PATHFINDER ATOMIC POWER PLANT

RECIRCULATION PIPING

DESIGN

MASTER

Submitted to
U. S. ATOMIC ENERGY COMMISSION
NORTHERN STATES POWER COMPANY
and
CENTRAL UTILITIES ATOMIC POWER ASSOCIATES

by

ALLIS-CHALMERS MANUFACTURING COMPANY
ATOMIC ENERGY DIVISION
Milwaukee 1, Wisconsin

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RECIRCULATION PIPING DESIGN

By: D. J. Nolan and J. F. Patterson

Submitted to

U. S. ATOMIC ENERGY COMMISSION
NORTHERN STATES POWER COMPANY

and

CENTRAL UTILITIES ATOMIC POWER ASSOCIATES

by

ALLIS-CHALMERS MANUFACTURING COMPANY

Under

Agreement dated 2nd Day of May 1957, as Amended
betweenAllis-Chalmers Mfg. Co. & Northern States Power Co.
under

AEC Contract No. AT(11-1)-589

January 10, 1964

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PATHFINDER ATOMIC POWER PLANT
RECIRCULATION PIPING DESIGN

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FOREWORD

One of a series of reports on research and development in connection with the design of the Pathfinder Atomic Power Plant, this particular report deals with the design of the recirculation piping. The Pathfinder plant will be located at a site near Sioux Falls, South Dakota, and is scheduled for operation in 1964. Owners and operators of the plant will be the Northern States Power Company of Minneapolis, Minnesota. Allis-Chalmers is performing the research, development, and design as well as being responsible for plant construction.

The U.S. Atomic Energy Commission, through Contract No. AT(11-1)-589 with Northern States Power Company, and Central Utilities Atomic Power Associates (CUAPA) are sponsors of the research and development program. The plant's reactor will be of the Controlled Recirculation Boiling Reactor type with Nuclear Superheater.

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1.0 INTRODUCTION

Forced recirculation of coolant in the Pathfinder reactor is accomplished by three mixed-flow centrifugal pumps. The coolant is forced, by the pumps, through piping to the reactor vessel. Fig. 1 shows one of the three recirculation loops that are located in the reactor building.

The Pathfinder's recirculation piping loops were designed to comply with the code for pressure piping ASA B31.1-1955 and applicable nuclear code cases. A piping flexibility analysis was performed using the Kellogg General Analytical Method. Other calculations were made to determine whether vibration was a significant problem. For vibration analysis refer to Appendix C.

2.0 OBJECTIVE

To report code calculations, flexibility and vibration analyses, and all piping stress calculations.

3.0 CONCLUSIONS

All stresses are within the allowable limits. Calculations indicate that there should be no resonance difficulties during any phase of the recirculation pump operation. Operation of the pumps in the field confirm this.

Calculations performed indicate that loop No. Two is the most critical. It also indicates that vibration should present no problem.

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Fig. 1... Reactor recirculation loop

4.0 DESCRIPTION OF RECIRCULATION PIPING COMPLEX

The recirculation piping is designed for 700 psig at a temperature of 500 F. The inside diameter of all piping is 20-1/2 in. The outside diameter of the piping varies, depending upon the piping material. A typical recirculation loop is shown in Fig. 1; Figs. 2 and 3 show the piping arrangement within the reactor building.

Part of the piping is fabricated of 5/8 in. thick ASTM A155 Gr KC70 type carbon steel with a 1/8 in. stainless steel cladding of Type 304L. The cladding is integrally bonded to the carbon steel backing material by a nickel interface layer. This type construction, both suction and discharge pipes, extends from the reactor vessel to a point one foot from the reactor side of the butterfly valves. The single foot of pipe before the butterfly valves, and the piping connecting recirculation pumps, is constructed of 13/16 in. thick stainless steel, Type TP304.

5.0 DESIGN CRITERIA

Design and construction of the recirculation piping system is in accordance with the Code for Pressure Piping ASA B31.1-1955 and applicable nuclear code cases. In order to fulfill code requirements, and ensure complete structural integrity, the following design criteria have been adopted:

5.1 Loading Conditions.

1. Internal pressure.
2. Mechanical forces including the dead weight of pipe, insulation

Fig. 2...Piping arrangement within Pathfinder reactor building.
(43-002-024, Sheet 1)

Fig. 3...Reactor with Pumps in Reactor Building

and contents, and the friction loads at the pump supports.

3. Thermal loads due to

- a). Differential axial thermal expansion of vessel, piping, and pump casing.
- b). Differential radial expansion at the junction of the clad carbon and solid stainless steel pipe.
- c). Differential axial expansion between the base metal and stainless steel cladding of the carbon steel pipe.

5.2 Steady-State Mechanical Stress Limits

Before summarizing the stress limits used in the Pathfinder design, it is necessary that certain terms be defined.

5.2.1 Stress Intensity...The stress quantity upon which all stress limitations are established here is "stress intensity." It may be defined as the equivalent intensity of combined stresses due to multi-axial stress conditions, and is the largest algebraic difference between two of the three principal stresses at a given point.

5.2.2 Primary Stress...A direct or shear stress developed by the imposed loading which is necessary to satisfy only the simple laws of equilibrium. The primary stresses are the longitudinal stresses due to the pressure, dead weight, and friction loads.

1. Basis of Stress Limits

Where multi-axial conditions exist, some theory must be used to correlate these conditions with available

uniaxial test data. Although some experimental data indicates that the "Maximum Distortion Energy Theory" is more accurate for most ductile materials (Ref. 2), there is limited experimental data for ductile materials in the plastic range. Because of this, and since the "Maximum Shear Stress Theory" is slightly more conservative and easier to apply, this theory is used to calculate maximum intensity of combined stress.

2. Primary Stress Limit

Primary stresses are limited by code. No primary stress shall exceed the allowable code "S" value at an operating temperature tabulated in Table I of special code case N-7.

5.3 Steady -State Thermal Stress Limits

Steady-state thermal stresses are limited by Para. 622(c), Section 6 of the Code for Pressure Piping ASA B31.1-1955. This paragraph states that the maximum computed expansion stress, based on 100 per cent of the expansion and "E" for the cold condition, shall not exceed the allowable stress range, S_A .

$$\text{Where } S_A = f(1.25 S_C + 0.25 S_h)$$

and S_C = allowable stress (S value) in the cold condition

S_h = allowable stress (S value) in the hot condition

f = stress range reduction factor for cyclic conditions (1 for 7000 cycles or less)

6.0 OPTIMIZING DESIGN OF RECIRCULATION LOOPS

The recirculation piping loops are very rigid since they consist mainly of straight lengths of pipe from the reactor vessel to the recirculation pumps. Restraining the thermal expansion would cause stresses well above the yield strength of the pipe materials. Therefore, the pumps and piping must be allowed to expand radially from the reactor vessel. This freedom of motion is provided by the pump support arrangement.

The recirculation pump support assembly will slide on Lubrite expansion plates to allow freedom of movement in a horizontal plane. The pump casing trunnions are mounted in Lubrite bushings to provide freedom of rotation about the trunnion axis.

Several piping flexibility analyses were made to determine the best support arrangement and combination of piping materials.

The types of support arrangement analyzed were:

1. Freedom in horizontal plane

Assumes:

- a. Pump can move in x and z directions on a frictionless surface (Fig. 1).
- b. Pump can rotate freely about the z-axis.

2. Freedom in horizontal plane and about the z-axis

Assumes:

- a. Pump can move in 'x' and 'z' directions on a frictionless surface.

- b. Pump can rotate freely about the y-axis.
- c. Pump mounted on trunnions which allows freedom of rotation about the z-axis.

3. Effects of friction considered for Case 2.

Assumes:

- a. Friction force of 5000 lbs resisting movement in the horizontal plane.
- b. Torque of 6500 ft-lbs on pump casing considered as external moment about the y-axis.
- c. Resisting friction moment of 1600 ft-lbs in trunnion bearings.

4. Pump considered as floating point.

Assumes:

- a. Freedom of movement in all planes.
- b. Freedom of rotation about x, y, and z-axis
- c. No friction or external restraints considered.

The material combinations studies were:

- 1. Piping for entire system-----Al55KC 70 with stainless steel cladding
- Pump Casing-----Carbon steel with stainless steel cladding
- 2. Piping for entire system-----Al55KC 70 with stainless steel cladding
- Pump Casing-----Solid stainless steel casting
- 3. Piping from reactor vessel-----Al55KC 70 with stainless steel cladding
to one foot on reactor side of
butterfly valves.

Piping from end of clad carbon-----	Type TP304 stainless
piping to pump casing	steel
Pump Casing-----	Solid stainless steel
	casting

These analyses were made using the Kellogg General Analytical Method, with each loop considered as a two anchor-point system having one intermediate restraint. The connections of the suction and discharge pipes at the vessel were considered as the two anchor points, with the pump as an intermediate restraint. This method was adapted for use in the piping flexibility program coded for the IBM 704 computer by Allis-Chalmers.

The Number Four pump support arrangement listed above gave the most favorable loop conditions regarding forces and moment magnitude. However, this type of pump support presents additional factors to be considered such as vibration, and feasibility of supporting the pumps on constant support hangers from above. The Number Two support arrangement gives the most favorable loop conditions for the design of the pump supports.

The results also show that the effects of friction and external moments did not affect the magnitude of forces and moments enough to necessitate a change in the design of the pipe loop or the pump supports.

The results of the stress analysis showing the maximum stresses anticipated in the pipe loops are in Table I. This data includes

TABLE I

Classification of Stress	Max. Axial Stress Near Dissimilar Joint		Max. Hoop Stress At Dissimilar Joint		Allowable Stress
	See Note 1	See Note 2	See Note 1	See Note 2	
1. Thermal Expansion and Discontinuity Stresses *					SALL
Clad pipe - inside	14,150	14,150	19,160	20,070	} 23,620
Clad pipe - outside	610	1,810	19,710	19,710	
Sta. St'l. pipe - inside	-8,040	-8,040	-12,820	-12,820	} 26,560
Sta. St'l. pipe - outside	8,040	9,020	-12,360	-13,130	
2. Pressure Stresses					Sh
Clad pipe - inside	4,610	5,310	9,930	10,630	} 15,750
Clad pipe - outside	4,610	4,610	9,230	9,230	
Sta. St'l. pipe - inside	4,250	4,950	9,200	9,900	} 12,500
Sta. St'l. pipe - outside	4,250	4,250	8,500	8,500	
3. Max. Total Stress					Sy
Clad pipe - inside	18,760	19,460	29,090	30,000	} 31,000
Clad pipe - outside	5,220	5,220	28,940	28,940	
Sta. St'l. pipe - inside	5,050	5,720	9,780	10,560	} 19,500
Sta. St'l. pipe - outside	12,290	13,270	10,130	10,130	

where $SALL = f (1.25 Sc + 0.25 Sh)$

Sc = Allowable stress in cold condition (ambient temperature)

Sh = Allowable stress in hot condition (operation temperature 490°F)

Sy = Yield strength of material at operating temperature.

* The stresses due to the dissimilar metal joint are included in these figures based on a butt joint of two dissimilar metals. (See Calculations in Appendix A, A3, Condition 2). However the joint design calls for a gradual metal transition resulting in a considerable reduction in stresses. (See Condition 3 Calculations).

1 = Max. Principal stress psi

2 = Equivalent Uniaxial Intensity of Combined Stress, psi,
Based on Maximum Shear Theory

stresses resulting from forces and moments due to the thermal expansion of all the recirculation loop components. These stresses are well within code allowances. Detailed calculations are given in Appendix A.

The maximum stress calculations are based on an assumption that the joint between the stainless steel and the clad pipe is a sharp butt joint. The actual joint design is shown in Fig. 4. The transition from Type 304L stainless steel pipe to the clad carbon steel pipe is very gradual. This is accomplished by machining a 15 degree taper on the end of the clad carbon steel pipe. The pipe is overlayed with stainless steel for the length of the taper.

A piping flexibility analysis of the final recirculation piping complex was run by Arthur D. Little, Inc. as a check. The moments, forces, deflections, rotations and stresses were calculated for the pertinent points in the systems. These calculations are tabulated in Appendix B.

7.0 VIBRATION ANALYSIS

In this analysis the vibratory characteristics of the recirculation pump and piping of Loop No. Two (the most critical from a vibration standpoint) were determined. The natural frequencies of vibration were ascertained and compared to the driving frequency of the rotating elements of the pump and motor assembly. Since there is unbalance in these rotating elements, vibration will be induced in the pump, motor mass, and the connected piping. Hence, the pump, motor masses, and the attached piping will be referred to as the vibration system.

Fig. 4...Weld Preparation of Austenitic Stainless Steel Clad Carbon Steel Pipe to Austenitic Stainless Steel Pipe

If the induced vibration is at the natural frequency or frequencies of the vibration system, resonance will result. Since the amplitude of vibration becomes very large at resonance, a resonant condition must be avoided, as this may result in structural failure. The natural frequencies can be shifted by changing the piping configuration or by introducing additional restraints. However, even if resonance is not present or can be avoided, the induced vibration present will impose loads on the piping, pump casing and vessel.

In the actual system there are two obvious sources of excitation which would induce vibration:

1. unbalance of the rotating elements,
2. hydraulic impulses produced by the impeller blades.

The natural frequencies of Loop No. Two were calculated as 414 cpm and 3850 cpm. The ratio of the operating speed to the lowest natural frequency is about 1.4. It was determined that the amplitude of vibration under operating conditions would be small. Actual operation of all three pump loops in the field has proven this to be the case.

In conclusion, vibration is not expected to be a problem. A detailed presentation of the analysis and calculations performed is included in Appendix C.

APPENDIX A

Thermal Expansion Stress Calculations

- A.1 Code Calculations
- A.2 Pressure Stresses
- A.3 Analysis of Dissimilar Weld Joint
- A.4 Analysis of Stresses Due to Clad Construction

A.1 CODE CALCULATIONS

THICKNESS OF CLAD CARBON PIPE

$$t_m = \frac{PD}{2S + 2YP} + C$$

Ref. 1, P. 13

where: t_m = minimum pipe wall base metal thickness, inches

P = design pressure, 700 psig

D = outside diameter of pipe, 22 inches

S = allowable stress at design temperature @500 F for A155KC70
15750 psi

Y = 0.4 Ref. 1 Para. 122(a)

C = allowable for threading, mechanical strength, and/or corrosion,
0.065

$$t_m = \frac{700 \times 22}{2 \times 15750 + 2 \times 0.4 \times 700} + 0.065$$

$$t_m = 0.480 + 0.065$$

$$t_m = 0.545 \text{ inches}$$

The actual thickness of 0.625 inches is adequate

THICKNESS OF SOLID STAINLESS STEEL

S = allowable stress at design temperature of 500 F for A

TP 304, 12,500 psi (Ref. 4 - Special Case N-7)

$$t_m = \frac{700 \times 22.125}{2 \times 12500 + 2 \times 0.4 \times 700} + 0.065$$

$$t_m = 0.607 + 0.065$$

$$t_m = 0.672 \text{ inches}$$

The actual pipe wall thickness of 0.8125 is adequate

A.2 STRESSES DUE TO PRESSURE ALONE

AXIAL STRESS

$$\sigma_a = \frac{P}{Y^2 - 1}$$

$$P = 700 \text{ psi}$$

Ref. 3, p. 52

$$\text{where: } Y = r_i = \frac{r_i + t}{r_i}$$

$$P = 700 \text{ psig}$$

$$\text{CYLINDER A} \quad r_i = 10.275, \quad t = .625$$

(Clad is included for strength)

$$\sigma_a = 4610 \text{ psi}$$

$$\text{CYLINDER B} \quad r_i = 10.25, \quad t = .812$$

$$\sigma_a = 4,250 \text{ psi}$$

CIRCUMFERENTIAL STRESS - OUTSIDE

$$\sigma_n = \frac{2P}{Y^2 - 1}$$

Ref. 3, p. 52

where: σ_n = circumferential or hoop stress, psi

CYLINDER A

$$\sigma_n = 9,230 \text{ psi}$$

CYLINDER B

$$\sigma_n = 8,500 \text{ psi}$$

CIRCUMFERENTIAL STRESS - INSIDE

$$\sigma_n = P \frac{Y^2 + 1}{Y^2 - 1}$$

Ref. 3, p. 52

CYLINDER A

$$\sigma_n = 9,930 \text{ psi}$$

CYLINDER B

$$\sigma_n = 9,200 \text{ psi}$$

A.3 DISCONTINUITY STRESSES AT DISSIMILAR METAL JOINTS CAUSED BY TEMPERATURE

TEMPERATURE 500 F

The pipe junction is considered at a steady temperature only.

Since cylinder "A", as shown, is difficult to treat accurately, three mathematical models are chosen to represent this component.

The three mathematical models will be treated in order of decreasing conservatism. The respective conditions are calculated and a graphical comparison presented.

CONDITION 1

Cylinder "A" can be treated simply as a solid carbon steel cylinder.

This is the most conservative assumption.

CONDITION 2

Cylinder "A" can be treated as a composite carbon steel cylinder with a stainless steel (cladding) liner.

A coefficient of thermal expansion is determined for the composite cylinder.

CONDITION 3

Cylinder "A" can be treated taking into account the bimetallic weld region. The bimetallic weld region can be considered as a short cylinder with a linearly varying coefficient of thermal expansion from that of the weld metal and stainless steel clad at one end to that composite (carbon steel - stainless steel clad) cylinder at the other and connecting to a semi-infinite composite cylinder.

FORMULAE FOR CYLINDERS

Influence Coefficients

Mean Radius of Cylinder = a

Thickness of Cylinder = h

Poissons Ratio = $\nu = 0.3$

Flexural Rigidity, D = $\frac{Eh^3}{12(1-\nu^2)}$

Ref. 5, p. 469-470

$$\beta^4 = \frac{3(1-\nu^2)}{a^2 h^2}$$

C_1, C_2, C_3, C_4 are constants of integration at any distance, x

$$\delta_x = w_x = e^{-\beta x} (C_1 \cos \beta x + C_2 \sin \beta x) + e^{\beta x} (C_3 \cos \beta x + C_4 \sin \beta x)$$

$$\theta_x = \frac{\partial w}{\partial x} = -\beta e^{-\beta x} \{C_1 - C_2\} \cos \beta x + (C_1 + C_2) \sin \beta x \\ + \beta e^{\beta x} \{(C_3 + C_4) \cos \beta x - (C_3 - C_4) \sin \beta x\}$$

$$M_x = -D \frac{\partial^2 w}{\partial x^2} = -2D\beta^2 e^{-\beta x} \{C_1 \sin \beta x - C_2 \cos \beta x\} \\ - 2D\beta^2 e^{\beta x} \{-C_3 \sin \beta x + C_4 \cos \beta x\}$$

$$Q_x = \frac{\partial}{\partial x} (M_x) = -2D\beta^3 e^{-\beta x} \{(C_1 + C_2) \cos \beta x - (C_1 - C_2) \sin \beta x\} \\ + 2D\beta^3 e^{\beta x} \{(C_3 - C_4) \cos \beta x + (C_3 + C_4) \sin \beta x\}$$

For an infinitely long cylinder, the second half of each equation has no meaning and two constants of integration are determined for each of two boundary conditions, M_1 and Q_1 . For a short cylinder we have four constants of integration determined for each of four boundary conditions, M_1, Q_1, M_2, Q_2 .

All influence coefficients below are calculated with the use of computer programs based on the four equations above.

