BNL-20480

SP 75-3

BROOKHAVEN NATIONAL LABORATORY
Associated Universities, Inc.
Upton, New York

ACCELERATOR DEPARTMENT
Informal Report

PRELIMINARY DESIGN AND COST ESTIMATE

FOR A

600 MeV, 20 MICROAMPERE PROTON SYNCHROTRON

J.P. Blewett, R. Chasman, J.G. Cottingham,
P. Grand, G.K. Green, and C. Lasky

September 3, 1975

ABSTRACT

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A design study is presented for a 600-GeV proton synchrotron to yield an average current of 20 microamperes. Its purpose is production of negative pions for use in cancer therapy. The machine consists of a 4 MeV electrostatic preinjector, a 20 MeV linac injector and a magnet ring about 20 meters in diameter.

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Introduction

Stanford University is proposing a pion irradiation facility using the superconducting pion concentrator already operating in the Stanford High Energy Physics Laboratory. To produce pions in sufficient numbers to permit reasonable dose times (ten minutes or so) an associated particle accelerator must produce a current of about 600 μ A of 500-MeV (or higher) electrons or a current of about 20 μ A of 500-MeV (or higher) protons.

Three possible accelerators appear to lie within the state of the art — the electron linear accelerator, the proton linear accelerator, and the proton synchrotron. Other machines such as microtrons and isochronous cyclotrons either need development or appear prohibitively large and expensive. The electron linac is under study at SLAC and the proton linac is to be evaluated at Los Alamos. The present report is a preliminary design study and cost estimate for a 600-MeV proton synchrotron prepared by a group at the Brookhaven National Laboratory.

Proton synchrotrons consist of an injector and a ring in which, during acceleration, a rising magnetic field keeps the protons travelling on a fixed path. The injector may consist of a high voltage preinjector and a short linear accelerator. We have considered two possibilities: a 4-MeV electrostatic machine and a combination of a 4-MeV electrostatic machine and a 20-MeV linear accelerator. The former proves to put unreasonable demands on the accelerator ring and has been abandoned. The parameters for both will, however, be tabulated below up to the point where the former system is shown to be unreasonable.

Basic Parameters

Below we tabulate initial and final values of β ($= v/c$, the ratio of proton velocity to the velocity of light), γ ($= (1 - \beta^2)^{-1/2}$), and various combinations of these parameters for later use in calculation of secondary parameters. For example, the ratio of final to initial values of β gives the range over which the accelerating frequency must swing during the accelerating cycle. For the final value of magnetic field in the ring we have chosen a value of 0.8 T (8000 G). The magnet will be cycled 60 times per second and higher peak fields would result in unreasonable hysteresis losses in the magnet cores. A magnet cycled at this rate to a peak field of about 0.8 T was run successfully for a number of years as a component of the Cambridge (Mass.) Electron Accelerator.

The basic parameters are as follows:

Injection energy	4 MeV	20 MeV
β_{inj}	0.09205	0.2032
γ_{inj}	1.00426	1.0213
$\beta\gamma_{inj}$	0.0924	0.2075
Final energy	600 MeV	
β_{final}	0.7924	
γ_{final}	1.6395	
$\beta\gamma_{final}$	1.2992	
$\beta_{final}/\beta_{inj}$	8.61	3.90
$\beta\gamma_{final}/\beta\gamma_{inj}$		
(= final momentum/initial momentum)	14.06	6.26
ρ (= radius of curvature of proton orbit in magnetic field)	5.09 m	
Circumference of machine (including field free straight sections)	66.4 m	
Effective radius (circumference/ 2π)	10.57 m	
B_{inj} (magnetic field at injection)	0.057 T	0.127 T
B_{final}	0.8 T	
$B\rho_{inj}$	0.289 T·m	0.649 T·m
$B\rho_{final}$	4.066 T·m	
Average accelerated proton current	20 μ A	
	(= 1.2×10^{14} protons/sec)	

Magnet Aperture and Space-Charge Limit

The period of injection and the rate of cycling of the machine to reach 20 μ A of proton current will be set by the space-charge limit at injection. This is the amount of charge that can be injected and retained in the magnet. Space-charge limits later in the cycle will always be higher than at injection. No limit is set in the injection system because the charge in the ring can always be brought toward the space-charge limit by injecting a number of turns of lower current. Procedures for such "stacking" in the magnet ring are well known.

