESS DISTRIBUTION AROUND LONG CYLINDRICAL CAVITY. ⁶⁵

FIG. 2-5. STRESS DISTRIBUTION AROUND SPHERICAL CAVITY ⁶⁵

The effect of opening the cavity upon the initial stress condition in salt becomes negligible at large distances from the cavity. By substituting $r = \infty$ in Eq. 2-8, the stresses in salt become:

$$S_r = -P_o$$

$$S_t = -P_o$$

The stress around the cavity boundary is obtained by assuming $r = a$,

$$S_r = -P_i$$

$$S_t = -\frac{3}{2} P_o + \frac{1}{2} P_i \quad (2-9)$$

The maximum shearing stress is:

$$Q_{\max} = \frac{S_t - S_r}{2} = -(P_o - P_i) \frac{a^3}{r^3} \quad (2-10)$$

Thus, as in the cylindrical tunnel, there occurs no shearing stress when the internal pressure of a cavity, P_i , is made equal to the overburden pressure of the salt formation, P_o . While the distance from the opening increases, $Q_{\max}$ decreases much faster in the spherical cavity than in the long cylindrical cavity. Stress distribution around the spherical cavity is plotted in Fig. 2-5, on the assumption that the internal pressure is zero. As seen in Figs. 2-4 and 2-5, the high concentration of shearing stress, due to opening a cavity in salt, is limited to close proximity of the opening.

For any other form of salt cavity, it is also concluded that the larger the cavity, the further the extent of influence on stress distribution. But the critical stress exerted around the cavity boundary with identical forms is the same regardless of the size of the cavity, at the same depth.

Stress Concentration Around Tunnel Opening of Various Forms of Cross Sections

In the previous discussion, it has been shown that maximum and minimum stresses and maximum shearing stress always appear on the boundary around a cavity opening, and also that the magnitude of these critical stresses is determined by the geometry of the cavity opening. It is clear from Eqs. 2-4 and 2-8 that the two principal stresses are always compressive as long as the external pressure surrounding the cavity is larger than the internal pressure in the cavity. Around the boundary of a cavity opening, the maximum compressive stress acts parallel to the boundary and the minimum compressive stress acts perpendicular to the boundary. This maximum stress around the boundary is called the critical stress, since the distribution of the maximum stress can indicate the critical condition in the cavity. The distribution of critical stress around the boundary depends mainly upon two factors, the initial stress condition of the formation and the form of cavity opening.

The critical stress distribution around a tunnel opening can be studied as a two-dimensional problem. A cylindrical cavity with a relatively large axial length compared to the diameter of the cross section can also be approximated to a two dimensional problem. This study can be directly applied to the problems encountered in tunnel openings in salt mines.

The following graphical presentations of the critical stress concentration around tunnel openings are the author's application of the works of Duvall¹⁴, Frocht¹⁷, Greenspan²¹, Heywood²⁹, Peterson⁵², Caudle and Clark¹⁰ in the field of machine design. The three fundamental conditions of the natural stress in salt formations, hydrostatic, non-hydrostatic, and uni-directional, are considered for reference even though conditions in most salt formations can be assumed to be hydrostatic.

1. Circular Opening

The distribution of the critical stress around the boundary of a circular salt tunnel is given in Fig. 2-6. As in the diagram for the initial hydrostatic stress condition, the critical stress is twice that of the overburden pressure and is the same at points equidistant from the boundary. There occurs no tensile stress unless the minor principal stress becomes less than one-third of the major stress of the formation. It should be noted that under a hydrostatic initial stress, a circle is the most stable form of tunnel cross section.

2. Rectangular Opening

As an example of a rectangular tunnel, the concentration of critical stress around a square tunnel opening is given in Fig. 2-6, where the three fundamental conditions of initial stress are applied. Attention must be paid to the higher stress concentration factors around the corner of the square. This corner effect upon the stress concentration factor is given in Fig. 2-8, where the degree of sharpness at the corner is expressed by the ratio of radius of curvature of the corner to the shorter width of the rectangular opening¹⁰. The greater the elongation of the opening form and the sharper the corners, the greater the stress concentration factor at the corner³⁷.

3. Ovaloid Opening

An ovaloid is an extreme case of a rectangle in which the curvature of the corner is half of the shorter width as shown in Fig. 2-7. This cross section has the advantage of stress concentration lower around the curvature than around any other rectangular cross section. Under the three fundamental conditions of the initial stress, the critical stress concentration around an ovaloid, whose ratio of longer to shorter width is four, is shown in Fig. 2-7. In the case of hydrostatic initial stress, the maximum stress concen-

FIG. 2-6. STRESS CONCENTRATION AROUND CIRCULAR AND SQUARE TUNNEL OPENINGS^{10, 14}

FIG. 2-7. EFFECT OF ORIENTATION OF OVALOID TUNNEL OPENING ON STRESS CONCENTRATION AROUND THE BOUNDARY.^{10, 14}

FIG. 2-8. HIGHER CONCENTRATION OF CRITICAL STRESS AT CORNER OF TUNNEL OPENING. ^{14, 37}

FIG. 2-9. CRITICAL STRESS CONCENTRATION AROUND ELLIPTICAL TUNNEL OPENING. ^{13, 35, 37}

FIG. 2-10. EFFECT OF MULTIPLE TUNNEL OPENINGS ON CONCENTRATION OF CRITICAL STRESS. ^{33, 35, 43, 58}

tration factor is observed to be three and the minimum to be slightly less than one; while no tensile stress occurs all around the opening. Under a non-hydrostatic initial stress condition, an ovaloid tunnel is sensitive to the orientation of the ovaloid. The ovaloid form is much stronger than a circular form, when the direction of the longer width of the ovaloid is parallel with that of the major initial stress and vice versa.

From Fig. 2-8, it is conceivable that the greater the ratio of the major initial stress to the minor stress, the greater should be the ratio of the longer width to the shorter width.

4. Elliptical Opening

The elliptical shape is the ideal tunnel cross section whenever the initial stress condition is non-hydrostatic. Even though this is the least practical form to be constructed, it still has theoretical significance. A mathematical solution for the stress concentration around an elliptical tunnel has been given by Inglis³⁷. When an elliptical opening is created in a formation, where the principal stresses S_x and S_y are existing initially, the maximum and minimum critical stresses occur at both ends of the principal axes of the ellipse.

These two critical stresses are expressed by the following equations:

$$S_A = S_y \left(1 + 2 \frac{a}{b} \right) - S_x \quad (2-11)$$

$$S_B = S_x \left(1 + 2 \frac{b}{a} \right) - S_y$$

where S_A, S_B = critical stresses at the ends of the two principal diameters of the ellipse

S_x, S_y = initial principal stresses in the plane perpendicular to the tunnel axis

a = major radius of ellipse

b = minor radius of ellipse

These critical stresses are plotted in Fig. 2-9 as a function of the ratio of the initial principal stresses, $\beta = S_x / S_y$, and the ratio of the two principal radii, $e = b/a$. The validity of these relationships has been confirmed by photoelastic investigation by Durelli and Murray¹³.

Under a hydrostatic condition in which $S_x = S_y = P_o$, the greatest stress will be produced at the ends of the major axis, as derived from Eq. 2-11, that is:

$$\text{if } a > b, S_{\max} = S_A = 2 P_o a/b$$

$$\text{if } b > a, S_{\max} = S_B = 2 P_o b/a$$

Equating the stresses at A and B in Eq. 2-11, the following relation is obtained:

$$S_x / S_y = a / b \quad (2-12)$$

Substituting this into Eq. 2-11,

$$S_A = S_B = S_x + S_y$$

This indicates that the stress all around an ellipse is uniform when the ratio of the initial principal stresses, β , is equal to the ratio of the principal diameters, e , and the stress is equal to a total of the initial principal stresses.

Effect of Multiple Tunnel Opening

The stress concentration around a tunnel opening is affected by the introduction of another tunnel opening close by. Stress concentration around tunnels is always increased by construction of more openings. Questions which arise here include the influence of the size of cavity, the thickness of partition, the initial stress condition, and the number of cavities upon the distribution of stress concentration factors. These problems have been mathematically solved by a number of investigators such as Ling⁴³, Schoutz⁵⁸, Hutter³⁵, Howland³³, and Heywood²⁹.

To answer these particular questions, a diagram is presented in Fig. 2-10. Multiple circular tunnels located in a row at an equal distance from each other are considered here. As seen in the diagram, the critical stress concentration is very sensitive to the initial stress condition. As the partition thickness increases, the critical stress concentration factor rapidly approaches the value for a single opening. The effect of the number of openings is much less important than the initial stress condition. When the thickness of partition is less than a few times the diameter of the opening, the increase of the critical stress concentration factor for additions of openings from one to two or from two to three is considerable. On the other hand, the increase diminishes very quickly as the number of openings increases. The critical stress concentration factor reaches a constant, which is the value for an infinite number of openings. In any other type of arrangement of openings, a reasonable assumption can be made from this result. For instance, the effect of the number and the relative thickness of partition would be identical, regardless of the shape of tunnel opening. In the case of random distribution of openings, only the openings with minimum partition thicknesses should be considered. In highly complex arrangements of cavity openings, experiments by a photo-elastic method are required. In general, a partition thickness twice the opening diameter would be sufficient to minimize the effect of openings upon stress distribution.

Cavities of Three Dimensions

There will be many instances where a storage cavity has to be considered as a three dimensional opening, rather than as a tunnel of a two-dimensional nature. Particularly, those cavity openings being made by the washing-out method will most likely require three dimensional analysis. Even in the case

of a long tunnel cavity, the vicinity around both ends of the cavity must be considered as a three dimensional problem. As the simplest example of three dimensional analysis, problems of the stress concentration around a spherical cavity in a stressed media have been solved mathematically by Southwell and Gough⁵⁹, Goodier²⁰, and Edwards¹⁵. The investigators have reached identical results for stress distribution. Since the radial component of the stress on the boundary is always zero or equal to the internal pressure of the cavity as found in Eq. 2-9, only the principal stress in a tangential plane to the boundary should be considered for analysis of the critical stress. The position, direction and magnitude of the critical stress depend on the initial stress condition of the formation. The critical stress concentrations under various initial stress conditions are summarized by Heywood from the original work of the above mentioned investigators, and are further adapted to a salt cavity opening as presented in Table 2-1. Careful study of this table along with the diagrams of stress analysis in this chapter will assist one to understand the concept of stress concentration around a three dimensional cavity opening. In the first place, a three dimensional cavity creates much less concentration of stress than a two-dimensional one. Under a hydrostatic stress condition, creation of a spherical cavity in a formation produces the constant stress concentration factor of 1.5 all around the opening, while the creation of a circular tunnel produces that of 2.0. If the two types of cavities are subjected to a unit shearing stress, the spherical cavity produces the critical stress concentration of 2.73 while the circular tunnel opening produces that of 4.0. The value of Poisson's ratio of formation does not affect these values very much.

TABLE 2-1

CRITICAL STRESS IN SPHERICAL SALT CAVITY

FROM SOUTHWELL AND GOUGH,⁵⁹ GOODIER,²⁰
AND HEYWOOD.²⁹

INITIAL PRINCIPAL STRESSES				DESCRIPTION OF CRITICAL STRESSES			
CONDITION	S_x	S_y	S_z	POSITION	DIRECTION	CONCENTRATION FACTOR	
						IN GENERAL	IN SALT
UNI-DIRECTIONAL	P_0	0	0	CIRCUMFERENCE ABCD	PARALLEL TO S_z	$-\frac{3(9-5\nu)}{2(7-5\nu)}$	-2.17
	P_0	0	0	CIRCUMFERENCE ABCD	TANGENT TO CIRCUMFERENCE	$\frac{3(7-5\nu)}{2(7-5\nu)}$	0.50
	P_0	0	0	POINTS E & F	NORMAL TO S_x	$\frac{3(1+5\nu)}{2(7-5\nu)}$	1.17
SHEARING	P_0	$-P_0$	0	POINTS B & D	PARALLEL TO S_x	$-\frac{15}{7-5\nu}$	-3.33
	P_0	$-P_0$	0	POINTS E & F	PARALLEL TO S_y	$\frac{15}{7-5\nu}$	3.33
BIDIRECTIONAL	P_0	P_0	0	CIRCUMFERENCE BEDF	TANGENT TO CIRCUMFERENCE	$-\frac{3(4-5\nu)}{7-5\nu}$	-1.0
HYDROSTATIC	P_0	P_0	P_0	ALL POINTS ON SPHERICAL SURF.	TANGENT TO SURFACE	$-\frac{3}{2}$	-1.5

The initial stress condition is of primary importance for determination of the strength of cavities of any shape. For determination of stress concentration in an irregular form of three dimensional cavity, the frozen stress technique of photoelastic experiment should be applied.

CHAPTER III

PLASTICITY ANALYSIS

Application of the Theory of Plasticity

If a salt cavity is to be used for disposal of reactor fuel wastes, plasticity of salt is probably the most important influential factor for structural stability of the cavity. The concentrated stress resulting from the creation of the cavity is reduced to a certain tolerable value by plastic deformation of salt around the cavity. This plastic deformation starts when the difference between the external and internal pressures acting on the salt cavity exceeds a certain value, yet the salt cavity will never collapse under an increased pressure difference. It will merely widen the plastic zone surrounding the cavity. Thermal stress produces a further extension of the plastic zone.

The mechanism of the plastic flow in crystal aggregate has been studied extensively in the field of metallurgy^{7, 56, 61}. Microscopic observation of metal crystals under plastic deformation reveals that the plastic flow of a crystal is due to displacement of the relative position of atoms in the crystal lattices⁶². From the analogy of the crystal aggregates of metals to salt, the following should be found in the process of plastic displacement⁴⁸:

1. Slip - numerous displacements parallel to many sliding planes in a crystal.
2. Twin formation - shifting as a whole, on a major crystal plane.
3. Structure breakdown - displacement of grains accompanied by destruction of their cohesive force.

In the plastic flow of salt, slip seems to dominate at elevated temperatures, while structure breakdown occurs at and below room temperature.

The theory of plasticity of a solid body has been developed since the introduction of the concept of "octahedral shearing stress" by A. Nadai⁴⁸, and has been experimentally verified by various workers^{46, 49, 53, 62} in the last few decades.

This octahedral shearing stress, τ_o , can be calculated^{31,53} as:

$$\tau_o = \frac{1}{3} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \quad (3-1)$$

where $\sigma_1, \sigma_2, \sigma_3$ = the principal stresses

The stress relation is graphically given in Fig. 3-1.

According to this theory of plasticity, all the conditions in salt can be described thus:

Elastic condition $0 < \tau_o < K_e$

Plastic condition $K_e \leq \tau_o \leq K_p$

Failure condition $K_p < \tau_o$

where K_e = elastic limit of octahedral shearing stress

K_p = plastic limit of octahedral shearing stress

The K_e and K_p values can be obtained from uniaxial compression tests in a laboratory only when the test material is ductile. For uniaxial compression, the principal stresses in Eq. 3-1 become:

$$\sigma_1 = \sigma_2 = \text{axial stress}$$

$$\sigma_3 = \sigma_3 = 0$$

Then the octahedral shearing stress, τ_o , is:

$$\tau_o = \frac{\sqrt{2}}{3} \sigma_2 \quad (3-2)$$

Thus K_e and K_p can be obtained from Eq. 3-2 as:

$$K_e = \frac{\sqrt{2}}{3} \sigma_e$$

$$K_p = \frac{\sqrt{2}}{3} \sigma_p$$

where σ_e = elastic limit in compression tests of ductile material

σ_p = plastic limit in compression tests of ductile material

In Figs. 3-2 and 3-3, a typical compression test result of ductile material, such as salt at elevated temperatures, is shown in two different ways, where

FIG. 3-1. OCTAHEDRAL SHEARING STRESS IN PRINCIPAL STRESS COORDINATE.

FIG. 3-2. TYPICAL STRESS-STRAIN RELATION OF SALT AT ELEVATED TEMPERATURES

FIG. 3-3. PLASTIC STRESS-STRAIN RELATION OF SALT AT ELEVATED TEMPERATURES

FIG. 3-4. DIMENSIONS AND STRESS EQUILIBRIUM OF VOLUME ELEMENTS AROUND SPHERICAL AND CYLINDRICAL CAVITIES.³¹

the octahedral shearing stresses, K_e and K_p , correspond to the compressive stresses σ_e and σ_p respectively.

The octahedral shearing stress for salt at ordinary temperature can not be determined from Eq. 3-2 because the salt is brittle. However, Eq. 3-2 can be used by introducing a new concept of "the equivalent plastic limit", σ_o , of salt in a brittle condition. This σ_o can be determined experimentally with respect to a cavity stress condition as discussed later in Chap. VIII. The equivalent plastic limit is defined such that the σ_o can be used instead of σ_p in the plasticity analysis regardless of brittle or ductile characteristics.

The plastic stress-strain curve in Fig. 3-3 begins at $\tau_o = K_e$ and rises as the plastic strain increases until it becomes $\tau_o = K_p$. This increase of stress with increase of strain is called "limiting strain hardening", and the value of K_e depends upon the previous strain hardening within the limit of K_p .

"Yielding octahedral shearing stress" is defined as the stress under which a solid body yields to plastic deformation without any structural failure.

The yielding octahedral shearing stress, τ_y , varies in the following range, depending upon the material's previous experience with the limiting strain hardening:

$$K_e < \tau_y < K_p$$

When the octahedral shearing stress, τ_o , in a stressed salt mass reaches K_e the salt starts to flow with a slight increase of stress. Then, the plastic mass continues to flow until a new stress equilibrium has been established. This plastic condition can be maintained if the rate of deformation, $\frac{\Delta \epsilon}{\Delta t}$, stays within a certain value and τ_o does not exceed K_p .

During this static flow of the salt, the following can be assumed⁴⁸:

1. The direction of the principal strains coincides with that of the principal stresses at all times.

2. The density of the volume of the mass does not change appreciably, thus Poisson's ratio, ν , remains 0.5.
3. The figure consisting of Mohr's three principal strain circles remains geometrically similar to the figure consisting of three principal stress circles.

These assumptions lead to the following expressions of the principal strains due to plastic flow:

$$\begin{aligned}\epsilon_1'' &= \frac{F(\tau_0)}{3\tau_0} \left[\sigma_1 - \frac{1}{2} (\sigma_2 + \sigma_3) \right] \\ \epsilon_2'' &= \frac{F(\tau_0)}{3\tau_0} \left[\sigma_2 - \frac{1}{2} (\sigma_3 + \sigma_1) \right] \\ \epsilon_3'' &= \frac{F(\tau_0)}{3\tau_0} \left[\sigma_3 - \frac{1}{2} (\sigma_1 + \sigma_2) \right]\end{aligned}\tag{3-3}$$

These relations have been verified experimentally by many investigators^{46,62}. The function, $F(\tau_0)$, can be obtained by mathematical interpretation of the stress-strain relation of salt.

There are a few major factors which affect the K_e and K_p values. They are elevated temperature, prolonged loading, and previous strain hardening. The elevated temperature in salt reduces these stress values, but their reduction is not simply proportional to the temperature. A longer duration of loading also reduces their value somewhat. The previous strain hardening increases K_e , but does not affect K_p , as shown in Figs. 3-2 and 3-3. These effects are determined from experiments which follow in Part Two. By applying the theory of plasticity to the ideal forms of a salt cavity, i. e., spherical and cylindrical, the thickness of the plastic zone and the stress distribution in the zone around the cavity were calculated under various possible conditions, as shown in the following sections.

Development of Plastic Zone Around a Spherical Salt Cavity

When a spherical salt cavity is created in an underground salt formation, stress equilibrium is obtained by producing a higher concentration of elastic stress around the cavity. A plastic zone, or shell in plastic state starts to form around the cavity whenever this stress concentration reaches a point where the octahedral shearing stress, τ_o , exceeds K_e . The formation of the plastic zone creates a change in the initial distribution of elastic stress. The stress redistribution in this plastic zone can be obtained by introducing the plastic limit of octahedral shearing stress into the differential equation of stress equilibrium in the zone, which is illustrated in Fig. 3-4.

$$\frac{d\sigma_r}{dr} + \frac{2}{r}(\sigma_r - \sigma_e) = 0 \quad (3-4)$$

The octahedral shearing stress around the cavity is obtained by substituting $\sigma_1 = \sigma_r$ and $\sigma_2 = \sigma_3 = \sigma_t$ in Eq. 3-1 as:

$$\begin{aligned} \tau_o &= \pm \frac{1}{3} \sqrt{(\sigma_r - \sigma_e)^2 + (\sigma_e - \sigma_e)^2 + (\sigma_e - \sigma_r)^2} \\ &= \pm \frac{\sqrt{2}}{3} (\sigma_r - \sigma_e) \end{aligned} \quad (3-5)$$

Taking $\sigma_t < 0$, $\sigma_r < 0$, $\sigma_t < \sigma_r$ and $\frac{d\sigma_r}{dr} < 0$, then Eq. 3-4 becomes:

$$\frac{d\sigma_r}{dr} = - \left| \frac{2}{r} (\sigma_r - \sigma_e) \right|$$

In a plastic zone, this equation states:

$$\frac{d\sigma_r}{dr} = - \frac{2}{r} \frac{3}{\sqrt{2}} |\tau_{o_{max}}|$$

where $|\tau_{o_{max}}|$ = absolute value of maximum octahedral shearing stress of salt

$$= |K_p| = \frac{\sqrt{2}}{3} \sigma_o \quad (3-5a)$$

Thus, Eqs. 3-4 and 3-5 are rewritten for the spherical cavity as:

$$\frac{d\sigma_r}{dr} = - \frac{2}{r} \sigma_o \quad (3-6)$$

$$\sigma_r - \sigma_t = \sigma_o$$

where σ_o = absolute value of the equivalent plastic limit of salt

By integrating Eq. 3-6, the general equations of stress distribution in the plastic zone are obtained:

$$\begin{aligned}\sigma_r &= C - 2 \sigma_0 \ln r \\ \sigma_t &= C - 2 \sigma_0 \left(\frac{1}{2} + \ln r \right)\end{aligned}\tag{3-7}$$

where C is a coefficient of integration.

Surrounding this plastic zone, there is a zone of elastic state extending to an infinite distance in the salt formation. The general equation of stress distribution in this elastic zone is obtained from Eq. 2-8.

$$\begin{aligned}S_r &= A - \frac{B}{r^3} \\ S_t &= A + \frac{B}{2r^3}\end{aligned}\tag{3-8}$$

The coefficients of integration, A, B and C, in the above equations can be determined from the boundary conditions of the salt cavity, assuming that the front of the plastic zone has developed to $r = \rho$. At the wall of the cavity ($r = a$), the radial stress is $\sigma_r = -P_i$

where P_i = internal pressure in the cavity.