CALCULATIONS

At Design Temperature, 500 F

$$E_{ss} = 26.4 \times 10^6 \text{ psi}$$

$$\alpha_{ss} = 9.4 \times 10^{-6} \text{ ins/in/}^\circ\text{F}$$

$$E_{cs} = 26.8 \times 10^6 \text{ psi}$$

$$\alpha_{cs} = 7.0 \times 10^{-6} \text{ ins/in/}^\circ\text{F}$$

CONDITION 1

Respective radial expansions due to temperature:

$$\text{Cylinder "A", } \delta_T = -7.0 \times 10^{-6} \times 10.625 \times 430$$

$$= \underline{-32,100 \times 10^{-6} \text{ ins}}$$

$$\text{Cylinder "B", } \delta_T = -9.4 \times 10^{-6} \times 10.656 \times 430$$

$$= \underline{-43,100 \times 10^{-6} \text{ ins}}$$

The following set of equations are then obtained:

$$\text{Cylinder "A", } 10^6 \delta_I = 2.331 M_I - 5.120 Q_I - 32,100$$

$$10^6 \rho_I = 2.122 M_I - 2.331 Q_I$$

$$\text{Cylinder "B", } 10^6 \delta_I = 2.019 M_I + 4.625 Q_I - 43,100$$

$$10^6 \rho_I = -1.763 M_I - 2.019 Q_I$$

CONDITION 2

Cylinder "A"

For the composite cylinder

The net temperature deflection is given by

$$\frac{\delta}{R} = -(\alpha_{ss} - \alpha_{cs}) \Delta T \left[\frac{1}{1 + \frac{t_{ss} E_{ss}}{t_{cs} E_{cs}}} \right] + \alpha_{ss} \Delta T$$

where: $\alpha_{ss} = 9.4 \times 10^{-6}$ IN/IN $^{\circ}$ F

$\alpha_{cs} = 7.0 \times 10^{-6}$ IN/IN $^{\circ}$ F

$E_{ss} = 26.4 \times 10^6$ PSI

$E_{cs} = 26.8 \times 10^6$ PSI

$$\frac{1}{(1 + \frac{t_{ss} E_{ss}}{t_{cs} E_{cs}})} = \frac{1}{(1 + \frac{0.125 \times 26.4}{0.625 \times 26.8})} = 0.836$$

$$\therefore \frac{\delta}{R} = \{-(9.4 - 7.0) 10^6 \times 0.836 + 9.4 \times 10^{-6}\} 430$$
$$= + 7.4 \times 10^{-6} \times 430$$

Hence, For Cylinder "A"

$$\begin{aligned}\delta_T &= -7.4 \times 10^{-6} \times 430 \times 10.625 \\ &= \underline{-33,800 \times 10^{-6} \text{ INS}}\end{aligned}$$

As before, for Cylinder "B"

$$\delta_T = -43,100 \times 10^{-6} \text{ INS}$$

Hence, the influence coefficients are:

Cylinder "A"

$$10^6 \delta_1 = 2.331 M_1 - 5.120 Q_1 - 33,800$$

$$10^6 \theta_1 = 2.122 M_1 - 2.331 Q_1$$

Cylinder "B"

$$10^6 \delta_1 = 2.019 M_1 + 4.625 Q_1 - 44,100$$

$$10^6 \theta_1 = -1.763 M_1 - 2.019 Q_1$$

CONDITION 3

Considering the bimetallic weld junction:

Considering cylinder "A"

At position (1), coefficient of thermal Exp = α_{ss}

$$= 9.4 \times 10^{-6} \text{ in/in/}^{\circ}\text{F}$$

$$EA^1 = 26.6 \times 10^6 \text{ PSI}$$

At position (2), coefficient of thermal Exp = $\alpha_{\text{composite cylinder}}$

$$= 7.4 \times 10^{-6} \text{ INS/IN/}^{\circ}\text{F}$$

Then, due to temp.,

$$\delta_{T1} = -43,000 \times 10^{-6} \text{ INS}$$

$$\delta_{T2} = -33,800 \times 10^{-6} \text{ INS}$$

also assuming a linear variation in coefficient of thermal expansion, the slope at each end, is:

$$\theta = -\frac{(43,000 - 33,800)}{2.33} 10^{-6}$$

$$\theta = -3950 \times 10^{-6} \text{ rads (see note 1)}$$

Hence influence coefficients are:

Cylinders "A"

$$10^6 \delta_2 = 2.331 M_2 - 5.120 Q_2 - 33,800$$

$$10^6 \theta_2 = 2.122 M_2 - 2.331 Q_2$$

Cylinder "B"

$$10^6 \delta_1 = 2.019 M_1 + 4.625 Q_1 - 44,100$$

$$10^6 \theta_1 = -1.763 M_1 - 2.019 Q_1$$

Cylinder "A"

$$10^6 \delta_1 = + 6.531 M_1 - 6.095 M_2 - 9.833 Q_1 - 4.772 Q_2 - 44,100$$

$$10^6 \theta_1 = + 6.208 M_1 - 5.082 M_2 - 6.531 Q_1 - 6.095 Q_2 - 3950$$

$$10^6 \delta_2 = - 6.095 M_1 + 6.531 M_2 + 4.772 Q_1 + 9.83 Q_2 - 33,800$$

$$10^6 \theta_2 = + 5.082 M_1 - 6.208 M_2 - 6.095 Q_1 - 6.531 Q_2 - 3950$$

Note 1: To be completely rigorous, bending moments and forces should be applied at each end of cylinder for continuity, but the effect of these is small.

DISCONTINUITY

STRESSES IN THE ASSEMBLY

The stresses tabulated on following pages were calculated with a computer using the following formulas:

δ'_x = displacement radially at x due to discontinuity

bending moments and shear forces only acting.

M_x = discontinuity bending moment at x

STRESSES AT

OUTSIDE SURFACE

$$\sigma_{hoop} = - E \frac{\delta'_x}{R_m} \vee 6 \frac{M_x}{h^2} = SHO$$

$$\sigma_{axial} = - 6 \frac{M_x}{h^2} = SAO$$

INSIDE SURFACE

$$\sigma_{\text{hoop}} = -E \frac{\delta_x}{R_m} + \nu \frac{6 M_x}{h^2} = SHI$$

$$\sigma_{\text{axial}} = \frac{6 M_x}{h^2} = SAI$$

CONDITION 1 CYLINDER A

STRESSES IN A SEMI-INFINITE CYLINDER

DESCRIPTION OF CYLINDER BEING ANALYZED

INSIDE RADIUS	10.250	INCHES
WALL THICKNESS	.750	INCHES
YOUNGS MODULUS	26.8000E+06	PSI
DESIGN PRESSURE		PSI
END BENDING MOMENT	-9.0885E+01	IN-LB/IN
END SHEAR FORCE	1.1317E+03	LB/IN

SHMO= .0000E-99
 SHMI= .0000E-99
 SAM = .0000E-99

	SHO	SHI	SAO	SAI
X= .00	1.4371E+04	1.3789E+04	9.6944E+02	-9.6944E+02
X= .25	1.2019E+04	1.3057E+04	-1.7296E+03	1.7296E+03
X= .50	9.8693E+03	1.2173E+04	-3.8393E+03	3.8393E+03
X= 1.00	6.1993E+03	1.0143E+04	-6.5730E+03	6.5730E+03
X= 2.00	1.2813E+03	6.0093E+03	-7.8801E+03	7.8801E+03
X= 3.00	-1.0345E+03	2.7661E+03	-6.3344E+03	6.3344E+03
X= 4.00	-1.6944E+03	7.3385E+02	-4.0472E+03	4.0472E+03
X= 5.00	-1.5145E+03	-2.7684E+02	-2.0628E+03	2.0628E+03
X= 6.00	-1.0439E+03	-6.1076E+02	-7.2203E+02	7.2203E+02
X= 8.00	-2.4102E+02	-4.2035E+02	2.9888E+02	-2.9888E+02
X= 10.00	5.0059E+01	-1.0861E+02	2.6446E+02	-2.6446E+02

CONDITION 1 CYLINDER B

STRESSES IN A SEMI-INFINITE CYLINDER

DESCRIPTION OF CYLINDER BEING ANALYZED

INSIDE RADIUS	10.250	INCHES
WALL THICKNESS	.812	INCHES
YOUNGS MODULUS	26.4000E+06	PSI
DESIGN PRESSURE	.	PSI
END BENDING MOMENT	-9.0885E+01	IN-LB/IN
END SHEAR FORCE	-1.1317E+03	LB/IN

SHMO= .0000E-99
 SHMI= .0000E-99
 SAM = .0000E-99

	SHO	SHI	SAO	SAI
X= .00	-1.3199E+04	-1.3695E+04	8.2704E+02	-8.2704E+02
X= .25	-1.1003E+04	-1.2876E+04	3.1215E+03	-3.1215E+03
X= .50	-9.0034E+03	-1.1942E+04	4.8991E+03	-4.8991E+03
X= 1.00	-5.6017E+03	-9.8944E+03	7.1544E+03	-7.1544E+03
X= 2.00	-1.0486E+03	-5.8676E+03	8.0315E+03	-8.0315E+03
X= 3.00	1.1105E+03	-2.7397E+03	6.4171E+03	-6.4171E+03
X= 4.00	1.7381E+03	-7.5837E+02	4.1607E+03	-4.1607E+03
X= 5.00	1.5714E+03	2.5573E+02	2.1928E+03	-2.1928E+03
X= 6.00	1.1143E+03	6.1600E+02	8.3060E+02	-8.3060E+02
X= 8.00	2.9441E+02	4.6233E+02	-2.7985E+02	2.7985E+02
X= 10.00	-3.6381E+01	1.4046E+02	-2.9474E+02	2.9474E+02

STRESSES IN A SEMI-INFINITE CYLINDER

DESCRIPTION OF CYLINDER BEING ANALYZED

INSIDE RADIUS	10.250	INCHES
WALL THICKNESS	.750	INCHES
YOUNGS MODULUS	26.8000E+06	PSI
DESIGN PRESSURE	.	PSI
END BENDING MOMENT	-8.5101E+01	IN-LB/IN
END SHEAR FORCE	1.0597E+03	LB/IN

SHMO= .0000E-99
 SHMI= .0000E-99
 SAM = .0000E-99

	SHO	SHI	SAO	SAI
X= .00	1.3456E+04	1.2912E+04	9.0774E+02	-9.0774E+02
X= .25	1.1255E+04	1.2226E+04	-1.6196E+03	1.6196E+03
X= .50	9.2414E+03	1.1398E+04	-3.5951E+03	3.5951E+03
X= 1.00	5.8049E+03	9.4979E+03	-6.1548E+03	6.1548E+03
X= 2.00	1.1997E+03	5.6270E+03	-7.3788E+03	7.3788E+03
X= 3.00	-9.6873E+02	2.5901E+03	-5.9314E+03	5.9314E+03
X= 4.00	-1.5866E+03	6.8716E+02	-3.7897E+03	3.7897E+03
X= 5.00	-1.4181E+03	-2.5923E+02	-1.9316E+03	1.9316E+03
X= 6.00	-9.7757E+02	-5.7191E+02	-6.7609E+02	6.7609E+02
X= 8.00	-2.2568E+02	-3.9360E+02	2.7986E+02	-2.7986E+02
X= 10.00	4.6875E+01	-1.0170E+02	2.4764E+02	-2.4764E+02

STRESSES IN A SEMI-INFINITE CYLINDER

DESCRIPTION OF CYLINDER BEING ANALYZED

INSIDE RADIUS	10.250	INCHES
WALL THICKNESS	.812	INCHES
YOUNGS MODULUS	26.4000E+06	PSI
DESIGN PRESSURE	.	PSI
END BENDING MOMENT	-8.5101E+01	IN-LB/IN
END SHEAR FORCE	-1.0597E+03	LB/IN

SHMO= .0000E-99
 SHMI= .0000E-99
 SAM = .0000E-99

	SHO	SHI	SAO	SAI
X= .00	-1.2359E+04	-1.2824E+04	7.7441E+02	-7.7441E+02
X= .25	-1.0303E+04	-1.2057E+04	2.9229E+03	-2.9229E+03
X= .50	-8.4306E+03	-1.1183E+04	4.5874E+03	-4.5874E+03
X= 1.00	-5.2453E+03	-9.2649E+03	6.6992E+03	-6.6992E+03
X= 2.00	-9.8197E+02	-5.4943E+03	7.5205E+03	-7.5205E+03
X= 3.00	1.0399E+03	-2.5654E+03	6.0088E+03	-6.0088E+03
X= 4.00	1.6275E+03	-7.1012E+02	3.8960E+03	-3.8960E+03
X= 5.00	1.4714E+03	2.3946E+02	2.0533E+03	-2.0533E+03
X= 6.00	1.0434E+03	5.7681E+02	7.7776E+02	-7.7776E+02
X= 8.00	2.7568E+02	4.3292E+02	-2.6205E+02	2.6205E+02
X= 10.00	-3.4066E+01	1.3152E+02	-2.7599E+02	2.7599E+02

CONDITION 3 CYLINDER "A"

STRESSES IN A SEMI-INFINITE CYLINDER

DESCRIPTION OF CYLINDER BEING ANALYZED

INSIDE RADIUS	10.250	INCHES
WALL THICKNESS	.750	INCHES
YOUNGS MODULUS	26.8000E+06	PSI
DESIGN PRESSURE	.	PSI
END BENDING MOMENT	4.7973E+02	IN-LB/IN
END SHEAR FORCE	2.7118E+02	LB/IN

SHMO= .0000E-99
 SHMI= .0000E-99
 SAM = .0000E-99

	SHO	SHI	SAO	SAI
X= .00	6.1086E+03	9.1789E+03	-5.1171E+03	5.1171E+03
X= .25	5.0747E+03	8.5242E+03	-5.7491E+03	5.7491E+03
X= .50	4.1344E+03	7.8620E+03	-6.2127E+03	6.2127E+03
X= 1.00	2.5243E+03	6.5518E+03	-6.7124E+03	6.7124E+03
X= 2.00	2.7720E+02	4.1668E+03	-6.4827E+03	6.4827E+03
X= 3.00	-9.2671E+02	2.2812E+03	-5.3466E+03	5.3466E+03
X= 4.00	-1.4029E+03	9.4950E+02	-3.9207E+03	3.9207E+03
X= 5.00	-1.4287E+03	1.1276E+02	-2.5691E+03	2.5691E+03
X= 6.00	-1.2177E+03	-3.3702E+02	-1.4678E+03	1.4678E+03
X= 8.00	-6.1870E+02	-5.2797E+02	-1.5122E+02	1.5122E+02
X= 10.00	-1.7969E+02	-3.4058E+02	2.6814E+02	-2.6814E+02

CONDITION 3 CYLINDER B

STRESSES IN A SEMI-INFINITE CYLINDER

DESCRIPTION OF CYLINDER BEING ANALYZED

INSIDE RADIUS	10.250	INCHES
WALL THICKNESS	.812	INCHES
YOUNGS MODULUS	26.4000E+06	PSI
DESIGN PRESSURE	.	PSI
END BENDING MOMENT	-5.5973E+02	IN-LB/IN
END SHEAR FORCE	-2.8741E+02	LB/IN

SHMO= .0000E-99
 SHMI= .0000E-99
 SAM = .0000E-99

	SHO	SHI	SAO	SAI
X= .00	-4.5785E+03	-7.6346E+03	5.0935E+03	-5.0935E+03
X= .25	-3.4812E+03	-6.8545E+03	5.6221E+03	-5.6221E+03
X= .50	-2.5248E+03	-6.0809E+03	5.9268E+03	-5.9268E+03
X= 1.00	-1.0043E+03	-4.6108E+03	6.0107E+03	-6.0107E+03
X= 2.00	7.3530E+02	-2.2115E+03	4.9114E+03	-4.9114E+03
X= 3.00	1.2802E+03	-6.6465E+02	3.2414E+03	-3.2414E+03
X= 4.00	1.1919E+03	1.4500E+02	1.7449E+03	-1.7449E+03
X= 5.00	8.6121E+02	4.4723E+02	6.8996E+02	-6.8996E+02
X= 6.00	5.1076E+02	4.6311E+02	7.9419E+01	-7.9419E+01
X= 8.00	6.6056E+01	2.2296E+02	-2.6150E+02	2.6150E+02
X= 10.00	-5.3600E+01	3.8804E+01	-1.5400E+02	1.5400E+02

CONDITION 3 SHORT CYLINDER "A"

STRESSES IN A SHORT CYLINDER

DESCRIPTION OF CYLINDER BEING ANALYZED

INSIDE RADIUS	10.250	INCHES
WALL THICKNESS	.750	INCHES
AXIAL LENGTH	2.330	INCHES
YOUNGS MODULUS	26.6000E+06	PSI
DESIGN PRESSURE	.	PSI
BENDING MOMENT-END 1	-5.5973E+02	IN-LB/IN
BENDING MOMENT-END 2	4.7973E+02	IN-LB/IN
SHEAR FORCE-END 1	2.8741E+02	LB/IN
SHEAR FORCE-END 2	2.7118E+02	LB/IN

SHMO= .0000E-99
 SHMI= .0000E-99
 SAM = .0000E-99

	SHO	SHI	SAO	SAI
X= .00	-4.3649E+03	-7.9472E+03	5.9704E+03	-5.9704E+03
X= .25	-3.1837E+03	-6.2262E+03	5.0708E+03	-5.0708E+03
X= .50	-2.1411E+03	-4.5109E+03	3.9496E+03	-3.9496E+03
X= .75	-1.2017E+03	-2.8047E+03	2.6716E+03	-2.6716E+03
X= 1.00	-3.2862E+02	-1.1082E+03	1.2992E+03	-1.2992E+03
X= 1.25	5.1579E+02	5.7997E+02	-1.0696E+02	1.0696E+02
X= 1.50	1.3694E+03	2.2619E+03	-1.4874E+03	1.4874E+03
X= 1.75	2.2698E+03	3.9393E+03	-2.7824E+03	2.7824E+03
X= 2.00	3.2536E+03	5.6123E+03	-3.9311E+03	3.9311E+03
X= 2.25	4.3560E+03	7.2786E+03	-4.8710E+03	4.8710E+03
X= 2.33	4.7392E+03	7.8095E+03	-5.1171E+03	5.1171E+03

END OF ANALYSIS

STOP

Axial Stresses - SAO = outside
SAI = inside

— condition 1
- - - condition 2
- . - condition 3

Hoop Stresses - SHO = outside
 SHI = inside

— condition 1
 --- condition 2
 - · - condition 3

A.4 STRESS IN CLAD CARBON STEEL PIPE DUE TO DIFFERENTIAL EXPANSION BETWEEN BASE METAL AND CLADDING

Axial and Circumferential Stress in Base Metal

$$\sigma_a = \sigma_c = \frac{t_c (\alpha_c - \alpha_b) \Delta T E_c}{t_b (1 - \nu) \left(1 + \frac{t_c E_c}{t_b E_b}\right)}$$

where σ_a = axial stress, psi

σ_c = circumferential stress

t_c = thickness of cladding, 0.125 in.

t_b = thickness of carbon steel base metal, 0.625 in.