The space-charge limit is, in turn, set by the aperture of the ring vacuum chamber and by the energy of the injected protons. It is given by the formula

$$N = K \Delta v \sqrt{A_v} (\sqrt{A_h} + \sqrt{A_v}) \beta^2 \gamma^3, \quad (1)$$

where N is the number of protons contained in the ring,

v is the number of oscillation periods around the ring,

Δv is the tolerable shift in v (usually taken to be 0.25),

K is a constant independent of machine radius, aperture, and v -value,

A_h and A_v are the "admittances" of the vacuum chamber in the horizontal and vertical directions (see below),

β and γ are as defined in the preceding section.

The admittances A are the areas in phase space that can be accepted by the ring. They are given by the formula

$$A = \pi a^2 / \beta_{\max}, \quad (2)$$

where a is the semi-aperture of the vacuum chamber in meters,

$\beta_{\max}$ regrettably is not the same as the β 's defined in the preceding section but is the maximum value (in meters) of the Courant-Snyder β -function which describes the excursions of the protons from the equilibrium orbit in an alternating-gradient focusing system.

A_h and A_v are measured in meter-radians.

The constant K can be calculated from theory but a more conservative procedure is to obtain it from an actual operation. When the AGS injector was

still a 50-MeV linac, the highest intensity captured was slightly over 3×10^{12} protons. In the AGS

$$\begin{aligned} A_v &= 4 \times 10^{-5} \pi \text{ m} \cdot \text{rad} \\ A_h &= 1.2 \times 10^{-4} \pi \text{ m} \cdot \text{rad} \\ \beta_{inj} &= 0.3141 \\ \gamma_{inj} &= 1.0533 \\ \text{Hence } K &= 3.03 \times 10^{17} . \end{aligned}$$

For the 600-MeV machine we have chosen an aperture of 10 cm in both the vertical and the horizontal direction. For the lattice chosen, θ_{max} in the horizontal plane is 12.71 m and in the vertical plane is 18.59 m. From Eq. (2) $A_h = 1.97 \times 10^{-4} \pi \text{ m} \cdot \text{rad}$ and $A_v = 1.34 \times 10^{-4} \pi \text{ m} \cdot \text{rad}$. Substituting in Eq. (1) we are now able to obtain:

$$\begin{aligned} \text{Space-charge limit for 4-MeV injection is } 6.1 \times 10^{11} \\ \text{for 20-MeV injection is } 3.1 \times 10^{12} . \end{aligned}$$

It is now clear what must be the cycling rate of the machine. With 4-MeV injection the maximum charge per acceleration cycle is 6×10^{11} , hence we shall need 200 pulses per second. With 20-MeV injection, less than 40 pulses per second will suffice. We shall choose a pulse rate of 60 per second, and 20-MeV injection.

The Injector

Conventional proton linear accelerators are provided with preinjectors operating at 500-750 kV usually taking the form of Cockcroft-Walton voltage multipliers. The linear accelerator itself is a resonant cavity operating in the TM_{010} waveguide mode (with axial electric fields). As the protons pass along the axis of the cavity they are accelerated by the electric field when it is in the correct phase. When the field is decelerating, the protons are shielded by "drift tubes" hung from the wall of the cavity. The protons are focused and restored to the cavity axis by magnetic quadrupole lenses imbedded in the drift tubes.

The frequency of excitation of a linac with 500-750 keV injection cannot be higher than about 200 MHz for reasons associated with rf field patterns. With injection at energies four times higher, the frequency of excitation can be raised by a factor of two to 400 MHz; this reduces the cross-sectional

dimensions of the linac tank by the same factor of two, from a diameter of about one meter to less than 50 centimeters.

Fortunately, present 3-4 MV electrostatic machines cost no more than 500-750 kV Cockcroft-Walton generators. Consequently we propose to inject into the linac at 4 MeV from an electrostatic machine. This can be either a Van de Graaff generator (manufactured by High Voltage Engineering) or a "pelletron" (manufactured by the National Electrostatic Corporation). Both machines carry charge on an insulated belt or chain to an insulated high voltage terminal. This structure is supported in a steel tank pressurized with nitrogen or carbon dioxide or a mixture. An ion source in the terminal provides protons which are accelerated down the axis of an accelerating tube to emerge through a hole in the end of the tank.