As it is in a plastic zone, Eq. 3-7 becomes

$$-P_i = C - 2 \sigma_0 \ln a$$

$$\text{therefore, } C = -P_i + 2 \sigma_0 \ln a$$

At an infinite distance from the cavity ($r = \infty$), the stress condition is

$$S_r = S_t = -P_o$$

where P_o = overburden load

As it is in an elastic zone, Eq. 3-8 becomes

$$-P_o = \lim_{r \rightarrow \infty} \left(A - \frac{B}{r^3} \right)$$

$$\text{therefore, } A = -P_o$$

At the border line between the plastic and elastic zones ($r = \rho$), it is found that $\sigma_r = S_r$. By substituting σ_r and S_r of Eqs. 3-7 and 3-8 into the above relation, it becomes:

$$-P_i + 2\sigma_o \ln a - 2\sigma_o \ln \rho = -P_o - \frac{B}{\rho^3}$$

therefore, $B = -\rho^3 (P_o - P_i - 2\sigma_o \ln \frac{\rho}{a})$

By using those coefficients A, B, and C, the following stress distribution around the cavity is obtained:

in the plastic zone ($a \leq r \leq \rho$),

$$\sigma_r = -P_i - 2\sigma_o \ln \frac{r}{a} \quad (3-9)$$

$$\sigma_t = -P_i - 2\sigma_o \left(\frac{1}{2} + \ln \frac{r}{a} \right)$$

in the elastic zone ($\rho \leq r \leq \infty$),

$$S_r = -P_o + \frac{\rho^3}{r^3} (P_o - P_i - 2\sigma_o \ln \frac{\rho}{a}) \quad (3-10)$$

$$S_t = -P_o - \frac{\rho^3}{r^3} (P_o - P_i - 2\sigma_o \ln \frac{\rho}{a})$$

The radius to the front of the plastic zone can be obtained as a function of the loading condition to the cavity. This function can be derived from Eq. 3-6, which becomes at $r = \rho$,

$$S_t - S_r = -\sigma_o$$

By substituting Eq. 3-10 in the above relation, it becomes:

$$-\frac{3\rho^3}{2\rho^3} (P_o - P_i - 2\sigma_o \ln \frac{\rho}{a}) = -\sigma_o$$

$$\ln \frac{\rho}{a} = \frac{P_o - P_i}{2\sigma_o} - \frac{1}{3}$$

The radius of the plastic front is thus obtained as:

$$\rho = a e^{\left(\frac{P_o - P_i}{2\sigma_o} - \frac{1}{3} \right)} \quad (3-11)$$

The stress condition with which the plastic zone first starts around the cavity is calculated by taking $P = a$ in Eq. 3-11.

$$e^{\left(\frac{P_0 - P_1}{2\sigma_0} - \frac{1}{3}\right)} = 1$$

$$\text{therefore, } P_0 - P_1 = \frac{2}{3} \sigma_0$$

In Fig. 3-5, the radius of the plastic front is given under various stress conditions of the salt cavity. The stress distribution in both the plastic and elastic zones is shown in Fig. 3-6 by assuming that:

$$\text{Equivalent plastic limit of salt } (\sigma_0) = 1,500 \text{ psi}$$

$$\text{Overburden load } (P_0) = 1 \text{ psi/ft} \times \text{depth of cavity}$$

$$\text{Internal pressure } (P_1) = 0$$

$$\text{Cavity temperature } (t) = 80^\circ\text{F.}$$

$$\text{Depth of the cavity} = 1,000, 2,000, 3,000, 4,000, 5,000 \text{ and } 6,000 \text{ ft.}$$

Development of Plastic Zone Around a Cylindrical Salt Cavity

Development of the plastic zone and the stress distribution around a cylindrical cavity was analyzed by a method similar to that used for a spherical cavity. The differential equation of stress equilibrium in a thick-walled cylinder is illustrated in Fig. 3-4 and given as:

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \quad (3-12)$$

The octahedral shearing stress around the cylindrical cavity is, from Eq. 3-1,

$$\tau_o = \frac{1}{3} \sqrt{(\sigma_r - \sigma_\theta)^2 + (\sigma_\theta - \sigma_z)^2 + (\sigma_z - \sigma_r)^2}$$

Since a displacement in the axial direction in the plastic zone is not probable, the axial plastic strain, ϵ_z'' , in Eq. 3-3, can be equated to zero.

$$\epsilon_z'' = \frac{F(\tau_o)}{3\tau_o} \left[\sigma_z - \frac{1}{2}(\sigma_\theta + \sigma_r) \right] = 0$$

$$\text{therefore, } \sigma_z = \frac{\sigma_\theta + \sigma_r}{2}$$

FIG. 3-5. DEVELOPMENT OF PLASTIC ZONE AROUND SPHERICAL AND CYLINDRICAL SALT CAVITIES.

By substituting this in the equation of octahedral shearing stress, the octahedral shearing stress in a plastic zone is obtained.

$$\tau_o = \pm \frac{1}{\sqrt{6}} (\sigma_r - \sigma_e) \quad (3-13)$$

Thus Eq. 3-12 can be rewritten for the stress condition in the plastic zone.

$$\begin{aligned} \frac{d\sigma_r}{dr} &= -\frac{1}{r} (\sigma_r - \sigma_e) \\ &= \pm \frac{\sqrt{6}}{r} \tau_o \max \end{aligned}$$

As in the case of a spherical cavity, the maximum octahedral shearing stress of salt can be determined by:

$$|\tau_o \max| = |K_p| = \frac{\sqrt{2}}{3} \sigma_o$$

At the same time the stress condition around the cavity is restricted to:

$$\sigma_t < 0, \quad \sigma_r < 0, \quad \sigma_t < \sigma_r, \quad \text{and} \quad \frac{d\sigma_r}{dr} < 0$$

Therefore, the fundamental stress equation of the plastic zone around a cylindrical cavity is expressed as:

$$\begin{aligned} \frac{d\sigma_r}{dr} &= -\frac{2}{\sqrt{3}} \frac{\sigma_o}{r} \\ \sigma_r - \sigma_t &= \frac{2}{\sqrt{3}} \sigma_o \\ \sigma_z &= \frac{\sigma_r + \sigma_t}{2} \end{aligned} \quad (3-14)$$

By integration, the general equations of stress distribution in the plastic zone are obtained:

$$\begin{aligned} \sigma_r &= C - \frac{2}{\sqrt{3}} \sigma_o \ln r \\ \sigma_t &= C - \frac{2}{\sqrt{3}} \sigma_o (1 + \ln r) \\ \sigma_z &= C - \frac{2}{\sqrt{3}} \sigma_o \left(\frac{1}{2} + \ln r \right) \end{aligned} \quad (3-15)$$

In an elastic zone, the general equations of stress distribution are, from Eq. 2-4,

$$\begin{aligned} S_r &= A + \frac{B}{r^2} \\ S_t &= A - \frac{B}{r^2} \\ S_z &= 2 \nu A \end{aligned} \quad (3-16)$$

The integration constants, A, B, and C, in the six equations (Eqs. 3-15 and 3-16) can be determined by application of the following boundary conditions:

at the wall of the cavity ($r = a$),

$$\sigma_r = -P_i$$

at an infinite distance away from the cavity ($r = \infty$),

$$S_r = -P_o$$

at the border line between plastic and elastic zones ($r = \rho$),

$$\sigma_r = S_r$$

The integration constants are obtained as:

$$\begin{aligned} A &= -P_o \\ B &= \rho^2 \left(P_o - P_i - \frac{2}{\sqrt{3}} \sigma_o \ln \frac{\rho}{a} \right) \\ C &= -P_i + \frac{2}{\sqrt{3}} \sigma_o \ln a \end{aligned}$$

Thus, the equations for the stress distribution in both the plastic and elastic zones around the cavity are found as:

in the plastic zone ($a \leq r \leq \rho$),

$$\begin{aligned} \sigma_r &= -P_i - \frac{2}{\sqrt{3}} \sigma_o \ln \frac{r}{a} \\ \sigma_t &= -P_i - \frac{2}{\sqrt{3}} \sigma_o \left(1 + \ln \frac{r}{a} \right) \\ \sigma_z &= -P_i - \frac{2}{\sqrt{3}} \sigma_o \left(\frac{1}{2} + \ln \frac{r}{a} \right) \end{aligned} \quad (3-17)$$

in the elastic zone ($\rho \leq r \leq \infty$):

$$\begin{aligned} S_r &= -P_o + \frac{\rho^2}{r^2} \left(P_o - P_i - \frac{2}{\sqrt{3}} \sigma_o \ln \frac{\rho}{a} \right) \\ S_t &= -P_o - \frac{\rho^2}{r^2} \left(P_o - P_i - \frac{2}{\sqrt{3}} \sigma_o \ln \frac{\rho}{a} \right) \\ S_z &= -P_o \end{aligned} \quad (3-18)$$

The radius to the front of the plastic zone is derived as a function of the stress condition of the cavity. Since $\sigma_r = S_r$ and $\sigma_t = S_t$ at $r = \rho$, Eq. 3-14 becomes:

$$S_r - S_t = \frac{2}{\sqrt{3}} \sigma_o$$

Substituting S_r and S_t of Eq. 3-18 into the above equation, the relation between the radius of the plastic front and the loading condition is found.

$$2 (P_o - P_i) - 2 \frac{2}{\sqrt{3}} \sigma_o \ln \frac{\rho}{a} = \frac{2}{\sqrt{3}} \sigma_o$$

Then,

$$\ln \frac{\rho}{a} = \frac{\sqrt{3}}{2} \frac{P_o - P_i}{\sigma_o} - \frac{1}{2}$$

Therefore,

$$\rho = a e^{\left(\frac{\sqrt{3}}{2} \frac{P_o - P_i}{\sigma_o} - \frac{1}{2} \right)} \quad (3-19)$$

The stress condition under which the plastic zone first appears around the cavity is calculated by substituting $\rho = a$ in Eq. 3-19.

$$e^{\left(\frac{\sqrt{3}}{2} \frac{P_o - P_i}{\sigma_o} - \frac{1}{2} \right)} = 1$$

Thus,

$$\frac{\sqrt{3}}{2} \frac{P_o - P_i}{\sigma_o} - \frac{1}{2} = 0$$

$$P_o - P_i = 0.577 \sigma_o$$

In Fig. 3-5, the radius of the plastic front under various loading conditions of the salt cavity is given as a function of the internal and external pressures and the equivalent plastic limit of salt.

The stress distribution in the plastic and elastic zones around the cylindrical cavity is given in Fig. 3-7 by using the same assumptions previously made.

FIG. 3-6. STRESS DISTRIBUTION IN PLASTIC AND ELASTIC ZONES AROUND SPHERICAL SALT CAVITY AT VARIOUS DEPTHS.

FIG. 3-7. STRESS DISTRIBUTION IN PLASTIC AND ELASTIC ZONES AROUND CYLINDRICAL SALT CAVITY AT VARIOUS DEPTHS.

Practical Considerations

From the previous analysis on the two ideal forms of a salt cavity, it is clear that the thickness of the plastic zone depends entirely on the structural loading factor, X , which is defined as:

$$X = \frac{P_o - P_c}{\sigma_o}$$

And the increase of the radius of the plastic front, ρ , is exponential to the factor, X , as shown in Fig. 3-5.

In Figs. 3-6 and 3-7, it is found that the maximum shearing stress, Q_{max} , remains a constant in the plastic zone and is expressed as:

$$Q_{max} = \frac{\sigma_t - \sigma_r}{2}$$

In conclusion of the foregoing analysis, a salt cavity can withstand unlimited structural loading by extending the size of its plastic zone, if the plastic zone can be extended to adjacent formations. However, the structural loading is limited by the following restrictions:

1. Reduction of the cavity volume due to thermal expansion of the plastic zone with cavity temperature rise.
2. Limitation of extension of the plastic zone due to the earth's surface.
3. Reduction of the cavity volume due to the relaxation of salt in the plastic zone.

As analyzed in the following discussion, the reduction of the cavity volume due to the thermal expansion will be an actual limit to the use of the cavity for waste storage.

Some observations in the Grand Saline salt mine seem to agree with the conclusion of this plasticity analysis, as it is discussed in Chap. X later.

The effect of elevated temperature in the cavity upon the development of a plastic zone and the reduction of cavity space due to the thermal expansion and plastic flow of salt are analyzed in the following chapters.

CHAPTER IV

THERMAL STRESS ANALYSIS

Thermal Stress

Thermal stress is produced whenever a local temperature rise occurs in an underground formation. This thermal stress is produced around a salt cavity when the cavity temperature rises above the surrounding earth temperature. The cavity temperature rise occurs as the result of radioactive decay of the fission products in the stored waste. This thermal stress can be determined by calculating the temperature distribution around the cavity which is a function of the cavity geometry and the cavity temperature rise.

As discussed in the work by Schechter and Gloyna⁵⁷, the nature of the temperature rise in the cavity depends upon three major factors: the nature and condition of the waste, the geometry of the cavity and the thermometric conductivity of salt. Because of the extremely slow change in the rate of heat emission from the waste, the heat flowing from the cavity to the surrounding environment approaches a state of thermal equilibrium; therefore, the temperature at any point around the cavity at any time can be assumed to be a constant.

Mathematical analysis of thermal stress distribution around a salt cavity with elevated temperature is easily obtainable only for very simple geometrical configurations such as a sphere or cylinder with a long axial length. In this section, the thermal stress distribution around spherical and cylindrical cavities is analyzed, and the results are presented in diagrams.

Flow of Heat

Heat in a salt formation propagates itself only by conduction. Such heat conduction in a homogeneous medium like salt can be expressed by rectilinear coordinates in the following equation, known as Fourier's conduction equation³⁶:

$$\frac{\partial T}{\partial t} = \lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) \quad (4-1)$$

$$= \lambda \nabla^2 T$$

where λ = thermometric conductivity = $K / \mu c$

K = thermal conductivity

C = specific heat

μ = density

T = temperature

t = time

Under steady state conditions, the temperature is independent of time, t , even though the temperature varies from point to point in the formation. Consequently, under steady state conditions the left hand term of Fourier's conduction equation equals zero. By applying Eq. 4-1, the temperature distribution around a spherical or cylindrical cavity can be analyzed. However, to investigate the temperature distribution around an ideal spherical salt cavity, Fourier's conduction equation must be rewritten, using polar coordinates, by substituting the following relations into Eq. 4-1:

$$r^2 = x^2 + y^2 + z^2$$

$$\frac{\partial T}{\partial x} = \frac{dT}{dr} \frac{\partial r}{\partial x} = \frac{dT}{dr} \frac{x}{r}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{d^2 T}{dr^2} \frac{x^2}{r^2} - \frac{dT}{dr} \frac{1}{r} - \frac{dT}{dr} \frac{x^2}{r^3}$$

Because of symmetry of the cavity, consolidation provides:

$$\frac{\partial T}{\partial r} = \frac{\lambda}{r} \frac{d^2(rT)}{dr^2} \quad (4-2)$$

For the steady state, Eq. 4-2 becomes:

$$\frac{d^2(rT)}{dr^2} = 0 \quad (4-3)$$

By integrating Eq. 4-3, a general relationship of temperature to distance from the center of the cavity can be obtained: $T = A + \frac{B}{r}$

Furthermore, by considering boundary conditions and assigning a as the cavity radius and b as the outside diameter of the spherical shell surrounding the cavity, it is possible to obtain:

$$T - T_2 = \frac{a}{r} \frac{(b - r)}{(b - a)} (T_1 - T_2) \quad (4-4)$$

where T = temperature at r

T_1 = temperature at a

T_2 = temperature at b

Eq. 4-4 describes the temperature distribution in a thick spherical shell with an inner radius (a) and an outer radius (b). To apply this equation to the case of a salt cavity, b is equal to ∞ and T_2 is equal to T_0 .

$$\text{or } T_r = \frac{a}{r} T_c \quad (4-5)$$

where T_0 = original temperature of the environment

$T_r = T - T_0$ = temperature rise at r

$T_c = T_1 - T_0$ = cavity temperature rise

In Fig. 4-1, the above equation is plotted by defining T_r / T_c as temperature ratio. The temperature rise around the cavity decreases with increase of the distance away from the cavity.

From Eq. 4-5, the rate of heat flow per unit area in a radial direction, w , can be expressed in terms of cavity temperature and of distance from the source. Likewise, the quantity of heat flowing out in a unit of time, q , is expressed as a function of w .

$$\begin{aligned} w &= -K \frac{dt}{dr} \\ &= K \frac{a}{r^2} (T_1 - T_0) \\ &= K \frac{a}{r^2} T_c \end{aligned} \quad (4-6)$$

FIG. 4-1. TEMPERATURE DISTRIBUTION AROUND SPHERICAL AND CYLINDRICAL CAVITIES WITH STEADY STATE HEAT FLOW.

FIG. 4-2. DISTRIBUTION OF UNIT THERMAL STRESS INCREASE AROUND SPHERICAL SALT CAVITY PER UNIT TEMPERATURE RISE IN CAVITY.

FIG. 4-3. DISTRIBUTION OF UNIT THERMAL STRESS INCREASE AROUND CYLINDRICAL SALT CAVITY PER UNIT TEMPERATURE RISE IN CAVITY.

$$\begin{aligned}
 q &= 4 \pi r^2 w \\
 &= 4 \pi a K (T_1 - T_0) = 4 \pi a K T_c
 \end{aligned}
 \tag{4-7}$$

Then from Eq. 4-5, T_r is obtained as a function of q and r .

$$T_r = T - T_0 = \frac{q}{4 \pi K r} \tag{4-8}$$

To investigate temperature distribution around an ideal cylindrical salt cavity, Eq. 4-1 must be rewritten using the two dimensional polar coordinates:

$$\frac{T}{r} = \frac{1}{r} \frac{d}{dr} \left(\frac{r dT}{dr} \right) = 0$$

$$\text{or} \quad T = A \ln r + B \tag{4-9}$$

By applying the following boundary conditions to Eq. 4-9, the temperature distribution in a thick-walled cylinder is obtained:

$$\text{at } r = a, T = T_1 \quad \text{at } r = b, T = T_2$$

$$T - T_2 = \frac{\ln b - \ln r}{\ln b - \ln a} (T_1 - T_2)$$

$$\text{or} \quad T_r = \frac{\ln b - \ln r}{\ln b - \ln a} T_c \tag{4-10}$$

This equation indicates that the temperature rise, T_r , around a cavity with an infinite length approaches the constant value T_c at any distance from the cavity. Considering a cylindrical cavity with a finite length, the temperature rise is seen to approach zero as the distance r increases. Therefore, if r is taken as 50 to 100 times the cavity length, the temperature rise at that distance is insignificant (much less than 2 %) compared to the cavity temperature rise. Disregarding this insignificant temperature rise will not affect the temperature distribution in the vicinity of the cavity.

So, temperature distribution in this cavity can be calculated the same as that in an extremely thick-walled cylinder, with zero temperature rise at the outer radius, b , assuming the following conditions:

$$50 \text{ (LENGTH OF CYLINDER)} < b < 100 \text{ (LENGTH OF CYLINDER)}$$

$$10 < \frac{\text{LENGTH OF CYLINDER}}{\text{RADIUS OF CYLINDER}} < 20 \quad (4-10a)$$

Therefore, $b = 1,000 a$

By substituting the relation $\ln b - \ln a = \ln 1,000 = 6.91$ into Eq. 4-10, Eq. 4-11 can be developed to calculate the temperature rise at any distance, r , provided $r < 100a$.

$$T - T_2 = \frac{\ln \left(\frac{1000a}{r} \right)}{6.91} (T_1 - T_0)$$

$$T_r = \frac{\ln \left(\frac{1000a}{r} \right)}{6.91} T_c = \left(1 - \frac{\ln \frac{r}{a}}{6.91} \right) T_c \quad (4-11)$$

It is apparent from Fig. 4-1 that the temperature gradients around the cylinder are steeper than those around a sphere. This effect is even more apparent if the length of the cylinder is increased.

Similarly to the case of a spherical cavity, the rate of heat flow per unit area on the radial direction, w , and the total quantity of heat flowing out from unit length, q , for a cylindrical cavity, are derived as:

$$w = \frac{K (T_1 - T_0)}{r (\ln b - \ln a)} = \frac{K}{6.91 r} T_c \quad (4-12)$$

$$q = 2 \pi r w = \frac{2 \pi K}{\ln b - \ln a} (T_1 - T_0) = \frac{2 \pi K T_c}{6.91} \quad (4-13)$$

Thermal Stress Distribution Around a Spherical Cavity

Thermal stress developed around a spherical salt cavity, due to the temperature rise in the cavity, can be mathematically calculated by applying thermal stress analysis of a thick-walled spherical shell to the cavity.

The equilibrium condition of an element in a stressed spherical body is illustrated⁶⁵ in Fig. 3-4, from which the following fundamental relationship is obtained:

$$\frac{d \sigma_r}{d r} + \frac{2}{r} (\sigma_r - \sigma_t) = 0 \quad (4-14)$$

where

r = radius to the element in the spherical body

σ_r = stress on radial direction at r

σ_t = stress on tangential direction at r

The stress-strain relation of the element to the temperature rise, T_r , is derived⁶⁵ by introducing thermal strain into Eq. 2-1.

$$\epsilon_r - \alpha T_r = \frac{1}{E} (\sigma_r - 2\nu \sigma_t) \quad (4-15)$$

$$\epsilon_t - \alpha T_r = \frac{1}{E} [\sigma_t - \nu(\sigma_r + \sigma_t)]$$

$$\epsilon_r = \frac{du}{dr} = \text{strain on radial direction} \quad (4-16)$$

$$\epsilon_t = \frac{u}{r} = \text{strain on tangential direction}$$

where u = radial displacement

α = coefficient of linear expansion

By substituting Eqs. 4-15 and 4-16 into Eq. 4-14, it is possible to develop Eq. 4-17⁶⁵:

$$\sigma_r = \frac{\alpha E T_c}{1-\nu} \frac{ab}{b^3 - a^3} \left[a + b - \frac{1}{r} (a^2 + ab + b^2) + \frac{a^2 b^2}{r^3} \right] \quad (4-17)$$

$$\sigma_t = \frac{\alpha E T_c}{1-\nu} \frac{ab}{b^3 - a^3} \left[a + b - \frac{1}{2r} (a^2 + ab + b^2) - \frac{a^2 b^2}{2r^3} \right]$$

In the case of a spherical salt cavity where the outer radius of the sphere approaches infinity, the following stress distribution equations can be derived:

$$S_r = \lim_{b \rightarrow \infty} \sigma_r = - \frac{\alpha E T_c}{1-\nu} \frac{a (r^2 - a^2)}{r^3} \quad (4-18)$$

$$S_t = \lim_{b \rightarrow \infty} \sigma_t = - \frac{\alpha E T_c}{1-\nu} \frac{a (r^2 + a^2)}{2 r^3}$$

The tangential stress, S_t , is therefore at its maximum at the boundary surface, decreasing as the distance from the cavity increases. As shown in Fig. 4-2, the radial stress, S_r , is zero at the boundary surface and reaches its maximum value at some distance from the boundary, then gradually decreases to zero as the distance increases.