α_c = thermal expansion of cladding, 9.4×10^{-6} in/in/°F

α_b = thermal expansion of base metal, 7.0×10^{-6} in/in/°F

ΔT = temperature change, 430 F

E_c = Young's Modulus for cladding 26.4×10^6 lbs/in²

E_b = Young's Modulus for base metal, 26.8×10^6 lbs/in²

ν = Poisons Ratio

$$\sigma_a = \sigma_c = \frac{0.125 (9.4 \times 10^{-6} - 7.0 \times 10^{-6}) 430 \times 26.4 \times 10^6}{0.625(1-0.3) \times \left(1 + \frac{0.125 \times 26.4 \times 10^6}{0.625 \times 26.8 \times 10^6}\right)}$$

$\sigma_a = \sigma_c = 6520$ psi tension on outer and inner surfaces should be added to the discontinuity stress for Conditions 2 & 3.

Axial and Circumferential stress in Cladding

$$\begin{aligned} \sigma_a = \sigma_c &= \frac{-(\alpha_c - \alpha_b) E_c \Delta T}{(1-\nu) \left(1 + \frac{t_c E_c}{t_b E_b}\right)} \\ &= - \frac{1.4 \times 10^{-6} \times 26.4 \times 10^6 \times 430}{0.7 \times \left(1 + \frac{0.25 \times 26.4 \times 10^6}{0.625 \times 26.8 \times 10^6}\right)} \end{aligned}$$

$$\sigma_a = \sigma_c = -19,000 \text{ psi}$$

APPENDIX B

Piping Flexibility Calculations

Performed by Arthur D. Little Inc.

NOTE

The computer printout sheets contained in this Section were reproduced from Arthur D. Little, Inc. originals. Some of these original sheets were hardly legible because of missing parts of figures and words. Nevertheless it was felt that the information presented here should be included as reference material. It is also felt that the desired information can be obtained even though all of the printout sheets are not completely legible.

This Appendix contains the results of work performed by Arthur D. Little Inc. on piping flexibility. A total of five cases were analyzed for each recirculation loop.

The five cases can be described as follows:

- Case I - The pump will be completely restrained from movement along the x axis and along the z axis, but will be free to rotate about the z axis.
- Case II - The pump will be restrained from movement along the x axis and along the z axis as in Case I, and a moment of 2500 foot-pounds will be applied about the z axis to oppose the rotations observed about the z axis in Case I.
- Case III - The moment defined in Case II will be applied about the z axis but the pump will be free to move in the x - z plane.
- Case IV - The moment defined in Case II will be applied about the z axis and a force of 2500 pounds will be applied to oppose the deflections observed at the pump in Case III.
- Case V - The conditions will be the same as in Case I except that the piping system will be analyzed for the effects of gravity in the absence of thermal expansion.

The following is a list of information which is to be found in this Appendix and explanation of that information.

- a. The printed results of the machine computations of cases 1 through 5 of each of the three loops.
- b. Isometric drawings with encircled numbers which correspond to the member numbers on the printed results.

The forces, motions, and other output are given at the ends of member.

The following convention is used to determine the direction of forces:

1. The force printed out at POINT 0 is that which the anchor exerts on the piping system.
2. The arrows along the section show the sign convention of the forces and moments; for example, the force printed out at POINT 2 is that which member 2 exerts on that part of the system which follows 2.
3. The force printed out at a restraint is that which the piping system exerts on the restraint.
4. External forces are forces exerted on the piping system.
5. The force printed out at A is that which the piping system exerts on the vessel.

Forces are designated as F_x , F_y & F_z , moments as M_x , M_y and M_z , stresses as SE, deflections as D_x and D_z , and rotations as R_x , R_y and R_z .

RECIRCULATION - PIPE LOOP N° 1

ARTHUR D. LITTLE INC. APPLIED THERMODYNAMICS GROUP 000000000000
 PREPARED FOR THE ALLIS-CHALMERS MFG. COMPANY 000000000000
 RECIRCULATION PIPE LOOP NO.1 CASE 1 JANUARY 25, 1960 000000000000

SECTION	MEMBER	FX LBS	FY LBS	FZ LBS	MX IN-LBS	MY IN-LBS	MZ IN-LBS	SE PSI	DX INS	DY INS	DZ INS	RX RADS	RY RADS	RZ RADS
1	0	125142	-8386	24800	-1428851	742031	7011241	64757	-0.142	-0.022	0.079	0.00000	0.00000	0.00000
1	1	125142	-8386	24800	-889583	-98292	4019922	90612	-0.952	-0.066	0.065	0.00163	-0.00270	-0.01298
1	2	125142	-8386	24800	-224900	-98292	662925	6799	-0.708	-0.163	0.016	0.00181	-0.00268	-0.01313
1	3	125142	-8386	24800	511876	-503701	-9203120	91323	-1.015	-0.588	0.064	0.00008	-0.00151	-0.00457
1	4	125142	-8386	24800	910067	-1597124	-2554519	28856	-0.770	-0.824	0.297	-0.00019	-0.00049	-0.00175
1	5	125142	-8386	24800	299021	-1602468	-2500587	28721	-0.747	-0.890	0.242	-0.00047	0.00046	-0.00028
1	6	125142	-8386	24800	175012	116087	-1299712	9095	-0.219	-0.595	0.094	-0.00090	0.00174	0.00300
1	6 CHANGE	125142	-8386	24800	175012	116087	-1299712	9095	-0.219	-0.595	0.094	-0.00090	0.00174	0.00300
1	7	125142	-8386	24800	164726	258694	-1199406	4791	-0.169	-0.557	0.078	-0.00092	0.00172	0.00318
1	8	125142	-8386	24800	151868	436819	-1068474	4526	-0.092	-0.508	0.060	-0.00092	0.00172	0.00318
1	8 CHANGE	125142	-8386	24800	151868	436819	-1068474	9349	-0.092	-0.508	0.060	-0.00092	0.00172	0.00318
1	9	125142	-8386	24800	127726	840704	-784841	9045	0.066	-0.391	0.019	-0.00097	0.00152	0.00346
1	10	125142	-8386	24800	-441796	1102041	2151819	17479	0.079	-0.201	0.000	-0.00000	0.00000	0.00265
1	0 FORCE	0	-0	0	0	-76800	0	0	0.079	-0.201	0.000	-0.00000	0.00000	0.00265
1	0	125121	-8602	24920	-499890	1025114	2142452	6973	0.079	-0.201	0.000	-0.00000	0.00000	0.00265
1	11	125121	-8602	24920	-1187697	1025114	5906076	17977	0.000	-0.064	0.000	-0.00000	0.00000	0.00264
1	0 RESTRT	1746074	-15622	193721	-272996	-7760575	-0	0	-0.000	-0.064	0.000	0.00000	0.00000	0.00264
1	0	-1621853	-24314	-168792	-915941	8785690	5906076	90563	-0.000	-0.064	0.000	0.00000	0.00000	0.00264
1	12	-1621853	-24314	-168792	-380431	-26895081	5906076	70210	0.000	-0.064	-0.121	0.00000	0.00001	0.00264
1	13	-1621853	-24314	-168792	-380431	-17442703	4544487	91897	-0.296	-0.212	-0.099	0.00000	0.00004	0.00263
1	14	-1621853	-24314	-168792	-380431	-14910816	6179775	60905	-0.324	-0.291	-0.099	0.00000	0.00004	0.00263
1	14 CHANGE	-1621853	-24314	-168792	-380431	-14910816	6179775	60226	-0.324	-0.291	-0.099	0.00000	0.00004	0.00263
1	15	-1621853	-24314	-168792	-380431	-12885813	9888079	92343	-0.345	-0.278	-0.078	0.00007	0.00210	0.00203
1	15 CHANGE	-1621853	-24314	-168792	-380431	-12885813	9888079	99343	-0.365	-0.278	-0.078	0.00007	0.00210	0.00203
1	16	-1621853	-24314	-168792	-380431	15948902	-265396	153490	-0.796	-0.175	0.951	0.00108	-0.00103	-0.00166
1	17	-1621853	-24314	-168792	432301	-32694683	-1067694	314656	-0.001	-0.116	0.102	-0.00000	0.00000	0.00000

ARTHUR D. LITTLE INC. APPLIED THERMODYNAMICS GROUP
 PREPARED FOR THE ALLYS-CHALMERS MFG. COMPANY
 RECIRCULATION PIPE LOOP NO.1 CASE 2 JANUARY 25, 1960

SECTION	MEMBER	FX LBS	FY LBS	FZ LBS	MX IN-LBS	MY IN-LBS	MZ IN-LBS	SE PSI	OX INS	OY INS	OZ INS	RX RADS	RY RADS	RZ RADS
1	0	125204	-8400	24811	-1429242	742513	7014433	64787	-0.142	-0.022	0.079	0.00000	0.00000	0.00000
1	1	125204	-8400	24811	-889768	-58137	4020953	39638	-0.352	-0.066	0.065	0.00163	-0.00270	-0.01239
1	2	125204	-8400	24811	-224171	-58137	662303	6732	-0.709	-0.163	0.016	0.00181	-0.00268	-0.01314
1	3	125204	-8400	24811	512246	-503854	-1204723	31339	-1.015	-0.588	0.064	0.00008	-0.00151	-0.00457
1	4	125204	-8400	24811	310090	-1597887	-2555009	28864	-0.770	-0.824	0.237	-0.00019	-0.00049	-0.00175
1	5	125204	-8400	24811	299025	-1603240	-2500984	28728	-0.747	-0.830	0.242	-0.00047	0.00044	-0.00028
1	6	125204	-8400	24811	174803	116028	-1292037	5088	-0.219	-0.595	0.094	-0.00090	0.00174	0.00300
1	6 CHANGE	125204	-8400	24811	174803	116028	-1292037	5088	-0.219	-0.595	0.094	-0.00090	0.00174	0.00300
1	7	125204	-8400	24811	164490	258635	-1191750	4784	-0.164	-0.557	0.078	-0.00092	0.00172	0.00318
1	8	125204	-8400	24811	151610	436894	-1066412	4510	-0.093	-0.508	0.060	-0.00092	0.00172	0.00318
1	8 CHANGE	125204	-8400	24811	151610	436894	-1066412	4510	-0.093	-0.508	0.060	-0.00092	0.00172	0.00318
1	9	125204	-8400	24811	122426	840946	-782292	9030	0.065	-0.391	0.019	-0.00097	0.00152	0.00346
1	10	125204	-8400	24811	-442316	1102392	2156033	17506	0.079	-0.201	0.000	-0.00000	0.00000	0.00264
1	0 FORCE	-0	-0	-0	0	-76800	-30000	0	0.079	-0.201	0.000	-0.00000	0.00000	0.00264
1	0	125225	-9192	24760	-443066	1025646	2125391	4907	0.079	-0.201	0.000	-0.00000	0.00000	0.00264
1	11	125225	-9192	24760	-1185867	1025646	5882147	17510	0.000	-0.064	-0.000	-0.00000	0.00000	0.00263
1	0 RESTRT	1747082	15083	193553	-274312	-7760099	-0	0	-0.000	-0.064	-0.000	-0.00000	-0.00000	0.00263
1	0	-1621857	-24215	-168793	-911555	8785746	5882147	30524	-0.000	-0.064	-0.000	-0.00000	-0.00000	0.00263
1	12	-1621857	-24215	-168793	-378831	-26895108	5882147	70195	0.000	-0.064	-0.101	0.00000	0.00001	0.00263
1	13	-1621857	-24215	-168793	-378831	-17442716	4526122	51844	-0.256	-0.211	-0.099	0.00000	0.00004	0.00262
1	14	-1621857	-24215	-168793	-378831	-14910825	4162901	44942	-0.324	-0.250	-0.099	0.00000	0.00004	0.00262
1	14 CHANGE	-1621857	-24215	-168793	-378831	-14910825	4162901	44148	-0.324	-0.250	-0.099	0.00000	0.00004	0.00262
1	15	-1621857	-24215	-168793	-378831	-12885312	3872324	52324	-0.365	-0.277	-0.078	0.00007	0.00210	0.00203
1	15 CHANGE	-1621857	-24215	-168793	-378831	-12885312	3872324	52324	-0.365	-0.277	-0.078	0.00007	0.00210	0.00203
1	16	-1621857	-24215	-168793	-378831	15968340	-264105	153490	-0.796	-0.175	0.051	0.00108	-0.00103	-0.00165
1	17	-1621857	-24215	-168793	430580	-32694762	-1063125	314656	-0.001	-0.116	0.102	0.00000	-0.00000	-0.00000

ARTHUR D. LITTLE INC. APPLIED THERMODYNAMICS GROUP
 PREPARED FOR THE ALLIS-CHALMERS MFG. COMPANY
 RECIRCULATION PIPE LOOP NO.1 CASE 3 JANUARY 25, 1960

SECTION	MEMBER	FY LRS	FY LRS	FZ LRS	MX IN-LRS	MY IN-LRS	MZ IN-LRS	SF PSI	DX INS	DY INS	DZ INS	RX RADS	RY RADS	RZ RADS
1	0	1714	-20	224	-17499	24482	90261	889	-0.142	-0.022	0.079	0.00000	0.00000	0.00000
1	1	1714	-20	224	-12411	14501	40000	499	-0.175	-0.106	0.096	0.00003	-0.00005	-0.00016
1	2	1714	-20	224	-6400	14501	4016	92	-0.179	-0.202	0.095	0.00003	-0.00005	-0.00017
1	3	1714	-20	224	688	4747	-91622	482	-0.071	-0.325	0.131	0.00000	-0.00006	-0.00004
1	4	1714	-20	224	-14	-19199	-49368	499	0.207	-0.126	0.229	0.00001	-0.00006	0.00001
1	5	1714	-20	224	-92	-20012	-49181	529	0.230	-0.126	0.228	0.00000	-0.00005	0.00004
1	6	1714	-20	224	-483	-19159	-44987	182	0.749	-0.314	0.286	0.00001	-0.00002	0.00012
1	6 CHANGE	1714	-20	224	-483	-19159	-44987	182	0.749	-0.314	0.286	0.00001	-0.00002	0.00012
1	7	1714	-20	224	-510	-12590	-44639	180	0.804	-0.312	0.291	0.00001	-0.00001	0.00019
1	8	1714	-20	224	-566	-11880	-44204	178	0.872	-0.310	0.299	0.00001	-0.00001	0.00019
1	8 CHANGE	1714	-20	224	-566	-11880	-44204	172	0.872	-0.310	0.299	0.00001	-0.00001	0.00019
1	9	1714	-20	224	-669	-10269	-43218	946	1.027	-0.306	0.315	0.00001	-0.00001	0.00014
1	10	1714	-20	224	-5652	-9227	-4876	64	1.122	-0.201	0.125	-0.00000	-0.00000	0.00022
1	0 FORCE	0	-0	0	0	-76800	-30000	0	1.122	-0.201	0.125	-0.00000	-0.00000	0.00022
1	0	1714	-28	224	-5652	-86027	-48876	267	1.122	-0.201	0.125	-0.00000	-0.00000	0.00022
1	11	1714	-28	224	-12366	-86027	16599	294	1.116	-0.064	0.125	-0.00000	-0.00000	0.00022
1	0 RESTRT	0	-169	0	-28764	-87984	0	0	1.116	-0.064	0.125	0.00000	-0.00000	0.00022
1	0	1714	141	224	16381	1998	16599	67	1.116	-0.064	0.125	0.00000	-0.00000	0.00022
1	12	1714	141	224	13283	99262	16599	128	1.116	-0.064	0.125	-0.00000	-0.00000	0.00022
1	13	1714	141	224	13283	26796	24424	111	0.860	-0.076	0.225	-0.00000	-0.00000	0.00022
1	14	1714	141	224	13283	23378	26537	109	0.791	-0.080	0.225	-0.00000	-0.00000	0.00022
1	14 CHANGE	1714	141	224	13283	23378	26537	147	0.791	-0.080	0.225	-0.00000	-0.00000	0.00022
1	15	1714	141	224	13283	20609	28226	146	0.736	-0.082	0.225	-0.00000	-0.00000	0.00022
1	15 CHANGE	1714	141	224	13283	20609	28226	146	0.736	-0.082	0.225	-0.00000	-0.00000	0.00022
1	16	1714	141	224	13283	-17523	52280	533	0.119	-0.119	0.229	-0.00000	-0.00001	0.00019
1	17	1714	141	224	8577	12382	96926	992	-0.001	-0.116	0.189	-0.00000	0.00000	0.00000

ARTHUR D. LITTLE INC. APPLIED THERMODYNAMICS GROUP
 PREPARED FOR THE ALLIS-CHALMERS MFG. COMPANY
 RECIRCULATION PIPE LOOP NO. 1 CASE 4 JANUARY 25, 1960

SECTION	MEMBER	FX LBS	FY LBS	FZ LBS	MX IN-LBS	MY IN-LBS	MZ IN-LBS	SE PSI	DX INS	DY INS	DZ INS	RX RADS	RY RADS	RZ RADS
1	0	2205	-58	462	-29128	10485	119445	1096	-0.142	-0.022	0.079	0.00000	0.00000	0.00000
1	1	2205	-58	462	-18629	-3822	67488	674	-0.175	-0.105	0.096	0.00003	-0.00005	-0.00021
1	2	2205	-58	462	-6246	-3822	8346	101	-0.181	-0.202	0.095	0.00004	-0.00004	-0.00022
1	3	2205	-58	462	8413	-10892	-62567	609	-0.074	-0.326	0.130	0.00001	-0.00001	-0.00005
1	4	2205	-58	462	7006	-28246	-58044	618	0.205	-0.328	0.217	0.00001	0.00000	0.00001
1	5	2205	-58	462	6929	-28182	-57668	621	0.228	-0.328	0.222	0.00000	0.00002	0.00004
1	6	2205	-58	462	6064	5648	-49252	194	0.749	-0.315	0.270	-0.00001	0.00004	0.00013
1	6 CHANGE	2205	-58	462	6064	5648	-49252	194	0.749	-0.315	0.270	-0.00001	0.00004	0.00013
1	7	2205	-58	462	5993	8454	-48554	193	0.803	-0.313	0.275	-0.00001	0.00004	0.00014
1	8	2205	-58	462	5903	11961	-47681	192	0.871	-0.311	0.281	-0.00001	0.00004	0.00014
1	8 CHANGE	2205	-58	462	5903	11961	-47681	142	0.871	-0.311	0.281	-0.00001	0.00004	0.00014
1	9	2205	-58	462	5700	19912	-45704	392	1.026	-0.306	0.296	-0.00001	0.00003	0.00016
1	10	2205	-58	462	-4587	25056	4080	87	1.121	-0.201	0.306	-0.00000	-0.00000	0.00023
1	0 FORCE	-4802	-0	-1398	-41940	-76800	114060	0	1.121	-0.201	0.306	-0.00000	-0.00000	0.00023
1	0	-2597	-58	-936	-46527	-51744	118140	394	1.121	-0.201	0.306	-0.00000	-0.00000	0.00023
1	11	-2597	-58	-936	-18435	-51744	40221	196	1.114	-0.064	0.306	-0.00000	-0.00000	0.00023
1	0 RESTRT	0	-100	0	-31070	93410	0	0	1.114	-0.064	0.306	0.00000	-0.00000	0.00023
1	0	-2597	42	-936	12635	-145154	40221	435	1.114	-0.064	0.306	0.00000	-0.00000	0.00023
1	12	-2597	42	-936	11700	-202294	40221	594	1.114	-0.064	0.205	-0.00000	0.00000	0.00023
1	13	-2597	42	-936	11700	-149856	42600	449	0.858	-0.077	0.205	-0.00000	0.00000	0.00023
1	14	-2597	42	-936	11700	-135810	43237	411	0.790	-0.080	0.205	-0.00000	0.00000	0.00023
1	14 CHANGE	-2597	42	-936	11700	-135810	43237	556	0.790	-0.080	0.205	-0.00000	0.00000	0.00023
1	15	-2597	42	-936	11700	-124574	43746	515	0.735	-0.083	0.205	-0.00000	0.00002	0.00022
1	15 CHANGE	-2597	42	-936	11700	-124574	43746	515	0.735	-0.083	0.205	-0.00000	0.00002	0.00022
1	16	-2597	42	-936	11700	35384	51002	599	0.118	-0.114	0.222	-0.00003	0.00011	0.00013
1	17	-2597	42	-936	10280	-20535	52404	302	-0.001	-0.116	0.102	-0.00000	-0.00000	0.00000