The average current capability of one of these machines is about 400 μ A. We would like to draw currents up to 200 mA during short injection pulses. The terminal capacity to ground can be about 100 pF; with a current of 200 mA the terminal voltage will drop at a rate of about 2000 V per microsecond. This will not prove to be troublesome and can be compensated, if desired, by a pulsed liner inside the tank of the electrostatic machine; this is standard practice.

With 200 mA entering the linac, about 100 mA will be captured and accelerated to 20 MeV. After this beam is injected into the synchrotron there will be a further loss of 40-50% in the rf capture process. The revolution time at 20 MeV will be 1.09 μ sec. Consequently for each revolution injected, a total of about 3.5×10^{11} protons will be captured. Hence, to reach the space-charge limit of 3.1×10^{12} it will be necessary to inject 9 full revolutions, which will require an injection time of 10 μ sec.

The droop in terminal voltage in 10 μ sec will be about 20 kV. A 10 μ sec pulse of a few amperes charging the tank liner will suffice to compensate for this. However, as can be seen from Appendix A, the linac can accept this one-half percent droop in proton energy without compensation.

The magnetic field gradients required for focusing in the linac are discussed in Appendix B.

Pertinent parameters of the electrostatic preinjector and the linear accelerator are as follows:

Preinjector:

Approximate tank diameter	2 m
Approximate tank length	5 m
Space required to roll the tank back and expose the terminal	6 m

Linear accelerator:

Diameter	45 cm
Length	8 m
Number of drift tubes	72
Diameter of drift tubes	8 cm
Length of first drift tube	6.9 cm
Length of last drift tube	15.2 cm
Gap to drift tube length ratio	1:4
Stable phase	30°
Rate of energy gain	2 MeV/m

Table of Parameters Derived in the Two Preceding Sections

K (constant in space-charge-limit formula)	3.03×10^{17}
A_v (vertical admittance)	2.5×10^{-4} m·rad
A_h (horizontal admittance)	2.5×10^{-4} m·rad
Space-charge limit (20-MeV injection)	3.1×10^{12} protons
Current injected into linac	200 mA
Current injected into synchrotron	100 mA
Charge captured in synchrotron in one-turn injection	3.5×10^{11}
Number of turns required to reach space-charge limit	9
Time required to reach space-charge limit	10 μ sec
Droop on terminal of electrostatic machine	20 kV
Machine cycling rate	60 Hz

Lattice

The main ring lattice on which this report is based is shown in Fig. 1. The lattice includes eight 20-cm-long quadrupoles defocusing in the horizontal plane and focusing in the vertical plane, and four 20-cm and eight 10-cm quadrupoles focusing in the horizontal plane and defocusing in the vertical plane. There will be 32 bending magnets each one meter long. Spaces between magnets will be 40 cm except for four, symmetrically located long straight sections each 3 m long. The sequence of magnets will be thus:

...Long straight section $\frac{1}{2}F$ B B D B B F B B D B B $\frac{1}{2}F$, long str. sec. $\frac{1}{2}F$ B B...

where $\frac{1}{2}F$ represents a 10-cm horizontally focusing quadrupole,

F represents a 20-cm horizontally focusing quadrupole,

D represents a 20-cm horizontally defocusing quadrupole,

B represents a bending magnet.

One of the long straight sections will be used for injection of the 20-MeV beam, one will be used for extraction of the 600-MeV beam, and the other two will be used for the accelerating rf system.

The ν -value for this lattice will be 2.4. Phase transition will lie above 1.3 GeV and so will not need special arrangements for passing through it.

It is probable that some changes will be necessary in this lattice to optimize orbit parameters. These changes, however, will make negligible modifications in the amount of iron and copper needed and in the cost of the magnet system.

We have used a separated-function lattice (a lattice in which the functions of bending and focusing are performed by different magnets) because design of the lattice is simpler (thin lens approximations can be used to evaluate quadrupole performance). Possibly further study will show a combined-function system preferable. The cost of the combined-function system will, however, not differ appreciably from that presented here.