The distance at which the radial stress reaches its maximum can be calculated in the following manner:

$$\frac{dS_r}{dr} = \frac{d}{dr} \left[\frac{\alpha E T_c}{1-\nu} \cdot \frac{a(r^2 - a^2)}{r^3} \right] = 0$$

$$\text{therefore, } r = \sqrt{3} a$$

The maximum stress along radial and tangential directions is:

$$S_{r_{\max}} = - 0.384 \frac{\alpha E T_c}{1-\nu}, \text{ at } r = \sqrt{3} a$$

(4-19)

$$S_{t_{\max}} = - \frac{\alpha E T_c}{1-\nu}, \text{ at } r = a$$

Thermal Stress Distribution Around a Cylindrical Cavity

Thermal stress distribution around a cylindrical cavity can be calculated similarly to that of the spherical cavity. The necessary assumptions are that the temperature does not change along the axial direction, that the temperature is distributed symmetrically about the axis, and that the axial displacement is zero throughout.

The differential equation defining the stress condition around a cylinder is⁶⁵:

$$\frac{d \sigma_r}{d r} + \frac{\sigma_r - \sigma_t}{r} = 0 \quad (4-20)$$

The stress-strain relations with respect to the temperature rise, T_r , are:

$$\begin{aligned}\epsilon_r - \alpha T_r &= \frac{1}{E} [\sigma_r - \nu(\sigma_t + \sigma_z)] \\ \epsilon_t - \alpha T_r &= \frac{1}{E} [\sigma_t - \nu(\sigma_z + \sigma_r)] \\ \epsilon_z - \alpha T_r &= \frac{1}{E} [\sigma_z - \nu(\sigma_r + \sigma_t)]\end{aligned}\quad (4-21)$$

Therefore, by taking $\epsilon_z = 0$, the thermal stress around a cylindrical cavity must be⁶⁵:

$$\begin{aligned}\sigma_r &= \frac{\alpha E}{1-\nu} \frac{1}{r^2} \left[\frac{r^2 - a^2}{b^2 - a^2} \int_a^b T_r r dr - \int_a^r T_r r dr \right] \\ \sigma_t &= \frac{\alpha E}{1-\nu} \frac{1}{r^2} \left[\frac{r^2 + a^2}{b^2 - a^2} \int_a^b T_r r dr + \int_a^r T_r r dr - T_r r^2 \right] \\ \sigma_z &= \frac{\alpha E}{1-\nu} \left[\frac{2}{b^2 - a^2} \int_a^b T_r r dr - T_r \right]\end{aligned}\quad (4-22)$$

Since T_r is a function of the radial distance from the center of the cavity, substituting Eq. 4-11 into the above relation provides an equation which relates thermal stress to cavity temperature rise.

$$\begin{aligned}\sigma_r &= \frac{\alpha E T_c}{2(1-\nu) \ln \frac{b}{a}} \left[-\ln \frac{b}{r} - \frac{a^2}{b^2 - a^2} \frac{r^2 - b^2}{r^2} \ln \frac{b}{a} \right] \\ \sigma_t &= \frac{\alpha E T_c}{2(1-\nu) \ln \frac{b}{a}} \left[1 - \frac{b}{r} - \frac{a^2}{b^2 - a^2} \frac{r^2 + b^2}{r^2} \ln \frac{b}{a} \right] \\ \sigma_z &= \frac{\alpha E T_c}{2(1-\nu) \ln \frac{b}{a}} \left[1 - 2 \ln \frac{b}{a} - \frac{2a}{b^2 - a^2} \ln \frac{b}{a} \right]\end{aligned}\quad (4-23)$$

If the assumptions as stated in Eq. 4-10a are applicable to Eq. 4-23, the thermal stress distribution around a cylindrical cavity with a finite length can be solved. Since the distance, r , at which the stress analysis

is required, is usually less than 1/100 of b , the ratio of $(r^2 - b^2)$ to $(b^2 - a^2)$ is approximately equal to minus 1. Eq. 4-23 then becomes:

$$\begin{aligned} S_r &= - \frac{\alpha E T_c}{2(1-\nu)} \left[\frac{\ln b - \ln r}{\ln b - \ln a} - \frac{a^2}{r^2} \right] \\ S_t &= - \frac{\alpha E T_c}{2(1-\nu)} \left[\frac{\ln b - \ln r - 1}{\ln b - \ln a} + \frac{a^2}{r^2} \right] \\ S_z &= - \frac{\alpha E T_c}{2(1-\nu)} \left[\frac{2(\ln b - \ln r) - 1}{\ln b - \ln a} \right] \end{aligned} \quad (4-24)$$

Since from Eq. 4-10a, $\ln b/a = \ln 1,000 = 6.91$, the final equation for thermal stress around the cavity having the assumed geometry is:

$$\begin{aligned} S_r &= - \frac{\alpha E T_c}{2(1-\nu)} \left[1 - \frac{\ln r - \ln a}{6.91} - \frac{a^2}{r^2} \right] \\ S_t &= - \frac{\alpha E T_c}{2(1-\nu)} \left[1 - \frac{1 + \ln r - \ln a}{6.91} + \frac{a^2}{r^2} \right] \\ S_z &= - \frac{\alpha E T_c}{2(1-\nu)} \left[2 - \frac{1 + 2(\ln r - \ln a)}{6.91} \right] \end{aligned} \quad (4-25)$$

Similarly to the spherical salt cavity, the maximum value for S_t appears at the boundary surface. The radius, r , for the maximum value of S_r can be obtained by taking the derivative of Eq. 4-25 and equating it to zero.

$$r = a \sqrt{2 \ln \frac{b}{a}}$$

The maximum thermal stresses can be computed from the modified relationships shown below:

$$\begin{aligned} \sigma_{r_{\max}} &= -0.371 \frac{\alpha E T_c}{1-\nu} \quad \text{at } r = a \sqrt{2 \ln \frac{b}{a}} \\ \sigma_{t_{\max}} &= \sigma_{z_{\max}} = -0.923 \frac{\alpha E T_c}{1-\nu} \quad \text{at } r = a \end{aligned} \quad (4-26)$$

These values of the maximum stress are fairly close to those for the spherical salt cavity, as compared in Figs. 4-2 and 4-3.

Practical Considerations

The actual thermal stress distribution around spherical and cylindrical salt cavities due to cavity temperature rise can be calculated by introducing into Eqs. 4-18 and 4-25 the proper values of the three coefficients of salt; namely, Young's modulus, Poisson's ratio and the coefficient of linear expansion.

In the test data reported herein, the Young's modulus of salt was found to be about 150,000 psi at room temperature and the Poisson's ratio of salt to be about 0.5. The coefficient of linear expansion of salt, $22.4 \times 10^{-6}/F^{\circ}$ at room temperature, was found from other sources^{9,42}

By using these figures, the thermal stress distribution around the cavity was computed as shown in Figs. 4-2 and 4-3. To use the diagrams, thermal stress can be determined by multiplying the temperature rise in the cavity with the unit thermal stress increase obtained from the diagrams.

To construct these diagrams it was assumed that the effect of temperature on the term $\frac{\alpha E}{1 - \nu}$ is negligible. In general, the value $\frac{\alpha E}{1 - \nu}$ remains relatively constant because the α decreases and the E increases while the temperature is elevated⁴⁸. The Poisson's ratio will remain constant at 0.5 for elevated temperatures. Laboratory investigations are needed here to determine more precise relations of α and E to temperature. The accuracy of this thermal stress analysis will be greatly improved when empirical relations of α and E to temperature can be introduced into Eqs. 4-18 and 4-26.

By comparing Fig. 4-3 with Fig. 4-2 it should be noted that the thermal stress distribution curves for the cylindrical cavity are influenced by the ratio of axial length to the radius of the cylinder (l/a). If this ratio is

reduced to one, the curves become similar in shape and magnitude to those developed for a spherical cavity. Conversely, the slope of the stress distribution curves approaches zero as the ratio $(1/a)$ increases.

In practice, the calculated thermal stress should be superimposed over the structural stress to obtain the overall stress distribution around a salt cavity. Thus, a rational estimate of the stress distribution around a salt cavity can be made from a thermal and structural analysis of spherical and cylindrical cavities.

CHAPTER V

DEVELOPMENT OF STRUCTURAL EQUILIBRIUM EQUATIONS FOR DESIGNING CAVITIES

Establishment of structural equilibrium of a salt cavity is possible at elevated temperatures as a result of development of a plastic zone and cavity volume reduction. For the purpose of designing cavities, the structural equilibrium equations of ideal spherical and cylindrical cavities have been developed in relation to cavity temperature rise, cavity volume reduction, strength of salt, and structural loading condition.

Stress Distribution Around Ideal Cavities with Elevated Temperature

Because thermal and structural stresses are independent of each other, they can be superimposed to obtain the combined stress distribution. For a spherical cavity, the general equation of elastic stress distribution around the cavity is obtained from superimposition of Eqs. 3-8 and 4-18.

$$\begin{aligned} S_r &= A - \frac{B}{r^3} - \frac{\alpha E T_c}{1-\nu} \frac{a(r^2 - a^2)}{r^3} \\ S_t &= A + \frac{B}{r^3} - \frac{\alpha E T_c}{1-\nu} \frac{a(r^2 + a^2)}{2r^3} \end{aligned} \quad (5-1)$$

Whenever the octahedral shearing stress, which can be calculated by S_r and S_t of Eq. 5-1, reaches the maximum value of salt, $\tau_{o \max}$, at a given temperature, a plastic zone starts to develop around the cavity. The stress distribution in the plastic zone has been described previously by Eq. 3-7. Since the boundary condition of the cavity is not changed with elevated temperatures, temperature rise in the cavity does not affect the equation of stress distribution in the plastic zone, Eq. 3-9. For the same reason the coefficient A in Eq. 5-1 is unchanged at elevated temperatures.

The coefficient B in Eq. 5-1 can be obtained from the following relation existing at the boundary between plastic and elastic zones:

$$\text{at } r = \rho, \quad \sigma_r = S_r$$

By substituting Eqs. 3-9 and 5-1 into the above relation, the coefficient B is determined:

$$-P_i - 2\sigma_0 \ln \frac{\rho}{a} = -P_o - \frac{B}{\rho^3} - \frac{\alpha E T_c}{1-\nu} \frac{a(\rho^2 - a^2)}{3}$$

Then, $B = -\rho^3 \left[P_o - P_i - 2\sigma_0 \ln \frac{\rho}{a} + \frac{\alpha E T_c}{1-\nu} \frac{a(\rho^2 - a^2)}{\rho^3} \right]$

Therefore, the stress distribution in the elastic zone around the spherical cavity with a temperature rise, T_c , is found as:

$$S_r = -P_o + \frac{\rho^3}{r^3} \left[P_o - P_i - 2\sigma_0 \ln \frac{\rho}{a} + \frac{\alpha E T_c}{1-\nu} \frac{a(\rho^2 - a^2)}{\rho^3} \right] - \frac{\alpha E T}{1-\nu} \frac{a(r^2 - a^2)}{r^3} \quad (5-2)$$

$$S_t = -P_o + \frac{\rho^3}{2r^3} \left[P_o - P_i - 2\sigma_0 \ln \frac{\rho}{a} + \frac{\alpha E T_c}{1-\nu} \frac{a(\rho^2 - a^2)}{\rho^3} \right] - \frac{\alpha E T_c}{1-\nu} \frac{a(r^2 + a^2)}{2r^3}$$

For the cylindrical cavity, the general equation of the elastic stress distribution around the cavity is similarly obtained by superimposition of Eqs. 3-16 and 4-24.

$$S_r = A + \frac{B}{r^2} - \frac{\alpha E T_c}{2(1-\nu)} \left[\frac{\ln b - \ln r}{\ln b - \ln a} - \frac{a^2}{r^2} \right]$$

$$S_t = A - \frac{B}{r^2} - \frac{\alpha E T_c}{2(1-\nu)} \left[\frac{\ln b - \ln r - 1}{\ln b - \ln a} + \frac{a^2}{r^2} \right] \quad (5-3)$$

$$S_z = 2\nu A - \frac{\alpha E T_c}{2(1-\nu)} \left[\frac{\ln b - \ln r - 1}{\ln b - \ln a} \right]$$

By following the same process used for a spherical cavity, it is found that the equations of the stress distribution in the plastic zone around the

cylindrical cavity are not affected by the cavity temperature rise. Therefore, the equation in the plastic zone is the same as Eq. 3-17. The equations in the elastic zone can be calculated by inserting the proper boundary conditions into Eq. 5-3.

$$\begin{aligned}
 S_r &= -P_o + \frac{\rho^2}{r^2} \left\{ P_o - P_1 - \frac{2}{\sqrt{3}} \sigma_o \ln \frac{\rho}{a} + \frac{\alpha E T_c}{2(1-\nu)} \left[\frac{\ln \frac{b}{\rho}}{\ln \frac{b}{a}} - \frac{a^2}{\rho^2} \right] \right\} \\
 &\quad - \frac{\alpha E T_c}{2(1-\nu)} \left[\frac{\ln \frac{b}{r}}{\ln \frac{b}{a}} - \frac{a^2}{r^2} \right] \\
 S_t &= -P_o + \frac{\rho^2}{r^2} \left\{ P_o - P_1 - \frac{2}{\sqrt{3}} \sigma_o \ln \frac{\rho}{a} + \frac{\alpha E T_c}{2(1-\nu)} \left[\frac{\ln \frac{b}{\rho}}{\ln \frac{b}{a}} - \frac{a^2}{\rho^2} \right] \right\} \\
 &\quad - \frac{\alpha E T_c}{2(1-\nu)} \left[\frac{\ln(\frac{b}{r}) - 1}{\ln \frac{b}{a}} + \frac{a^2}{r^2} \right] \quad (5-4) \\
 S_z &= -2\nu P_o - \frac{\alpha E T_c}{2(1-\nu)} \left[\frac{2 \ln(\frac{b}{r}) - 1}{\ln \frac{b}{a}} \right]
 \end{aligned}$$

Further Development of Plastic Front with Elevated Temperature

As a result of combining thermal and structural stresses, the radius of the plastic front will be further extended with any temperature rise in the cavity. The radii of the plastic zones around the spherical and cylindrical cavities are obtained here as a function of the cavity temperature rise and structural loading conditions.

The following conditions have to be satisfied for a spherical cavity: at the borderline between the plastic and elastic zones ($r = \rho$)

$$\sigma_r = S_r$$

and

$$\sigma_t = S_t$$

Therefore, Eq. 3-5 can be converted into $S_r - S_t = \sigma_o$. By substituting Eq. 5-2 into this equation, the following relation is obtained:

$$\frac{3}{2} \left[P_o - P_i - 2 \sigma_o \ln \frac{\rho}{a} + \frac{\alpha E T_c}{1-\nu} \frac{a (\rho^2 - a^2)}{\rho^3} \right] - \frac{\alpha E T_c}{2(1-\nu)} \left[\frac{a}{\rho} - \frac{3a^3}{\rho^3} \right] = \sigma_o$$

This relation is further derived as:

$$\frac{3}{2} (P_o - P_i) - 3 \sigma_o \ln \frac{\rho}{a} + \frac{\alpha E T_c}{1-\nu} \frac{a}{\rho} = \sigma_o$$

Then:

$$\frac{P_o - P_i}{\sigma_o} = 2 \ln \frac{\rho}{a} + \frac{2}{3} - \frac{2}{3} \frac{\alpha E T_c}{\sigma_o (1-\nu)} \frac{a}{\rho} \quad (5-5)$$

Eq. 5-5 expresses the relation between the radius of the plastic front in terms of the cavity radius $(\frac{\rho}{a})$ and the condition of the structural loading in terms of the pressure difference divided by the equivalent plastic limit, $\frac{P_o - P_i}{\sigma_o}$, for a given cavity temperature rise, T_c .

For the cylindrical cavity, at $r = \rho$, Eq. 3-14 can be converted to Eq. 5-6 in a similar manner. $S_r - S_t = \frac{2}{\sqrt{3}} \sigma_o$ (5-6)

By substituting Eq. 5-4 into the above equation, the following relation is obtained:

$$2 \left\{ P_o - P_i - \frac{2}{\sqrt{3}} \sigma_o \ln \frac{\rho}{a} + \frac{\alpha E T_c}{2(1-\nu)} \left[\frac{\ln \frac{b}{\rho}}{\ln \frac{b}{a}} - \frac{a^2}{\rho^2} \right] \right\} + \frac{\alpha E T_c}{2(1-\nu)} \frac{2a^2}{\rho^2} - \frac{1}{\ln \frac{b}{a}} = \frac{2}{\sqrt{3}} \sigma_o \quad (5-7)$$

Then:

$$\frac{P_o - P_i}{\sigma_o} = \frac{2}{\sqrt{3}} \ln \frac{\rho}{a} + \frac{1}{\sqrt{3}} - \frac{\alpha E T_c}{2\sigma_o(1-\nu)} \left[\frac{2 \ln(\frac{b}{\rho}) - 1}{2 \ln \frac{b}{a}} \right] \quad (5-8)$$

By taking the same assumptions given in Eq. 4-10a, Eq. 5-8 becomes:

$$\frac{P_o - P_i}{\sigma_o} = \frac{2}{\sqrt{3}} \ln \frac{P}{a} + \frac{1}{\sqrt{3}} - \frac{\alpha E T_c}{2\sigma_o(1-\nu)} \left[0.928 - \frac{\ln \frac{P}{a}}{6.91} \right] \quad (5-9)$$

Although Eqs. 5-5 and 5-9 seem to be complex, the functions which they represent can be readily used if properly plotted because they are essentially composed of the following three factors:

$$\text{Factor of structural loading: } X = \frac{P_o - P_i}{\sigma_o}$$

$$\text{Factor of plastic front radius: } Y = \frac{P}{a} \quad (5-10)$$

$$\text{Factor of temperature effect: } Z = \frac{\alpha E T_c}{\sigma_o(1-\nu)}$$

Introducing these factors, Eq. 5-5 and Eq. 5-9 can be rewritten as:

$$\text{Spherical cavity: } X = 2 \ln Y + \frac{2}{3} - \frac{2}{3} \frac{Z}{Y} \quad (5-11)$$

$$\text{Cylindrical cavity: } X = \frac{2}{\sqrt{3}} \ln Y + \frac{1}{\sqrt{3}} - \frac{Z}{2} \left[0.928 - \frac{\ln Y}{6.91} \right]$$

Since Z value is constant at any given temperature, the equations can best be plotted on an X - Y diagram as shown in Fig. 5-1, using the following assumptions:

$$\frac{\alpha E}{1-\nu} = 6.7 \text{ psi/F}^\circ \quad (5-12)$$

- $\sigma_o = 1500 \text{ psi}$ in temperature range up to 300°F.
- $= 1300 \text{ psi}$ in temperature range from 300°F. to 500°F.
- $= 1000 \text{ psi}$ in temperature range from 500°F. to 800°F.
- $= 800 \text{ psi}$ in temperature range from 800°F. to 1100°F.
- $= 600 \text{ psi}$ in temperature range from 1100°F. to 1475°F.

As seen in the diagram, a plastic zone will develop without any structural stress whenever cavity temperature is elevated over 200°F. for both cylindrical

FIG. 5-1. DEVELOPMENT OF PLASTIC ZONE AROUND SPHERICAL AND CYLINDRICAL CAVITIES WITH VARIOUS TEMPERATURE RISES.

and spherical cavities. Furthermore, the development of the plastic zone with elevation of the cavity temperature is much greater in terms of ρ / a , around a cylindrical cavity than a spherical one.

Structural Equilibrium Equations in Relation to Cavity Volume Reduction

Establishing structural equilibrium of the cavity by development of a plastic zone, at elevated cavity temperature, always results in cavity volume reduction. This volume reduction is due to the plastic flow of thermally expanded salt in the plastic zone. For completion of the structural equilibrium equations, the amount of cavity volume reduction has to be obtained in relation to the cavity temperature rise and the radius of plastic front.