ARTHUR D. LITTLE INC. APPLIED THERMODYNAMICS GROUP
 PREPARED FOR THE ALLIS-CHALMERS MFG. COMPANY
 RECIRCULATION PIPE LOOP NO.1 DEADWEIGHT JANUARY 25, 1960

SECTION	MEMBER	FX LBS	FY LBS	FZ LBS	MX IN-LBS	MY IN-LBS	MZ IN-LBS	SE PSI	DX INS	DY INS	DZ INS	RX RADS	RY RADS	RZ RADS
1	0	-3741	7381	-626	-49265	-37099	-158205	1634	0.000	0.000	0.000	0.00000	0.00000	0.00000
1	1	-3741	6659	-626	-30179	-14152	-10636	313	0.000	-0.001	-0.001	0.00007	0.00003	0.00018
1	2	-3741	5913	-626	-44963	-14152	89721	976	0.000	-0.001	-0.003	0.00008	0.00003	0.00017
1	3	-3741	4470	-626	-18646	2812	55811	367	0.000	-0.009	-0.010	0.00028	0.00006	-0.00033
1	4	-3741	2215	-626	61795	44450	-202721	2084	0.010	-0.041	-0.014	0.00026	0.00003	-0.00025
1	5	-3741	2032	-626	64612	45354	-216376	2185	0.010	-0.043	-0.014	0.00023	0.00001	-0.00012
1	6	-3741	-1994	-626	64894	10631	-219119	889	0.010	-0.030	-0.011	0.00009	-0.00004	0.00033
1	6 CHANGE	-3741	-1994	-626	64894	10631	-219119	889	0.010	-0.030	-0.011	0.00009	-0.00004	0.00033
1	7	-3741	-2328	-626	62243	7751	-193319	790	0.010	-0.026	-0.010	0.00007	-0.00004	0.00036
1	8	-3741	-4328	-626	57140	4151	-143656	601	0.010	-0.020	-0.010	0.00007	-0.00004	0.00036
1	8 CHANGE	-3741	-4328	-626	57140	4151	-143656	445	0.010	-0.020	-0.010	0.00007	-0.00004	0.00036
1	9	-3741	-5447	-626	40154	-4009	21661	185	0.010	-0.007	-0.008	0.00006	-0.00004	0.00038
1	10	-3741	-6585	-626	40739	-9290	67619	618	0.004	-0.000	-0.007	0.00000	-0.00000	0.00019
1	10 CHANGE	-3741	-6585	-626	40739	-9290	67619	229	0.004	-0.000	-0.007	0.00000	-0.00000	0.00019
1	11	-3741	-6585	-626	59510	-9290	-44614	216	-0.002	-0.000	-0.007	0.00000	-0.00000	0.00019
1	0 RESTRT	-0	-10072	-0	39731	26937	0	0	-0.002	0.000	-0.007	0.00000	-0.00000	0.00019
1	0	-3741	3487	-626	19779	-36226	-44615	175	-0.002	0.000	-0.007	0.00000	-0.00000	0.00019
1	12	-3741	3487	-626	-56940	-118531	-44615	399	-0.002	-0.000	-0.007	0.00000	0.00000	0.00019
1	13	-3741	3487	-626	-56940	-83493	150669	522	-0.002	-0.011	-0.007	0.00000	0.00000	0.00019
1	14	-3741	1487	-626	-56940	-74108	187978	604	-0.002	-0.014	-0.007	0.00000	0.00000	0.00019
1	14 CHANGE	-3741	1487	-626	-56940	-74108	187978	816	-0.002	-0.014	-0.007	0.00000	0.00000	0.00019
1	15	-3741	1153	-626	-56940	-66600	203820	862	-0.002	-0.016	-0.007	0.00001	0.00001	0.00017
1	15 CHANGE	-3741	1153	-626	-56940	-66600	203820	862	-0.002	-0.016	-0.007	0.00001	0.00001	0.00017
1	16	-3741	-3601	-626	-56940	40280	-5279	449	-0.002	-0.008	0.003	0.00016	0.00004	-0.00017
1	17	-3741	-5056	-626	94369	-64126	-141411	1217	0.000	0.000	-0.000	-0.00000	-0.00000	0.00000

RECIRCULATION - PIPE LOOP N° 2

W/

ARTHUR D. LITTLE INC. APPLIED THERMODYNAMICS GROUP
 PREPARED FOR THE ALLIS-CHALMERS MFG. COMPANY
 RECIRCULATION PIPE LOOP NO. 2 CASE 1 JANUARY 25, 1960

SECTION	MEMBER	FX LBS	FY LBS	FZ LBS	MX IN-LBS	MY IN-LBS	MZ IN-LBS	SE PSI	DX INS	DY INS	DZ INS	RX RADS	RY RADS	RZ RADS
1	0	151537	-14883	-1250	371994	-12881	7879375	71029	0.140	-0.022	0.081	0.00000	0.00000	0.00000
1	1	151537	-14883	-1250	270789	-756290	4460829	49109	-0.028	-0.155	0.094	0.00066	-0.00199	-0.01984
1	2	151537	-14883	-1250	237290	-756290	995782	4325	-0.427	-0.251	0.077	0.00058	-0.00162	-0.01462
1	3	151537	-14883	-1250	214221	-611738	-4114140	90046	-0.726	-0.663	0.084	0.00110	-0.00015	-0.00252
1	4	151537	-14883	-1250	295189	29659	-1996472	7616	-0.214	-0.586	0.039	0.00046	0.00036	0.00277
1	4 CHANGE	151537	-14883	-1250	295189	29659	-1996472	7616	-0.214	-0.586	0.039	0.00046	0.00036	0.00277
1	5	151537	-14883	-1250	301818	82218	-1798004	6941	-0.160	-0.551	0.027	0.00040	0.00035	0.00305
1	6	151537	-14883	-1250	310112	147929	-1594919	6114	-0.092	-0.509	0.019	0.00040	0.00035	0.00305
1	6 CHANGE	151537	-14883	-1250	310112	147929	-1594919	4524	-0.092	-0.509	0.019	0.00040	0.00035	0.00305
1	7	151537	-14883	-1250	328911	296856	-1029259	8949	0.060	-0.392	0.002	0.00027	0.00029	0.00344
1	8	151537	-14883	-1250	368584	393224	2631738	20818	0.071	-0.201	-0.000	0.00000	0.00000	0.00237
1	0 FORCE	-0	-0	0	-0	-76800	0	0	0.071	-0.201	-0.000	0.00000	0.00000	0.00237
1	0	151546	-15331	-1247	368642	316419	2631444	7697	0.071	-0.201	-0.000	0.00000	0.00000	0.00237
1	0	151546	-15331	-1247	406038	316419	7177897	20699	0.000	-0.064	-0.000	0.00000	0.00000	0.00236
1	0 RESTRT	8532000	20010	9348	1183471	-182160032	-0	0	-0.000	-0.064	-0.000	0.00000	0.00000	0.00236
1	0	-8380454	-35341	-10595	-777434	182476490	7177897	929279	-0.000	-0.064	-0.000	0.00000	0.00000	0.00236
1	10	-8380454	-35341	-10595	66	-1899590	7177897	21952	0.002	-0.064	-0.101	0.00000	-0.00005	0.00236
1	11	-8380454	-35341	-10595	66	-1300218	9198746	19414	-0.253	-0.196	-0.103	0.00000	-0.00005	0.00235
1	12	-8380454	-35341	-10595	66	-1141295	4668699	19824	-0.322	-0.231	-0.104	0.00000	-0.00005	0.00235
1	12 CHANGE	-8380454	-35341	-10595	66	-1141295	4668699	18683	-0.322	-0.231	-0.104	0.00000	-0.00005	0.00235
1	13	-8380454	-35341	-10595	66	-1014156	4244542	16964	-0.306	-0.254	-0.103	0.00000	0.00011	0.00169
1	13 CHANGE	-8380454	-35341	-10595	66	-1014156	4244542	16964	-0.306	-0.254	-0.103	0.00000	0.00011	0.00169
1	14	-8380454	-35341	-10595	66	1107274	-2831899	11830	0.103	-0.116	-0.000	0.00000	-0.00000	-0.00000

ARTHUR D. LITTLE INC. APPLIED THERMODYNAMICS GROUP
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 RECIRCULATION PIPE LOOP NO. 2 CASE 2 JANUARY 25, 1960

SECTION	MEMBER	DX INS	DY INS	DZ INS	RX RADS	RY RADS	RZ RADS	FX LBS	FY LBS	FZ LBS	MX IN-LBS	MY IN-LBS	MZ IN-LBS	SF PSI
1	0	0.140	-0.022	0.081	0.00000	0.00000	0.00000	151500	-14800	-1251	372109	-12812	7875862	71052
1	1	-0.028	-0.144	0.094	0.00066	-0.00103	-0.01984	151500	-14800	-1251	270899	-756490	4462172	49116
1	2	-0.428	-0.251	0.077	0.00058	-0.00182	-0.01463	151500	-14800	-1251	297262	-756490	395690	5325
1	3	-0.727	-0.663	0.084	0.00110	-0.00019	-0.00292	151500	-14800	-1251	214290	-611905	-4115473	30950
1	4	-0.214	-0.586	0.033	0.00046	0.00036	0.00277	151500	-14800	-1251	295270	29631	-1995449	7612
1	4 CHANGE	-0.214	-0.586	0.033	0.00046	0.00036	0.00277	151500	-14800	-1251	295270	29631	-1995449	7612
1	5	-0.161	-0.550	0.027	0.00040	0.00035	0.00305	151500	-14800	-1251	301922	82207	-1756788	6937
1	6	-0.092	-0.505	0.019	0.00040	0.00035	0.00305	151500	-14800	-1251	310224	147928	-1599462	6109
1	6 CHANGE	-0.092	-0.505	0.019	0.00040	0.00035	0.00305	151500	-14800	-1251	310224	147928	-1599462	4520
1	7	0.060	-0.392	0.002	0.00027	0.00029	0.00344	151500	-14800	-1251	329044	296894	-1027259	8335
1	8	0.071	-0.201	-0.000	0.00000	0.00000	0.00236	151500	-14800	-1251	368754	393284	2635278	20846
1	8 ENDCF	0.071	-0.201	-0.000	0.00000	0.00000	0.00236	0	-0	0	-0	-76800	-30000	0
1	9	0.071	-0.201	-0.000	0.00000	0.00000	0.00236	151600	-15947	-1248	368811	316478	2604984	7622
1	9	-0.000	-0.064	0.000	0.00000	0.00000	0.00236	151600	-15947	-1248	406237	316478	7152988	20628
1	9 RECYRT	-0.000	-0.064	-0.000	0.00000	0.00000	0.00236	8592054	19870	9947	1180934	-182159972	-0	0
1	10	-0.000	-0.064	-0.000	0.00000	0.00000	0.00236	-8380454	-95217	-10595	-774699	182476450	7152988	525210
1	10	0.002	-0.064	-0.101	0.00000	-0.00005	0.00236	-8380454	-95217	-10595	66	-1893530	7152988	21283
1	11	-0.259	-0.195	-0.103	0.00000	-0.00005	0.00236	-8380454	-95217	-10595	66	-1900218	9180860	19364
1	12	-0.322	-0.231	-0.104	0.00000	-0.00005	0.00236	-8380454	-95217	-10595	66	-1141295	4652612	19770
1	12 CHANGE	-0.322	-0.231	-0.104	0.00000	-0.00005	0.00236	-8380454	-95217	-10595	66	-1141295	4652612	18623
1	13	-0.306	-0.253	-0.103	0.00000	0.00011	0.00168	-8380454	-95217	-10595	66	-1014156	4230013	14910
1	13 CHANGE	-0.306	-0.253	-0.103	0.00000	0.00011	0.00168	-8380454	-95217	-10595	66	-1014156	4230013	14910
1	14	0.103	-0.116	0.000	-0.00000	-0.00000	-0.00000	-8380454	-95217	-10595	66	1107274	-2821471	11782

ARTHUR D. LITTLE INC. APPLIED THERMODYNAMICS GROUP
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 RECIRCULATION PIPE LOOP NO. 2 CASE 3 JANUARY 25, 1960

SECTION	MEMBER	FX LRS	FY LRS	FZ LRS	MX IN-LRS	MY IN-LRS	MZ IN-LRS	SE PSI	OX INS	OY INS	OZ INS	RX RADS	RY RADS	RZ RADS
1	0	-2133	364	-444	8949	47910	-116922	1059	0.140	-0.022	0.081	0.00000	0.00000	0.00000
1	1	-2133	364	-444	344	54110	-70175	307	0.179	-0.105	0.099	-0.00005	-0.00001	0.00020
1	2	-2133	364	-444	-11549	54110	-17948	269	0.179	-0.202	0.101	-0.00005	-0.00009	0.00021
1	3	-2133	364	-444	-24498	56896	49446	566	0.309	-0.316	0.101	-0.00006	-0.00013	0.00009
1	4	-2133	364	-444	-28698	-39701	-7822	193	0.833	-0.307	0.102	0.00000	-0.00019	0.00009
1	4 CHANGE	-2133	364	-444	-28698	-39701	-7822	193	0.833	-0.307	0.102	0.00000	-0.00019	0.00009
1	5	-2133	364	-444	-28801	-45975	-12108	216	0.890	-0.307	0.101	0.00001	-0.00012	0.00006
1	6	-2133	364	-444	-29004	-53818	-17645	247	0.958	-0.306	0.101	0.00001	-0.00012	0.00006
1	6 CHANGE	-2133	364	-444	-29004	-53818	-17645	183	0.958	-0.306	0.101	0.00001	-0.00012	0.00006
1	7	-2133	364	-444	-29469	-71595	-30014	616	1.114	-0.304	0.099	0.00002	-0.00010	0.00006
1	8	-2133	364	-444	-19995	-89097	-84990	729	1.211	-0.201	0.096	-0.00000	-0.00000	0.00021
1	8 FORCE	0	0	0	0	-76800	-30000	0	1.211	-0.201	0.096	-0.00000	-0.00000	0.00021
1	9	-2133	364	-444	-19995	-149897	-114990	569	1.211	-0.201	0.096	-0.00000	-0.00000	0.00021
1	9	-2133	364	-444	-6670	-149897	-178990	691	1.205	-0.064	0.096	-0.00000	-0.00000	0.00021
1	9 RESTRT	-0	-1107	-0	-30029	-128947	0	0	1.205	-0.064	0.096	0.00000	-0.00000	0.00021
1	10	-2133	1470	-444	32944	-91550	-178990	591	1.205	-0.064	0.096	0.00000	-0.00000	0.00021
1	10	-2133	1470	-444	-9	-78483	-178990	562	1.205	-0.064	-0.005	-0.00000	0.00000	0.00021
1	11	-2133	1470	-444	-9	-53625	-96607	318	0.949	-0.076	-0.005	-0.00000	0.00000	0.00021
1	12	-2133	1470	-444	-9	-46967	-74551	259	0.880	-0.079	-0.005	-0.00000	0.00000	0.00021
1	12 CHANGE	-2133	1470	-444	-9	-46967	-74551	349	0.880	-0.079	-0.005	-0.00000	0.00000	0.00021
1	13	-2133	1470	-444	-9	-41640	-56906	274	0.826	-0.081	-0.005	-0.00000	0.00001	0.00022
1	13 CHANGE	-2133	1470	-444	-9	-41640	-56906	274	0.826	-0.081	-0.005	-0.00000	0.00001	0.00022
1	14	-2133	1470	-444	-9	47241	297516	941	0.102	-0.116	-0.000	-0.00000	-0.00000	-0.00000

ARTHUR D. LITTLE INC. APPLIED THERMODYNAMICS GROUP
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 RECIRCULATION PIPE LOOP NO. 2 CASE 4 JANUARY 25, 1960

SECTION	MEMBER	FX LBS	FY LBS	FZ LBS	MX IN-LBS	MY IN-LBS	MZ IN-LBS	SE PSI	DX INS	DY INS	DZ INS	RX RADS	RY RADS	RZ RADS
1	0	-2050	357	-397	7665	42605	-112428	1007	0.140	-0.022	0.081	0.00000	0.00000	0.00000
1	1	-2050	357	-397	131	49195	-67572	678	0.173	-0.105	0.099	-0.00005	-0.00001	0.00019
1	2	-2050	357	-397	-10510	49195	-12587	248	0.179	-0.202	0.101	-0.00005	-0.00003	0.00020
1	3	-2050	357	-397	-24037	33601	43288	530	0.305	-0.316	0.100	-0.00006	-0.00012	0.00009
1	4	-2050	357	-397	-25978	-35591	-8916	175	0.834	-0.308	0.099	-0.00000	-0.00011	0.00006
1	4 CHANGE	-2050	357	-397	-25978	-35591	-8916	175	0.834	-0.308	0.099	-0.00000	-0.00011	0.00006
1	5	-2050	357	-397	-26137	-41262	-13194	197	0.889	-0.307	0.098	0.00000	-0.00011	0.00006
1	6	-2050	357	-397	-26336	-48350	-18542	228	0.958	-0.306	0.097	0.00000	-0.00011	0.00006
1	6 CHANGE	-2050	357	-397	-26336	-48350	-18542	167	0.958	-0.306	0.097	0.00000	-0.00011	0.00006
1	7	-2050	357	-397	-26786	-64417	-30664	546	1.113	-0.304	0.095	0.00002	-0.00009	0.00006
1	8	-2050	357	-397	-18351	-74813	-83601	703	1.211	-0.201	0.092	-0.00000	-0.00000	0.00021
1	0 FORCE	-4983	-0	-397	-11910	-76800	119490	0	1.211	-0.201	0.092	-0.00000	-0.00000	0.00021
1	0	-7033	357	-794	-30261	-151613	35889	457	1.211	-0.201	0.092	-0.00000	-0.00000	0.00021
1	9	-7033	357	-794	-6451	-151613	-175092	666	1.204	-0.064	0.092	-0.00000	-0.00000	0.00021
1	0 RESTRT	-0	-1095	-0	-38373	-165998	0	0	1.204	-0.064	0.092	0.00000	-0.00000	0.00021
1	0	-7033	1451	-794	31922	14385	-175092	514	1.204	-0.064	0.092	0.00000	-0.00000	0.00021
1	10	-7033	1451	-794	-3	-140334	-175092	645	1.204	-0.064	-0.008	-0.00000	0.00000	0.00021
1	11	-7033	1451	-794	-3	-95889	-93831	386	0.948	-0.076	-0.008	-0.00000	0.00000	0.00021
1	12	-7033	1451	-794	-3	-83984	-72064	318	0.880	-0.079	-0.008	-0.00000	0.00000	0.00021
1	12 CHANGE	-7033	1451	-794	-3	-83984	-72064	430	0.880	-0.079	-0.008	-0.00000	0.00000	0.00021
1	13	-7033	1451	-794	-3	-74460	-54651	359	0.825	-0.081	-0.008	-0.00000	0.00001	0.00022
1	13 CHANGE	-7033	1451	-794	-3	-74460	-54651	359	0.825	-0.081	-0.008	-0.00000	0.00001	0.00022
1	14	-7033	1451	-794	-3	84458	235907	974	0.102	-0.116	-0.000	-0.00000	-0.00000	0.00000