Magnet System

The bending magnets are shown in cross section in Fig. 2. The gap chosen is 11 cm to allow for insertion of a vacuum chamber with an inside diameter of 10 cm and a wall thickness of about a half centimeter. To obtain a horizontal extent of good field 10 cm wide the poles need to extend about a gap width on either side, making a total pole width of 30 cm. Modest cuts are possible by

shaping of the outer edges of the poles - specifically, by adding small bumps on the pole face at its outer edges. The return parts of the magnetic circuit are dimensioned to carry a return flux density of about 1.2 T. The magnets will be constructed of 0.014 in. high silicon steel laminations. Some special treatment will be needed at the magnet ends to limit eddy currents in the end laminations. Probably the simplest procedure is to terminate the pole in a "Rogowski pole" shape.

To magnetize an air gap of 11 cm to 0.8 T requires 70,000 ampere-turns. A slightly higher allowance than this will be assumed, to include magnetization of the magnet back leg; the coil will include 32 turns and will carry a maximum current of 2300 A, giving 73,600 ampere-turns.

The packing factor in this coil will be rather poor. Since it must perform at 60 Hz it will be made of stranded cable with built-in water cooling. We shall assume a packing factor of 50%.

With these assumptions one of the magnets illustrated in Fig. 2 will have the following parameters:

Weight of iron	8000 lb
Weight of copper	1400 lb
Stored energy	13 kJ
Inductance	5 mH
Resistance	3 mΩ
I^2R losses	16 kW peak 6 kW average (see next section)
Core losses	0.6 kW

The 20 quadrupoles occupy only one eighth as much of the circumference as do the bending magnets. Per unit weight, they will, however, be more costly because of their more complicated coils.

Magnet Power Supply

The bending magnets will be supplied with power by the circuit shown in Fig. 3. This system will supply a biased sine wave so that acceleration can be spread out over 180° of the wave, instead of 90° as would be the case with an unbiased wave. A sinusoidal wave, rather than a triangular wave is used so that a resonant supply can be used. The total stored energy of over 400 kJ is rather large for generation and reabsorption by a conventional polyphase rectifier. Resonant power supplies are in use at several large electron synchrotrons.

The 32 inductor-transformers will each have the following parameters:

Mid-field inductance	11 mH
Air gap dimensions	length 60 cm
	width 30 cm
	height 10.7 cm
Stored energy at peak current	14.5 kJ
Coil area	800 cm ²
Coil resistance	3.9 m Ω
I ² R losses	8.6 kW
Core losses	1.5 kW

Total dissipation in magnets and inductors is 630 kW. Total stored energy is 870 kJ. At 60 Hz the system has an overall Q of slightly over 500.

Stored energy in the quadrupoles will be very small compared with that in the bending magnets. A conventional power supply will provide current which will be computer-controlled to make the quadrupole gradients track the fields in the bending magnets.

Rf Accelerating System

During acceleration from 20 to 600 MeV the frequency of revolution of the protons changes from 0.92 to 3.58 MHz. Accordingly the frequency of the rf acceleration system must change over the same range, or a harmonic thereof. We have chosen to use the fundamental.

Acceleration will take place at eight accelerating gaps distributed along two of the long straight sections. The capacity across each gap will be resonated against the inductance of a ferrite-loaded cavity. To keep the system resonant as the frequency increases the ferrite will be saturated, thereby reducing the inductance of the cavity. This is the method used in the AGS and in other proton synchrotrons.

The maximum energy gain per turn is 47 keV, occurring at the point where the magnetic field has reached half of its peak value. Before and after that time energy gains per turn are lower, being proportional to $\sin \omega t$, where $\omega = 2\pi \times 60$ Hz. We shall take advantage of this fact in the lower frequency range where, for the same gap voltage, losses in the ferrite are higher. Hence in the lower range the gap voltage will be dropped to the point where ferrite losses are kept constant.

Parameters for the rf system are:

Number of stations	2
Number of gaps per station	4
Frequency range	0.92-3.58 MHz
Gap voltage	4.5-12 kV
Capacity per gap	450 pF
Peak flux in ferrite	0.01 T
Ferrite material	4C4
Ferrite core cross section per gap	600 cm ²
Total weight of ferrite	6000 lb
Cavity excitation power per station	130 kW
Beam power	24 kW
Total rf power	284 kW
Design drive power per station	200 kW

Vacuum System

At a 60-Hz cycling rate it is not possible to use a simple metal vacuum chamber because eddy currents induced in the chamber wall would distort the magnetic field patterns and overheat the chamber wall. These difficulties will be avoided by using a ceramic chamber. The chamber will be made in short sections terminated in metal flanges which will be located in field-free sectors of the synchrotron ring. The chamber sections will be joined by welding the flanges together. In some of the 10-cm clear sections between magnets, T-sections will be included for connections to vacuum pumps.

