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STUDY OF REAL-GAS AND HEAT-TRANSFER
EFFECTS ON THE DESIGN OF
HOTSHOT WIND TUNNELS

by

Vidya, Inc.

MARCH 1960

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STUDY OF REAL GAS AND HEAT TRANSFER EFFECTS
ON THE DESIGN OF HOTSHOT WIND TUNNELS

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SUMMARY

A study of heat transfer in the arc chamber and converging section of the nozzle of a "hotshot" wind tunnel was conducted to determine the effects of heat transfer on the performance of this type of wind tunnel at initial arc-chamber pressures from 100 to 6800 atmospheres and initial arc-chamber temperatures from 4000°K to 14,000°K.

The air flow from the arc chamber to the nozzle throat and air within the arc chamber were assumed to be in chemical equilibrium, and an isentropic expansion from the arc chamber to the throat was assumed.

Radiation from the hot gas was found to be governed by black-body emission laws for pressures above 100 atmospheres at the temperatures considered. Convection to the throat was evaluated using several relationships, and it was found that a method developed by Sibulkin for turbulent heat transfer at nozzle throats gives the best estimate of convective heat transfer on the basis of physical considerations of the flow conditions at the throat.

Tungsten, copper, and beryllium throat sections were considered. Preliminary evaluations of these materials indicated that tungsten would be the most durable, followed by copper and beryllium in order of relative durability. The thermal response of tungsten throats with diameters of 0.02, 0.10, and 0.30 inch was evaluated, assuming steady flow at the throat for the entire range of arc-chamber conditions. Constant thermal properties were assumed for tungsten, and exact analytic solutions for heat conduction, modified to account for two-dimensional conduction, were used to evaluate the time to melt the throat.

The time for a 10-percent loss of arc-chamber enthalpy was estimated using approximate integrals of the mass-balance and energy-balance equations for the arc chamber.

Transpiration cooling and cooling by injection of gases through slots were investigated as means of increasing the time to melt the throat. An empirical estimate of the effects of transpiration on boundary-layer stability was assumed to govern the range of applicability of transpiration. Film cooling to reduce radiation was considered.

The results of this investigation show that the time available for steady test-section flow in the "hotshot" wind tunnel is governed by melting of the nozzle throat at initial arc-chamber

pressures approaching 6800 atmospheres. At initial arc-chamber pressures near 100 atmospheres, radiation to the arc-chamber walls limits the time for steady test-section flow. Transpiration cooling of the throat was found to increase the time to melt the throat. Cooling by injection through slots has a smaller effect than distributed transpiration, but is probably not limited by the empirical stability criterion. However, the possible benefits attainable by slot cooling are cancelled by the effects of radiation to the arc-chamber walls.

The maximum time to melt the nozzle throat was found to be limited by gaseous radiation. At initial arc-chamber pressures approaching 100 atmospheres it was found that the time for steady flow in the test section is governed by radiation to the walls of the arc chamber. Film cooling was found to be of negligible benefit as a means of reducing radiative heat transfer.

INTRODUCTION

The time available for testing in a "hotshot" wind tunnel is limited due to melting and erosion of the nozzle throat caused by the very high rates of heat transfer by convection and radiation from the hot gas flow. In addition, radiation from the gas within the arc chamber causes an extremely rapid decay of the stagnation enthalpy, particularly at high pressures and temperatures, which also limits the usefulness of this type of facility. The investigation described in this report was conducted, in part, to determine the heat-transfer rates at the nozzle throat, and the time available for testing before melting occurs at the throat. Consideration was given to the feasibility of cooling the throat by transpiration of gases into the boundary layer, and by injection of light gases through slots upstream of the throat.

This study of heat-transfer effects was confined to the arc chamber, the converging section of the nozzle, and the throat of the nozzle. The divergent section of the nozzle was not considered since the most severe heat-transfer effects are encountered upstream of the throat.

SYMBOLS

A	area
a	speed of sound
C_p	specific heat of gas at constant pressure
C	specific heat
D	diameter
E	total energy
e	base of natural logarithms
erfc V	complementary error function of V
	$\text{erfc } V = \frac{2}{\sqrt{\pi}} \int_V^{\infty} e^{-u^2} du$
H	enthalpy
J_n	Bessel function of the first kind of order n
k	thermal conductivity
L	radius of curvature in the streamwise direction
M	Mach number
P	Pressure
Pr	Prandtl number = $\frac{C_p \mu}{k}$
q	heat-transfer rate per unit area
r	radius
St	Stanton number (eq. (6))
T	temperature
t	time
u	velocity
V	volume
X	length of boundary-layer run (eq. (5))

x	distance into the nozzle wall from the surface (eq. (11))
Z	variable of integration (eq. (23))
α	thermal diffusivity (eq. (11))
γ	isentropic exponent (eq. (27))
δ	boundary-layer thickness at the throat (eq. (9))
μ	absolute viscosity
ρ	density
ϕ	(eq. (18))

Subscripts

*	conditions evaluated at the nozzle throat
s	conditions in the arc chamber
R	adiabatic wall condition
w	properties evaluated at the local nozzle wall temperature and pressure
f	properties evaluated at the local reference state (eq. (7))

Superscript

(-)	average quantity
-----	------------------

INVESTIGATION

Flow Conditions

The state properties in the arc chamber were evaluated by assuming that the gas within the arc chamber, after being heated by an arc discharge, achieves chemical equilibrium instantaneously at the desired level of temperature and pressure. The temperature range considered was from 4000°K (7200°R) to $14,000^{\circ}\text{K}$ ($25,200^{\circ}\text{R}$), at pressures from 100 atmospheres to 6800 atmospheres. The gas within the arc chamber was assumed to be air, and state properties were evaluated with the aid of reference 1 and reference 2.

The enthalpy, density, and sonic velocity of the gas in the arc chamber are plotted as functions of temperature for pressures from 100 atmospheres to 6800 atmospheres in figure 1. The required energy addition, for a given initial-charge temperature, and the required initial-charge pressure, can be determined for the specified conditions of pressure and temperature after the arc is discharged if all reactions are assumed to be instantaneous and if chemical equilibrium is assumed to be established in the arc chamber before energy is dissipated by radiation to the arc-chamber walls. On this basis, the required charge pressure and energy addition per unit arc chamber volume were estimated for an initial charge temperature of 294.4°K (530°R) as shown in figure 2.