In the case of a spherical cavity, assuming that the original cavity radius, a , is reduced to c by the plastic flow, the total volume of this flow, V , is given as:

$$V = \frac{4\pi}{3} (a^3 - c^3)$$

The amount of thermal expansions of a differential spherical element, $d(\Delta V_r)$, with a radius r , is given as:

$$\begin{aligned} \Delta V_r &= 4\pi r^2 dr \\ d(\Delta V_r) &= 3\alpha T_r (4\pi r^2 dr) \end{aligned}$$

The total thermal expansion is obtained by integrating this unit thermal expansion from the cavity radius, a , to the plastic front, ρ . By equating both of the expanded volumes, the following relation is found:

$$V = \frac{4\pi}{3} (a^3 - c^3) = 3\alpha \int_a^\rho T_r 4\pi r^2 dr \quad (5-13)$$

With the assumption that the heat flow has reached its equilibrium state, the temperature, T_r , at the radius, r , can be given as a function of r from Eq. 4-5 as:

$$T_r = \frac{a}{r} T_c \quad (4-5)$$

By substituting this relation into Eq. 5-13, the relation becomes:

$$\frac{1}{3} (a^3 - c^3) = 3 \alpha a T_c \int_a^p r dr$$

Then,

$$c^3 = a^3 - 9 \alpha a T_c \frac{p^2 - a^2}{2} \quad (5-14)$$

The ratio of the reduced volume to the original volume, ξ , is obtained by modifying Eq. 5-14 as follows:

$$\begin{aligned} \xi &= \frac{V_c}{V_a} = \frac{\frac{4}{3} \pi c^3}{\frac{4}{3} \pi a^3} = \frac{c^3}{a^3} \\ &= 1 - \frac{9 \alpha T_c}{2 a^2} (p^2 - a^2) \\ &= 1 - \frac{9}{2} \alpha T_c \left[\left(\frac{p}{a} \right)^2 - 1 \right] \end{aligned} \quad (5-15)$$

From this equation, the reduction of the cavity volume is calculated if the radius of the plastic front is known by using Fig. 5-1. As the converse of this relation, the maximum allowable radius of the plastic front can be calculated for a certain reduction of the cavity volume. From Eq. 5-15 the radius of the plastic front can be expressed as a function of the reduction of cavity volume, $(1 - \xi)$, with a given cavity temperature rise, T_c , as:

$$\frac{p}{a} = \sqrt{\frac{2}{9 \alpha T_c} (1 - \xi) + 1} \quad (5-16)$$

In the case of a cylindrical cavity, a similar analysis is made. By assuming that the original radius of the cylinder, a , is reduced to c by the plastic flow, the total volume of the flow in a unit length of the cylinder, V , can be calculated as:

$$V = \pi (a^2 - c^2) = \int_a^p 3 \alpha T_r 2 \pi r dr \quad (5-17)$$

By substituting Eq. 4-10 into the above relation, it becomes:

$$\pi (a^2 - c^2) = 3\alpha \int_a^{\rho} \frac{\ln b - \ln r}{\ln b - \ln a} T_c 2\pi r dr$$

Then,

$$a^2 - c^2 = \frac{6\alpha T_c}{\ln \frac{b}{a}} \left[\ln b \int_a^{\rho} r dr - \int_a^{\rho} r \ln r dr \right]$$

By integration it becomes:

$$a^2 - c^2 = \frac{3\alpha T_c}{\ln \frac{b}{a}} \left[\rho^2 \ln \frac{b}{\rho} - a^2 \ln \frac{b}{a} + \frac{\rho^2 - a^2}{2} \right] \quad (5-18)$$

The ratio of the reduced volume to the original volume, ξ , is obtained by modification of Eq. 5-18:

$$\begin{aligned} \xi &= \frac{V_c}{V_a} = \frac{\pi c^2}{\pi a^2} = \frac{c^2}{a^2} \\ &= 1 - 3\alpha T_c \left[\left(\frac{\rho}{a} \right)^2 \frac{\ln \frac{b}{\rho}}{\ln \frac{b}{a}} - 1 + \frac{\left(\frac{\rho}{a} \right)^2 - 1}{2 \ln b/a} \right] \end{aligned} \quad (5-19)$$

By using the same assumption given in Eq. 4-25, the ratio, ξ , is obtained as a function of the factor $\frac{\rho}{a}$.

$$\xi = 1 - 3\alpha T_c \left[\left(\frac{\rho}{a} \right)^2 \left(1.072 - \frac{\ln \frac{\rho}{a}}{6.91} \right) - 1.072 \right]$$

Then, the reduction of cavity volume, $(1 - \xi)$, becomes:

$$1 - \xi = 3\alpha T_c \left[\left(\frac{\rho}{a} \right)^2 \left(1.072 - \frac{\ln \frac{\rho}{a}}{6.91} \right) - 1.072 \right] \quad (5-20)$$

Under a steady heat flow from the cavity, a direct relation between the factor of structural loading and the ultimate cavity volume reduction can be obtained by using the previous analysis. Any amount of cavity volume reduction causes a decrease of the cavity radius from a to c . Therefore, if the cavity volume is reduced from 1 to ξ , Eq. 5-5 (sphere) and Eq. 5-9 (cylinder) become:

for the spherical cavity

$$\frac{P_o - P_i}{\sigma_o} = 2 \ln \frac{P}{c} + \frac{2}{3} - \frac{2}{3} \frac{\alpha E T_c}{\sigma_o (1-\nu)} \frac{c}{P} \quad (5-21)$$

for the cylindrical cavity

$$\frac{P_o - P_i}{\sigma_o} = \frac{2}{\sqrt{3}} \ln \frac{P}{c} + \frac{1}{\sqrt{3}} - \frac{\alpha E T_c}{2\sigma_o (1-\nu)} \left(0.928 - \frac{\ln \frac{P}{c}}{6.91} \right) \quad (5-22)$$

Then, the following relations between the factor of structural loading $\left(\frac{P_o - P_i}{\sigma_o}\right)$ and the ratio of the reduced cavity volume to the original one (ξ) are obtained with any given cavity temperature rise (T_c) for both spherical and cylindrical cavities. In a spherical cavity, by eliminating P from Eqs. 5-16 and 5-21, the relation is found as:

$$\frac{P_o - P_i}{\sigma_o} = \ln \left[\xi^{-\frac{2}{3}} + \frac{2(\xi^{-\frac{2}{3}} - \xi^{\frac{1}{3}})}{9 \alpha T_c} \right] + \frac{2}{3} - \frac{2}{3} \frac{\alpha E T_c}{\sigma_o (1-\nu)} \times \left[\xi^{-\frac{2}{3}} + \frac{2(\xi^{-\frac{2}{3}} - \xi^{\frac{1}{3}})}{9 \alpha T_c} \right]^{-\frac{1}{2}} \quad (5-23)$$

In a cylindrical cavity, similarly from Eqs. 5-20 and 5-22, the relation is found as follows:

$$A = 1.072 - \frac{\ln \xi}{2} + \frac{6.91 (\xi - 1 - 3.22 \alpha T_c)}{3 \alpha T_c \xi} e^{-2A} \quad (5-24)$$

where

$$A = \frac{\frac{P_o - P_i}{\sigma_o} + \frac{0.928 \alpha E T_c}{2 \sigma_o (1-\nu)} - \frac{1}{\sqrt{3}}}{\frac{\alpha E T_c}{2 \times 6.91 \sigma_o (1-\nu)} + \frac{2}{\sqrt{3}}}$$

Eqs. 5-23 and 5-24 are computed for various possible cavity conditions, and plotted in Figs. 5-2 and 5-3 respectively, by taking the cavity volume reduction $(1 - \mathcal{L})$ on the abscissa. As seen in the diagrams, the reduction of cavity volume is accelerated with the increase of T_c and $\frac{P_o - P_i}{\sigma_o}$ until about 20% of the cavity is closed; then the reduction becomes almost linear until an 80% cavity closure, beyond which the reduction is decelerated. In theory, therefore, the cavity will not be completely closed under any given condition although the cavity volume approaches zero with an increase of T_c and $\frac{P_o - P_i}{\sigma_o}$.

However, the relations expressed by Eqs. 5-23 and 5-24 hold true if the rate of cavity reduction is small compared to the natural thermal value reduction of the stored radioactive waste, such that a steady state of heat flow can be maintained around the cavity, along with cavity volume reduction. If the rate of cavity volume reduction is greater, a steady state of heat flow cannot be maintained along with cavity volume reduction, in which case Eqs. 5-23 and 5-24 are no longer applicable.

In order to supplement Eqs. 5-23 and 5-24, the relations between $\frac{P_o - P_i}{\sigma_o}$ and $\mathcal{L}$ are obtained by assuming the other extreme condition in which the cavities close instantaneously, the thermal value of the waste remaining unchanged. Under this condition, a direct relation between the factor of structural loading and the ultimate reduction of volume can be obtained by eliminating the factor $\frac{\rho}{a}$ from Eqs. 5-5 and 5-16 for a spherical cavity and from Eqs. 5-9 and 5-20 for a cylindrical cavity.

In the case of the spherical cavity, the relation is found as:

$$\frac{P_o - P_i}{\sigma_o} = \ln \left[\frac{2(1-\mathcal{L})}{9 \alpha T_c} + 1 \right] + \frac{2}{3} - \frac{2}{3} \frac{E T_c}{\sigma_o (1-\mathcal{L})} \sqrt{\frac{2(1-\mathcal{L})}{9 \alpha T_c} + 1} \quad (5-25)$$

FIG. 5-2. RELATION BETWEEN ULTIMATE VOLUME REDUCTION AND STRUCTURAL LOADING WITH VARIOUS TEMPERATURE RISES IN SPHERICAL CAVITY.

FIG. 5-3. RELATION BETWEEN ULTIMATE VOLUME REDUCTION AND STRUCTURAL LOADING WITH VARIOUS TEMPERATURE RISES IN CYLINDRICAL CAVITY.

In the case of the cylindrical cavity, the relation is found as:

$$1 - \zeta = 3 \alpha T_c (A e^{+B} - 1.072) \quad (5-26)$$

where

$$A = 0.928 - \frac{B}{13.8}$$

$$B = \frac{-\frac{2}{\sqrt{3}} + 0.928 \frac{\alpha E T_c}{\sigma_o (1-\nu)} + 2 \frac{P_o - P_1}{\sigma_o}}{\frac{2}{\sqrt{3}} + 0.072 \frac{\alpha E T_c}{\sigma_o (1-\nu)}}$$

The above relations between the reduction of cavity volume, $1 - \zeta$, and the factor of structural loading, $\frac{P_o - P_1}{\sigma_o}$, are also plotted in Figs. 5-2 and 5-3. It should be noted in the diagram that the reduction of cavity volume is much more accelerated with the assumption of instantaneous closure, than under a steady state of heat flow.

Practical Consideration

These analyses have demonstrated the important fact that a salt cavity will virtually close as a result of the plastic flow and the thermal expansion of salt. A real closure is expected at some point in the range between the two extreme conditions as shown in Figs. 5-2 and 5-3. This range becomes greater as cavity temperature rises. To determine an exact point at which the cavity closure occurs, a study is required on the velocity of cavity closure in relation to the cavity temperature rise and to the reduction rate of the thermal value of the radioactive wastes. However, it is estimated that the actual closure would be similar to the one indicated as the instantaneous closure in Figs. 5-2 and 5-3, because of the highly increased velocity of creep at elevated temperatures.

The reduction of the cavity volume due to the elevated temperature is one of the more important practical limitations upon the use of the salt cavity for storage purposes. In other words, the cavity could be successfully used for waste disposal if the cavity temperature rise is controlled.

For construction of Figs. 5-1, 5-2 and 5-3, the assumptions given in Eq. 5-12 were used. The equivalent plastic limit, σ_0 , is taken as a function of temperature from the experimental results discussed later in Chap. XI. The $\frac{\alpha E}{1-\nu}$ value is assumed to be independent of temperature for the reason discussed in Chap. IV. In order to eliminate these assumptions and to make the equations in this analysis perfect, the structural coefficients, E , σ_0 , and α , should be determined as precise functions of temperature by laboratory study. Since the temperature around the cavity is also a function of distance from the cavity center, r , all the coefficients can be expressed as functions of the distance, r . These computations can best be made with the aid of an electronic computer.

PART TWO: EXPERIMENTAL INVESTIGATION

CHAPTER VI

DEVELOPMENT OF STANDARD PROCEDURE FOR TESTING SALT

Experimental test results are always influenced by the procedures used in testing the material. Unlike many other construction materials, no standard procedure for testing salt has been previously established; yet a unique testing procedure is required for salt because of its unusual characteristics. As seen in the following discussion, direct observation of any experimental data, without a rational standard testing procedure, is not only meaningless but may lead to serious mistakes. Because of this need, a special procedure for testing salt was developed. For purposes of presenting this paper, the author has used the term standard and standardized to mean the adopted procedure for this investigation.

Difficulties in Compression Test of Salt

As was discussed in Part One, only the properties of salt under compressive stress are involved here. In testing the compressive strength of salt, the testing procedure and the dimension of the test specimen are particularly significant because of the difficulties encountered in a compression test in addition to the effect of the grains. The major difficulties in a compression test of salt are summarized here as follows:

1. The difficulty of applying a truly concentric load.
2. Accidental irregularities occurring in the test specimen, such as grain size, impurities, and previous damage.
3. The difficulty of making precise specimens without disturbing the granular structure of salt.

4. Friction force created between the loading surface of salt and the head of the testing machine.

5. Necessity of obtaining a relatively large cross section of the specimen to insure the reliability of test results.

6. Influence upon testing result of grain size relative to specimen size.

To overcome these difficulties, some techniques were developed as the need arose and some ideas originated from a literature search^{1,12,19}. The following practical solutions were devised to satisfy the above six problems.

1. A practical test procedure, which will be elaborated upon in the next section, was set up by referring to testing specifications of the A. S. T. M.

2. A reasonable number of test specimens were tested, and the test results were evaluated by statistical means whenever necessary.

3. A large block of salt approximately 4 ft x 4 ft x 4 ft, called "the mother specimen", was cut from the floor of the Grand Saline Salt Mine, at a depth 700 ft. below the earth's surface, without the use of explosives. The mine tunnel and the specimen cutting are shown in Fig. 6-1 and 6-2. As seen in Fig. 6-3, this mother specimen was cut into smaller pieces by using a big hand saw, and finally, a mechanical band saw, and grinding disk, were used to obtain the desired dimensions.

4. The end friction effect was successfully eliminated by introducing a friction reducing technique.

5 & 6. As given in Table 6-1, a standardized specimen size was determined after evaluating extensive test results. A smaller specimen size was required because time and labor necessary for preparing specimens increase almost exponentially with increase of specimen size.

FIG. 6-1. MINE OPERATION AT 700 FT. BELOW SURFACE IN GRAND SALINE SALT DOME IN EAST TEXAS.

FIG. 6-2 SAW AND DRILL TECHNIQUE FOR REMOVING TEST SPECIMEN.

FIG. 6-3. HAND CUTTING PREPARATORY TO MACHINE CUTTING.

FIG. 6-4. FAILURE CHARACTERISTICS RESULTING FROM DIFFERENT END FRICTIONS.

FIG. 6-5. IMPURITY STRATA INDICATING PREVIOUS HISTORY OF DEFORMATION.

FIG. 6-6 DEVIATION OF IMPURITY STRATA FROM NORMAL VERTICAL AXIS.

TABLE 6-1

LABORATORY PROCEDURES FOR COMPRESSION TEST OF SALT

1. Capping	not applied
2. Accuracy of specimen cutting	not more than 1/32 in
3. Accuracy of specimen setting	nore more than 1/16 in
4. Speed of testing	
in strain	0.5 to 1.0 %/minute
in increase of stress	200 to 400 psi/minute
in relation to maximum stress	1/5 of max. stress/minute
5. Standard specimen size with application of the friction reducer	1.75 in x 1.75 in x 1.75 in
6. Specimen without application of the friction reducer.	
loading cross section area	not less than 2.5 in x 2.5 in
ratio of height to width	2 to 3
7. Strain measurement device	dial gage
8. Offset strain for yield strength in uniaxial compression	0.5 %
9. Offset strain for differential yield strength in triaxial compression	2.0 %

Standardized Procedure

None of the common construction materials for which testing procedures are established in the testing specifications of A. S. T. M. behave like salt. Although salt is brittle and fractures readily, it is also plastic and creeps.

The testing specifications for brick and concrete were used as references for brittle material and that of wood for plastic material^{1,19}, and in conjunction with testing experiences reported herein, a standardized procedure for testing salt was developed and is summarized in Table 6-1.

The dimension of the standard specimen and application of the friction reducer were determined according to analysis of the laboratory tests, which follows here. Use of SR-4 strain gages for measuring salt strain is not recommended except under certain circumstances. Thus, mechanical dial gages are to be used for the measurement.

In compression of salt, the strain measured by a dial gage is quite different from the same strain measured simultaneously by SR-4 strain gages which are attached to the sides of the specimen. This difference is compared in Fig. 6-7. This complex relation between the strain readings by the dial gage and the SR-4 strain gage indicates the complexity of intergranular movement and the nature of the test. The SR-4 strain gages and their measuring devices are more fully described in Chapter 9 and shown in Figs. 9-1, 9-2, 9-3 and 9-4.

Application of the End Friction Reducer

Friction on the end surfaces of a salt specimen creates a restraining force on the lateral expansion of the salt under compression, and sets up a complex stress condition near the ends of the specimen. If the specimen is short, this phenomenon expresses itself in a remarkable increase of testing strength which is no longer a real compressive strength. To eliminate this undesirable effect, a friction reducing layer, made of aluminum foil and plastic sheet sandwiched together with vaseline, was successfully introduced, as illustrated in Fig. 6-8. In this diagram the effect of the end friction

FIG. 6-7. COMPARISON OF STRAIN READINGS BY SR-4 GAGE AND DIAL GAGE.

FIG. 6-8. EFFECT OF END FRICTION REDUCER ON STRESS-STRAIN RELATION.

on the compressive strength of salt cube specimen is clearly shown by two groups of the stress-strain curves obtained, with and the other without the application of the friction reducer. Important findings in this diagram are summarized as follows:

1. The friction effect on the cube specimen changes the entire stress-strain curve of a specimen. The friction results in an increase of ultimate strength about twice as great as the real compressive strength of the cube specimen.
2. In each group of stress-strain curves, all the curves do not deviate appreciably from one another and remain in a narrow band as seen in Fig. 6-8. Judging from this fact, the author concludes that the mother salt specimen is homogeneous and has no evidence of previous structural disturbance.
3. With the friction reducer, the salt is broken by failure of the cohesive force between the grains, while without the friction reducer the specimen develops a hard core at each end of the specimen, and then finally fails by a sudden shearing at the middle as shown in Fig. 6-4.

Dimension of Test Specimen

The dimension of the test specimen has a particularly important effect on compression tests of coarse granular materials such as salt. Several series of compression tests were designed to investigate the effect of height, cross-section area, size and ratio of width to height of the specimen. In Fig. 6-9, a definite effect of specimen height to the stress-strain curves is demonstrated from testing specimens with a constant cross-section and different heights.

It is evident that the stress-strain curves of the specimens tested without the friction reducer are influenced by the specimen height, and this

FIG 6-9 EFFECT OF SPECIMEN HEIGHT ON STRESS-STRAIN RELATION.

FIG 6-10 EFFECT OF SPECIMEN CROSS-SECTION ON STRESS-STRAIN RELATION

FIG 6-11 EFFECT OF SIZE OF CUBE SPECIMEN ON STRESS-STRAIN RELATION

influence is lessened as the height exceeds twice that of the width. The influence of the end friction becomes enormous when the specimen cross-section increases, as seen in Fig. 6-10. However, this influence is completely eliminated in those specimens tested with the friction reducer as shown by comparison of the two groups of curves, in the diagram.

It is found that the size of specimen also has an important influence upon the stress-strain curves even when the proportion of specimen height and width is kept constant. In Fig. 6-11, this influence of specimen size is demonstrated. Since no effect from size is found among those specimens tested with the friction reducer, the author concludes that size effect can also be eliminated by use of the friction reducer.

Finally, the maximum stress of all the specimens tested are plotted in Fig. 6-12 by taking the ratio of width to height (w/h) on the abscissa. In this diagram all of the test results were clearly separated into two bands. The difference between the two bands is due to end friction. The width of the bands indicates the effect of specimen size in addition to the random nature of the testing procedure and specimen quality. In theory, these two bands should possess the same maximum stress as the ratio, w/h , approaches zero, where the end friction becomes nil. Therefore, the two bands were extended to a common maximum stress at the ratio, w/h , which is zero.

From these analyses, the true maximum compressive stress of the granular salt of the Grand Saline salt dome is determined:

The mean maximum stress = 2,300 psi

Standard deviation of the mean maximum stress = 200 psi

Coefficient of variation = 8.7 %

Further discussion on determination of strength of salt follows in the next section.

FIG. 6-12. EFFECT OF DIMENSION AND END FRICTION ON MAXIMUM STRESS.

CHAPTER VII

FUNDAMENTAL STRUCTURAL PROPERTIES OF SALT

The reaction of different materials to a given stress condition varies depending entirely upon their structural properties. The structural properties can usually be described sufficiently with a few fundamental coefficients, such as Young's modulus, Poisson's ratio, and yielding stress. These coefficients are essential for any application of the results of the theoretical analysis developed in Part One.

Under uniaxial compression tests, the following fundamental structural properties of salt were investigated: strength, Young's modulus, Poisson's ratio, and strain-hardening.

Strength

There are several ways to define the strength of a material. The maximum strength (or ultimate strength) has been the most popular way of describing the strength of a material. The maximum strength of salt was previously determined to be 2,300 psi with the standard deviation of 200 psi by statistical extrapolation. Attention is called to the wide range of maximum strength of salt reported by various investigators as presented in the following table.

Unfortunately, the maximum stresses given by these investigators are not comparable because their testing procedures were not specified. According to the conclusions drawn in the previous chapter on the effects of dimensions and end friction on the strength of salt, it is probable that the widely differing values of the maximum strength of salt reported in Table 7-1 are due to the different testing procedures used, in addition to the difference of the true strength of salt. It is understandable that the maximum strength determined in this laboratory is the lowest one among them, because the effect of

dimension and end friction were completely eliminated by statistical extrapolation for the determination of the value.

TABLE 7-1

MAXIMUM STRENGTH OF SALT AS DETERMINED BY VARIOUS INVESTIGATORS

Source of Salt	Form	Grain Size (mm)	Max.Strength (psi)	Investigator
Grand Saline Dome, in Texas	Aggregate	0.1 to 0.5	2,300	the author
Salt Dome in Louisiana	"	1.0 to 1.7	5,000	R.G. Wuerker ⁶⁸
Salt Mine in New Mexico	"	0.5 to 1.2	4,200	"
Hockley Dome, Texas	"	0.05 to 0.1	3,700	J. Handin ²⁴
Various Salt Domes in West Texas	-	-	2,600 to 4,000	D.F. Jahns ³⁸
Salt Mine in Rumania	-	-	4,700 to 5,300	M. Stamatiu ⁶⁰
Grand Saline Dome, Texas & artificial product by Harshaw Co.	Single Crystal. Loading parallel to crystal plane	-	4,000 to 4,200	the author

The proportional limit of salt has been arbitrarily established at 0.5 % offset because its stress-strain curve deviates very gradually from the beginning of loading. This proportional limit eventually coincides with the yield strength standardized in the previous chapter. This coincidence is due to the nature of the stress-strain curves found in Figs. 6-8 to 6-11. The yield

strength of the salt (also the proportional limit) is found from the diagrams to be in the range of 1,800 psi to 2,200 psi.

Young's Modulus

Young's modulus is one of the most important physical constants which indicates the structural behavior of an elastic body, since the modulus is a proportional constant of unit stress increase and unit strain increase. Therefore, salt deforms according to Young's modulus until the stress exceeds the yield strength.

For practical measurement of Young's modulus of salt, the modulus is defined here as the slope of a straight line which covers the most linear part of the stress-strain curve below its yield strength. According to this definition, Young's modulus varied from 0.1×10^6 to 1.0×10^6 , depending upon the testing procedure. Values of Young's modulus determined by different test procedures are compared in Table 7-2.