The ring will be evacuated by twenty 400 liter/sec Vac-ion pumps. Four roughing pumps will be included, one in each quadrant. Eight remotely-controlled valves will be available to isolate sectors of the ring from each other and from vacuum systems leading to the injector and to the pion target.

Injection and Extraction

The emittance of the 20-MeV beam from the linear accelerator will be about $1.6 \times 10^{-5} \pi$ m-rad. Nine turns with this emittance will fit easily into the horizontal admittance of the synchrotron (about $2 \times 10^{-4} \pi$ m-rad). The usual procedure for multiturn injection into proton synchrotrons is to introduce a magnetic bump in the orbit such that the equilibrium orbit passes through an

electrostatic inflector displaced horizontally from the center of the chamber. This procedure can be used in this machine without difficulty.

The emittance of the accelerated beam should be about 3×10^{-5} m·rad. The energy spread in the beam should be about 1%.

Several extraction processes are available. Probably the best for this machine will be the one which spreads out the extracted beam over the longest time — this is so-called "resonant" extraction in which the ν -value of the machine is shifted toward a value at which the beam becomes unstable. It can be made to cross an extraction septum at any desired rate. Thus extraction could be spread over about a millisecond on the flat top of the magnetic field wave.

Resonant extraction will not be as efficient as single-turn extraction. Some 10% of the beam will probably be lost in the extraction process whereas single-turn extraction can be almost 100% efficient.

Controls

Control will be by a small computer which will make all operations automatic. Most operating synchrotrons already have computer control; in most cases the systems include much more versatility than is required for this application since it is expected that this machine will always operate at full energy, full intensity and with extraction into only one beam line. The main function of the system will be to give rapid and precise indication of malfunction or component failure.

Shielding

Information in this section has been derived from data quoted in BNL Report AADD-127 (Design Study for a 1000-MeV Booster Synchrotron Injector for the Brookhaven AGS). Pages 51-3 of that report describe a shielding experiment performed with the Brookhaven Cosmotron operated at 1000 MeV.

From the data quoted, it can be shown that, if 10% of the beam is lost during extraction and if it is desired to reduce the radiation level to 5 mR/year (allowed level for general public), the shielding thickness must be between 5 and 6 meters of Ilmenite-loaded concrete (density 4.5 g/cm^3). This would correspond to a thickness of about 14 m of Long Island sand (density 1.7 g/cm^3).

Building

A building to house the accelerator is not included in this design study or cost estimate. It would be about 80 ft square and would include a tunnel tangential to the ring and extending back about 60 ft to house the electrostatic preinjector and linear accelerator.

Management, Time Schedule and Cost Estimate

It would seem that the best management for construction of a 600-MeV proton synchrotron would be at one of the National Laboratories that has built a proton synchrotron. This would include the Lawrence Berkeley Laboratory, the Argonne National Laboratory, the Fermi National Laboratory or the Brookhaven National Laboratory. Of these institutions Berkeley would seem to be preferable because of its convenient location for travel to Stanford. If none of these institutions is able to undertake construction, industry can be approached. A first choice would be Brobeck Associates (1011 Gilman St., Berkeley, California). Other possibilities would include High Voltage Engineering (Burlington, Mass.), and National Electrostatic Corporation (Madison, Wisconsin).

It should be possible to complete the machine in about three years. The first six months should be used in an intensive effort to establish and optimize all of the machine parameters. This probably will not result in much change in the cost estimate but is necessary for most effective functioning of the completed machine.

A summary of the cost estimate is as follows (details are given below):

Preinjector	\$ 400,000
Linear accelerator	900,000
Magnet system	826,000
Magnet power supply	1,900,000
Rf acceleration system	760,000
Vacuum system	257,000
Controls	277,000
Miscellaneous	500,000
Total for components	5,820,000
25% of above for Engineering, Design, Inspection and Administration	1,455,000
20% Contingency	1,455,000
20% Escalation	1,455,000
Grand Total	\$10,185,000

Details of Cost Estimate

Preinjector

Estimates from manufacturers of prices for 4-MV electrostatic machines run between \$250,000 and \$300,000. To adapt these machines to the required service by adding a pulsed ion source and a pulsed liner would probably bring the price to approximately \$400,000.