Determination of the properties of the flowing gas at the throat of a nozzle, through which the arc chamber is discharged, was made by assuming that the gas is locally in chemical equilibrium during the expansion, that the expansion is isentropic¹, and that the flow at the throat is one-dimensional. On this basis the mass-flow rate per unit area, velocity, static pressure, and static enthalpy were evaluated at the throat using the charts of reference 1 and reference 2. These properties are plotted in figure 3 as functions of the temperature and pressure in the arc chamber.

It will be noted in figures 1 through 3 that no properties are plotted at temperatures below about 8000°K to $10,000^{\circ}\text{K}$ at pressures above 4000 atmospheres. This is due to the fact that the state properties of air in chemical equilibrium are not published for these conditions. The lack of data in this region does not represent a serious problem insofar as the estimation of heat transfer is concerned since, as will be shown later, the convective

¹ Isentropic flow in the convergent portion of the nozzle is a valid assumption since only 0.01 percent of the total enthalpy at the initial arc chamber conditions is lost by the combined effects of convection and radiation to the converging nozzle walls.

heat transfer may be extrapolated by a very simple relationship, and the radiation from hot air is probably insensitive to pressure above 100 atmospheres at the gas temperatures considered here.

Radiation Heat-Transfer Estimates

Radiation from the hot gas to the walls of the arc chamber and to the nozzle throat is significant at the temperatures and pressures considered in this study. Heat losses by radiation from the gas within the arc chamber will cause a rapid decay in the enthalpy of the gas, and radiation to the nozzle throat decreases the time available for testing before melting occurs. The effect of radiation at the throat is not as significant as losses within the arc chamber. However, radiation does contribute significantly to the heat transfer at the throat and tends to limit the operational usefulness of the "hotshot" wind-tunnel nozzle.

Data for radiation from high temperature air is presented in reference 3 in terms of the effective total emissivity for radiation from a gas layer one centimeter thick, and as a function of the gas density and temperature. It is explicitly stated in reference 1 that the gas behaves as a black-body emitter whenever $\epsilon/L \geq 1$. A line representing the lower limit of applicability of black-body radiation is plotted as the line of minimum density for black-body radiation in figure 1(b). Radiation from the hot gas for all conditions above this line is adequately represented by the relationship

$$q_r = \sigma T^4$$

As indicated above the validity of assuming black-body radiation is fairly well established for arc chamber pressures greater than 100 atmospheres. This assumption leads to slightly conservative evaluations of radiation effects at $P_s = 100$ atmospheres. However, the conservatism introduced is not sufficiently large to warrant the use of more exact relationships for estimation of the effects of radiation on the time to melt the nozzle throat.

Transport Properties of Air

Before evaluations of convective heat transfer at the nozzle throat could be made, it was necessary to estimate the variations of the viscosity and Prandtl number of the high-temperature air as functions of the temperature and pressure. No published information for transport properties of air at pressures above 100 atmospheres was found. These properties are known to vary quite

widely as a function of the degree of dissociation and/or ionization of the constituent gases, and since neglect of these variations leads to heat transfer predictions which are probably too low, a rational extrapolation to higher pressures was attempted.

For pressures below 100 atmospheres, the absolute viscosity and the Prandtl number for air are tabulated in reference 4 as functions of temperature and pressure. These tabulations are reproduced in figures 4(a) and 5(a). In reference 5, a semi-empirical equation is presented which supposedly correlates the variation of viscosity with temperature and pressure as a function of a reduced temperature.

$$T' = T(1 + a \log_{10} P) \quad (1)$$

where a is an empirical constant and P is the pressure in atmospheres. It was found that the viscosity could be correlated quite well by the relationship

$$\frac{\mu}{\mu_0} = 1 + 0.024 \frac{T}{1000} \left[1 + \tanh \frac{\frac{T}{1000}(1 - \frac{1}{8} \log_{10} P) - 6.5}{1.5 + \frac{1}{8} \log_{10} P} \right] \quad (2)$$

where μ_0 is the viscosity determined by Sutherland's equation

$$\mu_0 = 9.83 \times 10^{-7} \frac{\sqrt{T}}{1 + \frac{112}{T}} \quad \frac{\text{lb}}{\text{sec-ft}} \quad (3)$$

The ratio $\frac{\mu}{\mu_0}$ from equation (2) is plotted in figure 4(b) for pressures from 100 to 8000 atmospheres. It was found that the Prandtl number curves of figure 5(a) could be superimposed by plotting

$$\text{Pr}(1 + 0.015 \log_{10} P) \quad \text{versus} \quad T(1 - 0.137 \log_{10} P)$$

As shown in figure 5(b), this yields a fair correlation, which was assumed to be representative for pressures much higher than 100 atmospheres.

Convective Heat-Transfer Estimates

Reynolds numbers at the nozzle throat were evaluated at the local free-stream temperature and pressure, using the estimates of viscosity described above. These Reynolds numbers are plotted in figure 6 as a function of arc-chamber temperature and pressure.

The usual criteria for transition from laminar to turbulent flow are based on the local Reynolds number, with the characteristic length equal to the passage diameter for fully developed pipe flow, or equal to the boundary-layer-run length for boundary-layer flow. In a fully developed pipe flow, transition usually occurs when

$$\frac{\rho u D}{\mu} > 2000$$

where D is the passage diameter. For boundary-layer flows, the critical Reynolds number may lie anywhere within the range

$$10^5 < \frac{\rho u x}{\mu} < 10^7$$

depending on the temperature of the wall, the streamwise pressure gradient, the degree of free-stream turbulence and the roughness of the wall. In general, a wall colder than free-stream stagnation conditions and a decreasing pressure in the direction of flow tend to delay transition, and high levels of surface roughness and free-stream turbulence will cause premature transition. Since nothing is known about the free-stream flow with regard to turbulence, except that it is probably disturbed, and since the lower limit on the transition Reynolds number is conservative for design, the transition criterion

$$\frac{\rho u x}{\mu} > 10^5$$

was assumed to apply for the case of boundary-layer flow.