In the first group of the table (No. 1 to 3), higher values of the Young's modulus were observed. They are not acceptable because of improper application of the SR-4 strain gages. In the second group (No. 4 to 7), the moduli were determined to be in a range between 0.3 and 0.4 by using dial strain gages. By comparison of these to the third group, it is clear that the end friction during the compression tests increases the ostensible value of Young's modulus. Now, it seems reasonable to determine the modulus from the experimental data of the third group. The modulus was thus calculated to be:

$$\text{Mean value of Young's modulus} = 0.14 \times 10^6 \text{ psi}$$

$$\text{with standard deviation} = 0.03 \times 10^6 \text{ psi}$$

$$\text{coefficient of variation} = 21 \%$$

TABLE 7-2

YOUNG'S MODULUS DETERMINED BY DIFFERENT TEST PROCEDURES

Spec. No.	Specimen Size (in.)	Frict. Red.	Gage	Max. Stress (psi)	Young's Modulus (10^6)	Stress Range (psi)
1	3 x 3 x 3	Not applied	SR-4	4,500	1.0	0-1,000
2	4 x 4 x 6	"	"	3,500	1.0	0-1,000
3	2 x 2 x 3.8	"	"	3,000	0.5	0-2,000
4	3 x 3 x 3	Not applied	Dial	5,000	0.3	0-2,500
5	3 x 3 x 3	"	"	4,000	0.4	0-3,000
6	1.75 x 1.75 x 4	"	"	2,000	0.4	0-1,500
7	1.75 x 1.75 x 3.5	"	"	2,500	0.3	0-1,500
8	3.5 x 3.5 x 3.5	Applied	Dial	2,500	0.13	0-1,500
9	3 x 3 x 3	"	"	2,200	0.18	"
10	2 x 2 x 2	"	"	2,500	0.15	"
11	1.75 x 1.75 x 1.75	"	"	2,300	0.11	"
12	3.5 x 3.5 x 1.75	"	"	2,900	0.13	"
13	3 x 3 x 1.75	"	"	2,500	0.11	"
14	2 x 2 x 1.75	"	"	2,900	0.10	"
15	1.75 x 1.75 x 4	"	"	2,800	0.18	"
16	1.75 x 1.75 x 2.5	"	"	2,700	0.20	"
17	1.75 x 1.75 x 2	"	"	2,100	0.13	"

Poisson's Ratio

Along with Young's modulus, Poisson's ratio is the other one of the two most important properties of materials for describing their elastic and plastic behavior as found in the theoretical analysis in Part One. According to the previous discussion in Chap. II, the initial stress naturally existing in any underground formation is determined as a function of the Poisson's ratio, as given in Eq. 2-2.

To determine the Poisson's ratio of salt, laboratory experiments were conducted first by using the SR-4 strain gages, since the Poisson's ratio is defined as the ratio of a lateral strain to a strain in the loading direction within the elastic range. The SR-4 strain gages (types AR-1 and A-8) used for the determination are shown later in Figs. 9-2 and 9-4. The Poisson's ratio thus determined exhibits much higher values than the theoretical limit of 0.5, as plotted in Fig. 7-1. This may indicate that the SR-4 strain gage is not adequate to measure the strain of salt since the strain is not truly elastic. The higher Poisson's ratio in the granular aggregate may be a reflection of the greater lateral displacement among the grains. The Poisson's ratio taken on a single crystal grain in the same aggregate reveals a value much closer to the maximum theoretical limit of 0.5. The Poisson's ratio remains almost unchanged after releasing the pressure as found in Fig. 7-1. This may indicate that the crystal is also plastic with a stress over 300 psi, and the lateral deformation in the crystal is less than that of an aggregate as a whole. This higher value of the observed Poisson's ratio may indicate that the Poisson's ratio becomes 0.5 with compressive stress of more than 300 psi regardless of whether the salt is aggregate or crystal.

Besides the direct measurement of strain, an entirely different method of determining the Poisson's ratio was sought because of the difficulty involved with accurate measurement of lateral strain of salt in the laboratory.

FIG. 7-1. POISSON'S RATIO FOR CRYSTAL GRAIN IN AGGREGATE AND AGGREGATE AS A WHOLE DETERMINED BY SR-4 STRAIN GAGES.

FIG. 7-2. STRAIN-HARDENING IN CRYSTAL AND AGGREGATE AS DETERMINED BY SR-4 STRAIN GAGES.

FIG. 7-3. ORIENTATIONAL ANALYSIS OF STRAIN HARDENING BY SR-4 STRAIN GAGES.

The Poisson's ratio can be computed if the propagation velocity of seismic waves in a salt formation is known. The compressional wave velocity in a salt formation has been reported, from field determination, to be 4.4 - 7.5 km/sec, at a depth of 10,000 ft^{18,32}. The Poisson's ratio can be thus calculated from the following equation¹⁸ of the propagation velocity, V_p :

$$V_p = \sqrt{\frac{E}{\mu} \frac{(1-\nu)}{(1+\nu)(1-2\nu)}} \quad (7-1)$$

where E = Young's modulus = 0.14×10^6 psi = 0.99×10^4 Kg/cm²

μ = density of salt = 2.2×10^{-3} Kg/cm³

V_p = propagation velocity = 5.5×10^5 cm/sec

ν = Poisson's ratio

By substituting these experimental values into Eq. 7-1, the Poisson's ratio is given as:

$$\nu = \frac{1}{2} \left[\frac{\beta-1}{2} \pm \sqrt{\left(\frac{\beta-1}{2}\right)^2 - 4\left(\frac{\beta-1}{2}\right)} \right]$$

where

$$\beta = \frac{E}{V_p^2 \mu} = \frac{0.99 \times 10^4}{(5.5 \times 10^5)^2 \times 2.2 \times 10^{-3}} = 1.5 \times 10^{-5}$$

then,

$$\nu \approx \frac{1}{2} - \frac{\beta}{6} = 0.499998$$

From this calculation, it seems that the actual Poisson's ratio at this depth is close to 0.5. The initial stress condition in this underground salt formation therefore must be hydrostatic.

Strain Hardening

A stress-strain curve of salt is strongly influenced by any previous compression the specimen has undergone. Prior compression raises the yield strength for further compression, but lowers it for tension, as seen previously in Fig. 3-2 and 6-7. This strain-hardening phenomenon is especially

pronounced at low temperatures; however, with increasing temperature, its effect diminishes, as will be shown later in the study of heat effect in Chap, XI. Unlike most common construction materials, salt exhibits its strain-hardening effect near the beginning of loading in both crystal and aggregate salt, as shown in Fig. 7-2.

The effect of time upon strain-hardening characteristics was studied by reloading a previously loaded and released specimen three months later. As shown in Fig. 7-2, resting the specimen for that period of time does not appreciably influence the previous strain-hardening. The fact that salt previously exposed to shearing stress retains this evidence in the stress-strain curve may be utilized for investigating the history of previous exposure of the salt to shearing stress.

According to this hypothesis, any orientational difference of the structural property of salt should be revealed by the loading of three specimens, which were cut from a common block of salt with a known orientation, with three mutually perpendicular loading directions. As shown in Fig. 7-3, the three stress-strain curves obtained by loading from the three different orientations appear to be all alike and show no trace of strain-hardening in any direction in the salt. This can be taken as experimental proof that salt is structurally isotropic. The question remains, however, of the existence of strain-hardening caused in the process of salt dome formation. The strata of impurities observed in the salt domes indicates a previous exposure to complex shearing stress acting from numerous directions, as seen previously in Fig. 6-5 and 6-6.

From consideration of the elevated temperature at the depth where the dome was started and the time which has elapsed since the creation of the

dome, the author concludes that recrystallization of the once strain-hardened or plastically deformed salt has taken place, and the trace of strain-hardening was absorbed in its process.

CHAPTER VIII

STRENGTH THEORY OF SALT

The determination of mechanical properties of a material is generally affected by the environmental conditions in which the material is found. The strength of salt is influenced by temperature and stress conditions in which the salt occurs. At room temperature, salt is brittle and fractures under uniaxial compression. Salt becomes more ductile, however, and yields plastically under high temperature or triaxial compression. The strength of ductile salt can be analyzed by using the theories of octahedral shearing stress and the Mohr's envelope technique. However, neither can explain every case of brittle failure at ordinary temperature when there is little or no triaxial compression.

The proper development of a "strength theory" of salt under various conditions was sought here because a salt cavity might be exposed to a wide range of temperature and stress conditions from the time of its creation until actual storage of radioactive wastes. The effect of triaxial compression on the strength of salt was studied by constructing Mohr's envelopes from various test data. The important effect of the mean principal stress was investigated by the confinement test and the cylindrical cavity test. All the experimental results were finally evaluated to determine the equivalent plastic limit (σ_0) in relation to the confining pressure.

Triaxial Compression

Mohr's theory of strength is most commonly used when establishing criteria for material failure. To establish the criteria for salt, a series of triaxial compression tests with various confining pressures is required.

FIG. 8-1. PLASTIC DEFORMATION OF SPECIMEN BY UNIAXIAL COMPRESSION AT 770°F.

FIG. 8-2. 400,000 POUND TEST APPARATUS FOR MEASURING CREEP RATES IN CYLINDRICAL CAVITY

FIG. 8-3. CYLINDRICAL CAVITY TEST APPARATUS.

FIG. 8-4. CLOSURES RESULTING FROM CREEP IN CYLINDRICAL SALT CAVITY.
(6" O.D X 2" I.D)

FIG. 8-5. PARTIALLY CLOSED CYLINDRICAL CAVITY.

FIG 8-6. THE 98 % CAVITY CLOSURE.

In the triaxial compression test, a specimen receives different stress in each of the three mutually perpendicular directions. Stress distribution at any point in an underground body is by nature triaxial, as previously discussed in Chap. II. The actual strength of any underground structure, therefore, has to be studied by this triaxial test procedure.

Fortunately, the Shell Development Laboratory of Houston has been conducting triaxial tests of many underground minerals, including salt. Extensive data of triaxial tests on aggregate and crystal salt were obtained from J. Handin²⁴ of that laboratory. Handin carried out some of the tests using aggregate salt from the Hockley salt dome near Houston, Texas, under both compression and extension, which he defined as:

"Compression - A triaxial compression in which the axial compression is larger than the confining compression.

Extension - A triaxial compression in which the confining compression is larger than the axial compression."

The three groups of test results, compression, extension and compression with specimen exposed to kerosene, were plotted in the differential stress-strain diagrams of Figs. 8-7, 8-8 and 8-9 respectively. As seen in these diagrams, the maximum differential stress increases as the confining pressure increases, finally approaching a certain value. Greater strain develops with greater confining pressure. Even though a distinct difference is noticed between the stress-strain diagrams of compression and extension, their ultimate differential strengths are about the same (16,000 psi). On the other hand, the ultimate differential strength of the compression with specimen exposed to kerosene is found to be about 7,000 psi, which is only 42 % of the value obtained when the samples are encased in copper jackets.

FIG. 8-7. TRIAXIAL COMPRESSION OF AGGREGATE SALT UNDER VARIOUS CONFINING PRESSURES.

FIG. 8-8. TRIAXIAL EXTENSION OF AGGREGATE SALT UNDER VARIOUS CONFINING PRESSURES.

FIG. 8-9. TRIAXIAL COMPRESSION OF AGGREGATE SALT EXPOSED TO CONFINING LIQUID; i.e., KEROSENE.

One possible explanation for this remarkable difference in the ultimate differential strength is the fact that the salt specimens in compression and extension are encased in a copper jacket while the salt specimens in the other group are exposed to kerosene. The copper jacketing increases end friction while kerosene exposure reduces end friction.

Besides this friction effect, the maximum strength observed at more than a few percent offset strain does not indicate the true strength, because of the increased cross section, the increased end friction, and the increased velocity of strain. Therefore, the differential yield strengths were determined from 2% offset strain for construction of Mohr's envelopes.

Mohr's Envelope

Mohr's envelope was constructed to establish criteria for failure or plastic yielding. It is a graphic presentation of experimental data indicating the maximum differential stress which is permissible between the major and minor principal stress. Consequently, the Mohr's envelope for salt should indicate the maximum stress permissible on the boundary of a salt cavity for any given internal pressure of the cavity.

Handin's work on the triaxial strength of salt given in Figs. 8-7, 8-8 and 8-9 was utilized to construct three Mohr's envelopes, two for compression and one for extension, as shown in Fig. 8-10. In comparing the two envelopes for compression and extension, the ultimate shearing strengths in both are found to be around 4,000 psi although a considerable difference is noted between the envelopes at relatively less confining pressures. This difference cannot be explained by Mohr's theory, because Mohr's theory does not take account of the mean principal stress, yet the magnitude of the mean principal stress is the only difference in the testing conditions of the two envelopes.

FIG. 8-10. MOHR'S ENVELOPES FOR 2% OFFSET YIELD UNDER COMPRESSION AND EXTENSION AT VARIOUS TEMPERATURES.

FIG. 8-11. PRINCIPLE OF REDUCING MAX. SHEARING STRESS BY ADJUSTMENT OF INTERNAL PRESSURE OF CAVITY UNDER NON-HYDROSTATIC INITIAL STRESS CONDITION

The theory of octahedral shearing stress also fails to explain this difference. The relation between the maximum octahedral shearing stress, $\tau_{o_{\max}}$, and the maximum differential stress, $\sigma_{\max} - \sigma_{\min}$, can be obtained by introducing $\sigma_2 = \sigma_{\max}$ for the triaxial extension, and $\sigma_2 = \sigma_{\min}$ for the triaxial compression, into Eq. 3-1. In the case of the extension Eq. 3-1 thus becomes:

$$\tau_{o_{\max}} = 0.471 (\sigma_{\max} - \sigma_{\min})$$

In the case of the compression, Eq. 3-1 also becomes:

$$\tau_{o_{\max}} = 0.471 (\sigma_{\max} - \sigma_{\min})$$

From the above calculations, the theory of octahedral shearing stress has indicated no difference between the two envelopes.

The discrepancy between the theory and the experimental data can be explained, however, from the different processes of failure observed. This failure manifests itself as slip and structural breakdown, as discussed in Chap. III. Under a great confining pressure, where no difference in the Mohr's envelopes for the extension and compression are found, salt yields mainly by slip. On the other hand, under little or no confining pressure, where a considerable difference is found in the envelopes, structural breakdown dominates. The transition point from the brittle to the ductile state can be found in the envelopes as the point where the two envelopes join together. To indicate this transition point, the mean of the three principal stresses, σ_m , can be conveniently used. The mean of the principal stresses is calculated as:

$$\sigma_m = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

where $\sigma_1, \sigma_2, \sigma_3$ = the three principal stresses of the Mohr's circle touching its envelope.

From the three Mohr's circles touching at transition points of the three different envelopes, the means of triaxial stresses are calculated as:

for the extension:

$$\sigma_m = \frac{8,000 + 8,000 + 0}{3} = 5,330 \text{ psi}$$

for the compression:

$$\sigma_m = \frac{10,500 + 3,000 + 3,000}{3} = 5,500 \text{ psi}$$

For the compression with reduced friction (submersion in kerosene):

$$\sigma_m = \frac{9,500 + 3,500 + 3,500}{3} = 5,500 \text{ psi}$$

Thus, it can be stated that salt becomes ductile, and that the theory of octahedral shearing stress can be applied for stress analysis when the mean of triaxial stresses exceeds 5,500 psi at room temperature, assuming that the triaxial tests of Handin are applicable.

Shown on the same diagram, Fig. 8-10, are the experimental results obtained in the laboratory by using the standardized test procedure under various temperatures. It is apparent that the Mohr's circle of the standardized specimen (the cross-hatched area) resembles the envelope obtained from the compression with the specimen exposed to kerosene. This resemblance could be explained by the fact that the end friction is reduced in both test specimens.

As observed in Fig. 8-10, the two significant changes in the Mohr's envelopes take place with the elevation of temperature. One is reduction of shearing strength, and the other is the increase in ductility. With increase of temperature, the ultimate shearing stress is reduced rapidly, up to 500°F.; however, this reduction becomes smaller with further increase of the temperature above 500°F., as discussed later in Chap. XI.

At a temperature around 500°F., salt becomes extremely ductile; a standardized specimen is no longer fractured, but plastically deforms even under uniaxial compression, as demonstrated in Figs. 8-1 and 11-3.

As the result of these effects from elevated temperature, the Mohr's envelope becomes smaller and flatter as seen in Fig. 8-10, indicating the insignificant effect of triaxial confinement as the temperature increases. Thus, the theory of octahedral shearing stress becomes freely applicable at a temperature above 500°F.

To illustrate the practical use of Mohr's envelope for analyzing the triaxial stress condition on the boundary of a cavity, a graphic explanation is presented in Fig. 8-11. A stress condition is presented by using three principal stresses, A, B, and C, in the diagram. A is the stress produced by the internal pressure of the cavity, which is equal to atmospheric pressure of the empty cavity; B and C are the major and minor principal stresses on the boundary of the cavity. Three Mohr's circles between the stresses AB, BC, and CA were constructed inside the Mohr's envelope. If any one of the three circles cross the envelope a failure or creep condition is indicated; if any touch the envelope the condition is critical; if all three stay inside the envelope the condition is stable. Therefore, the condition can be made more stable by increasing the internal pressure of the cavity, A, to A', which comes between the principal stress on the boundary B and C, assuming that the initial stress or external pressure is non-hydrostatic. This condition is indicated by the three circles, CB, A'C and A'B, in the same diagram. If the initial stress is hydrostatic, the shearing stress can be reduced to zero by increasing the internal pressure of the cavity to the initial stress, as indicated by the theory of elasticity described in Eqs. 2-6 and 2-10.

Effect of the Mean Principal Stress

The effect of the mean principal stress on the strength of salt was investigated here because neither Mohr's theory nor the theory of octahedral shearing stress is precisely applicable to the analysis of strength of a salt cavity at ordinary temperatures^{46, 48, 53}. For instance, contrary to what the theories indicate, the differential yield strength (at 2% offset strain) is more than doubled by increasing the mean principal stress from $\sigma_{\min}$ ($\alpha = 0$) to $\sigma_{\max}$ ($\alpha = 1$), as found in the triaxial test results presented in Fig. 8-10.

A further study on the effect of the mean principal stress was conducted in the laboratory by using the high pressure cell for confinement, which is illustrated later in Fig. 11-7. The friction on the four loading surfaces of the specimen was successfully reduced by using the friction reducing technique. The mean principal stress due to confinement in the pressure cell is calculated from the relation between stress and plastic strain in Eq. 3-3. Assuming a negligible elastic deformation of the pressure cell compared to the salt deformation, the lateral strain on the confined direction, ϵ_2 , is zero, and the lateral stress on the non-confined direction, σ_3 , is zero. By substituting these relations into Eq. 3-3, the mean principal stress produced by the confinement of the pressure cell, σ_2 , is calculated as follows:

$$\epsilon_2 = \frac{F(\tau_0)}{3\tau_0} \left[\sigma_2 - \frac{1}{2} (\sigma_3 + \sigma_1) \right] = 0 \quad (3-3)$$

therefore,

$$\sigma_2 = \frac{1}{2} (\sigma_1 + \sigma_3) \quad (8-1)$$

Thus, the mean principal stress becomes one-half of the maximum principal stress, since the stresses are taken as $\sigma_3 = \sigma_{\min} = 0$ and $\sigma_1 = \sigma_{\max}$.

In Fig. 8-12, the experimental results demonstrate the effect of mean principal stress upon the stress-strain characteristics of salt. Maximum stress is increased more than 50%, while strain before failure is also increased. Upon close inspection of the failure process of aggregate salt, it is observed that the failure of salt at ordinary temperature under uniaxial stress is always due to loss of cohesive force among the grains in the aggregate rather than to shearing fractures, as shown in Figs. 6-4 and 11-3. Therefore, any additional confinement such as mean principal stress will hold the grains closer together, helping to exert a certain resistance to failure of the aggregate.

The magnitude of the mean principal stress around an actual cavity depends upon the cavity geometry. For instance, α , which is defined in Fig. 8-12 as an index of the magnitude of mean principal stress, is 1.0 for an ideal spherical cavity and 0.5 for an ideal cylindrical cavity. This ratio for a sharp corner of an irregularly formed cavity is less than 0.5, and around the corners of a pillar this ratio is zero. It can be concluded from the experimental observations that the maximum differential stress of salt is influenced by the magnitude of the mean principal stress, increasing as the mean principal stress increases. However, this effect becomes insignificant when either the temperature rises above 500°F. or the mean of the principal stresses exceeds a few thousand psi. The influence of the elevated temperatures and the increased confining pressure are discussed further in Chap. XI.

Strength of a Cylindrical Cavity

The strength and plastic deformation of a cavity created in salt have been studied in the laboratory in cooperation with K. Brown of the University of Texas, who conducted these tests. A cylindrical hole 2 in. I.D. and 2 in. high was cored out of the center in a cylindrical salt specimen of 6 in. O.D. and

FIG. 8-12. EFFECT OF MEAN PRINCIPAL STRESS ON STRESS-STRAIN CURVE.

FIG. 8-13. CLOSURE RATE OF MODEL SALT CAVITY.

6 in. high. As seen in Fig. 8-14, this specimen was tightly fitted into a stainless steel confinement cylinder. A triaxial stress is created around the cylindrical salt cavity by simply applying axial pressure, since the lateral pressure is produced by the reaction of the steel cylinder against the lateral expansion of the salt. This lateral pressure was measured by strain gages attached to the outside of the steel cylinder as shown in Figs. 8-2, 8-3 and 8-14.

About 16,000 psi of axial compressive stress was produced around the salt cavity by applying a vertical load of 400,000 pounds on the top of the specimen. The rate of cavity closure was determined by measuring the displacement of mercury contained in the cavity before the loading, while the vertical load was kept constant for nine days. After this period, 98% of the mercury was displaced by creep closure of the cavity, shown as Experiment Nos. 2 and 4 in Fig. 8-13. In the same diagram two similar experiments, Nos. 5 and 6, are shown with less loading.

The stress distribution around the cavity was calculated from the axial and lateral pressures applied to the salt specimen, and then the process of cavity closure was analyzed. This analysis was made with the assumption that the cavity was an ideal cylinder. The lateral pressure was calculated by introducing the measured strain outside of the steel cylinder, ϵ_t , into the following equation, which was derived from Eqs. 2-1 and 2-3:

$$\epsilon_t = \frac{-1}{E(w^2 - 1)} \left\{ -P_i \left[(1 - \nu) + \frac{b^2}{r^2} (1 + \nu) \right] + P_o \left[(1 - \nu) w + \frac{b^2}{r^2} (1 + \nu) \right] \right\} - \frac{\nu \sigma_z}{E} \quad (8-2)$$

where w = ratio of the outside diameter to the inside diameter of the steel cylinder.

