Collective Enhancement of Inclusive Cross-Sections
at Large Transverse Momentum
in Stochastic-Field Multiparticle Theory

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COLLECTIVE ENHANCEMENT OF INCLUSIVE CROSS-SECTIONS
AT LARGE TRANSVERSE MOMENTUM
IN STOCHASTIC-FIELD MULTIPARTICLE THEORY*

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ABSTRACT

A stochastic-field calculus, previously discussed in connection with Regge intercepts and instability questions, is applied to inclusive cross-sections, and is shown to predict a growth with energy of large- $P_{\perp}$ inclusives.

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I. Formulation of Theory

In an earlier paper with G. Thomas, referred to as AT,⁽¹⁾ we formulated a "multi-Eikonal" theory of multiple hadron production which contained possibilities of many kinds of collective phenomena not seen in multiperipheral models. In two succeeding papers^(2, 3) the basics of that theory were reformulated in functional-integral terms, convenient for discussion of analog critical phenomena and phase transitions. The formalism was applied to the study of instabilities at ultrahigh energies⁽²⁾ (I) and to Regge intercepts⁽³⁾ (II), both of which required only rapidity dynamics. Here we will examine transverse momentum phenomena expected in such a theory, with emphasis on the implications of critical behavior for large- $P_\perp$ inclusives.

As in reference (1), we can discuss only the limit $Y \rightarrow \infty$ for fixed $P_\perp$ (Feynman-scaling limit), and the (slow Y -dependent) corrections to this limit. We consider $a + b \rightarrow a' + b' + h_1 + \dots + h_n$ amplitudes, using the A-T "multi-Eikonal" Ansatz⁽¹⁾:

$$A_n(r_1, \dots, r_n | R) = A_0(R) \prod_{j=1}^n G(r_j | R) \prod_{k \neq j} K(r_j, r_k | R) \quad (1)$$

Initial (ab) states are described by $R = (Y, \vec{B})$; $r = (y, \vec{b})$ describes secondaries. Here we will assume G and K independent of R ; slow Y -dependences arise in inclusives, but only through phase-space integrals; G is independent of y ; and K is translation-invariant. We make 1-dim. kinematic approximations as in AT, I, and II, and ignore quantum numbers for simplicity.

Observables are calculated from the generating functional

$$\mathcal{F}[\xi] = \sum_n \frac{1}{n!} \int \prod_j d^3 r_j \xi(r_j) \left| \prod_k \exp[V(r_j, r_k)] \right| \quad (2)$$

where $|K|^2 = \exp V$, and where $\xi(r)$ is any function of $(y, \vec{b})$. The limits of

y integrals are $\pm Y/2$; and the $\bar{b}$ integrals are damped by $|G|^2$.

As in I and II, we assume $V = V_S + V_L$ where V_S is short range in rapidity, while V_L is weak, slowly varying, translation-invariant in y and $\bar{b}$, and positive definite.

As a definite parametrization of V_L , we adopt here the following 3-parameter form:

$$V_L(\bar{r}) = \int \frac{d^3 k}{(2\pi)^3} \frac{e^{i \bar{k} \cdot \bar{r}}}{\lambda_y^{-\sigma} k_1^\sigma + \lambda_b^{-2} \bar{k}_\perp^2 + \mu^2} \quad (3)$$

where $\bar{k}_\perp = (k_2, k_3)$ is Fourier-conjugate to $\bar{b}$, and k_1 is Fourier-conjugate to y .

This parametrization exhibits a power-law rapidity dependence characteristic of branch point in the l -plane when $\sigma < 2$, and an exponential damping in $\bar{b}$.