Linear Accelerator

The linac mechanical structure is estimated to cost \$300,000

The rf system (including a \$70,000 klystron, a low-level rf system at \$30,000, and power supplies, etc. for \$200,000)

	300,000
Controls	100,000
Miscellany (prebuncher, matching beam transport, cooling, etc.)	<u>200,000</u>
Total	\$900,000

Magnet System

Bending magnets:

Material: Laminations	134,855
End plates and tie rods	33,300
Fabrication: Laminations	37,500
Die	25,000
End plates	57,600
Tie rods	14,000
Assembly	23,000
Coils	320,000

Quadrupoles:

Material: Laminations	16,800
End plates and tie rods	20,800
Fabrication: Laminations	9,400
Die	25,000
End plates	36,000
Tie rods	18,000
Assembly	14,400
Coils	<u>40,000</u>
Total	\$826,055

Magnet Power Supply

Items included are as follows:

1 bias supply, 275 V, 1500 A	\$ 40,000
32 ac excitation supplies, 500 V, 50 A @ \$3750	120,000
32 inductors, 1 mH, 300 A @ \$600	19,200
32 inductors, 120 μ H, 300 A @ \$300	9,600
32 SCR units, 1200 V, 1200 A peak @ \$1200	38,400
32 chokes: 5600 lb steel each	179,200
4100 lb copper each	984,000
32 capacitors, 3750 μ F, 600 V @ \$4500	144,000
32 capacitors, 2000 μ F, 2500 V @ \$7000	224,000
32 trigger systems @ \$2500	80,000
Protection and monitoring systems	10,000
1 spare excitation system	<u>56,200</u>
Total	\$1,904,600

Rf Accelerating System

6000 lb of ferrite @ \$20/lb	\$ 120,000
Cavity hardware @ \$20,000 per station	40,000
Rf power amplifier @ \$1/watt	400,000
Spare rf amplifier	<u>200,000</u>
Total	\$ 760,000

Vacuum System

66.4 m of vacuum chamber @ \$1500/m	\$ 99,600
20 400 liter/sec Vac-ion pumps @ \$4000	80,000
20 Vac-ion pump power supplies @ \$1300	26,000
4 roughing pumps @ \$6000	24,000
8 remotely-controlled valves @ \$900	7,200
Instrumentation	<u>20,000</u>
Total	\$ 256,800

Controls are estimated at 5% of component cost.

The "Miscellaneous" item includes beam transport and injection and extration gear.

Conclusion

It is probable that a cost estimate turned out in so short a time is somewhat low. When more detailed re-estimates have been done on the various components they have almost invariably come out slightly higher. But we believe that the estimates on the major items are reasonable. Wherever possible they have been checked with vendors and with other accelerator builders.

It seems likely that more careful attempts to optimize performance and cost will lead to the choice of a higher injection energy. The linac structure becomes cheaper for energies above 20 MeV; we believe that it can be extended for about \$15,000 per MeV. If the energy were raised to 40 MeV the space-charge limit in the synchrotron would more than double, the magnet could be cycled more slowly and the power demands on the magnet power supply and the rf accelerating system would ease materially.

Only three lattices have been studied with computation of the Courant-Snyder beta function and other pertinent parameters. It seems probable that variations in the arrangement of components can reduce the maximum value of the beta function and improve the momentum compaction. These changes will not, however, strongly affect the cost estimate.

APPENDIX A

Energy Spread in the Accelerated Beam

It will be assumed that we inject into the linac a 4-MeV beam with negligible energy spread. The following treatments follow from basic considerations to be found in Lloyd Smith's article in the Handbuch der Physik (Vol. XLIV) and in the paper by Courant and Snyder (Ann. Phys. 3, 1, 1958).

Our treatment of phase oscillations in the linac will be nonrelativistic. We assume that the rate of energy gain with distance is constant

$$\frac{d}{dz} \left(\frac{1}{2} m \beta^2 c^2 \right) = e E_0 T \cos \varphi \quad , \quad (1)$$

where E_0 is field in volts per meter = 2×10^6 V/m,
 T is transit time factor - assumed to be 0.8.