ARTHUR D. LITTLE INC. APPLIED THERMODYNAMICS GROUP
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 RECIRCULATION PIPE LOOP NO.2 DEADWEIGHT JANUARY 25, 1960

SECTION	MEMBER	FX LBS	FY LBS	FZ LBS	MX IN-LBS	MY IN-LBS	MZ IN-LBS	SE PSI	DX INS	DY INS	DZ INS	RX RADS	RY RADS	RZ RADS
1	0	-2882	6296	54	-29794	-5582	-49948	484	0.000	0.000	0.000	0.00000	0.00000	0.00000
1	1	-2882	5574	54	720	8806	-32959	319	0.002	0.000	-0.000	0.00001	0.00002	0.00011
1	2	-2882	4827	54	2158	8806	44352	429	0.005	0.000	-0.001	0.00001	0.00002	0.00011
1	3	-2882	3385	54	-865	7041	10554	122	0.005	-0.003	-0.001	-0.00001	-0.00000	-0.00015
1	4	-2882	-691	54	-8192	-792	-186519	726	0.005	-0.011	0.000	0.00000	-0.00001	0.00009
1	4 CHANGE	-2882	-691	54	-8192	-792	-186519	726	0.005	-0.011	0.000	0.00000	-0.00001	0.00009
1	5	-2882	-1025	54	-7809	-1434	-176231	686	0.006	-0.010	0.000	0.00000	-0.00001	0.00012
1	6	-2882	-3025	54	-6681	-2236	-145879	568	0.006	-0.008	0.000	0.00000	-0.00001	0.00012
1	6 CHANGE	-2882	-3025	54	-6681	-2236	-145879	420	0.006	-0.008	0.000	0.00000	-0.00001	0.00012
1	7	-2882	-4144	54	-2153	-4055	-24091	192	0.006	-0.003	0.000	0.00000	-0.00001	0.00015
1	8	-2882	-5282	54	393	-5232	12699	101	0.003	-0.000	0.001	-0.00000	-0.00000	0.00011
1	8 CHANGE	-2882	-5282	54	393	-5232	12699	40	0.003	-0.000	0.001	-0.00000	-0.00000	0.00011
1	9	-2882	-5282	54	-1216	-5232	-73760	213	-0.000	0.000	0.001	0.00000	0.00000	0.00011
1	0 RESTRT	0	-8997	0	-69752	-78108	-0	0	-0.000	-0.000	0.001	0.00000	-0.00000	0.00011
1	9	-2882	3116	54	68536	72876	-73760	358	-0.000	-0.000	0.001	0.00000	-0.00000	0.00011
1	10	-2882	3116	54	-6	9472	-73760	214	-0.000	-0.000	0.001	-0.00000	-0.00000	0.00011
1	11	-2882	3116	54	-6	6470	100709	290	-0.000	-0.006	0.001	-0.00000	-0.00000	0.00011
1	12	-2882	1116	54	-6	5665	132441	381	-0.000	-0.008	0.001	-0.00000	-0.00000	0.00011
1	12 CHANGE	-2882	1116	54	-6	5665	132441	515	-0.000	-0.008	0.001	-0.00000	-0.00000	0.00011
1	13	-2882	782	54	-6	5022	143824	559	-0.000	-0.009	0.001	-0.00000	-0.00000	0.00009
1	13 CHANGE	-2882	782	54	-6	5022	143824	559	-0.000	-0.009	0.001	-0.00000	-0.00000	0.00009
1	14	-2882	-4792	54	-6	-5714	-257650	1002	0.000	-0.000	-0.000	-0.00000	-0.00000	0.00000

APPENDIX C

Vibration Analysis

- C.1 Introduction
- C.2 Nomenclature
- C.3 Assumptions
- C.4 Free Vibration Analysis of an Idealized System
- C.5 Forced Vibration Analysis of an Idealized System
- C.6 Determination of Influence Coefficients
- C.7 Discussion
- C.8 Attached Data

Appendix C - VIBRATION ANALYSIS

C.1 INTRODUCTION

This appendix includes the thesis, "Vibration Characteristics of a Pump and Motor Mass Attached to a Piping Complex," written by Donald Jerome Nolan. This paper was submitted in partial fulfillment of the degree of Master of Science in Mechanical Engineering at the University of Wisconsin, 1962.

In this analysis the vibratory characteristics of the recirculation pump and piping of Loop Number Two (the most critical from a vibration standpoint) were determined. The natural frequencies of vibration were ascertained and compared to the driving frequency of the rotating elements of the pump and motor assembly. Since there is unbalance in these rotating elements, vibration will be induced in the pump, motor mass, and the connected piping. Hence, the pump, motor masses, and the attached piping will be referred to as the vibration system.

If the induced vibration is at the natural frequency or frequencies of the vibration system, resonance will result. Since the amplitude of vibration becomes very large at resonance, a resonant condition must be avoided, as this may result in structural failure. The natural frequencies can be shifted by changing the piping configuration or by introducing additional restraints. However, even if resonance is not present or can be avoided, the induced vibration present will impose loads on the piping, pump casing, and vessel.

In the actual system there are two obvious sources of excitation which would induce vibration:

1. unbalance of the rotating elements,
2. hydraulic impulses produced by the impeller blades.

The natural frequencies of Loop Number Two were calculated as 414 cpm and 3850 cpm. The ratio of the operating speed to the lowest natural frequency is about 1.4. It was determined that the amplitude of vibration under operating conditions would be small. Actual operation of all three pump loops in the field has proved this to be the case. In conclusion, vibration is not expected to be a problem.

C.2 NOMENCLATURE

UNITS

M	= mass of vibrating body	lb. sec ² /ft
I	= mass moment of inertia of the vibrating body about the center of gravity	lb. ft. sec ²
$\ddot{x}_o$	= linear acceleration of the center of gravity of the vibrating body	ft/sec ²
$\ddot{\theta}_o$	= angular acceleration of the center of gravity of the vibrating body	rad/sec ²
x_o	= linear displacement of the center of gravity of the vibrating body	ft
θ_o	= angular displacement of the center of gravity of the vibrating body	radians
K_{11}	= linear displacement influence coefficient representing the stiffness of piping complex	lb /in
K_{12}	= angular displacement coefficient representing the stiffness of piping complex	lb
K_{21}	= linear displacement influence coefficient representing the stiffness of piping complex	lb
K_{22}	= angular displacement coefficient representing the stiffness of piping complex	in-lb
M_s	= restoring moment imposed on the vibrating body due to the <u>entire</u> connected piping complex	in-lb

H_s = restoring force imposed on the vibrating body due to the <u>entire</u> connected piping complex	lb
W = weight of vibrating body	lb
g = gravitational constant	ft/sec ²
r = radius of gyration of the body about the center of gravity	
ω = frequency of vibration	rad/sec
ω_n = natural frequency of vibrating system	rad/sec
t = time	seconds
X_o = linear amplitude of vibration	ft
θ_o = angular amplitude of vibration	radians
P_o = exciting force acting on vibrating system	lb
L = vertical distance from center of gravity at which P_o acts	ft
α = magnification factor for linear displacement	
β = magnification factor for angular displacement	
F = Auxiliary force applied while actual load is acting	lb
M' = Auxiliary couple applied while actual load is acting	in-lb
θ = slope of beam at point auxiliary load is acting	radians
M_m = Bending moment in beams due to auxiliary loading	in-lb
M'' = variable bending moment in beam due to auxiliary loading	in-lb

M = bending moments in beams due to actual loading	in-lb
ds = incremental length of beam	in
E = Young's modulus	lb /in ²
I = bending moment of inertia of pipe cross section	in ⁴

C.3 ASSUMPTIONS

The recirculation pump and piping Loop No. Two, shown in Fig. 1, is considered to be contained in the vertical "xy" plane. Actually the discharge side of the pump forms an angle of about 2 degrees with the suction side. However, the projection of the piping into the "xy" plane would result in a negligible change in dimensions since the difference in the sine and cosine values for 0 degrees and 2 degrees is very slight.

The analysis is confined to motion in the x-direction and rotation about the z-axis. The other possible degrees of freedom, linear displacement in the z direction and rotation about the y-axis, are not considered here. The piping complex is fabricated of straight runs of piping and elbows. In the analysis, the elbows were replaced by square corners which increased the stiffness of the piping complex, causing a slight increase in the natural frequencies. However, this effect would be of an insignificant magnitude since the elbows have short radii and are a small part of the entire piping complex.

Any damping present in the system, i.e., internal hysteresis, energy dissipation due to contained fluid, and pad friction, will not significantly effect the natural frequencies of the system, and therefore are not included in the analysis. These factors would tend to reduce the amplitude of vibration, but their effects were not evaluated.

For vibration analysis, the piping complex is considered to have flexibility but no mass. Of course, the actual piping complex has mass and is vibrating along with the pump and motor masses. The corresponding kinetic energy would introduce additional modes of vibration, besides affecting the vibration characteristics of the pump and motor mass. The extent of this effect would depend on the ratio of pipe mass to pump and motor mass. For the system being analyzed, this ratio is about 25 per cent. However, it would be difficult to say whether the fundamental frequencies of the pump and motor mass change by the same percentage.

C.4 FREE VIBRATION ANALYSIS OF AN IDEALIZED SYSTEM

The vibration system is considered to have two degrees of freedom; therefore, there are two corresponding independent coordinates. They are linear displacement in the x-direction and angular displacement about the Z-axis. Orientation of the coordinates is shown in Fig. C-1. Since the particular vibration system being analyzed is considered to have two independent coordinates, two differential equations of motion will be required. The assumption that there are two independent coordinates does not imply the motions are independent. The motions are definitely coupled; that is, a force or moment applied at the center of gravity will cause both linear and angular motion.

The equations of motion for this system are found by summing moments and horizontal forces in the "xy" plane about the center of gravity of the oscillating mass. These equations are:

$$M \ddot{x}_0 = -K_{11} x_0 - K_{12} \phi_0 \quad (2-1)$$

$$I \ddot{\phi}_0 = -K_{21} x_0 - K_{22} \phi_0 \quad (2-2)$$

C-7

Fig. C-1...Orientation of Coordinates

The motions of the system, linear displacement in the x-direction and angular displacement about the z-axis, are represented by the following equivalent systems.

EQUIVALENT SYSTEM FOR LINEAR DISPLACEMENT

EQUIVALENT SYSTEM FOR ANGULAR DISPLACEMENT

The motion of the system causes a restoring force and moment exerted by the connected piping complex on the pump and motor mass. The restoring force and moment representing the action of the entire piping complex are expressed as follows:

$$H_s = -K_{11} x_0 - K_{12} \phi_0 \quad (2-3)$$

$$M_s = -K_{21} x_0 - K_{22} \phi_0 \quad (2-4)$$

K_{11} , K_{12} , K_{21} , and K_{22} are the influence coefficients representing the stiffness of the connected piping complex, and K_{12} will be numerically equal to K_{21} .

It is desirable to write Eqs. (2-1 and 2-2) in a more workable form. Letting

$I = mr^2$ and dividing through by m and mr^2 the following expressions are obtained:

$$\ddot{x}_0 + \frac{K_{11}}{m} x_0 + \frac{K_{12}}{m} \phi_0 = 0 \quad (2-5)$$

$$\ddot{\phi}_0 + \frac{K_{21}}{mr^2} x_0 + \frac{K_{22}}{mr^2} \phi_0 = 0 \quad (2-6)$$

Let

$$A = \frac{K_{11}}{m} ; \quad B = \frac{K_{12}}{m} ; \quad C = \frac{K_{22}}{mr^2}$$

Therefore Eqs. (2-5 and 2-6) become

$$\ddot{x}_0 + A x_0 + B \phi_0 = 0 \quad (2-7)$$

$$\ddot{\phi}_0 + \frac{B}{r^2} x_0 + C \phi_0 = 0 \quad (2-8)$$

A general solution for each of these equations is

$$x_0 = X_0 \sin \omega t \quad \text{and} \quad \phi_0 = \theta_0 \sin \omega t$$

The second differential of these solutions with respect to "t" is

$$\ddot{x}_0 = -\omega^2 X_0 \sin \omega t \quad \text{and} \quad \ddot{\phi}_0 = -\theta_0 \omega^2 \sin \omega t$$

Substituting these values into the differential equations of motion the following results are obtained:

$$-\omega^2 X_0 \sin \omega t + A X_0 \sin \omega t + B \theta_0 \sin \omega t = 0 \quad (2-9)$$

$$-\omega^2 \theta_0 \sin \omega t + \frac{B}{r^2} X_0 \sin \omega t + C \theta_0 \sin \omega t = 0 \quad (2-10)$$

Combining terms

$$(-\omega^2 X_0 + A X_0 + B \theta_0) \sin \omega t = 0 \quad (2-11)$$

$$(-\omega^2 \theta_0 + \frac{B}{r^2} X_0 + C \theta_0) \sin \omega t = 0 \quad (2-12)$$

Since $\sin \omega t$ does not equal zero for all values of " t " then

$$-\omega^2 X_0 + A X_0 + B \theta_0 = 0 \quad (2-13)$$

$$-\omega^2 \theta_0 + \frac{B}{r^2} X_0 + C \theta_0 = 0 \quad (2-14)$$

The magnitudes of linear amplitude X_0 and angular amplitude θ_0 cannot be determined for the free vibration case, but their ratios can be evaluated. These amplitudes define the two principal modes of free vibration. The natural frequencies will be found by determining the amplitude ratios and equating them.

From Eq. (2-13)

$$X_0 (-\omega^2 + A) = -B \theta_0$$

$$\frac{X_0}{\theta_0} = \frac{-B}{(-\omega^2 + A)} \quad (2-15)$$

and from Eq. (2-14)

$$\theta_0 (-\omega^2 + C) = -\frac{B}{r^2} X_0$$

$$\frac{X_0}{\theta_0} = \frac{(-\omega^2 + C)}{-B/r^2} \quad (2-16)$$

Equating the amplitude ratios

$$\frac{-B}{-\omega^2 + A} = \frac{-\omega^2 + C}{-B/r^2}$$

which becomes

$$\omega^4 - \omega^2(A+C) + AC - \frac{B}{r^2} = 0 \quad (2-17)$$

This quadratic is the frequency equation for the system whose roots are the natural frequencies corresponding to the principle modes of vibration.

$$\omega_{1,2}^2 = \frac{A+C}{2} \pm \sqrt{\frac{(A-C)^2}{4} + \frac{B^2}{r^2}} \quad (2-18)$$

With the values of the frequencies ascertained, the position of the modes can be determined using Eqs. (2-15 and 2-16).

C.5 FORCED VIBRATION ANALYSIS OF AN IDEALIZED SYSTEM

C.5.1 Linear and Angular Amplitudes of Vibration

Previously the equations of motion were evaluated for the case of free vibration, i.e., the right side of the equations were equal to zero. This represents part of the general solution called the complementary function. The complementary function procedure is followed to determine the natural frequencies of the system; however, the system is under the action of an exciting force $P = P_0 \sin \omega t$, which will maintain the vibratory motion at constant amplitude. The primary reason for evaluating a forced vibration system is to determine the amplitude at the driving frequency.

The assumptions and discussion for the free vibration analysis leading up to Eqs. (2-9 and 2-10) apply for the forced vibration system. Now, it is necessary to obtain a particular solution for the steady-state motion governed by the forcing function $P = P_0 \sin \omega t$. A particular solution is obtained by taking $x_0 = X_0 \sin (\omega t - \phi)$ and $\theta_0 = \Theta_0 \sin (\omega t - \phi)$, ϕ is the phase angle by which the motion lags the impressed force. Since damping is not considered in this analysis the phase angle is either zero or 180 degrees. The differential equations of motion for the forced vibration system are the same as the case of the free vibration system (Eqs. 2-9 and 2-10) except the equations are set equal to the exciting force $P_0 \sin \omega t$ instead of zero. Thus the equations are:

$$(-\omega^2 X_o + A X_o + B \theta_o) \sin \omega t = D \sin \omega t \quad (3-1)$$

$$(-\omega^2 \theta_o + \frac{B}{r^2} X_o + C \theta_o) \sin \omega t = (\frac{D}{r^2} \sin \omega t) L \quad (3-2)$$

where $D = \frac{P_o}{m}$

For the forced vibration system the amplitude of the linear and angular displacements are the desired quantities. By solving the above equations simultaneously the maximum amplitude values are determined.

$$X_o = \frac{P_o |(-\omega^2 + C) - LB/r^2|}{m |(-\omega^2 + A)(-\omega^2 + C) - B^2/r^2|} \quad (3-3)$$

and

$$\theta_o = \frac{P_o |L(-\omega^2 + A)/r^2 - B/r^2|}{m |(-\omega^2 + A)(-\omega^2 + C) - B^2/r^2|} \quad (3-4)$$

The detailed solution for these formulas are shown in the attached data section.

Ignoring free vibration, the steady-state forced vibration is defined by the following equations:

$$X_o = \frac{P_o |(-\omega^2 + C) - LB/r^2|}{m |(-\omega^2 + A)(-\omega^2 + C) - B^2/r^2|} \times \sin \omega t \quad (3-5)$$

$$\theta_o = \frac{P_o |L(-\omega^2 + A)/r^2 - B/r^2|}{m |(-\omega^2 + A)(-\omega^2 + C) - B^2/r^2|} \times \sin \omega t \quad (3-6)$$

At the natural frequencies the quantity $|(-\omega^2 + A)(-\omega^2 + C) - B^2/r^2|$ equals zero; therefore, the amplitude of the linear and angular displacements become infinitely large.

In describing the relationship between amplitude of vibration and frequency,

a non-dimensional expression called the magnification factor is used.

The magnification factor is the ratio of the dynamic displacements or amplitudes due to the exciting force P_0 to the zero frequency displacements produced if P_0 is acting statically.