In this paper we will assume for greatest simplicity that $V_S = 0$; i. e., no short-range rapidity correlations are present. Thus the short-range ensemble obtained by deleting V_L has a (generalized) Chew-Pignotti (or independent emission) form:

$$\mathcal{Z}_S[\xi] = \sum_n \frac{1}{n!} \int \prod_j d^3 r_j \xi(r_j) |G(r_j)|^2 = \exp \left\{ \int d^3 r \xi(r) |G(r)|^2 \right\}. \quad (4)$$

Since $|G|^2$ is independent of y , we obtain a simple Regge $\mathcal{Z}_S$ for $\xi = \text{const.}$

To calculate the full $\mathcal{Z}[\xi]$ we apply the functional-integral representation as discussed in I and II; here the expression is exact, for $Y \rightarrow \infty$, since $V_S \equiv 0$. Following Weigel's discussion⁽⁴⁾, we obtain as in II,

$$\mathcal{Z}[\xi] = \int \delta\phi \exp \left\{ - \int d^3 r \mathcal{L}[\phi(r); \xi(r)] \right\} / \int \delta\phi \exp \left\{ - \int d^3 r \mathcal{L}_0[\phi(r)] \right\} \quad (5)$$

where

$$\mathcal{L}_0[\phi] = \frac{1}{2} \lambda_y^{-\tau} (\nabla_y^{\tau/2} \phi)^2 + \frac{1}{2} \lambda_b^{-2} (\bar{\nabla}_b \phi)^2 + \frac{\mu^2}{2} \phi^2 \quad (6)$$

and

$$\mathcal{L}[\phi; \xi] = \mathcal{L}_0[\phi] - \xi(r) |G(r)|^2 e^{\phi(r)} \quad (7)$$

We choose $\tau = 1/2$, following the reasoning outlined in reference (2).

To calculate single-particle inclusive densities, according to the formalism in AT, we must evaluate $\mathcal{Z}[\xi]$ with $\xi(r') = e^{V(r', r)}$ and divide by $\mathcal{Z}[1]$. Specifically, from equations (5.4) and (5.7) of AT, we compute the density g given by equation (2.6) of AT:

$$g(r) = |G(r)|^2 \mathcal{Z}[e^{V(\cdot, r)}] / \mathcal{Z}[1] \quad , \quad (8)$$

which should be independent of y as $Y \rightarrow \infty$.

The inclusive density in $P_\perp$ is then given by the absolute square of the transverse Fourier transform of the square root of g :

$$\rho(\vec{P}_\perp) = |F(\vec{P}_\perp)|^2 \quad (9)$$

where

$$F(\vec{P}_\perp) = \int d^2b \, e^{i\vec{b} \cdot \vec{P}_\perp} \sqrt{g(\vec{b})} \quad .$$

[We note $g \geq 0$, from the formula given above.]

For large $P_\perp$, the function F can be computed by steepest-descents, as sketched in Appendix C of AT, once we have a definite form for $g(\vec{b})$.

The parameters of our calculation are to be set such that $\mathcal{Z}[1]$ describes a critical ensemble; this sets the normalization of $\int |G|^2 d^2b$, in terms of the three parameters of $V = V_L$.

We wish to investigate the consequences of this critical behavior for

$\rho(\vec{P}_\perp)$ at large $P_\perp$. We note $|G(b)|^2$ is, in principle, determined by single particle exclusive production in the central region. We will determine in practice the b -dependence of $|G|^2$ from data on low-multiplicity, or moderate energy inclusive, cross-sections. Experimentally one finds⁽⁵⁾ the shape of $\rho(\vec{P}_\perp)$ is very well approximated by $e^{-6P_\perp}$ (where $P_\perp$ is measured in GeV units) at laboratory collision energies below 100 GeV. Above that energy, an increase is seen at large $P_\perp$ compared to lower energy data; this increasing trend continues at least through ISR energies.⁽⁵⁾ We associate the relative increase at large $P_\perp$ with collective phenomena arising through V_L . Our goal here is to reach at least a qualitative understanding of this phenomenon, based on proximity of the ensemble to criticality. This was not possible in AT, as critical behavior was excluded at an early stage through reliance on a truncated cluster expansion for $\mathcal{G}$. Although we cannot give a complete solution to the problem as posed here, we can identify some characteristics, subject to experimental tests, following from our hypothesis that V_L is responsible for the large- $P_\perp$ enhancements.