FIG. 8 - 14. CROSS SECTION OF TEST CYLINDER.

By substituting the following conditions of the steel cylinder into Eq. 8-2:

ϵ_t = tangential strain obtained by SR-4 strain gage on the outside of the steel cylinder.

P'_i = radial stress exerted at the border between the salt and the steel cylinder.

P_o = pressure acting outside of the steel cylinder = 0

σ_z = axial stress in the steel cylinder = 0

r = radius at which the strain is measured = b

The confining pressure on salt, P'_i , is obtained as:

$$P'_i = \frac{E (w^2 - 1)}{2} \epsilon_t \quad (8-3)$$

By substituting the measured tangential strain, ϵ_t , of 730×10^{-6} , into Eq. 8-3, the confining pressure on salt was calculated to be 13,700 psi.

Should the salt behave elastically under this compression, stress distribution in the salt can be calculated by using Eq. 2-3. This elastic stress distribution is plotted in Fig. 8-15, in which the maximum differential stress was found to be 31,000 psi. Since such a great differential stress cannot exist in salt, the calculation for the stress distribution was tried once again by using Eq. 3-17, assuming that the salt is deforming plastically. As shown in the same diagram (Fig. 8-15) the difference between the plastic stress distribution and the elastic stress distribution is remarkable.

The maximum differential stress in the salt is now reduced from 31,000 psi to 13,000 psi, which is obviously still much higher than the differential stress that salt can withstand. This excess of the differential stress is the driving force of the creep closure of the cavity, until stress is reduced to the point at which the salt can withstand creep. It should be noted here that the salt

FIG.8-15. STRESS DISTRIBUTION AND REDUCTION OF RADIUS IN PROCESS OF CREEP CLOSURE OF CYLINDRICAL CAVITY.

cavity can withstand a great shock loading without any fractures around the cavity, as seen in the cross-section of the tested cavities shown in Figs. 8-4, 8-5 and 8-6.

From Eq. 3-17, the differential stress allowable in the salt around a cylindrical cavity is obtained as:

$$\sigma_t - \sigma_r = - \frac{2}{\sqrt{3}} \sigma_o = - 1.15 \sigma_o \quad (8-4)$$

where σ_o = the equivalent plastic limit of salt as defined in Chap. III
Therefore, the creep continues until the differential stress reaches $1.15 \sigma_o$.

From the relation between the strain reading outside of the steel cylinder, ϵ_t , and the final radius of the reduced cavity, a' , P_1' of Eq. 8-3 and σ_r at $r = b$ of Eq. 3-17 can be equated as:

$$-P_1 - \frac{2}{\sqrt{3}} \sigma_o \ln \frac{b}{a'} = - \frac{E (w^2 - 1)}{2} \epsilon_t$$

therefore,

$$a' = b e^{- \frac{\sqrt{3}}{2 \sigma_o} \left[-P_1 + \frac{E (w^2 - 1)}{2} \right]} \quad (8-5)$$

where P_1 = internal pressure of the salt cavity

b = outside diameter of the salt

E = Young's modulus of the steel cylinder

w = ratio of the outside diameter to the inside diameter of the steel cylinder

The radius of the reduced cavity, a' , can thus be calculated by Eq. 8-5 from SR-4 strain gage data, since all the other coefficients, P_1 , σ_o , b , and E , remain constant during the creep.

In Fig. 8-15, the reduced inside diameters of the cavity are shown at equilibrium with the assigned σ_o values (10,000 psi, 6,000 psi and 3,000 psi). In

obtaining the above equilibrium, the insignificant increase of the lateral confining pressure was not considered. The stress distribution with the σ_0 value of 3,000 psi at equilibrium is shown in the same diagram.

The process of actual creep closure of a salt cavity is shown in the cross-section of the tested cavity in Fig. 8-4. The flow of salt can be traced in the close-up of the reduced cavities in Figs. 8-5 and 8-6. As clearly demonstrated in these pictures, the salt flows into the cavity at the bottom of a cylindrical cavity as well as at the boundary. The crystals of salt which have crept into the cavity retain almost the original size, except at the corner between the cylindrical boundary and the bottom, where greater stress concentration is produced. The horizontal fracture lines seen in the pictures are produced during the process of removing the tested salt specimens from the steel cylinder.

The theoretical analyses presented in Eqs. 8-4 and 8-5 are not perfectly applicable to this experiment, since the cavity is not exactly cylindrical. However, the value σ_0 can be estimated to be less than 3,000 psi from a calculation using the reduced cavity radius shown in Fig. 8-6. Determination of a precise σ_0 value is discussed in the following section.

Experimental Determination of the Equivalent Plastic Limit, σ_0

The equivalent plastic limit, σ_0 , can be determined from the triaxial compression or from the triaxial extension tests. In the case of triaxial compression, the principal stresses ($\sigma_1, \sigma_2, \sigma_3$) are by definition:

$$\sigma_1 = \sigma_{\max}$$

$$\sigma_2 = \sigma_3 = \sigma_{\min}$$

By substituting the above relation into the equation of octahedral shearing stress (Eq. 3-1), the octahedral shearing stress is found to be:

$$\tau_o = \frac{\sqrt{2}}{3} (\sigma_{\max} - \sigma_{\min}) \quad (8-6)$$

In the case of the triaxial extension, the principal stresses are also by definition:

$$\sigma_1 = \sigma_{\min}$$

$$\sigma_2 = \sigma_3 = \sigma_{\max}$$

By a similar substitution, the octahedral shearing stress is found to be the same as Eq. 8-6. The relation between the octahedral shearing stress and the equivalent plastic limit is already given in Eq. 3-5a as:

$$|\tau_{o_{\max}}| = \frac{\sqrt{2}}{3} \sigma_o \quad (3-5a)$$

Therefore, from the above two equations, the following relation is derived under the assumption that salt behaves like a ductile material:

$$\sigma_o = \sigma_{\max} - \sigma_{\min} \quad (8-7)$$

Depending upon this equation, the σ_o is determined directly from the differential stress-strain diagram of the test data plotted in Figs. 8-7 to 8-9 and Figs. 11-10 to 11-13. Eq. 8-7 can be applied even for uniaxial test results obtained at elevated temperatures. The σ_o values thus determined are summarized in Table 8-1.

Considerable differences are found among the σ_o values presented in the Table. Only the σ_o in Test Group III agrees with the author's experimental results in Test Group IV. There should be no difference between the σ_o in Groups II and III, since the stress-strain diagrams of both the natural and artificial crystals are always identical in uniaxial compression tests. The σ_o obtained in Test Group I appears to be too high compared with that of Test Group IV.

TABLE 8-1

DETERMINATION OF σ_o FROM COMPRESSIVE STRENGTH TEST DATA

Test Group	Type of Salt	Fig. No.	Test Temp. (F°)	Testing Condition	σ_{min} (psi)	σ_{max} (psi)	σ_o (psi)
I	Fine Aggregate* (Hockley)	8-7	75	Compression	greater than 7,350	-	8,000
		8-8	75	Extension	greater than 7,350	-	8,000
		8-9	75	Compression Exposed to Kerosene	greater than 8,090	-	6,000
II	Natural Crystal* (Grand Saline)	11-11	75	Compression	14,700	17,600	2,900
		"	570	"	"	17,500	2,800
		"	930	"	"	17,400	2,700
III	Artificial Crystal* (Harshaw)	11-12	75	Compression	14,500	16,000	1,500
		"	"	"	29,000	30,900	1,900
		"	300	"	14,500	15,600	1,100
		"	"	"	29,000	30,200	1,200
		"	575	"	29,000	30,000	1,000
		11-13	75	Extension	14,500	16,000	1,500
		"	"	"	29,000	30,700	1,700
		"	300	"	29,000	30,500	1,500
		"	575	"	29,000	30,300	1,300
IV	Coarse Aggregate** (Grand Saline)	11-10	480	Uniaxial Compression	0	1,100	1,100
		"	770	"	"	1,000	1,000

* Experiments conducted by J. Handin

** Experiments conducted by the author

Those differences among the σ_0 values could be explained by two reasons: the different properties in different types of salt, and possible errors in the complex triaxial measurements. Therefore, the only conclusion here is that the aggregate salt of the Grand Saline salt dome and the synthetic crystal possess the following σ_0 values at various temperatures, as presented by arithmetical mean of the observed values:

<u>TEMPERATURE</u> (°F.)	<u>EQUIVALENT PLASTIC LIMIT, σ_0</u> (psi)
75	1,650
300	1,270
480 - 575	1,130
770	1,000

Since conducting the triaxial compression tests requires an elaborate installation as well as a prolonged laboratory effort, a new practical and precise test method for determination of the σ_0 value has been developed in the laboratory. This method uses an ideal hollow cylinder of salt which fits in the steel confinement cylinder as shown in Fig. 8-14.

With uniaxial stress of probably 5,000 psi, creep will reduce the initial radius, a , of the hollow salt cylinder considerably. Regardless of the magnitude of the loading, the creep always diminishes to zero as the internal radius of the hollow salt cylinder is reduced, because of the negative exponential relation found in Eq. 8-5. This reduction of the radius establishes a structural equilibrium in the cylinder.

Under equilibrium conditions σ_0 can be determined by using Eq. 8-5 with the strain data obtained from the external face of the steel cylinder and the reduced radius, a' . The latter can be determined by measuring mercury dis-

placement from the cavity. By this method, the σ_0 value can be determined under a number of triaxial stress conditions simply by shifting the structural equilibrium in the single test specimen. Temperature effect upon the σ_0 value can also be investigated by the same method. Under elevated temperature, a friction reducer at the ends of the test specimen should be made with layers of graphite.

An additional advantage of using this method for determining the σ_0 value is elimination of the necessity for determining the creep strength of salt, which ordinarily requires years of testing.

In conclusion, the following findings can be summarized regarding the strength of salt and the salt cavity:

1. The real cause of failure or creep of a salt cavity is not maximum stress, but mainly the difference between the maximum and minimum stress, acting on the cavity boundary.
2. This stress difference is proportional to the maximum shearing stress, which is the direct cause of failure or creep of the salt cavity.
3. When the maximum shearing stress reaches a critical point on the boundary of the cavity, the salt yields by plastic deformation rather than failing by brittle fracture.
4. Salt can withstand shock loading without suffering structural damage.
5. Mohr's theory of rupture cannot be applied to the strength of the salt under triaxial stress, because the theory does not consider the mean principal stress, which has a great influence upon the strength of salt.
6. The theory of octahedral shearing strength cannot be directly applied to the strength of salt if the salt is not ductile.

7. Both strength theories become applicable whenever temperature surpasses 500°F., or the mean of triaxial stresses exceeds 5,500 psi.

8. If the internal pressure in the cavity can be controlled, a salt cavity can be created and kept stable at any depth.

9. If the internal pressure of the cavity is atmospheric, the maximum depth at which no appreciable plastic flow occurs is determined by the form of the cavity and the σ_0 value. This σ_0 value is an equivalent of the plastic limit of salt derived from Eq. 3-1, assuming that salt is ductile.

10. The σ_0 value can be experimentally determined by testing the creep closure of an ideal hollow cylinder confined by a steel cylinder.

11. Although no precise experimental determination of the σ_0 has been completed, the interpretation of the previous experimental results indicates that the σ_0 value should occur between 1,000 and 2,000 psi, at a temperature up to 770°F.

12. At the elevated temperatures used in the uniaxial compression test, the σ_0 value can be determined.

13. Further investigation of the heat effect upon the σ_0 value will be presented in Chap. XI.

CHAPTER IX

CREEP

Salt undergoes continuous permanent deformation without fracture under a constant stress of less than its yielding point. This permanent deformation is called creep, also referred to as plastic flow. The nature and rate of creep must be understood as a function of the environmental conditions in order to calculate the rate of volume change in a salt cavity used for waste storage.

Grain and Crystal

Knowledge of a single crystal sheds little light on the mechanism of aggregate salt deformation. There are considerable differences between the single crystal and the aggregate in their fundamental structural properties, such as maximum strength, Young's modulus, and strain-hardening. These properties were compared previously in Fig. 7-2. The single crystal is sensitive to the orientation of stress conditions, while the aggregate is isotropic to stress.

A grain in the aggregate is constrained on all sides by neighboring grains and cannot change its shape in an arbitrary manner, while the single crystal is free to deform on its major slip plane, altering its shape in accordance with a given pressure condition. In Fig. 9-7, this difference of creep among a single crystal, an individual grain crystal in an aggregate, and the aggregate as a whole, is demonstrated by comparing their transient creeps as measured with SR-4 strain gages. The SR-4 strain indicating devices and various specimens tested are shown in Figs. 9-1 through 9-4. As shown in Fig. 9-7, the SR-4 strain gages attached to the single crystal indicates elongation under constant compressive loads. The cause of this peculiar creep of the single crystal is due to its fracture process through the longitudinal spreading of crystal planes and their consequent buckling as observed in the broken crystal in Fig. 9-3.

FIG. 9-1. HYDRAULIC TESTING MACHINE AND SR-4 STRAIN INDICATING DEVICES.

FIG. 9-2. TEST SPECIMEN USED FOR STUDYING POISSON'S RATIO, TRANSIENT CREEP, AND INTER-GRANULAR MOVEMENT.

(BROKEN) (ORIGINAL)

FIG. 9-3. TYPICAL FAILURE OF SINGLE CRYSTAL COMPRESSED PARALLEL TO CRYSTAL PLANE.

FIG. 9-4. SR-4 STRAIN GAGES; TYPE AR-1 FOR STRAIN IN AGGREGATE AND TYPE A-8 FOR STRAIN IN GRAINS.

FIG. 9-5. SELF-REGULATING PRESSURE SYSTEM.

FIG. 9-6. FURNACE ATTACHED TO CREEP TESTING STAND.

Fig. 9-7 shows that the aggregate is creeping a little more freely than the individual grain in the aggregate. Thus, the experimental evidence confirms the fact that the individual grain crystals in the aggregate do not behave like a single crystal but more like the aggregate as a whole, and further, that the creep of the aggregate does not occur from a single cause but as a result of complex processes inside the aggregate. In the process of creep in the aggregate, there may be a flow at grain boundaries in addition to slip and twinning of the individual grain crystals. Although formulation of the creep mechanism of the aggregate is yet unobtainable, enough evidence has been found to indicate that salt as an aggregate will show a definite creep characteristic of its own. As shown in Fig. 9-8, the transient creep of an aggregate under different creep pressures was measured by using SR-4 strain gages and plotted on a semi-log scale. A consistent increase of the coefficient of transient creep is observed with the increase of the load.

It has been found that many accepted rules for the creep characteristics of metals are generally applicable to salt^{2,23,40}. The creep curve of salt appears to be essentially identical to that of metals. Although conclusive proof is required, the following general rules for crystal aggregates of metals and alloys seem to be relevant for salt^{6,8,22,23}.

1. Fine-grained material is more creep resistant than coarse-grained material at lower temperatures but the elevation of temperature gradually reverses the case.
2. There is only a transition at the crystal aggregate boundary, where the atom positions represent a compromise between the crystalline arrangements in the two adjoining grains. Therefore, no amorphous cement needs to be assumed there.

FIG 9-7. COMPARISON OF TRANSIENT CREEP IN SINGLE CRYSTAL, GRAIN CRYSTAL IN AGGREGATE, AND AGGREGATE AS A WHOLE.

FIG 9-8. TRANSIENT CREEP UNDER VARIOUS LOADS

FIG. 9-9. SCHEMATIC AND CIRCUIT DIAGRAMS OF PRESSURE SELF-REGULATOR FOR CREEP TEST.

3. Boundary characteristics are usually influenced by impurities, a certain concentration of them increasing the cohesive force.

4. Conditions favoring recrystallization produce creep through continuous solution and recrystallization.

Test Procedure and Results

In the laboratory, a creep test device was developed to obtain the creep curve of salt. This uniaxial creep test involves the application of a constant load to the test specimen and the measurement of the deformation at regular intervals over a long period of time. The application of a constant load has been made possible by constructing a pressure self-regulator of which schematic and circuit diagrams are given in Fig. 9-9. Further details of the creep test device are discussed later in Chap. XI.

Some dissimilar creep curves were obtained under a constant pressure of 3,000 psi depending upon the end friction conditions of the specimens. A comparison is made in Fig. 9-10. The standardized test specimens were used and these were submerged in various solutions. The solutions in which the specimens were submerged did not affect the creep except to reduce the end friction. These test results indicate the importance of the end friction and the need for a standardized test procedure for creep studies. Thus, to keep the required test specimen size as small as possible, the previously standardized cubical specimen of 1.75 in x 1.75 in x 1.75 in with the friction reducer was also used for the creep test. All the following experiments were conducted by the standardized procedure unless otherwise mentioned. By using these standardized specimens, subsequent test results were found to be consistent.

The effect of the mean principal stress upon creep was studied under a partial confinement condition by using the high pressure cell which is illustrated later in Fig. 11-7. In Fig. 9-11, the creep curves of the standardized

FIG. 9-10. EFFECT OF END FRICTION ON CREEP OF STANDARD SPECIMENS UNDER COMPRESSION OF 3000 psi.

FIG. 9-11. EFFECT OF MEAN PRINCIPAL STRESS ON STEADY STATE CREEP.

specimen tested in this high pressure cell are given in comparison with curves of uniaxial creep. As derived in Eq. 8-1, this confinement provides a mean principal stress of one-half the maximum differential stress. Contrary to the compressive strength tests, under creep pressures up to 2,000 psi, no appreciable influence of this mean principal stress is observed among the two groups of curves shown in Fig. 9-11. The only difference is the fact that the salt under uniaxial compression fails at a pressure exceeding 2,000 psi, while the salt in the cell can withstand a creep load over 3,000 psi. Creep tests under pressures of 1,000 and 2,000 psi conducted over a period of 80 days are presented in Fig. 9-13. Fig. 11-16 is similar to Fig. 9-13 with the exception that it is plotted using arithmetical scales.

Analysis of Creep Curve

It has been found from the test results that the characteristics of all creep curves of salt have some common features. They may be expressed by the formula describing a normal curve. This normal curve, which has been the subject of considerable investigation in metallurgy^{6,22}, is the most common creep curve of aggregate materials. Generally, the curve can be divided into four stages of creep, as presented on both arithmetical and semi-log scales for comparison in Fig. 9-12.

The first stage represents the instantaneous creep observed at the time after loading is just completed. This time, δ , is the time period necessary to complete an observation, which can be less than a few seconds but must be more than one-tenth of a second. The amount of this instantaneous creep can be approximated from the standardized stress-strain diagrams of Fig. 6-8.

Unlike other construction materials, the instantaneous creep of salt is composed of three parts, which can be expressed as:

FIG. 9-12. IDEAL NORMAL CREEP CURVE COMPARED ON LINEAR AND SEMI-LOGARITHMIC SCALES.

FIG. 9-13. DETERMINATION OF COEFFICIENT OF TRANSIENT CREEP.

$$\begin{aligned}
 \epsilon_1 &= \alpha + \beta \log \delta \\
 &= \alpha_e + \alpha_p + \beta \log \delta
 \end{aligned}
 \tag{9-1}$$

where

ϵ_1 = apparent instantaneous creep

α = instantaneous creep at time δ

α_e = elastic instantaneous creep

α_p = plastic instantaneous creep

δ = time of the instantaneous creep reading

β = the coefficient of transient creep

Thus, the instantaneous creep is not a constant value but a function of the time at which this strain is observed.

The second stage represents the transient creep, where the rate of strain decreases with time. Although little is understood of the mechanism of this creep, the creep dominating in this stage can be best approximated from the linear relation between the transient strain, ϵ_2 , and the logarithm of time, t , as it is expressed in the following equation:

$$\epsilon_2 = \beta \log t - \beta \log \delta
 \tag{9-2}$$

where

β = coefficient of transient creep

δ = time at instantaneous creep reading

t = time which is not less than δ

The third stage represents the steady state creep, where the rate of strain remains almost constant at its minimum value. Variations in the rate of strain throughout this stage are so small that they are not measurable in

a relatively short creep test. This minimum rate is maintained for a longer period of time if the creep stress is smaller. The creep of this stage can usually be closely approximated by a linear relation of the strain and time, which is:

$$\epsilon_3 = \gamma t \quad (9-3)$$

where γ = coefficient of steady state creep

The coefficient of steady state creep can provide practical information regarding creep velocity even though this velocity does not necessarily indicate the creep strength of salt.

The fourth stage represents creep failure in which the rate of deformation is rapidly accelerated with time and finally leads to failure.

More detailed information concerning the mechanism of creep such as its strain-hardening, temperature softening, recovery and recrystallization and their interrelationships, is available in the works of Barret⁸, Stanford⁶, Kanter³ and Bailey⁴. Although no mathematical expression can describe the exact process of creep, many formulae can be adapted to establish empirical equations for creep. One of the most useful formulae can be obtained from summation of the first three stages of the creep as:

$$\begin{aligned} \epsilon_o &= \epsilon_1 + \epsilon_2 + \epsilon_3 \\ &= \alpha + \beta \log t + \gamma t \end{aligned} \quad (9-4)$$

where ϵ_o = total creep at time t which is not less than δ

The advantage of this empirical equation is that the three coefficients can be determined by simple graphical methods as explained in Fig. 9-12.

Under various testing conditions, the coefficients of the creep in salt are summarized in the following table.

As shown in Table 9-1, the coefficients of the creep are extremely sensitive to stress, temperature, and other testing conditions. For use of the

coefficients in this table, the units of the coefficients should be understood. The coefficient β is unitless; the unit of the coefficient γ is the reciprocal of time. Although the coefficient α is unitless, it varies with the particular unit. To convert the coefficient α from days to hours, the following relation can be applied from Eq. 9-1:

$$\begin{aligned}\alpha_h &= \alpha_d - \beta \log \frac{\delta_h}{\delta_d} \\ &= \alpha_d - \beta \log 24\end{aligned}\tag{9-5}$$

where

α_h = the α coefficient in terms of hours

α_d = the α coefficient in terms of days

β = coefficient of transient creep

δ_h/δ_d = ratio of hours to days

It is now possible to calculate the amount of creep which would occur for a given period of time provided that the three coefficients of creep for a given condition are known.