The transition criteria were evaluated by assuming that the minimum throat diameter would be 0.02 inch and the boundary-layer-run length, measured along the nozzle from the arc chamber to the throat, would be about one inch. On this basis the minimum Reynolds numbers, which occur at $P_s = 100$ atm. and $T_s = 14,000^\circ\text{K}$, are

$$\left(\frac{\rho_* a_* D_*}{\mu_*} \right)_{\min} = 4500$$

and

$$\left(\frac{\rho_* u_* X_*}{\mu_*} \right)_{\min} = 2.25 \times 10^5$$

On the basis of the assumed transition criteria, the flow at the throat may be considered turbulent over the range of arc-chamber conditions.

Three general methods for estimating convective heat transfer were considered in this study. They are

1. Correlations for fully developed turbulent pipe flow.
2. Relationships for a turbulent boundary layer on a flat plate.
3. A method developed by Sibulkin for specific application to heat-transfer determination at nozzle throats.

The convective heat transfer in fully developed turbulent pipe flow is given by the correlation equation (ref. 6)..

$$St_{pipe} = 0.023 (Pr_f)^{-0.6} \left(\frac{\rho_f u D}{\mu_f} \right)^{-0.2} \quad (4)$$

and for the turbulent boundary layer on a flat plate by the relationship (ref. 7)

$$St_{plate} = 0.0296 (Pr_f)^{-\frac{2}{3}} \left(\frac{\rho_f u x}{\mu_f} \right)^{-0.2} \quad (5)$$

where the Stanton number is defined as

$$St = \frac{q}{\rho_f u (H_R - H_w)} \quad (6)$$

In the above relationships, the density, viscosity and Prandtl number should be evaluated at the reference enthalpy defined in reference 7:

$$H_f = H_\infty (1 + 0.035 M_\infty^2) + 0.45 (H_w - H_\infty) \quad (7)$$

At the throat of a nozzle, $M_\infty = 1$ and $H_\infty = H_*$. Equation (7) therefore becomes

$$H_f = 0.585 H_* + 0.45 H_w \quad (7a)$$

An approximate method is developed in reference 8 for the estimation of the heat transfer to the throat of a nozzle in which the boundary layer is turbulent. This method considers the streamwise pressure gradient at the throat by assuming a circular streamwise contour at the throat. The essential results of this development yield the Stanton number

$$St_* = 0.0439 (Pr_f)^{-\frac{2}{3}} \left(\frac{r_*}{L_*} \right)^{0.10} \left(\frac{\rho_f a_*}{\mu_f} D_* \right)^{-0.20} \left(\frac{a_*}{a_s} \right)^{1.20} \quad (8)$$

where L_* is the streamwise radius of curvature at the throat. In addition, an expression is given for the boundary-layer thickness at the throat:

$$\frac{\delta}{r_*} = 0.1252 \left(\frac{L_*}{r_*} \right)^{0.40} \left(\frac{a_s}{a_*} \right)^{0.80} \left(\frac{\rho_f a_* D_*}{\mu_f} \right)^{-0.20} \quad (9)$$

Numerical comparisons of equations (4), (5), and (8) were made to determine which would be the most conservative for design use in this application. The dimensions assumed for this evaluation were

$$0.02 \text{ inch} \leq D_* \leq 1.00 \text{ inch}$$

$$L_* = 5 D_*$$

$$X_* = 1.00 \text{ inch}$$

The results of the comparison are as follows:

$$1.18 < \frac{St_*}{St_{\text{plate}}} < 2.57$$

$$1.43 < \frac{St_*}{St_{\text{pipe}}} < 1.47$$

The flat-plate relationship given by equation (5) was found to be the least conservative over most of the range of arc-chamber conditions, and was therefore dropped from further

consideration. The validity of equation (8) relative to equation (4) was evaluated by estimating the boundary layer thickness at the throat using equation (9). It was found that

$$\frac{\delta}{r_*} \leq 0.035$$

for $L/r_* = 10$ and $0.02 \text{ inch} < D_* < 1.00 \text{ inch}$ over the complete range of arc-chamber conditions considered. Since $\delta/r_* \ll 1$, equation (4) cannot be considered valid and was therefore eliminated on the basis that it has no physical significance in this application. Equation (8) was therefore used for all evaluations of convective heat transfer estimates at the throat.

For the purpose of evaluating the temperature response of the nozzle wall, it is convenient to evaluate the heat transfer coefficient

$$\frac{q_*}{H_R - H_w} = \rho_f a_* St_*$$

This was done using equation (8) with properties evaluated at reference enthalpies corresponding to nozzle wall temperatures of 300°K and 3600°K. These wall temperatures are approximately the same as the initial temperature of the nozzle and the melting temperature of Tungsten, respectively.

All of the heat transfer coefficients obtained from equation (8) are plotted in figure 7 as

$$\frac{q_* D_*^{0.2}}{H_R - H_w} \quad \text{versus arc-chamber pressure.}$$

As indicated in figure 7, all of the computed heat-transfer coefficients for the complete range of arc-chamber conditions lie along a band which is parallel to a line defined by $P_s^{0.80}$. Figure 8 shows representative calculations of

$$\frac{q_*}{H_R - H_w} \quad \text{versus pressure}$$

for arc-chamber temperatures of 4000°K and 14,000°K. As noted earlier, it was impossible to evaluate the properties of air at

temperatures below 8000°K for pressures greater than 4000 atmospheres. For these conditions, the heat-transfer coefficients are estimated by extrapolation parallel to $P_s^{0.80}$, as indicated in figure 8(b).

Recovery enthalpies at the throat (enthalpy of the gas at the wall for zero heat transfer) were evaluated using the equation

$$H_R = H_* + R \frac{a_*^2}{2} \quad (10)$$

where the recovery factor $R \approx (Pr_f)^{1/3}$ for turbulent flow (ref. 6).

Heat-transfer rates are plotted, using Sibulkin's method, equation (8), for 3 throat diameters and 2 wall temperatures, in figure 9.

Heat Conduction at the Nozzle Throat

The following solutions for conduction in a solid having constant thermodynamic properties, given in reference 9, are of direct applicability to this investigation:

- a. A semi-infinite solid with uniform initial temperature heated by convection from a constant recovery temperature with constant heat-transfer coefficient.
- b. A semi-infinite solid initially at a uniform temperature with constant heat flux at the surface.
- c. An infinite circular cylinder with a uniform initial temperature heated by a constant heat flux at the inner surface of a cylindrical hole through the solid.