We note here that associated-multiplicity distributions are easily obtained, once the solutions for (5) - (8) are found; it is only necessary to introduce $\xi = Ze^V$ (where Z is a positive real variable); then Z plays the role of a generating variable for associated multiplicities.

Instead of eq. (8), we might use the alternative expression, equation () of AT;

$$g(r) = \left. \frac{\delta \ln \mathcal{G}[\xi]}{\delta \xi(r)} \right|_{\xi=1} \quad (10)$$

This leads to a formally simple expression for $g/|G|^2$ in terms of the functional average of $\exp[\phi(r)]$. However, the features of g arising from V become

somewhat more concealed in the formalism. We prefer eq. (8) for this reason; explicit V dependence appears in the "classical" saddle-point approximation, even without the "loop" corrections from $\mathcal{L}_0$. If we use eq. (10), the V -dependence arises entirely from "nonclassical" terms in the functional integral.

II. Analytic Approximation Methods

Proceeding as in references (2) and (3), we evaluate $\mathcal{F}$ by saddle-point methods. The saddle-point function $\hat{\phi}(y, \bar{y})$ will satisfy $\frac{\delta \mathcal{L}}{\delta \hat{\phi}} = 0$. Note that $\hat{\phi}$ will now be a functional of ξ , as will be the coefficients C_n (defined in I) of the effective Lagrangian (expanded around $\hat{\phi}$).

Since we will need to evaluate $\hat{\phi}$ with only one choice of function $\xi(r) = e^{V(r-r_1)}$, we may write the dependence of $\hat{\phi}$ on r_1 (as well as r) explicitly. Then $\hat{\phi}(r; r_1)$ satisfies:

$$\lambda_y^{-\sigma} \nabla_y^\sigma \hat{\phi} + \lambda_b^{-2} \nabla_b^2 \hat{\phi} + \mu^2 \hat{\phi} = |G(r)|^2 \exp [\hat{\phi}(r; r_1) + V(r-r_1)] \quad (11)$$

(with $\sigma = 1/2$).

Given a solution $\hat{\phi}(r; r_1)$, we then expand $\mathcal{L}$ around $\hat{\phi}$, to the fourth order. Defining $\psi = \phi - \hat{\phi}$, we change functional-integral variable to ψ , as in I, and write:

$$\begin{aligned} \mathcal{L}[\psi; e^{V(\cdot, r_1)}] &= \frac{1}{2} \lambda_y^{-\sigma} (\nabla_y^{\sigma/2} \psi)^2 + \frac{1}{2} \lambda_b^{-2} (\bar{\nabla}_b \psi)^2 + \frac{\mu^2}{2} \psi^2 \\ &\quad - \left\{ \frac{C_2(r, r_1)}{2!} \psi^2 + \frac{C_3(r, r_1)}{3!} \psi^3 + \frac{C_4(r, r_1)}{4!} \psi^4 \right\} \\ &\quad + \hat{\mathcal{L}}(r, r_1) \end{aligned} \quad (12)$$

where

$$\hat{\mathcal{L}}(r, r_1) = \mathcal{L}[\hat{\phi}(r, r_1); e^{V(r, r_1)}] \quad (13)$$

is the saddle-point value of $\mathcal{L}$; and

$$C_n(r, r_1) = |G(r)|^2 \exp [\hat{\phi}(r, r_1) + V(r - r_1)] \quad (14)$$

for all n , particularly simple because of our assumption $V_S = 0$.

If we keep only quadratic terms in ψ , we can explicitly carry out the functional integrals over ψ , in terms of the eigenvalues of a linear differential equation for ψ . This will not lead to a satisfactory theory of behavior for $V > 0$, as the fluctuations ψ^3 , ψ^4 , etc. will become very large. However, we can identify qualitative phenomena and attempt to generalize. The mathematical problems are exactly those of a fluid at the critical point perturbed by a strong localized field, $V(r - r_1)$; we want to know the space-dependent density in the presence of this field.