Practical Considerations

At the end of the third stage of creep salt may fail, as shown in Fig. 9-10. The length of time required to reach this failure is increased as the load is reduced. Therefore, in creep, the strength of salt should be expressed in terms of time. It is possible to establish an empirical relation between the maximum creep stress and the period of tolerance by conducting a number of creep tests under various loads for a long time. A bold extrapolation of the laboratory test results is required if the creep strength for 500 years has to be estimated. It is probable that the salt will not fail but will creep continuously under a load less than a certain value. This can be called the fundamental strength.

TABLE 9-1
SUMMARY OF THE COEFFICIENTS OF CREEP

Testing Condition	Test Temp. (F°)	Creep Stress (psi)	Coeff. of Instant Creep ($\delta=1$ day)	Coeff. of Transient Creep	Coeff. of Steady State Creep (1/day)
Standardized Specimen (with Friction Reducer)	80	1,000	0.01-0.02	0.0008-	0.0005-
	"	2,000	0.03-0.04	0.0010 0.006- 0.010	0.0008 0.002- 0.003
Standardized Specimen under Different Temp.	80	700	0.015	0.0007	0.00004
	770	"	0.135	0.062	0.003
Aggregate of 3" x 3" x 3" No Friction Reducer	80	1,000	0.010	0.0004	-
	"	2,000	0.015	0.0011	0.001
	"	3,000	0.025	0.0020	0.003

Determination of the fundamental strength of salt was not attempted in the laboratory for two reasons: the inordinate amount of time and labor required and the relative insignificance of the fundamental strength of salt for evaluating the stability of a salt cavity.

When the creep is extended over 20 days the creep curve of the standardized specimen in Fig. 11-16 deviates away from the normal creep equation. An increase in the cross-sectional area and end friction can cause the decrease in the coefficient of steady state creep. As seen in Fig. 11-16, the total strain which consists of the elastic and plastic strains under a given load approaches an asymptote. However, the elastic strain decreases in magnitude

while the plastic strain increases as the loading period is prolonged. Therefore, the determination of the coefficient of steady state creep is difficult, particularly when using the long-term creep test.

It is to be noted that the creep equation developed as a result of the uniaxial creep measurement procedures cannot be applied directly in determining the creep velocity of a cavity. The experimental results obtained here are of value for understanding the basic mechanism of creep. For the study of creep rate in a salt cavity, creep under triaxial compression has to be investigated. One method of studying this is to use a hollow salt cylinder confined in the steel confinement cylinder as shown in Fig. 8-14. The creep rate of the cavity can be studied by determining the following relation:

$$\frac{d \tau_o}{dt} = F (\tau_o , T) \quad (9-6)$$

where τ_o = octahedral shearing stress

t = time

T = temperature

From the above relation, which can be experimentally determined, the time required to reach structural equilibrium of the cavity can be calculated.

CHAPTER X

ACTUAL CREEP MEASUREMENT IN THE GRAND SALINE SALT MINE

The purpose of measuring creep in the Grand Saline salt mine was to confirm previously developed theoretical conclusions on the structural equilibrium of a salt cavity, by long-term observation in the mine. Furthermore, certain characteristics of the mine opening from which actual design of a storage cavity could be developed were studied. The Grand Saline salt mine was selected for such measurement because the physical property of the salt and the geology of the salt dome had been previously investigated³. Also, the mine is relatively convenient to The University of Texas. Figs. 10-1 through 10-8 show various characteristics of the mine.

Test Procedure

The measurements were started with two different types of observation so that the test results could be checked against each other. A device equipped with a dial gage (Fig. 10-1), was designed to measure the rate of closure, while SR-4 strain gages (Figs. 10-3 and 10-4) were prepared to measure the rate of surface creep on the boundary of the opening.

Two locations in the mine, A and B, which were opened at different times, (A in 1947 and B in 1952) were chosen for measurement of the closure rate. By selecting these two stations, the effect of age upon the closure rate was studied.

At the same time, seven points in the mine were selected for measurement of the surface flow rate. In Fig. 10-7, a plan of the mine opening is given, in which two stations A and B, and seven points from Nos. 1 to 7 are shown. This mine opening is situated about 700 ft. below the surface, a little off of the central axis of the dome, as shown in the profile of the dome in Fig. 10-8.

FIG.10-1.
STATION B;
MEASURING
CREEP CLO-
SURE RATE OF
MINE OPENING
BY USING DIAL
GAGE DEVICE.

FIG.10-2.
TYPICAL SU-
PERFICIAL
SCALING WITH
SHARP ANGLE
OF FRACTURE
DEVELOPED AT
VERTICAL COR-
NER OF PILLAR.

FIG.10-3 POINT NO.1; SR-4 STRAIN GAGE
MEASUREMENT ON WALL OF MINE.

FIG 10-4. POINT NO.4 ; MEASURING STRAIN
ON FLOOR OF MINE OPENING
WITH SR-4 STRAIN GAGES.

FIG.10-5 BROKEN TIMBER FRAMES
RESULTING FROM CREEP.

FIG.10-6 TYPICAL FRACTURE OF SUPPORT-
ING TIMBERS.

FIG.10-7. PLAN OF THE GRAND SALINE SALT MINE OPENING.

FIG.10-8. PROFILE OF GRAND SALINE SALT DOME.

These seven points were grouped into three categories according to the following objective functions:

TABLE 10 - 1

OBJECTIVES OF SEVEN CREEP MEASURING POINTS BY USING SR-4 STRAIN GAGES

Partition	Sr-4 Gage Point	Position	Functions
Group I	No. 1	wall	Measuring creep on the wall of the pillar which has the smallest cross-section in the mine and consequently may be suffering the greatest creep.
	No. 2	wall	
	No. 3	wall	
Group II	No. 4	floor	Measuring creep at relatively old opening, where station A is located.
	No. 5	wall	
Group III	No. 6	wall	Measuring creep at the relatively new opening.
	No. 7	floor	

The device used for measuring the closure rates at Stations A and B was a regular 1/1000" graduated dial gage fixed on the top of a long aluminum tube with 1 3/4 in. O.D. As shown in Fig. 10-9, each station was made of two plug points which were forced into the floor and ceiling of the tunnel along the same perpendicular line. The distance between the two plug points was measured by using the dial gage device with reference to a standard scale. The standard scale was made of aluminum tubing of 2 1/2 in. O.D. An actual measurement at station B is shown in Fig. 10-1.

This portable device for the measurements was preferred over a fixed measuring station in order not to disturb the mine operation for the duration of creep observation. All the joints of the tubing were secured by the use of

FIG.10-9. SCHEMATIC DIAGRAM OF STATIONS AND MEASURING DEVICES FOR DETERMINATION OF CLOSURE RATE OF MINE OPENINGS.

welding, aluminum cement and clamps. The thermal expansion of the tubing is negligible due to the constant temperature of the mine, between 80°F. and 80.5°F. all year round.

The SR-4 strain gages of the 45° rosette (AR-1) were mounted on the boundary surface at the seven points as shown in Figs. 10-3 and 10-4. Two reference gages were prepared by attaching them to a small block of salt free from any stress or strain. Since heavy corrosion was expected, the metal parts of the devices were sealed with wax and plastic cement.

Experimental Results

The field data for the measurement of closure rate of the mine opening at stations A and B are presented in Table 10-2 and Fig. 10-10.

The measurement of creep on the boundary surface by using SR-4 strain gages resulted in complete failure because of serious corrosion found on the fine filament of the SR-4 strain gages. This corrosion was probably caused by the moisture-sensitive nature of the cement used to attach the gages to the salt. A new type of cement must be found before another attempt is made to attach SR-4 strain gages on the surface of salt.

Significance of Mine Observation

The test result of the measurement of mine creep for over a year has revealed some significant facts which support the theoretical analysis. The findings are analyzed and presented in the following summary.

1. Effect of Age

The creep rates varied between the two stations observed, as shown in Fig. 10-10. This variation is caused by the difference in cavity ages. Additional data⁴ to support this finding was obtained from Mr. M. Barker, a mine engineer

TABLE 10-2

SUMMARY OF MEASUREMENT AND COMPUTATION OF CLOSURE RATE OF MINE OPENING

Station A - Opening created in 1947							
Date of Observa- tion	(1) Station Reading (in.)	(2) Reference Scale (in.)	(3) Relative Distance (2)-(1) (in.)	(4) Total Deform. (in.)	(5) Unit Deform. (4)/22x12 (10 ⁻³)	(6) Duration (days)	(7) Cree Rate ⁺ (10 ⁻³ yr.)
(1952 measured by Mr. Barker)				-0.001	-0.004	daily	-1.40
4-1-57	0.0362	0.1853	0.1491	0	0	0	-
7-18-57	0.2221	0.3903	0.0692	-0.0799*	-0.290*	108	-0.02
9-10-57	0.2749	0.3539	0.0790	-0.0701	-0.253	162	-0.02
4-17-58	0.2243	0.2882	0.0639	-0.0852	-0.304	383	-0.02
8-4-58	0.2387	0.3068	0.0681	-0.0810	-0.293	493	-0.02
Station B - Opening created in 1952							
4-2-57	0.3345	0.0259	-0.3086	0	0	0	-
7-18-57	0.8210	0.4140	-0.4070	-0.0984*	-0.356*	108	-1.20
9-10-57	0.7849	0.3561	-0.4288	-0.1202	-0.436	163	-0.75
4-17-58	0.6130	0.0463	-0.5673	-0.2587	-0.937	383	-0.60
8-4-58	0.9001	0.2910	-0.6091	-0.3005	-1.080	493	-0.45

* Minus sign indicates reduction of the height of the tunnels

+ Obtained by tangents on the plotted curves in Fig. 10-10

FIG.10-10. CREEP CLOSURE RATE OF TUNNEL OPENINGS IN GRAND SALINE SALT DOME AT DEPTH OF 700 FEET.

of the Morton Salt Company, who measured the creep at station A by using a dial gage device in 1952. He recorded the total creep between the two plug points at about one-thousandth of an inch daily, at that time. As calculated in Table 10-2, the two stations have shown very similar creep rates at the same ages. Thus it is concluded that the closure rate of the mine opening decreases with its age.

2. Deceleration of Creep Rate

In Fig. 10-10, deceleration of the creep rate is observed. The creep rate can be expressed by an inverse exponential function of time as:

$$\text{Creep Rate} = \frac{d\epsilon}{dt} = A \times e^{-(t+B)} \quad (10-1)$$

where A and B are certain constants, and, t is the time in years

From the experimental data, the constants were determined and the following experimental formula was derived:

$$\frac{d\epsilon}{dt} = -0.82 e^{-(t+3.0)} \quad (10-2)$$

According to this equation, the creep will be extinguished soon. Further observation can improve the accuracy of this empirical formula.

3. Development of Plastic Zone

The deceleration of creep is positive proof of the theory of plasticity as applied to a salt cavity. The opening will reach structural equilibrium by developing plastic zones around the corners of the opening. Thus, the present openings of the mine are stable and will maintain their form without appreciable change.

4. Inadequacy of Elastic Theory for Direct Application

Inspection of the mine tunnel has proven the inadequacy of applying the elastic theory for the determination of the strength of the mine opening. In

the mine opening at 700 Ft., shown in Fig. 6-1, no failure at the horizontal corners of the rectangular tunnels has ever been reported. Assuming the stress concentration factor at the horizontal corners to be six (from Fig. 2-8), the stress there would be over 4,000 psi. If this stress is applied to the Mohr's envelope of maximum compressive strength in Fig. 8-10, the corners should have failed. This is another piece of evidence to support the theory of the development of the plastic zone, as well as the significance of the mean principal stress existing on the boundary.

5. Total Amount of Creep

The total amount of strain in the opening can be calculated from integration of Eq. 10-2 as:

$$\epsilon = \int_{t_1}^{t_2} d\epsilon = -0.82 \int_{t_1}^{t_2} e^{-(3.0 + t)} dt$$

The ultimate total strain can be obtained by taking $t_1 = 0$ and $t_2 = \infty$ as:

$$\epsilon = \int_0^{\infty} d\epsilon = -0.82 e^{-3.0} = -4.1 \%$$

Timber beams broken by the creep are often found in the older openings of the mine, as shown in Figs. 10-5 and 10-6. Measurement of the broken beams indicates that the openings have shrunk one to two percent over the past 15 to 20 years. This agrees with the above calculation if the beams were erected there several years after the mine was opened. Lateral shrinkage at the openings of similar magnitude was also observed. This indicates the existence of a hydrostatic stress condition in the salt formation at this depth.

6. Superficial Scaling

Superficial scalings with sharp fracture angle to the boundary surface are found everywhere in the mine. They appear most frequently around the sharp vertical corners of the pillars, as shown in Fig. 10-2.

This scaling is identical to the scaling phenomenon observed during compression of the standard specimen without application of the friction reducer. This scaling will stop as soon as the plastic zone develops around its vicinity. More scaling occurs around the sharp corners of the pillars because of less confinement and greater creep. Still, the scaling will be stopped by the eventual development of the plastic zone around the area. As a matter of fact, new scale development in the older mine sections was not found. If the present openings are enlarged, the scaling will again occur until a new structural equilibrium is reached.

All the evidence observed in the mine points favorably to the stability of salt cavity openings and their use for waste storage.

CHAPTER XI

VARIOUS FACTORS AFFECTING PROPERTIES OF SALT

Salt will be subjected to various factors which may affect its structural properties during the storage of reactor fuel wastes in a salt cavity. Investigations reported herein were conducted to find out the nature of such factors and their influence upon the stability of a salt cavity. The effects of heat generation resulting from radioactive decay, physico-chemical interaction, and gamma irradiation upon the structural properties of salt were studied. Some of the tested specimens and testing devices are depicted in Figs. 11-1 through 11-6.

Development of Test Device

The individual factors were first studied; then the comprehensive effects of those factors were correlated with the use of a high pressure test device. A cross-sectional view of the high pressure cell which was used for the comprehensive tests is shown in Fig. 11-7. This high pressure cell was developed so that a tri-axial stress condition could be created. If a standardized specimen is fitted tightly into the cell, the mean principal stress of $1/2 (\sigma_{\max} + \sigma_{\min})$ can be developed. Also, if the specimen is slightly smaller than the standardized size, the compression and extension effects can be studied.

In addition to this triaxial compression capability, the high pressure cell is also useful in studying the effect of solutions and heat on the structural property of salt. A test specimen can be exposed to a solution in the cell by replacing the confining oil with any solution. For studying the effect of elevated temperatures, heat is supplied by a 750 watt heating element attached to the cell. The temperature of the salt in the cell is measured by a thermocouple installed in the bottom plate. The heating element and temperature measuring devices are both shown in Figs. 11-8 and 11-9.

FIG. II-1 DARKENING EFFECT OF GAMMA IRRADIATION UPON NATURAL AND SYNTHETIC SALT CRYSTAL.

FIG. II-2 REMOVAL OF DARK COLOR FROM IRRADIATED SALT BY HEAT OR RECRYSTALLIZATION.

FIG. II-3 SPECIMEN TESTED BY UNIAXIAL COMPRESSION SHOWING INCREASE OF DUCTILITY WITH INCREASE OF TEMPERATURE.

FIG. II-4 CRYSTAL FROM GRAND SALINE SALT MINE TESTED WITH CONFINING PRESSURE OF 14,500 psi AT 300°F AND 570°F (LEFT, ORIGINAL; MIDDLE, 300°F, RIGHT, 570°F).

FIG II-5 CREEP TEST AND HEATING APPARATUS.

FIG II-6 CREEP TEST OF STANDARD SPECIMEN SUBMERGED IN REACTOR FUEL WASTE SUBSTITUTE. (ALUMINUM NITRATE SOLUTION)

FIG. II-7. HIGH PRESSURE CELL ASSEMBLY.

FIG. II-8. ARRANGEMENT FOR PERMEABILITY MEASUREMENT AND COMPREHENSIVE TEST OF SALT.

FIG. II-9. ASSEMBLY OF COMPREHENSIVE TEST DEVICE.

The dynamic high pressure O-rings placed around the loading piston are capable of containing a liquid confining pressure up to 10,000 psi. Details of the sealing system, which consisted of brass, teflon, and rubber O-rings, are shown in Fig. 11-7. In addition to the ceramic insulation plate in the piston, a cooling system was provided to protect these O-rings, thereby keeping the temperature below 300°F.

This high pressure cell was also designed to investigate the permeability of salt under various temperatures and loading conditions. As illustrated in Fig. 11-8, liquid pressures can be built up on one side of a specimen. The amount of fluid permeating salt can be measured by the burette connected to the other side of the specimen.

As sketched in Fig. 11-9, the loading pressure is given by the Black Hawk 100,000 pound capacity oil pump through three rams which are fixed on each of the three test stands. The testing stands and the loading system, also used for creep tests, were built to maintain constant creep loads automatically. Precise loading pressures were constantly maintained during the test period by use of the pressure self-regulator, which drives the oil pump whenever the needle of the oil pressure gage drops below a specified reading, as previously illustrated in Fig. 9-9. Also, as shown in Fig. 11-8, the high pressure cell can be installed in a furnace. The complete assembly is presented in Figs. 11-5 and 11-9, with a picture also shown in Fig. 9-6.

A special steel, Allegheny Alloy A-286, was chosen for the cell walls because of its excellent creep-resistant quality at elevated temperatures. The steel is an austenitic, iron-nickel-chromium-titanium alloy designed for high creep strength and corrosion-resistant service up to 1,300°F. Its minimum secondary creep rates are 81,000psi at 1,000°F. and 64,000psi at 1,100°F. for a creep rate of 0.1%/1000 hours. See Table 11-1 for test device capacity.

TABLE 11-1

SUMMARY OF COMPREHENSIVE TEST DEVICE CAPACITY

Maximum Creep Load	100,000 pounds
Maximum confining pressure	10,000 psi
Present testing temperature with 750 watts	770°F.
Probable maximum temperature with 1500 watts	1,100°F.
Maximum differential stress at the maximum confining pressure	13,000 psi

Effect of Heat

Heat is the most critical factor affecting the structural property of salt. As calculated by Schechter and Gloyna⁵⁷, the temperature of the waste in the cavity could under certain conditions rise above the melting point of salt (1,475°F.) depending upon the heat emission rate of the waste, size and form of the cavity, and duration of storage.

The major problems which would be produced by the elevation of temperature in a salt cavity are:

1. The creation of thermal stress around the cavity.
2. Development of a plastic zone around the cavity.
3. The reduction of cavity space due to the accelerated creep and thermal expansion of salt.
4. Internal pressure of the cavity required to prevent boiling of the waste.
5. The reduction of salt strength.
6. An increase of creep velocity.
7. The acceleration of corrosive action by the waste.

The first four problems have been previously analyzed in Part One. The next two problems will be investigated in this section. The last problem is beyond the scope of this study.

To investigate the problem of temperature effect upon the strength of salt, uniaxial compression tests were conducted with various temperatures up to 770°F. The effect of the elevated temperatures on the stress-strain relation is shown in Fig. 11-10. At elevated temperatures, both the maximum strength and the yielding strength are reduced, indicating a sudden yielding.

It should be noted that the compression tests were performed without use of the friction reducer; therefore, the maximum strength of the salt at 80°F. should be around 2,300 instead of over 5,000 psi. For the same reason, the stress-strain curve at 270°F. is not an indication of the true strength. However, the stress-strain curves at 480°F. and 770°F. are close approximations of their real values because the end friction effect is nullified by the increased ductility of salt at the elevated temperature. The specimens tested are shown in Fig. 11-3.

A temperature above 500°F. indicates a yielding strength around 1,000 to 1,500 psi. According to the test results reported by Theille⁶⁴, crystal salt indicates a nearly constant yielding strength of 1,000 psi at temperatures ranging from 1,020°F. to 1,120°F. Therefore, salt seems to follow the general rule of aggregates in that the strength is reduced with an increase in temperature. However, beyond a certain temperature the strength approaches a nearly constant value. This rule remains valid when the salt is subject to triaxial stress in addition to elevated temperature.

Figs. 11-11 to 11-13 are test results reported by Handin²⁵. Synthetic and natural crystals, from Harshaw Chemical Co. and the Grand Saline salt

FIG.II-10. EFFECT OF ELEVATED TEMPERATURE ON STRESS-STRAIN RELATION OF STANDARD SPECIMEN WITHOUT FRICTION REDUCER.

FIG. II-11. TRIAXIAL COMPRESSION OF NATURAL SALT CRYSTAL AT VARIOUS TEMPERATURES

FIG. II-12. TRIAXIAL COMPRESSION OF ARTIFICIAL SALT CRYSTAL AT VARIOUS TEMPERATURES

FIG. II-13. TRIAXIAL EXTENSION OF ARTIFICIAL SALT CRYSTAL AT VARIOUS TEMPERATURES.

mine respectively, were tested under triaxial stress and elevated temperature. Some of the tested specimens are shown in Fig. 11-4. In these figures, a sudden yielding appears in the differential stress between 1,000 psi and 2,500 psi. (The differential stresses, which are equal to σ_o , obtained from these figures are summarized previously in Table 8-1.)

A variation in the confining stress produced little difference in the stress-strain curves. This similarity probably exists because the salt is fully ductile under such triaxial compression. For the same reason, there is no difference in the stress-strain relations between the compression and the extension. Consequently, the aggregate behaves in a manner identical to the crystals under such a testing condition.

As previously discussed in Chap. VIII, considerable differences between the differential stresses found in Fig. 11-11 and those in Figs. 11-12 and 11-13 are probably due to some errors involved in the complex triaxial determinations. The test results presented in Figs. 11-12 and 11-13 agree with the experimental results of the granular salt from the Grand Saline salt mine. Compared with other experimental evidence, the test results given in Figs. 11-12 and 11-13 seem acceptable.

Thus, it is concluded that the equivalent plastic limit, σ_o , which is equal to the maximum differential strength under the triaxial compressions, is gradually reduced from about 1,700 psi to 1,000 psi with no corresponding increases in temperature up to about 500°F. The stress remains nearly constant for temperatures varying from 500°F. to near the melting point regardless of the confining stress.

Young's modulus of salt seems to be influenced to a small extent by the elevated temperatures as found in Figs. 11-11 to 11-13. The noticeable increase

of Young's modulus found in Fig. 11-10 is due to reduction of the end friction rather than the elevated temperature itself. Since the elevated temperature tends to increase Young's modulus slightly, precise determination of this value as a function of temperature is needed for improving the accuracy of the application of the theoretical analysis developed in Part One.