The solution for the semi-infinite solid heated by convection is

$$\frac{T - T_i}{T_E - T_i} = \operatorname{erfc} \frac{x}{2 \sqrt{\alpha t}} - e^{\frac{hx}{k_w} + \frac{h^2 \alpha t}{k_w^2}} \operatorname{erfc} \left[\frac{x}{2 \sqrt{\alpha t}} + \frac{h}{k_w} \sqrt{\alpha t} \right] \quad (11)$$

where the heat input at the surface is given by

$$q = -k_w \frac{\partial T}{\partial x} = h (T_E - T_w) \quad (12)$$

The initial temperature is T_i , and T_E is the equilibrium temperature. At the conditions under consideration in this study radiation from the hot gases is very nearly constant for steady arc-chamber conditions and is not affected appreciably by variations in wall temperature until the wall temperature approaches the local gas temperature. If it is assumed that radiation from the hot gas is adequately represented by a constant heat flux, then the total heat flux at the wall is given by

$$q = \frac{q_*}{H_R - H_w} (H_R - H_w) + q_r \quad (13)$$

where q_r is constant. For the range of wall temperatures from room temperature to the melting point of tungsten, the specific heat of air, at pressures to 10,000 atmospheres, varies between 0.24 and 0.35 Btu/lb.°F. The enthalpy of the air at the wall temperature, H_w , is much smaller than the recovery enthalpy, H_R , so that the heat flux, q , is not significantly affected by assuming an average specific heat $\bar{C}_p$ such that

$$H_w \approx \bar{C}_p T_w \quad (14)$$

A pseudo-recovery temperature is defined by

$$T_R' = \frac{H_R}{\bar{C}_p} \quad (15)$$

and equation (13) becomes

$$q = \frac{q_*}{H_R - H_w} \bar{C}_p (T_R' - T_w) + q_r \quad (13a)$$

For the conditions of this study $q_*/(H_R - H_w)$ and H_R are essentially constant for steady arc-chamber conditions. If

equation (12) and equation (13a) are set equal to one another

$$\begin{aligned} h(T_E - T_w) &= \frac{q_*}{H_R - H_w} \bar{C}_p (T_R' - T_w) + q_r \\ &= \frac{q_*}{H_R - H_w} \bar{C}_p \left[T_R' + \frac{q_r}{q_* \bar{C}_p / (H_R - H_w)} - T_w \right] \end{aligned} \quad (16)$$

Solving for h and T_E ,

$$\begin{aligned} h &= \frac{q_* \bar{C}_p}{H_R - H_w} \\ T_E &= T_R' + \frac{q_r}{q_* \bar{C}_p / (H_R - H_w)} = \left[H_R + \frac{q_r}{q_* / (H_R - H_w)} \right] \frac{1}{\bar{C}_p} \end{aligned} \quad (17)$$

Substitution of equation (17) into equation (11) yields

$$\begin{aligned} T &= T_i + \left\{ \left[H_R + \frac{q_r}{q_* / (H_R - H_w)} \right] \frac{1}{\bar{C}_p} - T_i \right\} \\ &\times \left[\operatorname{erfc} \frac{x}{2\sqrt{\alpha t}} - e^{-\frac{q_* \bar{C}_p x}{k_w (H_R - H_w)} \left(\frac{q_* \bar{C}_p}{H_R - H_w} \right)^2 \frac{\alpha t}{k_w^2}} \right. \\ &\left. \operatorname{erfc} \left(\frac{x}{2\sqrt{\alpha t}} + \frac{q_* \bar{C}_p}{H_R - H_w} \frac{\sqrt{\alpha t}}{k_w} \right) \right] \end{aligned} \quad (11a)$$

The last equation is an exact solution within the framework of the following assumptions:

- a. q_r is constant.
- b. $\bar{C}_p$ is constant to the melting point of the solid.
- c. $\frac{q_*}{H_R - H_W}$ and H_R are constant.
- d. α and k_w are constant.

At the surface of the solid, $x = 0$, and

$$T_w = T_i + \left\{ \left[H_R + \frac{q_r}{q_*/(H_R - H_W)} \right] \frac{1}{\bar{C}_p} - T_i \right\} \left(1 - e^{\phi^2} \operatorname{erfc} \phi \right) \quad (18)$$

where

$$\phi = \frac{q_* \bar{C}_p}{H_R - H_W} \frac{\sqrt{\alpha t}}{k_w}$$

If the surface temperature is set equal to the melting temperature, then the time to reach the melting point at the surface is

$$t_m = \frac{k_w \rho_w C_w}{\bar{C}_p^2} \left[\frac{\phi_m}{q_*/(H_R - H_W)} \right] \quad (19)$$

where ϕ_m is determined from the relationship

$$e^{\phi_m^2} \operatorname{erfc} \phi_m = 1 - \frac{\bar{C}_p (T_m - T_i)}{H_R + \left\{ q_r / [q_*/(H_R - H_W)] \right\} - \bar{C}_p T_i} \quad (19a)$$

The temperature distribution in a semi-infinite solid with constant heat flux at the surface is given by

$$T - T_i = \frac{2q}{k_w} \left\{ \sqrt{\frac{\alpha t}{\pi}} - \frac{x}{2} \operatorname{erfc} \frac{x}{2\sqrt{\alpha t}} \right\} \quad (20)$$

At $x = 0$

$$T_w - T_i = \frac{2}{\sqrt{\pi}} \frac{q\sqrt{\alpha t}}{k_w} \quad (21)$$

For the infinite hollow cylinder with a constant heat input at the surface $r = r_*$, the temperature distribution is

$$T - T_i = -\frac{2q}{\pi k_w} \int_0^\infty \left(1 - e^{-\alpha u^2 t} \right) \frac{J_0(ur)Y_1(ur_*) - Y_0(ur)J_1(ur_*)}{u^2 \left[J_1^2(ur_*) + Y_1^2(ur_*) \right]} du \quad (22)$$

