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The Defeat of Shear Stabilization by Peaking
of the Diamagnetic Frequency Profile

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The radial wave equation describing collisionless drift instabilities in plasma is solved including the effects of a peaked diamagnetic frequency profile. Radially localized fluctuations with no radial energy propagation are found in cases with sufficiently low shear and large peaking.

Recent estimates of prospects for the stabilization of trapped electron modes¹ have been based on the assumption that the radial mode structure is determined by the shear of the magnetic field. In this note we show that in many present generation toroidal confinement devices the peaking of the diamagnetic frequency profile is sufficient to introduce reflections which prevent the radial propagation of wave energy. The analysis is done in slab geometry with the instability mechanism treated as an arbitrary perturbative quantity.

In the regime of the conventional trapped electron instability,² neglecting ion temperature effects, the radial mode structure is determined by the equation

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(I) INTRODUCTION

There are three major energy loss mechanisms in a tokamak: radiation loss; energy loss associated with fast neutrals as a result of the charge-exchange process; and the energy loss associated with charged particle diffusion and thermal conduction. The first two mechanisms occur somewhat uniformly throughout the surface of the plasma, while the charged particle loss should occur mainly at the limiters. In our experiment on the ATC tokamak [1] the energy losses to the limiters and through the plasma surface were monitored. The temperature of the limiters was measured by thermocouple and thermistor, from which the total energy loss to the limiters could be calculated. However, there was no time resolution of the energy deposition because of the high noise level at the limiters. To measure the radiation and the charge exchange losses a bolometer was devised out of a thick film flake thermistor, which had a time resolution on the order of 1 millisecond. Several points were concluded from our measurements: (1) the total loss from the plasma surface (radiation and charge-exchange) in a normal discharge with or without disruption was always less than 40% of the total energy input; (2) for a normal discharge (40-45 msec duration) or a discharge with a short life (5-10 msec) the absolute level of the surface energy losses was relatively constant, i.e., within a factor of 2. On the other hand, the limiters detected either a major portion of the input energy for a normal discharge or a negligible portion of the input energy for a short-lived discharge

$$\frac{d^2 \phi}{dx^2} = \left[k_y^2 + \frac{\Omega_i^2}{c_s^2} \left(1 - \frac{\omega_{*e}}{\omega} - \frac{k_{||}^2 c_s^2}{\omega^2} - \frac{i \gamma_L(x)}{\omega} \frac{\omega_{*e}}{\omega} \right) \right] \phi \quad (1)$$

where k_y is the wave vector in the direction of symmetry (toroidal), Ω_i is the ion gyrofrequency, c_s the ion sound speed, ω the mode frequency, ω_{*e} the electron diamagnetic frequency, $k_{||}$ the parallel wave vector, and the local growth rate $\gamma_L(x)$ given by the electron response

$$\tilde{n}_e = \frac{ne\phi}{T_e} \left[1 - \frac{i \gamma_L(x)}{\omega} \frac{\omega_{*e}}{\omega} \right] \quad (2)$$

Specializing to an even trapped electron mode, neglecting corrections to adiabaticity for all except the trapped electron and using the collision operator

$$\left. \frac{d\tilde{f}}{dt} \right|_{\text{collision}} = -\nu_t \left(\tilde{f} + \frac{q\phi f_o}{T} \right) \quad (3)$$

where $\tilde{f}$ is the perturbed distribution function and ν_t the effective collision frequency we obtain

$$\gamma_L(x) = \frac{-\omega^2}{\omega_{*e}} i \int_{\text{trapped}} d^3v \frac{J_0^2(k_{||} L_c) e^{-v^2/v_T^2} (\omega - \omega_{*Te})}{\pi^{3/2} v_T^3 (\omega - \omega_{de} + i\nu_t)} \quad (4)$$

where the Bessel function term gives the parallel orbit average effect,³ L_c being the connection length which is a function of pitch angle, $v_T^2 = 2T_e/m_e$, and ω_{*Te} and ω_{de} being generalized diamagnetic and curvature drift frequencies.⁴

The character of the radial structure of the mode, as governed by Eq. (1), is determined by the radial dependences of ω_{*} , $k_{||}$, and γ_L . Treating the driving term as a perturbative quantity and approximating

$$k_{\parallel} = k_z + k_y \frac{x}{L_s} \quad (5)$$

and expanding ω_* around its maximum

$$\omega_{*e} = \omega_{*0} + \frac{x^2}{2} \omega_*'' , \quad \omega_*''/\omega < 0 \quad (6)$$

The nature of the solution changes at

$$\omega_*''/2\omega + k_y^2 c_s^2 / \omega^2 L_s^2 = 0 \quad (7)$$

The normal modes are given by⁵

$$\phi_n = H_n [\alpha_n (x - x_n)] e^{-\alpha_n^2 (x - x_n)^2 / 2} \quad (8)$$

where

$$\alpha_n^2 (2n+1) = -k_y^2 - \frac{\Omega_i^2}{c_s^2} \frac{(1 - \omega_{*0}/\omega) (\omega_*''/2 + k_y^2 c_s^2 / \omega^2 L_s^2) - \omega_*''/2\omega c_s^2 k_z^2 / \omega^2}{\omega_*''/2\omega + k_y^2 c_s^2 / \omega^2 L_s^2} \quad (9)$$

$$\alpha_n^4 = -\Omega_i^2 / c_s^2 (\omega_*''/2\omega + k_y^2 c_s^2 / \omega^2 L_s^2) \quad (10)$$

$$x_n = - \frac{c_s^2 k_y k_z / \omega^2 L_s^2}{\omega_*''/2\omega + k_y^2 c_s^2 / \omega^2 L_s^2} \quad (11)$$

and H_n is the Hermite polynomial of order n .

If $\omega_*''/2\omega + k_y^2 c_s^2 / \omega^2 L_s^2 > 0$ then $\alpha_n^4 < 0$ so that traveling waves result and the condition of outward energy propagation at $|x| \rightarrow \infty$ gives the traditional damping due to shear.

On the other hand, if $\omega_*''/2\omega + k_y^2 c_s^2 / \omega^2 L_s^2 < 0$ the solutions are undamped standing waves. In this case perturbation theory shows the overall growth rate is a weighted average of the local growth rate

$$\gamma_n = 1/\sqrt{\pi} 2^n n! \int_{-\infty}^{\infty} dx \alpha_n e^{-\alpha_n^2 (x-x_n)^2} H_n^2[\alpha_n (x-x_n)] \gamma_L(x) . \quad (12)$$

The stabilizing effect of ion Landau damping can be minimized by requiring that the mode be localized such that the parallel phase velocity does not pass through the ion thermal velocity;⁶ i.e., $|\alpha_n L_s| \gg (a/\rho_i)(T_i/T_e)$ where a is the density scale length and ρ_i the ion gyroradius.

In summary, we have found that with sufficient peaking of the diamagnetic frequency profile the radial transport of wave energy, which is one of the conventional roles of shear, is eliminated and that the recent analyses which neglect this effect are not applicable to machines of moderate dimensions.

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