The relations between the linear and angular displacements or amplitudes and the force P_0 acting statically are

$$P_0 = -K_{11}X_0 - K_{12}\theta_0 \quad (3-7)$$

$$P_0 L = -K_{21}X_0 - K_{22}\theta_0 \quad (3-8)$$

These equations are equivalent to Eqs. (2-3 and 2-4) where

$$P_0 = H_0 \quad \text{and} \quad P_0 L = M_0$$

By solving these two simultaneous equations the zero frequency displacements are

$$(X_0)_{static} = \frac{P_0 \left| 1 - L \times mB/mr^2c \right|}{mA - (mB)^2/mr^2c} \quad (3-9)$$

and

$$(\theta_0)_{static} = \frac{P_0 \left| 1 - AL/B \right|}{m \left| B - Ar^2c/B \right|} \quad (3-10)$$

The detailed calculation determining these values are given below. To obtain the magnification factor for linear displacement divide the numerator and denominator of Eqs. (3-3) by

$$\frac{1 - L \times mB/mr^2c}{mA - (mB)^2/mr^2c}$$

This gives

$$X_o = \frac{\left| \frac{P_o \left| 1 - L \times \frac{mB}{mr^2c} \right|}{\left| mA - (mB)^2/mr^2c \right|} \right| \times \left| (-\omega^2 + c) - \frac{LB}{r^2} \right|}{\left| \frac{m \left| 1 - L \times \frac{mB}{mr^2c} \right|}{\left| mA - (mB)^2/mr^2c \right|} \right| \times \left| (-\omega^2 + A)(-\omega^2 + c) - \frac{B^2}{r^2} \right|}$$

where

$$(X_o)_{static} = \frac{P_o \left| 1 - L \times \frac{mB}{mr^2c} \right|}{\left| mA - (mB)^2/mr^2c \right|}$$

and

$$\frac{m \left| 1 - L \times \frac{mB}{mr^2c} \right|}{\left| mA - (mB)^2/mr^2c \right|} = \frac{m \left| 1 - \frac{LB}{r^2c} \right|}{\left| m \left| A - \frac{B^2}{r^2c} \right| \right|}$$

Therefore

$$X_o = \frac{(X_o)_{static} \left| (-\omega^2 + c) - \frac{LB}{r^2} \right|}{\frac{1 - \frac{LB}{r^2c}}{A - \frac{B^2}{r^2c}} \times \left| (-\omega^2 + A)(-\omega^2 + c) - \frac{B^2}{r^2} \right|}$$

Rearranging terms the magnification factor for linear displacement is

$$\alpha = \frac{X_o}{(X_o)_{static}} = \frac{\left| A - \frac{B^2}{r^2c} \right| \times \left| (-\omega^2 + c) - \frac{LB}{r^2} \right|}{\left| 1 - \frac{LB}{r^2c} \right| \times \left| (-\omega^2 + A)(-\omega^2 + c) - \frac{B^2}{r^2} \right|} \quad (3-11)$$

Similarly the magnification factor for angular displacement is obtained by

dividing the numerator and denominator of Eq. (3-4) by $\frac{1 - AL/B}{m \left| B - Ar^2c/B \right|}$

This gives the following:

$$\Theta_o = \frac{\left| \frac{P_o \left| 1 - AL/B \right|}{m \left| B - Ar^2c/B \right|} \right| \times \left| \frac{L(-\omega^2 + A)}{r^2} - \frac{B}{r^2} \right|}{\left| \frac{m \left| 1 - AL/B \right|}{m \left| B - Ar^2c/B \right|} \right| \times \left| (-\omega^2 + A)(-\omega^2 + c) - \frac{B^2}{r^2} \right|}$$

where

$$(\theta_o)_{static} = \frac{P_o}{m} \frac{1 - AL/B}{B - Ay^2C/B}$$

Therefore

$$\theta_o = \frac{(\theta_o)_{static} \left(L(-\omega^2 + A)/r^2 - B/r^2 \right)}{\frac{(1 - AL/B)}{(B - Ay^2C/B)} (-\omega^2 + A)(-\omega^2 + C) - B^2/r^2}$$

Rearranging terms the magnification factor for angular displacement is

$$\beta = \frac{\theta_o}{(\theta_o)_{static}} = \frac{(B^2/A - r^2C) \left(L(-\omega^2 + A)/r^2 - B/r^2 \right)}{(B/A - L) (-\omega^2 + A)(-\omega^2 + C) - B^2/r^2}$$

The significance of the magnification factors is shown by plotting them against a frequency ratio. This ratio is arbitrarily defined as the frequency of the exciting force divided by the lowest frequency of the free vibration system.

The initial value of the magnification factor will be unity since the frequency of the exciting force is zero. In other words, the displacements are a result of the statical application of the exciting force. This can be shown by equating the frequency ω to zero in Eqs. (3-11 and 3-12).

$$\begin{aligned} \frac{X_o}{(X_o)_{static}} &= \frac{A - B^2/r^2C}{1 - LB/r^2C} \frac{C - LB/r^2}{AC - B^2/r^2} \\ &= \frac{AC - ALB/r^2 - B^2/r^2 + LB^3/r^2C}{AC - B^2/r^2 - ALB/r^2 + LB^3/r^2C} \\ &= 1 \end{aligned}$$

Similarly

$$\begin{aligned} \frac{\theta_0}{(\theta_0)_{\text{static}}} &= \frac{B^2/A - r^2 C \cdot LA/r^2 - B/r^2}{B/A - L \cdot AC - B^2/r^2} \\ &= \frac{B^2 L/r^2 - B^3/r^2 A - LAC + BC}{BC - B^3/r^2 A - LAC + LB^2/r^2} \\ &= 1 \end{aligned}$$

The magnification factors for linear and angular displacement were calculated for different values of the frequency ratio. These calculations are given by Tables C.I and C.II in the attached data. Fig. C.2 shows the values of the magnification factor plotted against the frequency ratio.

As the frequency of the disturbing force increases, both magnification factors increase in the positive direction. As the ratio of ω/ω_n approaches the first and lowest natural frequency of the free vibration system, the magnification factors and the amplitudes become infinite. This is a resonant condition. For this region, both the linear and angular motions are in phase with the exciting force P_0 . When the frequency of the exciting force goes beyond the first natural frequency, the magnification factors become finite but they now have negative values. Both types of motion are now 180 degrees out-of-phase with the exciting force, but in-phase with each other. With further increases in ω , the magnification factor for linear displacement goes through zero and becomes positive while the magnification factor for angular displacement remains negative. This means the linear displacement is again in-phase with the exciting force while the angular motion is out-of-phase with the exciting force. As the frequency of the exciting force approaches the second natural frequency of the free vibration, the magnification factors, and therefore the amplitudes of vibration, become infinite again. As the frequency of the exciting force goes beyond the second natural frequency, the linear displacement is positive and in-phase with the exciting force while the angular motion is negative and out-of-phase. Also, both the linear and angular motions decrease with less and less amplitude. As the driving frequency, ω , becomes very large, the amplitude approaches zero asymptotically.

**MAGNIFICATION FACTOR FOR LINEAR
AND ANGULAR DISPLACEMENT AS
A FUNCTION OF FREQUENCY RATIO.**

L GIVEN VALUE OF 1 FT.

$$\alpha = \frac{x_0}{(x_0)_{ST.}} \quad \text{AND} \quad \beta = \frac{\theta_0}{(\theta_0)_{ST.}}$$

Fig. C.2...Magnification factor plotted
against the frequency ratio

C.5.2 Vibratory Loads in the Piping of an Actual System

The differential equations of motion for the forced vibration system are expressed by

$$M\ddot{X}_0 + K_{11}X_0 + K_{12}\theta_0 = P_0 \sin \omega t \quad 3-13$$

$$Mr^2\ddot{\theta}_0 + K_{21}X_0 + K_{22}\theta_0 = (P_0 \sin \omega t)L \quad 3-14$$

where

$$\ddot{X}_0 = -\omega^2 X_0 \sin \omega t$$

$$\ddot{\theta}_0 = -\omega^2 \theta_0 \sin \omega t$$

$$K_{11}X_0 + K_{12}\theta_0 = H_3$$

$$K_{21}X_0 + K_{22}\theta_0 = M_3$$

$$X_0 = X_0 \sin \omega t$$

$$\theta_0 = \theta_0 \sin \omega t$$

Therefore Eqs. (3-13 and 3-14) become

$$(-m\omega^2 X_0 + H_3) \sin \omega t = P_0 \sin \omega t$$

$$(-mr^2\omega^2 \theta_0 + M_3) \sin \omega t = (P_0 \sin \omega t)L$$

The maximum value of the disturbing force occurs when $\sin \omega t = 1$, and the linear and angular amplitudes of vibration X_0 and θ_0 are given by Eqs. (C-1 and C-2) in the attached data. After substituting,

Eqs. (3-15 and 3-16) become

$$-m\omega^2 \left| \frac{P_o \left| (-\omega^2 + C) - LB/r^2 \right|}{m \left| (-\omega^2 + C)(-\omega^2 + A) - B^2/r^2 \right|} \right| + H_s = P_o \quad (3-15)$$

and

$$-mr^2\omega^2 \left| \frac{P_o \left| L(-\omega^2 + A)/r^2 - B/r^2 \right|}{m \left| (-\omega^2 + A)(-\omega^2 + C) - B^2/r^2 \right|} \right| + M_s = P_o L \quad (3-16)$$

Rearranging terms will give expressions for H_s and M_s as follows:

$$H_s = P_o \left| 1 + \frac{\omega^2 \left| (-\omega^2 + C) - LB/r^2 \right|}{\left| (-\omega^2 + A)(-\omega^2 + C) - B^2/r^2 \right|} \right| \quad (3-17)$$

$$M_s = P_o \left| L + \frac{r^2\omega^2 \left| L(-\omega^2 + A)/r^2 - B/r^2 \right|}{\left| (-\omega^2 + A)(-\omega^2 + C) - B^2/r^2 \right|} \right| \quad (3-18)$$

The exciting force P_o is given by

$$P_o = wR\omega^2/g \quad (3-19)$$

Therefore

$$H_s = wR\frac{\omega^2}{g} \left| 1 + \frac{\omega^2 \left| (-\omega^2 + C) - LB/r^2 \right|}{\left| (-\omega^2 + A)(-\omega^2 + C) - B^2/r^2 \right|} \right| \quad (3-20)$$

$$M_s = wR\frac{\omega^2}{g} \left| L + \frac{r^2\omega^2 \left| L(-\omega^2 + A)/r^2 - B/r^2 \right|}{\left| (-\omega^2 + A)(-\omega^2 + C) - B^2/r^2 \right|} \right| \quad (3-21)$$

Where wR is the amount of unbalance in the system.

For any pump and motor assembly, the amount of unbalance is specified;

the driving frequency (the speed of the rotating parts) is known;

the weight and radius of gyration for the assembly can be easily determined; the gravitational constant "g" is known and the influence coefficients for the attached piping complex can be determined. With these quantities the external or restoring force, H_s , and moment, M_s , can be calculated.

The reactions at the pertinent points in the piping complex are known in terms of H_s and M_s , and therefore, the maximum vibratory bending stresses are easily determined.

C.6 DETERMINATION OF INFLUENCE COEFFICIENTS

The elastic energy method is used to determine the influence coefficients K_{11} , K_{12} , K_{21} , and K_{22} which represent the stiffness of the piping complex.

This method is based on the principle of the conservation of energy.

On the basis of the above principle, energy stored in a structure is equated to work done by the actual load, because an auxiliary load is acting while the actual load is being applied.

The equations which express this principle are

$$\left. \begin{aligned} F\delta &= \int \frac{M'' M ds}{EI} \\ M'\theta &= \int \frac{M'' M ds}{EI} \end{aligned} \right\} \quad (4-1)$$

The left side of each equation represents the mechanical work done by an auxiliary force or moment, and the right side of the equations express elastic energy stored in the beam because an auxiliary force or moment is acting during application of the actual force.

Several assumptions must be made when using the elastic energy theory.

They are: 1) The structure being analyzed behaves as a homogeneous and elastic body... 2) Deformations are small... 3) The effect of direct shear on deformation is negligible... 4) The law of superposition is

* The effect of direct shear on deformation can be evaluated if it is of a significant magnitude.

applicable . . 5) The equations of equilibrium apply.

To explain the application of the method to this problem, a step by step procedure will be given for a typical auxiliary loading.

METHOD OF ANALYSIS

STEP 1. An auxiliary load is introduced, only for the sake of analysis, at a certain location and in a direction in which the displacement or rotation is desired.

The Auxiliary Force F_v is applied at Point 1 to obtain the vertical displacement at this location. Before applying Auxiliary Load F_v or any other auxiliary loading, it is permissible to remove sufficient restraints from the structure in order to make it statically determinate. Therefore, the piping complex is considered free at Point 1 and the vertical restraint at Point 0 is removed. This is allowed as long as the beam is stable and no additional supports or restraints are introduced into the system.

STEP 2. Determine the expression for the bending moment due to the auxiliary load for each member in the structure.

The elastic energy equation for the vertical displacement at POINT 1 is

$$\begin{aligned}
 F_v \delta_v = & \int_0^L \frac{M'' M dL}{EI_{0-L}} = \int_0^{l_1} \frac{F_v w M dw}{EI_{0-l_1}} + \int_0^{l_2} \frac{F_v (l_1 + d) M dx}{EI_{0-l_2}} \\
 & - \int_0^{l_3} \frac{F_v y M dy}{EI_{0-l_3}} + \int_0^{l_3} \frac{F_v (l_1 + d) M dy}{EI_{0-l_3}} \\
 & + \int_0^{l_4} \frac{F_v (l_1 + d - l_3) M dz}{EI_{0-l_4}} \quad (4-2)
 \end{aligned}$$

where $F_v \times \omega$ is the value of the auxiliary bending moment from $\omega = 0$ to $\omega = 1$.

$F_v(l_1 + d)$ is the value of the auxiliary bending moment from $x = 0$ to $x = l_2$.

$F_v(l_1 + d - y)$ is the value of the auxiliary bending moment from $y = 0$ to $y = l_3$. The total auxiliary bending moment is evaluated by two separate integrals, one expressing the variable moment $F_v \times(y)$ and the other the constant moment $F_v(l_1 + d)$.

$F_v(l_1 + d - l_3)$ is the value of the auxiliary bending moment from $z = 0$ to $z = l_4$.

FREE BODY DIAGRAM

AUXILIARY BENDING MOMENT DIAGRAM

Since Auxiliary Load F_v is common to both sides of the energy equation, it can be cancelled out. The bending moment of inertia for the entire beam is constant, and is factored out as a common term.

The energy equation reduces to

$$\delta_v = \frac{1}{EI_{0-L}} \left\{ \int_0^{l_1} w M dw + (l_1 + d) \int_0^{l_2} M dx - \int_0^{l_3} y M dy + (l_1 + d) \int_0^{l_3} M dy + (l_1 + d - l_3) \int_0^{l_4} M dz \right\} \quad (4-3)$$

The expression $\int_0^{l_1} w M dw$ represents the moment of the area of the actual bending moment diagram between POINT 0 and POINT 1, since "M" represents the actual bending moment. A similar analogy is drawn for $\int_0^{l_3} y M dy$. The expression $\int_0^{l_2} M dx$ represents the area of the actual bending moment diagram between the limits 0 to l_2 because "M" represents the actual bending moment. A similar analogy is true for $\int_0^{l_3} M dy$ and $\int_0^{l_4} M dz$. For the actual condition of loading, POINT 1 is completely restrained, i.e., displacement and rotation are zero. Therefore, the energy expression is equal to zero. The final expression for the elastic energy equation is

$$0 = \text{Area } \bar{w} \Big|_0^{l_1} + (l_1 + d) \Big| \text{Area} \Big|_0^{l_2} - \text{Area } \bar{y} \Big|_0^{l_3} + (l_1 + d) \times \Big| \text{Area} \Big|_0^{l_3} + (l_1 + d - l_3) \times \Big| \text{Area} \Big|_0^{l_4} \quad (4-4)$$

STEP 3. Determine the expression for the bending moment for each member in the structure due to the actual loads.

The actual loads referred to here are the horizontal and vertical

forces and the moment acting at the terminal points in the piping complex, i.e., H_1 , V_1 , M_1 , H_0 , V_0 , M_0 , etc.. However, all the forces and moments at these terminal points will be expressed in terms of reactions at POINT 1, the vertical reaction at POINT 0, and the restoring loads H_s and M_s . This introduces five unknown quantities, H_1 , V_1 , M_1 , V_0 and H_s or M_s depending upon which external load is considered acting.

The end result of this part of the analysis is to determine the unknowns in terms of the loads H_s and M_s . The energy equation being derived in detail for the Auxiliary Load F_v gives one equation in terms of the unknowns. Three additional energy equations are obtained by applying horizontal and moment auxiliary loadings at POINT 1 and a vertical auxiliary load at POINT 0. This gives four equations in terms of the five unknowns. The four equations are solved simultaneously to give H_1 , V_1 , M_1 , and V_0 in terms of the loads H_s or M_s . Where H_s and M_s have been defined as the restoring force and moment acting on the vibrating mass, but could be thought of as the loads acting on the piping complex due to the linear and angular displacement of the vibrating mass.

The unknown quantities are now evaluated in terms of the external or restoring force, H_s , for Auxiliary Load F_v .

LINE DIAGRAM WITH H_s ACTING

Now it is necessary to evaluate the quantities in Eq. (4-4)

$$\left. \begin{aligned} \text{Area } \bar{w} \Big|_0^{l_1} &= \frac{V_1 l_1^3}{3} - \frac{M_1 l_1^2}{2} \\ \text{Area} \Big|_0^{l_2} &= \frac{-H_2 l_2^2}{2} - M_2 l_2 \\ \text{Area } \bar{y} \Big|_0^{l_3} &= \frac{V_4 l_3^3}{3} - \frac{M_4 l_3^2}{2} \\ \text{Area} \Big|_0^{l_3} &= \frac{V_4 l_3^2}{2} - M_4 l_3 \\ \text{Area} \Big|_0^{l_4} &= \frac{H_5 l_4^2}{2} - M_5 l_4 \end{aligned} \right\} \quad (4-5)$$

Substituting these values in Eq. (4-4) yields

$$\begin{aligned} 0 = & \left| \frac{V_1 l_1^3}{3} - \frac{M_1 l_1^2}{2} \right| + (l_1 + d) \left| \frac{-H_2 l_2^2}{2} - M_2 l_2 \right| \\ & - \left| \frac{V_4 l_3^3}{3} - \frac{M_4 l_3^2}{2} \right| + (l_1 + d) \left| \frac{V_4 l_3^2}{2} - M_4 l_3 \right| \\ & + (l_1 + d - l_3) \left| \frac{H_5 l_4^2}{2} - M_5 l_4 \right| \end{aligned}$$

FREE BODY DIAGRAM

ACTUAL BENDING MOMENT DIAGRAM

The values of the horizontal and vertical forces and the bending moment at terminal points 2, 3, 4 and 5 in terms of H_1 , V_1 and M_1 are obtained by using the equations of static equilibrium. Substituting for these values and the values of the dimensions of the piping complex l_1, d, l_2, e, l_3 and l_4 yields.

$$0 = 11.22235 \times 10^6 V_1 - 0.06307 \times 10^6 M_1 = 2.08154 \times 10^6 H_1 + 2.84237 \times 10^6 V_0 - 3.02440 \times 10^6 H_s \quad (4-7)$$

Three additional energy equations can be obtained, giving four totals from which the quantities H_1 , V_1 , M_1 , and V_0 can be expressed in terms of H_s .

STEP 4. Apply an auxiliary load F_{H0} at the center of gravity of the pump mass in the direction of the H_s force.

The Auxiliary Load F_{H0} is applied at the center of gravity to obtain the horizontal displacement at this location. The piping complex is made statically determinate by removing restraints at POINT 1 and POINT 0.

STEP 5. Determine the expression for the bending moment due to the auxiliary load for each member in the structure.