The effect of elevated temperature is most prominent upon the rate of creep. The rate of creep was studied at 80°F. and 770°F. under a constant pressure of 700 psi. The test results are compared in Fig. 11-14 by plotting them on both linear and semi-log scales. By increasing the temperature from 80°F. to 770°F., the rate of the transient creep is increased 90 times and the rate of the steady state creep 75 times.

In actual practice, elevation of cavity temperature will result in a quick establishment of structural equilibrium in addition to a certain reduction of the σ_0 (not more than 50%) and the cavity volume.

Effect of Solutions

When liquid waste is introduced into a salt cavity, the salt around the cavity will be directly exposed to the liquid. Fresh water would also have direct contact with the salt if the cavity is created by using the washing-out method. To study any effect of solution exposure upon the structural property of salt, strength and creep experiments were conducted with the standardized specimen submerged in various solutions: light mineral oil, saturated salt water, and a substitute for one of the typical reactor fuel wastes.

The chemical composition of the waste substitute was prepared according to the Process which produces waste of the following major constituents:¹¹ 0.167 M. nitric acid, 1.67 M. aluminum, 0.02 M. sulfuric acid, 0.001 lb. silicon per gallon. Addition of NaCl to this solution does not result in an apparent chemical reaction except for the solution developing a yellowish-green color.

FIG. 11-14. EFFECT OF ELEVATED TEMPERATURE ON CREEP UNDER CONSTANT PRESSURE OF 770 psi.

FIG. 11-15. EFFECT OF SUBMERSION IN VARIOUS SOLUTIONS FOR ONE MONTH ON STRESS-STRAIN RELATION.

FIG. 11-16. EFFECT OF SUBMERSION IN SOLUTIONS ON CREEP.

The physical properties of the reactor fuel waste substitute and the saturated salt solution were determined in the laboratory as:

Boiling Points

the substitute	228°F.
the substitute saturated with NaCl	243°F.

Solubility of NaCl in the substitute

at temperature of 73°F.	13.5 gm/100 ml.
at temperature of 200°F.	16.6 gm/100 ml.
ratio of solubility to that of distilled water	37%

For testing the effect of the solution on the strength of salt, the standardized specimens were kept submerged in each of the three solutions for over one month. Then their stress-strain diagrams were determined by testing them while they were still submerged in the solutions. The test results plotted in Fig. 11-15 show no appreciable effect from any one of the solutions.

Creep tests with standardized specimens submerged in the two solutions were conducted for over 90 days as shown in Fig. 11-6. The test results are plotted in Fig. 11-16. Their cumulative creeps differ considerably, although their rates of transient and steady state creeps do not. A few points of discontinuity are found in the creep curve of the specimen submerged in the saturated salt water. No adequate explanation can be given to explain this unusual creep curve. However, the brine had to be supplemented occasionally, as it evaporated while no evaporation occurred in the reactor fuel waste substitute. Therefore, the sudden increase of the creep could be related to a process of solution and recrystallization of the salt. Further investigation is required before any further comment can be made of the effect of brine upon the creep of salt.

Any serious effect of solutions upon a salt cavity is unlikely because a cavity is different from the uniaxially compressed specimen soaked in the solutions with respect to the following points:

1. On the boundary of the cavity, the salt is subject to a two-dimensional compressive stress which is much greater than the liquid pressure. Therefore, penetration of the solutions through the salt boundary is not probable. This means that the solution effect, if any, is limited only to the boundary surface.
2. The exposed area of the salt surface compared to its mass is much smaller for the salt cavity than the specimen submerged in the solution.
3. No evaporation of the solution can take place in an actual cavity because the cavity has to be sealed. The solution would remain at chemical equilibrium in the cavity, which is the case of an artificial pond found in the Grand Saline salt mine.

For studying the solution effect upon a salt cavity, the ideal cylindrical cavity confined with the steel cylinder (the one discussed in Chap. VIII) should be employed. Since the above mentioned differences do not exist, this cylindrical cavity is the best approximation to an actual cavity. By using this cylindrical cavity, the effect of disposing fresh water into the cavity upon the structural property of the cavity should be investigated.

Effect of Gamma Irradiation

The radioactivity of reactor fuel waste is estimated to be at most 500 curies/gal, depending upon the processing method, cooling time, and dilution factor. The effect of gamma irradiation upon the structural property of salt was investigated by testing the salt with three different levels of radiation: 10^6 rep, 10^7 rep, and 2×10^7 rep. The creep and stress-strain characteristics of these irradiated specimens were studied in relation to their radiation doses.

In Fig. 11-1, the crystal specimens of the different radiation doses are shown. The color of the salt becomes increasingly dark with an increase in dose. Natural crystals become darker than the synthetic ones with the same dose. However, the color can be removed by either heat or recrystallization, as demonstrated in Fig. 11-2. These facts indicate that the dark color is produced by disturbance in the atom positions in the crystal lattices of salt, and the natural crystal being more disturbed compared to the synthetic or artificial crystal.

The stress-strain diagrams of the irradiated aggregates are compared in Fig. 11-17. No appreciable effect from the gamma irradiation was found upon the fundamental properties of salt, such as strength, Young's modulus, and strain hardening.

As shown in Figs. 11-18, the effect of the irradiation upon the creep rate of salt was studied by comparing the coefficient of the transient creep of the irradiated salt under different creep stresses of 1,000 psi, 2,000 psi, and 3,000 psi. Among the aggregates, it seems that there is a slight decrease in creep rate with an increase of radiation dosage.

The dissolving rate of the irradiated salt was studied by continuous measurement of salt concentration in the distilled water in which irradiated salt grains were immersed. As shown in Fig. 11-19, irradiation up to 2×10^7 rep has no appreciable effect upon the dissolving rate of the salt.

No change on the solubility of salt is expected since the radiation effect disappears when the salt is dissolved into water. It is unlikely that thermal conductivity is affected by irradiation. In general, gamma irradiation of the salt in a storage cavity seems to have little significance on its structural stability.

FIG. II-17 EFFECT OF GAMMA IRRADIATION ON STRESS-STRAIN RELATION.

FIG. II-18. EFFECT OF GAMMA IRRADIATION ON TRANSIENT CREEP OF AGGREGATE SALT.

FIG II-19 EFFECT OF GAMMA IRRADIATION ON DISSOLVING RATE OF GRANULAR SALT.

PART THREE: EVALUATION

CHAPTER XII

DESIGN PRINCIPLE

The principle of designing a salt cavity can be derived from the previously described theoretical and experimental investigations, and the structural equilibrium of a salt cavity can be evaluated from these analyses. The waste solution does not affect the structural equilibrium of the salt cavity if a chemical equilibrium between the salt and waste has been established. It is this structural and chemical equilibrium upon which the principle of designing a salt cavity for disposal of reactor fuel wastes is based.

Structural Equilibrium

The structural equilibrium of a salt cavity can be determined by three principal factors: the factor of structural loading, cavity volume reduction, and the temperature rise in the cavity. Any one of these three factors can be determined from Figs. 5-2 and 5-3, if the other two factors are known.

In using those diagrams for designing purposes, it is safer to use the curve of the instantaneous closure which is an extreme case of maximum cavity closure. The accuracy of this curve is provided in the diagrams as the range between the two extreme cases: instantaneous closure and closure under a steady state of heat flow. It is noted that the accuracy of the curves is very high until the cavity closure exceeds 40% and the cavity temperature rise exceeds 400°F.

The most practical application of this method will be to find the maximum allowable temperature rise of a cavity at a given depth when the reduction of cavity volume is limited to a particular value. For example, in a spherical cavity at a depth of 1,500 feet, the maximum allowable cavity temperature rise is

found to be around 500°F., with an expected cavity volume reduction of 20%; similarly, for a cylindrical cavity at the same depth, the maximum allowable cavity temperature rise is around 450°F., with the same amount of volume reduction. The factor of structural loading is about 1 for both types of cavity, assuming the equivalent plastic limit of salt to be 1,500 psi.

Similarly, at a given cavity depth, the ultimate cavity volume reduction can be obtained if the cavity temperature rise is known. Or, if the maximum cavity temperature rise is known, the maximum allowable depth of the cavity may be determined for any given ultimate cavity volume reduction.

The Factor of Structural Loading

The factor of structural loading is directly related to the allowable cavity temperature rise and the cavity volume reduction, given a structural equilibrium condition. This factor, X , is composed of three elements as previously defined in Eq. 5-10:

$$X = \frac{P_o - P_i}{\sigma_o} \quad (5-10)$$

where P_o = overburden pressure of a cavity = external pressure of a cavity

P_i = internal pressure of a cavity

σ_o = equivalent plastic limit

It should be noted that the degree of structural loading on a cavity is proportional to the external and internal pressure difference, and inversely proportional to the equivalent plastic limit of salt.

The structural load can be decreased by reducing the value, $P_o - P_i$. Since the P_o is the product of the cavity depth times the density of the overburden layer, the shallower the cavity depth the less the external load. Another method of reducing the load is to decrease the differential pressure,

$P_o - P_i$, by raising P_i until it approaches P_o . The value of σ_o has a rather minor role because it is simply a function of temperature ranging from 1,000 psi to 1,700 psi for temperatures nearly up to the melting point of salt, 1,475°F.

The cavity should be designed for the maximum loading condition at any given depth by assuming P_i as zero. In practice, however, the P_i value should be maintained above the vapor pressure of the waste in order to prevent boiling of the waste. The minimum internal pressure required to prevent boiling of the waste is a little less than the vapor pressure of water, which is as follows³⁰:

<u>TEMPERATURE</u> (°F)	<u>VAPOR PRESSURE OF WATER</u> (psi)
212	15
300	58
400	250
500	681
600	1,553
700	3,100

Increase of the internal pressure over the external pressure must not be allowed, because it will cause a cavity fracture³⁴.

Volume Reduction

A cavity should be designed with excess space so that the stored waste will not be forced out when the ultimate reduction of cavity volume is reached.

In the analysis presented in Figs. 5-2 and 5-3, the volume reduction caused by the structural expansion of salt is not considered because the amount of reduction is insignificant when compared with the reduction caused by thermal expansion. However, structural expansion becomes an important factor when a salt cavity has to be created at a depth deeper than around 2,000 ft. In this case, the reduction of the cavity due to structural expansion must be calculated.

Cavity Temperature

In general, when the ambient temperature of the salt formation is around 100°F., a cavity temperature rise of 400°F. is permissible. For example, a cavity of any form starts to close without any structural load with a cavity temperature rise exceeding 400°F. In designing a cavity, both the cavity temperature rise and the cavity temperature should remain within the above mentioned limits. If the ambient temperature is already around 200°F., the increase of the vapor pressure becomes most critical. On the other hand, if the ambient temperature is below zero, the cavity volume reduction becomes critical first.

The cavity temperature, which is the sum of the cavity temperature rise and the ambient temperature, is limited by an increase in the vapor pressure of the waste. Therefore, the cavity temperature should be maintained below the point where the vapor pressure of the waste is equal to the overburden pressure. Because of this vapor pressure increase with elevation of the cavity temperature, the temperature of cavities at 500 ft. is limited to 400°F., at 1,000 ft. to 500°F., and at 2,000 ft. to 600°F.

Other Considerations

The form of a cavity has material effect upon the volume reduction and heat emission of the cavity. The rate of heat emission from a cavity is higher and thus the cavity temperature is kept lower if the boundary surface per unit cavity volume is greater. Consequently, a cylindrical cavity will have less temperature rise than a spherical cavity of the same volume.

However, for the cavity with a greater boundary surface, the volume reduction is greater with the same temperature rise. Therefore, the volume reduction in a cylindrical cavity will be greater than that in a spherical cavity of equal volume, given the same temperature rise. In general, a spherical cavity is suited for waste with a lower thermal value and a cylindrical cavity for waste with a higher thermal value.

There is no structural restriction upon the size of a cavity, except that the cavity radius should not exceed one-third of the overburden depth. Therefore, in most cases, the size of the cavity should be determined in relation to the cavity temperature rise because the greater the cavity volume, the higher the cavity temperature rise.

The size of the cavity could be enlarged to the full extent of the salt formation if the adjacent formations are structurally sound and impermeable. Furthermore, the plastic zone around the cavity can be permitted to extend into the adjacent formations, provided that the formations are not subject to brittle fracture under triaxial loading conditions.

CHAPTER XIII

CONCLUSION

Based upon the theoretical analyses and experimental investigations discussed in this monograph, the following conclusions have been reached:

Structural Equilibrium

1. The disposal of reactor fuel wastes into a salt cavity is possible because a properly designed cavity establishes an ultimate structural equilibrium.

2. The ultimate structural equilibrium can be determined by the relationships found to exist among four major variables: the cavity temperature rise, the overburden pressure of the cavity, the internal pressure of the cavity, and the ultimate reduction of the cavity volume. Therefore, the cavity design should be restricted by the following relationships among the variables:

for a spherical cavity

$$\frac{P_o - P_i}{\sigma_o} = \ln \left[\frac{2(1-\xi)}{9\alpha T_c} + 1 \right] + \frac{2}{3} - \frac{2}{3} \frac{\alpha E T_c}{\sigma_o(1-\nu)} \sqrt{\frac{9\alpha T_c}{9\alpha T_c + 2(1-\xi)}}$$

for a cylindrical cavity

$$1 - \xi = 3\alpha T_c (A e^B - 1.072)$$

where $A = 0.928 - \frac{B}{13.8}$

$$B = \frac{\frac{2(P_o - P_i)}{\sigma_o} + 0.928 \frac{\alpha E T_c}{\sigma_o(1-\nu)} - \frac{2}{\sqrt{3}}}{0.072 \frac{\alpha E T_c}{\sigma_o(1-\nu)} + \frac{2}{\sqrt{3}}}$$

P_0 = overburden pressure of the cavity

P_1 = internal pressure of the cavity

σ_0 = equivalent plastic limit of salt

ζ = ratio of reduced cavity volume to original one

ν = Poisson's ratio of salt

α = coefficient of linear expansion of salt

T_c = cavity temperature rise above ambient temperature

E = Young's modulus of salt

$$10 < \frac{\text{LENGTH OF CYLINDER}}{\text{RADIUS OF CYLINDER}} < 20$$

3. Ultimate structural equilibrium occurs because a plastic zone may develop around the cavity. This plastic zone may possibly extend to contiguous formations. The radius of the plastic zone, ρ , can be calculated from the following equations:

for a spherical cavity

$$\frac{P_0 - P_1}{\sigma_0} = 2 \ln \frac{\rho}{a} + \frac{2}{3} - \frac{2}{3} \frac{\alpha E T_c}{\sigma_0 (1 - \nu)} \frac{a}{\rho}$$

for a cylindrical cavity:

$$\frac{P_0 - P_1}{\sigma_0} = \frac{2}{\sqrt{3}} \ln \frac{\rho}{a} + \frac{1}{\sqrt{3}} - \frac{\alpha E T_c}{2 \sigma_0 (1 - \nu)} \left(0.928 - \frac{\ln \rho - \ln a}{6.91} \right)$$

where a = radius of the cavity (spherical or cylindrical)

$$10 < \frac{\text{LENGTH OF CYLINDER}}{\text{RADIUS OF CYLINDER}} < 20$$

4. The reduction of cavity volume imposes a practical limitation upon the depth of the cavity and the permissible temperature rise in the cavity. This reduction in volume is mainly caused by thermal expansion rather than structural expansion of the salt in the plastic zone.

5. The ultimate structural equilibrium of an irregularly shaped cavity can be approximated by comparisons with ideal spherical and cylindrical cavities. A cavity of irregular form would probably be as stable as a cavity of ideal form since minor irregularities will not produce higher stress concentration because of the plastic nature of salt.

6. In general, salt cavity storage becomes impractical when the cavity temperature exceeds 500°F. The temperature rise in the cavity is restricted by two factors: One is the reduction of the cavity volume due to the thermal expansion of salt; the other is the increase of the waste vapor pressure beyond the overburden pressure.

7. The loading condition of a salt cavity is best described by the term "factor of structural loading" which is defined as:

$$X = \frac{P_o - P_i}{\sigma_o}$$

where X = factor of structural loading

P_o = overburden pressure of the cavity

P_i = internal pressure of the cavity

σ_o = equivalent plastic limit of salt

The structural load on the cavity is minimized if the internal pressure is equal to the overburden pressure. The cavity can be fractured if the internal pressure exceeds the overburden pressure.

8. The elastic theory can be applied for stress analysis around a salt cavity when the octahedral shearing stress around the cavity is less than its maximum value for salt.

9. The theory of plasticity must be introduced into the analysis whenever the stress around the cavity exceeds the maximum octahedral shearing stress of salt.

10. Increased temperatures not only create thermal stress but also extend the plastic zone around the cavity.

11. Stress concentration around the cavity is independent of the size of the cavity. However, the extent of the stress concentration away from the boundary is proportional to the size of the opening.

12. If no plastic zone develops, the geometry of a cavity determines the stress concentration around the boundary where critical stress and maximum shearing stress always occur.

13. Generally, the depth of the cavity determines the overburden pressure on the cavity; therefore, the shallower the depth, the less the external load. However, the thickness of the overburden layer above the cavity should be at least 3 times the cavity diameter.

14. The experimental results of creep measurement in the Grand Saline salt mine agree with the theoretical analysis. The empirical creep equation developed for the mine opening indicates that creep in the mine, at a depth of 700 ft., will practically disappear within a decade after the creation of the opening; and that the total amount of the closure resulting from creep will not exceed several percent of the opening diameter or height.

Structural Properties of Salt

1. Consistent compression test results can be obtained only when the end friction effect of the salt specimen is eliminated. By using the standardized test procedure, the following fundamental properties of the aggregate salt from the Grand Saline salt mine were determined:

The Maximum Compressive Stress	2,300 psi
with the standard deviation	200 psi
The 0.5 % offset yield strength	2,000 psi

Young's modulus	0.142×10^6 psi
with the standard deviation	0.03×10^6 psi
Poisson's ratio	
with the compressive stress up to 300 psi	0.25 - 0.5
with the compressive stress over 300 psi	0.5

The wide variation in the previously reported figures regarding the fundamental properties of salt is probably due to the varied testing procedures used by previous investigators, besides some difference in the properties of the tested salt.

2. The stress condition in a salt formation becomes hydrostatic at a depth approximately below 300 ft.

3. Salt specimens described herein are structurally isotropic and homogeneous regardless of their previous experience in stress and strain.

4. Neither Mohr's theory nor the theory of octahedral shearing stress is directly applicable for analyzing the strength of salt under a uniaxial stress condition at room temperature. These two theories, however, are useful when the temperature rises above 500° F or the mean of the principal stresses exceeds a limiting value between 2,000 to 6,000 psi.

5. The theory of octahedral shearing stress around a salt cavity can be applied to the creep analysis by introduction of the "equivalent plastic limit of salt" which is defined as:

$$\sigma_o = \frac{3}{\sqrt{2}} \tau_{o \max}$$

where σ_o = equivalent plastic limit

$\tau_{o \max}$ = the maximum octahedral shearing stress of salt under creep

The equivalent plastic limit of both Harshaw's synthetic crystal and the Grand Saline aggregate salt is approximately as follows:

at temperature 80° F 1,700 psi

at temperatures 500° F to 1100° F 1,000 psi

6. The total creep of salt consists of instantaneous, transient, and steady state creeps and can be formulated with respect to time:

$$\epsilon_o = \alpha + \beta \log t + \gamma t$$

where ϵ_o = total creep

α = instantaneous creep

β = the coefficient of transient creep

γ = the coefficient of steady state creep

7. Heat has the most significant effect upon the structural property of salt. The creep rate of salt is increased 75 times with temperature increase of 80° F to 770° F. This means that a salt cavity reaches its ultimate structural equilibrium much faster if the cavity temperature is elevated. The equivalent plastic limit of salt is reduced to 1,000 psi with temperature increase of 80° F to 500° F; thereafter, the equivalent plastic limit remains nearly constant at around 1,000 psi while the temperature approaches the melting point of salt. Heat affects Young's modulus and the coefficient of linear expansion of salt very slightly.

8. Gamma irradiation of salt up to 20×10^6 rep has no effect upon the fundamental structural properties of salt. However, the irradiated salt exhibits a slight reduction in the rate of transient creep.

9. The maximum strength of salt is not affected by its submersion for periods of one month in light mineral oil, reactor fuel waste substitute (nitrate waste), and saturated salt water.

10. Salt submerged in saturated water exhibited some increase in creep although no similar effect was found in salt submerged in the reactor fuel waste substitute.

APPENDIX

NATURE OF REACTOR FUEL WASTES

Reactor fuel wastes come from various types of reactor fuel reprocessing plants and may vary widely in their characteristics and quantity. They present difficult problems in handling and disposal because they possess large amounts of acid and dissolved chemicals in addition to high radioactivity. The radioactivity may range from 20 to 500 curies per gallon depending upon the concentration of fission products in the waste, the power produced per unit of fuel charged to the reactor, and the decay cooling time^{26,47}.

The total accumulation of stored reactor fuel wastes has been estimated at several billion gallons in the next 40 years. This estimate is plotted in Fig. A, assuming waste production at 500 gallons per ton of natural uranium used and a burnout rate of 10%⁴⁴. Among the hundreds of fission products found in reactor fuel wastes, several isotopes -- I^{131} , Sr^{90} , Cs^{137} , and Kr^{85} -- pose the most serious waste disposal problems, because of their longer half-lives and greater concentrations. The accumulated total quantities of these individual isotopes in the next 40 years are also estimated in Fig. A. As an example of the potential hazard of these isotopes, the accumulation of Sr^{90} alone would require about 5% of the entire ocean water of the world to dilute its concentration to the allowable level by the next 40 years.

A concentration of radioactivity more than 500 curies per gallon of waste is probably less economical for storage because removing heat from the waste becomes impractical and expensive⁵⁵. The rate of heat production in waste of the above concentration may range from 1 to 3 BTU/gal/hr; this rate would be sufficient to raise the waste above the boiling point in a few days at atmospheric pressure. If an economical method of removing critical isotopes could

be developed, the volume of waste might be reduced to a certain extent, but the waste disposal problem would remain the same. Disposal would not be greatly simplified if the waste were transformed into solid form because of the greater heat production rate and difficulty of handling, in addition to the expensive operation of solidifying the waste. Therefore, it is preferable to dispose of the waste in liquid form through shielded pipe systems⁶³.

FIG. A. TOTAL ACCUMULATION OF AMOUNTS OF REACTOR FUEL WASTE AND SELECTED FISSION PRODUCT ESTIMATED BY GENERAL ELECTRIC COMPANY RESEARCH GROUP (HANFORD).

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