At the surface $r = r_*$

$$J_0(ur_*) Y_1(ur_*) - Y_0(ur_*) J_1(ur_*) = -\frac{2}{\pi(ur_*)}$$

and the surface temperature is given by

$$T_w - T_i = \frac{4qr_*}{\pi^2 k_w} \int_0^\infty \frac{\left(1 - e^{-\frac{\alpha t}{r_*^2} z} \right)}{z^3 \left[J_1^2(z) + Y_1^2(z) \right]} dz \quad (23)$$

If equation (21) is divided on both sides by r_* , then the one-dimensional solution becomes

$$\left(k_w \frac{T_w - T_i}{q r_*} \right)_{1-d} = \frac{2}{\sqrt{\pi}} \sqrt{\frac{\alpha t}{r_*^2}} \quad (21a)$$

and the solution for the hollow cylinder is expressible as

$$\left(k_w \frac{T_w - T_i}{q r_*} \right)_{\text{cyl.}} = \frac{4}{\pi^2} \int_0^\infty \frac{\left(1 - e^{-\frac{\alpha t}{r_*^2} z} \right) dz}{z^3 \left[J_1^2(z) + Y_1^2(z) \right]}$$

$$= t \left(\frac{\alpha t}{r_*^2} \right) \quad (23a)$$

Equation (21a) and equation (23a) are plotted in figure 10. From this plot it is apparent that the time to reach a given surface temperature is grossly underestimated by the one-dimensional conduction solutions as the ratio $k_w/q r_*$ increases for a given value of α/r_*^2 . Since the heat-transfer rate to the nozzle throat is very nearly constant with time at most arc-chamber conditions, the time to melt the throat wall as determined by equation (19) can be corrected for axisymmetric conduction effects by use of equation (21a) and equation (23a).

The time required to reach the melting temperature of a given material relative to the time to melt another material can be estimated using equation (21). For a constant heat flux at the surface

$$t_m = \frac{\pi}{4} \frac{k_w \rho_w C_w (T_m - T_i)^2}{q^2} \quad (21b)$$

For given arc-chamber conditions the time to melt a given material is therefore

$$t_m \sim k_w \rho_w C_w (T_m - T_i)^2 \quad (24)$$

The relative times to reach the melting temperatures of tungsten, copper, and beryllium were evaluated using the above proportionality, and the results are shown in figure 11. Also shown in figure 11 are the results of estimates of the relative times to melt these materials as computed for the extreme heat-transfer environments of this study. This plot indicates that equation (24) is an optimistic estimate, and the most durable material of the three considered on the basis of the time available for testing before melting occurs is tungsten. Estimates

of the time to reach the melting temperature were therefore calculated only for tungsten.

To estimate the time to melt copper and beryllium it is only necessary to multiply the time to melt tungsten by an appropriate factor from figure 11.

The time to melt tungsten throats with diameters of 0.02 inch, 0.10 inch, and 0.30 inch is plotted in figure 13 for steady-flow conditions and thermodynamic properties evaluated at a temperature midway between the initial temperature (294.4°K) and the melting temperature (3550°K). Due to the small variation of the thermal properties of tungsten with temperature, as shown in figure 12, the assumption of constant thermal properties will not lead to large errors in the estimated time to melt this material.

In order to establish an upper limit on the time to melt the throat, the time to melt tungsten was estimated assuming that the only heat transfer to the throat was due to radiation from the hot gases. The results of this calculation are shown in figure 14.

Figure 13 indicates that the time to melt the throat decreases as either the arc-chamber pressure or the arc-chamber temperature increases. The time to melt the throat under steady flow conditions is significantly less than 25 milliseconds over most of the range of operating conditions considered, and for a large portion of these conditions the time to melt is on the order of microseconds rather than milliseconds. The curves of time to melt for radiation only (fig. 14) show that even if convection heat transfer is completely eliminated, the maximum time available before melting starts will be limited by radiation to less than 25 milliseconds for arc-chamber temperatures above approximately $10,000^{\circ}\text{K}$. (Note also that the results of figure 14 for a semi-infinite solid are indicative of the time that would be available for testing before the walls of the arc chamber start to melt under steady arc-chamber conditions.)

Decay of Arc-Chamber Conditions Due to Radiation and Flow

The estimates of heat transfer and melting time presented in the preceding section are based on the assumption of steady arc-chamber conditions and steady flow in the nozzle. In order to establish the validity of this assumption the rate of decay of enthalpy and density in the arc chamber should be known. It is possible to estimate the decay of arc chamber gas properties

as follows: the mass balance on the arc chamber is given by the differential equation

$$V_s \frac{d\rho_s}{dt} = - \rho_* a_* A_* \quad (25)$$

and the energy balance is given by

$$V_s \frac{d(\rho_s H_s)}{dt} = - A_s q_s - \rho_* a_* A_* H_s \quad (26)$$

Substitution of equation (25) into equation (26) and separation

of $\frac{d(\rho_s H_s)}{dt}$ yields

$$\rho_s V_s \frac{d H_s}{dt} = - A_s q_s \quad (26a)$$

During the course of the evaluation of flow conditions, it was found that the isentropic exponent of the flowing gas is very nearly constant from the arc chamber to the throat, as shown in figure 15. A negligible error in the determination of the mass-flow rate, $\rho_* a_*$, is introduced if $\gamma = \text{constant}$ is assumed. From isentropic flow relationships

$$\rho_* a_* = \frac{2}{\gamma+1} \rho_s a_s \quad (27)$$

if $\gamma = 1.24$, then

$$\rho_* a_* = 0.589 \rho_s a_s (\pm 2\%) \quad (27a)$$

Substitution of equation (27a) into equation (25) and equation (26a) yields differential equations which may be integrated directly, giving

$$\rho_s = \rho_{si} e^{-0.589 \frac{A_*}{V_s} \int_0^t a_s d\lambda} \quad (28)$$

$$\begin{aligned}
 H_s &= H_{s_i} - \frac{A_s}{V_s} \int_0^t \frac{q_s}{\rho_s} d\lambda \\
 &= H_{s_i} - \frac{A_s}{V_s \rho_{s_i}} \int_0^t q_s e^{0.589 \frac{A_*}{V_s} \int_0^\lambda a_s d\lambda} d\lambda \quad (29)
 \end{aligned}$$