The elastic energy equation for the horizontal displacement at POINT O, the center of gravity, is

$$\begin{aligned}
 F_{H_0} \delta_{H_0} = & \int_0^L \frac{M'' M dL}{EI_{0-L}} = - \int_0^{l_2} \frac{F_{H_0} x M dx}{EI_{0-l_2}} \\
 & - \int_0^{l_2} \frac{F_{H_0} e M dx}{EI_{0-l_2}} - \int_0^{l_3} \frac{F_{H_0} (l_2 + e) M dy}{EI_{0-l_3}} \\
 & + \int_0^{l_4} \frac{F_{H_0} z M dz}{EI_{0-l_4}} - \int_0^{l_4} \frac{F_{H_0} (e + l_2) M dz}{EI_{0-l_4}}
 \end{aligned}$$

This expression can be simplified by following a procedure similar to that given in STEP 2. However, the displacement will not be equal to zero and is given by the following equation.

$$\begin{aligned}
 \delta_{H_0} = & \frac{1}{EI_{0-L}} \left\{ - \int_0^{l_2} x M dx - \int_0^{l_2} e M dx \right. \\
 & - \int_0^{l_3} (e + l_2) M dy + \int_0^{l_4} z M dz \\
 & \left. - \int_0^{l_4} (e + l_2) M dz \right\} \quad (4-9)
 \end{aligned}$$

FREE BODY DIAGRAM

AUXILIARY BENDING MOMENT DIAGRAM

The quantities in Eq. (4-9) are expressed as follows:

$$\left. \begin{aligned} \int_0^{l_2} x M dx &= Area \bar{x} \Big|_0^{l_2} = \frac{-H_3 l_2^3}{3} - \frac{M_3 l_2^2}{2} \\ \int_0^{l_2} M dx &= Area \Big|_0^{l_2} = \frac{-H_3 l_2^2}{2} - M_3 l_2 \\ \int_0^{l_3} M dy &= Area \Big|_0^{l_3} = \frac{V_4 l_3^2}{2} - M_4 l_3 \\ \int_0^{l_4} z M dz &= Area \bar{z} \Big|_0^{l_4} = \frac{H_5 l_4^3}{3} - \frac{M_5 l_4^2}{2} \\ \int_0^{l_4} M dz &= Area \Big|_0^{l_4} = \frac{H_5 l_4^2}{2} - M_5 l_4 \end{aligned} \right\} (4-10)$$

Substitution of the above expression into Eq. (4-10) gives

$$\delta_{H_0} = \frac{1}{EI_{0-1}} \left\{ - \left| -\frac{H_3 l_2^3}{3} - \frac{M_3 l_2^2}{2} \right| - e \left| \frac{-H_3 l_2^2}{2} - M_3 l_2 \right| \right. \\ \left. - (e + l_2) \left| \frac{V_4 l_3^2}{2} - M_4 l_3 \right| + \left| \frac{H_5 l_4^3}{3} - \frac{M_5 l_4^2}{2} \right| \right. \\ \left. - (e + l_2) \left| \frac{H_5 l_4^2}{2} - M_5 l_4 \right| \right\} \quad (4-11)$$

The equations of static equilibrium give reactions at terminal points in terms of V_1 , H_1 , M_1 , V_0 and H_5 , and since the values of V_1 , H_1 , M_1 and V_0 have been found in terms of H_5 from the preceding work, Eq. (4-12) simplifies to

$$H_5 = (13.13 \times 10^6) \delta_{H_0}$$

Therefore, influence coefficient K_{11} equals 13.13×10^6 lb/in. By the same procedure, the other influence coefficients are determined.

C.7 DISCUSSION

In the actual system analyzed there are two obvious sources of excitation which would induce vibration: 1. the unbalance of the rotating elements; 2. the hydraulic impulses produced by the impeller blades. This means there are two driving frequencies; the operating speed of the rotating elements and the operating speed times the number of impeller blades.

The natural frequencies of the actual system were calculated as 414 cpm and 3850 cpm. The ratio of the operating speed to the lowest natural frequency is about 1.4. From the graph of magnification factor the amplitude of vibration at operating speed would be small. The same conclusion can be made due to the hydraulic impulses.

The loads imposed on the piping complex at operating speed were calculated and found to be insignificant. From this it can be concluded that the driving frequencies would have to be very close to the natural frequencies before the amplitudes of vibration would build up to a magnitude which could cause serious effects.

The nodal point corresponding to the lowest natural frequency is located at the centerline of the piping complex discharge pipe. Due to this fact, the rigidity of this member is greatly decreased. Restraint on the vibration system is thereby reduced, and as a result the pump and motor

mass oscillates about this point at the lowest natural frequency.

Since there is no experimental verification of the theoretical analysis, it is difficult to determine the accuracy of the results. Several simplifying assumptions were made and each will effect the correctness of the results to varying degrees. However, even though experimental verification is not possible there are several indications in the analysis which lend validity to the theory. This is readily apparent in the section on determination of influence coefficients. The coefficients on either side of the diagonal of the matrix formed by the four final energy equations are equal. This is true for the matrix formed by consideration of the restoring force and moment.

The final equation giving the influence coefficients representing stiffness of the piping complex show that influence coefficients K_{12} and K_{21} are equal within $2\frac{1}{2}$ per cent. This should be, since Maxwell's Law of Reciprocals says that the coefficient for rotation due to a force equals the coefficient for deflection due to a moment.

There are no other places in the analysis where the theory can be checked in a similar manner. However, if the remainder of the analysis is carefully executed the results should be correct within the limits of the assumptions.

C.8 ATTACHED DATA

C.8.1 Natural Frequencies

$$\omega_{1,2}^2 = \frac{A+C}{2} \pm \sqrt{\frac{(A-C)^2}{4} + \frac{B^2}{r^2}}$$

Where ω = natural frequencies of vibration system, rad/sec

r = radius of gyration of vibrating mass, 3.16 ft.

$$\text{and } A = \frac{K_{11}}{m} = \frac{13.13 \times 10^6 \times 12}{42620/32.2}$$

$$= 119000 \text{ sec}^{-2}$$

$$B = \frac{K_{12}}{m} \text{ or } \frac{K_{21}}{m} = \frac{301.5 \times 10^6}{42620/32.2}$$

$$= 227500 \text{ ft-sec}^{-2}$$

$$C = \frac{K_{22}}{mr^2} = \frac{7250 \times 10^6}{42620/32.2 \times 3.16^2 \times 12}$$

$$= 45750 \text{ sec}^{-2}$$

Substitution yields

$$\omega_{1,2} = \left. \begin{array}{l} 403 \text{ rad/sec} \\ 43.3 \text{ rad/sec} \end{array} \right\} = \left. \begin{array}{l} 64.3 \text{ cps} \\ 6.9 \text{ cps} \end{array} \right\} = \left. \begin{array}{l} 3850 \text{ cpm} \\ 414 \text{ cpm} \end{array} \right\}$$

C.8.2 NODAL POSITIONS

$$\frac{X_o}{\Theta_o} = \frac{-B}{(-\omega_1^2 + A)} = \frac{-227500}{(-162875 + 119000)}$$

$$= \underline{\underline{+5.2 \text{ ft.}}}$$

$$\frac{X_o}{\Theta_o} = \frac{-B}{(-\omega_2^2 + A)} = \frac{-227500}{(-1875 + 119000)}$$

$$= \underline{\underline{-1.94 \text{ ft.}}}$$

C.8.3 LINEAR AND ANGULAR AMPLITUDES OF VIBRATION

$$(-\omega^2 X_o + A X_o + B \Theta_o) \sin \omega t = D \sin \omega t$$

$$(-\omega^2 \Theta_o + \frac{B}{r^2} X_o + C \Theta_o) \sin \omega t = \left(\frac{D_x \sin \omega t}{r^2} \right) \times L$$

Assume $\sin \omega t = 1$ and rearrange terms

$$X_o (-\omega^2 + A) + \Theta_o B = D$$

$$X_o \frac{B}{r^2} + \Theta_o (-\omega^2 + C) = \frac{D_x}{r^2} \times L$$

Multiply first expression by $-\frac{(-\omega^2 + C)}{B}$

$$-X_o \frac{(-\omega^2 + A)(-\omega^2 + C)}{B} - \Theta_o \frac{B(-\omega^2 + C)}{B} = -\frac{D(-\omega^2 + C)}{B}$$

$$X_o \frac{B}{r^2} + \Theta_o (-\omega^2 + C) = \frac{D_x}{r^2} \times L$$

Adding yields

$$-X_0 \left[\frac{(-\omega^2 + A)(-\omega^2 + c)}{B} - \frac{B}{r^2} \right] = -D \left[\frac{(-\omega^2 + c)}{B} - \frac{L}{r^2} \right]$$

Simplifying and multiplying through by B yields

$$X_0 = \frac{D \left| (-\omega^2 + c) - \frac{LB}{r^2} \right|}{\left| (-\omega^2 + A)(-\omega^2 + c) - \frac{B^2}{r^2} \right|}$$

$$\text{since } D = \frac{P_0}{m}$$

$$X_0 = \frac{P_0 \left| (-\omega^2 + c) - \frac{LB}{r^2} \right|}{\left| m \left| (-\omega^2 + A)(-\omega^2 + c) - \frac{B^2}{r^2} \right| \right|} \quad (C-1)$$

To obtain expression for Θ_0 multiply

$$\left| X_0 \frac{B}{r^2} + \Theta_0 (-\omega^2 + c) = \frac{DL}{r^2} \right| \text{ by } \left| \frac{(-\omega^2 + A)}{B/r^2} \right|$$

This gives

$$X_0 (-\omega^2 + A) + \Theta_0 B = D$$

$$-X_0 \times \frac{B}{r^2} \times \frac{(-\omega^2 + A)r^2}{B} - \Theta_0 \frac{(-\omega^2 + c)(-\omega^2 + A)}{B/r^2} = \frac{DL(-\omega^2 + A)}{r^2 \frac{B}{r^2}}$$

Adding yields

$$-\Theta_0 \left| \frac{(-\omega^2 + c)(-\omega^2 + A)r^2}{B} - B \right| = -D \left| \frac{L(-\omega^2 + A) - 1}{B} \right|$$

Simplifying and multiplying through by $\frac{B}{r^2}$ yields

$$\Theta_0 = \frac{D \left| L(-\omega^2 + A) - \frac{B}{r^2} \right|}{\left| (-\omega^2 + A)(-\omega^2 + C) - \frac{B^2}{r^2} \right|}$$

since $D = \frac{P_0}{m}$

$$\Theta_0 = \frac{P_0 \left| \frac{L(-\omega^2 + A)}{r^2} - \frac{B}{r^2} \right|}{m \left| (-\omega^2 + C)(-\omega^2 + A) - \frac{B^2}{r^2} \right|} \quad (c-2)$$

C.8.4 SAMPLE CALCULATION OF MAGNIFICATION FACTORS

1. FOR LINEAR DISPLACEMENT

$$\frac{X_o}{(X_o)_{\text{static}}} = \frac{\left| A - \frac{B^2}{r^2 C} \right| \left| (-\omega^2 + C) - \frac{LB}{r^2} \right|}{\left| 1 - \frac{LB}{r^2 C} \right| \left| (-\omega^2 + A)(-\omega^2 + C) - \frac{B^2}{r^2} \right|}$$

$$\text{where } A = 119000 \text{ sec}^{-2}$$

$$B = 227500 \text{ ft-sec}^{-2}$$

$$C = 45750 \text{ sec}^{-2}$$

$$r = 3.16 \text{ ft.}$$

Assume $L = 1 \text{ ft.}$; $\omega = 10.8 \text{ rad/sec}$ and $\omega/\omega_n = 0.25$

$$\frac{B^2}{r^2 C} = \frac{227500}{(3.16)^2 (45750)} = 113400 \text{ sec}^{-2}$$

$$(-\omega^2 + C) = -(10.8)^2 + (45750) = 45633 \text{ sec}^{-2}$$

$$\frac{LB}{r^2} = \frac{1 \times 227500}{(3.16)^2} = 22750 \text{ sec}^{-2}$$

$$\frac{LB}{r^2 C} = \frac{1 \times 227500}{(3.16)^2 (45750)} = 0.4965$$

$$(-\omega^2 + A) = -(10.8)^2 + 119000 = 118883 \text{ sec}^{-2}$$

$$\frac{B^2}{r^2} = \frac{(227500)^2}{(3.16)^2} = 5.195 \times 10^9 \text{ sec}^{-4}$$

Therefore

$$\frac{X_o}{(X_o)_{static}} = \underline{\underline{+1.11}}$$

2. FOR ANGULAR DISPLACEMENT

$$\frac{\theta_o}{(\theta_o)_{static}} = \frac{\left| \frac{L(-\omega^2 + A)}{r^2} - \frac{B}{r^2} \right| \left| r^2 C - \frac{B^2}{A} \right|}{\left| L - \frac{B}{A} \right| \left| (-\omega^2 + A)(-\omega^2 + C) - \frac{B^2}{r^2} \right|}$$

Assume $L = 1 \text{ ft.}$; $\omega = 10.8 \text{ rad/sec}$; $\omega/\omega_n = 0.25$

$$\frac{L(-\omega^2 + A)}{r^2} = \frac{1 \times |-(10.8)^2 + 119000|}{(3.16)^2} = 11888 \text{ ft}^{-1} \text{ sec}^{-2}$$

$$\frac{B}{r^2} = \frac{227500}{(3.16)^2} = 22750 \text{ ft}^{-1} \text{ sec}^{-2}$$

$$r^2 C = (3.16)^2 (45750) = 457500 \text{ ft}^2 \text{ sec}^{-2}$$

$$\frac{B^2}{A} = \frac{(227500)^2}{119000} = 435000 \text{ ft}^2 \text{ sec}^{-2}$$

$$\frac{B}{A} = \frac{227500}{119000} = 1.91 \text{ ft}$$

Therefore

$$\frac{\theta_o}{(\theta_o)_{static}} = \underline{\underline{+1.17}}$$

TABLE C.1 MAGNIFICATION FACTOR FOR LINEAR DISPLACEMENT

RATIO w/w_n	w (rad./sec)	B^2/r^2	$(-w^2+c)$	LB/r^2	LB/r^2	$(-w^2+A)$	B^2/r^2
0.25	10.8	113400	45633	22750.	0.4965	118883	5.195
0.50	21.65	↑	45281	↑	↑	118531	↑
0.75	32.45		44700			117950	
1.05	45.5		43680			116930	
1.25	54.1		42820			116070	
1.50	65.0		41530			114780	
2.00	86.6		38250			111500	
4.00	173.2		15750			89000	
6.00	259.8		-21750			51500	
8.00	346.4		-74250			-1000	
9.00	390.0		-106250			-33000	
9.15	396.0	↓	-111250	↓	↓	-38000	↓
9.50	411.0		-123250			-49800	
10.00	433.0		-142050			-68800	

RATIO w/w_n	w (rad./sec)	$A - B^2/r^2$	$(-w^2+c) - LB/r^2$	$1 - LB/r^2$	$(-w^2+A)(-w^2+c) - B^2/r^2$	$\propto \frac{X_o}{(X_o)_{static}}$
0.25	10.8	5600	22883	0.5035	+0.23x10 ⁹	+1.11
0.50	21.65	↑	22531	↑	+0.16x10 ⁹	+1.57
0.75	32.45		21950		+0.07x10 ⁹	+3.5
1.05	45.5		20930		-0.09x10 ⁹	-2.58
1.25	54.1		20070		-0.23x10 ⁹	-0.965
1.50	65.0		18780		-0.43x10 ⁹	-0.485
2.00	86.6		15500		-0.94x10 ⁹	-0.183
4.00	173.2		-7000		-3.80x10 ⁹	+0.02
6.00	259.8		-44500		-6.313x10 ⁹	+0.078
8.00	346.4		-97000		-5.121x10 ⁹	+0.21
9.00	390.0		-129000		-1.69x10 ⁹	+0.85
9.15	396.0	↓	-134000	↓	-0.97x10 ⁹	+1.54
9.50	411.0		-146000		+0.93x10 ⁹	-1.72
10.00	433.0		-164800		+4.75x10 ⁹	-0.20

TABLE C.11 MAGNIFICATION FACTOR FOR ANGULAR DISPLACEMENT

RATIO ω/ω_n	ω (rad/sec)	$(-\omega^2 + A)$	$\frac{L}{r^2}(-\omega^2 + A)$	$-\frac{B}{r^2}$	$r^2 C$	$\frac{B^2}{A}$	$\frac{L(-\omega^2 + A)}{r^2} - \frac{B^2}{r^2}$
0.25	10.80	118883	11888	22750	457500	435000	-10862
0.50	21.65	118531	11853	↑	↑	↑	-10897
0.75	32.45	117950	11795	↑	↑	↑	-10955
1.05	45.50	116930	11693	↑	↑	↑	-11057
1.25	54.10	116070	11607	↑	↑	↑	-11143
1.50	65.00	114780	11478	↑	↑	↑	-11272
2.00	86.60	111500	11150	↑	↑	↑	-11600
4.00	173.2	89000	8900	↑	↑	↑	-13850
6.00	259.8	51500	5160	↑	↑	↑	-17590
8.00	346.4	-1000	-100	↑	↑	↑	-22850
9.00	390.0	-33000	-3300	↑	↑	↑	-26050
9.15	396.0	-38000	-3800	↑	↑	↑	-26550
9.20	398.0	-39500	-3950	↑	↑	↑	-26700
9.31	403.0	-43500	-4350	↑	↑	↑	-27100
9.50	411.0	-49800	-4980	↓	↓	↓	-27730
10.00	433.0	-68800	-6880	↓	↓	↓	-29630