Hence

$$\frac{d}{dz} (\Delta \beta^2) = - \frac{2eE_0 T \sin \varphi_s}{mc^2} \Delta \varphi \quad , \quad (2)$$

where $\Delta \beta^2$ is variation of β^2 from synchronous value,
 φ_s is synchronous phase,
 $\Delta \varphi$ is phase error.

The phase error is given by

$$\frac{d}{dz} (\Delta \varphi) = - \frac{\pi}{\lambda} \frac{\Delta \beta^2}{\beta^3} \quad . \quad (3)$$

From (2) and (3)

$$\frac{d}{dz} \left(\beta^3 \frac{d\Delta \varphi}{dz} \right) = \frac{2\pi}{\lambda} \frac{eE_0 T \sin \varphi_s}{mc^2} \Delta \varphi \quad . \quad (4)$$

From (1) the synchronous β is given by

$$\beta^2 - \beta_1^2 = \frac{2eE_0 T \cos \varphi_s}{mc^2} z \quad , \quad (5)$$

where β_1 is β at injection (≈ 0.09205).

We can now write

$$\frac{dz}{\beta^3} = \frac{mc^2}{2eE_0 T \cos \varphi_s} \frac{d\beta^2}{\beta^3} = - \frac{mc^2}{eE_0 T \cos \varphi_s} d(1/\beta) \quad (6)$$

We use this to change the variable in (4) from z to $1/\beta$ and obtain

$$\frac{d^2(\Delta\varphi)}{d(1/\beta)^2} = \frac{2\pi}{\lambda} \frac{mc^2 \sin \varphi_s}{eE_0 T \cos^2 \varphi_s} (1/\beta)^{-3} \Delta\varphi \quad (7)$$

The equation $y'' + bx^m y = 0$ is a modified Bessel's equation and has the solution

$$y = x^{\frac{1}{2}} Z_{\frac{1}{m+2}} \left\{ \frac{2b^{\frac{1}{2}}}{m+2} x^{(m+2)/2} \right\} \quad (8)$$

where Z is a Bessel function. In our case this results in

$$\begin{aligned} \Delta\varphi &= \beta^{-\frac{1}{2}} Z_{-1} \left\{ 2 \left(\frac{2\pi}{\lambda} \frac{mc^2 \sin \varphi_s}{eE_0 T \cos^2 \varphi_s} \beta \right)^{\frac{1}{2}} \right\} \\ &= \beta^{-\frac{1}{2}} Z_{-1} (114 \beta^{\frac{1}{2}}) \quad (9) \end{aligned}$$

The solution will be a combination of J and N functions which, in this case, can be represented by their asymptotic forms to give

$$\Delta\varphi = \Delta\varphi_i (\beta_i/\beta)^{3/4} \cos (114 \beta^{\frac{1}{2}} - 114 \beta_i^{\frac{1}{2}}) \quad (10)$$

The energy spread is given by (3), whence

$$\begin{aligned} \Delta\beta^2 &= (\lambda\beta^3/\pi) \frac{d\Delta\varphi}{dz} = \frac{\lambda\beta^2}{\pi} \frac{eE_0 T \cos \varphi_s}{mc^2} \frac{d\Delta\varphi}{d\beta} \\ &= 3.53 \times 10^{-4} \beta^2 \frac{d\Delta\varphi}{d\beta} \quad (11) \end{aligned}$$

The derivative of the $\beta^{3/4}$ term in (10) will be negligible compared with the derivative of the cosine term. Hence the amplitude of the $\Delta\beta^2$ oscillations is

$$\Delta\beta^2 = 3.36 \times 10^{-3} \beta^{3/4} \Delta\varphi_i$$

$$\text{and} \quad \Delta\beta^2/\beta^2 = \Delta E/E = 3.36 \times 10^{-3} \beta^{-5/4} \Delta\varphi_i$$

At 20 MeV $\beta = 0.2032$, hence $\Delta E/E = 0.025 \Delta\varphi_i$.

Thus for each radian of initial phase spread of the injected beam the energy spread at 20 MeV will be ± 1.2 percent. The momentum spread will be ± 0.6 percent.