For very small changes in the density and enthalpy a_s and q_s are nearly constant. Equation (28) and equation (29) can therefore be approximated by letting

$$a_s = a_{s_i}$$

$$q_s = q_{s_i}$$

Integration of equation (28) and equation (29), substitution of equation (27a) into equation (29) and rearranging terms yields

$$\frac{\rho_{s_i} - \rho_s}{\rho_{s_i}} \approx 1 - e^{-0.589 \frac{A_*}{V_s} a_{s_i} t} \quad (28a)$$

$$\frac{H_{s_i} - H_s}{H_{s_i}} \approx \frac{q_{s_i} A_s}{\rho_{s_i} a_{s_i} H_{s_i} A_*} \left[e^{0.589 \frac{A_*}{V_s} a_{s_i} t} - 1 \right] \quad (29a)$$

As a means of evaluating the validity of assuming steady-flow conditions for the heat conduction calculations, a criterion for the decay of enthalpy and density was established on the basis of acceptable errors in the computed heat-transfer rates. From equation (21b), the time to melt is given by the proportionality

$$t_m \sim \frac{1}{q^2}$$

and the relative error in the time to melt is

$$\frac{\Delta t_m}{t_m} \approx -2 \left(\frac{\Delta q}{q} \right) \quad (30)$$

where $\left(\frac{\Delta q}{q} \right)$ is the relative error in the heat flux, averaged with respect to time.

Combining equation (13a) and the correlation depicted in figure 7 gives

$$q_* \sim P_s^{0.80} H_s$$

and the pressure is related to the density and enthalpy by the approximate (perfect gas) relationship

$$P_s = \frac{\gamma-1}{\gamma} \rho_s H_s$$

so that

$$q_* \sim \rho_s^{0.80} H_s^{1.80}$$

and

$$\frac{\Delta q_*}{q_*} \approx 0.80 \frac{\Delta \rho_s}{\rho_s} + 1.80 \frac{\Delta H_s}{H_s} \quad (31)$$

It is reasonable to assume that the average error in q_* is about one half of the error at the time for which the steady flow assumption is no longer valid. The error in the time to melt is therefore approximately

$$\frac{\Delta t_m}{t_m} \approx -0.80 \frac{\Delta \rho_s}{\rho_s} - 1.80 \frac{\Delta H_s}{H_s} \quad (32)$$

The arc-chamber configuration, identical to that of reference 10, shown in figure 16, was assumed to be representative and was used for evaluation of the decay of properties with time according to the equations presented above. For the assumed configuration, the decay in density is negligible until after the throat would melt under steady-flow

conditions or until after the nominally required running time of 25 milliseconds is achieved. The error in the time to melt, due to the assumption of steady flow-conditions, is therefore approximately

$$\frac{\Delta t_m}{t_m} \approx -2 \frac{\Delta H_s}{H_s}$$

It was arbitrarily assumed that a decrease in enthalpy of 10 percent would be acceptable. On this basis, the uncertainty in the time to melt is on the order of at least 20 percent when the time to cause $\Delta H_s/H_s$ to equal 0.10 is less than the time to melt. For $\Delta H_s/H_s \leq 0.10$, equation (29a) is a very good approximation for the enthalpy change due to heat losses from the arc chamber. The time at which a 10 percent decrease in enthalpy will occur is found by letting

$$\frac{H_{s_i} - H_s}{H_{s_i}} = 0.10 \quad \text{and solving for } t$$

$$t \left(\frac{\Delta H}{H} = 0.10 \right) \approx \frac{\ln \left[1 + \frac{\rho_{*i} a_{*i} H_{s_i} A_*}{10 \sigma T_{s_i}^4 A_s} \right]}{0.589 \frac{A_*}{V_s} a_{s_i}} \quad (33)$$

Equation (33) is plotted in figure 17 for the range of arc-chamber conditions. It will be noted that the time for $\frac{\Delta H}{H} = 0.10$ increases as the pressure increases and is almost independent of throat diameter for $D_* < 0.30$ inch.

If figure 18 is compared with figure 13, it will be noted that $t \left(\frac{\Delta H}{H} = 0.10 \right)$ is always less than t_m for D_* from 0.02 inch to 0.30 inch with $P_s = 100$ atmospheres, and $t \left(\frac{\Delta H}{H} = 0.10 \right) \approx t_m$ over a large portion of the operating conditions for pressures below 1000 atmospheres. On the basis of this comparison, it is apparent that the assumption of steady flow at the throat is not appropriate for the estimation of the time to melt the throat. However, the estimates of

$t \left(\frac{\Delta H}{H} = 0.10 \right)$ are indicative of the time available for essentially steady flow in the test section and should be considered in establishing the design of the "hot shot" wind tunnel.

Evaluation of Cooling Techniques Using Foreign Gas Injection

Analytic estimates of the effects of transpiration of air into a turbulent boundary layer are described in reference 11, and transpiration of helium and hydrogen is described in reference 12. In reference 13 it is shown that the reduction in heat transfer due to transpiration can be correlated as a function of the flow rate of the transpiration coolant and the Stanton number without transpiration. This correlation is reproduced in figure 18 for a Reynolds number, based on local conditions and the length of boundary-layer run on a flat plate, of 10^6 . The dependency of this correlation on Reynolds number is weak, and the particular value chosen is representative of the correlation of reference 13. It is assumed in this investigation that the effects of streamwise pressure gradients are not significant, and the analyses of reference 11 and reference 12 for the case of zero streamwise pressure gradient are applicable. The influence of moderate pressure gradients on the turbulent boundary layer does not normally have a gross effect on heat transfer, so that the last assumption is not likely to be seriously deficient.

An experimental evaluation of transpiration of air into a turbulent boundary layer on a flat plate (ref. 14) indicates that the boundary-layer flow will be blown away from the wall when the coolant flow-rate parameter is approximately

$$\frac{\rho_w V_w}{q_{*}/(H_R - H_w)} \approx 2.7$$

The validity of this result is questioned on the basis of the experimental results shown in reference 13, which do not indicate that the boundary layer will separate for

$$\frac{\rho_w V_w}{q_{*}/(H_R - H_w)} < 3$$

However, on the basis of the possibility that the results of reference 14 are correct and that separation with transpiration could possibly occur when

$$\frac{\rho_w V_w}{q_*/(H_R - H_w)} \approx 2.7$$

it is assumed that this value of the flow-rate parameter limits the range of coolant flow rates for which unseparated flow can be expected. It is desirable to prevent separation of the boundary layer at the throat in order to avoid disturbance of the flow at the throat and the consequent propagation of this disturbance to the test section. For this reason, the flow rate of transpiration coolant is limited in this investigation as indicated in figure 18, and the maximum effect of transpiration of air is limited to $St/St_0 = 0.287$ corresponding to the limit on the flow rate.