RATIO ω/ω_n	$r^2 C - \frac{B^2}{A}$	$\frac{B}{A}$	$L - \frac{B}{A}$	$(-\omega^2 + C)$	$\frac{B^2}{r^2}$	$(-\omega^2 + A)(-\omega^2 + C) - \frac{B^2}{r^2}$	$\beta = \frac{e}{(e_0)_{static}}$
0.25	22500	1.91	-0.91	45633	5.19x10 ⁹	+0.23x10 ⁹	+1.17
0.50	↑	↑	↑	45281	↑	+0.16x10 ⁹	+1.58
0.75	↑	↑	↑	44700	↑	+0.07x10 ⁹	+3.86
1.05	↑	↑	↑	43680	↑	-0.09x10 ⁹	-3.06
1.25	↑	↑	↑	42820	↑	-0.23x10 ⁹	-1.15
1.50	↑	↑	↑	41530	↑	-0.43x10 ⁹	-0.635
2.00	↑	↑	↑	38250	↑	-0.94x10 ⁹	-0.305
4.00	↑	↑	↑	15750	↑	-3.80x10 ⁹	-0.090
6.00	↑	↑	↑	-21650	↑	-6.313x10 ⁹	-0.069
8.00	↑	↑	↑	-74250	↑	-5.121x10 ⁹	-0.110
9.00	↑	↑	↑	-106250	↑	-1.69x10 ⁹	-0.391
9.15	↑	↑	↑	-111250	↑	-0.97x10 ⁹	-0.676
9.20	↑	↑	↑	-112750	↑	-0.75x10 ⁹	-0.892
9.31	↑	↑	↑	-116750	↑		
9.50	↓	↓	↓	-123250	↓	+0.93x10 ⁹	+0.737
10.00	↓	↓	↓	-142050	↓	+4.75x10 ⁹	+0.16

$$H_s = wR \frac{\omega^2}{g} \times \left| 1 + \frac{\omega^2 \left| (-\omega^2 + C) - \frac{LB}{r^2} \right|}{\left| (-\omega^2 + A)(-\omega^2 + C) - \frac{B^2}{r^2} \right|} \right|$$

where wR = specific amount of unbalance, 0.0521 ft-lbs

ω = speed of rotating parts, 61.2 rad/sec

$$(-\omega^2 + C) = -(61.2)^2 + 46750 = 42010 \text{ sec}^{-2}$$

$$\frac{LB}{r^2} = \frac{1 \times 227500}{(3.16)^2} = 22750 \text{ sec}^{-2}$$

$$(-\omega^2 + A) = -(61.2)^2 + 119000 = 115260 \text{ sec}^{-2}$$

$$\frac{B^2}{r^2} = \frac{(227500)^2}{(3.16)^2} = 6.195 \times 10^9 \text{ sec}^{-2}$$

$$H_s = \underline{\underline{4.82 \text{ lbs}}}$$

$$M_s = wR \frac{\omega^2}{g} \times \left| L + \frac{r^2 \omega^2 \left| \frac{L(-\omega^2 + A)}{r^2} - \frac{B}{r^2} \right|}{\left| (-\omega^2 + A)(-\omega^2 + C) - \frac{B^2}{r^2} \right|} \right|$$

$$\text{where } \frac{B}{v^2} = \frac{227500}{(3.16)^2} = 22750 \text{ ft}^{-1} \text{ sec}^{-2}$$

$$\frac{L(-w^2+A)}{v^2} = \frac{1 - (10.8)^2 + 119000}{(3.16)^2} = 11526 \text{ ft}^{-1} \text{ sec}^{-2}$$

$$M_s = \underline{\underline{78.5 \text{ ft-lbs}}}$$

PART I:- DETERMINE REACTIONS AT TERMINAL POINTS IN TERMS OF H_S AND M_S

LINE DIAGRAM WITH LOADS H_S AND M_S ACTING

FREE BODY DIAGRAM OF PIPING COMPLEX WITH ONLY LOAD H_S ACTING

ACTUAL BENDING MOMENT DIAGRAM
WITH H_5 ACTING

FREE BODY DIAGRAM OF PIPING COMPLEX
WITH ONLY LOAD M_5 ACTING

ACTUAL BENDING MOMENT DIAGRAM WITH M_5 ACTING

EXPRESSIONS FOR THE AREAS AND THE MOMENT OF THE AREAS OF THE ACTUAL
BENDING MOMENT DIAGRAMS

For H_s acting only

$$\text{Area} \Big|_0^{l_1} = \frac{V_1 l_1^2}{2} - M_1 l_1 \quad \& \quad \text{Area} \bar{w} \Big|_0^{l_1} = \frac{V_1 l_1^3}{3} - \frac{M_1 l_1^2}{2}$$

$$\text{Area} \Big|_0^{l_2} = -\frac{H_2 l_2^2}{2} - M_2 l_2 \quad \& \quad \text{Area} \bar{x} \Big|_0^{l_2} = -\frac{H_2 l_2^3}{3} - \frac{M_2 l_2^2}{2}$$

$$\text{Area} \Big|_0^{l_3} = \frac{V_4 l_3^2}{2} - M_4 l_3 \quad \& \quad \text{Area} \bar{y} \Big|_0^{l_3} = \frac{V_4 l_3^3}{3} - \frac{M_4 l_3^2}{2}$$

$$\text{Area} \Big|_0^{l_4} = \frac{H_5 l_4^2}{2} - M_5 l_4 \quad \& \quad \text{Area} \bar{z} \Big|_0^{l_4} = \frac{H_5 l_4^3}{3} - \frac{M_5 l_4^2}{2}$$

For M_s acting only

$$\text{Area} \Big|_0^{l_1} = \frac{V_1 l_1^2}{2} - M_1 l_1 \quad \& \quad \text{Area} \bar{w} \Big|_0^{l_1} = \frac{V_1 l_1^3}{3} - \frac{M_1 l_1^2}{2}$$

$$\text{Area} \Big|_0^{l_2} = \frac{H_2 l_2^2}{2} - M_2 l_2 \quad \& \quad \text{Area} \bar{x} \Big|_0^{l_2} = \frac{H_2 l_2^3}{3} - \frac{M_2 l_2^2}{2}$$

$$\text{Area} \Big|_0^{l_3} = \frac{V_4 l_3^2}{2} - M_4 l_3 \quad \& \quad \text{Area} \bar{y} \Big|_0^{l_3} = \frac{V_4 l_3^3}{3} - \frac{M_4 l_3^2}{2}$$

$$\text{Area} \Big|_0^{l_4} = -\frac{H_5 l_4^2}{2} - M_5 l_4 \quad \& \quad \text{Area} \bar{z} \Big|_0^{l_4} = -\frac{H_5 l_4^3}{3} - \frac{M_5 l_4^2}{2}$$

The moments and forces at all terminal points in terms of H_1, V_1, M_1, V_0 and H_s are obtained by the use of equations of static equilibrium.

APPLICATION OF AUXILIARY LOADING

AUXILIARY VERTICAL FORCE APPLIED AT POINT 1

LINE DIAGRAM SHOWING
STATICALLY DETERMINATE SYSTEM

FREE BODY DIAGRAM OF EACH MEMBER
IN PIPING COMPLEX

AUXILIARY BENDING MOMENT DIAGRAM

AUXILIARY BENDING FORCE APPLIED AT POINT 1 LINE DIAGRAM SHOWING STATICALLY DETERMINATE SYSTEM

FREE BODY DIAGRAM OF EACH MEMBER IN PIPING COMPLEX

AUXILIARY BENDING MOMENT DIAGRAM

AUXILIARY COUPLE APPLIED AT POINT 1

LINE DIAGRAM SHOWING
STATICALLY DETERMINATE SYSTEM

FREE BODY DIAGRAM OF EACH MEMBER
IN PIPING COMPLEX

AUXILIARY BENDING MOMENT DIAGRAM

AUXILIARY VERTICAL FORCE APPLIED AT POINT O

LINE DIAGRAM SHOWING
STATICALLY DETERMINATE SYSTEM

FREE BODY DIAGRAM OF EACH MEMBER
IN PIPING COMPLEX

AUXILIARY BENDING MOMENT DIAGRAM

INITIAL ENERGY EQUATIONS

Work done by Aux. load	Integral between o-l ₁	Integral between o-l ₂
$F_v \delta_v =$	$+\frac{F_v w M dw}{EI_{o-l_1}}$	$+\frac{F_v(l_1+d)M dz}{EI_{o-l_2}}$
$F_H \delta_H =$		$+\frac{F_H x M dx}{EI_{o-l_1}} + \frac{F_H b M dx}{EI_{o-l_2}}$
$M' \theta =$	$-\frac{M_m M dw}{EI_{o-l_1}}$	$-\frac{M_m M dx}{EI_{o-l_2}}$
$F_v \delta_u =$		

Work done by Aux. load	Integral between o-l ₃	Integral between o-l ₄
$F_v \delta_v =$	$-\frac{F_v y M dy}{EI_{o-l_3}} + \frac{F_v(l_1+d)M dy}{EI_{o-l_3}}$	$+\frac{F_v(l_1+d-l_3)M dz}{EI_{o-l_4}}$
$F_H \delta_H =$	$+\frac{F_H(l_2+b)M dy}{EI_{o-l_3}}$	$+\frac{F_H(b+l_2)M dz}{EI_{o-l_4}} - \frac{F_H z M dz}{EI_{o-l_4}}$
$M' \theta =$	$-\frac{M_m M dy}{EI_{o-l_3}}$	$-\frac{M_m M dz}{EI_{o-l_4}}$
$F_v \delta_u =$	$\frac{F_v y M dy}{EI_{o-l_3}}$	$\frac{F_v l_3 M dz}{EI_{o-l_4}}$

SIMPLIFIED ENERGY EQUATIONS

Quantity	Integral between o-l ₁	Integral between o-l ₂
$EI\delta_v =$	$+\int w M dw$	$+(l_1+d)\int M dx$
$EI\delta_n =$		$+\int x M dx + b\int M dx$
$EI\theta =$	$-\int M dw$	$-\int M dx$
$EI\delta_{v_0} =$		

Quantity	Integral between o-l ₃	Integral between o-l ₄
$EI\delta_v =$	$-\int y M dy + (l_1+d)\int M dy$	$+(l_1+d-l_3)\int M dz$
$EI\delta_n =$	$+(l_2+b)\int M dy$	$+(b+l_2)\int M dz - \int z M dz$
$EI\theta =$	$-\int M dy$	$-\int M dz$
$EI\delta_{v_0} =$	$+\int y M dy$	$+l_3\int M dz$

EQUIVALENT ENERGY EQUATIONS

Quantity	Integral between 0- l_1	Integral between 0- l_2
$0 =$	$+ Area \bar{w} _0^{l_1}$	$+ l_1 + d Area _0^{l_2}$
		$+ Area \bar{x} + b Area _0^{l_2}$
	$- Area _0^{l_1}$	$- Area _0^{l_2}$

Quantity	Integral between 0- l_3	Integral between 0- l_4
$0 =$	$- Area \bar{y} _0^{l_3} + l_1 + d Area _0^{l_3}$	$+ l_1 + d - l_3 Area _0^{l_4}$
$0 =$	$+ b + l_2 Area _0^{l_3}$	$+ b + l_2 Area _0^{l_4} - Area \bar{z} _0^{l_4}$
$0 =$	$- Area _0^{l_3}$	$- Area _0^{l_4}$
$0 =$	$+ Area \bar{y} _0^{l_3}$	$+ l_3 Area _0^{l_4}$

ENERGY EQUATIONS IN TERMS OF THE REACTIONS WITH ONLY Hs ACTING

Work done by Aux. load	Integral between o-l ₁	Integral between o-l ₂
$0 = \left \frac{V_1 l_1^3}{3} - \frac{M_1 l_1^2}{2} \right $		$+ \left l_1 + d \right \left \frac{H_2 l_2^2}{2} - M_2 l_2 \right $
$0 =$		$+ \left \frac{H_2 l_2^3}{3} - \frac{M_2 l_2^2}{2} \right + b \left \frac{H_2 l_2^2}{2} - M_2 l_2 \right $
$0 = - \left \frac{V_1 l_1^2}{2} - M_1 l_1 \right $		$- \left \frac{H_2 l_2^2}{2} - M_2 l_2 \right $
$0 =$		

Work done by Aux. load	Integral between o-l ₃	Integral between o-l ₄
$0 = - \left \frac{V_4 l_3^3}{3} - \frac{M_4 l_3^2}{2} \right + \left l_1 + d \right \left \frac{V_4 l_3^2}{2} - M_4 l_3 \right + \left l_1 + d - l_3 \right \left \frac{H_5 l_4^2}{2} - M_5 l_4 \right $		
$0 = + \left b + l_2 \right \left \frac{V_4 l_3^2}{2} - M_4 l_3 \right $		$\left b + l_2 \right \left \frac{H_5 l_4^2}{2} - M_5 l_4 \right - \left \frac{H_5 l_4^3}{3} - \frac{M_5 l_4^2}{2} \right $
$0 = - \left \frac{V_4 l_3^3}{3} - \frac{M_4 l_3^2}{2} \right $		$- \left \frac{H_5 l_4^2}{2} - M_5 l_4 \right $
$0 = \left \frac{V_4 l_3^3}{3} - \frac{H_4 l_3^2}{2} \right $		$l_3 \left \frac{H_5 l_4^2}{2} - M_5 l_4 \right $

The values of H_3 , M_3 , V_4 , M_4 , H_5 and M_5 in terms of V_1 , H_1 , and M_1 , are obtained by the use of the equations of static equilibrium.

Substitution for these values and the dimensions of the piping complex l_1 , d , l_2 , e , l_3 , and l_4 , yields the following equations:

$$1: 0 = 11.2223V_1 - 0.06307M_1 + 2.08154H_1 + 2.84237V_0 - 3.02444H_s$$

$$2: 0 = 0.06308V_1 + 0.00059M_1 - 0.01704H_1 - 0.05949V_0 + 0.02571H_s$$

$$3: 0 = 2.08120V_1 - 0.01704M_1 + 0.85870H_1 + 2.4893V_0 - 1.25409H_s$$

$$4: 0 = 2.84237V_1 - 0.05949M_1 + 2.48930H_1 + 13.11464V_0 - 3.87245H_s$$

Solution of the simultaneous equation yields

$$V_1 = 0.04834H_s; M_1 = 3.88016H_s; H_1 = 1.21084H_s; V_0 = 0.07257H_s$$

The same auxiliary loading and diagrams used to determine the energy equations with only H_s acting are used to obtain the energy equations when only M_s is acting. There will be a few sign changes but the equations are essentially the same.

ENERGY EQUATIONS IN TERMS OF THE REACTIONS WITH ONLY M_s ACTING

Work done by Aux. load	Integral between o- l_1	Integral between o- l_2
$0 =$	$\left \frac{V_1 l_1^3}{3} - \frac{M_1 l_1^2}{2} \right $	$\left l_1 + d \left \frac{H_2 l_2^2}{2} - M_2 l_2 \right \right $
$0 =$		$\left \frac{H_2 l_2^3}{3} - \frac{M_2 l_2^2}{2} \right + b \left \frac{H_2 l_2^2}{2} - M_2 l_2 \right $
$0 =$	$\left -\frac{V_1 l_1^2}{2} - M_1 l_1 \right $	$\left -\frac{H_2 l_2^2}{2} - M_2 l_2 \right $
$0 =$		

Work done by Aux. load	Integral between o- l_3	Integral between o- l_4
$0 =$	$\left -\frac{V_4 l_3^3}{3} - \frac{M_4 l_3^2}{2} \right + l_1 + d \left \frac{V_4 l_3^2}{2} - M_4 l_3 \right $	$\left l_1 + d - l_3 \left \frac{H_5 l_4^2}{2} - M_5 l_4 \right \right $
$0 =$	$\left b + l_2 \left \frac{V_4 l_3^2}{2} - M_4 l_3 \right \right $	$\left b + l_2 \left \frac{H_5 l_4^2}{2} - M_5 l_4 \right \right - \left \frac{H_5 l_4^3}{3} - \frac{M_5 l_4^2}{2} \right $
$0 =$	$\left -\frac{V_4 l_3^2}{2} - M_4 l_3 \right $	$\left -\frac{H_5 l_4^2}{2} - M_5 l_4 \right $
$0 =$	$\left \frac{V_4 l_3^3}{3} - \frac{M_4 l_3^2}{2} \right $	$\left l_{3x} \left \frac{H_5 l_4^2}{2} - M_5 l_4 \right \right $

The equations in the preceding page after substitution for the various quantities and dimensions yield the following equations.

$$1: 0 = 11.22236V_I - 0.06307M_I - 2.08154H_I + 2.84227V_O - 0.04055Ms$$

$$2: 0 = -0.06308V_I + 0.00059M_I + 0.01704H_I - 0.05949V_O + 0.00037Ms$$

$$3: 0 = -2.08120V_I + 0.01704M_I + 0.095970H_I - 2.48930V_O + 0.01704Ms$$

$$4: 0 = 2.84237V_I - 0.05949M_I - 2.48930H_I + 13.11464V_O - 0.05949Ms$$

Solving the simultaneous equations produces

$$V_I = 0.00233Ms; M_I = 0.20964Ms; H_I = 0.06867Ms; V_O = 0.00333Ms$$

PART 2:- H_s AND M_s IN TERMS OF THE DISPLACEMENT OF THE CENTER OF GRAVITY

AUXILIARY HORIZONTAL FORCE APPLIED AT CENTER OF GRAVITY

LINE DIAGRAM SHOWING STATICALLY DETERMINATE SYSTEM

FREE BODY DIAGRAM OF EACH MEMBER IN PIPING COMPLEX

AUXILIARY BENDING MOMENT DIAGRAM

AUXILIARY COUPLE APPLIED AT
CENTER OF GRAVITY

LINE DIAGRAM SHOWING
STATICALLY DETERMINATE SYSTEM

FREE BODY DIAGRAM OF EACH MEMBER
IN PIPING COMPLEX

AUXILIARY BENDING MOMENT DIAGRAM

INITIAL ENERGY EQUATIONS

Work done by Aux. load	Integral between o-l ₁	Integral between o-l ₂
$F_H \delta_o =$		$-\frac{F_H x M dx}{EI_{o-l_2}} - \frac{F_H e M dx}{EI_{o-l_2}}$
$M_o \theta_o =$		$-\frac{M_o M dx}{EI_{o-l_2}}$
	Integral between o-l ₃	Integral between o-l ₄
$F_H \delta_o =$	$-\frac{F_H (e+l_2) M dy}{EI_{o-l_3}}$	$+\frac{F_H z M dz}{EI_{o-l_4}} - \frac{F_H (e+l_2) M dz}{EI_{o-l_4}}$
$M_o \theta_o =$	$-\frac{M_o M dy}{EI_{o-l_3}}$	$-\frac{M_o M dz}{EI_{o-l_4}}$

SIMPLIFIED ENERGY EQUATIONS

Quantity	Integral between o-l ₁	Integral between o-l ₂
$EI \delta_o =$		$-\int x M dx - c \int M dx$
$EI \theta_o =$		$-\int M dx$
	Integral between o-l ₃	Integral between o-l ₄
$EI \delta_o =$	$-(e+l_2) \int M dy$	$+\int z M dz - (e+l_2) \int M dz$
$EI \theta_o =$	$-\int M dy$	$-\int M dz$

EQUIVALENT ENERGY EQUATIONS

Quantity	Integral between o-l ₁	Integral between o-l ₂
$EI\delta_o =$		$-Area\bar{x}\Big _o^{l_2} - e\Big Area\Big _o^{l_2}$
$EIO_o =$		$-Area\Big _o^{l_2}$
	Integral between o-l ₃	Integral between o-l ₄
$EI\delta_o =$	$- e+l_2 Area\Big _o^{l_3}$	$Area\bar{z}\Big _o^{l_4} - e+l_2 Area\Big _o^{l_4}$
$EIO_o =$	$-Area\Big _o^{l_3}$	$-Area\Big _o^{l_4}$

ENERGY EQUATIONS IN TERMS OF THE REACTIONS

Quantity	Integral between o-l ₁	Integral between o-l ₂
$EI\delta_o =$		$- \frac{H_2 l_2^3}{3} - \frac{M_2 l_2^2}{2} - e \frac{H_2 l_2^2}{2} - M_2 l_2 $
$EIO_o =$		$- \frac{-H_2 l_2^2}{2} - M_2 l_2 $
	Integral between o-l ₃	Integral between o-l ₄
$EI\delta_o =$	$- e+l_2 \frac{V_4 l_3^2}{2} - M_4 l_3 $	$ \frac{H_5 l_4^3}{3} - \frac{M_5 l_4^2}{2} - e+l_2 \frac{H_5 l_4^2}{2} - M_5 l_4 $
$EIO_o =$	$- \frac{V_4 l_3^2}{2} - M_4 l_3 $	$- \frac{H_5 l_4^2}{2} - M_5 l_4 $

The influence coefficients can now be determined. The energy equations in terms of the reactions can be written as a function of H_1 , V_1 , M_1 , V_0 and H_s or M_s . From previous calculations the quantities H_1 , V_1 , M_1 , and V_0 were evaluated in terms of H_s and M_s . After the appropriate substitution, the linear and angular displacement of the center of gravity as a function of M_s and H_s are:

$$\begin{aligned} H_s &= 13.13 \times 10^6 & + 305.0 \times 10^6 \\ M_s &= 298.0 \times 10^6 & + 7250.0 \times 10^6 \end{aligned}$$

where

$$\begin{aligned} K_{11} &= 13.13 \times 10^6 & \text{lb/in} \\ K_{12} &= 305.0 \times 10^6 & \text{lb} \\ K_{21} &= 298.0 \times 10^6 & \text{lb} \\ K_{22} &= 7250.0 \times 10^6 & \text{in-lb} \end{aligned}$$

Theoretically, K_{12} should equal K_{21} . They are off by about 2½ per cent which is a reasonable error for the analysis. In other calculations where K_{12} and K_{21} are present, a value of 301.5×10^6 is used.

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