Unlike the usual theories, e. g., of magnetic systems with external applied field, the dependence of all the C_n 's on V means that some "internal" effective parameters (e. g., local temperature) in the analog system will vary with r_1 , as well as the "external field" (analog) shift in $\hat{\phi}$. Thus the phase (i. e., ordering) of the analog system can change as a function of r_1 , and a very rapid change in inclusive density with $\bar{b}_1$ can appear, which enhances the large- P_1 cross-sections. We can illustrate the appearance of this phenomenon in the ψ^2 approximation; we write

$$\mathcal{L}[\psi] \cong \hat{\mathcal{L}} + \mathcal{L}_2([\psi]; r, r_1) \quad (15)$$

where

$$\mathcal{L}_2 = \frac{1}{2} \lambda_y^{-\sigma} (\nabla_y^{\sigma/2} \psi)^2 + \frac{1}{2} \lambda_b^{-2} (\bar{\nabla}_b \psi)^2 + \frac{1}{2} [\mu^2 - C_2(r, r_1)] \psi^2. \quad (16)$$

Then we evaluate

$$\mathcal{Z}[e^{V(\cdot, r_1)}] = F(r_1) \cdot \exp \left\{ - \int d^3r \mathcal{L}(r, r_1) \right\} \quad (17)$$

where $\hat{\mathcal{L}}$ is given by eq. (13), and

$$F(r_1) = \int \delta \psi \exp \left\{ - \int d^3 r \mathcal{L}_2([\psi]; r, r_1) \right\} / \int \delta \phi \exp \left\{ - \int d^3 r \mathcal{L}_2[\phi] \right\}. \quad (18)$$

Since $\mathcal{L}_2$ is quadratic in ψ we can express the functional integral in terms of eigenvalues appropriate to the diagonalization of $\mathcal{L}_2$. Define eigenvalues $E_k(r_1)$ and eigenfunctions $\psi_k(r, r_1)$ of the differential equation:

$$[\lambda_y^{-\sigma} \nabla_y^{\sigma} + \lambda_b^{-2} \nabla_b^2 + u^2 - C_2(r, r_1)] \psi_k(r, r_1) = E_k(r_1) \psi_k(r, r_1) \quad (19)$$

Expanding an arbitrary ψ in terms of this basis,

$$\psi(r, r_1) = \sum_k f_k \psi_k(r, r_1), \quad (20)$$

we can diagonalize $\mathcal{L}_2$; and the functional integral in eq. (18) can be carried out over the coefficients f_k . The result⁽⁴⁾ is:

$$\frac{F(b_1)}{F(\infty)} = \exp \left[- \sum_k \frac{1}{2} \ln E_k(b_1) + \sum_k \frac{1}{2} \ln E_k(\infty) \right] \quad (21)$$

where we have used the property $V(b_1) \xrightarrow{b_1 \rightarrow \infty} 0$ to eliminate the denominator in eq. (18). The ratio (21) is that required in finding $g(b_1)$, which is independent of y_1 in the central region. (We may set $y_1 = 0$, therefore.) The spectrum $\{E_k\}$ of eq. (19) will be continuous for $E_k > 0$; and only if all $E_k > 0$ is the quadratic approximation (15) possible.

We obtain critical rapidity-fluctuation behavior in the densities at b_1 when

$$\lim_{k \rightarrow 0} E_k(b_1) = 0. \quad (22)$$

Note the eigenvalues E_k will, in general, decrease as V increases, from eq. (19). The form (3) assumed for V is monotonically increasing as b_1 decreases from ∞ , if G^2 is maximal near $b = 0$. Thus, if the densities are

near criticality for large b_1 , we pass through a critical condition as b_1 decreases. If we could follow $\mathcal{F}(b_1)$ further inward we would find the analog of a two-phase coexistence region; but this region is not accessible in the approximation (15). We consider the above discussion as sufficient to elucidate the character of the cooperative phenomena expected; a lowering of analog temperature as b_1 decreases together with a b_1 -dependent external field.