Transpiration flow rates corresponding to $\frac{\rho_w V_w}{\left(\frac{q_x}{H_R - H_w} \right)} = 2.70$

are plotted in figure 19, and the time to melt the throat for $St/St_0 = 0.287$ is shown in figure 20, again assuming steady flow at the throat. Comparison of figure 20 with figure 13 indicates that the time to melt the throat can be increased by factors varying from about 2 at the lowest pressure to 7 at the highest pressure if transpiration to obtain $St/St_0 = 0.287$ is used. At the lower pressures the effects of transpiration are much less than at high pressures due entirely to radiation from the hot gas.

Transpiration of light gases was not considered quantitatively here, but it is obvious from figure 18 that injection of helium could be used to achieve greater reductions in convective heat transfer at the throat. However, there appears to be very little advantage in using light-gas transpiration except at lower temperatures where the effects of radiation are less pronounced.

Injection of light gases through slots upstream of the throat has not been considered for the simple reason that this method of cooling is much less effective than distributed transpiration (see ref. 15). It probably can be expected that the injection of a gas such as helium through slots can be made as effective as distributed transpiration of air, and the

possibility of separating the boundary layer is practically eliminated since injection from slots can be in a direction nearly parallel to the wall rather than normal to the wall as in the case of distributed transpirations. However, unless radiation to the nozzle throat is eliminated or significantly reduced, the use of this scheme can not possibly result in sufficient increases in the time to melt the throat.

In order to prevent the rapid decay of properties in the arc chamber from limiting the time available before serious deterioration of test-section conditions occurs, it is also necessary to limit radiation to the walls of the arc chamber.

The possibility of injecting a gas, or other material, to reduce radiation transfer from the hot gas to the arc-chamber and nozzle walls was considered as a means of reducing the severity of radiation effects. The only accessible data on absorption of radiation by gases are those contained in reference 6. An approximate estimate of the reduction in radiative heat transfer which would be possible by the injection of water vapor or carbon dioxide was performed using the data of reference 6. It was concluded that the maximum possible reduction in radiative heat transfer, using these gases as absorbers, could not exceed 5 percent of the incident radiation. It is necessary to absorb no less than 60 percent of the incident radiation at the throat in order to prevent melting before 10 milliseconds at 14,000°K, and about 95 percent of energy loss due to radiation in the arc chamber must be retained in the arc chamber gas to prevent a 10-percent decay in enthalpy in 10 milliseconds. It hardly seems likely, on the basis of the data for CO_2 and H_2O , that such large reductions in radiation can be achieved with any gas.

RESULTS

The essential results of this study are summarized in figure 21, which shows the time available for testing before serious degradation of test-section properties can occur due either to melting of the throat or decay of stagnation conditions caused by radiation to the arc-chamber walls. This figure also shows the effects of varying the throat diameter and the effects of transpiration to reduce the convective heat transfer at the throat by 71.3 percent.

At initial arc-chamber pressures approaching 6800 atmospheres, the governing criterion for steady test-section flow is the time to melt the throat. Melting of the throat can be delayed by transpiration. However, the increase in the time to melt the throat is limited by gaseous radiation (see figure 14). At initial arc-chamber pressures approaching 100 atmospheres, the decay of arc-chamber enthalpy governs steady flow in the test section. Transpiration cooling will provide no relief of radiation.

The effects of changing the throat diameter are relatively insignificant for throat diameters from 0.02 inch to 0.30 inch.

CONCLUSIONS

1. Tungsten nozzle-throat inserts will operate without melting for periods about 2.5 times as long as copper and about 4 times as long as beryllium.
2. The maximum time available for testing in the "hotshot" wind tunnel before the occurrence of serious deterioration of test-section flow conditions is governed by radiation from the hot gases for arc-chamber pressures of 100 atmospheres or greater.
3. At arc-chamber pressures approaching 6800 atmospheres, the deterioration of flow conditions is governed by melting of the throat, and at pressures approaching 100 atmospheres, by the decay of energy caused by radiation to the arc-chamber walls.
4. At the highest pressures, transpiration cooling will increase the time to melt the throat by factors of as much as seven for 71.3-percent reduction in convective heating at the throat. At the low end of the pressure range, transpiration cooling will probably not be of much use in preventing deterioration of flow conditions due to the effects of radiation in the arc chamber.
5. It will be impossible to achieve testing times greater than 25 milliseconds without throat melting at any arc-chamber temperature greater than about 11,000° Kelvin with throat diameters greater than 0.02 inch and arc-chamber pressures above 100 atmospheres. This limit is achievable only if convective heat transfer can be completely eliminated.
6. The injection of coolants through slots upstream of the throat is generally not as effective as distributed transpiration on the basis of percentage reduction in convective heat transfer per pound of coolant. The reduction in heat transfer by slot injection of light gases may be equal to or greater than that attainable by transpiration; however, the gains which might be obtained by this method are offset by the effects of radiation.
7. Injection of gases for absorption of radiation does not appear to offer any relief from the performance-limiting effects of radiation.

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Figure 1(a).- Gas enthalpy in the arc chamber (air in chemical equilibrium).

Figure 1(b).- Gas density in the arc chamber (air in chemical equilibrium).

Figure 1(c).- Sonic velocity in the arc chamber (air in chemical equilibrium).

Figure 2(a).-- Required energy addition per unit arc-chamber volume to achieve specified conditions (air in chemical equilibrium), initial temperature, 294.4°K (530° R).

Figure 2(b).- Required initial charge pressure to achieve specified conditions (air in chemical equilibrium), initial temperature, 294.4°K (530°R).

Figure 3(a).- Mass-flow rate per unit area at the nozzle throat.

Figure 3(b).- Gas velocity at the nozzle throat.

Figure 3(c).- Static pressure at the nozzle throat.