- - - - -

We now consider the phase oscillation in the synchrotron ring. According to Courant and Snyder the momentum acceptance of the ring at 20 MeV is given by

$$\frac{\Delta p}{p} = \pm \left\{ \frac{eV [(\pi - 2\varphi_0) \sin \varphi_0 - 2 \cos \varphi_0]}{\pi h p \beta c \eta} \right\}^{\frac{1}{2}}, \quad (12)$$

where eV is the possible energy gain per revolution (20 keV),

φ_0 is the equilibrium phase (45°),

h is the harmonic order (1),

η is the quantity $1/\gamma^2 - 1/\gamma_{Tr}^2$,

γ_{Tr} for our lattice is 2.24 whence $\eta = 1/\gamma^2 - 0.1993$.

From these numbers, the momentum acceptance is ± 0.8 percent. Hence the ring can accept the ± 0.6 percent spread of the linac beam.

Courant and Snyder show that the damping of momentum oscillations in an AG synchrotron is proportional to $\eta^{-1/4} E^{-3/4} \beta^{-1}$. Initial and final values of these quantities are

$\eta_{\text{initial}} = 0.7594$	$\eta_{\text{final}} = 0.1728$
$E_{\text{initial}} = 958 \text{ MeV}$	$E_{\text{final}} = 1538 \text{ MeV}$ (E here is total energy)
$\beta_{\text{initial}} = 0.2032$	$\beta_{\text{final}} = 0.7924$

These parameters lead to the conclusion that the momentum spread will be damped by a factor of 0.26 to ± 0.156 percent. At 600 MeV the percent energy spread is approximately the same as the percent momentum spread.

If a smaller final momentum spread is desired, this can be accomplished by inclusion of a "debuncher" cavity between the linac and the synchrotron ring. Using a single rf cavity appropriately placed the energy spread can be reduced by a factor of about three.

APPENDIX B

Magnetic Fields Required in Linac Quadrupoles

The necessary parameters of a quadrupole focusing system for use in a proton linear accelerator are discussed in the paper "Focusing in Linear Ion Accelerators" by L. Smith and R. L. Gluckstern (Rev. Sci. Instr. 26, 220, 1955).

Two parameters are used in constructing stability diagrams. The first defines the focusing force in the quadrupole system:

$$\theta_0^2 = \frac{eB' \beta c}{mf^2} = 0.0165 B' .$$

The second describes the radially defocusing effect of the accelerating field:

$$\Delta = \frac{\pi e E \lambda T \sin \varphi}{mc^2 \beta} = 0.0437 \sin \varphi .$$

Here B' is the gradient of magnetic field in the quadrupoles in T/m,
 βc is the proton velocity (= 0.09205 c at 4 MeV),
 f is the frequency of the accelerating rf field (= 400×10^6 Hz),
 E is the peak accelerating rf field (= 2×10^6 V/m),
 λ is the free space wavelength of the accelerating rf field (= 0.75 m),
 T is a transit time factor (assumed here to be 0.8),
 φ is the phase of the proton on the rf wave.

We assume an equilibrium phase of -30° . The stable phase range will then run from about -90° to $+30^\circ$ and the value of Δ can range from about -0.04 to +0.02.

Smith and Gluckstern provide three graphs of θ_0^2 vs Δ on which the ranges of radial stability are indicated for the three cases of alternation of quadrupole from one drift tube to the next ($N = 1$), alternation every second drift tube ($N = 2$), and alternation every three drift tubes ($N = 3$). For Δ ranging from -0.04 to +0.02 permissible values of θ_0^2 and consequent values of B' are as follows

N = 1	$\theta_o^2 = 2.5 - 4.5$	B' = 151 - 272 T/m	(15 - 27 kilogauss/cm)
N = 2	$\theta_o^2 = 1.2 - 1.6$	B' = 72 - 97 T/m	(7 - 10 kilogauss/cm)
N = 3	$\theta_o^2 = 0.5 - 0.75$	B' = 30 - 45 T/m	(3 - 4.5 kilogauss/cm)

These fields are easily reached with electromagnet quadrupoles. They will not, however, as originally hoped, be achievable with permanent magnet quadrupoles. That might, however, be possible with N's higher than 3. Exploration of this possibility should be part of the final design of this accelerator.

FIG. 1
LATTICE

FIG. 2
MAGNET CROSS SECTION
(ONE FIFTH SCALE)

FIG. 3 MAGNET POWER SUPPLY