The mathematical difficulties encountered by the quadratic approximation of a stochastic-field formulation of first-order phase transitions have been discussed by Langer.⁽⁶⁾ His results show that including a quartic term in ψ (in $\mathcal{L} - \mathcal{L}$) would suffice to remove unphysical features from our functional integrals. There is at present no exact evaluation method for the integrals in that case, but simpler integrals of this type, with constant $|G|^2$ and V , have become the prototype for applications of renormalization-group methods in critical phenomena; cf. Wilson and Kogut.⁽⁷⁾

III. Analog-System Expectations and a Rough Approximation

Since we have no analytic method at present adequate for extracting the consequences of equations (5) - (9), we may try to reason on the basis of the expected behavior of a physical analog system. Using the Feynman-Wilson fluid analog⁽⁸⁾ the density g is regarded as a fluid density, critical at large b .

We have shown that for small b the system is then in a liquid-gas coexistence region, as if the temperature were lower than critical. Evaluation of inclusive spectra at large P_1 will sample only this small- b region, while at small P_1 we will see critical behavior. The structure of typical events in rapidity and b -space will be governed by functions $\phi(r)$ which reflect dominant ensemble configurations in the fluid, in y -space and b -space.

In the coexistence region of simple fluids we expect, instead of a homogeneous medium, small droplets of liquid in the gas and small bubbles of gas in the liquid.⁽⁹⁾ These structures have, near criticality, a small size, limited by surface energies (in our theory by the long-range forces). Thus one expects a "grainy" $\phi(r)$ is typical of the coexistence region. In our analog, this means the densities at small b are rapidly varying in b on a small scale, leading to a Fourier transform⁽¹⁰⁾ which for large P_L is much larger than that expected for $V = 0$. This enhancement can only appear when Y is sufficiently large to allow many particles to participate in the correlations at a given b .

We can illustrate how the change in P_L slopes, as Y increases, arises in a qualitative way from our formalism, provided we remain at "moderate" energies where the collective effects are not at full strength, and there is no necessity to discuss a phase transition. Suppose V is small; then μ^2 is large, and we can solve eq. (11) approximately; we also drop the derivative terms here, to simplify the argument. Then

$$\hat{\phi}(r, r_1) \cong \mu^{-2} |G(r)|^2 \exp[V(r - r_1)] \quad (23)$$

From eq. (8), using eq. (13) with a similar approximation to $\mathcal{L}$ (and dropping $\mathcal{L} - \hat{\mathcal{L}}$), we obtain

$$g(r) \cong |G(r)|^2 \exp \left\{ \int dr' |G(r')|^2 \left[e^{V(r' - r)} - 1 \right] \right\} \quad (24)$$

The integral over y' runs over the allowed phase space; $(-Y/2, +Y/2)$. Note G is independent of y . We should only keep terms linear in V in the exponent of g , consistent with $V \ll 1$. Defining

$$\bar{V}(\bar{b}; Y) = \int_{-Y/2}^{+Y/2} V(\bar{b}, y) dy \quad , \quad (25)$$

we see $\bar{V}$ is an increasing function of Y , for each $\bar{b}$; and we obtain the approximation

$$g(\bar{b}) = |G(\bar{b})|^2 \exp \left\{ \int d^2b' |G(\bar{b}')|^2 \bar{V}(\bar{b} - \bar{b}', Y) \right\} \quad (26)$$

which shows g is an increasing function of Y for fixed $\bar{b}$, especially near $\bar{b} = 0$, where $\bar{V}$ will have its maximum effect. At large $\bar{b}$ there will be no energy dependence since $\bar{V}$ drops off, leaving $g \approx |G|^2$. We recognize eq. (26) as an approximation to the g equation of AT, in the $Y \rightarrow \infty$ limit. [A somewhat better approximation is then suggested; in eq. (4.6) of AT, replace v and v'' by their finite- Y counterparts.] At general b , eq. (26) yields a multiple-scattering series in which large $P_{\perp}$ cross-sections are enhanced whenever $\bar{V} > 0$, as discussed in AT. We believe eq. (26) represents a reasonable extension of the formalism of AT to include the approach to the asymptotic limit.