Figure 4(a).- Viscosity ratio for high-temperature air at several pressures (NACA TN 4150); μ = viscosity, μ_0 = viscosity determined by Sutherland's equation.

Figure 4(b).- Viscosity ratio for high-temperature air at several pressures (extrapolation based on correlation of tabulations in TN 4150); μ = viscosity, μ_0 = viscosity determined by Sutherland's equation.

Figure 5(a).- Prandtl number of high-temperature air at several pressures (NACA TN 4150).

Figure 5(b).-- Reduced Prandtl number versus reduced temperature for high-temperature air.

Figure 6.- Free-stream Reynolds number at the nozzle throat.

Figure 7.- Computed heat-transfer coefficients versus arc-chamber pressure.

Figure 8(a).— Heat-transfer coefficients at the nozzle throat. Arc-chamber temperature: $T_s = 4000^\circ\text{K}$; throat-wall temperature: T_w .

Figure 8(b).- Heat-transfer coefficients at the nozzle throat. Arc-chamber temperature : $T_s = 14000^\circ\text{K}$; throat-wall temperature: T_w .

Figure 9(a).-- Heat-transfer rate at the nozzle throat;
 $D_* = 0.02$ inch; $T_w = 294^\circ\text{K}$

Figure 9(b).- Heat-transfer rate at the nozzle throat;
 $D_* = 0.10$ inch; $T_w = 294^\circ$ K.

Figure 9(c).-- Heat-transfer rate at the nozzle throat;
 $D_* = 0.30$ inch; $T_w = 294^\circ$ K.

Figure 9(d).-- Heat-transfer rate at the nozzle throat;
 $D_* = 0.02$ inch; $T_w = 3550^\circ$ K.

Figure 9(e).- Heat-transfer rate at the nozzle throat;
 $D_* = 0.10$ inch; $T_w, 3550^\circ \text{K}$.

Figure 9(f).- Heat-transfer rate at the nozzle throat;
 $D^* = 0.30$ inch; $T_w = 3550^\circ$ K.

Figure 10.- Temperature at the surface of a semi-infinite solid and an infinite hollow cylinder with constant heat-transfer rate at the surface, $r = r_*$.

Figure 11.- Relative time to reach melting temperature as a function of thermal properties for several heating conditions.

Figure 12.- Thermodynamic properties of tungsten.

Figure 13(a).- Time to melt tungsten throat; $D_* = 0.02$ inch.

Figure 13(b).- Time to melt tungsten throat; $D_* = 0.10$ inch.

Figure 13(c).- Time to melt tungsten throat; $D_* = 0.30$ inch.

Figure 14.- Time to melt tungsten throats with radiation only.

Figure 15.- Estimated variation of the isentropic exponent.

Figure 16.- Assumed configuration of arc chamber and nozzle.

Figure 17(a).- Time for 10-percent decrease in arc-chamber enthalpy due to radiation; $D_* = 0.02$ inch. Arc chamber: volume = 41.4 in³; surface = 66.3 in².

Figure 17(b).- Time for 10-percent decrease in arc-chamber enthalpy due to radiation; $D^* = 0.10$ inch. Arc chamber: volume = 41.4 in^3 ; surface = 66.3 in^2 .

Figure 17(c).-- Time for 10-percent decrease in arc-chamber enthalpy due to radiation; $D_* = 0.30$ inch. Arc chamber: volume = 41.4 in^3 ; surface = 66.3 in^2 .

Figure 18.- Stanton number ratio attainable by transpiration into a turbulent boundary layer with $\frac{u_{1x}}{v} > 10^6$.

Figure 19(a).- Transpiration flow rate corresponding to $\rho_w V_w = 2.70 q_o / (H_e - H_w)$; $D_* = 0.02$ inch.

Figure 19(b).- Transpiration flow rate corresponding to $\rho_w V_w = 2.70 q_o / (H_e - H_w)$; $D_* = 0.10$ inch.

Figure 19(c).- Transpiration flow rate corresponding to
 $\rho_w V_w = 2.70 \cdot q_o / (H_e - H_w)$; $D_* = 0.30$ inch.

Figure 20(a).- Time to melt tungsten throat with transpiration cooling; $D_* = 0$. inch. Transpiration flow rate: $\rho_w V_w = 2.70 q_o / (H_e - H_w)$; $St/St_o = 0.287$.

Figure 20(b).- Time to melt tungsten throat with transpiration cooling; $D_* = 0.10$ inch. Transpiration flow rate: $\rho_w V_w = 2.70 q_o / (H_e - H_w)$; $St/St_o = 0.287$.

Figure 20(c).- Time to melt tungsten throat with transpiration cooling; $D_* = 0.30$ inch. Transpiration flow rate: $\rho_w V_w = 2.70 q_o / (H_e - H_w)$; $St/St_o = 0.287$.

Figure 21(a).- Maximum time for steady flow in the "hot-shot" tunnel; $D_* = 0.02$ inch; no transpiration at the throat. Arc chamber: volume = 41.4 in^3 ; surface = 66.3 in^2 .

P_s , arc-chamber pressure, atmospheres

Figure 21(b).-- Maximum time for steady flow in the "hot-shot" tunnel; $D_* = 0.10$ inch.
No transpiration at the throat, arc chamber: volume = 4.14 in^3 ; surface = 66.3 in^2 .

Figure 21(c).- Maximum time for steady flow in the "hot-shot" tunnel; $D_* = 0.30$ inch.
 No transpiration at the throat, arc chamber: volume = 41.4 in^3 ; surface = 66.3 in^2 .

Figure 21(d).- Maximum time for steady flow in the "hot-shot" tunnel; $D_* = 0.02$ inch. Transpiration of air for 71.3-percent reduction in heat flux, arc chamber: volume = 41.4 in^3 ; surface = 66.3 in^2 .

Figure 21(e).-- Maximum time for steady flow in the "hot-shot" tunnel; $D_* = 0.10$ inch. Transpiration of air for 71.3-percent reduction in heat flux, arc chamber: volume = 41.4 in^3 ; surface = 66.3 in^2 .

Figure 21(f).- Maximum time for steady flow in the "hot-shot" tunnel; $D_* = 0.30$ inch. Transpiration of air for 71.3-percent reduction in heat flux, arc chamber: volume = 41.4 in³; surface = 66.3 in².