Some quantitative estimate of this enhancement can be reached if we examine the specific features of V , from eq. (3). At fixed b , as $Y \rightarrow \infty$, $\bar{V}$ approaches its limiting value from below with a correction term of order $Y^{-1/2}$. Thus, at fixed $P_{\perp}$, we expect the inclusive cross-section to approach its asymptotic limit in a similar way. The form (26) suggests $\ln \rho(P_{\perp})$ should be plotted against $Y^{-1/2}$ at each fixed $P_{\perp}$. The intercepts at $Y^{-1/2} = 0$ should be approached linearly.

These (approximate) scaling expectations can thus be checked against hadron data. As a preliminary indication, note that in the region 100-2000 GeV, $Y^{1/2}$ varies about the same as $S^{1/8}$. From fits given for a scaling variable $(P_{\perp}/S^{1/8})$ in the survey of large- $P_{\perp}$ cross-sections written by S. Ellis,⁽¹⁰⁾ we see that a scaling form using the variable $P_{\perp}/Y^{1/2}$ gives a reasonable representation of the data, for $P_{\perp} > 3$ GeV. (In our model, as $Y \rightarrow \infty$ for fixed $P_{\perp}$,

we expect to recover a function of $P_{\perp}$ alone; however, the cross-section at fixed $P_{\perp}$ should be a function of Y only through the scaling variable.)

We consider these results to support the general mechanism proposed, and encourage further work to elucidate directly the consequences of equations (5) - (9).

The associated-multiplicity distributions predicted by this approximate theory will be of Van der Waals type⁽¹¹⁾ (with the Van der Waals b -parameter equal to zero, with our specific assumption $V_S \equiv 0$), with $\frac{\bar{n}(P_{\perp})}{Y}$ increasing with $P_{\perp}$. Since $V \geq 0$ at all b , we also find an increase with Y in the inclusive density at $P_{\perp} \cong 0$, given by the integral of $g(b)$ over all b . In the model defined by eq. (3), in our "moderate-energy" approximation (26), the correction term to the asymptotic result decreases as $Y^{-1/2}$; the $Y = \infty$ limit is approached from below, as seen in ISR data. However, these functional dependences (as well as those in the preceding paragraph) are not necessarily characteristic of the true asymptotic regime, including derivative terms in the complete solution, (5).

A proper solution of the phase-transition problem posed in equations (5)-(8) may lead to very complex behavior of g as $Y \rightarrow \infty$, for small b . We may try to guess some expected features, but we must be careful to keep certain essential difficulties in mind, as explained below.

In solving the ϕ -field theory represented in (5), the ground-state or "vacuum" ϕ in the presence of the "external field" V , will play a dominant role in determining $g(b)$. Equation (11) is an approximate equation for this ground state. The b -dependence of g is characterized by its response to a small change in b . If the vacuum state is stable and isolated from other states by an energy gap, as a system above critical point, this response is smooth in b .

However, if there is a degeneracy of the vacuum, a phase-coexistence region is present in the analog, and a small change in b , as a small perturbation in the external field, yields a large change in density g , analogous to moving along a horizontal isotherm in the phase diagram.

These transitions between different vacua are generally very difficult to treat analytically. [Classical theories of phase transitions, e.g., Vander Waals, have introduced "externally" the Maxwell construction to give flat isotherms.] Any detailed prediction from eq. (5) must be based on a theoretical method for phase transitions which give analytic behavior for $Y < \infty$, but flat isotherms in the $Y \rightarrow \infty$ limit. In this way we connect the theoretical problems of large $P_{\perp}$ cross-sections with the instability properties, discussed in I, which may be responsible for the weak and electromagnetic interactions.⁽²⁾

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